Computer algebra independent integration tests

1_Algebraic_functions/1.1_Binomial_products/1.1.3General/1.1.3.8P(x)(cx)^m(a+bx^n)^p

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December 6, 2018 Compiled on December 6, 2018 at 4:45am

Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

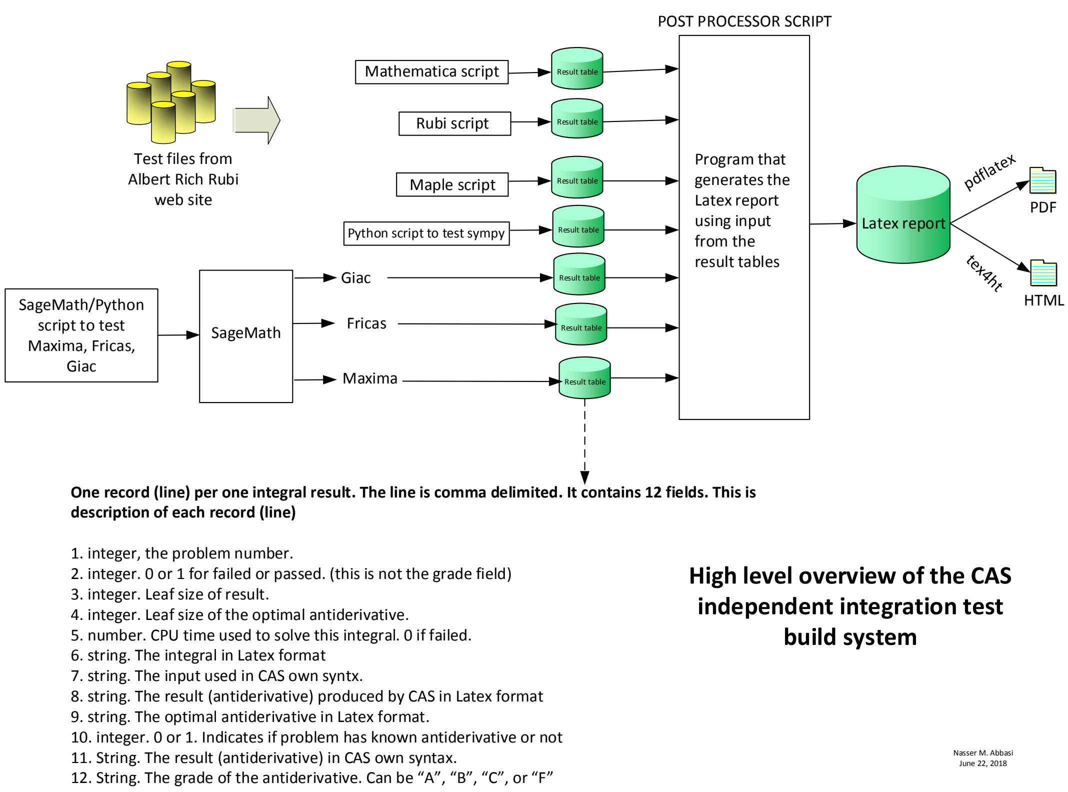
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.66 (580)	% 0.34 (2)
Rubi in Sympy	% 64.95 (378)	% 35.05 (204)
Mathematica	% 100. (582)	% 0. (0)
Maple	% 97.08 (565)	% 2.92 (17)
Maxima	% 33.51 (195)	% 66.49 (387)
Fricas	% 45.36 (264)	% 54.64 (318)
Sympy	% 74.05 (431)	% 25.95 (151)
Giac	% 69.76 (406)	% 30.24 (176)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

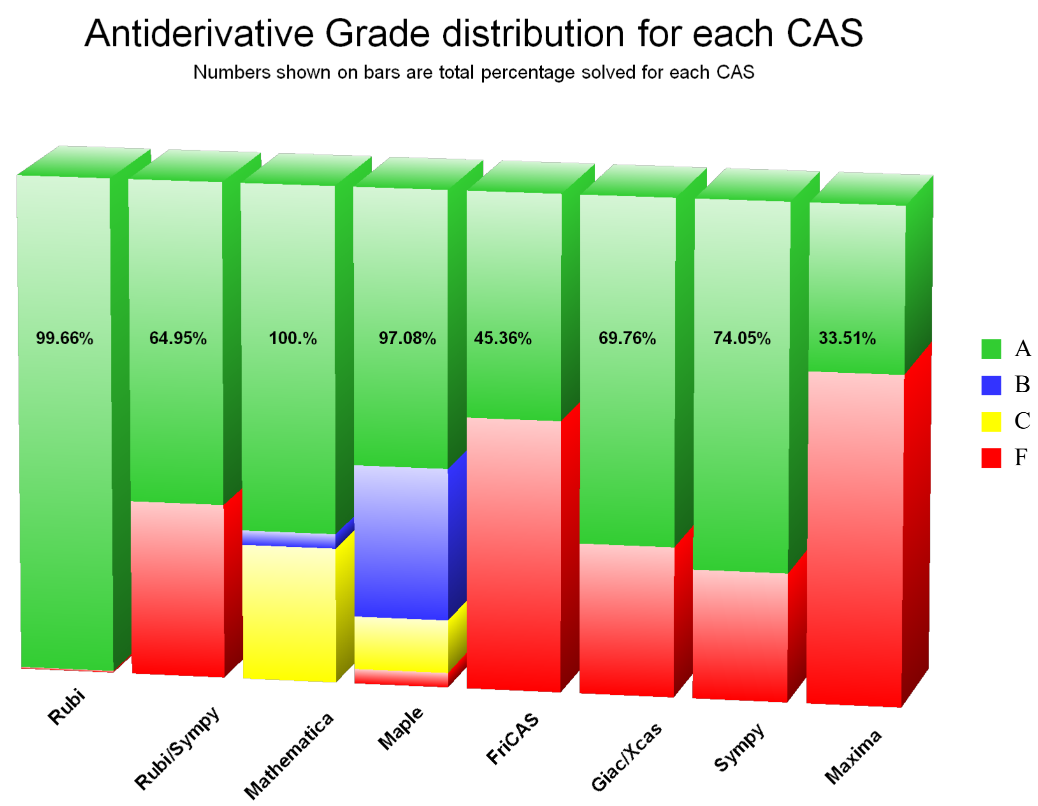
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

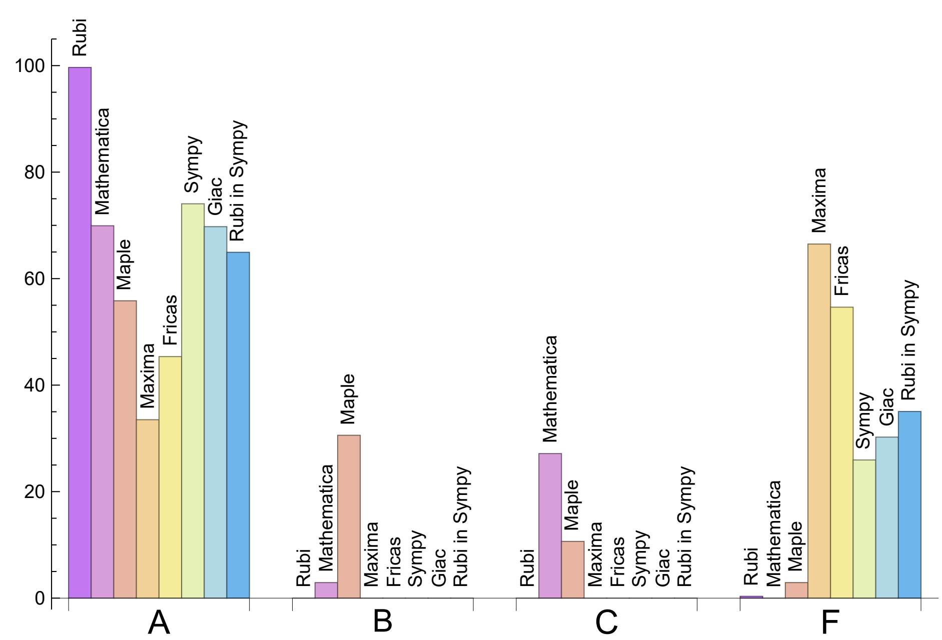
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.66	0.	0.	0.34
Rubi in Sympy	64.95	0.	0.	35.05
Mathematica	69.93	2.92	27.15	0.
Maple	55.84	30.58	10.65	2.92
Maxima	33.51	0.	0.	66.49
Fricas	45.36	0.	0.	54.64
Sympy	74.05	0.	0.	25.95
Giac	69.76	0.	0.	30.24

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.58	249.11	1.	225.	1.
Rubi in Sympy	55.17	199.3	0.88	178.	0.91
Mathematica	0.64	229.27	1.02	220.5	0.96
Maple	0.02	395.66	1.45	299.	1.28
Maxima	1.84	139.92	1.32	104.	1.25
Fricas	0.23	242.12	1.55	79.	1.39
Sympy	11.42	213.97	1.16	129.	0.93
Giac	0.22	298.74	1.69	273.5	1.42

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {221, 222}

Not solved by Rubi in Sympy {6, 53, 54, 55, 56, 71, 72, 73, 74, 75, 76, 77, 133, 134, 135, 137, 138, 139, 141, 142, 143, 145, 146, 147, 148, 171, 175, 181, 186, 187, 188, 189, 190, 191, 197, 202, 203, 207, 208, 209, 221, 222, 223, 224, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 251, 252, 253, 254, 255, 260, 261, 262, 263, 265, 267, 275, 276, 277, 278, 279, 280, 286, 287, 288, 289, 290, 291, 292, 297, 298, 299, 300, 301, 302, 303, 304, 305, 313, 314, 316, 317, 319, 320, 322, 323, 325, 326, 327, 364, 365, 366, 367, 368, 369, 370, 374, 375, 376, 377, 378, 379, 380, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 400, 401, 402, 409, 410, 425, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 465, 467, 469, 471, 474, 476, 492, 493, 494, 495, 496, 497, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 528, 547, 548, 549, 550, 553, 554, 556, 557, 558, 562, 565, 566, 567, 568, 573, 578, 582}

Not solved by Mathematica {}

Not solved by Maple {462, 463, 464, 539, 540, 541, 564, 569, 570, 571, 572, 573, 574, 575, 576, 578, 579}

Not solved by Maxima {7, 8, 9, 10, 11, 12, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 43, 44, 45, 46, 47, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 220, 221, 222, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 357, 358, 359, 360, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578}

Not solved by Fricas {7, 8, 9, 10, 11, 12, 24, 25, 26, 40, 41, 44, 45, 46, 47, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 154, 157, 158, 162, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 220, 221, 222, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 564, 569, 570, 571, 572, 574, 575, 576, 578}

Not solved by Sympy {40, 41, 44, 45, 46, 47, 64, 65, 68, 69, 75, 151, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 230, 231, 232, 246, 247, 248, 249, 250, 256, 257, 258, 259, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 329, 330, 331, 335, 336, 337, 338, 342, 343, 344, 345, 349, 350, 351, 352, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 462, 463, 464, 480, 481, 482, 539, 541, 564, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582}

Not solved by Giac {29, 31, 37, 38, 40, 41, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 210, 211, 212, 213, 214, 220, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 564, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {106}

Mathematica {41, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 555, 569, 578}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	53	53	104	72	223	117	68
normalized size	1	1.	0.74	0.74	1.44	1.	3.1	1.62	0.94
time (sec)	N/A	0.085	0.056	0.006	1.423	0.22	3.728	0.207	13.833

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	155	194	320	259	644	381	160
normalized size	1	1.	0.96	1.2	1.99	1.61	4.	2.37	0.99
time (sec)	N/A	0.223	0.159	0.01	1.424	0.216	20.86	0.213	37.17

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	355	495	709	617	1406	860	279
normalized size	1	1.	1.3	1.81	2.59	2.25	5.13	3.14	1.02
time (sec)	N/A	0.396	0.473	0.01	1.395	0.22	52.281	0.216	69.759

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	82	91	173	122	354	203	110
normalized size	1	1.	0.72	0.8	1.52	1.07	3.11	1.78	0.96
time (sec)	N/A	0.143	0.1	0.006	1.378	0.216	6.343	0.211	26.126

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	320	447	675	563	1365	846	348
normalized size	1	1.	1.	1.4	2.11	1.76	4.27	2.64	1.09
time (sec)	N/A	0.511	0.565	0.013	1.421	0.218	52.495	0.219	120.574

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	708	708	913	1417	1836	1648	3691	1	0
normalized size	1	1.	1.29	2.	2.59	2.33	5.21	0.	0.
time (sec)	N/A	1.498	4.768	0.015	1.436	0.228	174.121	0.232	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	0	0	76	216	150
normalized size	1	1.	0.77	1.16	0.	0.	0.47	1.34	0.93
time (sec)	N/A	0.243	0.11	0.005	0.	0.	1.029	0.214	31.419

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	180	238	0	0	105	252	172
normalized size	1	1.	0.95	1.26	0.	0.	0.56	1.33	0.91
time (sec)	N/A	0.266	0.332	0.004	0.	0.	1.873	0.215	41.773

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	205	272	0	0	146	279	201
normalized size	1	1.	0.95	1.27	0.	0.	0.68	1.3	0.93
time (sec)	N/A	0.356	0.326	0.007	0.	0.	2.451	0.218	51.912

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	306	0	0	185	312	228
normalized size	1	1.	0.95	1.27	0.	0.	0.77	1.3	0.95
time (sec)	N/A	0.423	0.44	0.007	0.	0.	3.935	0.214	64.529

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	125	186	0	0	76	197	150
normalized size	1	1.	0.78	1.16	0.	0.	0.47	1.22	0.93
time (sec)	N/A	0.254	0.105	0.007	0.	0.	1.042	0.213	31.535

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	125	188	0	0	78	155	150
normalized size	1	1.	0.78	1.17	0.	0.	0.48	0.96	0.93
time (sec)	N/A	0.216	0.088	0.007	0.	0.	1.063	0.213	31.727

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	22	26	22	22
normalized size	1	1.	1.	0.89	1.16	1.16	1.37	1.16	1.16
time (sec)	N/A	0.029	0.009	0.002	1.559	0.211	0.102	0.209	2.586

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	22	26	22	22
normalized size	1	1.	1.	0.89	1.16	1.16	1.37	1.16	1.16
time (sec)	N/A	0.029	0.008	0.003	1.603	0.211	0.101	0.207	4.681

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	22	17	23	17
normalized size	1	1.	1.	0.77	1.	1.	0.77	1.05	0.77
time (sec)	N/A	0.023	0.008	0.007	1.518	0.208	0.095	0.209	6.725

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	24	17	26	17
normalized size	1	1.	1.	0.86	1.09	1.09	0.77	1.18	0.77
time (sec)	N/A	0.023	0.007	0.007	1.51	0.21	0.088	0.211	6.929

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	43	53	44	45	41
normalized size	1	1.	1.	0.8	1.05	1.29	1.07	1.1	1.
time (sec)	N/A	0.057	0.014	0.007	1.522	0.22	0.155	0.209	9.735

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	35	46	38	54	38	31
normalized size	1	1.	1.07	1.21	1.59	1.31	1.86	1.31	1.07
time (sec)	N/A	0.047	0.019	0.009	1.525	0.217	0.2	0.211	5.161

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	45	35	53	35	29
normalized size	1	1.	1.	1.17	1.55	1.21	1.83	1.21	1.
time (sec)	N/A	0.042	0.015	0.006	1.579	0.218	0.198	0.211	9.357

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	195	0	1	88	65	44
normalized size	1	1.	0.9	5.	0.	0.03	2.26	1.67	1.13
time (sec)	N/A	0.061	0.037	0.005	0.	0.248	0.821	0.225	12.148

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	129	228	0	1	105	78	44
normalized size	1	1.	3.15	5.56	0.	0.02	2.56	1.9	1.07
time (sec)	N/A	0.101	0.097	0.016	0.	0.243	0.929	0.226	13.653

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	90	94	0	138	26	155	114
normalized size	1	1.	0.76	0.8	0.	1.17	0.22	1.31	0.97
time (sec)	N/A	0.231	0.024	0.006	0.	0.22	0.171	0.216	35.274

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	90	94	0	124	22	155	114
normalized size	1	1.	0.76	0.8	0.	1.05	0.19	1.31	0.97
time (sec)	N/A	0.198	0.021	0.004	0.	0.222	0.187	0.213	34.478

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	0	0	76	224	150
normalized size	1	1.	0.77	1.16	0.	0.	0.47	1.39	0.93
time (sec)	N/A	0.306	0.073	0.006	0.	0.	1.122	0.219	42.037

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	122	108	0	0	75	162	128
normalized size	1	1.	0.91	0.81	0.	0.	0.56	1.21	0.96
time (sec)	N/A	0.22	0.068	0.005	0.	0.	0.429	0.213	34.871

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	123	111	0	0	70	128	128
normalized size	1	1.	0.92	0.83	0.	0.	0.52	0.96	0.96
time (sec)	N/A	0.181	0.061	0.004	0.	0.	0.643	0.212	31.278

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	72	43	57	51	60	50	36
normalized size	1	1.	1.95	1.16	1.54	1.38	1.62	1.35	0.97
time (sec)	N/A	0.1	0.036	0.011	1.529	0.219	0.819	0.211	15.795

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	71	45	59	51	60	51	36
normalized size	1	1.	1.82	1.15	1.51	1.31	1.54	1.31	0.92
time (sec)	N/A	0.079	0.035	0.013	1.515	0.225	0.787	0.213	16.782

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	76	117	0	1	58	0	49
normalized size	1	1.	1.58	2.44	0.	0.02	1.21	0.	1.02
time (sec)	N/A	0.07	0.036	0.01	0.	0.248	0.881	0.	11.161

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	72	84	0	61	85	147	44
normalized size	1	1.	1.53	1.79	0.	1.3	1.81	3.13	0.94
time (sec)	N/A	0.065	0.046	0.009	0.	0.233	0.808	0.243	9.776

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	99	122	0	1	58	0	56
normalized size	1	1.	1.74	2.14	0.	0.02	1.02	0.	0.98
time (sec)	N/A	0.113	0.049	0.009	0.	0.245	0.977	0.	13.266

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	106	110	0	68	95	130	44
normalized size	1	1.	2.26	2.34	0.	1.45	2.02	2.77	0.94
time (sec)	N/A	0.101	0.069	0.011	0.	0.231	0.848	0.243	10.829

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	146	87	0	77	100	211	46
normalized size	1	1.	2.92	1.74	0.	1.54	2.	4.22	0.92
time (sec)	N/A	0.119	0.106	0.007	0.	0.233	0.95	0.241	11.551

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	150	135	0	81	110	204	49
normalized size	1	1.	2.83	2.55	0.	1.53	2.08	3.85	0.92
time (sec)	N/A	0.13	0.105	0.009	0.	0.234	0.989	0.241	13.092

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	149	132	0	80	109	123	49
normalized size	1	1.	2.76	2.44	0.	1.48	2.02	2.28	0.91
time (sec)	N/A	0.098	0.085	0.006	0.	0.236	0.985	0.218	12.613

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	147	90	0	78	102	115	46
normalized size	1	1.	2.77	1.7	0.	1.47	1.92	2.17	0.87
time (sec)	N/A	0.097	0.078	0.007	0.	0.236	1.047	0.217	13.439

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	95	117	0	1	70	0	63
normalized size	1	1.	1.56	1.92	0.	0.02	1.15	0.	1.03
time (sec)	N/A	0.083	0.042	0.005	0.	0.245	0.964	0.	13.152

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	116	122	0	1	73	0	70
normalized size	1	1.	1.66	1.74	0.	0.01	1.04	0.	1.
time (sec)	N/A	0.127	0.058	0.009	0.	0.25	1.065	0.	19.865

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	32	42	42	42	43	39
normalized size	1	1.	1.25	0.8	1.05	1.05	1.05	1.08	0.98
time (sec)	N/A	0.064	0.02	0.007	1.522	0.222	0.153	0.24	9.787

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	122	310	0	0	0	0	70
normalized size	1	1.	1.74	4.43	0.	0.	0.	0.	1.
time (sec)	N/A	0.116	0.099	0.009	0.	0.	0.	0.	20.793

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	F(-1)	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	238	345	0	0	0	0	78
normalized size	1	1.	2.7	3.92	0.	0.	0.	0.	0.89
time (sec)	N/A	0.185	1.239	0.01	0.	0.	0.	0.	27.39

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	12	12	16	16	7	18	7
normalized size	1	1.	1.09	1.09	1.45	1.45	0.64	1.64	0.64
time (sec)	N/A	0.02	0.003	0.003	1.365	0.23	0.063	0.234	6.538

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	218	0	23	20	22	19
normalized size	1	1.	1.	10.38	0.	1.1	0.95	1.05	0.9
time (sec)	N/A	0.033	0.005	0.007	0.	0.257	0.623	0.223	16.068

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F(-2)	F(-1)	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	247	121	0	0	0	351	54
normalized size	1	1.	3.48	1.7	0.	0.	0.	4.94	0.76
time (sec)	N/A	0.158	0.635	0.007	0.	0.	0.	0.248	17.376

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	F(-1)	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	288	345	0	0	0	344	60
normalized size	1	1.	3.79	4.54	0.	0.	0.	4.53	0.79
time (sec)	N/A	0.169	0.447	0.01	0.	0.	0.	0.248	18.592

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	F(-1)	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	253	340	0	0	0	180	60
normalized size	1	1.	3.24	4.36	0.	0.	0.	2.31	0.77
time (sec)	N/A	0.176	0.515	0.007	0.	0.	0.	0.223	19.466

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F(-2)	F(-1)	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	244	124	0	0	0	169	54
normalized size	1	1.	3.25	1.65	0.	0.	0.	2.25	0.72
time (sec)	N/A	0.174	0.608	0.008	0.	0.	0.	0.223	20.804

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	36	35	35	24	36	27
normalized size	1	1.	0.97	1.12	1.09	1.09	0.75	1.12	0.84
time (sec)	N/A	0.06	0.023	0.009	1.579	0.219	0.496	0.22	11.826

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	62	87	63	73	323	70	58
normalized size	1	1.	1.13	1.58	1.15	1.33	5.87	1.27	1.05
time (sec)	N/A	0.113	0.062	0.007	1.554	0.252	1.265	0.219	16.716

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	9	5
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.12	0.62
time (sec)	N/A	0.012	0.002	0.002	1.365	0.22	0.046	0.217	3.088

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	43	51	5	45	34
normalized size	1	1.	1.	1.1	1.43	1.7	0.17	1.5	1.13
time (sec)	N/A	0.057	0.015	0.006	1.52	0.227	0.146	0.217	12.42

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	22	15	23	15
normalized size	1	1.	1.	0.94	1.22	1.22	0.83	1.28	0.83
time (sec)	N/A	0.033	0.01	0.007	1.572	0.227	0.106	0.219	10.008

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	98	131	1	117	131	0
normalized size	1	1.	1.	0.87	1.16	0.01	1.04	1.16	0.
time (sec)	N/A	0.175	0.008	0.003	1.424	0.207	0.082	0.215	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	100	1	90	100	0
normalized size	1	1.	1.	0.85	1.14	0.01	1.02	1.14	0.
time (sec)	N/A	0.113	0.005	0.	1.382	0.193	0.068	0.215	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	1	58	68	0
normalized size	1	1.	1.	0.85	1.13	0.02	0.97	1.13	0.
time (sec)	N/A	0.071	0.004	0.003	1.448	0.192	0.056	0.213	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	14	0
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.17	0.
time (sec)	N/A	0.017	0.001	0.002	1.41	0.216	0.059	0.216	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	0	0	76	216	150
normalized size	1	1.	0.77	1.16	0.	0.	0.47	1.34	0.93
time (sec)	N/A	0.232	0.104	0.002	0.	0.	1.064	0.222	38.019

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	180	238	0	0	105	252	172
normalized size	1	1.	0.95	1.26	0.	0.	0.56	1.33	0.91
time (sec)	N/A	0.259	0.329	0.004	0.	0.	1.882	0.225	50.31

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	585	630	349	1618	0	0	265	0	592
normalized size	1	1.08	0.6	2.77	0.	0.	0.45	0.	1.01
time (sec)	N/A	1.125	1.325	0.007	0.	0.	6.506	0.	155.638

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	556	583	329	1546	0	0	170	0	534
normalized size	1	1.05	0.59	2.78	0.	0.	0.31	0.	0.96
time (sec)	N/A	0.861	1.814	0.007	0.	0.	3.935	0.	122.034

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	525	535	313	1480	0	0	163	0	478
normalized size	1	1.02	0.6	2.82	0.	0.	0.31	0.	0.91
time (sec)	N/A	0.659	1.269	0.007	0.	0.	3.74	0.	89.22

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	221	1536	0	0	78	0	430
normalized size	1	1.	0.45	3.13	0.	0.	0.16	0.	0.88
time (sec)	N/A	0.521	0.338	0.006	0.	0.	3.119	0.	60.691

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	305	1662	0	0	163	0	466
normalized size	1	1.	0.58	3.18	0.	0.	0.31	0.	0.89
time (sec)	N/A	0.671	1.37	0.007	0.	0.	20.116	0.	81.867

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	267	1782	0	0	0	0	498
normalized size	1	1.	0.48	3.22	0.	0.	0.	0.	0.9
time (sec)	N/A	0.847	0.769	0.108	0.	0.	0.	0.	103.388

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	287	1902	0	0	0	0	536
normalized size	1	1.	0.49	3.27	0.	0.	0.	0.	0.92
time (sec)	N/A	0.961	0.944	0.095	0.	0.	0.	0.	124.251

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	357	1491	0	0	187	0	527
normalized size	1	1.	0.61	2.53	0.	0.	0.32	0.	0.89
time (sec)	N/A	1.	2.107	0.01	0.	0.	4.324	0.	136.716

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	354	1547	0	0	189	0	524
normalized size	1	1.	0.6	2.6	0.	0.	0.32	0.	0.88
time (sec)	N/A	0.871	2.032	0.009	0.	0.	42.047	0.	98.265

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	329	1673	0	0	0	0	564
normalized size	1	1.	0.52	2.66	0.	0.	0.	0.	0.9
time (sec)	N/A	1.029	1.414	0.009	0.	0.	0.	0.	107.722

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	366	1793	0	0	0	0	612
normalized size	1	1.	0.54	2.65	0.	0.	0.	0.	0.91
time (sec)	N/A	1.292	1.482	0.008	0.	0.	0.	0.	135.762

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	200	211	0	0	156	262	175
normalized size	1	1.	1.08	1.13	0.	0.	0.84	1.41	0.94
time (sec)	N/A	0.332	0.165	0.004	0.	0.	1.987	0.216	42.178

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	214	325	0	0	245	327	0
normalized size	1	1.	0.96	1.46	0.	0.	1.1	1.47	0.
time (sec)	N/A	0.575	0.332	0.006	0.	0.	4.74	0.216	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	280	277	446	0	0	325	427	0
normalized size	1	0.99	0.98	1.58	0.	0.	1.15	1.51	0.
time (sec)	N/A	0.797	0.492	0.006	0.	0.	8.685	0.215	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	270	269	444	0	0	546	383	0
normalized size	1	0.99	0.99	1.63	0.	0.	2.01	1.41	0.
time (sec)	N/A	0.889	0.8	0.007	0.	0.	15.623	0.218	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	439	837	0	0	1314	648	0
normalized size	1	1.	1.06	2.01	0.	0.	3.16	1.56	0.
time (sec)	N/A	1.311	0.867	0.008	0.	0.	124.454	0.225	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	643	678	1339	0	0	0	1	0
normalized size	1	1.	1.05	2.08	0.	0.	0.	0.	0.
time (sec)	N/A	2.204	0.753	0.01	0.	0.	0.	0.222	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	54	38	50	65	44	51	0
normalized size	1	1.	1.26	0.88	1.16	1.51	1.02	1.19	0.
time (sec)	N/A	0.125	0.024	0.007	1.524	0.225	0.153	0.213	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	54	38	50	62	46	51	0
normalized size	1	1.	1.17	0.83	1.09	1.35	1.	1.11	0.
time (sec)	N/A	0.126	0.023	0.008	1.515	0.245	0.153	0.212	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	50	63	48	51	44
normalized size	1	1.	1.	0.86	1.14	1.43	1.09	1.16	1.
time (sec)	N/A	0.075	0.015	0.008	1.524	0.233	0.151	0.213	15.513

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	127	407	0	0	92	0	206
normalized size	1	1.	0.55	1.77	0.	0.	0.4	0.	0.9
time (sec)	N/A	0.126	0.187	0.02	0.	0.	1.983	0.	11.661

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	112	368	0	0	97	0	206
normalized size	1	1.	0.44	1.43	0.	0.	0.38	0.	0.8
time (sec)	N/A	0.15	0.124	0.028	0.	0.	2.474	0.	14.666

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	110	407	0	0	82	0	109
normalized size	1	1.	0.76	2.83	0.	0.	0.57	0.	0.76
time (sec)	N/A	0.078	0.114	0.015	0.	0.	2.4	0.	6.762

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	147	370	0	0	99	0	114
normalized size	1	1.	1.09	2.74	0.	0.	0.73	0.	0.84
time (sec)	N/A	0.075	0.2	0.015	0.	0.	2.026	0.	6.531

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	225	1003	0	0	122	0	410
normalized size	1	1.	0.48	2.14	0.	0.	0.26	0.	0.88
time (sec)	N/A	0.286	0.502	0.119	0.	0.	3.636	0.	25.042

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	182	949	0	0	128	0	413
normalized size	1	1.	0.38	1.97	0.	0.	0.27	0.	0.86
time (sec)	N/A	0.309	0.313	0.135	0.	0.	4.035	0.	28.02

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	257	952	0	0	112	0	224
normalized size	1	1.	0.95	3.51	0.	0.	0.41	0.	0.83
time (sec)	N/A	0.16	0.599	0.06	0.	0.	4.044	0.	13.768

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	227	1012	0	0	129	0	224
normalized size	1	1.	0.85	3.8	0.	0.	0.48	0.	0.84
time (sec)	N/A	0.148	0.518	0.046	0.	0.	3.953	0.	13.138

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	243	1004	0	0	0	0	450
normalized size	1	1.	0.47	1.93	0.	0.	0.	0.	0.87
time (sec)	N/A	0.452	0.462	0.056	0.	0.	1.326	0.	32.363

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	232	950	0	0	0	0	450
normalized size	1	1.	0.44	1.78	0.	0.	0.	0.	0.84
time (sec)	N/A	0.398	0.39	0.056	0.	0.	1.694	0.	35.861

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	267	953	0	0	0	0	449
normalized size	1	1.	1.04	3.72	0.	0.	0.	0.	1.75
time (sec)	N/A	0.18	0.56	0.029	0.	0.	1.72	0.	35.647

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	245	1013	0	0	0	0	449
normalized size	1	1.	0.98	4.04	0.	0.	0.	0.	1.79
time (sec)	N/A	0.149	0.511	0.019	0.	0.	1.452	0.	33.989

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	407	0	0	92	0	110
normalized size	1	1.	1.	3.2	0.	0.	0.72	0.	0.87
time (sec)	N/A	0.054	0.202	0.021	0.	0.	1.99	0.	6.459

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	112	368	0	0	97	0	110
normalized size	1	1.	0.79	2.59	0.	0.	0.68	0.	0.77
time (sec)	N/A	0.068	0.125	0.019	0.	0.	2.502	0.	8.305

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	110	407	0	0	82	0	202
normalized size	1	1.	0.42	1.54	0.	0.	0.31	0.	0.77
time (sec)	N/A	0.126	0.099	0.014	0.	0.	2.503	0.	13.081

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	147	370	0	0	97	0	211
normalized size	1	1.	0.6	1.5	0.	0.	0.39	0.	0.85
time (sec)	N/A	0.129	0.209	0.016	0.	0.	2.098	0.	13.131

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	129	407	0	0	92	0	110
normalized size	1	1.	1.02	3.23	0.	0.	0.73	0.	0.87
time (sec)	N/A	0.063	0.223	0.018	0.	0.	2.349	0.	6.269

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	112	368	0	0	97	0	110
normalized size	1	1.	0.78	2.57	0.	0.	0.68	0.	0.77
time (sec)	N/A	0.067	0.122	0.013	0.	0.	2.09	0.	7.017

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	110	407	0	0	82	0	202
normalized size	1	1.	0.42	1.55	0.	0.	0.31	0.	0.77
time (sec)	N/A	0.131	0.105	0.014	0.	0.	2.114	0.	12.246

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	146	370	0	0	97	0	211
normalized size	1	1.	0.59	1.49	0.	0.	0.39	0.	0.85
time (sec)	N/A	0.128	0.2	0.015	0.	0.	2.444	0.	13.076

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	225	1003	0	0	122	0	221
normalized size	1	1.	0.88	3.92	0.	0.	0.48	0.	0.86
time (sec)	N/A	0.126	0.474	0.059	0.	0.	3.757	0.	12.068

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	182	949	0	0	128	0	226
normalized size	1	1.	0.69	3.61	0.	0.	0.49	0.	0.86
time (sec)	N/A	0.116	0.327	0.062	0.	0.	4.118	0.	13.55

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	257	952	0	0	112	0	413
normalized size	1	1.	0.52	1.92	0.	0.	0.23	0.	0.83
time (sec)	N/A	0.311	0.663	0.02	0.	0.	4.324	0.	28.947

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	227	1012	0	0	128	0	415
normalized size	1	1.	0.47	2.07	0.	0.	0.26	0.	0.85
time (sec)	N/A	0.293	0.518	0.027	0.	0.	4.067	0.	27.67

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	243	1004	0	0	0	0	444
normalized size	1	1.	1.01	4.17	0.	0.	0.	0.	1.84
time (sec)	N/A	0.155	0.469	0.048	0.	0.	1.388	0.	32.738

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	232	950	0	0	0	0	444
normalized size	1	1.	0.94	3.83	0.	0.	0.	0.	1.79
time (sec)	N/A	0.153	0.397	0.05	0.	0.	1.756	0.	35.7

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	267	953	0	0	0	0	454
normalized size	1	1.	0.49	1.74	0.	0.	0.	0.	0.83
time (sec)	N/A	0.496	0.57	0.02	0.	0.	1.738	0.	37.389

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	540	540	245	1013	0	0	0	0	456
normalized size	1	1.	0.45	1.88	0.	0.	0.	0.	0.84
time (sec)	N/A	0.389	0.484	0.028	0.	0.	1.508	0.	35.492

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	221	720	0	0	78	0	430
normalized size	1	1.	0.45	1.47	0.	0.	0.16	0.	0.88
time (sec)	N/A	0.384	0.33	0.006	0.	0.	2.169	0.	28.495

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	208	681	0	0	82	0	430
normalized size	1	1.	0.41	1.35	0.	0.	0.16	0.	0.85
time (sec)	N/A	0.438	0.25	0.006	0.	0.	2.255	0.	30.417

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	236	683	0	0	73	0	434
normalized size	1	1.	0.46	1.33	0.	0.	0.14	0.	0.84
time (sec)	N/A	0.452	0.417	0.007	0.	0.	2.345	0.	31.174

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	223	726	0	0	83	0	435
normalized size	1	1.	0.44	1.43	0.	0.	0.16	0.	0.86
time (sec)	N/A	0.417	0.384	0.006	0.	0.	2.247	0.	30.472

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	136	291	0	0	61	0	221
normalized size	1	1.	0.55	1.18	0.	0.	0.25	0.	0.9
time (sec)	N/A	0.208	0.177	0.007	0.	0.	1.731	0.	12.465

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	121	267	0	0	65	0	221
normalized size	1	1.	0.45	0.99	0.	0.	0.24	0.	0.82
time (sec)	N/A	0.221	0.156	0.006	0.	0.	1.795	0.	14.913

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	119	291	0	0	56	0	218
normalized size	1	1.	0.43	1.06	0.	0.	0.2	0.	0.79
time (sec)	N/A	0.208	0.134	0.006	0.	0.	1.818	0.	13.105

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	152	269	0	0	66	0	226
normalized size	1	1.	0.58	1.03	0.	0.	0.25	0.	0.87
time (sec)	N/A	0.2	0.181	0.007	0.	0.	1.758	0.	12.933

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	134	101	0	0	126	304	82
normalized size	1	1.	1.54	1.16	0.	0.	1.45	3.49	0.94
time (sec)	N/A	0.151	0.063	0.008	0.	0.	1.365	0.217	21.593

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	184	151	0	0	124	288	207
normalized size	1	1.	0.84	0.69	0.	0.	0.57	1.32	0.95
time (sec)	N/A	0.398	0.14	0.009	0.	0.	1.354	0.216	58.573

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	168	142	0	0	155	343	102
normalized size	1	1.	1.53	1.29	0.	0.	1.41	3.12	0.93
time (sec)	N/A	0.189	0.343	0.007	0.	0.	2.413	0.222	32.959

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	224	188	0	0	155	321	231
normalized size	1	1.	0.93	0.78	0.	0.	0.64	1.33	0.96
time (sec)	N/A	0.44	0.396	0.007	0.	0.	2.412	0.218	74.773

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	193	180	0	0	194	367	128
normalized size	1	1.	1.42	1.32	0.	0.	1.43	2.7	0.94
time (sec)	N/A	0.234	0.339	0.01	0.	0.	4.34	0.222	42.288

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	249	222	0	0	192	346	257
normalized size	1	1.	0.94	0.83	0.	0.	0.72	1.3	0.97
time (sec)	N/A	0.507	0.41	0.008	0.	0.	4.266	0.222	86.427

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	217	184	0	0	231	400	151
normalized size	1	1.	1.34	1.14	0.	0.	1.43	2.47	0.93
time (sec)	N/A	0.292	0.432	0.022	0.	0.	16.203	0.224	52.323

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	274	224	0	0	231	378	280
normalized size	1	1.	0.94	0.77	0.	0.	0.79	1.3	0.96
time (sec)	N/A	0.581	0.511	0.024	0.	0.	16.195	0.219	98.427

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	42	44	47	47	313	50	20
normalized size	1	1.	1.75	1.83	1.96	1.96	13.04	2.08	0.83
time (sec)	N/A	0.045	0.026	0.007	1.52	0.228	0.772	0.209	7.258

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	99	68	116	0	83	116	88
normalized size	1	1.	1.01	0.69	1.18	0.	0.85	1.18	0.9
time (sec)	N/A	0.151	0.119	0.005	1.523	0.	0.745	0.213	19.05

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	187	161	0	0	471	396	105
normalized size	1	1.	1.61	1.39	0.	0.	4.06	3.41	0.91
time (sec)	N/A	0.211	0.113	0.003	0.	0.	10.556	0.222	26.758

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	229	280	0	0	466	371	258
normalized size	1	1.	0.83	1.01	0.	0.	1.68	1.34	0.93
time (sec)	N/A	0.441	0.279	0.005	0.	0.	10.449	0.22	70.434

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	211	228	0	0	507	440	131
normalized size	1	1.	1.45	1.56	0.	0.	3.47	3.01	0.9
time (sec)	N/A	0.271	0.542	0.007	0.	0.	13.667	0.221	41.703

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	305	344	0	0	505	413	287
normalized size	1	1.	0.99	1.12	0.	0.	1.64	1.34	0.93
time (sec)	N/A	0.568	0.814	0.006	0.	0.	13.705	0.221	88.05

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	244	286	0	0	563	481	167
normalized size	1	1.	1.36	1.6	0.	0.	3.15	2.69	0.93
time (sec)	N/A	0.354	0.662	0.007	0.	0.	18.722	0.225	53.563

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	337	396	0	0	558	454	326
normalized size	1	1.	0.99	1.16	0.	0.	1.64	1.33	0.96
time (sec)	N/A	0.693	0.631	0.007	0.	0.	18.332	0.225	103.555

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	276	281	0	0	612	531	197
normalized size	1	1.	1.31	1.33	0.	0.	2.9	2.52	0.93
time (sec)	N/A	0.442	0.532	0.019	0.	0.	34.237	0.222	69.456

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	369	393	0	0	610	504	357
normalized size	1	1.	0.99	1.06	0.	0.	1.64	1.35	0.96
time (sec)	N/A	0.804	0.963	0.022	0.	0.	34.119	0.224	122.227

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	24	34	1	27	34	0
normalized size	1	1.	0.96	0.86	1.21	0.04	0.96	1.21	0.
time (sec)	N/A	0.033	0.003	0.002	1.361	0.198	0.043	0.209	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	27	36	1	29	36	0
normalized size	1	1.	0.97	0.82	1.09	0.03	0.88	1.09	0.
time (sec)	N/A	0.043	0.002	0.001	7.477	0.196	0.048	0.207	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	1	58	68	0
normalized size	1	1.	1.	0.85	1.13	0.02	0.97	1.13	0.
time (sec)	N/A	0.121	0.003	0.001	6.077	0.194	0.058	0.208	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	36	1	31	36	31
normalized size	1	1.	1.	0.82	1.09	0.03	0.94	1.09	0.94
time (sec)	N/A	0.045	0.002	0.	1.377	0.197	0.05	0.211	6.007

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	1	60	68	0
normalized size	1	1.	1.	0.85	1.13	0.02	1.	1.13	0.
time (sec)	N/A	0.074	0.004	0.001	1.365	0.197	0.059	0.21	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	1	61	72	0
normalized size	1	1.	1.	0.83	1.11	0.02	0.94	1.11	0.
time (sec)	N/A	0.163	0.004	0.001	1.371	0.195	0.06	0.21	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	103	1	90	103	0
normalized size	1	1.	1.	0.84	1.12	0.01	0.98	1.12	0.
time (sec)	N/A	0.105	0.006	0.	1.408	0.207	0.064	0.21	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	27	20	1	29	22	12
normalized size	1	1.	1.94	1.59	1.18	0.06	1.71	1.29	0.71
time (sec)	N/A	0.018	0.001	0.001	1.41	0.204	0.049	0.21	2.565

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	68	1	58	68	0
normalized size	1	1.	1.33	1.13	1.51	0.02	1.29	1.51	0.
time (sec)	N/A	0.062	0.004	0.001	1.442	0.204	0.059	0.209	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	72	1	60	72	0
normalized size	1	1.	1.3	1.08	1.44	0.02	1.2	1.44	0.
time (sec)	N/A	0.142	0.005	0.001	1.392	0.202	0.06	0.209	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	103	1	88	103	0
normalized size	1	1.	1.19	1.	1.34	0.01	1.14	1.34	0.
time (sec)	N/A	0.144	0.006	0.001	1.369	0.204	0.066	0.208	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	1	61	72	44
normalized size	1	1.	1.	0.83	1.11	0.02	0.94	1.11	0.68
time (sec)	N/A	0.193	0.005	0.002	1.384	0.201	0.06	0.21	14.356

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	103	1	90	103	0
normalized size	1	1.	1.19	1.	1.34	0.01	1.17	1.34	0.
time (sec)	N/A	0.117	0.006	0.001	1.374	0.202	0.066	0.208	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	1	92	107	0
normalized size	1	1.	1.	0.82	1.1	0.01	0.95	1.1	0.
time (sec)	N/A	0.166	0.007	0.001	6.911	0.203	0.067	0.209	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	138	1	121	142	0
normalized size	1	1.	1.14	0.94	1.27	0.01	1.11	1.3	0.
time (sec)	N/A	0.163	0.012	0.001	5.935	0.198	0.075	0.215	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	203	1	180	208	0
normalized size	1	1.	1.19	1.	1.34	0.01	1.19	1.38	0.
time (sec)	N/A	0.224	0.009	0.001	1.369	0.198	0.087	0.209	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	220	248	0	0	518	452	138
normalized size	1	1.	1.42	1.6	0.	0.	3.34	2.92	0.89
time (sec)	N/A	0.299	0.288	0.007	0.	0.	30.846	0.223	44.39

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	253	326	0	0	583	505	173
normalized size	1	1.	1.35	1.73	0.	0.	3.1	2.69	0.92
time (sec)	N/A	0.359	0.518	0.008	0.	0.	126.867	0.225	56.985

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	286	287	0	0	0	555	204
normalized size	1	1.	1.3	1.3	0.	0.	0.	2.52	0.93
time (sec)	N/A	0.437	0.628	0.02	0.	0.	0.	0.223	72.586

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	101	78	114	166	366	88	131	90
normalized size	1	0.85	0.66	0.96	1.39	3.08	0.74	1.1	0.76
time (sec)	N/A	0.208	0.056	0.004	1.527	0.237	0.784	0.217	18.4

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	16	20	20	19	20	19
normalized size	1	1.	1.	0.73	0.91	0.91	0.86	0.91	0.86
time (sec)	N/A	0.036	0.018	0.002	1.515	0.22	0.103	0.213	2.874

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	123	107	129	198	0	88	155	110
normalized size	1	0.87	0.76	0.91	1.4	0.	0.62	1.1	0.78
time (sec)	N/A	0.227	0.11	0.004	1.53	0.	0.796	0.222	23.508

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	101	78	114	166	385	88	131	90
normalized size	1	0.85	0.66	0.96	1.39	3.24	0.74	1.1	0.76
time (sec)	N/A	0.173	0.03	0.004	1.555	0.236	0.762	0.225	19.346

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	113	226	225	4479	68	177	124
normalized size	1	1.	0.8	1.6	1.6	31.77	0.48	1.26	0.88
time (sec)	N/A	0.234	0.092	0.003	1.532	0.41	0.611	0.225	21.986

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	123	99	129	198	0	85	154	110
normalized size	1	0.87	0.7	0.91	1.4	0.	0.6	1.09	0.78
time (sec)	N/A	0.24	0.092	0.004	1.536	0.	0.805	0.223	27.042

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	129	241	252	0	292	193	144
normalized size	1	1.	0.79	1.48	1.55	0.	1.79	1.18	0.88
time (sec)	N/A	0.285	0.165	0.003	1.536	0.	6.004	0.228	30.059

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	15	10	15	10
normalized size	1	1.	1.	0.92	1.15	1.15	0.77	1.15	0.77
time (sec)	N/A	0.011	0.005	0.002	1.396	0.217	0.083	0.217	2.178

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	114	108	125	201	389	51	147	102
normalized size	1	0.86	0.82	0.95	1.52	2.95	0.39	1.11	0.77
time (sec)	N/A	0.209	0.061	0.004	1.545	0.246	0.504	0.224	23.796

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	65	28	153	42	53	126	31
normalized size	1	1.	1.81	0.78	4.25	1.17	1.47	3.5	0.86
time (sec)	N/A	0.074	0.057	0.004	1.525	0.228	0.497	0.221	9.553

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	136	128	140	231	0	199	169	122
normalized size	1	0.88	0.83	0.91	1.5	0.	1.29	1.1	0.79
time (sec)	N/A	0.262	0.13	0.003	1.552	0.	2.269	0.226	31.156

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	114	108	125	205	440	70	147	102
normalized size	1	0.86	0.82	0.95	1.55	3.33	0.53	1.11	0.77
time (sec)	N/A	0.24	0.049	0.003	1.536	0.24	0.428	0.223	28.371

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	148	237	263	4869	148	185	136
normalized size	1	1.	0.96	1.54	1.71	31.62	0.96	1.2	0.88
time (sec)	N/A	0.284	0.276	0.003	1.552	0.412	2.012	0.226	30.249

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	136	125	140	235	0	189	167	122
normalized size	1	0.88	0.81	0.91	1.53	0.	1.23	1.08	0.79
time (sec)	N/A	0.324	0.127	0.005	1.617	0.	2.248	0.225	36.817

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	164	252	279	0	580	201	156
normalized size	1	1.	0.93	1.43	1.59	0.	3.3	1.14	0.89
time (sec)	N/A	0.339	0.259	0.004	1.561	0.	12.961	0.23	38.979

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	9	5
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.12	0.62
time (sec)	N/A	0.014	0.002	0.002	1.378	0.214	0.046	0.21	3.659

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	102	103	217	73	95	48
normalized size	1	1.	0.94	1.92	1.94	4.09	1.38	1.79	0.91
time (sec)	N/A	0.112	0.064	0.004	1.519	0.242	0.676	0.212	16.612

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	203	171	0	0	187	392	109
normalized size	1	1.	1.64	1.38	0.	0.	1.51	3.16	0.88
time (sec)	N/A	0.242	0.092	0.005	0.	0.	1.955	0.218	30.322

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	283	286	0	0	187	365	255
normalized size	1	1.	1.02	1.03	0.	0.	0.68	1.32	0.92
time (sec)	N/A	0.475	0.449	0.004	0.	0.	1.906	0.22	73.588

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	249	244	0	0	0	539	0
normalized size	1	1.	1.68	1.65	0.	0.	0.	3.64	0.
time (sec)	N/A	0.436	0.167	0.006	0.	0.	0.	0.223	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	221	305	0	0	0	528	158
normalized size	1	1.	1.28	1.77	0.	0.	0.	3.07	0.92
time (sec)	N/A	0.343	1.026	0.013	0.	0.	0.	0.223	63.325

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	263	344	0	0	0	595	212
normalized size	1	1.	1.19	1.56	0.	0.	0.	2.69	0.96
time (sec)	N/A	0.541	0.992	0.02	0.	0.	0.	0.226	96.444

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	313	384	0	0	0	662	255
normalized size	1	1.	1.18	1.44	0.	0.	0.	2.49	0.96
time (sec)	N/A	0.649	0.629	0.021	0.	0.	0.	0.221	119.523

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	311	429	0	0	0	459	0
normalized size	1	1.	0.97	1.34	0.	0.	0.	1.44	0.
time (sec)	N/A	0.778	0.768	0.007	0.	0.	0.	0.222	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	319	484	0	0	0	493	325
normalized size	1	1.	0.94	1.42	0.	0.	0.	1.45	0.95
time (sec)	N/A	0.667	0.375	0.013	0.	0.	0.	0.222	115.932

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	366	521	0	0	0	562	386
normalized size	1	1.	0.93	1.32	0.	0.	0.	1.43	0.98
time (sec)	N/A	0.93	0.596	0.018	0.	0.	0.	0.226	149.134

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	411	562	0	0	0	629	428
normalized size	1	1.	0.94	1.29	0.	0.	0.	1.44	0.98
time (sec)	N/A	1.092	0.695	0.021	0.	0.	0.	0.222	177.693

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	20	20	15	20	7
normalized size	1	1.	0.82	0.73	1.82	1.82	1.36	1.82	0.64
time (sec)	N/A	0.013	0.003	0.001	1.423	0.207	0.073	0.211	4.08

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	14	8	16	16	8	16	7
normalized size	1	1.	1.27	0.73	1.45	1.45	0.73	1.45	0.64
time (sec)	N/A	0.014	0.001	0.002	1.371	0.214	0.065	0.209	4.064

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	9	5	9	0
normalized size	1	1.	1.	0.89	1.	1.	0.56	1.	0.
time (sec)	N/A	0.012	0.001	0.003	1.388	0.21	0.043	0.21	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	9	5
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.12	0.62
time (sec)	N/A	0.013	0.002	0.	1.368	0.213	0.045	0.21	3.648

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	9	5	9	3
normalized size	1	1.	1.	1.14	1.29	1.29	0.71	1.29	0.43
time (sec)	N/A	0.013	0.002	0.002	6.695	0.214	0.085	0.21	4.167

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	16	16	10	9	7
normalized size	1	1.	0.82	0.73	1.45	1.45	0.91	0.82	0.64
time (sec)	N/A	0.016	0.003	0.003	5.949	0.221	0.111	0.211	4.297

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	23	23	17	9	7
normalized size	1	1.	0.82	0.73	2.09	2.09	1.55	0.82	0.64
time (sec)	N/A	0.015	0.002	0.002	1.398	0.21	0.123	0.21	4.3

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	256	296	0	0	0	539	0
normalized size	1	1.	1.55	1.79	0.	0.	0.	3.27	0.
time (sec)	N/A	0.526	0.551	0.009	0.	0.	0.	0.226	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	301	367	0	0	0	807	0
normalized size	1	1.	1.6	1.95	0.	0.	0.	4.29	0.
time (sec)	N/A	0.65	0.8	0.009	0.	0.	0.	0.23	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	318	393	0	0	0	828	0
normalized size	1	1.	1.55	1.92	0.	0.	0.	4.04	0.
time (sec)	N/A	0.658	1.031	0.009	0.	0.	0.	0.23	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	342	462	0	0	0	506	0
normalized size	1	1.	1.01	1.37	0.	0.	0.	1.5	0.
time (sec)	N/A	0.874	0.743	0.007	0.	0.	0.	0.223	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	427	603	0	0	0	759	0
normalized size	1	1.	1.11	1.57	0.	0.	0.	1.98	0.
time (sec)	N/A	1.235	0.587	0.007	0.	0.	0.	0.228	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	445	627	0	0	0	780	0
normalized size	1	1.	1.11	1.56	0.	0.	0.	1.94	0.
time (sec)	N/A	1.248	0.627	0.007	0.	0.	0.	0.227	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	257	372	0	0	0	574	167
normalized size	1	1.	1.4	2.02	0.	0.	0.	3.12	0.91
time (sec)	N/A	0.462	0.303	0.013	0.	0.	0.	0.225	79.126

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	302	441	0	0	0	848	194
normalized size	1	1.	1.49	2.17	0.	0.	0.	4.18	0.96
time (sec)	N/A	0.593	0.447	0.013	0.	0.	0.	0.23	86.02

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	338	466	0	0	0	884	212
normalized size	1	1.	1.5	2.07	0.	0.	0.	3.93	0.94
time (sec)	N/A	0.744	0.459	0.015	0.	0.	0.	0.23	131.579

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	359	519	0	0	0	537	333
normalized size	1	1.	1.02	1.47	0.	0.	0.	1.52	0.94
time (sec)	N/A	0.779	0.472	0.013	0.	0.	0.	0.224	131.176

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	415	658	0	0	0	795	364
normalized size	1	1.	1.05	1.67	0.	0.	0.	2.01	0.92
time (sec)	N/A	1.164	0.715	0.016	0.	0.	0.	0.232	139.125

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	460	682	0	0	0	833	0
normalized size	1	1.	1.1	1.64	0.	0.	0.	2.	0.
time (sec)	N/A	1.278	0.708	0.016	0.	0.	0.	0.229	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	309	418	0	0	0	656	226
normalized size	1	1.	1.28	1.73	0.	0.	0.	2.72	0.94
time (sec)	N/A	0.71	0.555	0.017	0.	0.	0.	0.229	117.367

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	359	501	0	0	0	942	252
normalized size	1	1.	1.34	1.87	0.	0.	0.	3.51	0.94
time (sec)	N/A	0.959	0.674	0.017	0.	0.	0.	0.236	127.972

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	380	517	0	0	0	986	267
normalized size	1	1.	1.33	1.81	0.	0.	0.	3.46	0.94
time (sec)	N/A	0.924	0.543	0.017	0.	0.	0.	0.232	141.004

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	411	562	0	0	0	620	400
normalized size	1	1.	1.	1.36	0.	0.	0.	1.5	0.97
time (sec)	N/A	1.122	0.674	0.019	0.	0.	0.	0.225	170.297

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	473	717	0	0	0	892	0
normalized size	1	1.	1.02	1.55	0.	0.	0.	1.93	0.
time (sec)	N/A	1.566	1.126	0.019	0.	0.	0.	0.23	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	500	732	0	0	0	936	0
normalized size	1	1.	1.04	1.52	0.	0.	0.	1.95	0.
time (sec)	N/A	1.559	0.973	0.016	0.	0.	0.	0.234	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	360	463	0	0	0	738	274
normalized size	1	1.	1.23	1.58	0.	0.	0.	2.52	0.94
time (sec)	N/A	0.9	0.764	0.021	0.	0.	0.	0.224	145.162

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	422	551	0	0	0	1	304
normalized size	1	1.	1.27	1.66	0.	0.	0.	0.	0.92
time (sec)	N/A	1.278	0.742	0.02	0.	0.	0.	0.225	155.188

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	439	567	0	0	0	1	321
normalized size	1	1.	1.26	1.62	0.	0.	0.	0.	0.92
time (sec)	N/A	1.247	0.744	0.02	0.	0.	0.	0.226	168.366

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	461	608	0	0	0	703	0
normalized size	1	1.	1.	1.32	0.	0.	0.	1.52	0.
time (sec)	N/A	1.327	0.867	0.02	0.	0.	0.	0.225	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	516	530	768	0	0	0	992	0
normalized size	1	1.	1.03	1.49	0.	0.	0.	1.92	0.
time (sec)	N/A	1.909	1.383	0.023	0.	0.	0.	0.23	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	555	784	0	0	0	1035	0
normalized size	1	1.	1.04	1.47	0.	0.	0.	1.94	0.
time (sec)	N/A	1.861	1.124	0.02	0.	0.	0.	0.228	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	107	96	0	0	61	0	109
normalized size	1	1.	0.88	0.79	0.	0.	0.5	0.	0.9
time (sec)	N/A	0.154	0.266	0.006	0.	0.	2.812	0.	13.384

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	106	90	0	0	95	0	78
normalized size	1	1.	1.22	1.03	0.	0.	1.09	0.	0.9
time (sec)	N/A	0.137	0.269	0.006	0.	0.	2.995	0.	14.944

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	108	95	0	0	90	0	78
normalized size	1	1.	1.21	1.07	0.	0.	1.01	0.	0.88
time (sec)	N/A	0.138	0.246	0.023	0.	0.	3.013	0.	15.576

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	113	101	0	0	66	0	112
normalized size	1	1.	0.89	0.8	0.	0.	0.52	0.	0.88
time (sec)	N/A	0.156	0.269	0.023	0.	0.	2.921	0.	14.866

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	201	193	0	0	102	0	233
normalized size	1	1.	0.78	0.75	0.	0.	0.4	0.	0.91
time (sec)	N/A	0.312	0.349	0.006	0.	0.	3.364	0.	30.6

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	16	80	16	12
normalized size	1	1.	1.	0.93	1.14	1.14	5.71	1.14	0.86
time (sec)	N/A	0.012	0.025	0.006	1.555	0.216	13.529	0.215	6.023

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	34	46	104	43	22
normalized size	1	1.	1.	0.83	1.17	1.59	3.59	1.48	0.76
time (sec)	N/A	0.05	0.041	0.007	1.453	0.217	19.287	0.215	12.11

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	24	31	45	109	30	22
normalized size	1	1.	1.08	0.96	1.24	1.8	4.36	1.2	0.88
time (sec)	N/A	0.054	0.044	0.005	1.46	0.218	20.008	0.217	14.008

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	59	59	133	42	37
normalized size	1	1.	1.	0.92	1.55	1.55	3.5	1.11	0.97
time (sec)	N/A	0.061	0.055	0.005	7.141	0.223	26.051	0.219	16.536

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	58	14	10
normalized size	1	1.	1.	0.92	1.17	1.17	4.83	1.17	0.83
time (sec)	N/A	0.009	0.014	0.007	5.83	0.215	3.786	0.211	2.95

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	275	516	0	0	260	0	335
normalized size	1	1.	0.71	1.34	0.	0.	0.68	0.	0.87
time (sec)	N/A	0.946	1.039	0.01	0.	0.	8.943	0.	125.926

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	C	B	F	F(-2)	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	0	51	173	0	0	36	136	0
normalized size	1	0.	0.47	1.59	0.	0.	0.33	1.25	0.
time (sec)	N/A	0.049	0.017	0.033	0.	0.	0.799	0.22	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	C	B	F	F(-2)	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	0	47	169	0	0	36	136	0
normalized size	1	0.	0.43	1.55	0.	0.	0.33	1.25	0.
time (sec)	N/A	0.053	0.017	0.024	0.	0.	0.789	0.221	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	187	266	282	284	192	332	0
normalized size	1	1.	0.9	1.28	1.36	1.37	0.92	1.6	0.
time (sec)	N/A	0.609	0.164	0.007	1.401	0.232	2.422	0.213	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	218	228	230	155	266	0
normalized size	1	1.	0.91	1.28	1.34	1.35	0.91	1.56	0.
time (sec)	N/A	0.473	0.138	0.006	1.456	0.228	2.322	0.214	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	119	170	174	176	117	200	0
normalized size	1	1.	0.9	1.29	1.32	1.33	0.89	1.52	0.
time (sec)	N/A	0.347	0.101	0.005	1.416	0.226	2.234	0.214	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	88	124	123	124	83	136	0
normalized size	1	1.	0.92	1.29	1.28	1.29	0.86	1.42	0.
time (sec)	N/A	0.279	0.076	0.004	1.399	0.213	2.055	0.214	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	97	104	108	68	107	0
normalized size	1	1.	0.94	1.21	1.3	1.35	0.85	1.34	0.
time (sec)	N/A	0.221	0.055	0.009	7.327	0.247	9.66	0.21	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	94	104	115	70	128	0
normalized size	1	1.	0.95	1.16	1.28	1.42	0.86	1.58	0.
time (sec)	N/A	0.211	0.075	0.013	1.422	0.239	23.063	0.214	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	116	126	136	85	170	87
normalized size	1	1.	0.93	1.22	1.33	1.43	0.89	1.79	0.92
time (sec)	N/A	0.243	0.139	0.011	1.385	0.248	92.422	0.216	44.451

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	128	161	169	171	0	248	117
normalized size	1	1.	1.	1.26	1.32	1.34	0.	1.94	0.91
time (sec)	N/A	0.307	0.13	0.012	1.434	0.237	0.	0.214	50.163

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	164	210	224	227	0	317	153
normalized size	1	1.	1.	1.28	1.37	1.38	0.	1.93	0.93
time (sec)	N/A	0.358	0.132	0.014	1.486	0.261	0.	0.214	56.396

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	194	260	281	284	0	387	190
normalized size	1	1.	0.95	1.27	1.37	1.39	0.	1.89	0.93
time (sec)	N/A	0.444	0.407	0.013	1.442	0.284	0.	0.215	67.121

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	351	592	0	481	450	613	0
normalized size	1	1.	1.01	1.7	0.	1.38	1.29	1.76	0.
time (sec)	N/A	0.774	0.145	0.008	0.	0.223	3.698	0.218	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	311	554	0	462	496	595	0
normalized size	1	1.	0.98	1.75	0.	1.46	1.57	1.88	0.
time (sec)	N/A	0.697	0.183	0.006	0.	0.221	2.937	0.22	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	306	544	0	429	411	541	0
normalized size	1	1.	0.98	1.74	0.	1.38	1.32	1.73	0.
time (sec)	N/A	0.664	0.171	0.007	0.	0.219	3.671	0.215	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	266	502	0	410	459	521	0
normalized size	1	1.	0.95	1.8	0.	1.47	1.65	1.87	0.
time (sec)	N/A	0.615	0.165	0.006	0.	0.221	2.842	0.219	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	264	492	0	355	371	467	0
normalized size	1	1.	0.96	1.8	0.	1.3	1.35	1.7	0.
time (sec)	N/A	0.593	0.17	0.005	0.	0.223	3.449	0.219	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	231	450	0	313	422	466	0
normalized size	1	1.	0.94	1.84	0.	1.28	1.72	1.9	0.
time (sec)	N/A	0.48	0.296	0.005	0.	0.222	2.894	0.219	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	442	0	312	340	414	0
normalized size	1	1.	0.95	1.84	0.	1.3	1.42	1.72	0.
time (sec)	N/A	0.342	0.297	0.004	0.	0.219	3.601	0.217	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	224	419	0	312	406	428	0
normalized size	1	1.	0.99	1.85	0.	1.37	1.79	1.89	0.
time (sec)	N/A	0.413	0.264	0.009	0.	0.242	5.126	0.219	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	218	414	0	304	326	378	0
normalized size	1	1.	0.97	1.85	0.	1.36	1.46	1.69	0.
time (sec)	N/A	0.379	0.204	0.007	0.	0.245	6.096	0.217	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	220	412	0	312	411	417	0
normalized size	1	1.	0.97	1.81	0.	1.37	1.81	1.84	0.
time (sec)	N/A	0.429	0.198	0.01	0.	0.246	16.625	0.219	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	220	410	0	313	328	370	0
normalized size	1	1.	0.98	1.82	0.	1.39	1.46	1.64	0.
time (sec)	N/A	0.381	0.154	0.012	0.	0.239	20.54	0.218	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	231	440	0	333	432	444	224
normalized size	1	1.	0.95	1.82	0.	1.38	1.79	1.83	0.93
time (sec)	N/A	0.427	0.207	0.01	0.	0.239	62.55	0.221	63.913

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	231	441	0	319	348	401	228
normalized size	1	1.	0.95	1.81	0.	1.31	1.43	1.64	0.93
time (sec)	N/A	0.41	0.208	0.01	0.	0.222	106.427	0.217	65.842

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	266	491	0	389	0	508	257
normalized size	1	1.	0.96	1.77	0.	1.4	0.	1.83	0.93
time (sec)	N/A	0.502	0.205	0.013	0.	0.226	0.	0.221	78.67

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	266	493	0	427	0	456	260
normalized size	1	1.	0.95	1.76	0.	1.52	0.	1.63	0.93
time (sec)	N/A	0.472	0.215	0.011	0.	0.224	0.	0.218	84.07

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	308	546	0	464	0	566	291
normalized size	1	1.	0.98	1.74	0.	1.48	0.	1.81	0.93
time (sec)	N/A	0.56	0.183	0.012	0.	0.219	0.	0.221	101.679

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	311	548	0	481	0	531	294
normalized size	1	1.	0.99	1.74	0.	1.53	0.	1.69	0.93
time (sec)	N/A	0.515	0.189	0.012	0.	0.224	0.	0.217	113.806

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	346	600	0	514	0	640	326
normalized size	1	1.	0.99	1.71	0.	1.46	0.	1.82	0.93
time (sec)	N/A	0.599	0.203	0.014	0.	0.219	0.	0.219	155.236

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	205	288	300	409	224	405	0
normalized size	1	1.	0.93	1.31	1.36	1.86	1.02	1.84	0.
time (sec)	N/A	0.698	0.362	0.018	1.385	0.204	19.494	0.215	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	167	240	243	347	180	335	0
normalized size	1	1.	0.93	1.33	1.35	1.93	1.	1.86	0.
time (sec)	N/A	0.532	0.236	0.019	1.409	0.205	19.188	0.215	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	192	186	273	138	293	0
normalized size	1	1.	0.92	1.37	1.33	1.95	0.99	2.09	0.
time (sec)	N/A	0.37	0.181	0.016	1.375	0.233	18.429	0.217	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	142	132	193	97	278	0
normalized size	1	1.	0.9	1.38	1.28	1.87	0.94	2.7	0.
time (sec)	N/A	0.299	0.107	0.016	9.605	0.216	16.574	0.216	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	125	135	196	95	169	0
normalized size	1	1.	0.95	1.25	1.35	1.96	0.95	1.69	0.
time (sec)	N/A	0.249	0.229	0.023	1.387	0.243	54.843	0.214	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	97	132	157	232	0	177	102
normalized size	1	1.	0.89	1.21	1.44	2.13	0.	1.62	0.94
time (sec)	N/A	0.278	0.137	0.021	1.381	0.233	0.	0.216	48.479

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	118	167	186	281	0	271	124
normalized size	1	1.	0.91	1.28	1.43	2.16	0.	2.08	0.95
time (sec)	N/A	0.316	0.219	0.023	1.375	0.229	0.	0.215	51.091

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	160	229	244	352	0	371	170
normalized size	1	1.	0.91	1.31	1.39	2.01	0.	2.12	0.97
time (sec)	N/A	0.426	0.197	0.034	1.387	0.243	0.	0.216	61.851

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	198	282	305	419	0	447	216
normalized size	1	1.	0.93	1.32	1.43	1.96	0.	2.09	1.01
time (sec)	N/A	0.53	0.462	0.029	1.395	0.278	0.	0.22	72.915

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	364	622	0	678	490	609	0
normalized size	1	1.	0.99	1.69	0.	1.84	1.33	1.65	0.
time (sec)	N/A	0.974	0.88	0.015	0.	0.225	18.41	0.216	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	319	584	0	645	530	597	0
normalized size	1	1.	0.95	1.74	0.	1.93	1.58	1.78	0.
time (sec)	N/A	1.42	0.325	0.017	0.	0.264	54.349	0.22	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	315	567	0	590	440	532	0
normalized size	1	1.	0.96	1.73	0.	1.8	1.34	1.62	0.
time (sec)	N/A	0.803	0.517	0.016	0.	0.241	18.548	0.216	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	282	529	0	541	484	537	0
normalized size	1	1.	0.95	1.78	0.	1.82	1.62	1.8	0.
time (sec)	N/A	0.969	0.303	0.015	0.	0.239	51.18	0.219	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	277	514	0	531	398	471	284
normalized size	1	1.	0.96	1.78	0.	1.84	1.38	1.64	0.99
time (sec)	N/A	0.679	0.299	0.015	0.	0.236	17.627	0.219	141.744

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	255	495	0	513	461	494	0
normalized size	1	1.	0.94	1.83	0.	1.89	1.7	1.82	0.
time (sec)	N/A	0.618	0.275	0.014	0.	0.236	28.368	0.219	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	251	482	0	494	376	433	258
normalized size	1	1.	0.95	1.83	0.	1.87	1.42	1.64	0.98
time (sec)	N/A	0.597	0.282	0.013	0.	0.236	11.754	0.217	105.473

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	255	474	0	510	457	477	0
normalized size	1	1.	0.96	1.79	0.	1.92	1.72	1.8	0.
time (sec)	N/A	0.595	0.29	0.018	0.	0.238	49.411	0.219	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	250	463	0	522	381	417	250
normalized size	1	1.	0.96	1.78	0.	2.01	1.47	1.6	0.96
time (sec)	N/A	0.573	0.283	0.016	0.	0.239	107.283	0.217	132.498

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	255	486	0	532	0	483	270
normalized size	1	1.	0.95	1.81	0.	1.98	0.	1.8	1.
time (sec)	N/A	0.656	0.301	0.022	0.	0.237	0.	0.22	139.282

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	253	477	0	518	0	429	274
normalized size	1	1.	0.94	1.77	0.	1.92	0.	1.59	1.01
time (sec)	N/A	0.646	0.293	0.019	0.	0.246	0.	0.22	141.426

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	281	529	0	570	0	522	299
normalized size	1	1.	0.95	1.78	0.	1.92	0.	1.76	1.01
time (sec)	N/A	0.785	0.341	0.021	0.	0.248	0.	0.219	147.304

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	280	520	0	555	0	468	303
normalized size	1	1.	0.94	1.75	0.	1.87	0.	1.58	1.02
time (sec)	N/A	0.759	0.335	0.02	0.	0.252	0.	0.217	153.905

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	319	575	0	632	0	590	326
normalized size	1	1.	0.96	1.72	0.	1.89	0.	1.77	0.98
time (sec)	N/A	0.949	0.343	0.025	0.	0.247	0.	0.218	162.534

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	317	566	0	670	0	528	330
normalized size	1	1.	0.95	1.69	0.	2.	0.	1.58	0.99
time (sec)	N/A	0.966	0.345	0.023	0.	0.233	0.	0.218	172.126

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	370	631	0	721	0	651	0
normalized size	1	1.	0.99	1.68	0.	1.92	0.	1.74	0.
time (sec)	N/A	1.134	0.794	0.024	0.	0.229	0.	0.218	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	246	361	371	535	0	471	0
normalized size	1	1.	0.92	1.36	1.39	2.01	0.	1.77	0.
time (sec)	N/A	0.867	0.272	0.021	1.379	0.205	0.	0.219	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	208	313	315	477	0	402	0
normalized size	1	1.	0.92	1.38	1.39	2.11	0.	1.78	0.
time (sec)	N/A	0.699	0.231	0.022	1.444	0.206	0.	0.216	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	170	266	258	398	0	319	0
normalized size	1	1.	0.91	1.43	1.39	2.14	0.	1.72	0.
time (sec)	N/A	0.549	0.189	0.02	1.446	0.207	0.	0.218	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	145	213	198	304	0	197	0
normalized size	1	1.	0.99	1.46	1.36	2.08	0.	1.35	0.
time (sec)	N/A	0.388	0.135	0.017	1.382	0.207	0.	0.215	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	105	156	147	213	0	135	0
normalized size	1	1.	0.96	1.43	1.35	1.95	0.	1.24	0.
time (sec)	N/A	0.304	0.102	0.016	1.382	0.203	0.	0.216	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	104	147	174	252	0	173	105
normalized size	1	1.	0.91	1.29	1.53	2.21	0.	1.52	0.92
time (sec)	N/A	0.298	0.199	0.02	1.389	0.223	0.	0.215	54.648

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	121	163	194	338	0	234	124
normalized size	1	1.	0.9	1.22	1.45	2.52	0.	1.75	0.93
time (sec)	N/A	0.344	0.172	0.025	1.39	0.273	0.	0.216	57.161

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	149	213	246	427	0	255	156
normalized size	1	1.	0.91	1.31	1.51	2.62	0.	1.56	0.96
time (sec)	N/A	0.402	0.212	0.025	1.384	0.28	0.	0.216	61.975

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	200	293	313	535	0	437	212
normalized size	1	1.	0.92	1.34	1.44	2.45	0.	2.	0.97
time (sec)	N/A	0.543	0.28	0.03	1.431	0.31	0.	0.218	79.393

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	238	349	378	605	0	513	262
normalized size	1	1.	0.92	1.35	1.47	2.34	0.	1.99	1.02
time (sec)	N/A	0.671	0.416	0.03	1.393	0.299	0.	0.217	91.934

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	411	706	0	919	0	675	0
normalized size	1	1.	0.99	1.7	0.	2.21	0.	1.62	0.
time (sec)	N/A	1.461	0.973	0.021	0.	0.246	0.	0.217	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	380	668	0	887	0	663	0
normalized size	1	1.	0.99	1.74	0.	2.31	0.	1.73	0.
time (sec)	N/A	2.052	0.978	0.021	0.	0.241	0.	0.224	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	362	651	0	832	0	598	0
normalized size	1	1.	0.97	1.74	0.	2.22	0.	1.59	0.
time (sec)	N/A	1.253	0.96	0.02	0.	0.242	0.	0.217	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	329	611	0	780	0	601	0
normalized size	1	1.	0.95	1.77	0.	2.26	0.	1.74	0.
time (sec)	N/A	1.528	0.416	0.019	0.	0.238	0.	0.221	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	323	596	0	779	0	539	0
normalized size	1	1.	0.96	1.77	0.	2.32	0.	1.6	0.
time (sec)	N/A	1.038	0.756	0.019	0.	0.234	0.	0.218	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	300	574	0	747	0	558	0
normalized size	1	1.	0.95	1.82	0.	2.36	0.	1.77	0.
time (sec)	N/A	1.018	0.394	0.017	0.	0.237	0.	0.222	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	294	561	0	728	0	495	0
normalized size	1	1.	0.96	1.83	0.	2.37	0.	1.61	0.
time (sec)	N/A	0.88	0.373	0.017	0.	0.23	0.	0.219	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	284	550	0	711	0	522	291
normalized size	1	1.	0.94	1.83	0.	2.36	0.	1.73	0.97
time (sec)	N/A	0.805	0.38	0.017	0.	0.222	0.	0.221	128.405

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	279	539	0	710	0	463	289
normalized size	1	1.	0.96	1.85	0.	2.43	0.	1.59	0.99
time (sec)	N/A	0.685	0.356	0.017	0.	0.219	0.	0.219	116.6

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	286	547	0	741	0	525	286
normalized size	1	1.	0.94	1.81	0.	2.45	0.	1.73	0.94
time (sec)	N/A	0.805	0.417	0.02	0.	0.222	0.	0.222	145.367

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	283	539	0	737	0	486	287
normalized size	1	1.	0.94	1.79	0.	2.45	0.	1.61	0.95
time (sec)	N/A	0.786	0.409	0.02	0.	0.266	0.	0.218	148.259

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	303	574	0	765	0	547	0
normalized size	1	1.	0.96	1.81	0.	2.41	0.	1.73	0.
time (sec)	N/A	0.899	0.432	0.023	0.	0.24	0.	0.221	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	299	566	0	752	0	491	0
normalized size	1	1.	0.95	1.79	0.	2.38	0.	1.55	0.
time (sec)	N/A	0.88	0.427	0.023	0.	0.227	0.	0.218	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	328	611	0	809	0	586	0
normalized size	1	1.	0.96	1.78	0.	2.36	0.	1.71	0.
time (sec)	N/A	1.148	0.461	0.024	0.	0.223	0.	0.221	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	324	603	0	794	0	532	0
normalized size	1	1.	0.95	1.77	0.	2.33	0.	1.56	0.
time (sec)	N/A	1.12	0.481	0.025	0.	0.224	0.	0.22	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	366	659	0	873	0	656	0
normalized size	1	1.	0.96	1.73	0.	2.29	0.	1.72	0.
time (sec)	N/A	1.406	0.85	0.025	0.	0.231	0.	0.22	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	376	651	0	911	0	594	0
normalized size	1	1.	0.99	1.71	0.	2.4	0.	1.56	0.
time (sec)	N/A	1.395	0.869	0.026	0.	0.222	0.	0.218	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	419	716	0	963	0	717	0
normalized size	1	1.	0.99	1.69	0.	2.27	0.	1.69	0.
time (sec)	N/A	1.702	1.088	0.03	0.	0.244	0.	0.222	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	45	59	77	53	61	0
normalized size	1	1.	1.09	0.83	1.09	1.43	0.98	1.13	0.
time (sec)	N/A	0.136	0.026	0.009	1.522	0.216	0.159	0.212	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	32	24	34	0
normalized size	1	1.	1.	0.83	1.07	1.07	0.8	1.13	0.
time (sec)	N/A	0.069	0.007	0.007	1.525	0.208	0.094	0.211	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	38	50	65	44	51	41
normalized size	1	1.	1.2	0.86	1.14	1.48	1.	1.16	0.93
time (sec)	N/A	0.112	0.015	0.008	1.534	0.213	0.172	0.21	17.797

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	50	35	46	55	42	47	39
normalized size	1	1.	1.22	0.85	1.12	1.34	1.02	1.15	0.95
time (sec)	N/A	0.083	0.013	0.007	1.52	0.214	0.167	0.212	12.45

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	37	49	65	46	51	42
normalized size	1	1.	1.26	0.88	1.17	1.55	1.1	1.21	1.
time (sec)	N/A	0.1	0.014	0.01	1.518	0.215	0.27	0.212	14.542

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	60	44	58	81	49	61	46
normalized size	1	1.	1.22	0.9	1.18	1.65	1.	1.24	0.94
time (sec)	N/A	0.102	0.028	0.011	1.526	0.217	0.279	0.211	14.789

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	38	45	27	39	27
normalized size	1	1.	1.	0.84	1.19	1.41	0.84	1.22	0.84
time (sec)	N/A	0.062	0.008	0.011	1.526	0.223	0.132	0.211	12.334

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	47	35	46	57	42	47	39
normalized size	1	1.	1.15	0.85	1.12	1.39	1.02	1.15	0.95
time (sec)	N/A	0.081	0.014	0.007	1.582	0.235	0.167	0.211	11.52

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	53	33	43	53	41	45	37
normalized size	1	1.	1.36	0.85	1.1	1.36	1.05	1.15	0.95
time (sec)	N/A	0.083	0.021	0.008	1.523	0.239	0.156	0.211	11.117

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	54	1	46	57	0
normalized size	1	1.	1.	0.82	1.08	0.02	0.92	1.14	0.
time (sec)	N/A	0.059	0.006	0.002	1.377	0.204	0.047	0.207	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	58	1	49	61	0
normalized size	1	1.	1.	0.8	1.05	0.02	0.89	1.11	0.
time (sec)	N/A	0.077	0.004	0.001	1.398	0.2	0.049	0.208	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	58	1	49	61	49
normalized size	1	1.	1.	0.8	1.05	0.02	0.89	1.11	0.89
time (sec)	N/A	0.101	0.004	0.002	1.422	0.192	0.05	0.211	13.18

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	92	92	77	103	1	88	107	0
normalized size	1	1.19	1.19	1.	1.34	0.01	1.14	1.39	0.
time (sec)	N/A	0.114	0.005	0.002	1.42	0.192	0.066	0.208	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	1	94	111	0
normalized size	1	1.	1.	0.82	1.1	0.01	0.97	1.14	0.
time (sec)	N/A	0.145	0.006	0.001	1.377	0.189	0.069	0.209	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	1	92	111	75
normalized size	1	1.	1.	0.82	1.1	0.01	0.95	1.14	0.77
time (sec)	N/A	0.161	0.006	0.	1.376	0.183	0.066	0.209	35.103

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	134	113	151	1	134	157	0
normalized size	1	1.	1.	0.84	1.13	0.01	1.	1.17	0.
time (sec)	N/A	0.184	0.006	0.002	1.404	0.186	0.076	0.209	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	116	155	1	138	161	0
normalized size	1	1.	1.	0.83	1.12	0.01	0.99	1.16	0.
time (sec)	N/A	0.213	0.006	0.001	1.413	0.186	0.078	0.21	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	116	155	1	138	161	107
normalized size	1	1.	1.	0.83	1.12	0.01	0.99	1.16	0.77
time (sec)	N/A	0.251	0.008	0.001	1.371	0.185	0.079	0.209	34.641

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	173	148	198	1	178	205	0
normalized size	1	1.	1.	0.86	1.14	0.01	1.03	1.18	0.
time (sec)	N/A	0.26	0.008	0.002	1.415	0.185	0.089	0.21	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	181	152	204	1	185	211	0
normalized size	1	1.	1.	0.84	1.13	0.01	1.02	1.17	0.
time (sec)	N/A	0.306	0.008	0.001	1.425	0.182	0.089	0.21	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	181	152	204	1	184	211	136
normalized size	1	1.	1.	0.84	1.13	0.01	1.02	1.17	0.75
time (sec)	N/A	0.333	0.008	0.002	1.377	0.185	0.089	0.209	42.262

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	191	231	0	0	178	302	0
normalized size	1	1.	0.93	1.13	0.	0.	0.87	1.47	0.
time (sec)	N/A	0.529	0.227	0.005	0.	0.	2.889	0.216	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	184	221	0	0	150	285	0
normalized size	1	1.	0.95	1.15	0.	0.	0.78	1.48	0.
time (sec)	N/A	0.497	0.166	0.006	0.	0.	2.82	0.215	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	200	209	0	0	160	257	0
normalized size	1	1.	1.09	1.14	0.	0.	0.87	1.4	0.
time (sec)	N/A	0.469	0.126	0.004	0.	0.	2.959	0.216	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	176	200	0	0	160	246	163
normalized size	1	1.	0.99	1.13	0.	0.	0.9	1.39	0.92
time (sec)	N/A	0.291	0.189	0.003	0.	0.	2.25	0.217	43.474

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	176	207	0	0	0	267	170
normalized size	1	1.	0.96	1.12	0.	0.	0.	1.45	0.92
time (sec)	N/A	0.433	0.187	0.008	0.	0.	0.	0.216	72.579

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	184	216	0	0	0	279	175
normalized size	1	1.	0.96	1.12	0.	0.	0.	1.45	0.91
time (sec)	N/A	0.46	0.501	0.01	0.	0.	0.	0.217	77.108

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	192	225	0	0	0	285	184
normalized size	1	1.	0.95	1.11	0.	0.	0.	1.4	0.91
time (sec)	N/A	0.427	0.374	0.01	0.	0.	0.	0.217	67.291

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	174	219	0	0	109	258	175
normalized size	1	1.	0.92	1.15	0.	0.	0.57	1.36	0.92
time (sec)	N/A	0.364	0.275	0.012	0.	0.	4.315	0.217	49.22

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	186	228	0	0	124	273	178
normalized size	1	1.	0.93	1.14	0.	0.	0.62	1.36	0.89
time (sec)	N/A	0.366	0.455	0.01	0.	0.	3.505	0.215	54.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	189	253	0	0	116	266	178
normalized size	1	1.	0.95	1.27	0.	0.	0.58	1.34	0.89
time (sec)	N/A	0.315	0.383	0.006	0.	0.	2.611	0.217	46.514

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	199	274	0	0	0	311	175
normalized size	1	1.	0.9	1.23	0.	0.	0.	1.4	0.79
time (sec)	N/A	0.617	0.329	0.017	0.	0.	0.	0.218	58.777

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	213	275	0	0	0	328	139
normalized size	1	1.	0.92	1.19	0.	0.	0.	1.42	0.6
time (sec)	N/A	0.68	0.522	0.021	0.	0.	0.	0.217	49.63

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	221	276	0	0	0	344	24
normalized size	1	1.	0.91	1.14	0.	0.	0.	1.42	0.1
time (sec)	N/A	0.687	0.329	0.019	0.	0.	0.	0.217	13.58

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	225	289	0	0	0	373	26
normalized size	1	1.	0.86	1.1	0.	0.	0.	1.42	0.1
time (sec)	N/A	0.793	0.352	0.02	0.	0.	0.	0.216	13.796

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	198	255	0	0	148	288	192
normalized size	1	1.	0.92	1.19	0.	0.	0.69	1.34	0.89
time (sec)	N/A	0.442	0.444	0.013	0.	0.	10.673	0.218	61.871

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	214	256	0	0	170	306	216
normalized size	1	1.	0.9	1.07	0.	0.	0.71	1.28	0.9
time (sec)	N/A	0.46	0.557	0.014	0.	0.	7.141	0.218	73.794

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	213	308	0	0	163	301	207
normalized size	1	1.	0.95	1.37	0.	0.	0.72	1.34	0.92
time (sec)	N/A	0.415	0.562	0.006	0.	0.	4.378	0.22	59.817

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	229	331	0	0	0	359	204
normalized size	1	1.	0.89	1.29	0.	0.	0.	1.4	0.79
time (sec)	N/A	0.816	0.357	0.019	0.	0.	0.	0.218	73.925

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	248	334	0	0	0	377	160
normalized size	1	1.	0.93	1.25	0.	0.	0.	1.41	0.6
time (sec)	N/A	0.909	0.437	0.023	0.	0.	0.	0.22	57.43

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	253	337	0	0	0	390	26
normalized size	1	1.	0.92	1.22	0.	0.	0.	1.41	0.09
time (sec)	N/A	0.976	0.413	0.025	0.	0.	0.	0.218	14.184

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	255	351	0	0	0	421	27
normalized size	1	1.	0.86	1.18	0.	0.	0.	1.41	0.09
time (sec)	N/A	1.112	0.698	0.025	0.	0.	0.	0.222	14.122

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	230	275	0	0	201	333	228
normalized size	1	1.	0.93	1.11	0.	0.	0.81	1.34	0.92
time (sec)	N/A	0.534	0.478	0.017	0.	0.	27.395	0.218	75.874

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	241	278	0	0	214	346	250
normalized size	1	1.	0.89	1.03	0.	0.	0.79	1.28	0.93
time (sec)	N/A	0.548	0.713	0.016	0.	0.	15.017	0.218	91.282

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	239	360	0	0	202	333	235
normalized size	1	1.	0.96	1.44	0.	0.	0.81	1.33	0.94
time (sec)	N/A	0.499	0.446	0.007	0.	0.	8.36	0.218	71.005

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	259	394	0	0	0	409	231
normalized size	1	1.	0.89	1.35	0.	0.	0.	1.41	0.79
time (sec)	N/A	1.005	0.45	0.023	0.	0.	0.	0.22	86.48

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	279	397	0	0	0	427	178
normalized size	1	1.	0.93	1.32	0.	0.	0.	1.42	0.59
time (sec)	N/A	1.142	0.554	0.027	0.	0.	0.	0.22	63.224

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	284	400	0	0	0	441	26
normalized size	1	1.	0.92	1.29	0.	0.	0.	1.42	0.08
time (sec)	N/A	1.231	0.608	0.028	0.	0.	0.	0.218	13.948

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	284	415	0	0	0	459	27
normalized size	1	1.	0.84	1.22	0.	0.	0.	1.35	0.08
time (sec)	N/A	1.4	0.99	0.028	0.	0.	0.	0.218	14.19

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	35	41	54	36	31
normalized size	1	1.	1.97	1.	1.21	1.41	1.86	1.24	1.07
time (sec)	N/A	0.101	0.024	0.01	1.532	0.255	0.589	0.212	11.071

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	35	41	54	36	31
normalized size	1	1.	1.97	1.	1.21	1.41	1.86	1.24	1.07
time (sec)	N/A	0.069	0.01	0.005	1.519	0.249	0.593	0.212	9.621

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	38	43	54	39	31
normalized size	1	1.	1.87	0.94	1.23	1.39	1.74	1.26	1.
time (sec)	N/A	0.098	0.026	0.012	1.54	0.259	0.599	0.211	11.638

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	38	43	54	39	31
normalized size	1	1.	1.87	0.94	1.23	1.39	1.74	1.26	1.
time (sec)	N/A	0.073	0.011	0.007	1.569	0.246	0.59	0.214	10.752

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	146	87	0	77	100	220	46
normalized size	1	1.	2.92	1.74	0.	1.54	2.	4.4	0.92
time (sec)	N/A	0.16	0.092	0.008	0.	0.294	0.848	0.24	13.702

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	149	135	0	81	110	209	51
normalized size	1	1.	2.81	2.55	0.	1.53	2.08	3.94	0.96
time (sec)	N/A	0.172	0.134	0.007	0.	0.291	0.882	0.242	14.025

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	148	132	0	80	109	131	49
normalized size	1	1.	2.74	2.44	0.	1.48	2.02	2.43	0.91
time (sec)	N/A	0.141	0.088	0.005	0.	0.279	0.841	0.216	14.151

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	147	90	0	78	102	122	48
normalized size	1	1.	2.77	1.7	0.	1.47	1.92	2.3	0.91
time (sec)	N/A	0.144	0.099	0.008	0.	0.275	0.862	0.217	15.122

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	1	90	117	90
normalized size	1	1.	1.	0.82	1.1	0.01	0.93	1.21	0.93
time (sec)	N/A	0.231	0.051	0.002	1.379	0.23	0.071	0.21	28.039

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	1	90	117	90
normalized size	1	1.	1.	0.82	1.1	0.01	0.93	1.21	0.93
time (sec)	N/A	0.208	0.051	0.001	1.381	0.214	0.068	0.209	31.598

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	1	90	117	90
normalized size	1	1.	1.	0.82	1.1	0.01	0.93	1.21	0.93
time (sec)	N/A	0.21	0.055	0.001	1.384	0.229	0.068	0.209	28.004

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	1	90	117	0
normalized size	1	1.	1.	0.82	1.1	0.01	0.93	1.21	0.
time (sec)	N/A	0.192	0.038	0.002	1.422	0.207	0.067	0.208	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	103	1	87	113	0
normalized size	1	1.	1.	0.84	1.12	0.01	0.95	1.23	0.
time (sec)	N/A	0.17	0.039	0.001	1.376	0.207	0.065	0.207	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	81	100	100	85	112	0
normalized size	1	1.	1.	0.92	1.14	1.14	0.97	1.27	0.
time (sec)	N/A	0.122	0.058	0.004	1.378	0.245	0.699	0.21	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	81	100	109	82	112	0
normalized size	1	1.	1.	0.94	1.16	1.27	0.95	1.3	0.
time (sec)	N/A	0.149	0.077	0.009	1.394	0.24	0.734	0.209	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	78	100	109	82	108	0
normalized size	1	1.	0.91	0.91	1.16	1.27	0.95	1.26	0.
time (sec)	N/A	0.155	0.141	0.009	4.444	0.234	0.86	0.211	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	76	101	109	82	107	0
normalized size	1	1.	0.88	0.88	1.17	1.27	0.95	1.24	0.
time (sec)	N/A	0.155	0.148	0.01	1.372	0.241	1.517	0.211	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	76	101	109	82	104	0
normalized size	1	1.	0.9	0.88	1.17	1.27	0.95	1.21	0.
time (sec)	N/A	0.161	0.118	0.01	1.39	0.24	5.308	0.213	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	204	1	167	216	151
normalized size	1	1.	1.	0.93	1.25	0.01	1.02	1.33	0.93
time (sec)	N/A	0.422	0.082	0.002	1.375	0.223	0.1	0.21	44.964

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	204	1	167	216	151
normalized size	1	1.	1.	0.93	1.25	0.01	1.02	1.33	0.93
time (sec)	N/A	0.389	0.074	0.001	1.415	0.213	0.095	0.209	48.591

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	150	152	204	1	167	216	146
normalized size	1	1.	0.95	0.96	1.29	0.01	1.06	1.37	0.92
time (sec)	N/A	0.476	0.191	0.002	1.435	0.214	0.094	0.209	64.079

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	163	152	204	1	167	216	0
normalized size	1	1.	1.03	0.96	1.29	0.01	1.06	1.37	0.
time (sec)	N/A	0.4	0.067	0.001	1.42	0.216	0.094	0.209	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	125	149	200	1	163	212	0
normalized size	1	1.	0.82	0.97	1.31	0.01	1.07	1.39	0.
time (sec)	N/A	0.35	0.173	0.	1.379	0.217	0.092	0.208	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	154	153	197	197	162	211	0
normalized size	1	1.	1.03	1.03	1.32	1.32	1.09	1.42	0.
time (sec)	N/A	0.243	0.113	0.004	1.381	0.244	0.941	0.209	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	152	152	197	207	156	209	0
normalized size	1	1.	1.03	1.03	1.34	1.41	1.06	1.42	0.
time (sec)	N/A	0.332	0.159	0.009	1.38	0.246	0.983	0.209	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	127	150	197	207	156	207	0
normalized size	1	1.	0.86	1.02	1.34	1.41	1.06	1.41	0.
time (sec)	N/A	0.338	0.185	0.01	1.384	0.251	1.112	0.212	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	149	198	207	156	207	0
normalized size	1	1.	0.81	0.98	1.3	1.36	1.03	1.36	0.
time (sec)	N/A	0.308	0.183	0.011	7.458	0.249	1.868	0.22	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	125	149	198	207	155	205	0
normalized size	1	1.	0.82	0.98	1.3	1.36	1.02	1.35	0.
time (sec)	N/A	0.305	0.176	0.011	5.934	0.247	6.239	0.219	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	223	224	293	1	246	315	211
normalized size	1	1.	1.	1.	1.31	0.	1.1	1.41	0.95
time (sec)	N/A	0.591	0.112	0.003	1.437	0.224	0.122	0.214	58.572

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	223	224	293	1	246	315	211
normalized size	1	1.	1.	1.	1.31	0.	1.1	1.41	0.95
time (sec)	N/A	0.585	0.105	0.	1.416	0.222	0.117	0.216	61.647

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	223	224	293	1	246	315	199
normalized size	1	1.	1.05	1.06	1.38	0.	1.16	1.49	0.94
time (sec)	N/A	0.613	0.121	0.001	1.428	0.218	0.115	0.214	79.94

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	223	224	293	1	246	315	0
normalized size	1	1.	1.05	1.06	1.38	0.	1.16	1.49	0.
time (sec)	N/A	0.57	0.091	0.002	1.375	0.222	0.114	0.216	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	170	221	289	1	243	311	0
normalized size	1	1.	0.82	1.07	1.4	0.	1.17	1.5	0.
time (sec)	N/A	0.478	0.221	0.002	1.374	0.22	0.111	0.216	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	214	224	286	286	240	308	0
normalized size	1	1.	1.07	1.12	1.43	1.43	1.2	1.54	0.
time (sec)	N/A	0.316	0.196	0.007	1.395	0.243	1.252	0.22	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	172	224	286	296	236	308	0
normalized size	1	1.	0.87	1.13	1.44	1.49	1.19	1.56	0.
time (sec)	N/A	0.464	0.347	0.01	1.384	0.233	1.326	0.218	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	174	222	286	296	236	305	0
normalized size	1	1.	0.88	1.12	1.44	1.49	1.19	1.54	0.
time (sec)	N/A	0.465	0.5	0.011	1.383	0.234	1.426	0.218	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	172	220	286	296	235	304	0
normalized size	1	1.	0.82	1.05	1.37	1.42	1.12	1.45	0.
time (sec)	N/A	0.485	0.238	0.01	1.384	0.246	2.194	0.218	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	170	220	286	296	233	302	0
normalized size	1	1.	0.81	1.05	1.37	1.42	1.11	1.44	0.
time (sec)	N/A	0.479	0.328	0.011	1.384	0.247	6.369	0.221	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	334	533	0	0	874	513	0
normalized size	1	1.	1.01	1.61	0.	0.	2.64	1.55	0.
time (sec)	N/A	2.026	1.24	0.009	0.	0.	50.485	0.226	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	299	505	0	0	842	477	0
normalized size	1	1.	0.96	1.61	0.	0.	2.69	1.52	0.
time (sec)	N/A	1.97	0.515	0.007	0.	0.	47.21	0.224	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	290	483	0	0	789	450	0
normalized size	1	1.	0.99	1.64	0.	0.	2.68	1.53	0.
time (sec)	N/A	1.852	0.443	0.007	0.	0.	95.203	0.224	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	272	455	0	0	811	428	0
normalized size	1	1.	0.99	1.65	0.	0.	2.95	1.56	0.
time (sec)	N/A	1.82	1.263	0.006	0.	0.	56.423	0.227	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	257	254	429	0	0	804	397	0
normalized size	1	0.99	0.98	1.66	0.	0.	3.1	1.53	0.
time (sec)	N/A	0.801	0.74	0.006	0.	0.	49.168	0.224	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	256	258	426	0	0	0	409	0
normalized size	1	0.99	1.	1.65	0.	0.	0.	1.59	0.
time (sec)	N/A	0.968	0.444	0.01	0.	0.	0.	0.225	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	257	423	0	0	0	404	0
normalized size	1	1.	1.02	1.67	0.	0.	0.	1.6	0.
time (sec)	N/A	0.986	0.588	0.009	0.	0.	0.	0.226	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	258	257	423	0	0	0	393	228
normalized size	1	0.99	0.99	1.63	0.	0.	0.	1.51	0.88
time (sec)	N/A	0.814	0.782	0.008	0.	0.	0.	0.226	106.028

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	274	264	442	0	0	0	423	243
normalized size	1	0.99	0.96	1.6	0.	0.	0.	1.53	0.88
time (sec)	N/A	0.959	1.084	0.01	0.	0.	0.	0.227	121.797

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	334	562	0	0	0	512	0
normalized size	1	1.	0.99	1.67	0.	0.	0.	1.52	0.
time (sec)	N/A	1.476	1.279	0.015	0.	0.	0.	0.23	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	294	533	0	0	0	475	0
normalized size	1	1.	0.95	1.71	0.	0.	0.	1.53	0.
time (sec)	N/A	1.29	0.397	0.015	0.	0.	0.	0.227	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	288	280	506	0	0	0	444	0
normalized size	1	0.99	0.97	1.74	0.	0.	0.	1.53	0.
time (sec)	N/A	0.995	0.368	0.014	0.	0.	0.	0.227	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	285	504	0	0	0	451	265
normalized size	1	1.	0.99	1.74	0.	0.	0.	1.56	0.92
time (sec)	N/A	1.048	0.386	0.014	0.	0.	0.	0.227	160.671

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	268	465	0	0	0	429	252
normalized size	1	1.	0.97	1.68	0.	0.	0.	1.55	0.91
time (sec)	N/A	0.807	0.34	0.013	0.	0.	0.	0.225	118.282

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	287	269	509	0	0	0	452	262
normalized size	1	0.99	0.93	1.76	0.	0.	0.	1.56	0.91
time (sec)	N/A	1.103	0.373	0.019	0.	0.	0.	0.227	149.517

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	285	519	0	0	0	473	250
normalized size	1	1.	0.95	1.72	0.	0.	0.	1.57	0.83
time (sec)	N/A	1.182	0.558	0.019	0.	0.	0.	0.226	138.149

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	304	292	527	0	0	0	483	241
normalized size	1	0.99	0.95	1.72	0.	0.	0.	1.58	0.79
time (sec)	N/A	1.144	1.084	0.021	0.	0.	0.	0.226	110.265

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	336	303	561	0	0	0	520	255
normalized size	1	0.99	0.9	1.66	0.	0.	0.	1.54	0.75
time (sec)	N/A	1.453	1.146	0.022	0.	0.	0.	0.227	128.198

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	342	621	0	0	0	541	0
normalized size	1	1.	0.99	1.8	0.	0.	0.	1.57	0.
time (sec)	N/A	1.717	0.594	0.019	0.	0.	0.	0.229	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	315	520	0	0	0	512	0
normalized size	1	1.	0.97	1.6	0.	0.	0.	1.58	0.
time (sec)	N/A	1.322	0.539	0.017	0.	0.	0.	0.226	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	287	490	0	0	0	454	279
normalized size	1	1.	0.97	1.65	0.	0.	0.	1.53	0.94
time (sec)	N/A	0.901	0.461	0.016	0.	0.	0.	0.227	114.252

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	297	498	0	0	0	481	299
normalized size	1	1.	0.92	1.54	0.	0.	0.	1.49	0.93
time (sec)	N/A	1.018	0.577	0.015	0.	0.	0.	0.228	144.401

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	295	506	0	0	0	467	287
normalized size	1	1.	0.94	1.62	0.	0.	0.	1.49	0.92
time (sec)	N/A	0.916	0.458	0.014	0.	0.	0.	0.226	120.495

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	345	311	620	0	0	0	529	284
normalized size	1	0.99	0.9	1.79	0.	0.	0.	1.52	0.82
time (sec)	N/A	1.409	0.51	0.029	0.	0.	0.	0.232	136.863

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	336	624	0	0	0	556	219
normalized size	1	1.	0.93	1.72	0.	0.	0.	1.54	0.6
time (sec)	N/A	1.631	1.158	0.025	0.	0.	0.	0.23	96.027

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	357	337	626	0	0	0	568	83
normalized size	1	0.99	0.94	1.74	0.	0.	0.	1.58	0.23
time (sec)	N/A	1.562	1.3	0.027	0.	0.	0.	0.229	51.389

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	392	352	680	0	0	0	612	88
normalized size	1	0.99	0.89	1.72	0.	0.	0.	1.55	0.22
time (sec)	N/A	1.976	1.88	0.028	0.	0.	0.	0.229	49.522

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	583	583	329	793	0	0	129	0	522
normalized size	1	1.	0.56	1.36	0.	0.	0.22	0.	0.9
time (sec)	N/A	1.337	2.899	0.027	0.	0.	4.152	0.	163.345

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	319	773	0	0	107	0	498
normalized size	1	1.	0.57	1.38	0.	0.	0.19	0.	0.89
time (sec)	N/A	0.964	1.42	0.01	0.	0.	3.308	0.	117.422

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	537	537	314	753	0	0	107	0	474
normalized size	1	1.	0.58	1.4	0.	0.	0.2	0.	0.88
time (sec)	N/A	0.668	2.758	0.01	0.	0.	3.087	0.	70.836

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	305	735	0	0	105	0	447
normalized size	1	1.	0.6	1.44	0.	0.	0.21	0.	0.88
time (sec)	N/A	0.43	1.615	0.006	0.	0.	2.62	0.	39.099

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	493	740	0	0	105	0	457
normalized size	1	1.	0.95	1.43	0.	0.	0.2	0.	0.88
time (sec)	N/A	0.494	1.698	0.01	0.	0.	3.522	0.	45.7

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	513	759	0	0	107	0	479
normalized size	1	1.	0.94	1.39	0.	0.	0.2	0.	0.88
time (sec)	N/A	0.728	1.903	0.011	0.	0.	3.721	0.	78.042

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	525	778	0	0	112	0	495
normalized size	1	1.	0.92	1.37	0.	0.	0.2	0.	0.87
time (sec)	N/A	0.95	3.49	0.012	0.	0.	4.033	0.	111.172

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	334	836	0	0	129	0	0
normalized size	1	1.	0.56	1.41	0.	0.	0.22	0.	0.
time (sec)	N/A	1.236	1.792	0.031	0.	0.	60.099	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	330	817	0	0	129	0	517
normalized size	1	1.	0.57	1.42	0.	0.	0.22	0.	0.9
time (sec)	N/A	0.969	2.382	0.012	0.	0.	39.837	0.	134.209

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	319	800	0	0	129	0	483
normalized size	1	1.	0.59	1.48	0.	0.	0.24	0.	0.89
time (sec)	N/A	0.73	1.274	0.011	0.	0.	30.574	0.	76.853

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	305	779	0	0	109	0	466
normalized size	1	1.	0.58	1.49	0.	0.	0.21	0.	0.89
time (sec)	N/A	0.6	2.275	0.01	0.	0.	25.472	0.	50.42

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	317	782	0	0	109	0	495
normalized size	1	1.	0.57	1.39	0.	0.	0.19	0.	0.88
time (sec)	N/A	0.704	1.591	0.008	0.	0.	25.366	0.	71.48

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	314	785	0	0	107	0	468
normalized size	1	1.	0.59	1.48	0.	0.	0.2	0.	0.88
time (sec)	N/A	0.589	1.57	0.006	0.	0.	24.972	0.	48.263

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	518	810	0	0	265	0	462
normalized size	1	1.	0.89	1.4	0.	0.	0.46	0.	0.8
time (sec)	N/A	0.839	3.502	0.009	0.	0.	40.682	0.	57.643

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	542	825	0	0	267	0	214
normalized size	1	1.	0.89	1.36	0.	0.	0.44	0.	0.35
time (sec)	N/A	1.093	4.751	0.012	0.	0.	56.884	0.	31.669

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	733	733	433	1674	0	0	238	0	0
normalized size	1	1.	0.59	2.28	0.	0.	0.32	0.	0.
time (sec)	N/A	3.319	2.789	0.028	0.	0.	5.988	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	399	1197	0	0	223	0	0
normalized size	1	1.	0.59	1.76	0.	0.	0.33	0.	0.
time (sec)	N/A	2.492	2.087	0.012	0.	0.	5.413	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	667	667	390	1311	0	0	223	0	0
normalized size	1	1.	0.58	1.97	0.	0.	0.33	0.	0.
time (sec)	N/A	1.926	2.231	0.01	0.	0.	4.898	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	639	639	393	1557	0	0	194	0	0
normalized size	1	1.	0.62	2.44	0.	0.	0.3	0.	0.
time (sec)	N/A	1.441	2.358	0.009	0.	0.	4.506	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	620	620	714	1118	0	0	235	0	568
normalized size	1	1.	1.15	1.8	0.	0.	0.38	0.	0.92
time (sec)	N/A	1.114	3.467	0.01	0.	0.	6.667	0.	142.471

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	810	1248	0	0	236	0	580
normalized size	1	1.	1.27	1.96	0.	0.	0.37	0.	0.91
time (sec)	N/A	1.294	2.708	0.014	0.	0.	6.687	0.	168.208

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	640	640	962	1529	0	0	255	0	0
normalized size	1	1.	1.5	2.39	0.	0.	0.4	0.	0.
time (sec)	N/A	1.472	3.092	0.014	0.	0.	7.294	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	637	637	769	1114	0	0	265	0	0
normalized size	1	1.	1.21	1.75	0.	0.	0.42	0.	0.
time (sec)	N/A	1.597	2.648	0.015	0.	0.	9.147	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	855	1286	0	0	274	0	0
normalized size	1	1.	1.23	1.85	0.	0.	0.39	0.	0.
time (sec)	N/A	2.06	3.704	0.014	0.	0.	9.382	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	934	1571	0	0	240	0	0
normalized size	1	1.	1.43	2.41	0.	0.	0.37	0.	0.
time (sec)	N/A	1.558	3.921	0.014	0.	0.	8.933	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	800	1180	0	0	304	0	0
normalized size	1	1.	1.21	1.79	0.	0.	0.46	0.	0.
time (sec)	N/A	1.865	4.472	0.014	0.	0.	13.916	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	711	711	892	1376	0	0	308	0	0
normalized size	1	1.	1.25	1.94	0.	0.	0.43	0.	0.
time (sec)	N/A	2.149	4.208	0.013	0.	0.	14.755	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	743	743	979	1679	0	0	304	0	0
normalized size	1	1.	1.32	2.26	0.	0.	0.41	0.	0.
time (sec)	N/A	2.47	4.149	0.016	0.	0.	14.307	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	466	1764	0	0	512	0	0
normalized size	1	1.	0.59	2.23	0.	0.	0.65	0.	0.
time (sec)	N/A	3.646	2.473	0.029	0.	0.	14.618	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	742	742	436	1269	0	0	525	0	0
normalized size	1	1.	0.59	1.71	0.	0.	0.71	0.	0.
time (sec)	N/A	2.773	2.092	0.011	0.	0.	12.897	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	723	723	429	1383	0	0	525	0	0
normalized size	1	1.	0.59	1.91	0.	0.	0.73	0.	0.
time (sec)	N/A	2.352	2.179	0.01	0.	0.	11.371	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	429	1629	0	0	444	0	0
normalized size	1	1.	0.62	2.35	0.	0.	0.64	0.	0.
time (sec)	N/A	1.757	2.487	0.008	0.	0.	9.166	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	753	1188	0	0	473	0	0
normalized size	1	1.	1.11	1.76	0.	0.	0.7	0.	0.
time (sec)	N/A	1.334	3.426	0.013	0.	0.	14.686	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	817	1317	0	0	474	0	0
normalized size	1	1.	1.18	1.9	0.	0.	0.68	0.	0.
time (sec)	N/A	1.522	3.727	0.015	0.	0.	14.317	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	952	1613	0	0	462	0	0
normalized size	1	1.	1.37	2.32	0.	0.	0.67	0.	0.
time (sec)	N/A	1.686	4.095	0.012	0.	0.	14.479	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	813	1193	0	0	484	0	0
normalized size	1	1.	1.17	1.72	0.	0.	0.7	0.	0.
time (sec)	N/A	1.805	3.866	0.014	0.	0.	17.153	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	741	741	878	1342	0	0	495	0	0
normalized size	1	1.	1.18	1.81	0.	0.	0.67	0.	0.
time (sec)	N/A	2.278	3.241	0.014	0.	0.	17.543	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	949	1606	0	0	476	0	0
normalized size	1	1.	1.38	2.33	0.	0.	0.69	0.	0.
time (sec)	N/A	1.765	4.654	0.015	0.	0.	17.375	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	805	1196	0	0	524	0	0
normalized size	1	1.	1.16	1.73	0.	0.	0.76	0.	0.
time (sec)	N/A	1.903	4.587	0.014	0.	0.	24.127	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	897	1375	0	0	536	0	0
normalized size	1	1.	1.2	1.84	0.	0.	0.72	0.	0.
time (sec)	N/A	2.373	4.166	0.015	0.	0.	25.241	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	705	705	978	1663	0	0	527	0	0
normalized size	1	1.	1.39	2.36	0.	0.	0.75	0.	0.
time (sec)	N/A	1.855	4.476	0.015	0.	0.	24.442	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	714	714	844	1273	0	0	573	0	0
normalized size	1	1.	1.18	1.78	0.	0.	0.8	0.	0.
time (sec)	N/A	2.031	5.326	0.041	0.	0.	38.868	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	764	764	930	1470	0	0	576	0	0
normalized size	1	1.	1.22	1.92	0.	0.	0.75	0.	0.
time (sec)	N/A	2.484	4.589	0.015	0.	0.	41.519	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	796	796	1017	1773	0	0	541	0	0
normalized size	1	1.	1.28	2.23	0.	0.	0.68	0.	0.
time (sec)	N/A	2.835	4.694	0.046	0.	0.	38.622	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	120	129	0	0	0	0	0	95
normalized size	1	1.18	1.26	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.161	0.156	0.053	0.	0.	0.	0.	23.066

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	125	131	0	0	0	0	0	99
normalized size	1	1.17	1.22	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.192	0.124	0.057	0.	0.	0.	0.	23.897

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	125	131	0	0	0	0	0	99
normalized size	1	1.17	1.22	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.225	0.149	0.065	0.	0.	0.	0.	26.816

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	73	1	63	76	0
normalized size	1	1.	1.	0.81	1.07	0.01	0.93	1.12	0.
time (sec)	N/A	0.088	0.008	0.001	1.389	0.187	0.053	0.215	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	77	1	66	80	66
normalized size	1	1.	1.	0.79	1.05	0.01	0.9	1.1	0.9
time (sec)	N/A	0.126	0.005	0.001	1.374	0.192	0.059	0.223	17.99

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	138	1	121	142	0
normalized size	1	1.	1.14	0.94	1.27	0.01	1.11	1.3	0.
time (sec)	N/A	0.145	0.006	0.	1.37	0.187	0.079	0.229	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	129	106	142	1	124	146	107
normalized size	1	1.	1.13	0.93	1.25	0.01	1.09	1.28	0.94
time (sec)	N/A	0.319	0.009	0.001	1.379	0.19	0.081	0.223	42.356

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	203	1	180	208	0
normalized size	1	1.	1.19	1.	1.34	0.01	1.19	1.38	0.
time (sec)	N/A	0.219	0.007	0.	7.007	0.189	0.096	0.223	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	185	154	207	1	184	212	151
normalized size	1	1.	1.19	0.99	1.33	0.01	1.18	1.36	0.97
time (sec)	N/A	0.407	0.009	0.002	1.37	0.192	0.101	0.228	51.008

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	236	199	267	1	241	274	0
normalized size	1	1.	1.22	1.03	1.38	0.01	1.25	1.42	0.
time (sec)	N/A	0.299	0.009	0.002	1.373	0.185	0.116	0.228	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	241	202	271	1	245	278	201
normalized size	1	1.	1.22	1.02	1.37	0.01	1.24	1.4	1.02
time (sec)	N/A	0.496	0.01	0.003	1.378	0.188	0.114	0.226	64.736

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	214	177	0	0	952	419	119
normalized size	1	1.	1.61	1.33	0.	0.	7.16	3.15	0.89
time (sec)	N/A	0.282	0.111	0.006	0.	0.	25.638	0.242	38.705

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	221	208	0	0	887	443	0
normalized size	1	1.	1.36	1.28	0.	0.	5.48	2.73	0.
time (sec)	N/A	0.484	0.15	0.006	0.	0.	24.244	0.242	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	296	294	0	0	950	392	272
normalized size	1	1.	1.01	1.	0.	0.	3.24	1.34	0.93
time (sec)	N/A	0.52	0.571	0.005	0.	0.	24.979	0.231	82.734

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	311	325	0	0	886	416	0
normalized size	1	1.	0.97	1.01	0.	0.	2.76	1.3	0.
time (sec)	N/A	0.78	0.309	0.009	0.	0.	24.143	0.231	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	315	362	0	0	517	427	294
normalized size	1	1.	0.99	1.14	0.	0.	1.63	1.34	0.92
time (sec)	N/A	0.583	0.635	0.011	0.	0.	32.189	0.23	94.134

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	294	333	0	0	508	409	291
normalized size	1	1.	0.95	1.07	0.	0.	1.64	1.32	0.94
time (sec)	N/A	0.581	0.531	0.021	0.	0.	64.074	0.232	100.513

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	347	432	0	0	578	478	333
normalized size	1	1.	0.99	1.23	0.	0.	1.65	1.36	0.95
time (sec)	N/A	0.688	0.587	0.01	0.	0.	130.201	0.232	112.38

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	329	371	0	0	0	456	320
normalized size	1	1.	0.97	1.09	0.	0.	0.	1.34	0.94
time (sec)	N/A	0.689	0.8	0.018	0.	0.	0.	0.233	116.304

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	379	399	0	0	0	528	364
normalized size	1	1.	0.99	1.04	0.	0.	0.	1.38	0.95
time (sec)	N/A	0.823	0.93	0.022	0.	0.	0.	0.238	137.466

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	366	401	0	0	0	513	359
normalized size	1	1.	0.96	1.06	0.	0.	0.	1.35	0.94
time (sec)	N/A	0.845	0.789	0.022	0.	0.	0.	0.232	137.143

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	296	390	0	0	252	0	386
normalized size	1	1.	0.71	0.93	0.	0.	0.6	0.	0.92
time (sec)	N/A	1.079	0.86	0.041	0.	0.	10.266	0.	102.131

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	275	380	0	0	212	0	362
normalized size	1	1.	0.7	0.96	0.	0.	0.54	0.	0.92
time (sec)	N/A	0.978	0.833	0.013	0.	0.	8.801	0.	94.692

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	280	361	0	0	212	0	338
normalized size	1	1.	0.76	0.98	0.	0.	0.57	0.	0.92
time (sec)	N/A	0.82	0.725	0.015	0.	0.	8.43	0.	75.307

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	266	337	0	0	158	0	325
normalized size	1	1.	0.75	0.95	0.	0.	0.45	0.	0.92
time (sec)	N/A	0.736	0.719	0.013	0.	0.	5.443	0.	69.572

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	257	313	0	0	156	0	303
normalized size	1	1.	0.78	0.95	0.	0.	0.47	0.	0.92
time (sec)	N/A	0.518	0.763	0.011	0.	0.	5.158	0.	50.837

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	280	339	0	0	204	0	316
normalized size	1	1.	0.81	0.98	0.	0.	0.59	0.	0.92
time (sec)	N/A	0.687	2.358	0.019	0.	0.	7.372	0.	69.077

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	355	339	0	0	206	0	308
normalized size	1	1.	1.04	0.99	0.	0.	0.6	0.	0.9
time (sec)	N/A	0.719	6.18	0.02	0.	0.	7.46	0.	73.262

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	296	360	0	0	230	0	308
normalized size	1	1.	0.87	1.05	0.	0.	0.67	0.	0.9
time (sec)	N/A	0.724	1.095	0.02	0.	0.	6.757	0.	71.901

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	295	362	0	0	235	0	325
normalized size	1	1.	0.83	1.01	0.	0.	0.66	0.	0.91
time (sec)	N/A	0.824	0.891	0.023	0.	0.	7.05	0.	81.207

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	267	385	0	0	211	0	0
normalized size	1	1.	0.81	1.17	0.	0.	0.64	0.	0.
time (sec)	N/A	0.596	3.396	0.02	0.	0.	8.312	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	314	404	0	0	216	0	0
normalized size	1	1.	0.87	1.12	0.	0.	0.6	0.	0.
time (sec)	N/A	0.818	1.164	0.027	0.	0.	8.722	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	277	361	0	0	189	0	0
normalized size	1	1.	0.79	1.03	0.	0.	0.54	0.	0.
time (sec)	N/A	0.802	0.848	0.022	0.	0.	7.982	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	283	385	0	0	192	0	0
normalized size	1	1.	0.75	1.03	0.	0.	0.51	0.	0.
time (sec)	N/A	0.932	0.81	0.027	0.	0.	8.687	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	293	408	0	0	246	0	0
normalized size	1	1.	0.73	1.02	0.	0.	0.62	0.	0.
time (sec)	N/A	1.029	0.938	0.023	0.	0.	12.525	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	305	429	0	0	246	0	0
normalized size	1	1.	0.72	1.01	0.	0.	0.58	0.	0.
time (sec)	N/A	1.185	0.873	0.028	0.	0.	14.027	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	327	462	0	0	462	0	442
normalized size	1	1.	0.69	0.97	0.	0.	0.97	0.	0.93
time (sec)	N/A	1.311	1.096	0.047	0.	0.	28.921	0.	116.903

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	431	434	0	0	398	0	418
normalized size	1	1.	0.95	0.96	0.	0.	0.88	0.	0.92
time (sec)	N/A	1.19	6.132	0.013	0.	0.	23.888	0.	108.96

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	325	413	0	0	398	0	394
normalized size	1	1.	0.76	0.97	0.	0.	0.93	0.	0.92
time (sec)	N/A	1.024	0.807	0.017	0.	0.	22.199	0.	91.517

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	302	392	0	0	396	0	382
normalized size	1	1.	0.74	0.96	0.	0.	0.97	0.	0.93
time (sec)	N/A	0.91	0.836	0.019	0.	0.	14.897	0.	85.49

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	294	368	0	0	394	0	357
normalized size	1	1.	0.77	0.96	0.	0.	1.03	0.	0.93
time (sec)	N/A	0.639	0.797	0.011	0.	0.	13.824	0.	61.301

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	319	411	0	0	405	0	376
normalized size	1	1.	0.79	1.02	0.	0.	1.	0.	0.93
time (sec)	N/A	0.911	0.895	0.025	0.	0.	17.093	0.	90.477

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	328	411	0	0	406	0	376
normalized size	1	1.	0.81	1.02	0.	0.	1.	0.	0.93
time (sec)	N/A	0.838	0.973	0.02	0.	0.	16.869	0.	96.596

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	326	409	0	0	377	0	377
normalized size	1	1.	0.8	1.01	0.	0.	0.93	0.	0.93
time (sec)	N/A	0.826	1.153	0.027	0.	0.	13.385	0.	93.471

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	327	408	0	0	381	0	381
normalized size	1	1.	0.8	1.	0.	0.	0.93	0.	0.93
time (sec)	N/A	0.848	1.07	0.022	0.	0.	13.299	0.	91.759

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	329	409	0	0	379	0	0
normalized size	1	1.	0.85	1.06	0.	0.	0.98	0.	0.
time (sec)	N/A	0.776	1.513	0.028	0.	0.	15.644	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	331	409	0	0	386	0	0
normalized size	1	1.	0.86	1.06	0.	0.	1.	0.	0.
time (sec)	N/A	0.81	1.515	0.024	0.	0.	16.048	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	331	408	0	0	406	0	0
normalized size	1	1.	0.84	1.04	0.	0.	1.04	0.	0.
time (sec)	N/A	0.784	1.617	0.027	0.	0.	15.213	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	330	411	0	0	415	0	0
normalized size	1	1.	0.8	1.	0.	0.	1.01	0.	0.
time (sec)	N/A	0.914	1.126	0.026	0.	0.	16.307	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	309	416	0	0	444	0	0
normalized size	1	1.	0.82	1.1	0.	0.	1.18	0.	0.
time (sec)	N/A	0.679	4.975	0.027	0.	0.	21.266	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	351	437	0	0	449	0	0
normalized size	1	1.	0.87	1.08	0.	0.	1.11	0.	0.
time (sec)	N/A	0.874	1.337	0.03	0.	0.	23.087	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	314	417	0	0	398	0	0
normalized size	1	1.	0.79	1.05	0.	0.	1.	0.	0.
time (sec)	N/A	0.865	1.062	0.026	0.	0.	24.639	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	317	441	0	0	401	0	0
normalized size	1	1.	0.75	1.04	0.	0.	0.95	0.	0.
time (sec)	N/A	0.984	0.995	0.031	0.	0.	27.676	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	328	462	0	0	403	0	0
normalized size	1	1.	0.73	1.03	0.	0.	0.9	0.	0.
time (sec)	N/A	1.068	1.174	0.03	0.	0.	37.674	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	339	483	0	0	403	0	0
normalized size	1	1.	0.72	1.02	0.	0.	0.85	0.	0.
time (sec)	N/A	1.208	1.099	0.034	0.	0.	42.455	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	259	335	0	0	177	0	330
normalized size	1	1.	0.72	0.93	0.	0.	0.49	0.	0.91
time (sec)	N/A	0.81	1.017	0.022	0.	0.	8.133	0.	85.718

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	241	325	0	0	156	0	304
normalized size	1	1.	0.72	0.97	0.	0.	0.46	0.	0.9
time (sec)	N/A	0.708	0.877	0.01	0.	0.	6.909	0.	79.002

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	245	248	0	0	156	0	275
normalized size	1	1.	0.8	0.81	0.	0.	0.51	0.	0.89
time (sec)	N/A	0.582	0.659	0.01	0.	0.	6.532	0.	61.082

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	235	229	0	0	129	0	267
normalized size	1	1.	0.79	0.77	0.	0.	0.43	0.	0.89
time (sec)	N/A	0.496	0.513	0.008	0.	0.	4.431	0.	56.105

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	225	208	0	0	128	0	248
normalized size	1	1.	0.82	0.75	0.	0.	0.46	0.	0.9
time (sec)	N/A	0.353	0.415	0.007	0.	0.	3.683	0.	41.245

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	235	222	0	0	126	0	258
normalized size	1	1.	0.82	0.78	0.	0.	0.44	0.	0.91
time (sec)	N/A	0.376	0.919	0.01	0.	0.	4.566	0.	50.601

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	250	299	0	0	128	0	275
normalized size	1	1.	0.81	0.97	0.	0.	0.41	0.	0.89
time (sec)	N/A	0.564	3.775	0.012	0.	0.	4.663	0.	62.906

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	242	293	0	0	126	0	265
normalized size	1	1.	0.81	0.98	0.	0.	0.42	0.	0.88
time (sec)	N/A	0.547	0.761	0.021	0.	0.	4.409	0.	60.438

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	249	316	0	0	131	0	286
normalized size	1	1.	0.77	0.98	0.	0.	0.41	0.	0.89
time (sec)	N/A	0.655	0.653	0.014	0.	0.	4.871	0.	74.643

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	259	335	0	0	158	0	306
normalized size	1	1.	0.75	0.97	0.	0.	0.46	0.	0.88
time (sec)	N/A	0.75	0.893	0.015	0.	0.	7.135	0.	81.234

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	268	354	0	0	163	0	342
normalized size	1	1.	0.71	0.94	0.	0.	0.43	0.	0.91
time (sec)	N/A	0.865	0.905	0.015	0.	0.	7.779	0.	98.68

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	267	378	0	0	202	0	0
normalized size	1	1.	0.73	1.04	0.	0.	0.55	0.	0.
time (sec)	N/A	0.879	0.901	0.019	0.	0.	72.375	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	255	358	0	0	172	0	316
normalized size	1	1.	0.74	1.04	0.	0.	0.5	0.	0.92
time (sec)	N/A	0.714	0.649	0.011	0.	0.	50.538	0.	140.344

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	243	340	0	0	172	0	287
normalized size	1	1.	0.77	1.08	0.	0.	0.55	0.	0.91
time (sec)	N/A	0.577	0.584	0.011	0.	0.	38.204	0.	92.624

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	224	331	0	0	156	0	270
normalized size	1	1.	0.75	1.11	0.	0.	0.53	0.	0.91
time (sec)	N/A	0.421	0.541	0.011	0.	0.	31.075	0.	51.655

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	242	331	0	0	156	0	294
normalized size	1	1.	0.73	0.99	0.	0.	0.47	0.	0.88
time (sec)	N/A	0.561	0.91	0.012	0.	0.	28.343	0.	90.412

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	197	250	0	0	133	0	265
normalized size	1	1.	0.65	0.83	0.	0.	0.44	0.	0.87
time (sec)	N/A	0.403	0.434	0.014	0.	0.	26.559	0.	56.388

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	195	250	0	0	131	0	241
normalized size	1	1.	0.71	0.91	0.	0.	0.48	0.	0.88
time (sec)	N/A	0.273	0.393	0.007	0.	0.	27.356	0.	35.616

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	225	336	0	0	289	0	238
normalized size	1	1.	0.7	1.04	0.	0.	0.89	0.	0.74
time (sec)	N/A	0.559	5.225	0.011	0.	0.	39.947	0.	44.148

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	245	355	0	0	291	0	109
normalized size	1	1.	0.71	1.03	0.	0.	0.85	0.	0.32
time (sec)	N/A	0.696	0.734	0.015	0.	0.	69.358	0.	23.936

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	259	363	0	0	316	0	110
normalized size	1	1.	0.71	0.99	0.	0.	0.86	0.	0.3
time (sec)	N/A	0.838	0.71	0.024	0.	0.	84.43	0.	23.931

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	267	383	0	0	321	0	32
normalized size	1	1.	0.69	0.99	0.	0.	0.83	0.	0.08
time (sec)	N/A	1.045	0.719	0.014	0.	0.	114.883	0.	15.818

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	174	0	0	0	0	0	218
normalized size	1	1.	0.65	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.521	0.346	0.099	0.	0.	0.	0.	58.2

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	170	184	0	0	0	141	0	136
normalized size	1	1.19	1.29	0.	0.	0.	0.99	0.	0.95
time (sec)	N/A	0.275	0.163	0.056	0.	0.	142.287	0.	36.259

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	160	0	0	0	0	0	139
normalized size	1	1.	0.91	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.403	0.239	0.077	0.	0.	0.	0.	46.973

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	9	5
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.12	0.62
time (sec)	N/A	0.013	0.001	0.002	1.429	0.201	0.046	0.218	4.346

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	11	7	12	7
normalized size	1	1.	1.	0.9	1.1	1.1	0.7	1.2	0.7
time (sec)	N/A	0.016	0.002	0.002	1.421	0.201	0.059	0.216	8.256

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	11	8	12	8
normalized size	1	1.	1.	0.9	1.1	1.1	0.8	1.2	0.8
time (sec)	N/A	0.016	0.002	0.001	1.408	0.201	0.063	0.217	7.471

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	18	23	23	15	20	7
normalized size	1	1.	2.1	1.8	2.3	2.3	1.5	2.	0.7
time (sec)	N/A	0.012	0.004	0.008	1.365	0.206	0.1	0.219	3.89

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	22	22	24	22	22
normalized size	1	1.	1.	0.71	0.92	0.92	1.	0.92	0.92
time (sec)	N/A	0.035	0.011	0.006	1.545	0.204	0.129	0.216	6.388

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	51	61	46	53	0
normalized size	1	1.	1.	0.78	1.02	1.22	0.92	1.06	0.
time (sec)	N/A	0.083	0.028	0.014	1.564	0.211	0.25	0.221	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	39	51	61	46	53	0
normalized size	1	1.	0.92	0.78	1.02	1.22	0.92	1.06	0.
time (sec)	N/A	0.085	0.028	0.01	1.551	0.211	0.265	0.218	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	47	62	74	56	65	0
normalized size	1	1.	0.93	0.78	1.03	1.23	0.93	1.08	0.
time (sec)	N/A	0.088	0.02	0.01	1.522	0.212	0.284	0.221	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	47	62	74	56	65	0
normalized size	1	1.	0.87	0.78	1.03	1.23	0.93	1.08	0.
time (sec)	N/A	0.088	0.019	0.01	1.513	0.211	0.282	0.219	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	51	61	48	47	42
normalized size	1	1.	1.	0.78	1.02	1.22	0.96	0.94	0.84
time (sec)	N/A	0.052	0.009	0.008	1.516	0.21	0.205	0.22	7.667

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	51	61	48	53	44
normalized size	1	1.	1.	0.78	1.02	1.22	0.96	1.06	0.88
time (sec)	N/A	0.073	0.017	0.008	1.529	0.21	0.207	0.218	20.417

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	100	85	113	169	105	116	0
normalized size	1	1.	0.91	0.77	1.03	1.54	0.95	1.05	0.
time (sec)	N/A	0.218	0.173	0.016	1.512	0.214	0.595	0.219	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	85	113	169	105	116	0
normalized size	1	1.	0.88	0.77	1.03	1.54	0.95	1.05	0.
time (sec)	N/A	0.207	0.136	0.015	1.501	0.215	0.593	0.219	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	122	68	82	131	70	85	65
normalized size	1	1.	1.51	0.84	1.01	1.62	0.86	1.05	0.8
time (sec)	N/A	0.127	0.771	0.017	1.507	0.205	0.272	0.22	21.96

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	73	100	190	82	103	0
normalized size	1	1.	0.91	0.79	1.09	2.07	0.89	1.12	0.
time (sec)	N/A	0.207	0.059	0.019	1.581	0.213	0.463	0.22	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	119	115	142	362	124	150	0
normalized size	1	1.	0.8	0.78	0.96	2.45	0.84	1.01	0.
time (sec)	N/A	0.319	0.097	0.027	1.646	0.214	0.878	0.222	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	121	115	142	363	124	150	0
normalized size	1	1.	0.83	0.79	0.97	2.49	0.85	1.03	0.
time (sec)	N/A	0.314	0.137	0.023	1.728	0.214	0.865	0.223	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	111	111	128	269	116	143	117
normalized size	1	1.	0.78	0.78	0.9	1.89	0.82	1.01	0.82
time (sec)	N/A	0.268	0.118	0.022	1.556	0.214	0.833	0.22	48.127

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	111	111	128	269	116	143	117
normalized size	1	1.	0.78	0.78	0.9	1.89	0.82	1.01	0.82
time (sec)	N/A	0.262	0.105	0.021	1.519	0.215	0.834	0.221	55.487

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	103	102	117	193	110	120	100
normalized size	1	1.	0.91	0.9	1.04	1.71	0.97	1.06	0.88
time (sec)	N/A	0.167	0.093	0.019	1.521	0.212	0.679	0.22	25.849

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	111	102	128	269	119	134	0
normalized size	1	1.	0.85	0.78	0.98	2.05	0.91	1.02	0.
time (sec)	N/A	0.264	0.11	0.019	1.543	0.214	0.824	0.22	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	91	76	101	116	102	93	92
normalized size	1	1.	0.92	0.77	1.02	1.17	1.03	0.94	0.93
time (sec)	N/A	0.136	0.032	0.013	1.524	0.213	0.574	0.22	19.244

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	150	0	0	0	0	0	190
normalized size	1	1.	0.93	0.	0.	0.	0.	0.	1.17
time (sec)	N/A	0.312	0.492	0.083	0.	0.	0.	0.	53.302

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	162	130	0	412	1251	572	0
normalized size	1	1.	1.93	1.55	0.	4.9	14.89	6.81	0.
time (sec)	N/A	0.111	0.186	0.029	0.	0.228	10.984	0.227	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	99	87	0	216	552	289	0
normalized size	1	1.	1.62	1.43	0.	3.54	9.05	4.74	0.
time (sec)	N/A	0.082	0.109	0.024	0.	0.227	5.457	0.224	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	45	0	76	163	99	0
normalized size	1	1.	1.02	1.1	0.	1.85	3.98	2.41	0.
time (sec)	N/A	0.046	0.092	0.021	0.	0.224	2.412	0.22	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	23	15	16	0
normalized size	1	1.	1.	1.08	0.	1.92	1.25	1.33	0.
time (sec)	N/A	0.01	0.003	0.002	0.	0.227	0.037	0.213	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	0	0	0	0	0	32
normalized size	1	1.	1.14	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.063	0.07	0.086	0.	0.	0.	0.	14.76

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	56	0	0	0	0	0	32
normalized size	1	1.	1.27	0.	0.	0.	0.	0.	0.73
time (sec)	N/A	0.06	0.088	0.081	0.	0.	0.	0.	8.996

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	108	0	0	0	0	0	36
normalized size	1	1.	2.35	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.06	0.137	0.096	0.	0.	0.	0.	9.021

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	399	0	0	0	0	0	292
normalized size	1	1.	1.31	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.494	2.76	0.26	0.	0.	0.	0.	57.742

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	89	0	0	0
normalized size	1	1.	1.	0.	0.	1.98	0.	0.	0.
time (sec)	N/A	0.655	0.14	0.125	0.	0.233	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	178	0	0	0	0	0	214
normalized size	1	1.	0.65	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.377	0.338	0.108	0.	0.	0.	0.	52.157

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	204	0	0	0	0	0	272
normalized size	1	1.	0.69	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.413	1.555	0.114	0.	0.	0.	0.	55.442

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	151	0	0	0	0	0	122
normalized size	1	1.	0.93	0.	0.	0.	0.	0.	0.75
time (sec)	N/A	0.264	0.322	0.139	0.	0.	0.	0.	39.191

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	27	0	0	20
normalized size	1	1.	1.	0.88	1.12	1.12	0.	0.	0.83
time (sec)	N/A	0.023	0.076	0.013	1.542	0.218	0.	0.	31.219

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	166	0	0	0	0	0	0
normalized size	1	1.	1.61	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.411	0.317	0.068	0.	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	41	82	0	354	20
normalized size	1	1.	1.	0.	1.46	2.93	0.	12.64	0.71
time (sec)	N/A	0.15	0.202	0.393	2.751	0.241	0.	0.257	18.305

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	138	104	161	0	323	36
normalized size	1	1.	1.02	3.07	2.31	3.58	0.	7.18	0.8
time (sec)	N/A	0.242	0.266	0.677	2.205	0.246	0.	0.257	38.047

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	80	73	0	196	26
normalized size	1	1.	1.	1.68	2.58	2.35	0.	6.32	0.84
time (sec)	N/A	0.314	0.292	0.185	1.939	0.249	0.	0.265	75.372

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	136	124	119	0	244	0
normalized size	1	1.	0.91	3.02	2.76	2.64	0.	5.42	0.
time (sec)	N/A	0.84	0.423	0.605	2.112	0.254	0.	0.274	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [578] had the largest ratio of [0.7917]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	20	0.05
2	A	2	1	1.	22	0.045
3	A	2	1	1.	22	0.045
4	A	2	1	1.	25	0.04
5	A	2	1	1.	27	0.037
6	A	2	1	1.	27	0.037
7	A	6	6	1.	15	0.4
8	A	7	7	1.	15	0.467
9	A	8	7	1.	15	0.467
10	A	9	7	1.	15	0.467
11	A	6	6	1.	15	0.4
12	A	6	6	1.	16	0.375
13	A	3	3	1.	11	0.273
14	A	3	3	1.	15	0.2
15	A	3	3	1.	13	0.231
16	A	3	3	1.	13	0.231
17	A	6	6	1.	15	0.4
18	A	3	3	1.	19	0.158
19	A	3	3	1.	21	0.143
20	A	4	4	1.	31	0.129
21	A	3	3	1.	36	0.083
22	A	12	9	1.	35	0.257
23	A	11	8	1.	33	0.242
24	A	10	7	1.	36	0.194
25	A	10	9	1.	19	0.474
26	A	9	8	1.	18	0.444

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	4	4	1.	27	0.148
28	A	4	4	1.	28	0.143
29	A	4	4	1.	24	0.167
30	A	4	4	1.	24	0.167
31	A	4	4	1.	26	0.154
32	A	4	4	1.	26	0.154
33	A	4	4	1.	28	0.143
34	A	4	4	1.	30	0.133
35	A	4	4	1.	29	0.138
36	A	4	4	1.	29	0.138
37	A	4	4	1.	29	0.138
38	A	4	4	1.	32	0.125
39	A	6	6	1.	13	0.462
40	A	4	4	1.	49	0.082
41	A	4	4	1.	57	0.07
42	A	2	2	1.	31	0.065
43	A	3	3	1.	42	0.071
44	A	4	4	1.	42	0.095
45	A	4	4	1.	45	0.089
46	A	4	4	1.	45	0.089
47	A	4	4	1.	44	0.091
48	A	3	3	1.	20	0.15
49	A	6	6	1.	20	0.3
50	A	2	2	1.	16	0.125
51	A	5	5	1.	20	0.25
52	A	3	3	1.	18	0.167
53	A	2	1	1.	30	0.033
54	A	2	1	1.	30	0.033
55	A	2	1	1.	28	0.036
56	A	2	1	1.	30	0.033
57	A	7	7	1.	30	0.233
58	A	8	8	1.	30	0.267
59	A	7	5	1.08	32	0.156
60	A	6	5	1.05	32	0.156
61	A	5	4	1.02	32	0.125
62	A	4	4	1.	32	0.125
63	A	5	5	1.	32	0.156
64	A	6	5	1.	32	0.156
65	A	7	5	1.	32	0.156
66	A	7	6	1.	32	0.188
67	A	6	6	1.	32	0.188
68	A	5	5	1.	32	0.156
69	A	6	6	1.	32	0.188
70	A	8	7	1.	17	0.412
71	A	10	8	1.	17	0.471
72	A	10	8	0.99	17	0.471
73	A	10	8	0.99	22	0.364
74	A	10	8	1.	22	0.364
75	A	10	8	1.	22	0.364

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
76	A	9	8	1.	17	0.471
77	A	9	8	1.	19	0.421
78	A	8	7	1.	18	0.389
79	A	3	3	1.	18	0.167
80	A	3	3	1.	22	0.136
81	A	1	1	1.	20	0.05
82	A	1	1	1.	20	0.05
83	A	3	3	1.	33	0.091
84	A	3	3	1.	35	0.086
85	A	1	1	1.	36	0.028
86	A	1	1	1.	36	0.028
87	A	3	3	1.	30	0.1
88	A	3	3	1.	32	0.094
89	A	1	1	1.	33	0.03
90	A	1	1	1.	33	0.03
91	A	1	1	1.	20	0.05
92	A	1	1	1.	24	0.042
93	A	3	3	1.	22	0.136
94	A	3	3	1.	22	0.136
95	A	1	1	1.	20	0.05
96	A	1	1	1.	20	0.05
97	A	3	3	1.	18	0.167
98	A	3	3	1.	22	0.136
99	A	1	1	1.	35	0.029
100	A	1	1	1.	37	0.027
101	A	3	3	1.	38	0.079
102	A	3	3	1.	38	0.079
103	A	1	1	1.	32	0.031
104	A	1	1	1.	34	0.029
105	A	3	3	1.	35	0.086
106	A	3	3	1.	35	0.086
107	A	3	3	1.	17	0.176
108	A	3	3	1.	18	0.167
109	A	3	3	1.	19	0.158
110	A	3	3	1.	20	0.15
111	A	3	3	1.	15	0.2
112	A	3	3	1.	17	0.176
113	A	3	3	1.	15	0.2
114	A	3	3	1.	17	0.176
115	A	7	5	1.	16	0.312
116	A	13	9	1.	15	0.6
117	A	8	6	1.	16	0.375
118	A	14	10	1.	15	0.667
119	A	9	6	1.	16	0.375
120	A	15	10	1.	15	0.667
121	A	10	6	1.	16	0.375
122	A	16	10	1.	15	0.667
123	A	7	5	1.	15	0.333
124	A	13	9	1.	13	0.692

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	7	5	1.	21	0.238
126	A	13	9	1.	20	0.45
127	A	8	6	1.	21	0.286
128	A	14	10	1.	20	0.5
129	A	9	6	1.	21	0.286
130	A	15	10	1.	20	0.5
131	A	10	6	1.	21	0.286
132	A	16	10	1.	20	0.5
133	A	3	2	1.	11	0.182
134	A	3	2	1.	12	0.167
135	A	2	1	1.	15	0.067
136	A	3	2	1.	14	0.143
137	A	2	1	1.	17	0.059
138	A	3	2	1.	19	0.105
139	A	2	1	1.	20	0.05
140	A	2	2	1.	14	0.143
141	A	4	3	1.	17	0.176
142	A	5	4	1.	19	0.21
143	A	3	2	1.	20	0.1
144	A	3	2	1.	21	0.095
145	A	3	2	1.	22	0.091
146	A	3	2	1.	24	0.083
147	A	3	2	1.	25	0.08
148	A	3	2	1.	25	0.08
149	A	8	6	1.	26	0.231
150	A	9	7	1.	26	0.269
151	A	10	7	1.	26	0.269
152	A	10	7	0.85	11	0.636
153	A	3	3	1.	12	0.25
154	A	13	9	0.87	15	0.6
155	A	10	7	0.85	14	0.5
156	A	9	6	1.	17	0.353
157	A	14	10	0.87	19	0.526
158	A	13	9	1.	20	0.45
159	A	2	2	1.	14	0.143
160	A	12	7	0.86	17	0.412
161	A	5	5	1.	19	0.263
162	A	15	10	0.88	20	0.5
163	A	13	8	0.86	21	0.381
164	A	12	7	1.	22	0.318
165	A	16	11	0.88	24	0.458
166	A	15	10	1.	25	0.4
167	A	2	2	1.	19	0.105
168	A	11	8	1.	17	0.471
169	A	9	7	1.	20	0.35
170	A	15	10	1.	19	0.526
171	A	11	8	1.	31	0.258
172	A	8	6	1.	31	0.194
173	A	9	7	1.	31	0.226

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
174	A	10	8	1.	31	0.258
175	A	17	11	1.	30	0.367
176	A	14	10	1.	30	0.333
177	A	15	11	1.	30	0.367
178	A	16	12	1.	30	0.4
179	A	2	2	1.	21	0.095
180	A	2	2	1.	21	0.095
181	A	2	1	1.	19	0.053
182	A	2	2	1.	19	0.105
183	A	2	2	1.	21	0.095
184	A	2	2	1.	21	0.095
185	A	2	2	1.	21	0.095
186	A	13	9	1.	36	0.25
187	A	13	9	1.	41	0.22
188	A	13	9	1.	46	0.196
189	A	19	12	1.	35	0.343
190	A	19	12	1.	40	0.3
191	A	19	12	1.	45	0.267
192	A	8	6	1.	36	0.167
193	A	8	6	1.	41	0.146
194	A	10	8	1.	46	0.174
195	A	14	10	1.	35	0.286
196	A	14	10	1.	40	0.25
197	A	16	11	1.	45	0.244
198	A	9	7	1.	36	0.194
199	A	9	7	1.	41	0.171
200	A	9	7	1.	46	0.152
201	A	15	11	1.	35	0.314
202	A	15	11	1.	40	0.275
203	A	15	11	1.	45	0.244
204	A	10	8	1.	36	0.222
205	A	10	8	1.	41	0.195
206	A	10	8	1.	46	0.174
207	A	16	12	1.	35	0.343
208	A	16	12	1.	40	0.3
209	A	16	12	1.	45	0.267
210	A	6	5	1.	17	0.294
211	A	7	6	1.	18	0.333
212	A	7	6	1.	19	0.316
213	A	6	5	1.	20	0.25
214	A	8	7	1.	22	0.318
215	A	1	1	1.	23	0.043
216	A	1	1	1.	26	0.038
217	A	1	1	1.	28	0.036
218	A	1	1	1.	31	0.032
219	A	1	1	1.	15	0.067
220	A	12	10	1.	42	0.238
221	F	0	0	N/A	0	N/A
222	F	0	0	N/A	0	N/A

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
223	A	3	2	1.	30	0.067
224	A	3	2	1.	30	0.067
225	A	3	2	1.	30	0.067
226	A	3	2	1.	30	0.067
227	A	3	2	1.	30	0.067
228	A	3	2	1.	30	0.067
229	A	3	2	1.	30	0.067
230	A	3	2	1.	30	0.067
231	A	3	2	1.	30	0.067
232	A	3	2	1.	30	0.067
233	A	9	8	1.	30	0.267
234	A	9	8	1.	30	0.267
235	A	9	8	1.	30	0.267
236	A	9	8	1.	30	0.267
237	A	9	8	1.	30	0.267
238	A	9	8	1.	28	0.286
239	A	8	7	1.	27	0.259
240	A	8	7	1.	30	0.233
241	A	8	7	1.	30	0.233
242	A	8	7	1.	30	0.233
243	A	8	7	1.	30	0.233
244	A	8	7	1.	30	0.233
245	A	8	7	1.	30	0.233
246	A	8	7	1.	30	0.233
247	A	8	7	1.	30	0.233
248	A	8	7	1.	30	0.233
249	A	8	7	1.	30	0.233
250	A	8	7	1.	30	0.233
251	A	3	2	1.	30	0.067
252	A	3	2	1.	30	0.067
253	A	3	2	1.	30	0.067
254	A	3	2	1.	30	0.067
255	A	3	2	1.	30	0.067
256	A	3	2	1.	30	0.067
257	A	3	2	1.	30	0.067
258	A	3	2	1.	30	0.067
259	A	3	2	1.	30	0.067
260	A	9	8	1.	30	0.267
261	A	12	10	1.	30	0.333
262	A	9	8	1.	30	0.267
263	A	11	10	1.	30	0.333
264	A	9	8	1.	30	0.267
265	A	10	9	1.	28	0.321
266	A	9	9	1.	27	0.333
267	A	9	8	1.	30	0.267
268	A	9	8	1.	30	0.267
269	A	9	8	1.	30	0.267
270	A	9	8	1.	30	0.267
271	A	9	8	1.	30	0.267

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	9	8	1.	30	0.267
273	A	9	8	1.	30	0.267
274	A	9	8	1.	30	0.267
275	A	9	8	1.	30	0.267
276	A	3	2	1.	30	0.067
277	A	3	2	1.	30	0.067
278	A	3	2	1.	30	0.067
279	A	3	2	1.	30	0.067
280	A	3	2	1.	30	0.067
281	A	3	2	1.	30	0.067
282	A	3	2	1.	30	0.067
283	A	3	2	1.	30	0.067
284	A	3	2	1.	30	0.067
285	A	3	2	1.	30	0.067
286	A	10	9	1.	30	0.3
287	A	14	10	1.	30	0.333
288	A	10	9	1.	30	0.3
289	A	13	10	1.	30	0.333
290	A	10	9	1.	30	0.3
291	A	12	10	1.	30	0.333
292	A	10	10	1.	30	0.333
293	A	10	10	1.	28	0.357
294	A	9	9	1.	27	0.333
295	A	9	9	1.	30	0.3
296	A	9	9	1.	30	0.3
297	A	10	9	1.	30	0.3
298	A	10	9	1.	30	0.3
299	A	10	8	1.	30	0.267
300	A	10	8	1.	30	0.267
301	A	10	8	1.	30	0.267
302	A	10	8	1.	30	0.267
303	A	10	8	1.	30	0.267
304	A	8	7	1.	16	0.438
305	A	5	4	1.	16	0.25
306	A	8	7	1.	16	0.438
307	A	6	6	1.	14	0.429
308	A	6	5	1.	16	0.312
309	A	6	5	1.	16	0.312
310	A	3	2	1.	16	0.125
311	A	6	6	1.	14	0.429
312	A	6	6	1.	16	0.375
313	A	2	1	1.	18	0.056
314	A	2	1	1.	19	0.053
315	A	2	1	1.	21	0.048
316	A	2	1	1.19	20	0.05
317	A	2	1	1.	21	0.048
318	A	2	1	1.	23	0.043
319	A	2	1	1.	20	0.05
320	A	2	1	1.	21	0.048

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
321	A	2	1	1.	23	0.043
322	A	2	1	1.	20	0.05
323	A	2	1	1.	21	0.048
324	A	2	1	1.	23	0.043
325	A	10	8	1.	23	0.348
326	A	10	8	1.	23	0.348
327	A	10	8	1.	21	0.381
328	A	8	7	1.	20	0.35
329	A	10	8	1.	23	0.348
330	A	10	8	1.	23	0.348
331	A	10	8	1.	23	0.348
332	A	7	7	1.	23	0.304
333	A	7	7	1.	21	0.333
334	A	7	7	1.	20	0.35
335	A	11	9	1.	23	0.391
336	A	11	9	1.	23	0.391
337	A	11	9	1.	23	0.391
338	A	11	9	1.	23	0.391
339	A	8	8	1.	23	0.348
340	A	8	8	1.	21	0.381
341	A	8	8	1.	20	0.4
342	A	12	9	1.	23	0.391
343	A	12	9	1.	23	0.391
344	A	12	9	1.	23	0.391
345	A	12	9	1.	23	0.391
346	A	9	8	1.	23	0.348
347	A	9	9	1.	21	0.429
348	A	9	8	1.	20	0.4
349	A	13	9	1.	23	0.391
350	A	13	9	1.	23	0.391
351	A	13	9	1.	23	0.391
352	A	13	9	1.	23	0.391
353	A	5	5	1.	20	0.25
354	A	4	4	1.	18	0.222
355	A	5	5	1.	20	0.25
356	A	4	4	1.	18	0.222
357	A	4	4	1.	27	0.148
358	A	4	4	1.	29	0.138
359	A	4	4	1.	28	0.143
360	A	4	4	1.	28	0.143
361	A	2	1	1.	36	0.028
362	A	2	1	1.	36	0.028
363	A	2	1	1.	36	0.028
364	A	2	1	1.	34	0.029
365	A	2	1	1.	33	0.03
366	A	2	1	1.	36	0.028
367	A	2	1	1.	36	0.028
368	A	2	1	1.	36	0.028
369	A	2	1	1.	36	0.028

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
370	A	2	1	1.	36	0.028
371	A	2	1	1.	38	0.026
372	A	2	1	1.	38	0.026
373	A	3	2	1.	38	0.053
374	A	3	2	1.	36	0.056
375	A	3	2	1.	35	0.057
376	A	3	2	1.	38	0.053
377	A	3	2	1.	38	0.053
378	A	3	2	1.	38	0.053
379	A	2	1	1.	38	0.026
380	A	2	1	1.	38	0.026
381	A	2	1	1.	38	0.026
382	A	2	1	1.	38	0.026
383	A	3	2	1.	38	0.053
384	A	3	2	1.	36	0.056
385	A	3	2	1.	35	0.057
386	A	3	2	1.	38	0.053
387	A	3	2	1.	38	0.053
388	A	3	2	1.	38	0.053
389	A	2	1	1.	38	0.026
390	A	2	1	1.	38	0.026
391	A	13	9	1.	38	0.237
392	A	13	9	1.	38	0.237
393	A	13	9	1.	38	0.237
394	A	13	9	1.	36	0.25
395	A	10	8	0.99	35	0.229
396	A	10	8	0.99	38	0.21
397	A	10	8	1.	38	0.21
398	A	10	8	0.99	38	0.21
399	A	10	8	0.99	38	0.21
400	A	11	9	1.	38	0.237
401	A	11	9	1.	38	0.237
402	A	11	9	0.99	38	0.237
403	A	11	9	1.	36	0.25
404	A	9	8	1.	35	0.229
405	A	11	9	0.99	38	0.237
406	A	11	9	1.	38	0.237
407	A	11	9	0.99	38	0.237
408	A	11	9	0.99	38	0.237
409	A	12	10	1.	38	0.263
410	A	10	9	1.	38	0.237
411	A	8	8	1.	38	0.21
412	A	8	8	1.	36	0.222
413	A	8	8	1.	35	0.229
414	A	12	9	0.99	38	0.237
415	A	12	9	1.	38	0.237
416	A	12	9	0.99	38	0.237
417	A	12	9	0.99	38	0.237
418	A	10	7	1.	25	0.28

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
419	A	8	7	1.	25	0.28
420	A	6	6	1.	23	0.261
421	A	5	5	1.	22	0.227
422	A	7	7	1.	25	0.28
423	A	8	8	1.	25	0.32
424	A	9	8	1.	25	0.32
425	A	8	7	1.	25	0.28
426	A	7	7	1.	25	0.28
427	A	6	6	1.	25	0.24
428	A	4	4	1.	25	0.16
429	A	6	6	1.	23	0.261
430	A	4	4	1.	22	0.182
431	A	10	10	1.	25	0.4
432	A	11	11	1.	25	0.44
433	A	13	9	1.	35	0.257
434	A	11	9	1.	35	0.257
435	A	9	8	1.	33	0.242
436	A	8	7	1.	32	0.219
437	A	11	11	1.	35	0.314
438	A	11	11	1.	35	0.314
439	A	10	9	1.	35	0.257
440	A	11	9	1.	35	0.257
441	A	12	9	1.	35	0.257
442	A	10	10	1.	35	0.286
443	A	11	10	1.	35	0.286
444	A	12	10	1.	35	0.286
445	A	13	10	1.	35	0.286
446	A	14	9	1.	35	0.257
447	A	12	9	1.	35	0.257
448	A	10	8	1.	33	0.242
449	A	9	7	1.	32	0.219
450	A	12	11	1.	35	0.314
451	A	12	11	1.	35	0.314
452	A	11	9	1.	35	0.257
453	A	12	9	1.	35	0.257
454	A	13	9	1.	35	0.257
455	A	11	11	1.	35	0.314
456	A	12	11	1.	35	0.314
457	A	13	11	1.	35	0.314
458	A	11	10	1.	35	0.286
459	A	12	10	1.	35	0.286
460	A	13	10	1.	35	0.286
461	A	14	10	1.	35	0.286
462	A	8	7	1.18	20	0.35
463	A	7	4	1.17	21	0.19
464	A	7	4	1.17	23	0.174
465	A	2	1	1.	23	0.043
466	A	2	1	1.	26	0.038
467	A	3	2	1.	25	0.08

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
468	A	4	3	1.	28	0.107
469	A	3	2	1.	25	0.08
470	A	4	3	1.	28	0.107
471	A	3	2	1.	25	0.08
472	A	4	3	1.	28	0.107
473	A	9	7	1.	26	0.269
474	A	12	9	1.	29	0.31
475	A	15	10	1.	25	0.4
476	A	18	12	1.	28	0.429
477	A	14	10	1.	25	0.4
478	A	14	10	1.	28	0.357
479	A	15	11	1.	25	0.44
480	A	15	11	1.	28	0.393
481	A	16	11	1.	25	0.44
482	A	16	11	1.	28	0.393
483	A	14	12	1.	30	0.4
484	A	13	11	1.	30	0.367
485	A	12	11	1.	30	0.367
486	A	12	11	1.	28	0.393
487	A	11	10	1.	27	0.37
488	A	14	13	1.	30	0.433
489	A	14	13	1.	30	0.433
490	A	14	13	1.	30	0.433
491	A	15	14	1.	30	0.467
492	A	13	13	1.	30	0.433
493	A	14	14	1.	30	0.467
494	A	12	12	1.	30	0.4
495	A	13	12	1.	30	0.4
496	A	14	13	1.	30	0.433
497	A	15	13	1.	30	0.433
498	A	16	12	1.	30	0.4
499	A	15	11	1.	30	0.367
500	A	14	11	1.	30	0.367
501	A	14	11	1.	28	0.393
502	A	13	10	1.	27	0.37
503	A	16	13	1.	30	0.433
504	A	16	14	1.	30	0.467
505	A	16	15	1.	30	0.5
506	A	16	14	1.	30	0.467
507	A	15	15	1.	30	0.5
508	A	15	15	1.	30	0.5
509	A	15	15	1.	30	0.5
510	A	16	16	1.	30	0.533
511	A	14	13	1.	30	0.433
512	A	15	14	1.	30	0.467
513	A	13	12	1.	30	0.4
514	A	14	12	1.	30	0.4
515	A	15	13	1.	30	0.433
516	A	16	13	1.	30	0.433

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
517	A	12	10	1.	30	0.333
518	A	11	9	1.	30	0.3
519	A	10	9	1.	30	0.3
520	A	10	9	1.	28	0.321
521	A	9	8	1.	27	0.296
522	A	12	11	1.	30	0.367
523	A	13	12	1.	30	0.4
524	A	11	10	1.	30	0.333
525	A	12	10	1.	30	0.333
526	A	13	11	1.	30	0.367
527	A	14	11	1.	30	0.367
528	A	12	11	1.	30	0.367
529	A	11	10	1.	30	0.333
530	A	10	9	1.	30	0.3
531	A	9	8	1.	30	0.267
532	A	10	9	1.	30	0.3
533	A	7	6	1.	28	0.214
534	A	4	4	1.	27	0.148
535	A	11	10	1.	30	0.333
536	A	13	12	1.	30	0.4
537	A	15	12	1.	30	0.4
538	A	17	13	1.	30	0.433
539	A	14	4	1.	30	0.133
540	A	12	8	1.19	25	0.32
541	A	13	7	1.	28	0.25
542	A	2	2	1.	22	0.091
543	A	2	2	1.	35	0.057
544	A	2	2	1.	35	0.057
545	A	2	2	1.	22	0.091
546	A	3	3	1.	25	0.12
547	A	6	5	1.	15	0.333
548	A	6	5	1.	15	0.333
549	A	7	6	1.	20	0.3
550	A	7	6	1.	20	0.3
551	A	7	7	1.	17	0.412
552	A	7	6	1.	25	0.24
553	A	11	6	1.	35	0.171
554	A	11	6	1.	35	0.171
555	A	8	5	1.	22	0.227
556	A	11	7	1.	25	0.28
557	A	17	7	1.	15	0.467
558	A	17	7	1.	15	0.467
559	A	14	7	1.	20	0.35
560	A	14	7	1.	20	0.35
561	A	15	9	1.	17	0.529
562	A	14	7	1.	25	0.28
563	A	13	7	1.	18	0.389
564	A	7	4	1.	36	0.111
565	A	4	3	1.	19	0.158

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	4	3	1.	19	0.158
567	A	4	2	1.	17	0.118
568	A	1	0	1.	9	0.
569	A	3	3	1.	19	0.158
570	A	3	3	1.	19	0.158
571	A	3	3	1.	19	0.158
572	A	13	4	1.	38	0.105
573	A	2	2	1.	58	0.034
574	A	10	3	1.	30	0.1
575	A	13	4	1.	36	0.111
576	A	4	4	1.	35	0.114
577	A	1	1	1.	46	0.022
578	A	46	19	1.	24	0.792
579	A	1	1	1.	48	0.021
580	A	1	1	1.	45	0.022
581	A	1	1	1.	69	0.014
582	A	1	1	1.	86	0.012

3 Listing of integrals

3.1 $\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{a+bx}\,(a^2e-abd+b^2c)}{b^3}+\frac{2(a+bx)^{3/2}(bd-2ae)}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

```
[Out] (2*(b^2*c - a*b*d + a^2*e)*Sqrt[a + b*x])/b^3 + (2*(b*d - 2*a*e)*
(a + b*x)^(3/2))/(3*b^3) + (2*e*(a + b*x)^(5/2))/(5*b^3)
```

Rubi [A] time = 0.085047, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2\sqrt{a+bx}\,(a^2e-abd+b^2c)}{b^3}+\frac{2(a+bx)^{3/2}(bd-2ae)}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)*Sqrt[a + b*x])/b^3 + (2*(b*d - 2*a*e)*
(a + b*x)^(3/2))/(3*b^3) + (2*e*(a + b*x)^(5/2))/(5*b^3)
```

Rubi in Sympy [A] time = 13.8333, size = 68, normalized size = 0.94

$$\frac{2e\,(a+bx)^{\frac{5}{2}}}{5b^3}-\frac{2\,(a+bx)^{\frac{3}{2}}\,(2ae-bd)}{3b^3}+\frac{2\sqrt{a+bx}\,\left(a^2e-abd+b^2c\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] 2*e*(a + b*x)**(5/2)/(5*b**3) - 2*(a + b*x)**(3/2)*(2*a*e - b*d)/
(3*b**3) + 2*sqrt(a + b*x)*(a**2*e - a*b*d + b**2*c)/b**3
```

Mathematica [A] time = 0.0555634, size = 53, normalized size = 0.74

$$\frac{2\sqrt{a+bx}\,\left(8a^2e-2ab(5d+2ex)+b^2(15c+x(5d+3ex))\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2*e - 2*a*b*(5*d + 2*e*x) + b^2*(15*c + x*(
5*d + 3*e*x))))/(15*b^3)
```

Maple [A] time = 0.006, size = 53, normalized size = 0.7

$$\frac{6\,ex^2b^2-8\,abex+10\,b^2dx+16\,a^2e-20\,abd+30\,b^2c}{15\,b^3}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*e*x^2-4*a*b*e*x+5*b^2*d*x+8*a^2*e-10*a*
b*d+15*b^2*c)/b^3
```

Maxima [A] time = 1.42257, size = 104, normalized size = 1.44

$$\frac{2\left(15\sqrt{bx+a}c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a
)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
 a)*a^2)*e/b^2)/b
```

Fricas [A] time = 0.21953, size = 72, normalized size = 1.

$$\frac{2\left(3\,b^2ex^2+15\,b^2c-10\,abd+8\,a^2e+\left(5\,b^2d-4\,abe\right)x\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*e*x^2 + 15*b^2*c - 10*a*b*d + 8*a^2*e + (5*b^2*d - 4*
a*b*e)*x)*sqrt(b*x + a)/b^3
```

Sympy [A] time = 3.72752, size = 223, normalized size = 3.1

$$\begin{cases} -\frac{\frac{2ac}{\sqrt{a+bx}}+\frac{2ad\left(-\frac{a}{\sqrt{a+bx}}-\sqrt{a+bx}\right)}{b}+\frac{2ae\left(\frac{a^2}{\sqrt{a+bx}}+2a\sqrt{a+bx}-\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b^2}+2c\left(-\frac{a}{\sqrt{a+bx}}-\sqrt{a+bx}\right)+\frac{2d\left(\frac{a^2}{\sqrt{a+bx}}+2a\sqrt{a+bx}-\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2e\left(-\frac{a^3}{\sqrt{a+bx}}-3a^2\sqrt{a+bx}+a(a+bx)^{\frac{3}{2}}-(a+\cdots\right)}{b^2}}{b} \\ \frac{cx+\frac{dx^2}{2}+\frac{ex^3}{3}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c/sqrt(a + b*x) + 2*a*d*(-a/sqrt(a + b*x) - sqrt
(a + b*x))/b + 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a
 + b*x)**(3/2)/3)/b**2 + 2*c*(-a/sqrt(a + b*x) - sqrt(a + b*x)) +
 2*d*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3
)/b + 2*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*
x)**(3/2) - (a + b*x)**(5/2)/5)/b**2)/b, Ne(b, 0)), ((c*x + d*x**
2/2 + e*x**3/3)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.206862, size = 117, normalized size = 1.62

$$\frac{2\left(15\sqrt{bx+a}c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}b^{8}-10\,(bx+a)^{\frac{3}{2}}ab^{8}+15\sqrt{bx+a}a^{2}b^{8}\right)e}{b^{10}}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a
)*d/b + (3*(b*x + a)^(5/2)*b^8 - 10*(b*x + a)^(3/2)*a*b^8 + 15*sq
rt(b*x + a)*a^2*b^8)*e/b^10)/b
```

3.2 $\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=161

$$-\frac{2(a+bx)^{5/2}\left(-6a^2e^2+6abde+b^2\left(-\left(2ce+d^2\right)\right)\right)}{5b^5}+\frac{4(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)}{3b^5}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^2}{b^5}+\frac{4e(a+bx)^{7/2}(bd-2ae)}{7b^5}+\frac{2e^2(a+bx)^{9/2}}{9b^5}$$

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e
)*(b^2*c - a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*
e - 6*a^2*e^2 - b^2*(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*
e*(b*d - 2*a*e)*(a + b*x)^(7/2))/(7*b^5) + (2*e^2*(a + b*x)^(9/2)
)/(9*b^5)
```

Rubi [A] time = 0.222523, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2(a+bx)^{5/2}\left(-6a^2e^2+6abde+b^2\left(-\left(2ce+d^2\right)\right)\right)}{5b^5}+\frac{4(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)}{3b^5}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^2}{b^5}+\frac{4e(a+bx)^{7/2}(bd-2ae)}{7b^5}+\frac{2e^2(a+bx)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e
)*(b^2*c - a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*
e - 6*a^2*e^2 - b^2*(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*
e*(b*d - 2*a*e)*(a + b*x)^(7/2))/(7*b^5) + (2*e^2*(a + b*x)^(9/2)
)/(9*b^5)
```

Rubi in Sympy [A] time = 37.1698, size = 160, normalized size = 0.99

$$\frac{2e^2(a+bx)^{\frac{9}{2}}}{9b^5}-\frac{4e(a+bx)^{\frac{7}{2}}(2ae-bd)}{7b^5}+\frac{2(a+bx)^{\frac{5}{2}}\left(6a^2e^2-6abde+2b^2ce+b^2d^2\right)}{5b^5}-\frac{4(a+bx)^{\frac{3}{2}}(2ae-bd)\left(a^2e-abd+b^2c\right)}{3b^5}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] 2*e**2*(a + b*x)**(9/2)/(9*b**5) - 4*e*(a + b*x)**(7/2)*(2*a*e -
b*d)/(7*b**5) + 2*(a + b*x)**(5/2)*(6*a**2*e**2 - 6*a*b*d*e + 2*b
**2*c*e + b**2*d**2)/(5*b**5) - 4*(a + b*x)**(3/2)*(2*a*e - b*d)*
(a**2*e - a*b*d + b**2*c)/(3*b**5) + 2*sqrt(a + b*x)*(a**2*e - a*
b*d + b**2*c)**2/b**5
```

Mathematica [A] time = 0.158772, size = 155, normalized size = 0.96

$$\frac{2\sqrt{a+bx}\left(128a^4e^2-32a^3be(9d+2ex)+24a^2b^2\left(2e\left(7c+ex^2\right)+7d^2+6dex\right)-4ab^3\left(21c(5d+2ex)+x\left(21d^2+27dex+1\right.\right.\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4*e^2 - 32*a^3*b*e*(9*d + 2*e*x) + 24*a^2
*b^2*(7*d^2 + 6*d*e*x + 2*e*(7*c + e*x^2)) - 4*a*b^3*(21*c*(5*d +
 2*e*x) + x*(21*d^2 + 27*d*e*x + 10*e^2*x^2)) + b^4*(315*c^2 + 42
*c*x*(5*d + 3*e*x) + x^2*(63*d^2 + 90*d*e*x + 35*e^2*x^2))))/(315
*b^5)
```

Maple [A] time = 0.01, size = 194, normalized size = 1.2

$$\frac{70\,e^2x^4b^4 - 80\,ab^3e^2x^3 + 180\,b^4dex^3 + 96\,a^2b^2e^2x^2 - 216\,ab^3dex^2 + 252\,b^4cex^2 + 126\,b^4d^2x^2 - 128\,a^3be^2x + 288\,a^2b^2dex -}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/315*(b*x+a)^(1/2)*(35*b^4*e^2*x^4-40*a*b^3*e^2*x^3+90*b^4*d*e*x
^3+48*a^2*b^2*e^2*x^2-108*a*b^3*d*e*x^2+126*b^4*c*e*x^2+63*b^4*d^
2*x^2-64*a^3*b*e^2*x+144*a^2*b^2*d*e*x-168*a*b^3*c*e*x-84*a*b^3*d
^2*x+210*b^4*c*d*x+128*a^4*e^2-288*a^3*b*d*e+336*a^2*b^2*c*e+168*
a^2*b^2*d^2-420*a*b^3*c*d+315*b^4*c^2)/b^5
```

Maxima [A] time = 1.42389, size = 320, normalized size = 1.99

$$\frac{2\left(315\,\sqrt{bx+a}c^2 + 42\,c\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)d}{b} + \frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)e}{b^2}\right) + \frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)d^2}{b^2} +}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)^2/sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 42*c*(5*((b*x + a)^(3/2) - 3*sqrt(
b*x + a)*a)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*
sqrt(b*x + a)*a^2)*e/b^2) + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^
(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) -
 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)
*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378
*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a
)*a^4)*e^2/b^4)/b
```

Fricas [A] time = 0.215871, size = 259, normalized size = 1.61

$$\frac{2\left(35\,b^4e^2x^4 + 315\,b^4c^2 - 420\,ab^3cd + 168\,a^2b^2d^2 + 128\,a^4e^2 + 10\left(9\,b^4de - 4\,ab^3e^2\right)x^3 + 3\left(21\,b^4d^2 + 16\,a^2b^2e^2 + 6\left(7\,b^4c -\right.\right.\right.}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)^2/sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*e^2*x^4 + 315*b^4*c^2 - 420*a*b^3*c*d + 168*a^2*b^2
*d^2 + 128*a^4*e^2 + 10*(9*b^4*d*e - 4*a*b^3*e^2)*x^3 + 3*(21*b^4
*d^2 + 16*a^2*b^2*e^2 + 6*(7*b^4*c - 6*a*b^3*d)*e)*x^2 + 48*(7*a^
2*b^2*c - 6*a^3*b*d)*e + 2*(105*b^4*c*d - 42*a*b^3*d^2 - 32*a^3*b
*e^2 - 12*(7*a*b^3*c - 6*a^2*b^2*d)*e)*x)*sqrt(b*x + a)/b^5
```

Sympy [A] time = 20.8599, size = 644, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c**2/sqrt(a + b*x) + 4*a*c*d*(-a/sqrt(a + b*x) -
 sqrt(a + b*x))/b + 4*a*c*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*
x) - (a + b*x)**(3/2)/3)/b**2 + 2*a*d**2*(a**2/sqrt(a + b*x) + 2*
a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 4*a*d*e*(-a**3/sqrt(
a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)*
*(5/2)/5)/b**3 + 2*a*e**2*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b
*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x
)**(7/2)/7)/b**4 + 2*c**2*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 4*
c*d*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)
/b + 4*c*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b
*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 2*d**2*(-a**3/sqrt(a + b*
x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)
/5)/b**2 + 4*d*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a
**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/
7)/b**3 + 2*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10
*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x
)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4)/b, Ne(b, 0)), ((c**2*x + c
*d*x**2 + d*e*x**4/2 + e**2*x**5/5 + x**3*(2*c*e + d**2)/3)/sqrt(
a), True))
```

GIAC/XCAS [A] time = 0.212521, size = 381, normalized size = 2.37

$$2\left(315\,\sqrt{bx+a}c^2+\frac{210\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)cd}{b}+\frac{21\left(3\,(bx+a)^{\frac{5}{2}}b^8-10\,(bx+a)^{\frac{3}{2}}ab^8+15\,\sqrt{bx+a}a^2b^8\right)d^2}{b^{10}}+\frac{42\left(3\,(bx+a)^{\frac{5}{2}}b^8-10\,(bx+a)^{\frac{3}{2}}ab^8+15\,\sqrt{bx}\right.}{b^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)^2/sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 210*((b*x + a)^(3/2) - 3*sqrt(b*x
+ a)*a)*c*d/b + 21*(3*(b*x + a)^(5/2)*b^8 - 10*(b*x + a)^(3/2)*a*
b^8 + 15*sqrt(b*x + a)*a^2*b^8)*d^2/b^10 + 42*(3*(b*x + a)^(5/2)*
b^8 - 10*(b*x + a)^(3/2)*a*b^8 + 15*sqrt(b*x + a)*a^2*b^8)*c*e/b^
10 + 18*(5*(b*x + a)^(7/2)*b^18 - 21*(b*x + a)^(5/2)*a*b^18 + 35*
(b*x + a)^(3/2)*a^2*b^18 - 35*sqrt(b*x + a)*a^3*b^18)*d*e/b^21 +
(35*(b*x + a)^(9/2)*b^32 - 180*(b*x + a)^(7/2)*a*b^32 + 378*(b*x
+ a)^(5/2)*a^2*b^32 - 420*(b*x + a)^(3/2)*a^3*b^32 + 315*sqrt(b*x
 + a)*a^4*b^32)*e^2/b^36)/b
```

3.3 $\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=274

$$-\frac{2e(a+bx)^{9/2}\left(-5a^2e^2+5abde+b^2\left(-\left(ce+d^2\right)\right)\right)}{3b^7}-\frac{2(a+bx)^{7/2}(bd-2ae)\left(-10a^2e^2+10abde+b^2\left(-\left(6ce+d^2\right)\right)\right)}{7b^7}-\frac{6(a+bx)^{5/2}\left(a^2e-abd+b^2c\right)\left(-5a^2e^2+5abde+b^2\left(-\left(ce+d^2\right)\right)\right)}{5b^7}+\frac{2(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)^2}{b^7}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^3}{b^7}+\frac{6e^2(a+bx)^{11/2}(bd-2ae)}{11b^7}+\frac{2e^3(a+bx)^{13/2}}{13b^7}$$

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^3*Sqrt[a + b*x])/b^7 + (2*(b*d - 2*a*e
)*(b^2*c - a*b*d + a^2*e)^2*(a + b*x)^(3/2))/b^7 - (6*(b^2*c - a*
b*d + a^2*e)*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^
(5/2))/(5*b^7) - (2*(b*d - 2*a*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*
(d^2 + 6*c*e))*(a + b*x)^(7/2))/(7*b^7) - (2*e*(5*a*b*d*e - 5*a^2
*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(9/2))/(3*b^7) + (6*e^2*(b*d -
2*a*e)*(a + b*x)^(11/2))/(11*b^7) + (2*e^3*(a + b*x)^(13/2))/(13*
b^7)
```

Rubi [A] time = 0.396205, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$-\frac{2e(a+bx)^{9/2}\left(-5a^2e^2+5abde+b^2\left(-\left(ce+d^2\right)\right)\right)}{3b^7}-\frac{2(a+bx)^{7/2}(bd-2ae)\left(-10a^2e^2+10abde+b^2\left(-\left(6ce+d^2\right)\right)\right)}{7b^7}-\frac{6(a+bx)^{5/2}\left(a^2e-abd+b^2c\right)\left(-5a^2e^2+5abde+b^2\left(-\left(ce+d^2\right)\right)\right)}{5b^7}+\frac{2(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)^2}{b^7}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^3}{b^7}+\frac{6e^2(a+bx)^{11/2}(bd-2ae)}{11b^7}+\frac{2e^3(a+bx)^{13/2}}{13b^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^3*Sqrt[a + b*x])/b^7 + (2*(b*d - 2*a*e
)*(b^2*c - a*b*d + a^2*e)^2*(a + b*x)^(3/2))/b^7 - (6*(b^2*c - a*
b*d + a^2*e)*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^
(5/2))/(5*b^7) - (2*(b*d - 2*a*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*
(d^2 + 6*c*e))*(a + b*x)^(7/2))/(7*b^7) - (2*e*(5*a*b*d*e - 5*a^2
*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(9/2))/(3*b^7) + (6*e^2*(b*d -
2*a*e)*(a + b*x)^(11/2))/(11*b^7) + (2*e^3*(a + b*x)^(13/2))/(13*
b^7)
```

Rubi in Sympy [A] time = 69.7588, size = 279, normalized size = 1.02

$$\frac{2e^3\left(a+bx\right)^{\frac{13}{2}}}{13b^7}-\frac{6e^2\left(a+bx\right)^{\frac{11}{2}}\left(2ae-bd\right)}{11b^7}+\frac{2e\left(a+bx\right)^{\frac{9}{2}}\left(5a^2e^2-5abde+b^2ce+b^2d^2\right)}{3b^7}-\frac{2\left(a+bx\right)^{\frac{7}{2}}\left(2ae-bd\right)\left(10a^2e^2-10abde+6b^2ce+b^2d^2\right)}{7b^7}+\frac{6\left(a+bx\right)^{\frac{5}{2}}\left(a^2e-abd+b^2c\right)\left(5a^2e^2-5abde+b^2ce+b^2d^2\right)}{5b^7}-\frac{2\left(a+bx\right)^{\frac{3}{2}}\left(2ae-bd\right)\left(a^2e-abd+b^2c\right)^2}{b^7}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^3}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] 2*e**3*(a + b*x)**(13/2)/(13*b**7) - 6*e**2*(a + b*x)**(11/2)*(2*
a*e - b*d)/(11*b**7) + 2*e*(a + b*x)**(9/2)*(5*a**2*e**2 - 5*a*b*
d*e + b**2*c*e + b**2*d**2)/(3*b**7) - 2*(a + b*x)**(7/2)*(2*a*e
- b*d)*(10*a**2*e**2 - 10*a*b*d*e + 6*b**2*c*e + b**2*d**2)/(7*b*
*7) + 6*(a + b*x)**(5/2)*(a**2*e - a*b*d + b**2*c)*(5*a**2*e**2 -
 5*a*b*d*e + b**2*c*e + b**2*d**2)/(5*b**7) - 2*(a + b*x)**(3/2)*
(2*a*e - b*d)*(a**2*e - a*b*d + b**2*c)**2/b**7 + 2*sqrt(a + b*x)
*(a**2*e - a*b*d + b**2*c)**3/b**7
```

Mathematica [A] time = 0.472864, size = 355, normalized size = 1.3

$$\frac{2\sqrt{a+bx}\left(5120a^6e^3-1280a^5be^2(13d+2ex)+128a^4b^2e\left(e\left(143c+15ex^2\right)+143d^2+65dex\right)-16a^3b^3\left(78de\left(33c+5ex^2\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(5120*a^6*e^3 - 1280*a^5*b*e^2*(13*d + 2*e*x) +
128*a^4*b^2*e*(143*d^2 + 65*d*e*x + e*(143*c + 15*e*x^2)) - 16*a^
3*b^3*(429*d^3 + 572*d^2*e*x + 78*d*e*(33*c + 5*e*x^2) + 4*e^2*x*
(143*c + 25*e*x^2)) + 8*a^2*b^4*(3003*c^2*e + 429*c*(7*d^2 + 6*d*
e*x + 2*e^2*x^2) + x*(429*d^3 + 858*d^2*e*x + 650*d*e^2*x^2 + 175
*e^3*x^3)) + b^6*(15015*c^3 + 3003*c^2*x*(5*d + 3*e*x) + 143*c*x^
2*(63*d^2 + 90*d*e*x + 35*e^2*x^2) + 5*x^3*(429*d^3 + 1001*d^2*e*
x + 819*d*e^2*x^2 + 231*e^3*x^3)) - 2*a*b^5*(3003*c^2*(5*d + 2*e*
x) + 286*c*x*(21*d^2 + 27*d*e*x + 10*e^2*x^2) + x^2*(1287*d^3 + 2
860*d^2*e*x + 2275*d*e^2*x^2 + 630*e^3*x^3))))/(15015*b^7)
```

Maple [A] time = 0.01, size = 495, normalized size = 1.8

$$\frac{2310\,e^3x^6b^6-2520\,ab^5e^3x^5+8190\,b^6de^2x^5+2800\,a^2b^4e^3x^4-9100\,ab^5de^2x^4+10010\,b^6ce^2x^4+10010\,b^6d^2ex^4-3200\,a^3b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x)
```

```
[Out] 2/15015*(b*x+a)^(1/2)*(1155*b^6*e^3*x^6-1260*a*b^5*e^3*x^5+4095*b
^6*d*e^2*x^5+1400*a^2*b^4*e^3*x^4-4550*a*b^5*d*e^2*x^4+5005*b^6*c
*e^2*x^4+5005*b^6*d^2*e*x^4-1600*a^3*b^3*e^3*x^3+5200*a^2*b^4*d*e
^2*x^3-5720*a*b^5*c*e^2*x^3-5720*a*b^5*d^2*e*x^3+12870*b^6*c*d*e*
x^3+2145*b^6*d^3*x^3+1920*a^4*b^2*e^3*x^2-6240*a^3*b^3*d*e^2*x^2+
6864*a^2*b^4*c*e^2*x^2+6864*a^2*b^4*d^2*e*x^2-15444*a*b^5*c*d*e*x
^2-2574*a*b^5*d^3*x^2+9009*b^6*c^2*e*x^2+9009*b^6*c*d^2*x^2-2560*
a^5*b*e^3*x+8320*a^4*b^2*d*e^2*x-9152*a^3*b^3*c*e^2*x-9152*a^3*b^
3*d^2*e*x+20592*a^2*b^4*c*d*e*x+3432*a^2*b^4*d^3*x-12012*a*b^5*c^
2*e*x-12012*a*b^5*c*d^2*x+15015*b^6*c^2*d*x+5120*a^6*e^3-16640*a^
5*b*d*e^2+18304*a^4*b^2*c*e^2+18304*a^4*b^2*d^2*e-41184*a^3*b^3*c
*d*e-6864*a^3*b^3*d^3+24024*a^2*b^4*c^2*e+24024*a^2*b^4*c*d^2-300
30*a*b^5*c^2*d+15015*b^6*c^3)/b^7
```

Maxima [A] time = 1.39522, size = 709, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)^3/sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 3003*c^2*(5*((b*x + a)^(3/2) -
 3*sqrt(b*x + a)*a)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)
*a + 15*sqrt(b*x + a)*a^2)*e/b^2) + 143*c*(21*(3*(b*x + a)^(5/2)
- 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 18*(5*(b
*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a
)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 3
15*sqrt(b*x + a)*a^4)*e^2/b^4) + 429*(5*(b*x + a)^(7/2) - 21*(b*x
 + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^
3/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*
x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^
4)*d^2*e/b^4 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a +
990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x +
a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 5*(231*(b*x + a
)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8
580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x +
a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

Fricas [A] time = 0.220168, size = 617, normalized size = 2.25

$$2\left(1155\,b^6e^3x^6 + 15015\,b^6c^3 - 30030\,ab^5c^2d + 24024\,a^2b^4cd^2 - 6864\,a^3b^3d^3 + 5120\,a^6e^3 + 315\left(13\,b^6de^2 - 4\,ab^5e^3\right)x^5 + 35\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)^3/sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*e^3*x^6 + 15015*b^6*c^3 - 30030*a*b^5*c^2*d + 2
4024*a^2*b^4*c*d^2 - 6864*a^3*b^3*d^3 + 5120*a^6*e^3 + 315*(13*b^
6*d*e^2 - 4*a*b^5*e^3)*x^5 + 35*(143*b^6*d^2*e + 40*a^2*b^4*e^3 +
 13*(11*b^6*c - 10*a*b^5*d)*e^2)*x^4 + 5*(429*b^6*d^3 - 320*a^3*b
^3*e^3 - 104*(11*a*b^5*c - 10*a^2*b^4*d)*e^2 + 286*(9*b^6*c*d - 4
*a*b^5*d^2)*e)*x^3 + 1664*(11*a^4*b^2*c - 10*a^5*b*d)*e^2 + 3*(30
03*b^6*c*d^2 - 858*a*b^5*d^3 + 640*a^4*b^2*e^3 + 208*(11*a^2*b^4*
c - 10*a^3*b^3*d)*e^2 + 143*(21*b^6*c^2 - 36*a*b^5*c*d + 16*a^2*b
^4*d^2)*e)*x^2 + 1144*(21*a^2*b^4*c^2 - 36*a^3*b^3*c*d + 16*a^4*b
^2*d^2)*e + (15015*b^6*c^2*d - 12012*a*b^5*c*d^2 + 3432*a^2*b^4*d
^3 - 2560*a^5*b*e^3 - 832*(11*a^3*b^3*c - 10*a^4*b^2*d)*e^2 - 572
*(21*a*b^5*c^2 - 36*a^2*b^4*c*d + 16*a^3*b^3*d^2)*e)*x)*sqrt(b*x
+ a)/b^7
```

Sympy [A] time = 52.2806, size = 1406, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c**3/sqrt(a + b*x) + 6*a*c**2*d*(-a/sqrt(a + b*x
) - sqrt(a + b*x))/b + 6*a*c**2*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(
a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 6*a*c*d**2*(a**2/sqrt(a + b
*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 12*a*c*d*e*(
-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) -
 (a + b*x)**(5/2)/5)/b**3 + 2*a*d**3*(-a**3/sqrt(a + b*x) - 3*a**
2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 +
 6*a*c*e**2*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(
a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b*
*4 + 6*a*d**2*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a*
*2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7
)/b**4 + 6*a*d*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) +
 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a +
```

```
b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**5 + 2*a*e**3*(a**6/sqrt(a
+ b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*
(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9
/2)/3 - (a + b*x)**(11/2)/11)/b**6 + 2*c**3*(-a/sqrt(a + b*x) - s
qrt(a + b*x)) + 6*c**2*d*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x)
- (a + b*x)**(3/2)/3)/b + 6*c**2*e*(-a**3/sqrt(a + b*x) - 3*a**2*
sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 6
*c*d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)
**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 12*c*d*e*(a**4/sqrt(a + b*x)
 + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)
**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3 + 2*d**3*(a**4/sqrt(a + b*x)
 + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)
**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3 + 6*c*e**2*(-a**5/sqrt(a + b
*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*
(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b
**4 + 6*d**2*e*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a
**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)*
*(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 + 6*d*e**2*(a**6/sqrt(a + b*x
) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a +
b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3
 - (a + b*x)**(11/2)/11)/b**5 + 2*e**3*(-a**7/sqrt(a + b*x) - 7*a
**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3/2) - 7*a**4*(a + b*x)**(
5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a + b*x)**(9/2)/3 + 7*a*
(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)/13)/b**6)/b, Ne(b, 0)),
((c**3*x + 3*c**2*d*x**2/2 + d*e**2*x**6/2 + e**3*x**7/7 + x**5*(
3*c*e**2 + 3*d**2*e)/5 + x**4*(6*c*d*e + d**3)/4 + x**3*(3*c**2*e
 + 3*c*d**2)/3)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.216208, size = 860, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)^3/sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 15015*((b*x + a)^(3/2) - 3*sqr
t(b*x + a)*a)*c^2*d/b + 3003*(3*(b*x + a)^(5/2)*b^8 - 10*(b*x + a
)^(3/2)*a*b^8 + 15*sqrt(b*x + a)*a^2*b^8)*c*d^2/b^10 + 3003*(3*(b
*x + a)^(5/2)*b^8 - 10*(b*x + a)^(3/2)*a*b^8 + 15*sqrt(b*x + a)*a
^2*b^8)*c^2*e/b^10 + 429*(5*(b*x + a)^(7/2)*b^18 - 21*(b*x + a)^(
5/2)*a*b^18 + 35*(b*x + a)^(3/2)*a^2*b^18 - 35*sqrt(b*x + a)*a^3*
b^18)*d^3/b^21 + 2574*(5*(b*x + a)^(7/2)*b^18 - 21*(b*x + a)^(5/2
)*a*b^18 + 35*(b*x + a)^(3/2)*a^2*b^18 - 35*sqrt(b*x + a)*a^3*b^1
8)*c*d*e/b^21 + 143*(35*(b*x + a)^(9/2)*b^32 - 180*(b*x + a)^(7/2
)*a*b^32 + 378*(b*x + a)^(5/2)*a^2*b^32 - 420*(b*x + a)^(3/2)*a^3
*b^32 + 315*sqrt(b*x + a)*a^4*b^32)*d^2*e/b^36 + 143*(35*(b*x + a
)^(9/2)*b^32 - 180*(b*x + a)^(7/2)*a*b^32 + 378*(b*x + a)^(5/2)*a
^2*b^32 - 420*(b*x + a)^(3/2)*a^3*b^32 + 315*sqrt(b*x + a)*a^4*b^
32)*c*e^2/b^36 + 65*(63*(b*x + a)^(11/2)*b^50 - 385*(b*x + a)^(9/
2)*a*b^50 + 990*(b*x + a)^(7/2)*a^2*b^50 - 1386*(b*x + a)^(5/2)*a
^3*b^50 + 1155*(b*x + a)^(3/2)*a^4*b^50 - 693*sqrt(b*x + a)*a^5*b
^50)*d*e^2/b^55 + 5*(231*(b*x + a)^(13/2)*b^72 - 1638*(b*x + a)^(
11/2)*a*b^72 + 5005*(b*x + a)^(9/2)*a^2*b^72 - 8580*(b*x + a)^(7/
2)*a^3*b^72 + 9009*(b*x + a)^(5/2)*a^4*b^72 - 6006*(b*x + a)^(3/2
)*a^5*b^72 + 3003*sqrt(b*x + a)*a^6*b^72)*e^3/b^78)/b
```

3.4 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=114

$$\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{2(a+bx)^{5/2}(be-3af)}{5b^4}+\frac{2f(a+bx)^{7/2}}{7b^4}$$

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b
^2*d - 2*a*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*
a*f)*(a + b*x)^(5/2))/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

Rubi [A] time = 0.143349, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{2(a+bx)^{5/2}(be-3af)}{5b^4}+\frac{2f(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b
^2*d - 2*a*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*
a*f)*(a + b*x)^(5/2))/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

Rubi in Sympy [A] time = 26.1262, size = 110, normalized size = 0.96

$$\frac{2f\left(a+bx\right)^{\frac{7}{2}}}{7b^4}-\frac{2\left(a+bx\right)^{\frac{5}{2}}\left(3af-be\right)}{5b^4}+\frac{2\left(a+bx\right)^{\frac{3}{2}}\left(3a^2f-2abe+b^2d\right)}{3b^4}-\frac{2\sqrt{a+bx}\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] 2*f*(a + b*x)**(7/2)/(7*b**4) - 2*(a + b*x)**(5/2)*(3*a*f - b*e)/
(5*b**4) + 2*(a + b*x)**(3/2)*(3*a**2*f - 2*a*b*e + b**2*d)/(3*b*
*4) - 2*sqrt(a + b*x)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/b**
4
```

Mathematica [A] time = 0.0995704, size = 82, normalized size = 0.72

$$\frac{2\sqrt{a+bx}\left(-48a^3f+8a^2b(7e+3fx)-2ab^2(35d+x(14e+9fx))+b^3(105c+x(35d+3x(7e+5fx)))\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x) - 2*a*b^2*(35
*d + x*(14*e + 9*f*x)) + b^3*(105*c + x*(35*d + 3*x*(7*e + 5*f*x)
```

```
))))/(105*b^4)
```

Maple [A] time = 0.006, size = 91, normalized size = 0.8

$$-\frac{-30\,f x^3 b^3+36\,a b^2 f x^2-42\,b^3 e x^2-48\,a^2 b f x+56\,a b^2 e x-70\,b^3 d x+96\,a^3 f-112\,a^2 b e+140\,a b^2 d-210\,b^3 c}{105\,b^4}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x)
```

```
[Out] -2/105*(b*x+a)^(1/2)*(-15*b^3*f*x^3+18*a*b^2*f*x^2-21*b^3*e*x^2-2
4*a^2*b*f*x+28*a*b^2*e*x-35*b^3*d*x+48*a^3*f-56*a^2*b*e+70*a*b^2*
d-105*b^3*c)/b^4
```

Maxima [A] time = 1.37849, size = 173, normalized size = 1.52

$$\frac{2\left(105\sqrt{bx+a}c+\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}+\frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}a\right.}{b^3}\right.}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a
)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(
b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
 + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3)/b
```

Fricas [A] time = 0.216486, size = 122, normalized size = 1.07

$$\frac{2\left(15\,b^3 f x^3+105\,b^3 c-70\,a b^2 d+56\,a^2 b e-48\,a^3 f+3\left(7\,b^3 e-6\,a b^2 f\right)x^2+\left(35\,b^3 d-28\,a b^2 e+24\,a^2 b f\right)x\right)\sqrt{bx+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*f*x^3 + 105*b^3*c - 70*a*b^2*d + 56*a^2*b*e - 48*a^
3*f + 3*(7*b^3*e - 6*a*b^2*f)*x^2 + (35*b^3*d - 28*a*b^2*e + 24*a
^2*b*f)*x)*sqrt(b*x + a)/b^4
```

Sympy [A] time = 6.34342, size = 354, normalized size = 3.11

$$\begin{cases} -\frac{\frac{2ac}{\sqrt{a+bx}}+\frac{2ad\left(-\frac{a}{\sqrt{a+bx}}-\sqrt{a+bx}\right)}{b}+\frac{2ae\left(\frac{a^2}{\sqrt{a+bx}}+2a\sqrt{a+bx}-\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b^2}+\frac{2af\left(-\frac{a^3}{\sqrt{a+bx}}-3a^2\sqrt{a+bx}+a(a+bx)^{\frac{3}{2}}-\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^3}+2c\left(-\frac{a}{\sqrt{a+bx}}-\sqrt{a+bx}\right)+\frac{2d\left(\frac{a^2}{\sqrt{a+bx}}+2a\sqrt{a+bx}-\right.}{b}}{b} \\ \frac{cx+\frac{dx^2}{2}+\frac{ex^3}{3}+\frac{fx^4}{4}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c/sqrt(a + b*x) + 2*a*d*(-a/sqrt(a + b*x) - sqrt
(a + b*x))/b + 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a
 + b*x)**(3/2)/3)/b**2 + 2*a*f*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt
(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 2*c*(
-a/sqrt(a + b*x) - sqrt(a + b*x)) + 2*d*(a**2/sqrt(a + b*x) + 2*a
*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 2*e*(-a**3/sqrt(a + b*x)
 - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5
)/b**2 + 2*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*
(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b
**3)/b, Ne(b, 0)), ((c*x + d*x**2/2 + e*x**3/3 + f*x**4/4)/sqrt(a
), True))
```

GIAC/XCAS [A] time = 0.210928, size = 203, normalized size = 1.78

$$\frac{2\left(105\sqrt{bx+a}c+\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{7\left(3(bx+a)^{\frac{5}{2}}b^8-10(bx+a)^{\frac{3}{2}}ab^8+15\sqrt{bx+a}a^2b^8\right)e}{b^{10}}+\frac{3\left(5(bx+a)^{\frac{7}{2}}b^{18}-21(bx+a)^{\frac{5}{2}}ab^{18}+35(bx+a)^{\frac{3}{2}}a\right.}{b^{21}}}{105b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a
)*a)*d/b + 7*(3*(b*x + a)^(5/2)*b^8 - 10*(b*x + a)^(3/2)*a*b^8 +
15*sqrt(b*x + a)*a^2*b^8)*e/b^10 + 3*(5*(b*x + a)^(7/2)*b^18 - 21
*(b*x + a)^(5/2)*a*b^18 + 35*(b*x + a)^(3/2)*a^2*b^18 - 35*sqrt(b
*x + a)*a^3*b^18)*f/b^21)/b
```

3.5 $\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=320

$$-\frac{2(a+bx)^{9/2}\left(-15a^2f^2+10abef+b^2\left(-\left(2df+e^2\right)\right)\right)}{9b^7}+\frac{4(a+bx)^{7/2}\left(-10a^3f^2+10a^2bef-2ab^2\left(2df+e^2\right)+b^3(cf+de)\right)}{7b^7}+\frac{4(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^7}+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2}{b^7}+\frac{2(a+bx)^{5/2}\left(15a^4f^2-20a^3bef+6a^2b^2\left(2df+e^2\right)-6ab^3(cf+de)+b^4\left(2ce+d^2\right)\right)}{5b^7}+\frac{4f(a+bx)^{11/2}(be-3af)}{11b^7}+\frac{2f^2(a+bx)^{13/2}}{13b^7}$$

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x])/b^7 + (4*
(b^2*d - 2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(
a + b*x)^(3/2))/(3*b^7) + (2*(b^4*(d^2 + 2*c*e) - 20*a^3*b*e*f +
15*a^4*f^2 - 6*a*b^3*(d*e + c*f) + 6*a^2*b^2*(e^2 + 2*d*f))*(a +
b*x)^(5/2))/(5*b^7) + (4*(10*a^2*b*e*f - 10*a^3*f^2 + b^3*(d*e +
c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/(7*b^7) - (2*(10*a
*b*e*f - 15*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/(9*b^7)
 + (4*f*(b*e - 3*a*f)*(a + b*x)^(11/2))/(11*b^7) + (2*f^2*(a + b*
x)^(13/2))/(13*b^7)
```

Rubi [A] time = 0.511365, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$

$$-\frac{2(a+bx)^{9/2}\left(-15a^2f^2+10abef+b^2\left(-\left(2df+e^2\right)\right)\right)}{9b^7}+\frac{4(a+bx)^{7/2}\left(-10a^3f^2+10a^2bef-2ab^2\left(2df+e^2\right)+b^3(cf+de)\right)}{7b^7}+\frac{4(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^7}+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2}{b^7}+\frac{2(a+bx)^{5/2}\left(15a^4f^2-20a^3bef+6a^2b^2\left(2df+e^2\right)-6ab^3(cf+de)+b^4\left(2ce+d^2\right)\right)}{5b^7}+\frac{4f(a+bx)^{11/2}(be-3af)}{11b^7}+\frac{2f^2(a+bx)^{13/2}}{13b^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x])/b^7 + (4*
(b^2*d - 2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(
a + b*x)^(3/2))/(3*b^7) + (2*(b^4*(d^2 + 2*c*e) - 20*a^3*b*e*f +
15*a^4*f^2 - 6*a*b^3*(d*e + c*f) + 6*a^2*b^2*(e^2 + 2*d*f))*(a +
b*x)^(5/2))/(5*b^7) + (4*(10*a^2*b*e*f - 10*a^3*f^2 + b^3*(d*e +
c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/(7*b^7) - (2*(10*a
*b*e*f - 15*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/(9*b^7)
 + (4*f*(b*e - 3*a*f)*(a + b*x)^(11/2))/(11*b^7) + (2*f^2*(a + b*
x)^(13/2))/(13*b^7)
```

Rubi in Sympy [A] time = 120.574, size = 348, normalized size = 1.09

$$\frac{2f^2(a+bx)^{\frac{13}{2}}}{13b^7}-\frac{4f(a+bx)^{\frac{11}{2}}(3af-be)}{11b^7}+\frac{2(a+bx)^{\frac{9}{2}}(15a^2f^2-10abef+2b^2df+b^2e^2)}{9b^7}$$
$$-\frac{4(a+bx)^{\frac{7}{2}}(10a^3f^2-10a^2bef+4ab^2df+2ab^2e^2-b^3cf-b^3de)}{7b^7}$$
$$+\frac{2(a+bx)^{\frac{5}{2}}(15a^4f^2-20a^3bef+12a^2b^2df+6a^2b^2e^2-6ab^3cf-6ab^3de+2b^4ce+b^4d^2)}{5b^7}$$
$$-\frac{4(a+bx)^{\frac{3}{2}}(3a^2f-2abe+b^2d)(a^3f-a^2be+ab^2d-b^3c)}{3b^7}$$
$$+\frac{2\sqrt{a+bx}(a^3f-a^2be+ab^2d-b^3c)^2}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] 2*f**2*(a + b*x)**(13/2)/(13*b**7) - 4*f*(a + b*x)**(11/2)*(3*a*f
 - b*e)/(11*b**7) + 2*(a + b*x)**(9/2)*(15*a**2*f**2 - 10*a*b*e*f
 + 2*b**2*d*f + b**2*e**2)/(9*b**7) - 4*(a + b*x)**(7/2)*(10*a**3
*f**2 - 10*a**2*b*e*f + 4*a*b**2*d*f + 2*a*b**2*e**2 - b**3*c*f -
 b**3*d*e)/(7*b**7) + 2*(a + b*x)**(5/2)*(15*a**4*f**2 - 20*a**3*
b*e*f + 12*a**2*b**2*d*f + 6*a**2*b**2*e**2 - 6*a*b**3*c*f - 6*a*
b**3*d*e + 2*b**4*c*e + b**4*d**2)/(5*b**7) - 4*(a + b*x)**(3/2)*
(3*a**2*f - 2*a*b*e + b**2*d)*(a**3*f - a**2*b*e + a*b**2*d - b**
3*c)/(3*b**7) + 2*sqrt(a + b*x)*(a**3*f - a**2*b*e + a*b**2*d - b
**3*c)**2/b**7
```

Mathematica [A] time = 0.565278, size = 320, normalized size = 1.

$$\frac{2\sqrt{a+bx}\left(15360a^6f^2-2560a^5bf(13e+3fx)+128a^4b^2\left(f\left(286d+45fx^2\right)+143e^2+130efx\right)-32a^3b^3\left(1287cf+143d(9\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(15360*a^6*f^2 - 2560*a^5*b*f*(13*e + 3*f*x) + 1
28*a^4*b^2*(143*e^2 + 130*e*f*x + f*(286*d + 45*f*x^2)) - 32*a^3*
b^3*(1287*c*f + 143*d*(9*e + 4*f*x) + 2*x*(143*e^2 + 195*e*f*x +
75*f^2*x^2)) + 8*a^2*b^4*(3003*d^2 + 858*d*x*(3*e + 2*f*x) + 858*
c*(7*e + 3*f*x) + x^2*(858*e^2 + 1300*e*f*x + 525*f^2*x^2)) + b^6
*(45045*c^2 + 858*c*x*(35*d + 3*x*(7*e + 5*f*x)) + x^2*(9009*d^2
+ 1430*d*x*(9*e + 7*f*x) + 35*x^2*(143*e^2 + 234*e*f*x + 99*f^2*x
^2))) - 4*a*b^5*(429*c*(35*d + x*(14*e + 9*f*x)) + x*(3003*d^2 +
143*d*x*(27*e + 20*f*x) + 5*x^2*(286*e^2 + 455*e*f*x + 189*f^2*x^
2)))))/(45045*b^7)
```

Maple [A] time = 0.013, size = 447, normalized size = 1.4

$$\frac{6930\,f^2x^6b^6-7560\,ab^5f^2x^5+16380\,b^6efx^5+8400\,a^2b^4f^2x^4-18200\,ab^5efx^4+20020\,b^6dfx^4+10010\,b^6e^2x^4-9600\,a^3b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/45045*(b*x+a)^(1/2)*(3465*b^6*f^2*x^6-3780*a*b^5*f^2*x^5+8190*b
^6*e*f*x^5+4200*a^2*b^4*f^2*x^4-9100*a*b^5*e*f*x^4+10010*b^6*d*f*
x^4+5005*b^6*e^2*x^4-4800*a^3*b^3*f^2*x^3+10400*a^2*b^4*e*f*x^3-1
```

```
1440*a*b^5*d*f*x^3-5720*a*b^5*e^2*x^3+12870*b^6*c*f*x^3+12870*b^6
*d*e*x^3+5760*a^4*b^2*f^2*x^2-12480*a^3*b^3*e*f*x^2+13728*a^2*b^4
*d*f*x^2+6864*a^2*b^4*e^2*x^2-15444*a*b^5*c*f*x^2-15444*a*b^5*d*e
*x^2+18018*b^6*c*e*x^2+9009*b^6*d^2*x^2-7680*a^5*b*f^2*x+16640*a^
4*b^2*e*f*x-18304*a^3*b^3*d*f*x-9152*a^3*b^3*e^2*x+20592*a^2*b^4*
c*f*x+20592*a^2*b^4*d*e*x-24024*a*b^5*c*e*x-12012*a*b^5*d^2*x+300
30*b^6*c*d*x+15360*a^6*f^2-33280*a^5*b*e*f+36608*a^4*b^2*d*f+1830
4*a^4*b^2*e^2-41184*a^3*b^3*c*f-41184*a^3*b^3*d*e+48048*a^2*b^4*c
*e+24024*a^2*b^4*d^2-60060*a*b^5*c*d+45045*b^6*c^2)/b^7
```

Maxima [A] time = 1.42126, size = 675, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)^2/sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 858*c*(35*((b*x + a)^(3/2) - 3
*sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)
*a + 15*sqrt(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x
 + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/
b^3) + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b
*x + a)*a^2)*d^2/b^2 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7
/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*s
qrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/2)*f + 45*(b*e -
 4*a*f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a)^(5/2) +
 105*(3*a^2*b*e - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f
)*sqrt(b*x + a))*d/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)
^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1
155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*e*f/b^5 + 15*(23
1*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/
2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 60
06*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

Fricas [A] time = 0.218249, size = 563, normalized size = 1.76

$$2\left(3465\,b^6f^2x^6 + 45045\,b^6c^2 - 60060\,ab^5cd + 24024\,a^2b^4d^2 + 18304\,a^4b^2e^2 + 15360\,a^6f^2 + 630\left(13\,b^6ef - 6\,ab^5f^2\right)x^5 + 35\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)^2/sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*b^6*f^2*x^6 + 45045*b^6*c^2 - 60060*a*b^5*c*d + 240
24*a^2*b^4*d^2 + 18304*a^4*b^2*e^2 + 15360*a^6*f^2 + 630*(13*b^6*
e*f - 6*a*b^5*f^2)*x^5 + 35*(143*b^6*e^2 + 120*a^2*b^4*f^2 + 26*(
11*b^6*d - 10*a*b^5*e)*f)*x^4 + 10*(1287*b^6*d*e - 572*a*b^5*e^2
- 480*a^3*b^3*f^2 + 13*(99*b^6*c - 88*a*b^5*d + 80*a^2*b^4*e)*f)*
x^3 + 3*(3003*b^6*d^2 + 2288*a^2*b^4*e^2 + 1920*a^4*b^2*f^2 + 858
*(7*b^6*c - 6*a*b^5*d)*e - 52*(99*a*b^5*c - 88*a^2*b^4*d + 80*a^3
*b^3*e)*f)*x^2 + 6864*(7*a^2*b^4*c - 6*a^3*b^3*d)*e - 416*(99*a^3
*b^3*c - 88*a^4*b^2*d + 80*a^5*b*e)*f + 2*(15015*b^6*c*d - 6006*a
*b^5*d^2 - 4576*a^3*b^3*e^2 - 3840*a^5*b*f^2 - 1716*(7*a*b^5*c -
6*a^2*b^4*d)*e + 104*(99*a^2*b^4*c - 88*a^3*b^3*d + 80*a^4*b^2*e)
*f)*x)*sqrt(b*x + a)/b^7
```

Sympy [A] time = 52.4947, size = 1365, normalized size = 4.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c**2/sqrt(a + b*x) + 4*a*c*d*(-a/sqrt(a + b*x) -
 sqrt(a + b*x))/b + 4*a*c*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*
x) - (a + b*x)**(3/2)/3)/b**2 + 2*a*d**2*(a**2/sqrt(a + b*x) + 2*
a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 4*a*c*f*(-a**3/sqrt(
a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)*
*(5/2)/5)/b**3 + 4*a*d*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b
*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 4*a*d*f*(a*
*4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2)
 + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 2*a*e**2*(
a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/
2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 4*a*e*f*
(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**
(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a +
 b*x)**(9/2)/9)/b**5 + 2*a*f**2*(a**6/sqrt(a + b*x) + 6*a**5*sqrt
(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 1
5*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(
11/2)/11)/b**6 + 2*c**2*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 4*c*
d*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b
 + 4*c*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x
)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 2*d**2*(-a**3/sqrt(a + b*x)
 - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5
)/b**2 + 4*c*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**
2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)
/b**3 + 4*d*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2
*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/
b**3 + 4*d*f*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**
3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(
7/2)/7 - (a + b*x)**(9/2)/9)/b**4 + 2*e**2*(-a**5/sqrt(a + b*x) -
 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a +
b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 +
 4*e*f*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b
*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7
 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**5 + 2*f**2*(
-a**7/sqrt(a + b*x) - 7*a**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3
/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*
(a + b*x)**(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)
/13)/b**6)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + e*f*x**6/3 + f**2*
x**7/7 + x**5*(2*d*f + e**2)/5 + x**4*(2*c*f + 2*d*e)/4 + x**3*(2
*c*e + d**2)/3)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.218871, size = 846, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)^2/sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 30030*((b*x + a)^(3/2) - 3*sqr
t(b*x + a)*a)*c*d/b + 3003*(3*(b*x + a)^(5/2)*b^8 - 10*(b*x + a)^
(3/2)*a*b^8 + 15*sqrt(b*x + a)*a^2*b^8)*d^2/b^10 + 6006*(3*(b*x +
 a)^(5/2)*b^8 - 10*(b*x + a)^(3/2)*a*b^8 + 15*sqrt(b*x + a)*a^2*b
^8)*c*e/b^10 + 2574*(5*(b*x + a)^(7/2)*b^18 - 21*(b*x + a)^(5/2)*
a*b^18 + 35*(b*x + a)^(3/2)*a^2*b^18 - 35*sqrt(b*x + a)*a^3*b^18)
*c*f/b^21 + 2574*(5*(b*x + a)^(7/2)*b^18 - 21*(b*x + a)^(5/2)*a*b
^18 + 35*(b*x + a)^(3/2)*a^2*b^18 - 35*sqrt(b*x + a)*a^3*b^18)*d*
e/b^21 + 286*(35*(b*x + a)^(9/2)*b^32 - 180*(b*x + a)^(7/2)*a*b^3
2 + 378*(b*x + a)^(5/2)*a^2*b^32 - 420*(b*x + a)^(3/2)*a^3*b^32 +
 315*sqrt(b*x + a)*a^4*b^32)*d*f/b^36 + 143*(35*(b*x + a)^(9/2)*b
^32 - 180*(b*x + a)^(7/2)*a*b^32 + 378*(b*x + a)^(5/2)*a^2*b^32 -
 420*(b*x + a)^(3/2)*a^3*b^32 + 315*sqrt(b*x + a)*a^4*b^32)*e^2/b
^36 + 130*(63*(b*x + a)^(11/2)*b^50 - 385*(b*x + a)^(9/2)*a*b^50
+ 990*(b*x + a)^(7/2)*a^2*b^50 - 1386*(b*x + a)^(5/2)*a^3*b^50 +
1155*(b*x + a)^(3/2)*a^4*b^50 - 693*sqrt(b*x + a)*a^5*b^50)*f*e/b
^55 + 15*(231*(b*x + a)^(13/2)*b^72 - 1638*(b*x + a)^(11/2)*a*b^7
2 + 5005*(b*x + a)^(9/2)*a^2*b^72 - 8580*(b*x + a)^(7/2)*a^3*b^72
 + 9009*(b*x + a)^(5/2)*a^4*b^72 - 6006*(b*x + a)^(3/2)*a^5*b^72
```

```
+ 3003*sqrt(b*x + a)*a^6*b^72)*f^2/b^78)/b
```

3.6 $\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=708

$$
\begin{aligned}
&-\frac{2f(a+bx)^{15/2}\left(-12a^2f^2+8abef+b^2\left(-\left(df+e^2\right)\right)\right)}{5b^{10}} \\
&+\frac{2(a+bx)^{13/2}\left(-84a^3f^3+84a^2bef^2-21ab^2f\left(df+e^2\right)+b^3\left(3cf^2+6def+e^3\right)\right)}{13b^{10}} \\
&+\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2}{b^{10}} \\
&+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^3}{b^{10}} \\
&+\frac{6(a+bx)^{5/2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)\left(12a^4f^2-16a^3bef+a^2b^2\left(9df+5e^2\right)-ab^3(3cf+5de)+b^4\left(ce+d^2\right)\right)}{5b^{10}} \\
&-\frac{6(a+bx)^{11/2}\left(-42a^4f^3+56a^3bef^2-21a^2b^2f\left(df+e^2\right)+2ab^3\left(3cf^2+6def+e^3\right)-b^4\left(2cef+d^2f+de^2\right)\right)}{11b^{10}} \\
&+\frac{2(a+bx)^{9/2}\left(-42a^5f^3+70a^4bef^2-35a^3b^2f\left(df+e^2\right)+5a^2b^3\left(3cf^2+6def+e^3\right)-5ab^4\left(2cef+d^2f+de^2\right)+b^5\left(2cd\right.\right.}{3b^{10}} \\
&-\frac{2(a+bx)^{7/2}\left(-84a^6f^3+168a^5bef^2-105a^4b^2f\left(df+e^2\right)+20a^3b^3\left(3cf^2+6def+e^3\right)-30a^2b^4\left(2cef+d^2f+de^2\right)+1\right.}{7b^{10}} \\
&+\frac{6f^2(a+bx)^{17/2}(be-3af)}{17b^{10}}+\frac{2f^3(a+bx)^{19/2}}{19b^{10}}
\end{aligned}
$$

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x])/b^10 + (2
*(b^2*d - 2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^
2*(a + b*x)^(3/2))/b^10 + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
(b^4*(d^2 + c*e) - 16*a^3*b*e*f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c
*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)^(5/2))/(5*b^10) - (2*(16
8*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d*e + 3*c^2*f) - 105*
a^4*b^2*f*(e^2 + d*f) + 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) - 30*a
^2*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 20*a^3*b^3*(e^3 + 6*d*e*f + 3*
c*f^2))*(a + b*x)^(7/2))/(7*b^10) + (2*(70*a^4*b*e*f^2 - 42*a^5*f
^3 - 35*a^3*b^2*f*(e^2 + d*f) + b^5*(d^2*e + c*e^2 + 2*c*d*f) - 5
*a*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*e*f + 3*c
*f^2))*(a + b*x)^(9/2))/(3*b^10) - (6*(56*a^3*b*e*f^2 - 42*a^4*f^
3 - 21*a^2*b^2*f*(e^2 + d*f) - b^4*(d*e^2 + d^2*f + 2*c*e*f) + 2*
a*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(11/2))/(11*b^10) + (2
*(84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a*b^2*f*(e^2 + d*f) + b^3*(e^3
 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/(13*b^10) - (2*f*(8*a*b*
e*f - 12*a^2*f^2 - b^2*(e^2 + d*f))*(a + b*x)^(15/2))/(5*b^10) +
(6*f^2*(b*e - 3*a*f)*(a + b*x)^(17/2))/(17*b^10) + (2*f^3*(a + b*
x)^(19/2))/(19*b^10)
```

Rubi [A] time = 1.49768, antiderivative size = 708, normalized size of antiderivative = 1., number of

steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$

$$-\frac{2f(a+bx)^{15/2}\left(-12a^2f^2+8abef+b^2\left(-\left(df+e^2\right)\right)\right)}{5b^{10}}+\frac{2(a+bx)^{13/2}\left(-84a^3f^3+84a^2bef^2-21ab^2f\left(df+e^2\right)+b^3\left(3cf^2+6def+e^3\right)\right)}{13b^{10}}+\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2}{b^{10}}+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^3}{b^{10}}+\frac{6(a+bx)^{5/2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)\left(12a^4f^2-16a^3bef+a^2b^2\left(9df+5e^2\right)-ab^3(3cf+5de)+b^4\left(ce+d^2\right)\right)}{5b^{10}}-\frac{6(a+bx)^{11/2}\left(-42a^4f^3+56a^3bef^2-21a^2b^2f\left(df+e^2\right)+2ab^3\left(3cf^2+6def+e^3\right)-b^4\left(2cef+d^2f+de^2\right)\right)}{11b^{10}}+\frac{2(a+bx)^{9/2}\left(-42a^5f^3+70a^4bef^2-35a^3b^2f\left(df+e^2\right)+5a^2b^3\left(3cf^2+6def+e^3\right)-5ab^4\left(2cef+d^2f+de^2\right)+b^5\left(2cd\right.\right.}{3b^{10}}-\frac{2(a+bx)^{7/2}\left(-84a^6f^3+168a^5bef^2-105a^4b^2f\left(df+e^2\right)+20a^3b^3\left(3cf^2+6def+e^3\right)-30a^2b^4\left(2cef+d^2f+de^2\right)+1\right.}{7b^{10}}+\frac{6f^2(a+bx)^{17/2}(be-3af)}{17b^{10}}+\frac{2f^3(a+bx)^{19/2}}{19b^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x])/b^10 + (2
*(b^2*d - 2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^
2*(a + b*x)^(3/2))/b^10 + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
(b^4*(d^2 + c*e) - 16*a^3*b*e*f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c
*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)^(5/2))/(5*b^10) - (2*(16
8*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d*e + 3*c^2*f) - 105*
a^4*b^2*f*(e^2 + d*f) + 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) - 30*a
^2*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 20*a^3*b^3*(e^3 + 6*d*e*f + 3*
c*f^2))*(a + b*x)^(7/2))/(7*b^10) + (2*(70*a^4*b*e*f^2 - 42*a^5*f
^3 - 35*a^3*b^2*f*(e^2 + d*f) + b^5*(d^2*e + c*e^2 + 2*c*d*f) - 5
*a*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*e*f + 3*c
*f^2))*(a + b*x)^(9/2))/(3*b^10) - (6*(56*a^3*b*e*f^2 - 42*a^4*f^
3 - 21*a^2*b^2*f*(e^2 + d*f) - b^4*(d*e^2 + d^2*f + 2*c*e*f) + 2*
a*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(11/2))/(11*b^10) + (2
*(84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a*b^2*f*(e^2 + d*f) + b^3*(e^3
 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/(13*b^10) - (2*f*(8*a*b*
e*f - 12*a^2*f^2 - b^2*(e^2 + d*f))*(a + b*x)^(15/2))/(5*b^10) +
(6*f^2*(b*e - 3*a*f)*(a + b*x)^(17/2))/(17*b^10) + (2*f^3*(a + b*
x)^(19/2))/(19*b^10)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.76785, size = 913, normalized size = 1.29

$$\frac{2\sqrt{a+bx}\left(-1376256f^3a^9+229376bf^2(19e+3fx)a^8-14336b^2f\left(323e^2+152fxe+f\left(36fx^2+323d\right)\right)a^7+1024b^3\left(1615\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-1376256*a^9*f^3 + 229376*a^8*b*f^2*(19*e + 3*f
*x) - 14336*a^7*b^2*f*(323*e^2 + 152*e*f*x + f*(323*d + 36*f*x^2)
) + 1024*a^6*b^3*(1615*e^3 + 2261*e^2*f*x + 114*e*f*(85*d + 14*f*
x^2) + f^2*(4845*c + 2261*d*x + 420*f*x^3)) - 256*a^5*b^4*(20995*
d^2*f + 3230*c*f*(13*e + 3*f*x) + 323*d*(65*e^2 + 60*e*f*x + 21*f
^2*x^2) + x*(3230*e^3 + 6783*e^2*f*x + 5320*e*f^2*x^2 + 1470*f^3*
x^3)) + 128*a^4*b^5*(4199*d^2*(11*e + 5*f*x) + 323*c*(143*e^2 + 1
30*e*f*x + 45*f^2*x^2) + x^2*(4845*e^3 + 11305*e^2*f*x + 9310*e*f
^2*x^2 + 2646*f^3*x^3) + 323*d*(286*c*f + 5*x*(13*e^2 + 18*e*f*x
+ 7*f^2*x^2))) - 16*a^3*b^6*(138567*d^3 + 415701*c^2*f + 8398*d^2
*x*(22*e + 15*f*x) + 1292*c*x*(143*e^2 + 195*e*f*x + 75*f^2*x^2)
+ x^3*(32300*e^3 + 79135*e^2*f*x + 67032*e*f^2*x^2 + 19404*f^3*x^
3) + 323*d*(286*c*(9*e + 4*f*x) + 5*x^2*(78*e^2 + 120*e*f*x + 49*
f^2*x^2))) + b^9*(4849845*c^3 + 138567*c^2*x*(35*d + 3*x*(7*e + 5
*f*x)) + 323*c*x^2*(9009*d^2 + 1430*d*x*(9*e + 7*f*x) + 35*x^2*(1
43*e^2 + 234*e*f*x + 99*f^2*x^2)) + x^3*(692835*d^3 + 146965*d^2*
x*(11*e + 9*f*x) + 6783*d*x^2*(195*e^2 + 330*e*f*x + 143*f^2*x^2)
 + 231*x^3*(1615*e^3 + 4199*e^2*f*x + 3705*e*f^2*x^2 + 1105*f^3*x
^3))) + 8*a^2*b^7*(138567*c^2*(7*e + 3*f*x) + 323*c*(3003*d^2 + 8
58*d*x*(3*e + 2*f*x) + x^2*(858*e^2 + 1300*e*f*x + 525*f^2*x^2))
+ x*(138567*d^3 + 8398*d^2*x*(33*e + 25*f*x) + 323*d*x^2*(650*e^2
 + 1050*e*f*x + 441*f^2*x^2) + 7*x^3*(8075*e^3 + 20349*e^2*f*x +
17556*e*f^2*x^2 + 5148*f^3*x^3))) - 2*a*b^8*(138567*c^2*(35*d + x
*(14*e + 9*f*x)) + 646*c*x*(3003*d^2 + 143*d*x*(27*e + 20*f*x) +
5*x^2*(286*e^2 + 455*e*f*x + 189*f^2*x^2)) + x^2*(415701*d^3 + 20
995*d^2*x*(44*e + 35*f*x) + 2261*d*x^2*(325*e^2 + 540*e*f*x + 231
*f^2*x^2) + 21*x^3*(9690*e^3 + 24871*e^2*f*x + 21736*e*f^2*x^2 +
6435*f^3*x^3)))))/(4849845*b^10)
```

Maple [B] time = 0.015, size = 1417, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x)
```

```
[Out] -2/4849845*(b*x+a)^(1/2)*(-255255*b^9*f^3*x^9+270270*a*b^8*f^3*x^
8-855855*b^9*e*f^2*x^8-288288*a^2*b^7*f^3*x^7+912912*a*b^8*e*f^2*
x^7-969969*b^9*d*f^2*x^7-969969*b^9*e^2*f*x^7+310464*a^3*b^6*f^3*
x^6-983136*a^2*b^7*e*f^2*x^6+1044582*a*b^8*d*f^2*x^6+1044582*a*b^
8*e^2*f*x^6-1119195*b^9*c*f^2*x^6-2238390*b^9*d*e*f*x^6-373065*b^
9*e^3*x^6-338688*a^4*b^5*f^3*x^5+1072512*a^3*b^6*e*f^2*x^5-113954
4*a^2*b^7*d*f^2*x^5-1139544*a^2*b^7*e^2*f*x^5+1220940*a*b^8*c*f^2
*x^5+2441880*a*b^8*d*e*f*x^5+406980*a*b^8*e^3*x^5-2645370*b^9*c*e
*f*x^5-1322685*b^9*d^2*f*x^5-1322685*b^9*d*e^2*x^5+376320*a^5*b^4
*f^3*x^4-1191680*a^4*b^5*e*f^2*x^4+1266160*a^3*b^6*d*f^2*x^4+1266
160*a^3*b^6*e^2*f*x^4-1356600*a^2*b^7*c*f^2*x^4-2713200*a^2*b^7*d
*e*f*x^4-452200*a^2*b^7*e^3*x^4+2939300*a*b^8*c*e*f*x^4+1469650*a
*b^8*d^2*f*x^4+1469650*a*b^8*d*e^2*x^4-3233230*b^9*c*d*f*x^4-1616
615*b^9*c*e^2*x^4-1616615*b^9*d^2*e*x^4-430080*a^6*b^3*f^3*x^3+13
61920*a^5*b^4*e*f^2*x^3-1447040*a^4*b^5*d*f^2*x^3-1447040*a^4*b^5
*e^2*f*x^3+1550400*a^3*b^6*c*f^2*x^3+3100800*a^3*b^6*d*e*f*x^3+51
6800*a^3*b^6*e^3*x^3-3359200*a^2*b^7*c*e*f*x^3-1679600*a^2*b^7*d^
2*f*x^3-1679600*a^2*b^7*d*e^2*x^3+3695120*a*b^8*c*d*f*x^3+1847560
*a*b^8*c*e^2*x^3+1847560*a*b^8*d^2*e*x^3-2078505*b^9*c^2*f*x^3-41
57010*b^9*c*d*e*x^3-692835*b^9*d^3*x^3+516096*a^7*b^2*f^3*x^2-163
4304*a^6*b^3*e*f^2*x^2+1736448*a^5*b^4*d*f^2*x^2+1736448*a^5*b^4*
e^2*f*x^2-1860480*a^4*b^5*c*f^2*x^2-3720960*a^4*b^5*d*e*f*x^2-620
160*a^4*b^5*e^3*x^2+4031040*a^3*b^6*c*e*f*x^2+2015520*a^3*b^6*d^2
*f*x^2+2015520*a^3*b^6*d*e^2*x^2-4434144*a^2*b^7*c*d*f*x^2-221707
2*a^2*b^7*c*e^2*x^2-2217072*a^2*b^7*d^2*e*x^2+2494206*a*b^8*c^2*f
*x^2+4988412*a*b^8*c*d*e*x^2+831402*a*b^8*d^3*x^2-2909907*b^9*c^2
*e*x^2-2909907*b^9*c*d^2*x^2-688128*a^8*b*f^3*x+2179072*a^7*b^2*e
*f^2*x-2315264*a^6*b^3*d*f^2*x-2315264*a^6*b^3*e^2*f*x+2480640*a^
5*b^4*c*f^2*x+4961280*a^5*b^4*d*e*f*x+826880*a^5*b^4*e^3*x-537472
0*a^4*b^5*c*e*f*x-2687360*a^4*b^5*d^2*f*x-2687360*a^4*b^5*d*e^2*x
```

```
+5912192*a^3*b^6*c*d*f*x+2956096*a^3*b^6*c*e^2*x+2956096*a^3*b^6*
d^2*e*x-3325608*a^2*b^7*c^2*f*x-6651216*a^2*b^7*c*d*e*x-1108536*a
^2*b^7*d^3*x+3879876*a*b^8*c^2*e*x+3879876*a*b^8*c*d^2*x-4849845*
b^9*c^2*d*x+1376256*a^9*f^3-4358144*a^8*b*e*f^2+4630528*a^7*b^2*d
*f^2+4630528*a^7*b^2*e^2*f-4961280*a^6*b^3*c*f^2-9922560*a^6*b^3*
d*e*f-1653760*a^6*b^3*e^3+10749440*a^5*b^4*c*e*f+5374720*a^5*b^4*
d^2*f+5374720*a^5*b^4*d*e^2-11824384*a^4*b^5*c*d*f-5912192*a^4*b^
5*c*e^2-5912192*a^4*b^5*d^2*e+6651216*a^3*b^6*c^2*f+13302432*a^3*
b^6*c*d*e+2217072*a^3*b^6*d^3-7759752*a^2*b^7*c^2*e-7759752*a^2*b
^7*c*d^2+9699690*a*b^8*c^2*d-4849845*b^9*c^3)/b^10
```

Maxima [A] time = 1.43626, size = 1836, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)^3/sqrt(b*x + a),x, algorithm="maxima")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 138567*c^2*(35*((b*x + a)^
(3/2) - 3*sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x +
 a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2)
- 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a
)*a^3)*f/b^3) + 323*c*(3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/
2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(35*(b*x + a)^(9/2) -
180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(
3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/
2)*f + 45*(b*e - 4*a*f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(
b*x + a)^(5/2) + 105*(3*a^2*b*e - 4*a^3*f)*(b*x + a)^(3/2) - 315*
(a^3*b*e - a^4*f)*sqrt(b*x + a))*d/b^4 + 130*(63*(b*x + a)^(11/2)
 - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x +
a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*
e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 50
05*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a
)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*
f^2/b^6) + 138567*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*
(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 4199*(315*(
b*x + a)^(11/2)*f + 385*(b*e - 5*a*f)*(b*x + a)^(9/2) - 990*(2*a*
b*e - 5*a^2*f)*(b*x + a)^(7/2) + 1386*(3*a^2*b*e - 5*a^3*f)*(b*x
+ a)^(5/2) - 1155*(4*a^3*b*e - 5*a^4*f)*(b*x + a)^(3/2) + 3465*(a
^4*b*e - a^5*f)*sqrt(b*x + a))*d^2/b^5 + 1615*(231*(b*x + a)^(13/
2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b
*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/
2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6 + 2261*(429*(b*x + a)^(1
5/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 250
25*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x +
 a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^
7)*e^2*f/b^7 + 133*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2
)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 +
 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612
*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b
*x + a)*a^8)*e*f^2/b^8 + 323*(3003*(b*x + a)^(15/2)*f^2 + 3465*(2
*b*e*f - 7*a*f^2)*(b*x + a)^(13/2) + 4095*(b^2*e^2 - 12*a*b*e*f +
 21*a^2*f^2)*(b*x + a)^(11/2) - 25025*(a*b^2*e^2 - 6*a^2*b*e*f +
7*a^3*f^2)*(b*x + a)^(9/2) + 32175*(2*a^2*b^2*e^2 - 8*a^3*b*e*f +
 7*a^4*f^2)*(b*x + a)^(7/2) - 9009*(10*a^3*b^2*e^2 - 30*a^4*b*e*f
 + 21*a^5*f^2)*(b*x + a)^(5/2) + 15015*(5*a^4*b^2*e^2 - 12*a^5*b*
e*f + 7*a^6*f^2)*(b*x + a)^(3/2) - 45045*(a^5*b^2*e^2 - 2*a^6*b*e
*f + a^7*f^2)*sqrt(b*x + a))*d/b^7 + 21*(12155*(b*x + a)^(19/2) -
 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 149226
0*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(
b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x +
 a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)
*a^9)*f^3/b^9)/b
```

Fricas [A] time = 0.227951, size = 1648, normalized size = 2.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)^3/sqrt(b*x + a),x, algorithm="fricas")
```

```
[Out] 2/4849845*(255255*b^9*f^3*x^9 + 4849845*b^9*c^3 - 9699690*a*b^8*c
^2*d + 7759752*a^2*b^7*c*d^2 - 2217072*a^3*b^6*d^3 + 1653760*a^6*
b^3*e^3 - 1376256*a^9*f^3 + 45045*(19*b^9*e*f^2 - 6*a*b^8*f^3)*x^
8 + 3003*(323*b^9*e^2*f + 96*a^2*b^7*f^3 + 19*(17*b^9*d - 16*a*b^
8*e)*f^2)*x^7 + 231*(1615*b^9*e^3 - 1344*a^3*b^6*f^3 + 19*(255*b^
9*c - 238*a*b^8*d + 224*a^2*b^7*e)*f^2 + 646*(15*b^9*d*e - 7*a*b^
8*e^2)*f)*x^6 + 63*(20995*b^9*d*e^2 - 6460*a*b^8*e^3 + 5376*a^4*b
^5*f^3 - 76*(255*a*b^8*c - 238*a^2*b^7*d + 224*a^3*b^6*e)*f^2 + 3
23*(65*b^9*d^2 + 56*a^2*b^7*e^2 + 10*(13*b^9*c - 12*a*b^8*d)*e)*f
)*x^5 + 35*(46189*b^9*d^2*e + 12920*a^2*b^7*e^3 - 10752*a^5*b^4*f
^3 + 4199*(11*b^9*c - 10*a*b^8*d)*e^2 + 152*(255*a^2*b^7*c - 238*
a^3*b^6*d + 224*a^4*b^5*e)*f^2 + 646*(143*b^9*c*d - 65*a*b^8*d^2
- 56*a^3*b^6*e^2 - 10*(13*a*b^8*c - 12*a^2*b^7*d)*e)*f)*x^4 + 5*(
138567*b^9*d^3 - 103360*a^3*b^6*e^3 + 86016*a^6*b^3*f^3 - 33592*(
11*a*b^8*c - 10*a^2*b^7*d)*e^2 - 1216*(255*a^3*b^6*c - 238*a^4*b^
5*d + 224*a^5*b^4*e)*f^2 + 92378*(9*b^9*c*d - 4*a*b^8*d^2)*e + 32
3*(1287*b^9*c^2 - 2288*a*b^8*c*d + 1040*a^2*b^7*d^2 + 896*a^4*b^5
*e^2 + 160*(13*a^2*b^7*c - 12*a^3*b^6*d)*e)*f)*x^3 + 537472*(11*a
^4*b^5*c - 10*a^5*b^4*d)*e^2 + 19456*(255*a^6*b^3*c - 238*a^7*b^2
*d + 224*a^8*b*e)*f^2 + 3*(969969*b^9*c*d^2 - 277134*a*b^8*d^3 +
206720*a^4*b^5*e^3 - 172032*a^7*b^2*f^3 + 67184*(11*a^2*b^7*c - 1
0*a^3*b^6*d)*e^2 + 2432*(255*a^4*b^5*c - 238*a^5*b^4*d + 224*a^6*
b^3*e)*f^2 + 46189*(21*b^9*c^2 - 36*a*b^8*c*d + 16*a^2*b^7*d^2)*e
 - 646*(1287*a*b^8*c^2 - 2288*a^2*b^7*c*d + 1040*a^3*b^6*d^2 + 89
6*a^5*b^4*e^2 + 160*(13*a^3*b^6*c - 12*a^4*b^5*d)*e)*f)*x^2 + 369
512*(21*a^2*b^7*c^2 - 36*a^3*b^6*c*d + 16*a^4*b^5*d^2)*e - 5168*(
1287*a^3*b^6*c^2 - 2288*a^4*b^5*c*d + 1040*a^5*b^4*d^2 + 896*a^7*
b^2*e^2 + 160*(13*a^5*b^4*c - 12*a^6*b^3*d)*e)*f + (4849845*b^9*c
^2*d - 3879876*a*b^8*c*d^2 + 1108536*a^2*b^7*d^3 - 826880*a^5*b^4
*e^3 + 688128*a^8*b*f^3 - 268736*(11*a^3*b^6*c - 10*a^4*b^5*d)*e^
2 - 9728*(255*a^5*b^4*c - 238*a^6*b^3*d + 224*a^7*b^2*e)*f^2 - 18
4756*(21*a*b^8*c^2 - 36*a^2*b^7*c*d + 16*a^3*b^6*d^2)*e + 2584*(1
287*a^2*b^7*c^2 - 2288*a^3*b^6*c*d + 1040*a^4*b^5*d^2 + 896*a^6*b
^3*e^2 + 160*(13*a^4*b^5*c - 12*a^5*b^4*d)*e)*f)*x)*sqrt(b*x + a)
/b^10
```

Sympy [A] time = 174.121, size = 3691, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c**3/sqrt(a + b*x) + 6*a*c**2*d*(-a/sqrt(a + b*x
) - sqrt(a + b*x))/b + 6*a*c**2*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(
a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 6*a*c*d**2*(a**2/sqrt(a + b
*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 6*a*c**2*f*(
-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) -
 (a + b*x)**(5/2)/5)/b**3 + 12*a*c*d*e*(-a**3/sqrt(a + b*x) - 3*a
**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3
 + 2*a*d**3*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a +
b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 12*a*c*d*f*(a**4/sqrt(a
+ b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a
+ b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 6*a*c*e**2*(a**4/sqr
t(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a
*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 6*a*d**2*e*(a**4
/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) +
 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 12*a*c*e*f*(
-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(
3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a +
b*x)**(9/2)/9)/b**5 + 6*a*d**2*f*(-a**5/sqrt(a + b*x) - 5*a**4*sq
rt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2
) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**5 + 6*a*d*e**
2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)
```

```
**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a
 + b*x)**(9/2)/9)/b**5 + 6*a*c*f**2*(a**6/sqrt(a + b*x) + 6*a**5*
sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2)
 - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x
)**(11/2)/11)/b**6 + 12*a*d*e*f*(a**6/sqrt(a + b*x) + 6*a**5*sqrt
(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 1
5*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(
11/2)/11)/b**6 + 2*a*e**3*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(a + b
*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**2
*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/
11)/b**6 + 6*a*d*f**2*(-a**7/sqrt(a + b*x) - 7*a**6*sqrt(a + b*x)
 + 7*a**5*(a + b*x)**(3/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(a
+ b*x)**(7/2) - 7*a**2*(a + b*x)**(9/2)/3 + 7*a*(a + b*x)**(11/2)
/11 - (a + b*x)**(13/2)/13)/b**7 + 6*a*e**2*f*(-a**7/sqrt(a + b*x
) - 7*a**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3/2) - 7*a**4*(a +
b*x)**(5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a + b*x)**(9/2)/3
 + 7*a*(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)/13)/b**7 + 6*a*e*
f**2*(a**8/sqrt(a + b*x) + 8*a**7*sqrt(a + b*x) - 28*a**6*(a + b*
x)**(3/2)/3 + 56*a**5*(a + b*x)**(5/2)/5 - 10*a**4*(a + b*x)**(7/
2) + 56*a**3*(a + b*x)**(9/2)/9 - 28*a**2*(a + b*x)**(11/2)/11 +
8*a*(a + b*x)**(13/2)/13 - (a + b*x)**(15/2)/15)/b**8 + 2*a*f**3*
(-a**9/sqrt(a + b*x) - 9*a**8*sqrt(a + b*x) + 12*a**7*(a + b*x)**
(3/2) - 84*a**6*(a + b*x)**(5/2)/5 + 18*a**5*(a + b*x)**(7/2) - 1
4*a**4*(a + b*x)**(9/2) + 84*a**3*(a + b*x)**(11/2)/11 - 36*a**2*
(a + b*x)**(13/2)/13 + 3*a*(a + b*x)**(15/2)/5 - (a + b*x)**(17/2
)/17)/b**9 + 2*c**3*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 6*c**2*d
*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b
+ 6*c**2*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b
*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 6*c*d**2*(-a**3/sqrt(a +
b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/
2)/5)/b**2 + 6*c**2*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x)
 - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(
7/2)/7)/b**3 + 12*c*d*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x
) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)*
*(7/2)/7)/b**3 + 2*d**3*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x
) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)*
*(7/2)/7)/b**3 + 12*c*d*f*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a +
b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a
*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 + 6*c*e**2*(-a**5/
sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3
 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**
(9/2)/9)/b**4 + 6*d**2*e*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b
*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*
(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 + 12*c*e*f*(a**6/sq
rt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*
a**3*(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x
)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**5 + 6*d**2*f*(a**6/sqrt(a +
 b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(
a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/
2)/3 - (a + b*x)**(11/2)/11)/b**5 + 6*d*e**2*(a**6/sqrt(a + b*x)
+ 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*
x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 -
 (a + b*x)**(11/2)/11)/b**5 + 6*c*f**2*(-a**7/sqrt(a + b*x) - 7*a
**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3/2) - 7*a**4*(a + b*x)**(
5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a + b*x)**(9/2)/3 + 7*a*
(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)/13)/b**6 + 12*d*e*f*(-a*
*7/sqrt(a + b*x) - 7*a**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3/2)
 - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a
+ b*x)**(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)/13
)/b**6 + 2*e**3*(-a**7/sqrt(a + b*x) - 7*a**6*sqrt(a + b*x) + 7*a
**5*(a + b*x)**(3/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(a + b*x)
**(7/2) - 7*a**2*(a + b*x)**(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 -
(a + b*x)**(13/2)/13)/b**6 + 6*d*f**2*(a**8/sqrt(a + b*x) + 8*a**
7*sqrt(a + b*x) - 28*a**6*(a + b*x)**(3/2)/3 + 56*a**5*(a + b*x)*
*(5/2)/5 - 10*a**4*(a + b*x)**(7/2) + 56*a**3*(a + b*x)**(9/2)/9
- 28*a**2*(a + b*x)**(11/2)/11 + 8*a*(a + b*x)**(13/2)/13 - (a +
b*x)**(15/2)/15)/b**7 + 6*e**2*f*(a**8/sqrt(a + b*x) + 8*a**7*sqr
t(a + b*x) - 28*a**6*(a + b*x)**(3/2)/3 + 56*a**5*(a + b*x)**(5/2
)/5 - 10*a**4*(a + b*x)**(7/2) + 56*a**3*(a + b*x)**(9/2)/9 - 28*
a**2*(a + b*x)**(11/2)/11 + 8*a*(a + b*x)**(13/2)/13 - (a + b*x)*
*(15/2)/15)/b**7 + 6*e*f**2*(-a**9/sqrt(a + b*x) - 9*a**8*sqrt(a
+ b*x) + 12*a**7*(a + b*x)**(3/2) - 84*a**6*(a + b*x)**(5/2)/5 +
18*a**5*(a + b*x)**(7/2) - 14*a**4*(a + b*x)**(9/2) + 84*a**3*(a
+ b*x)**(11/2)/11 - 36*a**2*(a + b*x)**(13/2)/13 + 3*a*(a + b*x)*
```

```
*(15/2)/5 - (a + b*x)**(17/2)/17)/b**8 + 2*f**3*(a**10/sqrt(a + b
*x) + 10*a**9*sqrt(a + b*x) - 15*a**8*(a + b*x)**(3/2) + 24*a**7*
(a + b*x)**(5/2) - 30*a**6*(a + b*x)**(7/2) + 28*a**5*(a + b*x)**
(9/2) - 210*a**4*(a + b*x)**(11/2)/11 + 120*a**3*(a + b*x)**(13/2
)/13 - 3*a**2*(a + b*x)**(15/2) + 10*a*(a + b*x)**(17/2)/17 - (a
+ b*x)**(19/2)/19)/b**9)/b, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2
 + e*f**2*x**9/3 + f**3*x**10/10 + x**8*(3*d*f**2 + 3*e**2*f)/8 +
 x**7*(3*c*f**2 + 6*d*e*f + e**3)/7 + x**6*(6*c*e*f + 3*d**2*f +
3*d*e**2)/6 + x**5*(6*c*d*f + 3*c*e**2 + 3*d**2*e)/5 + x**4*(3*c*
*2*f + 6*c*d*e + d**3)/4 + x**3*(3*c**2*e + 3*c*d**2)/3)/sqrt(a),
 True))
```

GIAC/XCAS [A] time = 0.231854, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)^3/sqrt(b*x + a),x, algorithm="giac")
```

```
[Out] Done
```

3.7 $\int \frac{c+dx}{a+bx^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*
d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(1/3))
```

Rubi [A] time = 0.243177, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*
d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(1/3))
```

Rubi in Sympy [A] time = 31.4186, size = 150, normalized size = 0.93

$$-\frac{\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}+\frac{\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**3+a),x)
```

```
[Out] -(a**(1/3)*d - b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)
*b**(2/3)) + (a**(1/3)*d - b**(1/3)*c)*log(a**(2/3) - a**(1/3)*b*
*(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*b**(2/3)) - sqrt(3)*(a**(1/
3)*d + b**(1/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**
(1/3))/(3*a**(2/3)*b**(2/3))
```

Mathematica [A] time = 0.110094, size = 124, normalized size = 0.77

$$\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)
*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)
*b^(2/3))
```

Maple [A] time = 0.005, size = 186, normalized size = 1.2

$$\frac{c}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} +\frac{c\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} +\frac{d}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*c/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*c/b/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*c/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*d/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))+1/6*d/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*d*3^
(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.02857, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3a^2b^2 + 9tabcd + ad^3 - bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd + 3tabc^2 + 2acd^2}{ad^3 + bc^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lamb
da(_t, _t*log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)
/(a*d**3 + b*c**3))))
```

GIAC/XCAS [A] time = 0.214083, size = 216, normalized size = 1.34

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*(d*(-a/b)^(1/3) + c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/
a + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b
^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*ln(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.8 $\int \frac{c+dx}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=189

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcT
an[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/
(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3))
```

Rubi [A] time = 0.265621, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcT
an[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/
(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3))
```

Rubi in Sympy [A] time = 41.7733, size = 172, normalized size = 0.91

$$\frac{x\left(c+dx\right)}{3a\left(a+bx^3\right)}+\frac{\left(\frac{\sqrt[3]{a}d}{2}-\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(c + d*x)/(3*a*(a + b*x**3)) + (a**(1/3)*d/2 - b**(1/3)*c)*log(
a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(9*a**(5/3)*b**(2
/3)) - (a**(1/3)*d - 2*b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x)/(9*
a**(5/3)*b**(2/3)) - sqrt(3)*(a**(1/3)*d + 2*b**(1/3)*c)*atan(sqr
t(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*b**(2/3)
)
```

Mathematica [A] time = 0.332047, size = 180, normalized size = 0.95

$$\frac{\frac{\left(a^{2/3}d-2\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6ax(c+dx)}{a+bx^3}}{18a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c
+ a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3)
 + (2*(2*a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])
/b^(2/3) + ((-2*a^(1/3)*b^(1/3)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^2)
```

Maple [A] time = 0.004, size = 238, normalized size = 1.3

$$\frac{cx}{3a(bx^3+a)}+\frac{2c}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{9ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{2c\sqrt{3}}{9ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{dx^2}{3a(bx^3+a)}-\frac{d}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{d}{18ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d\sqrt{3}}{9ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*c*x/a/(b*x^3+a)+2/9*c/a/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9*c
/a/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9*c/a/b/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d*x^2
/a/(b*x^3+a)-1/9*d/a/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18*d/a/b/(
a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/9*d/a*3^(1/2)/b/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.8731, size = 105, normalized size = 0.56

$$\operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**
3, Lambda(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 +
 4*a*c*d**2)/(a*d**3 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*
a*b*x**3)
```

GIAC/XCAS [A] time = 0.214971, size = 252, normalized size = 1.33

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2 + cx}{3\,(bx^3 + a)a} + \frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(d*(-a/b)^(1/3) + 2*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3))
)/a^2 + 1/3*(d*x^2 + c*x)/((b*x^3 + a)*a) + 1/9*sqrt(3)*(2*(-a*b^
2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/18*(2*(-a*b^2)^(1/3)*a*b^3*c
+ (-a*b^2)^(2/3)*a*b^2*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^3*b^4)
```

3.9 $\int \frac{c+dx}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=215

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

```
[Out] (x*(c + d*x))/(6*a*(a + b*x^3)^2) + (x*(5*c + 4*d*x))/(18*a^2*(a
+ b*x^3)) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1
/3)*c - 2*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3
)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rubi [A] time = 0.355545, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x))/(6*a*(a + b*x^3)^2) + (x*(5*c + 4*d*x))/(18*a^2*(a
+ b*x^3)) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1
/3)*c - 2*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3
)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rubi in Sympy [A] time = 51.9125, size = 201, normalized size = 0.93

$$\frac{x\left(c+dx\right)}{6a\left(a+bx^3\right)^2}+\frac{x\left(5c+4dx\right)}{18a^2\left(a+bx^3\right)}-\frac{\left(2\sqrt[3]{a}d-5\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{8}{3}}b^{\frac{2}{3}}}+\frac{\left(2\sqrt[3]{a}d-5\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{8}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{8}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] x*(c + d*x)/(6*a*(a + b*x**3)**2) + x*(5*c + 4*d*x)/(18*a**2*(a +
 b*x**3)) - (2*a**(1/3)*d - 5*b**(1/3)*c)*log(a**(1/3) + b**(1/3)
*x)/(27*a**(8/3)*b**(2/3)) + (2*a**(1/3)*d - 5*b**(1/3)*c)*log(a*
*(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(54*a**(8/3)*b**(2/
3)) - sqrt(3)*(2*a**(1/3)*d + 5*b**(1/3)*c)*atan(sqrt(3)*(a**(1/3
)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a**(8/3)*b**(2/3))
```

Mathematica [A] time = 0.326277, size = 205, normalized size = 0.95

$$\frac{\frac{\left(2a^{2/3}d-5\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{9a^2x(c+dx)}{(a+bx^3)^2}-\frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}}{54a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] ((9*a^2*x*(c + d*x))/(a + b*x^3)^2 + (3*a*x*(5*c + 4*d*x))/(a + b
*x^3) - (2*Sqrt[3]*a^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(5*a^(1/3)*b^(1/3
)*c - 2*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/
3)*b^(1/3)*c + 2*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/b^(2/3))/(54*a^3)
```

Maple [A] time = 0.007, size = 272, normalized size = 1.3

$$\frac{cx}{6\,a\,(bx^3+a)^2}+\frac{5\,cx}{18\,a^2\,(bx^3+a)}+\frac{5\,c}{27\,a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{5\,c}{54\,a^2b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5\,c\sqrt{3}}{27\,a^2b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{dx^2}{6\,a\,(bx^3+a)^2}+\frac{2\,dx^2}{9\,a^2\,(bx^3+a)}-\frac{2\,d}{27\,a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{d}{27\,a^2b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{2\,d\sqrt{3}}{27\,a^2b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*c/a*x/(b*x^3+a)^2+5/18*c/a^2*x/(b*x^3+a)+5/27*c/a^2/b/(a/b)^(
2/3)*ln(x+(a/b)^(1/3))-5/54*c/a^2/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))+5/27*c/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))+1/6*d/a*x^2/(b*x^3+a)^2+2/9*d/a^2*x^2/(b
*x^3+a)-2/27*d/a^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27*d/a^2/b/(
a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/27*d/a^2*3^(1/2)/b
/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.45084, size = 146, normalized size = 0.68

$$\operatorname{RootSum}\left(19683t^3a^8b^2 + 810ta^3bcd + 8ad^3 - 125bc^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^6bd + 675ta^3bc^2 + 40acd^2}{8ad^3 + 125bc^3}\right)\right)\right) + \frac{8acx + 7adx^2 + 5bcx^4 + 4bdx^5}{18a^4 + 36a^3bx^3 + 18a^2b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 12
5*b*c**3, Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**
3*b*c**2 + 40*a*c*d**2)/(8*a*d**3 + 125*b*c**3)))) + (8*a*c*x + 7
*a*d*x**2 + 5*b*c*x**4 + 4*b*d*x**5)/(18*a**4 + 36*a**3*b*x**3 +
18*a**2*b**2*x**6)
```

GIAC/XCAS [A] time = 0.21817, size = 279, normalized size = 1.3

$$-\frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}bc - 2\,\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2} + \frac{4\,bdx^5 + 5\,bcx^4 + 7\,adx^2 + 8\,acx}{18\,(bx^3 + a)^2a^2} + \frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ab^3c + 2\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/
3)))/a^3 + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(2/3)*
d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2
) + 1/18*(4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/((b*x^3 +
a)^2*a^2) + 1/54*(5*(-a*b^2)^(1/3)*a*b^3*c + 2*(-a*b^2)^(2/3)*a*b
^2*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^4)
```

3.10 $\int \frac{c+dx}{(a+bx^3)^4}\,dx$

Optimal. Leaf size=240

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

```
[Out] (x*(c + d*x))/(9*a*(a + b*x^3)^3) + (x*(8*c + 7*d*x))/(54*a^2*(a
+ b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (2*(20*
b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a
^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3)*b^(2/3)) - ((20
*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rubi [A] time = 0.42287, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] (x*(c + d*x))/(9*a*(a + b*x^3)^3) + (x*(8*c + 7*d*x))/(54*a^2*(a
+ b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (2*(20*
b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a
^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3)*b^(2/3)) - ((20
*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rubi in Sympy [A] time = 64.5289, size = 228, normalized size = 0.95

$$\frac{x\left(c+dx\right)}{9a\left(a+bx^3\right)^3}+\frac{x\left(8c+7dx\right)}{54a^2\left(a+bx^3\right)^2}+\frac{x\left(40c+28dx\right)}{162a^3\left(a+bx^3\right)}-\frac{2\left(7\sqrt[3]{a}d-20\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{\frac{11}{3}}b^{\frac{2}{3}}}+\frac{\left(7\sqrt[3]{a}d-20\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{243a^{\frac{11}{3}}b^{\frac{2}{3}}}-\frac{2\sqrt{3}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{243a^{\frac{11}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] x*(c + d*x)/(9*a*(a + b*x**3)**3) + x*(8*c + 7*d*x)/(54*a**2*(a +
 b*x**3)**2) + x*(40*c + 28*d*x)/(162*a**3*(a + b*x**3)) - 2*(7*a
**(1/3)*d - 20*b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x)/(243*a**(11
/3)*b**(2/3)) + (7*a**(1/3)*d - 20*b**(1/3)*c)*log(a**(2/3) - a**
```

```
(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(243*a**(11/3)*b**(2/3)) - 2*sq
rt(3)*(7*a**(1/3)*d + 20*b**(1/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2
*b**(1/3)*x/3)/a**(1/3))/(243*a**(11/3)*b**(2/3))
```

Mathematica [A] time = 0.440452, size = 229, normalized size = 0.95

$$\frac{\frac{2\left(7a^{2/3}d-20\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{54a^3x(c+dx)}{(a+bx^3)^3}+\frac{9a^2x(8c+7dx)}{(a+bx^3)^2}-\frac{4\sqrt{3}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\right.}{b}}{486a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] ((54*a^3*x*(c + d*x))/(a + b*x^3)^3 + (9*a^2*x*(8*c + 7*d*x))/(a
+ b*x^3)^2 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3) - (4*Sqrt[3]*a^(
1/3)*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/b^(2/3) + (4*(20*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*L
og[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*c + 7*
a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3
))/(486*a^4)
```

Maple [A] time = 0.007, size = 306, normalized size = 1.3

$$\frac{cx}{9\,a\,(bx^3+a)^3}+\frac{4\,cx}{27\,a^2\,(bx^3+a)^2}+\frac{20\,cx}{81\,a^3\,(bx^3+a)}+\frac{40\,c}{243\,a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{20\,c}{243\,a^3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{40\,c\sqrt{3}}{243\,a^3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{dx^2}{9\,a\,(bx^3+a)^3}+\frac{7\,dx^2}{54\,a^2\,(bx^3+a)^2}+\frac{14\,dx^2}{81\,a^3\,(bx^3+a)}-\frac{14\,d}{243\,a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{7\,d}{243\,a^3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{14\,d\sqrt{3}}{243\,a^3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] 1/9*c/a*x/(b*x^3+a)^3+4/27*c/a^2*x/(b*x^3+a)^2+20/81*c/a^3*x/(b*x
^3+a)+40/243*c/a^3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-20/243*c/a^3/b
/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+40/243*c/a^3/b/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9*d/a*
x^2/(b*x^3+a)^3+7/54*d/a^2*x^2/(b*x^3+a)^2+14/81*d/a^3*x^2/(b*x^3
+a)-14/243*d/a^3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/243*d/a^3/b/(a
/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+14/243*d/a^3*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.93502, size = 185, normalized size = 0.77

$$\operatorname{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t\log\left(x + \frac{413343t^2a^8bd + 194400ta^4bc^2 + 7840a}{1372ad^3 + 32000bc^3}\right.\right.\right.$$
$$+ \frac{82a^2cx + 67a^2dx^2 + 104abcx^4 + 77abdx^5 + 40b^2cx^7 + 28b^2dx^8}{162a^6 + 486a^5bx^3 + 486a^4b^2x^6 + 162a^3b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a
*d**3 - 64000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*
d + 194400*_t*a**4*b*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b
*c**3)))) + (82*a**2*c*x + 67*a**2*d*x**2 + 104*a*b*c*x**4 + 77*a
*b*d*x**5 + 40*b**2*c*x**7 + 28*b**2*d*x**8)/(162*a**6 + 486*a**5
*b*x**3 + 486*a**4*b**2*x**6 + 162*a**3*b**3*x**9)
```

GIAC/XCAS [A] time = 0.214368, size = 312, normalized size = 1.3

$$-\frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4} + \frac{2\sqrt{3}\left(20\left(-ab^2\right)^{\frac{1}{3}}bc - 7\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^4b^2} + \frac{28\,b^2dx^8 + 40\,b^2cx^7 + 77\,abdx^5 + 104\,abcx^4 + 67\,a^2dx^2 + 82\,a^2cx}{162\left(bx^3 + a\right)^3a^3} + \frac{\left(20\left(-ab^2\right)^{\frac{1}{3}}ab^3c + 7\left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^3 + a)^4,x, algorithm="giac")
```

```
[Out] -2/243*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(
1/3)))/a^4 + 2/243*sqrt(3)*(20*(-a*b^2)^(1/3)*b*c - 7*(-a*b^2)^(2
/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4
*b^2) + 1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a
*b*c*x^4 + 67*a^2*d*x^2 + 82*a^2*c*x)/((b*x^3 + a)^3*a^3) + 1/243
*(20*(-a*b^2)^(1/3)*a*b^3*c + 7*(-a*b^2)^(2/3)*a*b^2*d)*ln(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^4)
```

3.11 $\int \frac{a+bx}{d+ex^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

```
[Out] -(((b*d^(1/3) + a*e^(1/3))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3
]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) - a*e^(1/3)
)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(2/3)) - ((a - (b*d^(1/3
))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^
(2/3)*e^(1/3))
```

Rubi [A] time = 0.254427, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)/(d + e*x^3),x]
```

```
[Out] -(((b*d^(1/3) + a*e^(1/3))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3
]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) - a*e^(1/3)
)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(2/3)) - ((a - (b*d^(1/3
))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^
(2/3)*e^(1/3))
```

Rubi in Sympy [A] time = 31.5348, size = 150, normalized size = 0.93

$$\frac{\left(a\sqrt[3]{e}-b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{\frac{2}{3}}e^{\frac{2}{3}}}-\frac{\left(a\sqrt[3]{e}-b\sqrt[3]{d}\right)\log\left(d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2\right)}{6d^{\frac{2}{3}}e^{\frac{2}{3}}}-\frac{\sqrt{3}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{d}}{3}-\frac{2\sqrt[3]{e}x}{3}\right)}{\sqrt[3]{d}}\right)}{3d^{\frac{2}{3}}e^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)/(e*x**3+d),x)
```

```
[Out] (a*e**(1/3) - b*d**(1/3))*log(d**(1/3) + e**(1/3)*x)/(3*d**(2/3)*
e**(2/3)) - (a*e**(1/3) - b*d**(1/3))*log(d**(2/3) - d**(1/3)*e**
(1/3)*x + e**(2/3)*x**2)/(6*d**(2/3)*e**(2/3)) - sqrt(3)*(a*e**(1
/3) + b*d**(1/3))*atan(sqrt(3)*(d**(1/3)/3 - 2*e**(1/3)*x/3)/d**(
1/3))/(3*d**(2/3)*e**(2/3))
```

Mathematica [A] time = 0.104646, size = 125, normalized size = 0.78

$$\frac{-\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\right)-2\sqrt{3}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(
1/3))/Sqrt[3]] - (b*d^(1/3) - a*e^(1/3))*(2*Log[d^(1/3) + e^(1/3)
*x] - Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)
*e^(2/3))
```

Maple [A] time = 0.007, size = 186, normalized size = 1.2

$$\frac{a}{3e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{a}{6e}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}$$
$$+\frac{a\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{b}{3e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}$$
$$+\frac{b}{6e}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}+\frac{b\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)/(e*x^3+d),x)
```

```
[Out] 1/3*a/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))-1/6*a/e/(d/e)^(2/3)*ln(x^2-
x*(d/e)^(1/3)+(d/e)^(2/3))+1/3*a/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(d/e)^(1/3)*x-1))-1/3*b/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3
))+1/6*b/e/(d/e)^(1/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))+1/3*b*3^
(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)/(e*x^3 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)/(e*x^3 + d),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.04167, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3d^2e^2 + 9tabde - a^3e + b^3d, \left(t \mapsto t\log\left(x + \frac{9t^2bd^2e + 3ta^2de + 2ab^2d}{a^3e + b^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)/(e*x**3+d),x)
```

```
[Out] RootSum(27*_t**3*d**2*e**2 + 9*_t*a*b*d*e - a**3*e + b**3*d, Lamb
da(_t, _t*log(x + (9*_t**2*b*d**2*e + 3*_t*a**2*d*e + 2*a*b**2*d)
/(a**3*e + b**3*d))))
```

GIAC/XCAS [A] time = 0.21255, size = 197, normalized size = 1.22

$$\frac{\sqrt{3}\left(\left(-de^2\right)^{\frac{1}{3}}ae - \left(-de^2\right)^{\frac{2}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-2)}}{3d} - \frac{\left(-de^{(-1)}\right)^{\frac{1}{3}}\left(\left(-de^{(-1)}\right)^{\frac{1}{3}}b + a\right)\ln\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3d} + \frac{\left(\left(-de^2\right)^{\frac{2}{3}}bde^2 + \left(-de^2\right)^{\frac{1}{3}}ade^3\right)e^{(-4)}\ln\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}}x + \left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)/(e*x^3 + d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-d*e^2)^(1/3)*a*e - (-d*e^2)^(2/3)*b)*arctan(1/3*sq
rt(3)*(2*x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-2)/d - 1/3
*(-d*e^(-1))^(1/3)*((-d*e^(-1))^(1/3)*b + a)*ln(abs(x - (-d*e^(-1
))^(1/3)))/d + 1/6*((-d*e^2)^(2/3)*b*d*e^2 + (-d*e^2)^(1/3)*a*d*e
^3)*e^(-4)*ln(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/d^2
```

3.12 $\int \frac{a+bx}{d-ex^3}\,dx$

Optimal. Leaf size=161

$$\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

```
[Out] -(((b*d^(1/3) - a*e^(1/3))*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3
]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) + a*e^(1/3)
)*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(2/3)) + ((b*d^(1/3) + a
*e^(1/3))*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2
/3)*e^(2/3))
```

Rubi [A] time = 0.215671, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)/(d - e*x^3),x]
```

```
[Out] -(((b*d^(1/3) - a*e^(1/3))*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3
]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) + a*e^(1/3)
)*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(2/3)) + ((b*d^(1/3) + a
*e^(1/3))*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2
/3)*e^(2/3))
```

Rubi in Sympy [A] time = 31.7272, size = 150, normalized size = 0.93

$$\frac{\sqrt{3}\left(a\sqrt[3]{e}-b\sqrt[3]{d}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{d}}{3}+\frac{2\sqrt[3]{e}x}{3}\right)}{\sqrt[3]{d}}\right)}{3d^{\frac{2}{3}}e^{\frac{2}{3}}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{\frac{2}{3}}e^{\frac{2}{3}}}+\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(d^{\frac{2}{3}}+\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2\right)}{6d^{\frac{2}{3}}e^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)/(-e*x**3+d),x)
```

```
[Out] sqrt(3)*(a*e**(1/3) - b*d**(1/3))*atan(sqrt(3)*(d**(1/3)/3 + 2*e*
*(1/3)*x/3)/d**(1/3))/(3*d**(2/3)*e**(2/3)) - (a*e**(1/3) + b*d**
(1/3))*log(d**(1/3) - e**(1/3)*x)/(3*d**(2/3)*e**(2/3)) + (a*e**(
1/3) + b*d**(1/3))*log(d**(2/3) + d**(1/3)*e**(1/3)*x + e**(2/3)*
x**2)/(6*d**(2/3)*e**(2/3))
```

Mathematica [A] time = 0.0878635, size = 125, normalized size = 0.78

$$\frac{-\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\left(2\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)-\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\right)-2\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)/(d - e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(
1/3))/Sqrt[3]] - (b*d^(1/3) + a*e^(1/3))*(2*Log[d^(1/3) - e^(1/3)
*x] - Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)
*e^(2/3))
```

Maple [A] time = 0.007, size = 188, normalized size = 1.2

$$-\frac{a}{3e}\ln\left(x-\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{a}{6e}\ln\left(x^2+x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}$$
$$+\frac{a\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}+1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{b}{3e}\ln\left(x-\sqrt[3]{\frac{d}{e}}\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}$$
$$+\frac{b}{6e}\ln\left(x^2+x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}-\frac{b\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}+1\right)\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)/(-e*x^3+d),x)
```

```
[Out] -1/3*a/e/(d/e)^(2/3)*ln(x-(d/e)^(1/3))+1/6*a/e/(d/e)^(2/3)*ln(x^2
+x*(d/e)^(1/3)+(d/e)^(2/3))+1/3*a/e/(d/e)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(d/e)^(1/3)*x+1))-1/3*b/e/(d/e)^(1/3)*ln(x-(d/e)^(1/
3))+1/6*b/e/(d/e)^(1/3)*ln(x^2+x*(d/e)^(1/3)+(d/e)^(2/3))-1/3*b*3
^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x + a)/(e*x^3 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x + a)/(e*x^3 - d),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.06266, size = 78, normalized size = 0.48

$$-\operatorname{RootSum}\left(27t^3d^2e^2 - 9tabde - a^3e - b^3d, \left(t \mapsto t\log\left(x + \frac{9t^2bd^2e - 3ta^2de - 2ab^2d}{a^3e - b^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**2 - 9*_t*a*b*d*e - a**3*e - b**3*d, Lam
bda(_t, _t*log(x + (9*_t**2*b*d**2*e - 3*_t*a**2*d*e - 2*a*b**2*d
)/(a**3*e - b**3*d))))
```

GIAC/XCAS [A] time = 0.213404, size = 155, normalized size = 0.96

$$-\frac{\sqrt{3}\left(bd^{\frac{2}{3}}e^{\frac{4}{3}} - ad^{\frac{1}{3}}e^{\frac{5}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2\,x\right)e^{\frac{1}{3}}}{3\,d^{\frac{1}{3}}}\right)e^{(-2)}}{3\,d} - \frac{\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + a\right)e^{\left(-\frac{1}{3}\right)}\ln\left(\left|-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + x\right|\right)}{3\,d^{\frac{2}{3}}} + \frac{\left(bd^{\frac{2}{3}}e^{\frac{4}{3}} + ad^{\frac{1}{3}}e^{\frac{5}{3}}\right)e^{(-2)}\ln\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)} + x^2 + d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x + a)/(e*x^3 - d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*d^(2/3)*e^(4/3) - a*d^(1/3)*e^(5/3))*arctan(1/3*s
qrt(3)*(d^(1/3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*e^(-2)/d - 1/3*(
b*d^(1/3)*e^(-1/3) + a)*e^(-1/3)*ln(abs(-d^(1/3)*e^(-1/3) + x))/d
^(2/3) + 1/6*(b*d^(2/3)*e^(4/3) + a*d^(1/3)*e^(5/3))*e^(-2)*ln(d^
(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d
```

3.13 $\int \frac{1+x}{1+x^3}\,dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] time = 0.0288778, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/(1 + x^3),x]

[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi in Sympy [A] time = 2.58611, size = 22, normalized size = 1.16

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(x**3+1),x)

[Out] 2*sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/3
```

Mathematica [A] time = 0.00913423, size = 19, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/(1 + x^3),x]

[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$\frac{2\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(x^3+1),x)
```

```
[Out] 2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.55926, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Fricas [A] time = 0.210897, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Sympy [A] time = 0.101568, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.209195, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^3 + 1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

3.14 $\int \frac{1-x}{1-x^3} \, dx$

Optimal. Leaf size=19

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] time = 0.0290193, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x)/(1 - x^3),x]

[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi in Sympy [A] time = 4.68088, size = 22, normalized size = 1.16

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3} + \frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)/(-x**3+1),x)

[Out] 2*sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/3
```

Mathematica [A] time = 0.00801525, size = 19, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x)/(1 - x^3),x]

[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.003, size = 17, normalized size = 0.9

$$\frac{2\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)/(-x^3+1),x)
```

```
[Out] 2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.60304, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 0.211303, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Sympy [A] time = 0.101319, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)/(-x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.206852, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

3.15 $\int \frac{1+x}{1-x^3}\,dx$

Optimal. Leaf size=22

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)$$

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Rubi [A] time = 0.0226266, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/(1 - x^3),x]
```

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Rubi in Sympy [A] time = 6.72534, size = 17, normalized size = 0.77

$$-\frac{2\log\left(-x+1\right)}{3}+\frac{\log\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(-x**3+1),x)
```

```
[Out] -2*log(-x + 1)/3 + log(x**2 + x + 1)/3
```

Mathematica [A] time = 0.00754296, size = 22, normalized size = 1.

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/(1 - x^3),x]
```

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Maple [A] time = 0.007, size = 17, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{3}-\frac{2\,\ln\left(-1+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(-x^3+1),x)
```

```
[Out] 1/3*ln(x^2+x+1)-2/3*ln(-1+x)
```

Maxima [A] time = 1.51777, size = 22, normalized size = 1.

$$\frac{1}{3}\,\log\left(x^2 + x + 1\right) - \frac{2}{3}\,\log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 1)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Fricas [A] time = 0.207835, size = 22, normalized size = 1.

$$\frac{1}{3}\,\log\left(x^2 + x + 1\right) - \frac{2}{3}\,\log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 1)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Sympy [A] time = 0.095287, size = 17, normalized size = 0.77

$$-\frac{2\log\left(x - 1\right)}{3} + \frac{\log\left(x^2 + x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(-x**3+1),x)
```

```
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3
```

GIAC/XCAS [A] time = 0.209055, size = 23, normalized size = 1.05

$$\frac{1}{3}\ln\left(x^2 + x + 1\right) - \frac{2}{3}\ln\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 1)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] 1/3*ln(x^2 + x + 1) - 2/3*ln(abs(x - 1))
```

3.16 $\int \frac{1-x}{1+x^3} \, dx$

Optimal. Leaf size=22

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

```
[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Rubi [A] time = 0.0228269, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x)/(1 + x^3),x]

[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Rubi in Sympy [A] time = 6.92861, size = 17, normalized size = 0.77

$$\frac{2\log\left(x+1\right)}{3}-\frac{\log\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)/(x**3+1),x)

[Out] 2*log(x + 1)/3 - log(x**2 - x + 1)/3
```

Mathematica [A] time = 0.00694171, size = 22, normalized size = 1.

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x)/(1 + x^3),x]

[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Maple [A] time = 0.007, size = 19, normalized size = 0.9

$$\frac{2\ln\left(1+x\right)}{3}-\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)/(x^3+1),x)

[Out] 2/3*ln(1+x)-1/3*ln(x^2-x+1)
```

Maxima [A] time = 1.51038, size = 24, normalized size = 1.09

$$-\frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{2}{3}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Fricas [A] time = 0.210305, size = 24, normalized size = 1.09

$$-\frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{2}{3}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Sympy [A] time = 0.088051, size = 17, normalized size = 0.77

$$\frac{2\log\left(x + 1\right)}{3} - \frac{\log\left(x^2 - x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)/(x**3+1),x)
```

```
[Out] 2*log(x + 1)/3 - log(x**2 - x + 1)/3
```

GIAC/XCAS [A] time = 0.210853, size = 26, normalized size = 1.18

$$-\frac{1}{3}\ln\left(x^2 - x + 1\right) + \frac{2}{3}\ln\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/3*ln(x^2 - x + 1) + 2/3*ln(abs(x + 1))
```

3.17 $\int \frac{3-x}{1-x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1
+ x + x^2]/3
```

Rubi [A] time = 0.0567307, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - x)/(1 - x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1
+ x + x^2]/3
```

Rubi in Sympy [A] time = 9.73538, size = 41, normalized size = 1.

$$-\frac{2\log\left(-x+1\right)}{3}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{4\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3-x)/(-x**3+1),x)
```

```
[Out] -2*log(-x + 1)/3 + log(x**2 + x + 1)/3 + 4*sqrt(3)*atan(sqrt(3)*(
2*x/3 + 1/3))/3
```

Mathematica [A] time = 0.0143004, size = 41, normalized size = 1.

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - x)/(1 - x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1
+ x + x^2]/3
```

Maple [A] time = 0.007, size = 33, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{3}+\frac{4\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{2\ln\left(-1+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3-x)/(-x^3+1),x)
```

```
[Out] 1/3*ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/3*ln(-1
+x)
```

Maxima [A] time = 1.52214, size = 43, normalized size = 1.05

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1)
- 2/3*log(x - 1)
```

Fricas [A] time = 0.219667, size = 53, normalized size = 1.29

$$\frac{1}{9}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)-2\sqrt{3}\log\left(x-1\right)+12\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) - 2*sqrt(3)*log(x - 1) + 12
*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.154906, size = 44, normalized size = 1.07

$$-\frac{2\log\left(x-1\right)}{3}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3-x)/(-x**3+1),x)
```

```
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3 + 4*sqrt(3)*atan(2*sqrt(3)*
x/3 + sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.209048, size = 45, normalized size = 1.1

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\ln\left(x^2+x+1\right)-\frac{2}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*ln(x^2 + x + 1) -
 2/3*ln(abs(x - 1))
```

3.18 $\int \frac{c+dx}{c^3+d^3x^3} \, dx$

Optimal. Leaf size=29

$$-\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[Out] (-2*ArcTan[(c - 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rubi [A] time = 0.0467412, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(c^3 + d^3*x^3),x]
```

```
[Out] (-2*ArcTan[(c - 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rubi in Sympy [A] time = 5.16106, size = 31, normalized size = 1.07

$$-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{c}{3}-\frac{2dx}{3}\right)}{c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(d**3*x**3+c**3),x)
```

```
[Out] -2*sqrt(3)*atan(sqrt(3)*(c/3 - 2*d*x/3)/c)/(3*c*d)
```

Mathematica [A] time = 0.0186217, size = 31, normalized size = 1.07

$$\frac{2\tan^{-1}\left(\frac{2dx-c}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(c^3 + d^3*x^3),x]
```

```
[Out] (2*ArcTan[(-c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Maple [A] time = 0.009, size = 35, normalized size = 1.2

$$\frac{2\sqrt{3}}{3\,cd}\arctan\left(\frac{\left(2\,d^2x-cd\right)\sqrt{3}}{3\,cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(d^3*x^3+c^3),x)
```

```
[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x-c*d)*3^(1/2)/c/d)
```

Maxima [A] time = 1.52522, size = 46, normalized size = 1.59

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,d^2x-cd)}{3\,cd}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(d^3*x^3 + c^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x - c*d)/(c*d))/(c*d)
```

Fricas [A] time = 0.217404, size = 38, normalized size = 1.31

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(d^3*x^3 + c^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

Sympy [A] time = 0.200274, size = 54, normalized size = 1.86

$$\frac{-\frac{\sqrt{3}i\log\left(x+\frac{-c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(d**3*x**3+c**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (-c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x
 + (-c + sqrt(3)*I*c)/(2*d))/3)/(c*d)
```

GIAC/XCAS [A] time = 0.21067, size = 38, normalized size = 1.31

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(d^3*x^3 + c^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

3.19 $\int \frac{c-dx}{c^3-d^3x^3}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rubi [A] time = 0.0424429, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x)/(c^3 - d^3*x^3),x]
```

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rubi in Sympy [A] time = 9.35742, size = 29, normalized size = 1.

$$\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{c}{3}+\frac{2dx}{3}\right)}{c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x+c)/(-d**3*x**3+c**3),x)
```

```
[Out] 2*sqrt(3)*atan(sqrt(3)*(c/3 + 2*d*x/3)/c)/(3*c*d)
```

Mathematica [A] time = 0.0152082, size = 29, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c - d*x)/(c^3 - d^3*x^3),x]
```

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Maple [A] time = 0.006, size = 34, normalized size = 1.2

$$\frac{2\sqrt{3}}{3cd}\arctan\left(\frac{\left(2d^2x+cd\right)\sqrt{3}}{3cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x+c)/(-d^3*x^3+c^3),x)
```

```
[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x+c*d)*3^(1/2)/c/d)
```

Maxima [A] time = 1.57928, size = 45, normalized size = 1.55

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,d^2x+cd)}{3\,cd}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x - c)/(d^3*x^3 - c^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x + c*d)/(c*d))/(c*d)
```

Fricas [A] time = 0.217605, size = 35, normalized size = 1.21

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x - c)/(d^3*x^3 - c^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

Sympy [A] time = 0.198037, size = 53, normalized size = 1.83

$$\frac{-\frac{\sqrt{3}i\log\left(x+\frac{c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x+c)/(-d**3*x**3+c**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x
+ (c + sqrt(3)*I*c)/(2*d))/3)/(c*d)
```

GIAC/XCAS [A] time = 0.210651, size = 35, normalized size = 1.21

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x - c)/(d^3*x^3 - c^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

3.20 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx$

Optimal. Leaf size=39

$$-\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[Out] (-2*B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*a^(1/3))
```

Rubi [A] time = 0.0609289, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$-\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (-2*B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*a^(1/3))
```

Rubi in Sympy [A] time = 12.1477, size = 44, normalized size = 1.13

$$-\frac{2\sqrt{3}B\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**(1/3)*b**(1/3)*B+b**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] -2*sqrt(3)*B*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))
/(3*a**(1/3))
```

Mathematica [A] time = 0.0368662, size = 35, normalized size = 0.9

$$-\frac{2B\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/
3))
```

Maple [B] time = 0.005, size = 195, normalized size = 5.

$$\frac{B}{3}\sqrt[3]{a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{6}\sqrt[3]{a}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B\sqrt{3}}{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{b}}\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B}{6}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{b}}\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{b}}\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x)
```

```
[Out] 1/3*B/b^(2/3)*a^(1/3)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*B/b^(2/3)
*a^(1/3)/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*B/b^(2
/3)*a^(1/3)/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))-1/3*B/b^(1/3)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*B/b^(1/3)/
(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*B/b^(1/3)*3^(1/
2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*b^(2/3)*x + B*a^(1/3)*b^(1/3))/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247734, size = 1, normalized size = 0.03

$$\left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,a^{\frac{1}{3}}bx^{2}-2\,a^{\frac{2}{3}}b^{\frac{2}{3}}x+3\,\sqrt{\frac{1}{3}}\left(2\,ab^{\frac{2}{3}}x-a^{\frac{4}{3}}b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-ab^{\frac{1}{3}}}{a^{\frac{1}{3}}bx^{2}-a^{\frac{2}{3}}b^{\frac{2}{3}}x+ab^{\frac{1}{3}}}\right),\ -\frac{2\,\sqrt{\frac{1}{3}}B\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}bx-ab^{\frac{2}{3}}\right)}{ab^{\frac{2}{3}}}\right)}{a^{\frac{1}{3}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*b^(2/3)*x + B*a^(1/3)*b^(1/3))/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*a^(1/3)*b*x^2 - 2*a^(2/3)*b^
(2/3)*x + 3*sqrt(1/3)*(2*a*b^(2/3)*x - a^(4/3)*b^(1/3))*sqrt(-1/a
^(2/3)) - a*b^(1/3))/(a^(1/3)*b*x^2 - a^(2/3)*b^(2/3)*x + a*b^(1/
3))), -2*sqrt(1/3)*B*arctan(-sqrt(1/3)*(2*a^(2/3)*b*x - a*b^(2/3)
)/(a*b^(2/3)))/a^(1/3)]
```

Sympy [A] time = 0.821074, size = 88, normalized size = 2.26

$$\frac{B\left(-\frac{\sqrt{3}i\log\left(x+\frac{-B\sqrt[3]{a}-\sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-B\sqrt[3]{a}+\sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(1/3)*b**(1/3)*B+b**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] B*(-sqrt(3)*I*log(x + (-B*a**(1/3) - sqrt(3)*I*B*a**(1/3))/(2*B*b
**(1/3)))/3 + sqrt(3)*I*log(x + (-B*a**(1/3) + sqrt(3)*I*B*a**(1/
3))/(2*B*b**(1/3)))/3)/a**(1/3)
```

GIAC/XCAS [A] time = 0.225475, size = 65, normalized size = 1.67

$$\frac{2\sqrt{3}Bb^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2b^{\frac{2}{3}}x-a^{\frac{1}{3}}b^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*b^(2/3)*x + B*a^(1/3)*b^(1/3))/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*B*b^(1/3)*arctan(1/3*sqrt(3)*(2*b^(2/3)*x - a^(1/3)*b
^(1/3))/sqrt(a^(2/3)*b^(2/3)))/sqrt(a^(2/3)*b^(2/3))
```

3.21 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx$

Optimal. Leaf size=41

$$\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[Out] (2*B*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[
3]*a^(1/3))
```

Rubi [A] time = 0.101456, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (2*B*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[
3]*a^(1/3))
```

Rubi in Sympy [A] time = 13.6531, size = 44, normalized size = 1.07

$$\frac{2\sqrt{3}B\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\frac{2x\sqrt[3]{-b}}{3}\right)}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(1/3)*(-b)**(1/3)*B-(-b)**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] 2*sqrt(3)*B*atan(sqrt(3)*(a**(1/3)/3 + 2*x*(-b)**(1/3)/3)/a**(1/3
))/(3*a**(1/3))
```

Mathematica [B] time = 0.0973164, size = 129, normalized size = 3.15

$$\frac{\sqrt[3]{-b}B\left(\left(\sqrt[3]{-b}+\sqrt[3]{b}\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)+2\sqrt{3}\left(\sqrt[3]{-b}-\sqrt[3]{b}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] ((-b)^(1/3)*B*(2*Sqrt[3]*((-b)^(1/3) - b^(1/3))*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] + ((-b)^(1/3) + b^(1/3))*(2*Log[a^(1/3
) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))
```

```
)/(6*a^(1/3)*b^(2/3))
```

Maple [B] time = 0.016, size = 228, normalized size = 5.6

$$\frac{B\sqrt[3]{-1}}{3}\sqrt[3]{a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B\sqrt[3]{-1}}{6}\sqrt[3]{a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{B\sqrt[3]{-1}\sqrt{3}}{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{B\sqrt[3]{-1}}{3}\sqrt[3]{-b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)b^{-\frac{2}{3}}\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{B\sqrt[3]{-1}}{6}\sqrt[3]{-b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{-\frac{2}{3}}\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{B\sqrt[3]{-1}\sqrt{3}}{3}\sqrt[3]{-b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)b^{-\frac{2}{3}}\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x)
```

```
[Out] 1/3*B/b^(2/3)*(-1)^(1/3)*a^(1/3)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/
6*B/b^(2/3)*(-1)^(1/3)*a^(1/3)/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(
a/b)^(2/3))+1/3*B/b^(2/3)*(-1)^(1/3)*a^(1/3)/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*B/b^(2/3)*(-1)^(1/3)*
(-b)^(1/3)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*B/b^(2/3)*(-1)^(1/3)
*(-b)^(1/3)/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1/3*B/b
^(2/3)*(-1)^(1/3)*(-b)^(1/3)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3))/(b*x^3 + a),x, algorithm="max
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242509, size = 1, normalized size = 0.02

$$\left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2a^{\frac{1}{3}}bx^2-2a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x-3\sqrt{\frac{1}{3}}\left(2a(-b)^{\frac{2}{3}}x+a^{\frac{4}{3}}(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}+a(-b)^{\frac{1}{3}}}{a^{\frac{1}{3}}bx^2-a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x-a(-b)^{\frac{1}{3}}}\right),\right.$$
$$\left.-\frac{2\sqrt{\frac{1}{3}}B\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}bx-a(-b)^{\frac{2}{3}}\right)}{a(-b)^{\frac{2}{3}}}\right)}{a^{\frac{1}{3}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3))/(b*x^3 + a),x, algorithm="fri
```

```
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*a^(1/3)*b*x^2 - 2*a^(2/3)*(-
b)^(2/3)*x - 3*sqrt(1/3)*(2*a*(-b)^(2/3)*x + a^(4/3)*(-b)^(1/3))*
sqrt(-1/a^(2/3)) + a*(-b)^(1/3))/(a^(1/3)*b*x^2 - a^(2/3)*(-b)^(2
/3)*x - a*(-b)^(1/3))), -2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*a^(2/3
)*b*x - a*(-b)^(2/3))/(a*(-b)^(2/3)))/a^(1/3)]
```

Sympy [A] time = 0.928607, size = 105, normalized size = 2.56

$$-\frac{B\left(-\frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}-\frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+\frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(1/3)*(-b)**(1/3)*B-(-b)**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] -B*(-sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) - sqrt(3)*I*a**(1/
3)*(-b)**(2/3)/(2*b) + x)/3 + sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)
/(2*b) + sqrt(3)*I*a**(1/3)*(-b)**(2/3)/(2*b) + x)/3)/a**(1/3)
```

GIAC/XCAS [A] time = 0.225563, size = 78, normalized size = 1.9

$$\frac{2\sqrt{3}Bb\arctan\left(-\frac{\sqrt{3}\left(2(-b)^{\frac{2}{3}}x+a^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}(-b)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3))/(b*x^3 + a),x, algorithm="gia
```

```
[Out] 2/3*sqrt(3)*B*b*arctan(-1/3*sqrt(3)*(2*(-b)^(2/3)*x + a^(1/3)*(-b
)^(1/3))/sqrt(a^(2/3)*(-b)^(2/3)))/(sqrt(a^(2/3)*(-b)^(2/3))*(-b)
^(2/3))
```

3.22 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx$

Optimal. Leaf size=118

$$\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

```
[Out] -((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(1/3)*b^(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/
3)) + (B*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/
3)*b^(2/3))
```

Rubi [A] time = 0.230538, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257

$$\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] -((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(1/3)*b^(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/
3)) + (B*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/
3)*b^(2/3))
```

Rubi in Sympy [A] time = 35.2745, size = 114, normalized size = 0.97

$$-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}}+\frac{B\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6\sqrt[3]{a}b^{\frac{2}{3}}}-\frac{\sqrt{3}B\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x)/(b*x**3+a),x)
```

```
[Out] -B*log(a**(1/3) + b**(1/3)*x)/(3*a**(1/3)*b**(2/3)) + B*log(a**(2
/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(1/3)*b**(2/3))
- sqrt(3)*B*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/
(3*a**(1/3)*b**(2/3))
```

Mathematica [A] time = 0.0238403, size = 90, normalized size = 0.76

$$\frac{B\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (B*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2]))/(6*a^(1/3)*b^(2/3))
```

Maple [A] time = 0.006, size = 94, normalized size = 0.8

$$-\frac{B}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B}{6\,b}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B\sqrt{3}}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x)
```

```
[Out] -1/3*B/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*B/b/(a/b)^(1/3)*ln(x^2
-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-C*x^2/(b*x^3 + a) + (C*x^2 + B*x)/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219612, size = 138, normalized size = 1.17

$$-\frac{\sqrt{3}\left(\sqrt{3}B\log\left(\left(-ab^{2}\right)^{\frac{1}{3}}bx^{2}-ab+\left(-ab^{2}\right)^{\frac{2}{3}}x\right)-2\sqrt{3}B\log\left(ab+\left(-ab^{2}\right)^{\frac{2}{3}}x\right)+6\,B\arctan\left(-\frac{\sqrt{3}ab-2\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}x}{3\,ab}\right)\right)}{18\,\left(-ab^{2}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-C*x^2/(b*x^3 + a) + (C*x^2 + B*x)/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*B*log((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2
)^(2/3)*x) - 2*sqrt(3)*B*log(a*b + (-a*b^2)^(2/3)*x) + 6*B*arctan
(-1/3*(sqrt(3)*a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)))/(-a*b^2)
^(1/3)
```

Sympy [A] time = 0.171269, size = 26, normalized size = 0.22

$$B\operatorname{RootSum}\left(27t^{3}ab^{2}+1,\left(t\mapsto t\log\left(9t^{2}ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x)/(b*x**3+a),x)
```

```
[Out] B*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)
))
```

GIAC/XCAS [A] time = 0.21612, size = 155, normalized size = 1.31

$$-\frac{B\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}B\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}B\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3 + a) + (C*x^2 + B*x)/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*B*(-a/b)^(2/3)*ln(abs(x - (-a/b)^(1/3)))/a - 1/3*sqrt(3)*(-a
*b^2)^(2/3)*B*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(a*b^2) + 1/6*(-a*b^2)^(2/3)*B*ln(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/(a*b^2)
```

3.23 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx$

Optimal. Leaf size=118

$$-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}$$

```
[Out] -((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(2/3)*b^(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/
3)) - (A*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/
3)*b^(1/3))
```

Rubi [A] time = 0.198355, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242

$$-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(2/3)*b^(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/
3)) - (A*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/
3)*b^(1/3))
```

Rubi in Sympy [A] time = 34.4784, size = 114, normalized size = 0.97

$$\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}\sqrt[3]{b}}-\frac{A\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}\sqrt[3]{b}}-\frac{\sqrt{3}A\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-C*x**2/(b*x**3+a)+(C*x**2+A)/(b*x**3+a),x)
```

```
[Out] A*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(1/3)) - A*log(a**(2/
3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*b**(1/3)) -
 sqrt(3)*A*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(
3*a**(2/3)*b**(1/3))
```

Mathematica [A] time = 0.0206194, size = 90, normalized size = 0.76

$$-\frac{A\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -(A*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2]))/(6*a^(2/3)*b^(1/3))
```

Maple [A] time = 0.004, size = 94, normalized size = 0.8

$$\frac{A}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{A}{6\,b}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{A\sqrt{3}}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x)
```

```
[Out] 1/3*A/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*A/b/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*A/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3 + a) + (C*x^2 + A)/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221999, size = 124, normalized size = 1.05

$$-\frac{\sqrt{3}\left(\sqrt{3}A\log\left(\left(a^{2}b\right)^{\frac{2}{3}}x^{2}-\left(a^{2}b\right)^{\frac{1}{3}}ax+a^{2}\right)-2\sqrt{3}A\log\left(\left(a^{2}b\right)^{\frac{1}{3}}x+a\right)-6\,A\arctan\left(\frac{2\sqrt{3}\left(a^{2}b\right)^{\frac{1}{3}}x-\sqrt{3}a}{3\,a}\right)\right)}{18\left(a^{2}b\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3 + a) + (C*x^2 + A)/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*A*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*
x + a^2) - 2*sqrt(3)*A*log((a^2*b)^(1/3)*x + a) - 6*A*arctan(1/3*
(2*sqrt(3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a))/(a^2*b)^(1/3)
```

Sympy [A] time = 0.186806, size = 22, normalized size = 0.19

$$A\,\mathrm{RootSum}\left(27t^{3}a^{2}b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-C*x**2/(b*x**3+a)+(C*x**2+A)/(b*x**3+a),x)
```

```
[Out] A*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

GIAC/XCAS [A] time = 0.213477, size = 155, normalized size = 1.31

$$-\frac{A\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}A\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(-ab^2\right)^{\frac{1}{3}}A\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3 + a) + (C*x^2 + A)/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*A*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a
*b^2)^(1/3)*A*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(a*b) + 1/6*(-a*b^2)^(1/3)*A*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^
(2/3))/(a*b)
```

3.24 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx$

Optimal. Leaf size=161

$$-\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

```
[Out] -(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*
B)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(1/3))
```

Rubi [A] time = 0.306013, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194

$$-\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] -(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*
B)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(1/3))
```

Rubi in Sympy [A] time = 42.0368, size = 150, normalized size = 0.93

$$\frac{\left(A\sqrt[3]{b}-B\sqrt[3]{a}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}-\frac{\left(A\sqrt[3]{b}-B\sqrt[3]{a}\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(A\sqrt[3]{b}+B\sqrt[3]{a}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x+A)/(b*x**3+a),x)
```

```
[Out] (A*b**(1/3) - B*a**(1/3))*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*
b**(2/3)) - (A*b**(1/3) - B*a**(1/3))*log(a**(2/3) - a**(1/3)*b**
(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*b**(2/3)) - sqrt(3)*(A*b**(1
/3) + B*a**(1/3))*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(
1/3))/(3*a**(2/3)*b**(2/3))
```

Mathematica [A] time = 0.0731088, size = 124, normalized size = 0.77

$$\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(A*b^(1/3) + a^(1/3)*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] + (A*b^(1/3) - a^(1/3)*B)*(2*Log[a^(1/3) + b^(1/3)
*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)
*b^(2/3))
```

Maple [A] time = 0.006, size = 186, normalized size = 1.2

$$\frac{A}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{A}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{A\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{B}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x)
```

```
[Out] 1/3*A/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*A/b/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*A/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*B/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))+1/6*B/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*B*3^
(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3 + a) + (C*x^2 + B*x + A)/(b*x^3 + a),x, algorithm="maxima'
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3 + a) + (C*x^2 + B*x + A)/(b*x^3 + a),x, algorithm="fricas'
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.12174, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3a^2b^2 + 9tABab - A^3b + B^3a, \left(t \mapsto t\log\left(x + \frac{9t^2Ba^2b + 3tA^2ab + 2AB^2a}{A^3b + B^3a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x+A)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*A*B*a*b - A**3*b + B**3*a, Lamb
da(_t, _t*log(x + (9*_t**2*B*a**2*b + 3*_t*A**2*a*b + 2*A*B**2*a)
/(A**3*b + B**3*a))))
```

GIAC/XCAS [A] time = 0.219074, size = 224, normalized size = 1.39

$$-\frac{\left(Bb\left(-\frac{a}{b}\right)^{\frac{1}{3}} + Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ab - \left(-ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Aab^3 + \left(-ab^2\right)^{\frac{2}{3}}Bab^2\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3 + a) + (C*x^2 + B*x + A)/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*(B*b*(-a/b)^(1/3) + A*b)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3
)))/(a*b) + 1/3*sqrt(3)*((-a*b^2)^(1/3)*A*b - (-a*b^2)^(2/3)*B)*a
rctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/
6*((-a*b^2)^(1/3)*A*a*b^3 + (-a*b^2)^(2/3)*B*a*b^2)*ln(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.25 $\int \frac{bx+cx^2}{d+ex^3}\,dx$

Optimal. Leaf size=134

$$\frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}}-\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}}+\frac{c\log\left(d+ex^3\right)}{3e}$$

```
[Out] -((b*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*
d^(1/3)*e^(2/3))) - (b*Log[d^(1/3) + e^(1/3)*x])/(3*d^(1/3)*e^(2/
3)) + (b*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(1/
3)*e^(2/3)) + (c*Log[d + e*x^3])/(3*e)
```

Rubi [A] time = 0.219905, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$\frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}}-\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}}+\frac{c\log\left(d+ex^3\right)}{3e}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] -((b*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*
d^(1/3)*e^(2/3))) - (b*Log[d^(1/3) + e^(1/3)*x])/(3*d^(1/3)*e^(2/
3)) + (b*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(1/
3)*e^(2/3)) + (c*Log[d + e*x^3])/(3*e)
```

Rubi in Sympy [A] time = 34.8705, size = 128, normalized size = 0.96

$$-\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{\frac{2}{3}}}+\frac{b\log\left(d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2\right)}{6\sqrt[3]{d}e^{\frac{2}{3}}}-\frac{\sqrt{3}b\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{d}}{3}-\frac{2\sqrt[3]{e}x}{3}\right)}{\sqrt[3]{d}}\right)}{3\sqrt[3]{d}e^{\frac{2}{3}}}+\frac{c\log\left(d+ex^3\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)/(e*x**3+d),x)
```

```
[Out] -b*log(d**(1/3) + e**(1/3)*x)/(3*d**(1/3)*e**(2/3)) + b*log(d**(2
/3) - d**(1/3)*e**(1/3)*x + e**(2/3)*x**2)/(6*d**(1/3)*e**(2/3))
- sqrt(3)*b*atan(sqrt(3)*(d**(1/3)/3 - 2*e**(1/3)*x/3)/d**(1/3))/
(3*d**(1/3)*e**(2/3)) + c*log(d + e*x**3)/(3*e)
```

Mathematica [A] time = 0.0678758, size = 122, normalized size = 0.91

$$\frac{b\sqrt[3]{e}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)-2b\sqrt[3]{e}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)-2\sqrt{3}b\sqrt[3]{e}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+2c\sqrt[3]{d}\log\left(d+ex^3\right)}{6\sqrt[3]{d}e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*b*e^(1/3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]]
 - 2*b*e^(1/3)*Log[d^(1/3) + e^(1/3)*x] + b*e^(1/3)*Log[d^(2/3) -
 d^(1/3)*e^(1/3)*x + e^(2/3)*x^2] + 2*c*d^(1/3)*Log[d + e*x^3])/(
6*d^(1/3)*e)
```

Maple [A] time = 0.005, size = 108, normalized size = 0.8

$$-\frac{b}{3e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}+\frac{b}{6e}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}$$
$$+\frac{b\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}+\frac{c\ln\left(ex^3+d\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)/(e*x^3+d),x)
```

```
[Out] -1/3*b/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6*b/e/(d/e)^(1/3)*ln(x^2
-x*(d/e)^(1/3)+(d/e)^(2/3))+1/3*b*3^(1/2)/e/(d/e)^(1/3)*arctan(1/
3*3^(1/2)*(2/(d/e)^(1/3)*x-1))+1/3*c*ln(e*x^3+d)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)/(e*x^3 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)/(e*x^3 + d),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.428813, size = 75, normalized size = 0.56

$$\operatorname{RootSum}\left(27t^3de^3-27t^2cde^2+9tc^2de+b^3e-c^3d,\left(t\mapsto t\log\left(x+\frac{9t^2de^2-6tcde+c^2d}{b^2e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)/(e*x**3+d),x)
```

```
[Out] RootSum(27*_t**3*d*e**3 - 27*_t**2*c*d*e**2 + 9*_t*c**2*d*e + b**
3*e - c**3*d, Lambda(_t, _t*log(x + (9*_t**2*d*e**2 - 6*_t*c*d*e
```

```
+ c**2*d)/(b**2*e))))
```

GIAC/XCAS [A] time = 0.213358, size = 162, normalized size = 1.21

$$-\frac{\sqrt{3}\left(-de^{2}\right)^{\frac{2}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-2)}}{3\,d}+\frac{1}{3}\,ce^{(-1)}\ln\left(\left|x^{3}e+d\right|\right)+\frac{\left(-de^{2}\right)^{\frac{2}{3}}be^{(-2)}\ln\left(x^{2}+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\,d}-\frac{\left(-de^{(-1)}\right)^{\frac{2}{3}}b\ln\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)/(e*x^3 + d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(-d*e^2)^(2/3)*b*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1
))^(1/3))/(-d*e^(-1))^(1/3))*e^(-2)/d + 1/3*c*e^(-1)*ln(abs(x^3*e
 + d)) + 1/6*(-d*e^2)^(2/3)*b*e^(-2)*ln(x^2 + (-d*e^(-1))^(1/3)*x
 + (-d*e^(-1))^(2/3))/d - 1/3*(-d*e^(-1))^(2/3)*b*ln(abs(x - (-d*
e^(-1))^(1/3)))/d
```

3.26 $\int \frac{a+cx^2}{d-ex^3}\,dx$

Optimal. Leaf size=134

$$\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

```
[Out] (a*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^
(2/3)*e^(1/3)) - (a*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(1/3))
 + (a*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*
e^(1/3)) - (c*Log[d - e*x^3])/(3*e)
```

Rubi [A] time = 0.181389, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] (a*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^
(2/3)*e^(1/3)) - (a*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(1/3))
 + (a*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*
e^(1/3)) - (c*Log[d - e*x^3])/(3*e)
```

Rubi in Sympy [A] time = 31.2776, size = 128, normalized size = 0.96

$$-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{\frac{2}{3}}\sqrt[3]{e}}+\frac{a\log\left(d^{\frac{2}{3}}+\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2\right)}{6d^{\frac{2}{3}}\sqrt[3]{e}}+\frac{\sqrt{3}a\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{d}}{3}+\frac{2\sqrt[3]{e}x}{3}\right)}{\sqrt[3]{d}}\right)}{3d^{\frac{2}{3}}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)/(-e*x**3+d),x)
```

```
[Out] -a*log(d**(1/3) - e**(1/3)*x)/(3*d**(2/3)*e**(1/3)) + a*log(d**(2
/3) + d**(1/3)*e**(1/3)*x + e**(2/3)*x**2)/(6*d**(2/3)*e**(1/3))
+ sqrt(3)*a*atan(sqrt(3)*(d**(1/3)/3 + 2*e**(1/3)*x/3)/d**(1/3))/
(3*d**(2/3)*e**(1/3)) - c*log(d - e*x**3)/(3*e)
```

Mathematica [A] time = 0.0612387, size = 123, normalized size = 0.92

$$\frac{ae^{2/3}\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)-2ae^{2/3}\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)+2\sqrt{3}ae^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)-2cd^{2/3}\log\left(d-ex^3\right)}{6d^{2/3}e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] (2*Sqrt[3]*a*e^(2/3)*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]]
- 2*a*e^(2/3)*Log[d^(1/3) - e^(1/3)*x] + a*e^(2/3)*Log[d^(2/3) +
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2] - 2*c*d^(2/3)*Log[d - e*x^3])/(6
*d^(2/3)*e)
```

Maple [A] time = 0.004, size = 111, normalized size = 0.8

$$-\frac{a}{3e}\ln\left(x-\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{a}{6e}\ln\left(x^2+x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}$$
$$+\frac{a\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}+1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{c\ln\left(ex^3-d\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)/(-e*x^3+d),x)
```

```
[Out] -1/3*a/e/(d/e)^(2/3)*ln(x-(d/e)^(1/3))+1/6*a/e/(d/e)^(2/3)*ln(x^2
+x*(d/e)^(1/3)+(d/e)^(2/3))+1/3*a/e/(d/e)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(d/e)^(1/3)*x+1))-1/3*c/e*ln(e*x^3-d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + a)/(e*x^3 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + a)/(e*x^3 - d),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.643426, size = 70, normalized size = 0.52

$$-\operatorname{RootSum}\left(27t^3d^2e^3-27t^2cd^2e^2+9tc^2d^2e-a^3e^2-c^3d^2,\left(t\mapsto t\log\left(x+\frac{-3tde+cd}{ae}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**3 - 27*_t**2*c*d**2*e**2 + 9*_t*c**2*d*
*2*e - a**3*e**2 - c**3*d**2, Lambda(_t, _t*log(x + (-3*_t*d*e +
```

```
c*d)/(a*e))))
```

GIAC/XCAS [A] time = 0.212121, size = 128, normalized size = 0.96

$$-\frac{1}{3}\,ce^{(-1)}\ln\left(\left|x^3e-d\right|\right)+\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2\,x\right)e^{\frac{1}{3}}}{3\,d^{\frac{1}{3}}}\right)e^{\left(-\frac{1}{3}\right)}}{3\,d^{\frac{2}{3}}}+\frac{ae^{\left(-\frac{1}{3}\right)}\ln\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^2+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6\,d^{\frac{2}{3}}}-\frac{ae^{\left(-\frac{1}{3}\right)}\ln\left(\left|-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right|\right)}{3\,d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + a)/(e*x^3 - d),x, algorithm="giac")
```

```
[Out] -1/3*c*e^(-1)*ln(abs(x^3*e - d)) + 1/3*sqrt(3)*a*arctan(1/3*sqrt(
3)*(d^(1/3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*e^(-1/3)/d^(2/3) + 1
/6*a*e^(-1/3)*ln(d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(
2/3) - 1/3*a*e^(-1/3)*ln(abs(-d^(1/3)*e^(-1/3) + x))/d^(2/3)
```

3.27 $\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx$

Optimal. Leaf size=37

$$\frac{\log(a+bx)}{b}-\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}$$

```
[Out] (-2*ArcTan[(a - 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) + Log[a + b*x]/b
```

Rubi [A] time = 0.100479, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$\frac{\log(a+bx)}{b}-\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) + Log[a + b*x]/b
```

Rubi in Sympy [A] time = 15.7949, size = 36, normalized size = 0.97

$$\frac{\log{(a+bx)}}{b}-\frac{2\sqrt{3}\operatorname{atan}{\left(\frac{\sqrt{3}\left(\frac{a}{3}-\frac{2bx}{3}\right)}{a}\right)}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**2+2*a**2)/(b**3*x**3+a**3),x)
```

```
[Out] log(a + b*x)/b - 2*sqrt(3)*atan(sqrt(3)*(a/3 - 2*b*x/3)/a)/(3*b)
```

Mathematica [A] time = 0.0356842, size = 72, normalized size = 1.95

$$\frac{\log\left(a^3+b^3x^3\right)-\log\left(a^2-abx+b^2x^2\right)+2\log(a+bx)+2\sqrt{3}\tan^{-1}\left(\frac{2bx-a}{\sqrt{3}a}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*b*x)/(Sqrt[3]*a)] + 2*Log[a + b*x] - Lo
g[a^2 - a*b*x + b^2*x^2] + Log[a^3 + b^3*x^3])/(3*b)
```

Maple [A] time = 0.011, size = 43, normalized size = 1.2

$$\frac{\ln{(bx+a)}}{b}+\frac{2\sqrt{3}}{3b}\arctan\left(\frac{(2b^2x-ab)\sqrt{3}}{3ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x)
```

```
[Out] ln(b*x+a)/b+2/3*3^(1/2)/b*arctan(1/3*(2*b^2*x-a*b)*3^(1/2)/a/b)
```

Maxima [A] time = 1.52875, size = 57, normalized size = 1.54

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2b^2x-ab)}{3ab}\right)}{3b}+\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^2 + 2*a^2)/(b^3*x^3 + a^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x - a*b)/(a*b))/b + log(b*x
 + a)/b
```

Fricas [A] time = 0.219281, size = 51, normalized size = 1.38

$$\frac{\sqrt{3}\left(\sqrt{3}\log(bx+a)+2\arctan\left(\frac{\sqrt{3}(2bx-a)}{3a}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^2 + 2*a^2)/(b^3*x^3 + a^3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*log(b*x + a) + 2*arctan(1/3*sqrt(3)*(2*b*x -
 a)/a))/b
```

Sympy [A] time = 0.819181, size = 60, normalized size = 1.62

$$\frac{-\frac{\sqrt{3}i\log\left(x+\frac{-a-\sqrt{3}ia}{2b}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-a+\sqrt{3}ia}{2b}\right)}{3}+\log\left(\frac{a}{b}+x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**2+2*a**2)/(b**3*x**3+a**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (-a - sqrt(3)*I*a)/(2*b))/3 + sqrt(3)*I*log(x
 + (-a + sqrt(3)*I*a)/(2*b))/3 + log(a/b + x))/b
```

GIAC/XCAS [A] time = 0.211481, size = 50, normalized size = 1.35

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx-a)}{3a}\right)}{3b}+\frac{\ln(|bx+a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^2 + 2*a^2)/(b^3*x^3 + a^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a)/b + ln(abs(b*x + a)
)/b
```

3.28 $\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx$

Optimal. Leaf size=39

$$\frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

```
[Out] (2*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) - Log[a - b*x]/b
```

Rubi [A] time = 0.0788608, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]

[Out] (2*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) - Log[a - b*x]/b
```

Rubi in Sympy [A] time = 16.7815, size = 36, normalized size = 0.92

$$-\frac{\log\left(a-bx\right)}{b}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}+\frac{2bx}{3}\right)}{a}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**2+2*a**2)/(-b**3*x**3+a**3),x)

[Out] -log(a - b*x)/b + 2*sqrt(3)*atan(sqrt(3)*(a/3 + 2*b*x/3)/a)/(3*b)
```

Mathematica [A] time = 0.0348964, size = 71, normalized size = 1.82

$$\frac{-\log\left(a^3-b^3x^3\right)+\log\left(a^2+abx+b^2x^2\right)-2\log(a-bx)+2\sqrt{3}\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]

[Out] (2*Sqrt[3]*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)] - 2*Log[a - b*x] + Log
[a^2 + a*b*x + b^2*x^2] - Log[a^3 - b^3*x^3])/(3*b)
```

Maple [A] time = 0.013, size = 45, normalized size = 1.2

$$-\frac{\ln\left(bx-a\right)}{b}+\frac{2\,\sqrt{3}}{3\,b}\arctan\left(\frac{\left(2\,b^2x+ab\right)\sqrt{3}}{3\,ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x)
```

```
[Out] -1/b*ln(b*x-a)+2/3*3^(1/2)/b*arctan(1/3*(2*b^2*x+a*b)*3^(1/2)/a/b
)
```

Maxima [A] time = 1.51534, size = 59, normalized size = 1.51

$$\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,b^2x+ab)}{3\,ab}\right)}{3\,b}-\frac{\log\left(bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b^2*x^2 + 2*a^2)/(b^3*x^3 - a^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x + a*b)/(a*b))/b - log(b*x
 - a)/b
```

Fricas [A] time = 0.224573, size = 51, normalized size = 1.31

$$-\frac{\sqrt{3}\left(\sqrt{3}\log\left(bx-a\right)-2\,\arctan\left(\frac{\sqrt{3}(2\,bx+a)}{3\,a}\right)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b^2*x^2 + 2*a^2)/(b^3*x^3 - a^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*log(b*x - a) - 2*arctan(1/3*sqrt(3)*(2*b*x
+ a)/a))/b
```

Sympy [A] time = 0.786517, size = 60, normalized size = 1.54

$$-\frac{\frac{\sqrt{3}i\log\left(x+\frac{a-\sqrt{3}ia}{2b}\right)}{3}-\frac{\sqrt{3}i\log\left(x+\frac{a+\sqrt{3}ia}{2b}\right)}{3}+\log\left(-\frac{a}{b}+x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**2+2*a**2)/(-b**3*x**3+a**3),x)
```

```
[Out] -(sqrt(3)*I*log(x + (a - sqrt(3)*I*a)/(2*b))/3 - sqrt(3)*I*log(x
+ (a + sqrt(3)*I*a)/(2*b))/3 + log(-a/b + x))/b
```

GIAC/XCAS [A] time = 0.213391, size = 51, normalized size = 1.31

$$\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx+a)}{3\,a}\right)}{3\,b}-\frac{\ln\left(|bx-a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b^2*x^2 + 2*a^2)/(b^3*x^3 - a^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a)/b - ln(abs(b*x - a)
)/b
```

3.29 $\int \frac{8C+b^{2/3}Cx^2}{8+bx^3} \, dx$

Optimal. Leaf size=48

$$\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

```
[Out] (-2*C*ArcTan[(1 - b^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*Log
[2 + b^(1/3)*x])/b^(1/3)
```

Rubi [A] time = 0.0697781, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - b^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*Log
[2 + b^(1/3)*x])/b^(1/3)
```

Rubi in Sympy [A] time = 11.1613, size = 49, normalized size = 1.02

$$\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}-\frac{2\sqrt{3}C\operatorname{atan}\left(\sqrt{3}\left(-\frac{\sqrt[3]{b}x}{3}+\frac{1}{3}\right)\right)}{3\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*C+b**(2/3)*C*x**2)/(b*x**3+8),x)
```

```
[Out] C*log(b**(1/3)*x + 2)/b**(1/3) - 2*sqrt(3)*C*atan(sqrt(3)*(-b**(1
/3)*x/3 + 1/3))/(3*b**(1/3))
```

Mathematica [A] time = 0.0362854, size = 76, normalized size = 1.58

$$\frac{C\left(-\log\left(b^{2/3}x^2-2\sqrt[3]{b}x+4\right)+\log\left(bx^3+8\right)+2\log\left(\sqrt[3]{b}x+2\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}x-1}{\sqrt{3}}\right)\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*ArcTan[(-1 + b^(1/3)*x)/Sqrt[3]] + 2*Log[2 + b^(1/3
)*x] - Log[4 - 2*b^(1/3)*x + b^(2/3)*x^2] + Log[8 + b*x^3]))/(3*b
^(1/3))
```

Maple [B] time = 0.01, size = 117, normalized size = 2.4

$$\frac{C\sqrt[3]{8}}{3\,b}\ln\left(x+\sqrt[3]{8}\sqrt[3]{b^{-1}}\right)\left(b^{-1}\right)^{-\frac{2}{3}}-\frac{C\sqrt[3]{8}}{6\,b}\ln\left(x^{2}-x\sqrt[3]{8}\sqrt[3]{b^{-1}}+8^{\frac{2}{3}}\left(b^{-1}\right)^{\frac{2}{3}}\right)\left(b^{-1}\right)^{-\frac{2}{3}}+\frac{C\sqrt[3]{8}\sqrt{3}}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{8^{\frac{2}{3}}x}{4}\frac{1}{\sqrt[3]{b^{-1}}}-1\right)\right)\left(b^{-1}\right)^{-\frac{2}{3}}+\frac{C\ln\left(bx^{3}+8\right)}{3}\frac{1}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x)
```

```
[Out] 1/3*C/b*8^(1/3)/(1/b)^(2/3)*ln(x+8^(1/3)*(1/b)^(1/3))-1/6*C/b*8^(
1/3)/(1/b)^(2/3)*ln(x^2-x*8^(1/3)*(1/b)^(1/3)+8^(2/3)*(1/b)^(2/3)
)+1/3*C/b*8^(1/3)/(1/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(
2/3)/(1/b)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+8)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*b^(2/3)*x^2 + 8*C)/(b*x^3 + 8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247513, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{bx^{2}+6\sqrt{\frac{1}{3}}\left(bx-b^{\frac{2}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-2\,b^{\frac{2}{3}}x-2\,b^{\frac{1}{3}}}{bx^{2}-2\,b^{\frac{2}{3}}x+4\,b^{\frac{1}{3}}}\right)+Cb^{\frac{2}{3}}\log\left(bx+2\,b^{\frac{2}{3}}\right)}{b},\ -\frac{2\sqrt{\frac{1}{3}}Cb^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(bx-b^{\frac{2}{3}}\right)}{b^{\frac{2}{3}}}\right)-Cb^{\frac{2}{3}}\log\left(bx+2\,b^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*b^(2/3)*x^2 + 8*C)/(b*x^3 + 8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((b*x^2 + 6*sqrt(1/3)*(b*x -
b^(2/3))*sqrt(-1/b^(2/3)) - 2*b^(2/3)*x - 2*b^(1/3))/(b*x^2 - 2*b
^(2/3)*x + 4*b^(1/3))) + C*b^(2/3)*log(b*x + 2*b^(2/3)))/b, -(2*s
qrt(1/3)*C*b^(2/3)*arctan(-sqrt(1/3)*(b*x - b^(2/3))/b^(2/3)) - C
*b^(2/3)*log(b*x + 2*b^(2/3)))/b]
```

Sympy [A] time = 0.880901, size = 58, normalized size = 1.21

$$\operatorname{RootSum}\left(3t^{3}b^{\frac{5}{3}}-3t^{2}Cb^{\frac{4}{3}}+tC^{2}b-C^{3}b^{\frac{2}{3}},\left(t\mapsto t\log\left(x+\frac{3t\sqrt[3]{b}-C}{C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*C+b**(2/3)*C*x**2)/(b*x**3+8),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*
b**(2/3), Lambda(_t, _t*log(x + (3*_t*b**(1/3) - C)/(C*b**(1/3)))
))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*b^(2/3)*x^2 + 8*C)/(b*x^3 + 8),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.30 $\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx$

Optimal. Leaf size=47

$$\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)-\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

```
[Out] -(C*ArcTan[(a^(1/3) - 4*x)/(Sqrt[3]*a^(1/3))])/(2*Sqrt[3]) + (C*L
og[a^(1/3) + 2*x])/4
```

Rubi [A] time = 0.0651268, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)-\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

```
[Out] -(C*ArcTan[(a^(1/3) - 4*x)/(Sqrt[3]*a^(1/3))])/(2*Sqrt[3]) + (C*L
og[a^(1/3) + 2*x])/4
```

Rubi in Sympy [A] time = 9.77557, size = 44, normalized size = 0.94

$$\frac{C\log\left(\sqrt[3]{a}+2x\right)}{4}-\frac{\sqrt{3}C\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{4x}{3}\right)}{\sqrt[3]{a}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**(2/3)*C+2*C*x**2)/(8*x**3+a),x)
```

```
[Out] C*log(a**(1/3) + 2*x)/4 - sqrt(3)*C*atan(sqrt(3)*(a**(1/3)/3 - 4*
x/3)/a**(1/3))/6
```

Mathematica [A] time = 0.0459204, size = 72, normalized size = 1.53

$$\frac{1}{12}C\left(-\log\left(a^{2/3}-2\sqrt[3]{a}x+4x^2\right)+\log\left(a+8x^3\right)+2\log\left(\sqrt[3]{a}+2x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3
) + 2*x] - Log[a^(2/3) - 2*a^(1/3)*x + 4*x^2] + Log[a + 8*x^3]))/
12
```

Maple [B] time = 0.009, size = 84, normalized size = 1.8

$$\frac{C8^{\frac{2}{3}}}{24}\ln\left(x+\frac{8^{\frac{2}{3}}}{8}\sqrt[3]{a}\right)-\frac{C8^{\frac{2}{3}}}{48}\ln\left(x^2-\frac{x8^{\frac{2}{3}}}{8}\sqrt[3]{a}+\frac{\sqrt[3]{8}}{8}a^{\frac{2}{3}}\right)+\frac{C8^{\frac{2}{3}}\sqrt{3}}{24}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{8}x}{\sqrt[3]{a}}-1\right)\right)+\frac{C\ln\left(8x^3+a\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x)
```

```
[Out] 1/24*C*8^(2/3)*ln(x+1/8*8^(2/3)*a^(1/3))-1/48*C*8^(2/3)*ln(x^2-1/
8*x*8^(2/3)*a^(1/3)+1/8*8^(1/3)*a^(2/3))+1/24*C*8^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x-1))+1/12*C*ln(8*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*C*x^2 + C*a^(2/3))/(8*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.232747, size = 61, normalized size = 1.3

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}C\log\left(2a^{\frac{2}{3}}x+a\right)+2C\arctan\left(\frac{4\sqrt{3}a^{\frac{2}{3}}x-\sqrt{3}a}{3a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*C*x^2 + C*a^(2/3))/(8*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*C*log(2*a^(2/3)*x + a) + 2*C*arctan(1/3*(4*
sqrt(3)*a^(2/3)*x - sqrt(3)*a)/a))
```

Sympy [A] time = 0.807761, size = 85, normalized size = 1.81

$$C\left(\frac{\log\left(\frac{\sqrt[3]{a}}{2}+x\right)}{4}-\frac{\sqrt{3}i\log\left(x+\frac{-C\sqrt[3]{a}-\sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12}+\frac{\sqrt{3}i\log\left(x+\frac{-C\sqrt[3]{a}+\sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((a**(2/3)*C+2*C*x**2)/(8*x**3+a),x)
```

```
[Out] C*(log(a**(1/3)/2 + x)/4 - sqrt(3)*I*log(x + (-C*a**(1/3) - sqrt(
3)*I*C*a**(1/3))/(4*C))/12 + sqrt(3)*I*log(x + (-C*a**(1/3) + sqr
t(3)*I*C*a**(1/3))/(4*C))/12)
```

GIAC/XCAS [A] time = 0.243111, size = 147, normalized size = 3.13

$$\frac{\sqrt{3}\left(\sqrt{3}ai + a\right)C\arctan\left(\frac{\sqrt{3}\left(4\,x+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{12\,a} - \frac{\left(\sqrt{3}ai - 3\,a\right)C\ln\left(x^2 + \frac{1}{2}\,(-a)^{\frac{1}{3}}\,x + \frac{1}{4}\,(-a)^{\frac{2}{3}}\right)}{24\,a} - \frac{\left(C\,(-a)^{\frac{2}{3}} + 2\,Ca^{\frac{2}{3}}\right)(-a)^{\frac{1}{3}}\ln\left(\left|x - \frac{1}{2}\,(-a)^{\frac{1}{3}}\right|\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*C*x^2 + C*a^(2/3))/(8*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*a*i + a)*C*arctan(1/3*sqrt(3)*(4*x + (-a)^(
1/3))/(-a)^(1/3))/a - 1/24*(sqrt(3)*a*i - 3*a)*C*ln(x^2 + 1/2*(-a
)^(1/3)*x + 1/4*(-a)^(2/3))/a - 1/12*(C*(-a)^(2/3) + 2*C*a^(2/3))
*(-a)^(1/3)*ln(abs(x - 1/2*(-a)^(1/3)))/a
```

3.31 $\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx$

Optimal. Leaf size=57

$$\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}x+2\right)}{\sqrt[3]{-b}}$$

```
[Out] (2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (
C*Log[2 + (-b)^(1/3)*x])/(-b)^(1/3)
```

Rubi [A] time = 0.112951, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}x+2\right)}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (
C*Log[2 + (-b)^(1/3)*x])/(-b)^(1/3)
```

Rubi in Sympy [A] time = 13.266, size = 56, normalized size = 0.98

$$-\frac{C\log\left(x\sqrt[3]{-b}+2\right)}{\sqrt[3]{-b}}+\frac{2\sqrt{3}C\operatorname{atan}\left(\sqrt{3}\left(-\frac{x\sqrt[3]{-b}}{3}+\frac{1}{3}\right)\right)}{3\sqrt[3]{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*C+(-b)**(2/3)*C*x**2)/(b*x**3-8),x)
```

```
[Out] -C*log(x*(-b)**(1/3) + 2)/(-b)**(1/3) + 2*sqrt(3)*C*atan(sqrt(3)*
(-x*(-b)**(1/3)/3 + 1/3))/(3*(-b)**(1/3))
```

Mathematica [A] time = 0.049111, size = 99, normalized size = 1.74

$$\frac{C\left(-b^{2/3}\log\left(b^{2/3}x^2+2\sqrt[3]{b}x+4\right)+2b^{2/3}\log\left(2-\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}x+1}{\sqrt{3}}\right)+(-b)^{2/3}\log\left(8-bx^3\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 + b^(1/3)*x)/Sqrt[3]] + 2*b^(2/3
)*Log[2 - b^(1/3)*x] - b^(2/3)*Log[4 + 2*b^(1/3)*x + b^(2/3)*x^2]
 + (-b)^(2/3)*Log[8 - b*x^3]))/(3*b)
```

Maple [B] time = 0.009, size = 122, normalized size = 2.1

$$\frac{C\sqrt[3]{8}}{3\,b}\ln\left(x-\sqrt[3]{8}\sqrt[3]{b^{-1}}\right)\left(b^{-1}\right)^{-\frac{2}{3}}-\frac{C\sqrt[3]{8}}{6\,b}\ln\left(x^{2}+x\sqrt[3]{8}\sqrt[3]{b^{-1}}+8^{\frac{2}{3}}\left(b^{-1}\right)^{\frac{2}{3}}\right)\left(b^{-1}\right)^{-\frac{2}{3}}-\frac{C\sqrt[3]{8}\sqrt{3}}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{8^{\frac{2}{3}}x}{4}\frac{1}{\sqrt[3]{b^{-1}}}+1\right)\right)\left(b^{-1}\right)^{-\frac{2}{3}}+\frac{C\ln\left(bx^{3}-8\right)}{3\,b}(-b)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x)
```

```
[Out] 1/3*C/b*8^(1/3)/(1/b)^(2/3)*ln(x-8^(1/3)*(1/b)^(1/3))-1/6*C/b*8^(
1/3)/(1/b)^(2/3)*ln(x^2+x*8^(1/3)*(1/b)^(1/3)+8^(2/3)*(1/b)^(2/3)
)-1/3*C/b*8^(1/3)/(1/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(
2/3)/(1/b)^(1/3)*x+1))+1/3*C*(-b)^(2/3)/b*ln(b*x^3-8)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*(-b)^(2/3)*x^2 + 8*C)/(b*x^3 - 8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244853, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{bx^{2}-6\sqrt{\frac{1}{3}}\left(bx+(-b)^{\frac{2}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2\,(-b)^{\frac{2}{3}}x+2\,(-b)^{\frac{1}{3}}}{bx^{2}+2\,(-b)^{\frac{2}{3}}x-4\,(-b)^{\frac{1}{3}}}\right)+C\,(-b)^{\frac{2}{3}}\log\left(bx-2\,(-b)^{\frac{2}{3}}\right)}{b},\ -\frac{2\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(bx+(-b)^{\frac{2}{3}}\right)}{b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}\right)-C\,(-b)^{\frac{2}{3}}\log\left(bx-2\,(-b)^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*(-b)^(2/3)*x^2 + 8*C)/(b*x^3 - 8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((b*x^2 - 6*sqrt(1/3)*(b*x
+ (-b)^(2/3))*sqrt((-b)^(1/3)/b) + 2*(-b)^(2/3)*x + 2*(-b)^(1/3))
/(b*x^2 + 2*(-b)^(2/3)*x - 4*(-b)^(1/3))) + C*(-b)^(2/3)*log(b*x
- 2*(-b)^(2/3)))/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(1/3)/b)*arctan(
sqrt(1/3)*(b*x + (-b)^(2/3))/(b*sqrt(-(-b)^(1/3)/b))) - C*(-b)^(2
/3)*log(b*x - 2*(-b)^(2/3)))/b]
```

Sympy [A] time = 0.976785, size = 58, normalized size = 1.02

$$\operatorname{RootSum}\left(3t^{3}b^{2}-3t^{2}Cb\,(-b)^{\frac{2}{3}}+tC^{2}\,(-b)^{\frac{4}{3}}-C^{3}b,\left(t\mapsto t\log\left(-\frac{3t}{C}+x+\frac{(-b)^{\frac{2}{3}}}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*C+(-b)**(2/3)*C*x**2)/(b*x**3-8),x)
```

```
[Out] RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4
/3) - C**3*b, Lambda(_t, _t*log(-3*_t/C + x + (-b)**(2/3)/b)))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*(-b)^(2/3)*x^2 + 8*C)/(b*x^3 - 8),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.32 $\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx$

Optimal. Leaf size=47

$$\frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

```
[Out] (C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(
-a)^(1/3) + 2*x])/4
```

Rubi [A] time = 0.101419, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] (C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(
-a)^(1/3) + 2*x])/4
```

Rubi in Sympy [A] time = 10.8287, size = 44, normalized size = 0.94

$$-\frac{C\log\left(2x+\sqrt[3]{-a}\right)}{4}+\frac{\sqrt{3}C\operatorname{atan}\left(\sqrt{3}\left(-\frac{4x}{3\sqrt[3]{-a}}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((-a)**(2/3)*C+2*C*x**2)/(-8*x**3+a),x)
```

```
[Out] -C*log(2*x + (-a)**(1/3))/4 + sqrt(3)*C*atan(sqrt(3)*(-4*x/(3*(-a
)**(1/3)) + 1/3))/6
```

Mathematica [B] time = 0.0688808, size = 106, normalized size = 2.26

$$\frac{C\left(-a^{2/3}\log\left(8x^3-a\right)+(-a)^{2/3}\log\left(a^{2/3}+2\sqrt[3]{a}x+4x^2\right)-2(-a)^{2/3}\log\left(\sqrt[3]{a}-2x\right)+2\sqrt{3}(-a)^{2/3}\tan^{-1}\left(\frac{\frac{4x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{12a^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(-a)^(2/3)*ArcTan[(1 + (4*x)/a^(1/3))/Sqrt[3]] - 2*
(-a)^(2/3)*Log[a^(1/3) - 2*x] + (-a)^(2/3)*Log[a^(2/3) + 2*a^(1/3
)*x + 4*x^2] - a^(2/3)*Log[-a + 8*x^3]))/(12*a^(2/3))
```

Maple [B] time = 0.011, size = 110, normalized size = 2.3

$$-\frac{C8^{\frac{2}{3}}}{24}(-a)^{\frac{2}{3}}\ln\left(x-\frac{8^{\frac{2}{3}}}{8}\sqrt[3]{a}\right)a^{-\frac{2}{3}}+\frac{C8^{\frac{2}{3}}}{48}(-a)^{\frac{2}{3}}\ln\left(x^{2}+\frac{x8^{\frac{2}{3}}}{8}\sqrt[3]{a}+\frac{\sqrt[3]{8}}{8}a^{\frac{2}{3}}\right)a^{-\frac{2}{3}}$$
$$+\frac{C8^{\frac{2}{3}}\sqrt{3}}{24}(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{8}x}{\sqrt[3]{a}}+1\right)\right)a^{-\frac{2}{3}}-\frac{C\ln\left(8\,x^{3}-a\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x)
```

```
[Out] -1/24*C*(-a)^(2/3)*8^(2/3)/a^(2/3)*ln(x-1/8*8^(2/3)*a^(1/3))+1/48
*C*(-a)^(2/3)*8^(2/3)/a^(2/3)*ln(x^2+1/8*x*8^(2/3)*a^(1/3)+1/8*8^
(1/3)*a^(2/3))+1/24*C*(-a)^(2/3)*8^(2/3)/a^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x+1))-1/12*C*ln(8*x^3-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*C*x^2 + C*(-a)^(2/3))/(8*x^3 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.231279, size = 68, normalized size = 1.45

$$-\frac{1}{12}\sqrt{3}\left(\sqrt{3}C\log\left(2\,(-a)^{\frac{2}{3}}\,x-a\right)-2\,C\arctan\left(\frac{4\,\sqrt{3}\,(-a)^{\frac{2}{3}}\,x+\sqrt{3}a}{3\,a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*C*x^2 + C*(-a)^(2/3))/(8*x^3 - a),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*(sqrt(3)*C*log(2*(-a)^(2/3)*x - a) - 2*C*arctan(1/3
*(4*sqrt(3)*(-a)^(2/3)*x + sqrt(3)*a)/a))
```

Sympy [A] time = 0.847554, size = 95, normalized size = 2.02

$$-C\left(\frac{\log\left(-\frac{a}{2(-a)^{\frac{2}{3}}}+x\right)}{4}+\frac{\sqrt{3}i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}}+x\right)}{12}-\frac{\sqrt{3}i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}}+x\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-a)**(2/3)*C+2*C*x**2)/(-8*x**3+a),x)
```

```
[Out] -C*(log(-a/(2*(-a)**(2/3)) + x)/4 + sqrt(3)*I*log(a/(4*(-a)**(2/3
)) - sqrt(3)*I*a/(4*(-a)**(2/3)) + x)/12 - sqrt(3)*I*log(a/(4*(-a
)**(2/3)) + sqrt(3)*I*a/(4*(-a)**(2/3)) + x)/12)
```

GIAC/XCAS [A] time = 0.242872, size = 130, normalized size = 2.77

$$\frac{\sqrt{3}\left(\sqrt{3}ai - a\right)C\arctan\left(\frac{\sqrt{3}\left(4\,x+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{12\,a} - \frac{\left(\sqrt{3}ai + 3\,a\right)C\ln\left(x^{2} + \frac{1}{2}\,a^{\frac{1}{3}}x + \frac{1}{4}\,a^{\frac{2}{3}}\right)}{24\,a} - \frac{\left(2\,C\,(-a)^{\frac{2}{3}} + Ca^{\frac{2}{3}}\right)\ln\left(\left|x - \frac{1}{2}\,a^{\frac{1}{3}}\right|\right)}{12\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*C*x^2 + C*(-a)^(2/3))/(8*x^3 - a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*a*i - a)*C*arctan(1/3*sqrt(3)*(4*x + a^(1/3
))/a^(1/3))/a - 1/24*(sqrt(3)*a*i + 3*a)*C*ln(x^2 + 1/2*a^(1/3)*x
 + 1/4*a^(2/3))/a - 1/12*(2*C*(-a)^(2/3) + C*a^(2/3))*ln(abs(x -
1/2*a^(1/3)))/a^(2/3)
```

3.33 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=50

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*L
og[(a/b)^(1/3) + x])/b
```

Rubi [A] time = 0.11925, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*L
og[(a/b)^(1/3) + x])/b
```

Rubi in Sympy [A] time = 11.5506, size = 46, normalized size = 0.92

$$\frac{C\log\left(x+\sqrt[3]{\frac{a}{b}}\right)}{b}-\frac{2\sqrt{3}C\operatorname{atan}\left(\sqrt{3}\left(-\frac{2x}{3\sqrt[3]{\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*(a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(x + (a/b)**(1/3))/b - 2*sqrt(3)*C*atan(sqrt(3)*(-2*x/(3*(a/
b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.10607, size = 146, normalized size = 2.92

$$\frac{C\left(-b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^{2/3}\log\left(a+bx^3\right)+2b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}\left(\frac{a}{b}\right)^{2/3}\tan^{-1}\right.}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] + 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] -
(a/b)^(2/3)*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
] + a^(2/3)*Log[a + b*x^3]))/(3*a^(2/3)*b)
```

Maple [A] time = 0.007, size = 87, normalized size = 1.7

$$\frac{2C}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)-\frac{C}{3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)+\frac{C\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*ln(x+(a/b)^(1/3))/b-1/3*C/b*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3
))+2/3*C/b*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/
b*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + 2*C*(a/b)^(2/3))/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233033, size = 77, normalized size = 1.54

$$\frac{\sqrt{3}\left(\sqrt{3}C\log\left(bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+a\right)+2C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + 2*C*(a/b)^(2/3))/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*C*log(b*x*(a/b)^(2/3) + a) + 2*C*arctan(1/3*
(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a))/b
```

Sympy [A] time = 0.94988, size = 100, normalized size = 2.

$$\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*(a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/
3)) - sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2
*b*(a/b)**(2/3)) + sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

GIAC/XCAS [A] time = 0.241073, size = 211, normalized size = 4.22

$$\frac{\sqrt{3}\left(\sqrt{3}ab^2 i + ab^2\right) C \arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3} - \frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}} + 2\,\left(ab^2\right)^{\frac{2}{3}} C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2} - \frac{\left(\sqrt{3}ab^2 i - 3\,ab^2\right) C \ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + 2*C*(a/b)^(2/3))/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*a*b^2*i + a*b^2)*C*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) - 1/3*(C*b^2*(-a/b)^(2/3) +
2*(a*b^2)^(2/3)*C)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^2)
 - 1/6*(sqrt(3)*a*b^2*i - 3*a*b^2)*C*ln(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a*b^3)
```

3.34 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] (2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C
*Log[(-(a/b))^(1/3) + x])/b
```

Rubi [A] time = 0.12987, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C
*Log[(-(a/b))^(1/3) + x])/b
```

Rubi in Sympy [A] time = 13.0925, size = 49, normalized size = 0.92

$$-\frac{C\log\left(x+\sqrt[3]{-\frac{a}{b}}\right)}{b}+\frac{2\sqrt{3}C\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*(-a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*log(x + (-a/b)**(1/3))/b + 2*sqrt(3)*C*atan(sqrt(3)*(-2*x/(3*(
-a/b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.105479, size = 150, normalized size = 2.83

$$\frac{C\left(b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-a^{2/3}\log\left(a-bx^3\right)-2b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+2\sqrt{3}b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\tan\right.}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] - 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*
x] + (-(a/b))^(2/3)*b^(2/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2] - a^(2/3)*Log[a - b*x^3]))/(3*a^(2/3)*b)
```

Maple [B] time = 0.009, size = 135, normalized size = 2.6

$$-\frac{2C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x^2+x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{2C\sqrt{3}}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C\ln\left(bx^3-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C*(-a/b)^(2/3)/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/3*C*(-a/b)^
(2/3)/b/(a/b)^(2/3)*ln(x^2+x*(a/b)^(1/3)+(a/b)^(2/3))+2/3*C*(-a/b
)^(2/3)/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1
/2))-1/3*C/b*ln(b*x^3-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*x^2 + 2*C*(-a/b)^(2/3))/(b*x^3 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233563, size = 81, normalized size = 1.53

$$-\frac{\sqrt{3}\left(\sqrt{3}C\log\left(bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-a\right)-2C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*x^2 + 2*C*(-a/b)^(2/3))/(b*x^3 - a),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*C*log(b*x*(-a/b)^(2/3) - a) - 2*C*arctan(1/
3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a))/b
```

Sympy [A] time = 0.989133, size = 110, normalized size = 2.08

$$-\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*(-a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(-a/b)**
(2/3)) - sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a
/(2*b*(-a/b)**(2/3)) + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

GIAC/XCAS [A] time = 0.24064, size = 204, normalized size = 3.85

$$\frac{\sqrt{3}\left(\sqrt{3}ab^2i - ab^2\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2} - \frac{\left(\sqrt{3}ab^2i + 3\,ab^2\right)C\ln\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*x^2 + 2*C*(-a/b)^(2/3))/(b*x^3 - a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*a*b^2*i - a*b^2)*C*arctan(1/3*sqrt(3)*(2*x +
 (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/3*(C*b^2*(a/b)^(2/3) + 2*(
-a*b^2)^(2/3)*C)*(a/b)^(1/3)*ln(abs(x - (a/b)^(1/3)))/(a*b^2) - 1
/6*(sqrt(3)*a*b^2*i + 3*a*b^2)*C*ln(x^2 + x*(a/b)^(1/3) + (a/b)^(
2/3))/(a*b^3)
```

3.35 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=54

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (
C*Log[(-(a/b))^(1/3) - x])/b
```

Rubi [A] time = 0.097917, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```



```
[Out] (-2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (
C*Log[(-(a/b))^(1/3) - x])/b
```

Rubi in Sympy [A] time = 12.6133, size = 49, normalized size = 0.91

$$\frac{C\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{b}-\frac{2\sqrt{3}C\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*(-a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(x - (-a/b)**(1/3))/b - 2*sqrt(3)*C*atan(sqrt(3)*(2*x/(3*(-a
/b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.0845229, size = 149, normalized size = 2.76

$$\frac{C\left(-b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^{2/3}\log\left(a+bx^3\right)+2b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\text{ta}\right.}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)
*x] - (-(a/b))^(2/3)*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2] + a^(2/3)*Log[a + b*x^3]))/(3*a^(2/3)*b)
```

Maple [B] time = 0.006, size = 132, normalized size = 2.4

$$\frac{2C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} +\frac{2C\sqrt{3}}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C\ln\left(bx^{3}+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*(-a/b)^(2/3)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/3*C*(-a/b)^(
2/3)/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/3*C*(-a/b)
^(2/3)/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))+1/3*C/b*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + 2*C*(-a/b)^(2/3))/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236429, size = 80, normalized size = 1.48

$$\frac{\sqrt{3}\left(\sqrt{3}C\log\left(bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+a\right)+2\,C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + 2*C*(-a/b)^(2/3))/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*C*log(b*x*(-a/b)^(2/3) + a) + 2*C*arctan(1/3
*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a))/b
```

Sympy [A] time = 0.985062, size = 109, normalized size = 2.02

$$\frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*(-a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(-a/b)**(
2/3)) - sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a
/(2*b*(-a/b)**(2/3)) + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

GIAC/XCAS [A] time = 0.217677, size = 123, normalized size = 2.28

$$-\frac{2\,\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{\left(Cb^{2}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\,\left(-ab^{2}\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + 2*C*(-a/b)^(2/3))/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/b - 1/3*(C*b^2*(-a/b)^(2/3) + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3
)*ln(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

3.36 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] (2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Lo
g[(a/b)^(1/3) - x])/b
```

Rubi [A] time = 0.0968086, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Lo
g[(a/b)^(1/3) - x])/b
```

Rubi in Sympy [A] time = 13.4389, size = 46, normalized size = 0.87

$$-\frac{C\log\left(x-\sqrt[3]{\frac{a}{b}}\right)}{b}+\frac{2\sqrt{3}C\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3\sqrt[3]{\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*(a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*log(x - (a/b)**(1/3))/b + 2*sqrt(3)*C*atan(sqrt(3)*(2*x/(3*(a/
b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.077881, size = 147, normalized size = 2.77

$$\frac{C\left(b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-a^{2/3}\log\left(a-bx^3\right)-2b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+2\sqrt{3}b^{2/3}\left(\frac{a}{b}\right)^{2/3}\tan^{-1}\left(\right.\right.}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] - 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (
a/b)^(2/3)*b^(2/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
 - a^(2/3)*Log[a - b*x^3]))/(3*a^(2/3)*b)
```

Maple [A] time = 0.007, size = 90, normalized size = 1.7

$$-\frac{2C}{3b}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)+\frac{C}{3b}\ln\left(x^2+x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)-\frac{C\ln\left(bx^3-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C/b*ln(x-(a/b)^(1/3))+1/3*C/b*ln(x^2+x*(a/b)^(1/3)+(a/b)^(2/
3))+2/3*C*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2))/b*3^(1/2)-1/3*C
/b*ln(b*x^3-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*x^2 + 2*C*(a/b)^(2/3))/(b*x^3 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23558, size = 78, normalized size = 1.47

$$-\frac{\sqrt{3}\left(\sqrt{3}C\log\left(bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-a\right)-2C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*x^2 + 2*C*(a/b)^(2/3))/(b*x^3 - a),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*C*log(b*x*(a/b)^(2/3) - a) - 2*C*arctan(1/3
*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a))/b
```

Sympy [A] time = 1.04674, size = 102, normalized size = 1.92

$$-\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*(a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(a/b)**(2
/3)) - sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2
*b*(a/b)**(2/3)) + sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

GIAC/XCAS [A] time = 0.216543, size = 115, normalized size = 2.17

$$\frac{2\,\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{\left(Cb^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,\left(ab^{2}\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*x^2 + 2*C*(a/b)^(2/3))/(b*x^3 - a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))
/b - 1/3*(C*b^2*(a/b)^(2/3) + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*ln(a
bs(x - (a/b)^(1/3)))/(a*b^2)
```

3.37 $\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=61

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

```
[Out] (-2*C*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*b^(1/3)) + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi [A] time = 0.0833869, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*b^(1/3)) + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi in Sympy [A] time = 13.1518, size = 63, normalized size = 1.03

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\sqrt{3}C\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*a**(2/3)*C+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(a**(1/3) + b**(1/3)*x)/b**(1/3) - 2*sqrt(3)*C*atan(sqrt(3)*
(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*b**(1/3))
```

Mathematica [A] time = 0.0416317, size = 95, normalized size = 1.56

$$\frac{C\left(-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\log\left(a+bx^3\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*Lo
g[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2] + Log[a + b*x^3]))/(3*b^(1/3))
```

Maple [B] time = 0.005, size = 117, normalized size = 1.9

$$\frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C\ln\left(bx^3+a\right)}{3}\frac{1}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*a^(2/3)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/3*C*a^(2/3)/b/(a/
b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/3*C*a^(2/3)/b/(a/b)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b^(1/
3)*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*b^(2/3)*x^2 + 2*C*a^(2/3))/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244809, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2a^{\frac{1}{3}}bx^2-2a^{\frac{2}{3}}b^{\frac{2}{3}}x+3\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}bx-ab^{\frac{2}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-ab^{\frac{1}{3}}}{a^{\frac{1}{3}}bx^2-a^{\frac{2}{3}}b^{\frac{2}{3}}x+ab^{\frac{1}{3}}}\right)+Cb^{\frac{2}{3}}\log\left(a^{\frac{2}{3}}bx+ab^{\frac{2}{3}}\right)}{b},\ -\frac{2\sqrt{\frac{1}{3}}Cb^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}bx-ab^{\frac{2}{3}}\right)}{ab^{\frac{2}{3}}}\right)-Cb^{\frac{2}{3}}\log\left(a^{\frac{2}{3}}bx+ab^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*b^(2/3)*x^2 + 2*C*a^(2/3))/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((2*a^(1/3)*b*x^2 - 2*a^(2/3)
*b^(2/3)*x + 3*sqrt(1/3)*(2*a^(2/3)*b*x - a*b^(2/3))*sqrt(-1/b^(2
/3)) - a*b^(1/3))/(a^(1/3)*b*x^2 - a^(2/3)*b^(2/3)*x + a*b^(1/3))
) + C*b^(2/3)*log(a^(2/3)*b*x + a*b^(2/3)))/b, -(2*sqrt(1/3)*C*b^
(2/3)*arctan(-sqrt(1/3)*(2*a^(2/3)*b*x - a*b^(2/3))/(a*b^(2/3)))
- C*b^(2/3)*log(a^(2/3)*b*x + a*b^(2/3)))/b]
```

Sympy [A] time = 0.964292, size = 70, normalized size = 1.15

$$\operatorname{RootSum}\left(3t^3b^{\frac{5}{3}} - 3t^2Cb^{\frac{4}{3}} + tC^2b - C^3b^{\frac{2}{3}}, \left(t \mapsto t\log\left(x + \frac{3t\sqrt[3]{a}\sqrt[3]{b} - C\sqrt[3]{a}}{2C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a**(2/3)*C+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*
b**(2/3), Lambda(_t, _t*log(x + (3*_t*a**(1/3)*b**(1/3) - C*a**(1
/3))/(2*C*b**(1/3)))))
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*b^(2/3)*x^2 + 2*C*a^(2/3))/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.38 $\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

```
[Out] (-2*C*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*(-b)^(1/3)) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rubi [A] time = 0.12748, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*(-b)^(1/3)) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rubi in Sympy [A] time = 19.8648, size = 70, normalized size = 1.

$$\frac{C\log\left(\sqrt[3]{a}-x\sqrt[3]{-b}\right)}{\sqrt[3]{-b}}-\frac{2\sqrt{3}C\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\frac{2x\sqrt[3]{-b}}{3}\right)}{\sqrt[3]{a}}\right)}{3\sqrt[3]{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*a**(2/3)*C-(-b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(a**(1/3) - x*(-b)**(1/3))/(-b)**(1/3) - 2*sqrt(3)*C*atan(sq
rt(3)*(a**(1/3)/3 + 2*x*(-b)**(1/3)/3)/a**(1/3))/(3*(-b)**(1/3))
```

Mathematica [A] time = 0.057948, size = 116, normalized size = 1.66

$$-\frac{C\left(-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+(-b)^{2/3}\log\left(a+bx^3\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] -(C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + (-b)^(2/3)*Log[a + b*x^3]))/(3*
b)
```

Maple [B] time = 0.009, size = 122, normalized size = 1.7

$$-\frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{2C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C\ln\left(bx^{3}+a\right)}{3b}(-b)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] -2/3*C*a^(2/3)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/3*C*a^(2/3)/b/(a
/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-2/3*C*a^(2/3)/b/(a/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*C*(-b)
^(2/3)/b*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*(-b)^(2/3)*x^2 + 2*C*a^(2/3))/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249605, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{2a^{\frac{1}{3}}bx^{2}-2a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x-3\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}bx-a(-b)^{\frac{2}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+a(-b)^{\frac{1}{3}}}{a^{\frac{1}{3}}bx^{2}-a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x-a(-b)^{\frac{1}{3}}}\right)-C(-b)^{\frac{2}{3}}\log\left(a^{\frac{2}{3}}bx+a(-b)^{\frac{2}{3}}\right)}{b},\right.$$
$$\left.-\frac{2\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}bx-a(-b)^{\frac{2}{3}}\right)}{ab\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}\right)+C(-b)^{\frac{2}{3}}\log\left(a^{\frac{2}{3}}bx+a(-b)^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*(-b)^(2/3)*x^2 + 2*C*a^(2/3))/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((2*a^(1/3)*b*x^2 - 2*a^(2/
3)*(-b)^(2/3)*x - 3*sqrt(1/3)*(2*a^(2/3)*b*x - a*(-b)^(2/3))*sqrt
((-b)^(1/3)/b) + a*(-b)^(1/3))/(a^(1/3)*b*x^2 - a^(2/3)*(-b)^(2/3
)*x - a*(-b)^(1/3))) - C*(-b)^(2/3)*log(a^(2/3)*b*x + a*(-b)^(2/3
)))/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
a^(2/3)*b*x - a*(-b)^(2/3))/(a*b*sqrt(-(-b)^(1/3)/b))) + C*(-b)^(
2/3)*log(a^(2/3)*b*x + a*(-b)^(2/3)))/b]
```

Sympy [A] time = 1.06539, size = 73, normalized size = 1.04

$$-\operatorname{RootSum}\left(3t^3b^2-3t^2Cb\left(-b\right)^{\frac{2}{3}}+tC^2\left(-b\right)^{\frac{4}{3}}-C^3b,\left(t\mapsto t\log\left(\frac{3t\sqrt[3]{a}}{2C}-\frac{\sqrt[3]{a}\left(-b\right)^{\frac{2}{3}}}{2b}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*a**(2/3)*C-(-b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] -RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(
4/3) - C**3*b, Lambda(_t, _t*log(3*_t*a**(1/3)/(2*C) - a**(1/3)*(
-b)**(2/3)/(2*b) + x)))
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*(-b)^(2/3)*x^2 + 2*C*a^(2/3))/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.39 $\int \frac{-3+x^2}{-1+x^3} \, dx$

Optimal. Leaf size=40

$$\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - (2*Log[1 - x])/3 + (5*Log[1 +
 x + x^2])/6
```

Rubi [A] time = 0.0639313, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462

$$\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - (2*Log[1 - x])/3 + (5*Log[1 +
 x + x^2])/6
```

Rubi in Sympy [A] time = 9.78696, size = 39, normalized size = 0.98

$$-\frac{2\log\left(-x+1\right)}{3}+\frac{5\log\left(x^2+x+1\right)}{6}+\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-3)/(x**3-1),x)
```

```
[Out] -2*log(-x + 1)/3 + 5*log(x**2 + x + 1)/6 + sqrt(3)*atan(sqrt(3)*(
2*x/3 + 1/3))
```

Mathematica [A] time = 0.0197222, size = 50, normalized size = 1.25

$$\frac{1}{3}\log\left(1-x^3\right)+\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[1 - x] + Log[1 + x + x^2]
/2 + Log[1 - x^3]/3
```

Maple [A] time = 0.007, size = 32, normalized size = 0.8

$$\frac{5\ln\left(x^2+x+1\right)}{6}+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}-\frac{2\ln\left(-1+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-3)/(x^3-1),x)
```

```
[Out] 5/6*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/3*ln(-1+x)
```

Maxima [A] time = 1.52211, size = 42, normalized size = 1.05

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/
3*log(x - 1)
```

Fricas [A] time = 0.22236, size = 42, normalized size = 1.05

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/
3*log(x - 1)
```

Sympy [A] time = 0.152633, size = 42, normalized size = 1.05

$$-\frac{2\log\left(x-1\right)}{3}+\frac{5\log\left(x^2+x+1\right)}{6}+\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3)/(x**3-1),x)
```

```
[Out] -2*log(x - 1)/3 + 5*log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*
x/3 + sqrt(3)/3)
```

GIAC/XCAS [A] time = 0.239933, size = 43, normalized size = 1.08

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\ln\left(x^2+x+1\right)-\frac{2}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*ln(x^2 + x + 1) - 2/3
*ln(abs(x - 1))
```

3.40 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

```
[Out] (-2*(B/a^(1/3) + C/b^(1/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/Sqrt[3] + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi [A] time = 0.116095, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(a + b*x^3),x
```

```
[Out] (-2*(B/a^(1/3) + C/b^(1/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/Sqrt[3] + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi in Sympy [A] time = 20.7928, size = 70, normalized size = 1.

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\sqrt{3}\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**(1/3)*b**(1/3)*B+2*a**(2/3)*C+b**(2/3)*B*x+b**(2/3)*C*x**2)
```

```
[Out] C*log(a**(1/3) + b**(1/3)*x)/b**(1/3) - 2*sqrt(3)*(B/a**(1/3) + C
/b**(1/3))*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/3
```

Mathematica [A] time = 0.0986351, size = 122, normalized size = 1.74

$$\frac{\sqrt[3]{a}C\left(-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\log\left(a+bx^3\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}C+\sqrt[3]{b}B\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(a + b*
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*B + a^(1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] + a^(1/3)*C*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + Log[a + b*x^3]))/(3*a^(1
```

```
/3)*b^(1/3))
```

Maple [B] time = 0.009, size = 310, normalized size = 4.4

$$\frac{B}{3}\sqrt[3]{a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{B}{6}\sqrt[3]{a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{B\sqrt{3}}{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{B}{3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{b}}\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B}{6}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{b}}\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{B\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{b}}\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{C\ln\left(bx^3+a\right)}{3}\frac{1}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] 1/3*B/b^(2/3)*a^(1/3)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/3*C*a^(2/3)
/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*B/b^(2/3)*a^(1/3)/(a/b)^(2/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1/3*C*a^(2/3)/b/(a/b)^(2/3)*l
n(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*B/b^(2/3)*a^(1/3)/(a/b)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/3*C*a^(2/3)/b
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*
B/b^(1/3)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*B/b^(1/3)/(a/b)^(1/3)
*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*B/b^(1/3)*3^(1/2)/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^
3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*b^(2/3)*x^2 + B*b^(2/3)*x + 2*C*a^(2/3) + B*a^(1/3)*b^(1/3))/(b*x^3
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*b^(2/3)*x^2 + B*b^(2/3)*x + 2*C*a^(2/3) + B*a^(1/3)*b^(1/3))/(b*x^3
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*b**(1/3)*B+2*a**(2/3)*C+b**(2/3)*B*x+b**(2/3)*C*x**2)/(b*x
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*b^(2/3)*x^2 + B*b^(2/3)*x + 2*C*a^(2/3) + B*a^(1/3)*b^(1/3))/(b*x^3
```

```
[Out] Timed out
```

3.41 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=88

$$\frac{2\left(\sqrt[3]{a}(-b)^{2/3}C+bB\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

```
[Out] (2*(b*B + a^(1/3)*(-b)^(2/3)*C)*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b) + (C*Log[a^(1/3) - (-b)^
(1/3)*x])/(-b)^(1/3)
```

Rubi [A] time = 0.185266, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.07

$$\frac{2\left(\sqrt[3]{a}(-b)^{2/3}C+bB\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In]    Int[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-
b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*(b*B + a^(1/3)*(-b)^(2/3)*C)*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b) + (C*Log[a^(1/3) - (-b)^
(1/3)*x])/(-b)^(1/3)
```

Rubi in Sympy [A] time = 27.39, size = 78, normalized size = 0.89

$$\frac{C\log\left(\sqrt[3]{a}-x\sqrt[3]{-b}\right)}{\sqrt[3]{-b}}+\frac{2\sqrt{3}\left(\frac{B}{\sqrt[3]{a}}+\frac{C(-b)^{\frac{2}{3}}}{b}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\frac{2x\sqrt[3]{-b}}{3}\right)}{\sqrt[3]{a}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**(1/3)*(-b)**(1/3)*B-2*a**(2/3)*C-(-b)**(2/3)*B*x-
(-b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(a**(1/3) - x*(-b)**(1/3))/(-b)**(1/3) + 2*sqrt(3)*(B/a**(1/
3) + C*(-b)**(2/3)/b)*atan(sqrt(3)*(a**(1/3)/3 + 2*x*(-b)**(1/3)/
3)/a**(1/3))/3
```

Mathematica [B] time = 1.23865, size = 238, normalized size = 2.7

$$\frac{\frac{\left(2\sqrt[3]{a}b\sqrt[3]{-b}C+b^{5/3}B+(-b)^{5/3}B\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2b\left(2\sqrt[3]{a}\sqrt[3]{-b}C+\left(b^{2/3}-(-b)^{2/3}\right)B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt[3]{a}(-b)^{2/3}\sqrt[3]{-b^2}C\log\left(a+bx^3\right)}{\sqrt[3]{-b^2}}+}{6\sqrt[3]{a}b}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-
b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*(((-b)^(2/3) - (-b^2)^(1/3))*B + 2*a^(1/3)*b^(
1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (-2*b*((-(-
b)^(2/3) + b^(2/3))*B + 2*a^(1/3)*(-b)^(1/3)*C)*Log[a^(1/3) + b^(
1/3)*x] + ((-b)^(5/3)*B + b^(5/3)*B + 2*a^(1/3)*(-b)^(1/3)*b*C)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a^(1/3)*(-b)^(2
/3)*(-b^2)^(1/3)*C*Log[a + b*x^3])/(-b^2)^(1/3))/(6*a^(1/3)*b)
```

Maple [B] time = 0.01, size = 345, normalized size = 3.9

$$
\begin{aligned}
&\frac{B}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\sqrt[3]{a}\sqrt[3]{-b}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&-\frac{B}{6b}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\sqrt[3]{a}\sqrt[3]{-b}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&+\frac{\sqrt{3}B}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\sqrt[3]{a}\sqrt[3]{-b}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{2C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&+\frac{B}{3b}(-b)^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{B}{6b}(-b)^{\frac{2}{3}}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\
&-\frac{\sqrt{3}B}{3b}(-b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{C\ln\left(bx^{3}+a\right)}{3b}(-b)^{\frac{2}{3}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*x^3+
```

```
[Out] 1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^(1/3)*(-b)^(1/3)*B-2/3*C*a^
(2/3)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))*a^(1/3)*(-b)^(1/3)*B+1/3*C*a^(2/3)/b/(a/
b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^(1/3)*(-b)^(1/3)*B-
2/3*C*a^(2/3)/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))+1/3*B*(-b)^(2/3)/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*B
*(-b)^(2/3)/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1/3*B
*(-b)^(2/3)*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))-1/3*C*(-b)^(2/3)/b*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*(-b)^(2/3)*x^2 + B*(-b)^(2/3)*x + 2*C*a^(2/3) - B*a^(1/3)*(-
b)^(1/3))/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*(-b)^(2/3)*x^2 + B*(-b)^(2/3)*x + 2*C*a^(2/3) - B*a^(1/3)*(-
b)^(1/3))/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-2*a**(2/3)*C-(-b)**(2/3)*B*x-(-
b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*(-b)^(2/3)*x^2 + B*(-b)^(2/3)*x + 2*C*a^(2/3) - B*a^(1/3)*(-
b)^(1/3))/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.42 $\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3} \, dx$

Optimal. Leaf size=11

$$\frac{\log(B - Cx)}{C}$$

```
[Out] Log[B - C*x]/C
```

Rubi [A] time = 0.0200754, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$\frac{\log(B - Cx)}{C}$$

Antiderivative was successfully verified.

```
[In]  Int[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]

[Out] Log[B - C*x]/C
```

Rubi in Sympy [A] time = 6.53793, size = 7, normalized size = 0.64

$$\frac{\log (B - Cx)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C**2*x**2+B*C*x+B**2)/(C**3*x**3-B**3),x)

[Out] log(B - C*x)/C
```

Mathematica [A] time = 0.00257394, size = 12, normalized size = 1.09

$$\frac{\log(Cx - B)}{C}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]

[Out] Log[-B + C*x]/C
```

Maple [A] time = 0.003, size = 12, normalized size = 1.1

$$\frac{\ln (-Cx + B)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x)

[Out] ln(-C*x+B)/C
```

Maxima [A] time = 1.3649, size = 16, normalized size = 1.45

$$\frac{\log\left(Cx - B\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2 + B*C*x + B^2)/(C^3*x^3 - B^3),x, algorithm="maxima")
```

```
[Out] log(C*x - B)/C
```

Fricas [A] time = 0.229778, size = 16, normalized size = 1.45

$$\frac{\log\left(Cx - B\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2 + B*C*x + B^2)/(C^3*x^3 - B^3),x, algorithm="fricas")
```

```
[Out] log(C*x - B)/C
```

Sympy [A] time = 0.062702, size = 7, normalized size = 0.64

$$\frac{\log\left(-B + Cx\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C**2*x**2+B*C*x+B**2)/(C**3*x**3-B**3),x)
```

```
[Out] log(-B + C*x)/C
```

GIAC/XCAS [A] time = 0.233571, size = 18, normalized size = 1.64

$$\frac{\ln\left(|Cx - B|\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2 + B*C*x + B^2)/(C^3*x^3 - B^3),x, algorithm="giac")
```

```
[Out] ln(abs(C*x - B))/C
```

3.43 $\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=21

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

```
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi [A] time = 0.0333973, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi in Sympy [A] time = 16.0681, size = 19, normalized size = 0.9

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**(2/3)*C-a**(1/3)*b**(1/3)*C*x+b**(2/3)*C*x**2)/(b*x**3+a),x
```

```
[Out] C*log(a**(1/3) + b**(1/3)*x)/b**(1/3)
```

Mathematica [A] time = 0.00519268, size = 21, normalized size = 1.

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Maple [B] time = 0.007, size = 218, normalized size = 10.4

$$\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{6b}a^{\frac{2}{3}}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3}\sqrt[3]{a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)b^{-\frac{2}{3}}\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{C}{6}\sqrt[3]{a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{-\frac{2}{3}}\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{C\sqrt{3}}{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)b^{-\frac{2}{3}}\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{C\ln\left(bx^3+a\right)}{3}\frac{1}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] 1/3*C*a^(2/3)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*C*a^(2/3)/b/(a/
b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*C*a^(2/3)/b/(a/b)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C*a^(1/
3)/b^(2/3)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*C*a^(1/3)/b^(2/3)/(a
/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1/3*C*a^(1/3)/b^(2/3)
*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*
C/b^(1/3)*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*b^(2/3)*x^2 - C*a^(1/3)*b^(1/3)*x + C*a^(2/3))/(b*x^3 + a),x, algor
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256511, size = 23, normalized size = 1.1

$$\frac{C\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*b^(2/3)*x^2 - C*a^(1/3)*b^(1/3)*x + C*a^(2/3))/(b*x^3 + a),x, algor
```

```
[Out] C*log(b*x + a^(1/3)*b^(2/3))/b^(1/3)
```

Sympy [A] time = 0.623161, size = 20, normalized size = 0.95

$$\frac{C\log\left(\sqrt[3]{a}b^{\frac{2}{3}}+bx\right)}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(2/3)*C-a**(1/3)*b**(1/3)*C*x+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(a**(1/3)*b**(2/3) + b*x)/b**(1/3)
```

GIAC/XCAS [A] time = 0.223282, size = 22, normalized size = 1.05

$$\frac{C\ln\left(\left|b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*b^(2/3)*x^2 - C*a^(1/3)*b^(1/3)*x + C*a^(2/3))/(b*x^3 + a),x, algor
```

```
[Out] C*ln(abs(b^(1/3)*x + a^(1/3)))/b^(1/3)
```

3.44 $\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=71

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(C\sqrt[3]{\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}$$

```
[Out] (-2*(a/b)^(2/3)*(B + (a/b)^(1/3)*C)*ArcTan[(1 - (2*x)/(a/b)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*a) + (C*Log[(a/b)^(1/3) + x])/b
```

Rubi [A] time = 0.158123, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(C\sqrt[3]{\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}$$

Antiderivative was successfully verified.

```
[In]  Int[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B + (a/b)^(1/3)*C)*ArcTan[(1 - (2*x)/(a/b)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*a) + (C*Log[(a/b)^(1/3) + x])/b
```

Rubi in Sympy [A] time = 17.3761, size = 54, normalized size = 0.76

$$\frac{C\log\left(x+\sqrt[3]{\frac{a}{b}}\right)}{b}-\frac{2\sqrt{3}\left(\frac{B}{\sqrt[3]{\frac{a}{b}}}+C\right)\operatorname{atan}\left(\sqrt{3}\left(-\frac{2x}{3\sqrt[3]{\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(x + (a/b)**(1/3))/b - 2*sqrt(3)*(B/(a/b)**(1/3) + C)*atan(s
qrt(3)*(-2*x/(3*(a/b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.634627, size = 247, normalized size = 3.48

$$\frac{\sqrt[3]{b}\left(a^{2/3}B-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}}+B\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}}+B\right)-a^{2/3}B\right)\log\left(\sqrt[3]{a}+\cdots\right.}{6ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B +
2*(a/b)^(1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))] + 2*b^(1/3)*(-(a^(2/3)*B) + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B +
2*(a/b)^(1/3)*C))*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(2/3)*B -
 a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1/3)*C))*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*C*Log[a + b*x^3])/(6*a*b)
```

Maple [A] time = 0.007, size = 121, normalized size = 1.7

$$\frac{2C}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)-\frac{C}{3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)$$
$$+\frac{2B\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{C\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*ln(x+(a/b)^(1/3))/b-1/3*C/b*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3
))+2/3*C/b*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/3*B*
3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3
*C/b*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + 2*C*(a/b)^(2/3) + B*(a/b)^(1/3))/(b*x^3 + a),x, algorith
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + 2*C*(a/b)^(2/3) + B*(a/b)^(1/3))/(b*x^3 + a),x, algorith
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

GIAC/XCAS [A] time = 0.248145, size = 351, normalized size = 4.94

$$-\frac{\left(Cb^{2}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(ab^{2}\right)^{\frac{1}{3}}Bb+2\left(ab^{2}\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{2}}+\frac{\sqrt{3}\left(\left(9\left(-a^{2}b^{4}\right)^{\frac{1}{3}}ab^{2}-27^{\frac{5}{6}}\left(-a^{2}b^{4}\right)^{\frac{5}{6}}\right)B+18\left(\sqrt{3}a^{2}b^{3}i+a^{2}b^{3}\right)C\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{54\,a^{2}b^{4}}+\frac{\left(\left(27\left(-a^{2}b^{4}\right)^{\frac{1}{3}}ab^{2}+27^{\frac{5}{6}}\left(-a^{2}b^{4}\right)^{\frac{5}{6}}\right)B-18\left(\sqrt{3}a^{2}b^{3}i-3\,a^{2}b^{3}\right)C\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{108\,a^{2}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + 2*C*(a/b)^(2/3) + B*(a/b)^(1/3))/(b*x^3 + a),x, algorith
```

```
[Out] -1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) + (a*b^2)^(1/3)*B*b
 + 2*(a*b^2)^(2/3)*C)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b
^2) + 1/54*sqrt(3)*((9*(-a^2*b^4)^(1/3)*a*b^2 - 27^(5/6)*(-a^2*b^
4)^(5/6))*B + 18*(sqrt(3)*a^2*b^3*i + a^2*b^3)*C)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) + 1/108*((27*(-a
^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-a^2*b^4)^(5/6))*B - 18*(sqrt(3)*
a^2*b^3*i - 3*a^2*b^3)*C)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^2*b^4)
```

3.45 $\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=76

$$\frac{2\left(C\sqrt[3]{-\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] (2*(B + (-(a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[
3]])/(Sqrt[3]*(-(a/b))^(1/3)*b) - (C*Log[(-(a/b))^(1/3) + x])/b
```

Rubi [A] time = 0.169493, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089

$$\frac{2\left(C\sqrt[3]{-\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*(B + (-(a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[
3]])/(Sqrt[3]*(-(a/b))^(1/3)*b) - (C*Log[(-(a/b))^(1/3) + x])/b
```

Rubi in Sympy [A] time = 18.5922, size = 60, normalized size = 0.79

$$-\frac{C\log\left(x+\sqrt[3]{-\frac{a}{b}}\right)}{b}+\frac{2\sqrt{3}\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\operatorname{atan}\left(\sqrt{3}\left(-\frac{2x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(((-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(-
b*x**3+a),x)
```

```
[Out] -C*log(x + (-a/b)**(1/3))/b + 2*sqrt(3)*(B/(-a/b)**(1/3) + C)*ata
n(sqrt(3)*(-2*x/(3*(-a/b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.446999, size = 288, normalized size = 3.79

$$-\frac{\left(-a^{2/3}B-\sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}}-2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6ab^{2/3}}-\frac{\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}}+2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right)\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3ab^{2/3}}-\frac{\left(a^{2/3}B-\sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}}-2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}ab^{2/3}}-\frac{C\log\left(a-bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] -(((a^(2/3)*B - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a
/b))^(2/3)*b^(1/3)*C)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a*b^(2/3))) - ((a^(2/3)*B + a^(1/3)*(-(a/b))^(1/
3)*b^(1/3)*B + 2*a^(1/3)*(-(a/b))^(2/3)*b^(1/3)*C)*Log[a^(1/3) -
b^(1/3)*x])/(3*a*b^(2/3)) - ((-(a^(2/3)*B) - a^(1/3)*(-(a/b))^(1/
3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)*b^(1/3)*C)*Log[a^(2/3) +
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(2/3)) - (C*Log[a - b*x^
3])/(3*b)
```

Maple [B] time = 0.01, size = 345, normalized size = 4.5

$$-\frac{2C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3b}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)\sqrt[3]{-\frac{a}{b}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x^2+x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B}{6b}\ln\left(x^2+x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\sqrt[3]{-\frac{a}{b}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2C\sqrt{3}}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}B}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)\sqrt[3]{-\frac{a}{b}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3b}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B}{6b}\ln\left(x^2+x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{\sqrt{3}B}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{C\ln\left(bx^3-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C*(-a/b)^(2/3)/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))-1/3/b/(a/b)^(
2/3)*ln(x-(a/b)^(1/3))*(-a/b)^(1/3)*B+1/3*C*(-a/b)^(2/3)/b/(a/b)^
(2/3)*ln(x^2+x*(a/b)^(1/3)+(a/b)^(2/3))+1/6/b/(a/b)^(2/3)*ln(x^2+
x*(a/b)^(1/3)+(a/b)^(2/3))*(-a/b)^(1/3)*B+2/3*C*(-a/b)^(2/3)/b/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2))+1/3/b/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2))*(-a/b)
^(1/3)*B-1/3*B/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6*B/b/(a/b)^(1/3
)*ln(x^2+x*(a/b)^(1/3)+(a/b)^(2/3))-1/3*B*3^(1/2)/b/(a/b)^(1/3)*a
rctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2))-1/3*C/b*ln(b*x^3-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*x^2 + B*x + 2*C*(-a/b)^(2/3) + B*(-a/b)^(1/3))/(b*x^3 - a),x, algor
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*x^2 + B*x + 2*C*(-a/b)^(2/3) + B*(-a/b)^(1/3))/(b*x^3 - a),x, algor
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

GIAC/XCAS [A] time = 0.247676, size = 344, normalized size = 4.53

$$-\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}Bb+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}+\frac{\sqrt{3}\left(\left(9\left(-a^2b^4\right)^{\frac{1}{3}}ab^2+27^{\frac{5}{6}}\left(-a^2b^4\right)^{\frac{5}{6}}\right)B+18\left(\sqrt{3}a^2b^3i-a^2b^3\right)C\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{54\,a^2b^4}+\frac{\left(\left(27\left(-a^2b^4\right)^{\frac{1}{3}}ab^2-27^{\frac{5}{6}}\left(-a^2b^4\right)^{\frac{5}{6}}\right)B-18\left(\sqrt{3}a^2b^3i+3\,a^2b^3\right)C\right)\ln\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{108\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*x^2 + B*x + 2*C*(-a/b)^(2/3) + B*(-a/b)^(1/3))/(b*x^3 - a),x, algor
```

```
[Out] -1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3) + (-a*b^2)^(1/3)*B*b
+ 2*(-a*b^2)^(2/3)*C)*(a/b)^(1/3)*ln(abs(x - (a/b)^(1/3)))/(a*b^2
) + 1/54*sqrt(3)*((9*(-a^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-a^2*b^4)
^(5/6))*B + 18*(sqrt(3)*a^2*b^3*i - a^2*b^3)*C)*arctan(1/3*sqrt(3
)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4) + 1/108*((27*(-a^2*b
^4)^(1/3)*a*b^2 - 27^(5/6)*(-a^2*b^4)^(5/6))*B - 18*(sqrt(3)*a^2*
b^3*i + 3*a^2*b^3)*C)*ln(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*
b^4)
```

3.46 $\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=78

$$\frac{2\left(B-C\sqrt[3]{-\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[Out] (2*(B - (-(a/b))^(1/3)*C)*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[
3]])/(Sqrt[3]*(-(a/b))^(1/3)*b) + (C*Log[(-(a/b))^(1/3) - x])/b
```

Rubi [A] time = 0.175848, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089

$$\frac{2\left(B-C\sqrt[3]{-\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*(B - (-(a/b))^(1/3)*C)*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[
3]])/(Sqrt[3]*(-(a/b))^(1/3)*b) + (C*Log[(-(a/b))^(1/3) - x])/b
```

Rubi in Sympy [A] time = 19.4657, size = 60, normalized size = 0.77

$$\frac{C\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{b}-\frac{2\sqrt{3}\left(-\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-(-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(x - (-a/b)**(1/3))/b - 2*sqrt(3)*(-B/(-a/b)**(1/3) + C)*ata
n(sqrt(3)*(2*x/(3*(-a/b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.51506, size = 253, normalized size = 3.24

$$\frac{\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\left(B-2C\sqrt[3]{-\frac{a}{b}}\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\left(B-2C\sqrt[3]{-\frac{a}{b}}\right)\right)\log\left(\right.}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (-(a/b))^(1/3)*b^(1/3)*(-
B + 2*(-(a/b))^(1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))] - 2*b^(1/3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3
)*(B - 2*(-(a/b))^(1/3)*C))*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a
^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*(B - 2*(-(a/b))^(1/3)*C
))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*C*Log[a +
 b*x^3])/(6*a*b)
```

Maple [B] time = 0.007, size = 340, normalized size = 4.4

$$\frac{2C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\sqrt[3]{-\frac{a}{b}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\sqrt[3]{-\frac{a}{b}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{2C\sqrt{3}}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{\sqrt{3}B}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\sqrt[3]{-\frac{a}{b}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{B}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\sqrt{3}B}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{C\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*(-a/b)^(2/3)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/3/b/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))*(-a/b)^(1/3)*B-1/3*C*(-a/b)^(2/3)/b/(a/b)^(
2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/6/b/(a/b)^(2/3)*ln(x^2-x
*(a/b)^(1/3)+(a/b)^(2/3))*(-a/b)^(1/3)*B+2/3*C*(-a/b)^(2/3)/b/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/b/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*(-a/b)^
(1/3)*B-1/3*B/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*B/b/(a/b)^(1/3)
*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*B*3^(1/2)/b/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + 2*C*(-a/b)^(2/3) - B*(-a/b)^(1/3))/(b*x^3 + a),x, algori
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + 2*C*(-a/b)^(2/3) - B*(-a/b)^(1/3))/(b*x^3 + a),x, algori
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

GIAC/XCAS [A] time = 0.223189, size = 180, normalized size = 2.31

$$-\frac{2\sqrt{3}\left(Cab+\left(-ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}Bb+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + 2*C*(-a/b)^(2/3) - B*(-a/b)^(1/3))/(b*x^3 + a),x, algori
```

```
[Out] -2/3*sqrt(3)*(C*a*b + (-a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(-a/b)^(2/3) +
B*b^2*(-a/b)^(1/3) - (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(-a
/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

3.47 $\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=75

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-C\sqrt[3]{\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] (-2*(a/b)^(2/3)*(B - (a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(a/b)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*a) - (C*Log[(a/b)^(1/3) - x])/b
```

Rubi [A] time = 0.174261, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-C\sqrt[3]{\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B - (a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(a/b)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*a) - (C*Log[(a/b)^(1/3) - x])/b
```

Rubi in Sympy [A] time = 20.804, size = 54, normalized size = 0.72

$$-\frac{C\log\left(x-\sqrt[3]{\frac{a}{b}}\right)}{b}+\frac{2\sqrt{3}\left(-\frac{B}{\sqrt[3]{\frac{a}{b}}}+C\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3\sqrt[3]{\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-(a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*log(x - (a/b)**(1/3))/b + 2*sqrt(3)*(-B/(a/b)**(1/3) + C)*atan
(sqrt(3)*(2*x/(3*(a/b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.608016, size = 244, normalized size = 3.25

$$\frac{\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}}-B\right)\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}}-B\right)\right)\log\left(\sqrt[3]{a}-\right.}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B -
 2*(a/b)^(1/3)*C))*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] -
2*b^(1/3)*(a^(2/3)*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^
(1/3)*C))*Log[a^(1/3) - b^(1/3)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)
*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)*C))*Log[a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*C*Log[a - b*x^3])/(6*a*b)
```

Maple [A] time = 0.008, size = 124, normalized size = 1.7

$$-\frac{2C}{3b}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)+\frac{C}{3b}\ln\left(x^2+x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)-\frac{2B\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{C\ln\left(bx^3-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C/b*ln(x-(a/b)^(1/3))+1/3*C/b*ln(x^2+x*(a/b)^(1/3)+(a/b)^(2/
3))+2/3*C*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2))/b*3^(1/2)-2/3*B
*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2))-1/
3*C/b*ln(b*x^3-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*x^2 + B*x + 2*C*(a/b)^(2/3) - B*(a/b)^(1/3))/(b*x^3 - a),x, algorit
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*x^2 + B*x + 2*C*(a/b)^(2/3) - B*(a/b)^(1/3))/(b*x^3 - a),x, algorit
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

GIAC/XCAS [A] time = 0.223387, size = 169, normalized size = 2.25

$$\frac{2\sqrt{3}\left(Cab-\left(ab^{2}\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{2}}-\frac{\left(Cb^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}+Bb^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}-\left(ab^{2}\right)^{\frac{1}{3}}Bb+2\left(ab^{2}\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*x^2 + B*x + 2*C*(a/b)^(2/3) - B*(a/b)^(1/3))/(b*x^3 - a),x, algorit
```

```
[Out] 2/3*sqrt(3)*(C*a*b - (a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (
a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2
*(a/b)^(1/3) - (a*b^2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)
*ln(abs(x - (a/b)^(1/3)))/(a*b^2)
```

3.48 $\int \frac{a+ax+cx^2}{1-x^3}\,dx$

Optimal. Leaf size=32

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

```
[Out] -((2*a + c)*Log[1 - x])/3 + ((a - c)*Log[1 + x + x^2])/3
```

Rubi [A] time = 0.060001, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + a*x + c*x^2)/(1 - x^3),x]

[Out] -((2*a + c)*Log[1 - x])/3 + ((a - c)*Log[1 + x + x^2])/3
```

Rubi in Sympy [A] time = 11.8263, size = 27, normalized size = 0.84

$$\left(\frac{a}{3}-\frac{c}{3}\right)\log\left(x^2+x+1\right)-\left(\frac{2a}{3}+\frac{c}{3}\right)\log\left(-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a*x+a)/(-x**3+1),x)

[Out] (a/3 - c/3)*log(x**2 + x + 1) - (2*a/3 + c/3)*log(-x + 1)
```

Mathematica [A] time = 0.0234506, size = 31, normalized size = 0.97

$$\frac{1}{3}\left((a-c)\log\left(x^2+x+1\right)-(2a+c)\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + a*x + c*x^2)/(1 - x^3),x]

[Out] (-((2*a + c)*Log[1 - x]) + (a - c)*Log[1 + x + x^2])/3
```

Maple [A] time = 0.009, size = 36, normalized size = 1.1

$$\frac{\ln\left(x^2+x+1\right)a}{3}-\frac{\ln\left(x^2+x+1\right)c}{3}-\frac{\ln\left(-1+x\right)c}{3}-\frac{2\ln\left(-1+x\right)a}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a*x+a)/(-x^3+1),x)

[Out] 1/3*ln(x^2+x+1)*a-1/3*ln(x^2+x+1)*c-1/3*ln(-1+x)*c-2/3*ln(-1+x)*a
```

Maxima [A] time = 1.57875, size = 35, normalized size = 1.09

$$\frac{1}{3}\left(a-c\right)\log\left(x^2+x+1\right)-\frac{1}{3}\left(2\,a+c\right)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + a*x + a)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Fricas [A] time = 0.21926, size = 35, normalized size = 1.09

$$\frac{1}{3}\left(a-c\right)\log\left(x^2+x+1\right)-\frac{1}{3}\left(2\,a+c\right)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + a*x + a)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Sympy [A] time = 0.496307, size = 24, normalized size = 0.75

$$\frac{(a-c)\log\left(x^2+x+1\right)}{3}-\frac{(2a+c)\log\left(x-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a*x+a)/(-x**3+1),x)
```

```
[Out] (a - c)*log(x**2 + x + 1)/3 - (2*a + c)*log(x - 1)/3
```

GIAC/XCAS [A] time = 0.219611, size = 36, normalized size = 1.12

$$\frac{1}{3}\left(a-c\right)\ln\left(x^2+x+1\right)-\frac{1}{3}\left(2\,a+c\right)\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + a*x + a)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] 1/3*(a - c)*ln(x^2 + x + 1) - 1/3*(2*a + c)*ln(abs(x - 1))
```

3.49 $\int \frac{a+bx+cx^2}{1-x^3}\,dx$

Optimal. Leaf size=55

$$\frac{1}{6}\log\left(x^2+x+1\right)(a+b-2c)-\frac{1}{3}\log(1-x)(a+b+c)+\frac{(a-b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ((a - b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a + b + c)*Log[1
- x])/3 + ((a + b - 2*c)*Log[1 + x + x^2])/6
```

Rubi [A] time = 0.112597, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{1}{6}\log\left(x^2+x+1\right)(a+b-2c)-\frac{1}{3}\log(1-x)(a+b+c)+\frac{(a-b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)/(1 - x^3),x]
```

```
[Out] ((a - b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a + b + c)*Log[1
- x])/3 + ((a + b - 2*c)*Log[1 + x + x^2])/6
```

Rubi in Sympy [A] time = 16.7158, size = 58, normalized size = 1.05

$$\frac{\sqrt{3}\,(a-b)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}+\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}\right)\log\left(x^2+x+1\right)-\left(\frac{a}{3}+\frac{b}{3}+\frac{c}{3}\right)\log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)/(-x**3+1),x)
```

```
[Out] sqrt(3)*(a - b)*atan(sqrt(3)*(2*x/3 + 1/3))/3 + (a/6 + b/6 - c/3)
*log(x**2 + x + 1) - (a/3 + b/3 + c/3)*log(-x + 1)
```

Mathematica [A] time = 0.0619036, size = 62, normalized size = 1.13

$$\frac{1}{6}\left((a+b)\log\left(x^2+x+1\right)-2(a+b)\log(1-x)+2\sqrt{3}(a-b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2c\log\left(1-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)/(1 - x^3),x]
```

```
[Out] (2*Sqrt[3]*(a - b)*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*(a + b)*Log[1 -
x] + (a + b)*Log[1 + x + x^2] - 2*c*Log[1 - x^3])/6
```

Maple [A] time = 0.007, size = 87, normalized size = 1.6

$$\frac{\ln\left(x^2+x+1\right)a}{6}+\frac{\ln\left(x^2+x+1\right)b}{6}-\frac{\ln\left(x^2+x+1\right)c}{3}+\frac{a\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}b}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln(-1+x)c}{3}-\frac{\ln(-1+x)b}{3}-\frac{\ln(-1+x)a}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)/(-x^3+1),x)
```

```
[Out] 1/6*ln(x^2+x+1)*a+1/6*ln(x^2+x+1)*b-1/3*ln(x^2+x+1)*c+1/3*3^(1/2)
*arctan(1/3*(1+2*x)*3^(1/2))*a-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(
1/2))*b-1/3*ln(-1+x)*c-1/3*ln(-1+x)*b-1/3*ln(-1+x)*a
```

Maxima [A] time = 1.55429, size = 63, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right)-\frac{1}{3}(a+b+c)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b -
2*c)*log(x^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

Fricas [A] time = 0.252444, size = 73, normalized size = 1.33

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}(a+b-2c)\log\left(x^2+x+1\right)-2\sqrt{3}(a+b+c)\log(x-1)+6(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*(a + b - 2*c)*log(x^2 + x + 1) - 2*sqrt(3)*
(a + b + c)*log(x - 1) + 6*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 1.26524, size = 323, normalized size = 5.87

$$-\frac{(a+b+c)\log\left(x+\frac{a^2c-a^2(a+b+c)-2ab^2+bc^2-2bc(a+b+c)+b(a+b+c)^2}{a^3-b^3}\right)}{3}-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right.$$
$$\left.-\frac{\sqrt{3}i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}i(a-b)}{6}\right)+9b\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right.}{a^3-b^3}\right.$$
$$-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right.$$
$$\left.+\frac{\sqrt{3}i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}i(a-b)}{6}\right)+9b\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\right.}{a^3-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)/(-x**3+1),x)
```

```
[Out] -(a + b + c)*log(x + (a**2*c - a**2*(a + b + c) - 2*a*b**2 + b*c*
*2 - 2*b*c*(a + b + c) + b*(a + b + c)**2)/(a**3 - b**3))/3 - (-a
/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-
a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*
c*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/6) + 9*b*(-a/6 - b/6 + c/
3 - sqrt(3)*I*(a - b)/6)**2)/(a**3 - b**3)) - (-a/6 - b/6 + c/3 +
```

```
 sqrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 + c/3
+ sqrt(3)*I*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 +
c/3 + sqrt(3)*I*(a - b)/6) + 9*b*(-a/6 - b/6 + c/3 + sqrt(3)*I*(a
 - b)/6)**2)/(a**3 - b**3))
```

GIAC/XCAS [A] time = 0.218793, size = 70, normalized size = 1.27

$$\frac{1}{3}\left(\sqrt{3}a - \sqrt{3}b\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + \frac{1}{6}(a + b - 2c)\ln\left(x^2 + x + 1\right) - \frac{1}{3}(a + b + c)\ln\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(3)*a - sqrt(3)*b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(
a + b - 2*c)*ln(x^2 + x + 1) - 1/3*(a + b + c)*ln(abs(x - 1))
```

3.50 $\int \frac{1+x+x^2}{1-x^3} \, dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -Log[1 - x]
```

Rubi [A] time = 0.0117235, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2)/(1 - x^3),x]

[Out] -Log[1 - x]
```

Rubi in Sympy [A] time = 3.08812, size = 5, normalized size = 0.62

$$-\log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x+1)/(-x**3+1),x)

[Out] -log(-x + 1)
```

Mathematica [A] time = 0.00173559, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2)/(1 - x^3),x]

[Out] -Log[1 - x]
```

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x+1)/(-x^3+1),x)

[Out] -ln(-1+x)
```

Maxima [A] time = 1.36511, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + x + 1)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

Fricas [A] time = 0.219915, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + x + 1)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A] time = 0.045601, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x+1)/(-x**3+1),x)
```

```
[Out] -log(x - 1)
```

GIAC/XCAS [A] time = 0.217015, size = 9, normalized size = 1.12

$$-\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + x + 1)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] -ln(abs(x - 1))
```

3.51 $\int \frac{1-x+3x^2}{1-x^3}\,dx$

Optimal. Leaf size=30

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Rubi [A] time = 0.0569621, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x + 3*x^2)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Rubi in Sympy [A] time = 12.4202, size = 34, normalized size = 1.13

$$-\log\left((x-1)\left(x^2+x+1\right)\right) + \frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2-x+1)/(-x**3+1),x)
```

```
[Out] -log((x - 1)*(x**2 + x + 1)) + 2*sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/
3))/3
```

Mathematica [A] time = 0.0153144, size = 30, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x + 3*x^2)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Maple [A] time = 0.006, size = 33, normalized size = 1.1

$$-\ln\left(x^2+x+1\right) + \frac{2\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) - \ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2-x+1)/(-x^3+1),x)
```

```
[Out] -ln(x^2+x+1)+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-ln(-1+x)
```

Maxima [A] time = 1.51984, size = 43, normalized size = 1.43

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\log\left(x^2+x+1\right)-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 - x + 1)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - lo
g(x - 1)
```

Fricas [A] time = 0.227213, size = 51, normalized size = 1.7

$$-\frac{1}{3}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)+\sqrt{3}\log\left(x-1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 - x + 1)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) + sqrt(3)*log(x - 1) - 2*a
rctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.146447, size = 5, normalized size = 0.17

$$-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2-x+1)/(-x**3+1),x)
```

```
[Out] -log(x - 1)
```

GIAC/XCAS [A] time = 0.217373, size = 45, normalized size = 1.5

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\ln\left(x^2+x+1\right)-\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^2 - x + 1)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - ln(x^2 + x + 1) - ln(
abs(x - 1))
```

3.52 $\int \frac{1+x+4x^2}{1-x^3} \, dx$

Optimal. Leaf size=18

$$-\log \left(x^2+x+1\right)-2 \log (1-x)$$

```
[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Rubi [A] time = 0.0329192, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\log \left(x^2+x+1\right)-2 \log (1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + 4*x^2)/(1 - x^3),x]

[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Rubi in Sympy [A] time = 10.0078, size = 15, normalized size = 0.83

$$-2 \log (-x+1)-\log \left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2+x+1)/(-x**3+1),x)

[Out] -2*log(-x + 1) - log(x**2 + x + 1)
```

Mathematica [A] time = 0.00953197, size = 18, normalized size = 1.

$$-\log \left(x^2+x+1\right)-2 \log (1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + 4*x^2)/(1 - x^3),x]

[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Maple [A] time = 0.007, size = 17, normalized size = 0.9

$$-\ln \left(x^2+x+1\right)-2 \ln (-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2+x+1)/(-x^3+1),x)

[Out] -ln(x^2+x+1)-2*ln(-1+x)
```

Maxima [A] time = 1.57186, size = 22, normalized size = 1.22

$$-\log \left(x^2+x+1\right)-2 \log (x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x^2 + x + 1)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

Fricas [A] time = 0.22663, size = 22, normalized size = 1.22

$$-\log\left(x^2 + x + 1\right) - 2\,\log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x^2 + x + 1)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

Sympy [A] time = 0.105992, size = 15, normalized size = 0.83

$$-2\log\left(x - 1\right) - \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+x+1)/(-x**3+1),x)
```

```
[Out] -2*log(x - 1) - log(x**2 + x + 1)
```

GIAC/XCAS [A] time = 0.218783, size = 23, normalized size = 1.28

$$-\ln\left(x^2 + x + 1\right) - 2\ln\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x^2 + x + 1)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] -ln(x^2 + x + 1) - 2*ln(abs(x - 1))
```

3.53 $\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right)\,dx$

Optimal. Leaf size=113

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^
2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*
b^3*d*x^11)/11 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14
```

Rubi [A] time = 0.174886, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^
2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*
b^3*d*x^11)/11 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4d\int x\,dx+a^4\int c\,dx+a^3bcx^4+\frac{4a^3bdx^5}{5}+\frac{6a^2b^2cx^7}{7}+\frac{3a^2b^2dx^8}{4}+\frac{2ab^3cx^{10}}{5}+\frac{4ab^3dx^{11}}{11}+\frac{b^4cx^{13}}{13}+\frac{b^4dx^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**3*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**4*d*Integral(x, x) + a**4*Integral(c, x) + a**3*b*c*x**4 + 4*a
**3*b*d*x**5/5 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*
a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11 + b**4*c*x**13/13 + b**4*d
*x**14/14
```

Mathematica [A] time = 0.00787382, size = 113, normalized size = 1.

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^
2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*
b^3*d*x^11)/11 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14
```

Maple [A] time = 0.003, size = 98, normalized size = 0.9

$$a^4cx+\frac{a^4dx^2}{2}+a^3bcx^4+\frac{4\,a^3bdx^5}{5}+\frac{6\,a^2b^2cx^7}{7}+\frac{3\,a^2b^2dx^8}{4}+\frac{2\,ab^3cx^{10}}{5}+\frac{4\,ab^3dx^{11}}{11}+\frac{b^4cx^{13}}{13}+\frac{b^4dx^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x
^7+3/4*a^2*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*
c*x^13+1/14*b^4*d*x^14
```

Maxima [A] time = 1.42368, size = 131, normalized size = 1.16

$$\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3
*c*x^10 + 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5
 + a^3*b*c*x^4 + 1/2*a^4*d*x^2 + a^4*c*x
```

Fricas [A] time = 0.206957, size = 1, normalized size = 0.01

$$\frac{1}{14}x^{14}db^4+\frac{1}{13}x^{13}cb^4+\frac{4}{11}x^{11}db^3a+\frac{2}{5}x^{10}cb^3a+\frac{3}{4}x^8db^2a^2+\frac{6}{7}x^7cb^2a^2+\frac{4}{5}x^5dba^3+x^4cba^3+\frac{1}{2}x^2da^4+xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d*b^4 + 1/13*x^13*c*b^4 + 4/11*x^11*d*b^3*a + 2/5*x^10*
c*b^3*a + 3/4*x^8*d*b^2*a^2 + 6/7*x^7*c*b^2*a^2 + 4/5*x^5*d*b*a^3
 + x^4*c*b*a^3 + 1/2*x^2*d*a^4 + x*c*a^4
```

Sympy [A] time = 0.082327, size = 117, normalized size = 1.04

$$a^4cx+\frac{a^4dx^2}{2}+a^3bcx^4+\frac{4a^3bdx^5}{5}+\frac{6a^2b^2cx^7}{7}+\frac{3a^2b^2dx^8}{4}+\frac{2ab^3cx^{10}}{5}+\frac{4ab^3dx^{11}}{11}+\frac{b^4cx^{13}}{13}+\frac{b^4dx^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**3*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/5 + 6*
a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a*b**3*c*x**10/5 +
4*a*b**3*d*x**11/11 + b**4*c*x**13/13 + b**4*d*x**14/14
```

GIAC/XCAS [A] time = 0.214945, size = 131, normalized size = 1.16

$$\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3
*c*x^10 + 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5
 + a^3*b*c*x^4 + 1/2*a^4*d*x^2 + a^4*c*x
```

3.54 $\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right) \, dx$

Optimal. Leaf size=88

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 +
 (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d
*x^11)/11
```

Rubi [A] time = 0.112911, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 +
 (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d
*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3d\int x\,dx+a^3\int c\,dx+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**3+a)**2*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**3*d*Integral(x, x) + a**3*Integral(c, x) + 3*a**2*b*c*x**4/4 +
 3*a**2*b*d*x**5/5 + 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3
*c*x**10/10 + b**3*d*x**11/11
```

Mathematica [A] time = 0.00537283, size = 88, normalized size = 1.

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 +
 (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d
*x^11)/11
```

Maple [A] time = 0., size = 75, normalized size = 0.9

$$a^3cx+\frac{a^3dx^2}{2}+\frac{3\,a^2bcx^4}{4}+\frac{3\,a^2bdx^5}{5}+\frac{3\,ab^2cx^7}{7}+\frac{3\,ab^2dx^8}{8}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c
*x^7+3/8*a*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

Maxima [A] time = 1.38181, size = 100, normalized size = 1.14

$$\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c
*x^7 + 3/5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*
x
```

Fricas [A] time = 0.192989, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}db^3+\frac{1}{10}x^{10}cb^3+\frac{3}{8}x^8db^2a+\frac{3}{7}x^7cb^2a+\frac{3}{5}x^5dba^2+\frac{3}{4}x^4cba^2+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d*b^3 + 1/10*x^10*c*b^3 + 3/8*x^8*d*b^2*a + 3/7*x^7*c*b
^2*a + 3/5*x^5*d*b*a^2 + 3/4*x^4*c*b*a^2 + 1/2*x^2*d*a^3 + x*c*a^
3
```

Sympy [A] time = 0.068102, size = 90, normalized size = 1.02

$$a^3cx+\frac{a^3dx^2}{2}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x**5/5
+ 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3*c*x**10/10 + b**3*
d*x**11/11
```

GIAC/XCAS [A] time = 0.215463, size = 100, normalized size = 1.14

$$\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c
*x^7 + 3/5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*
x
```

3.55 $\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx$

Optimal. Leaf size=60

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*
c*x^7)/7 + (b^2*d*x^8)/8
```

Rubi [A] time = 0.0713776, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*
c*x^7)/7 + (b^2*d*x^8)/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2d\int x\,dx+a^2\int c\,dx+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**2*d*Integral(x, x) + a**2*Integral(c, x) + a*b*c*x**4/2 + 2*a*
b*d*x**5/5 + b**2*c*x**7/7 + b**2*d*x**8/8
```

Mathematica [A] time = 0.00360621, size = 60, normalized size = 1.

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*
c*x^7)/7 + (b^2*d*x^8)/8
```

Maple [A] time = 0.003, size = 51, normalized size = 0.9

$$a^2cx+\frac{a^2dx^2}{2}+\frac{abcx^4}{2}+\frac{2\,abdx^5}{5}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1
/8*b^2*d*x^8
```

Maxima [A] time = 1.44795, size = 68, normalized size = 1.13

$$\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1
/2*a^2*d*x^2 + a^2*c*x
```

Fricas [A] time = 0.192193, size = 1, normalized size = 0.02

$$\frac{1}{8}x^8db^2+\frac{1}{7}x^7cb^2+\frac{2}{5}x^5dba+\frac{1}{2}x^4cba+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d*b^2 + 1/7*x^7*c*b^2 + 2/5*x^5*d*b*a + 1/2*x^4*c*b*a + 1
/2*x^2*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.055978, size = 58, normalized size = 0.97

$$a^2cx+\frac{a^2dx^2}{2}+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 + b**2*c
*x**7/7 + b**2*d*x**8/8
```

GIAC/XCAS [A] time = 0.21294, size = 68, normalized size = 1.13

$$\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1
/2*a^2*d*x^2 + a^2*c*x
```

3.56 $\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx$

Optimal. Leaf size=12

$$cx + \frac{dx^2}{2}$$

```
[Out] c*x + (d*x^2)/2
```

Rubi [A] time = 0.0165543, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]

[Out] c*x + (d*x^2)/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\int x\,dx + \int c\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a),x)

[Out] d*Integral(x, x) + Integral(c, x)
```

Mathematica [A] time = 0.00143704, size = 12, normalized size = 1.

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]

[Out] c*x + (d*x^2)/2
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x)

[Out] c*x+1/2*d*x^2
```

Maxima [A] time = 1.40989, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A] time = 0.216175, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + c*x
```

Sympy [A] time = 0.059286, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a),x)
```

```
[Out] c*x + d*x**2/2
```

GIAC/XCAS [A] time = 0.216064, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x
```

3.57 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2} dx$

Optimal. Leaf size=161

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*
d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(1/3))
```

Rubi [A] time = 0.232199, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*
d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(1/3))
```

Rubi in Sympy [A] time = 38.0195, size = 150, normalized size = 0.93

$$-\frac{\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}+\frac{\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**2,x)
```

```
[Out] -(a**(1/3)*d - b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)
*b**(2/3)) + (a**(1/3)*d - b**(1/3)*c)*log(a**(2/3) - a**(1/3)*b*
*(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*b**(2/3)) - sqrt(3)*(a**(1/
3)*d + b**(1/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**
(1/3))/(3*a**(2/3)*b**(2/3))
```

Mathematica [A] time = 0.103558, size = 124, normalized size = 0.77

$$\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)
*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)
*b^(2/3))
```

Maple [A] time = 0.002, size = 186, normalized size = 1.2

$$\frac{c}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} +\frac{c\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} +\frac{d}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*c/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*c/b/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*c/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*d/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))+1/6*d/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*d*3^
(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.06377, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3a^2b^2 + 9tabcd + ad^3 - bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd + 3tabc^2 + 2acd^2}{ad^3 + bc^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lamb
da(_t, _t*log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)
/(a*d**3 + b*c**3))))
```

GIAC/XCAS [A] time = 0.222472, size = 216, normalized size = 1.34

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(d*(-a/b)^(1/3) + c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/
a + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b
^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*ln(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.58 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcT
an[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/
(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3))
```

Rubi [A] time = 0.258876, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcT
an[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/
(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3))
```

Rubi in Sympy [A] time = 50.3101, size = 172, normalized size = 0.91

$$\frac{x\left(c+dx\right)}{3a\left(a+bx^3\right)}+\frac{\left(\frac{\sqrt[3]{a}d}{2}-\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**3,x)
```

```
[Out] x*(c + d*x)/(3*a*(a + b*x**3)) + (a**(1/3)*d/2 - b**(1/3)*c)*log(
a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(9*a**(5/3)*b**(2
/3)) - (a**(1/3)*d - 2*b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x)/(9*
a**(5/3)*b**(2/3)) - sqrt(3)*(a**(1/3)*d + 2*b**(1/3)*c)*atan(sqr
t(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*b**(2/3)
)
```

Mathematica [A] time = 0.32869, size = 180, normalized size = 0.95

$$\frac{\frac{\left(a^{2/3}d-2\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6ax(c+dx)}{a+bx^3}}{18a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c
+ a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3)
 + (2*(2*a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])
/b^(2/3) + ((-2*a^(1/3)*b^(1/3)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^2)
```

Maple [A] time = 0.004, size = 238, normalized size = 1.3

$$\frac{cx}{3\,a\,(bx^3+a)}+\frac{2\,c}{9\,ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{9\,ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{2\,c\sqrt{3}}{9\,ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{dx^2}{3\,a\,(bx^3+a)}-\frac{d}{9\,ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{d}{18\,ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d\sqrt{3}}{9\,ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x)
```

```
[Out] 1/3*c*x/a/(b*x^3+a)+2/9*c/a/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9*c
/a/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9*c/a/b/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d*x^2
/a/(b*x^3+a)-1/9*d/a/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18*d/a/b/(
a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/9*d/a*3^(1/2)/b/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.88176, size = 105, normalized size = 0.56

$$\operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**
3, Lambda(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 +
 4*a*c*d**2)/(a*d**3 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*
a*b*x**3)
```

GIAC/XCAS [A] time = 0.22452, size = 252, normalized size = 1.33

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2 + cx}{3\,(bx^3 + a)a} + \frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(d*(-a/b)^(1/3) + 2*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3))
)/a^2 + 1/3*(d*x^2 + c*x)/((b*x^3 + a)*a) + 1/9*sqrt(3)*(2*(-a*b^
2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/18*(2*(-a*b^2)^(1/3)*a*b^3*c
+ (-a*b^2)^(2/3)*a*b^2*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^3*b^4)
```

3.59 $\int \left(a+bx^3\right)^{3/2}\left(ac+adx+bcx^3+bdx^4\right)\,dx$

Optimal. Leaf size=585

$$-\frac{405\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{10/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{1729b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{54a^2\sqrt{a+bx^3}\left(1729cx+935dx^2\right)}{323323}$$
$$+\frac{54\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(1729\sqrt[3]{b}c-935\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{323323b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{30a\left(a+bx^3\right)^{3/2}\left(247cx+187dx^2\right)}{46189}+\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

```
[Out] (810*a^3*d*Sqrt[a + b*x^3])/(1729*b^(2/3)*((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)) + (54*a^2*(1729*c*x + 935*d*x^2)*Sqrt[a + b*x^3])/3
23323 + (30*a*(247*c*x + 187*d*x^2)*(a + b*x^3)^(3/2))/46189 + (2
*(19*c*x + 17*d*x^2)*(a + b*x^3)^(5/2))/323 - (405*3^(1/4)*Sqrt[2
 - Sqrt[3]]*a^(10/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*
b^(1/3)*c - 935*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
323323*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.12499, antiderivative size = 630, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$-\frac{405\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{10/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{1729b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{54}{935}a^2cx\sqrt{a+bx^3}+\frac{54a^2dx^2\sqrt{a+bx^3}}{1729}$$
$$+\frac{54\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(1729\sqrt[3]{b}c-935\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{323323b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{18a\sqrt{a+bx^3}\left(29393acx+17765adx^2+8645bcx^4+6545bdx^5\right)}{1616615}$$
$$+\frac{2\left(a+bx^3\right)^{3/2}\left(4199acx+3553adx^2+2717bcx^4+2431bdx^5\right)}{46189}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (54*a^2*c*x*Sqrt[a + b*x^3])/935 + (54*a^2*d*x^2*Sqrt[a + b*x^3])
/1729 + (810*a^3*d*Sqrt[a + b*x^3])/(1729*b^(2/3)*((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)) + (2*(a + b*x^3)^(3/2)*(4199*a*c*x + 3553*a
*d*x^2 + 2717*b*c*x^4 + 2431*b*d*x^5))/46189 + (18*a*Sqrt[a + b*x
^3]*(29393*a*c*x + 17765*a*d*x^2 + 8645*b*c*x^4 + 6545*b*d*x^5))/
1616615 - (405*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*d*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(1729*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (54*3^(3/4)
*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*c - 935*(1 - Sqrt[3])*a^(1/3
)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(323323*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b
*x^3])
```

Rubi in Sympy [A] time = 155.638, size = 592, normalized size = 1.01

$$-\frac{405\sqrt[4]{3}a^{\frac{10}{3}}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{1729b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{810a^3d\sqrt{a+bx^3}}{1729b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$
$$+\frac{54\cdot 3^{\frac{3}{4}}a^3\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-935\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+1729\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\right.}{323323b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{54a^2cx\sqrt{a+bx^3}}{935}+\frac{54a^2dx^2\sqrt{a+bx^3}}{1729}+\frac{9a\sqrt{a+bx^3}\left(\frac{4acx}{55}+\frac{4adx^2}{91}+\frac{4bcx^4}{187}+\frac{4bdx^5}{247}\right)}{2}$$
$$+\left(a+bx^3\right)^{\frac{3}{2}}\left(\frac{2acx}{11}+\frac{2adx^2}{13}+\frac{2bcx^4}{17}+\frac{2bdx^5}{19}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] -405*3**(1/4)*a**(10/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x +
b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sq
rt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1
 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)),
 -7 - 4*sqrt(3))/(1729*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3
)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
+ 810*a**3*d*sqrt(a + b*x**3)/(1729*b**(2/3)*(a**(1/3)*(1 + sqrt(
3)) + b**(1/3)*x)) + 54*3**(3/4)*a**3*sqrt((a**(2/3) - a**(1/3)*b
**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)*
*2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-935*a**(1/3)*d*(-
sqrt(3) + 1) + 1729*b**(1/3)*c)*elliptic_f(asin((-a**(1/3)*(-1 +
sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
 - 4*sqrt(3))/(323323*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)
*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) +
 54*a**2*c*x*sqrt(a + b*x**3)/935 + 54*a**2*d*x**2*sqrt(a + b*x**
```

```
3)/1729 + 9*a*sqrt(a + b*x**3)*(4*a*c*x/55 + 4*a*d*x**2/91 + 4*b*
c*x**4/187 + 4*b*d*x**5/247)/2 + (a + b*x**3)**(3/2)*(2*a*c*x/11
+ 2*a*d*x**2/13 + 2*b*c*x**4/17 + 2*b*d*x**5/19)
```

Mathematica [C] time = 1.32459, size = 349, normalized size = 0.6

$$54i3^{3/4}a^{10/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(935\sqrt[3]{a}d+1729\sqrt[3]{-b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-1514$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (2*(-b)^(2/3)*x*(a + b*x^3)*(1001*b^2*x^6*(19*c + 17*d*x) + 7*a*b
*x^3*(9139*c + 7667*d*x) + a^2*(91637*c + 61897*d*x)) - 151470*(-
1)^(2/3)*3^(1/4)*a^(11/3)*d*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/
a^(1/3))]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2
/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)
]/3^(1/4)], (-1)^(1/3)] + (54*I)*3^(3/4)*a^(10/3)*(1729*(-b)^(1/3
)*c + 935*a^(1/3)*d)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/
a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/
3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]
/3^(1/4)], (-1)^(1/3)])/(323323*(-b)^(2/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.007, size = 1618, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a*c*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I
*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+
b*d*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1
729/b*a^2*x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2)*(-a*b^2)^
(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+a*d*
(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
```

```
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2))))+b*c*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b
*x^3+a)^(1/2)+54/935/b*a^2*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bdx^4 + bcx^3 + adx + ac\right)\left(bx^3 + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2),x, algorithm="maxim
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2dx^7 + b^2cx^6 + 2\,abdx^4 + 2\,abcx^3 + a^2dx + a^2c\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2),x, algorithm="frica
```

```
[Out] integral((b^2*d*x^7 + b^2*c*x^6 + 2*a*b*d*x^4 + 2*a*b*c*x^3 + a^2
*d*x + a^2*c)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 6.50644, size = 265, normalized size = 0.45

$$\frac{a^{\frac{5}{2}}cx\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{5}{2}}dx^2\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{5}{3}\right)} + \frac{2a^{\frac{3}{2}}bcx^4\,\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{3}{2}}bdx^5\,\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}b^2cx^7\,\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}b^2dx^8\,\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**(5/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(4/3)) + a**(5/2)*d*x**2*gamma(2/3)*hyper((-1
/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*
*(3/2)*b*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(3/2)*b*d*x**5*gamma(5/3)*hy
per((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
 + sqrt(a)*b**2*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b**2*d*x**8*gamm
a(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(11/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bdx^4 + bcx^3 + adx + ac\right)\left(bx^3 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2),x, algorithm="giac"
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

3.60 $\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx$

Optimal. Leaf size=556

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(91\sqrt[3]{b}c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{18a\sqrt{a+bx^3}\left(91cx+55dx^2\right)}{5005}+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

```
[Out] (54*a^2*d*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)) + (18*a*(91*c*x + 55*d*x^2)*Sqrt[a + b*x^3])/5005 + (2
*(13*c*x + 11*d*x^2)*(a + b*x^3)^(3/2))/143 - (27*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(7/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(91*b^(1/3
)*c - 55*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.861246, antiderivative size = 583, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(91\sqrt[3]{b}c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}\left(1001acx+715adx^2+455bcx^4+385bdx^5\right)}{5005}+\frac{6}{55}acx\sqrt{a+bx^3}+\frac{6}{91}adx^2\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (6*a*c*x*Sqrt[a + b*x^3])/55 + (6*a*d*x^2*Sqrt[a + b*x^3])/91 + (
54*a^2*d*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)) + (2*Sqrt[a + b*x^3]*(1001*a*c*x + 715*a*d*x^2 + 455*b*
c*x^4 + 385*b*d*x^5))/5005 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3
)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(91*b^(1/3)*c - 55*(1 - Sqr
t[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 122.034, size = 534, normalized size = 0.96

$$-\frac{27\sqrt[4]{3}a^{\frac{7}{3}}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{91b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2d\sqrt{a+bx^3}}{91b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{18\cdot 3^{\frac{3}{4}}a^2\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-55\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+91\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{5005b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6acx\sqrt{a+bx^3}}{55}+\frac{6adx^2\sqrt{a+bx^3}}{91}+\sqrt{a+bx^3}\left(\frac{2acx}{5}+\frac{2adx^2}{7}+\frac{2bcx^4}{11}+\frac{2bdx^5}{13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(1/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] -27*3**(1/4)*a**(7/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b*
*(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt
(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 +
 sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -
7 - 4*sqrt(3))/(91*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)
/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 54
*a**2*d*sqrt(a + b*x**3)/(91*b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b
**(1/3)*x)) + 18*3**(3/4)*a**2*sqrt((a**(2/3) - a**(1/3)*b**(1/3)
*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqr
t(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-55*a**(1/3)*d*(-sqrt(3)
+ 1) + 91*b**(1/3)*c)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) +
 b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(
3))/(5005*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3
)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 6*a*c*x*sqr
t(a + b*x**3)/55 + 6*a*d*x**2*sqrt(a + b*x**3)/91 + sqrt(a + b*x*
*3)*(2*a*c*x/5 + 2*a*d*x**2/7 + 2*b*c*x**4/11 + 2*b*d*x**5/13)
```

Mathematica [C] time = 1.81421, size = 329, normalized size = 0.59

$$18i3^{3/4}a^{7/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(55\sqrt[3]{a}d+91\sqrt[3]{-b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-2970(-1)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (2*(-b)^(2/3)*x*(a + b*x^3)*(1274*a*c + 880*a*d*x + 455*b*c*x^3 +
 385*b*d*x^4) - 2970*(-1)^(2/3)*3^(1/4)*a^(8/3)*d*Sqrt[(-1)^(5/6)
*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) +
 ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I
*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + (18*I)*3^(3/4)*a^
(7/3)*(91*(-b)^(1/3)*c + 55*a^(1/3)*d)*Sqrt[((-1)^(5/6)*(-a^(1/3)
 + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b
)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)
^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(5005*(-b)^(2/3)*Sqrt[a
 + b*x^3])
```

Maple [B] time = 0.007, size = 1546, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a*c*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)))+b*d*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*
a/b*x^2*(b*x^3+a)^(1/2)+8/91*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+a*d*(2/7*x^2*
(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+b*c*(2/11*x^4*(b*x^3+a)^(1
/2)+6/55*a/b*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1
```

```
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bdx^4+bcx^3+adx+ac\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a),x, algorithm="maxima"
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bdx^4+bcx^3+adx+ac\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a),x, algorithm="fricas"
```

```
[Out] integral((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.93493, size = 170, normalized size = 0.31

$$\frac{a^{\frac{3}{2}}cx\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{3}{2}}dx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}bcx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}bdx^5\,\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(1/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1
/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt
(a)*b*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**5*gamma(5/3)*hyper((-
1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bdx^4+bcx^3+adx+ac\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

3.61 $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=525

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

```
[Out] (6*a*d*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (2*(7*c*x + 5*d*x^2)*Sqrt[a + b*x^3])/35 - (3*3^(1/4)*Sq
rt[2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3
)*c - 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.658647, antiderivative size = 535, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2}{5}cx\sqrt{a+bx^3}+\frac{2}{7}dx^2\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*c*x*Sqrt[a + b*x^3])/5 + (2*d*x^2*Sqrt[a + b*x^3])/7 + (6*a*d*
Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
```

```
- (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[
3]]*a*(7*b^(1/3)*c - 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(35*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 89.2202, size = 478, normalized size = 0.91

$$-\frac{3\sqrt[4]{3}a^{\frac{4}{3}}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{7b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ad\sqrt{a+bx^3}}{7b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}a\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+7\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{35b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2cx\sqrt{a+bx^3}}{5}+\frac{2dx^2\sqrt{a+bx^3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(1/2),x)
```

```
[Out] -3*3**(1/4)*a**(4/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**
(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(
3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 +
sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
 - 4*sqrt(3))/(7*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 6*a*
d*sqrt(a + b*x**3)/(7*b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)
*x)) + 2*3**(3/4)*a*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/
3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) +
 2)*(a**(1/3) + b**(1/3)*x)*(-5*a**(1/3)*d*(-sqrt(3) + 1) + 7*b**
(1/3)*c)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/
(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(35*b**(2
/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3))
 + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 2*c*x*sqrt(a + b*x**3)/5 +
 2*d*x**2*sqrt(a + b*x**3)/7
```

Mathematica [C] time = 1.2688, size = 313, normalized size = 0.6

$$\frac{2i3^{3/4}a^{4/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(5\sqrt[3]{a}d+7\sqrt[3]{-b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-30(-1)^{2/3}\sqrt[4]{3}\cdots}{35(-b)^{2/3}\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(-b)^(2/3)*x*(7*c + 5*d*x)*(a + b*x^3) - 30*(-1)^(2/3)*3^(1/4)
*a^(5/3)*d*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1
+ ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[Ar
cSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^
(1/3)] + (2*I)*3^(3/4)*a^(4/3)*(7*(-b)^(1/3)*c + 5*a^(1/3)*d)*Sqr
t[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^
(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqr
t[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/
(35*(-b)^(2/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.007, size = 1480, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*a*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+b*d*(2/7/b*x^2*(b*x^3+a)^(1/2)+8/21*I*a/b^2*3^(1/2)*(-a*b^2)
^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))-2/3
*I*a*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*
b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)))+b*c*(2/5/b*x*(b*x^3+a)^(1/2)+4/15*I*a/b^2*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a),x, algorithm="maxima"
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a),x, algorithm="fricas"
```

```
[Out] integral((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.73983, size = 163, normalized size = 0.31

$$\frac{\sqrt{a}cx\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\left(\frac{4}{3}\right)}+\frac{\sqrt{a}dx^2\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\left(\frac{5}{3}\right)}+\frac{bcx^4\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\left(\frac{7}{3}\right)}+\frac{bdx^5\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((1/2,
2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + b*c*x**4
*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*sqrt(a)*gamma(7/3)) + b*d*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/
3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

3.62 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=490

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.520593, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
```

```
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 60.6912, size = 430, normalized size = 0.88

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(3/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3)
 + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 + sq
rt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 -
 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 2*d*sqrt
(a + b*x**3)/(b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 2
*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1
/3) + b**(1/3)*x)*(-a**(1/3)*d*(-sqrt(3) + 1) + b**(1/3)*c)*ellip
tic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 +
 sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/
3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)*
*2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 0.338392, size = 221, normalized size = 0.45

$$-\frac{2\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left((-1)^{2/3}\sqrt{3}\sqrt[3]{a}dE\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-i\left(\sqrt[3]{a}d+\sqrt[3]{-b}c\right)F\left(\right.\right.}{\sqrt[4]{3}(-b)^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*a^(1/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*
Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*((-1)
```

```
^(2/3)*Sqrt[3]*a^(1/3)*d*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(
-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] - I*((-b)^(1/3)*c + a
^(1/3)*d)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^
(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(1/4)*(-b)^(2/3)*Sqrt[a + b*x^3
])
```

Maple [B] time = 0.006, size = 1536, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x)
```

```
[Out] a*c*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+b*d*(-2/3/b*x^2/((x^3+a/b)*b)
^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+
1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))))+a*d*(2/3/a*x^2/((x^3+a/b)*b)^(1/2)+
2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))))+b*c*(-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^
2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2),x, algorithm="maxim
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx+c}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2),x, algorithm="frica
```

```
[Out] integral((d*x + c)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.1191, size = 78, normalized size = 0.16

$$\frac{cx\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5
/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2),x, algorithm="giac"
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

3.63 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=522

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

```
[Out] (2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^3])/(3*
a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3
]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 - Sqrt[3])
*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.671094, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```



```
[Out] (2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^3])/(3*
a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3
```

```
]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 - Sqrt[3])
*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 81.8667, size = 466, normalized size = 0.89

$$\frac{4x\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{27a\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{9ab^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt[4]{3}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(5/2),x)
```

```
[Out] 4*x*(9*c/2 + 9*d*x/2)/(27*a*sqrt(a + b*x**3)) - 2*d*sqrt(a + b*x*
*3)/(3*a*b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 2*3**(
3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1
/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) +
 b**(1/3)*x)*(a**(1/3)*d*(-sqrt(3) + 1) + b**(1/3)*c)*elliptic_f(
asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(
3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(9*a*b**(2/3)*sqrt(a**(1/3)*(
a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*
sqrt(a + b*x**3)) + 3**(1/4)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)
*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqr
t(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3
)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)
*x)), -7 - 4*sqrt(3))/(3*a**(2/3)*b**(2/3)*sqrt(a**(1/3)*(a**(1/3
) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a
+ b*x**3))
```

Mathematica [C] time = 1.36972, size = 305, normalized size = 0.58

$$\frac{2i3^{3/4}\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-bx}-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-bx}}{\sqrt[3]{a}}+1}\left(\sqrt[3]{-b}c-\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+6(-1)^{2/3}\sqrt[4]{3}a^{2/3}c}{9a(-b)^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (6*(-b)^(2/3)*x*(c + d*x) + 6*(-1)^(2/3)*3^(1/4)*a^(2/3)*d*Sqrt[(
-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-b)^(1/3)*x)/
a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(
5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + (2*I)*3^
(3/4)*a^(1/3)*((-b)^(1/3)*c - a^(1/3)*d)*Sqrt[((-1)^(5/6)*(-a^(1/
3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((
-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-
b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(9*a*(-b)^(2/3)*Sqrt[
a + b*x^3])
```

Maple [B] time = 0.007, size = 1662, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x)
```

```
[Out] a*c*(2/9/a*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+14/27/a^2*x/((x^3+a/
b)*b)^(1/2)-14/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)))+b*d*(-2/9*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+
8/27/b/a*x^2/((x^3+a/b)*b)^(1/2)+8/81*I/b^2/a*3^(1/2)*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+a*d*(2
/9/a*x^2/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+10/27/a^2*x^2/((x^3+a/b)
*b)^(1/2)+10/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+b*c*(-2/9*x/b^3*(b*x^3+a)^(1
/2)/(x^3+a/b)^2+4/27/b/a*x/((x^3+a/b)*b)^(1/2)-4/81*I/b^2/a*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2),x, algorithm="maxim
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{(bx^3 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2),x, algorithm="frica
```

```
[Out] integral((d*x + c)/(b*x^3 + a)^(3/2), x)
```

Sympy [A] time = 20.1159, size = 163, normalized size = 0.31

$$\frac{cx\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\,\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\,\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(5/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 5/2), (
5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + b*c*x*
*4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)
/(3*a**(5/2)*gamma(7/3)) + b*d*x**5*gamma(5/3)*hyper((5/3, 5/2),
(8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2),x, algorithm="giac"
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

3.64 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx$

Optimal. Leaf size=554

$$\frac{5\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}d+7\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}}$$

```
[Out] (2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + (2*x*(7*c + 5*d*x))/(27
*a^2*Sqrt[a + b*x^3]) - (10*d*Sqrt[a + b*x^3])/(27*a^2*b^(2/3)*((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*d*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*c + 5*(1 - Sqrt[3])*a^(
1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(2/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.846545, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{5\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}d+7\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + (2*x*(7*c + 5*d*x))/(27
*a^2*Sqrt[a + b*x^3]) - (10*d*Sqrt[a + b*x^3])/(27*a^2*b^(2/3)*((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*d*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*c + 5*(1 - Sqrt[3])*a^(
1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(2/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 103.388, size = 498, normalized size = 0.9

$$\frac{2x(c+dx)}{9a(a+bx^3)^{\frac{3}{2}}}+\frac{8x\left(\frac{105c}{4}+\frac{75dx}{4}\right)}{405a^2\sqrt{a+bx^3}}-\frac{10d\sqrt{a+bx^3}}{27a^2b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$
$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}(1+\sqrt{3})+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+7\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{b}x}{\sqrt[3]{a}(1+\sqrt{3})+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{81a^2b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}(1+\sqrt{3})+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{5\sqrt[4]{3}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}(1+\sqrt{3})+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{b}x}{\sqrt[3]{a}(1+\sqrt{3})+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{27a^{\frac{5}{3}}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}(1+\sqrt{3})+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(7/2),x)
```

```
[Out] 2*x*(c + d*x)/(9*a*(a + b*x**3)**(3/2)) + 8*x*(105*c/4 + 75*d*x/4
)/(405*a**2*sqrt(a + b*x**3)) - 10*d*sqrt(a + b*x**3)/(27*a**2*b*
*(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 2*3**(3/4)*sqrt((
a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sq
rt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x
)*(5*a**(1/3)*d*(-sqrt(3) + 1) + 7*b**(1/3)*c)*elliptic_f(asin((-
a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b
**(1/3)*x)), -7 - 4*sqrt(3))/(81*a**2*b**(2/3)*sqrt(a**(1/3)*(a**
(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqr
t(a + b*x**3)) + 5*3**(1/4)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*
x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt
(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)
*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*
x)), -7 - 4*sqrt(3))/(27*a**(5/3)*b**(2/3)*sqrt(a**(1/3)*(a**(1/3
) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a
+ b*x**3))
```

Mathematica [C] time = 0.768676, size = 267, normalized size = 0.48

$$\frac{2\left(3(-b)^{2/3}\left(2ax(5c+4dx)+bx^4(7c+5dx)\right)+3^{3/4}\sqrt{(-1)^{5/6}\left(\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}-1\right)}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(a+bx^3\right)\left(5(-1)^{2/3}\sqrt{3}a^{2/}\right.\right.}{81a^2(-b)^{2/3}\left(a+bx^3\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (2*(3*(-b)^(2/3)*(2*a*x*(5*c + 4*d*x) + b*x^4*(7*c + 5*d*x)) + 3^
(3/4)*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-
b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*(a + b*x^3)*(5*(-
1)^(2/3)*Sqrt[3]*a^(2/3)*d*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I
*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + I*a^(1/3)*(7*(-b)
^(1/3)*c - 5*a^(1/3)*d)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-
b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])))/(81*a^2*(-b)^(2/3)*
(a + b*x^3)^(3/2))
```

Maple [B] time = 0.108, size = 1782, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x)
```

```
[Out] a*c*(2/15/a*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^3+26/135/a^2*x/b^2*(b
*x^3+a)^(1/2)/(x^3+a/b)^2+182/405/a^3*x/((x^3+a/b)*b)^(1/2)-182/1
215*I/a^3*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))+b*d*(-2/15*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^3+8/135/a*x^2/b
^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+8/81/b/a^2*x^2/((x^3+a/b)*b)^(1/2)
+8/243*I/b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+
1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))))+a*d*(2/15/a*x^2/b^3*(b*x^3+a)^(1/2)
/(x^3+a/b)^3+22/135/a^2*x^2/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+22/81
/a^3*x^2/((x^3+a/b)*b)^(1/2)+22/243*I/a^3*3^(1/2)/b*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
```

```
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+b*c*(-2/
15*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^3+4/135/a*x/b^3*(b*x^3+a)^(1/2
)/(x^3+a/b)^2+28/405/b/a^2*x/((x^3+a/b)*b)^(1/2)-28/1215*I/b^2/a^
2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2),x, algorithm="maxim
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{(b^2x^6 + 2\,abx^3 + a^2)\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2),x, algorithm="frica
```

```
[Out] integral((d*x + c)/((b^2*x^6 + 2*a*b*x^3 + a^2)*sqrt(b*x^3 + a)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2),x, algorithm="giac"
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

3.65 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx$

Optimal. Leaf size=581

$$\frac{11\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\,3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}+\frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(55\left(1-\sqrt{3}\right)\sqrt[3]{a}d+91\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

```
[Out] (2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + (2*x*(13*c + 11*d*x))/
(135*a^2*(a + b*x^3)^(3/2)) + (2*x*(91*c + 55*d*x))/(405*a^3*Sqrt
[a + b*x^3]) - (22*d*Sqrt[a + b*x^3])/(81*a^3*b^(2/3)*((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)) + (11*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(27*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 + (2*Sqrt[2 + Sqrt[3]]*(91*b^(1/3)*c + 55*(1 - Sqrt[3])*a^(1/3)*
d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(2/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rubi [A] time = 0.961401, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{11\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\,3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}+\frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(55\left(1-\sqrt{3}\right)\sqrt[3]{a}d+91\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

```
[Out] (2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + (2*x*(13*c + 11*d*x))/
(135*a^2*(a + b*x^3)^(3/2)) + (2*x*(91*c + 55*d*x))/(405*a^3*Sqrt
[a + b*x^3]) - (22*d*Sqrt[a + b*x^3])/(81*a^3*b^(2/3)*((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)) + (11*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(27*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 + (2*Sqrt[2 + Sqrt[3]]*(91*b^(1/3)*c + 55*(1 - Sqrt[3])*a^(1/3)*
d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(2/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rubi in Sympy [A] time = 124.251, size = 536, normalized size = 0.92

$$\frac{4x\left(\frac{21c}{2}+\frac{21dx}{2}\right)}{315a\left(a+bx^3\right)^{\frac{5}{2}}}+\frac{8x\left(\frac{273c}{4}+\frac{231dx}{4}\right)}{2835a^2\left(a+bx^3\right)^{\frac{3}{2}}}+\frac{16x\left(\frac{1911c}{8}+\frac{1155dx}{8}\right)}{8505a^3\sqrt{a+bx^3}}-\frac{22d\sqrt{a+bx^3}}{81a^3b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$
$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(55\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+91\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{1215a^3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{11\sqrt[4]{3}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{81a^{\frac{8}{3}}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(9/2),x)
```

```
[Out] 4*x*(21*c/2 + 21*d*x/2)/(315*a*(a + b*x**3)**(5/2)) + 8*x*(273*c/
4 + 231*d*x/4)/(2835*a**2*(a + b*x**3)**(3/2)) + 16*x*(1911*c/8 +
 1155*d*x/8)/(8505*a**3*sqrt(a + b*x**3)) - 22*d*sqrt(a + b*x**3)
/(81*a**3*b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 2*3**
(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3)
+ b**(1/3)*x)*(55*a**(1/3)*d*(-sqrt(3) + 1) + 91*b**(1/3)*c)*elli
ptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1
+ sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(1215*a**3*b**(2/3)*sq
rt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**
(1/3)*x)**2)*sqrt(a + b*x**3)) + 11*3**(1/4)*d*sqrt((a**(2/3) - a
**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**
(1/3)*x)**2)*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_
e(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqr
t(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(81*a**(8/3)*b**(2/3)*sqrt(
a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/
3)*x)**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 0.943734, size = 287, normalized size = 0.49

$$\frac{2\left(3(-b)^{2/3}\left(a^2x(157c+115dx)+13abx^4(17c+11dx)+b^2x^7(91c+55dx)\right)+3^{3/4}\sqrt{(-1)^{5/6}\left(\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}-1\right)}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}}\right.}{1215a^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

```
[Out] (2*(3*(-b)^(2/3)*(13*a*b*x^4*(17*c + 11*d*x) + b^2*x^7*(91*c + 55
*d*x) + a^2*x*(157*c + 115*d*x)) + 3^(3/4)*Sqrt[(-1)^(5/6)*(-1 +
((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^
(2/3)*x^2)/a^(2/3)]*(a + b*x^3)^2*(55*(-1)^(2/3)*Sqrt[3]*a^(2/3)*
d*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3
^(1/4)], (-1)^(1/3)] + I*a^(1/3)*(91*(-b)^(1/3)*c - 55*a^(1/3)*d)
*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^
(1/4)], (-1)^(1/3)])))/(1215*a^3*(-b)^(2/3)*(a + b*x^3)^(5/2))
```

Maple [B] time = 0.095, size = 1902, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x)
```

```
[Out] a*c*(2/21/a*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^4+38/315/a^2*x/b^3*(b
*x^3+a)^(1/2)/(x^3+a/b)^3+494/2835/a^3*x/b^2*(b*x^3+a)^(1/2)/(x^3
+a/b)^2+494/1215/a^4*x/((x^3+a/b)*b)^(1/2)-494/3645*I/a^4*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+b*d*(-2/21*x
^2/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^4+8/315/a*x^2/b^4*(b*x^3+a)^(1/2
)/(x^3+a/b)^3+88/2835/a^2*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+88/
1701/b/a^3*x^2/((x^3+a/b)*b)^(1/2)+88/5103*I/b^2/a^3*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
+a*d*(2/21/a*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^4+34/315/a^2*x^2/b
^3*(b*x^3+a)^(1/2)/(x^3+a/b)^3+374/2835/a^3*x^2/b^2*(b*x^3+a)^(1/
2)/(x^3+a/b)^2+374/1701/a^4*x^2/((x^3+a/b)*b)^(1/2)+374/5103*I/a^
4*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^
```

```
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))))+b*c*(-2/21/b^5*x*(b*x^3+a)^(1/2)/(x^3+a/b)^4+4/3
15/a*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^3+52/2835/a^2*x/b^3*(b*x^3+a
)^(1/2)/(x^3+a/b)^2+52/1215/b/a^3*x/((x^3+a/b)*b)^(1/2)-52/3645*I
/b^2/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2),x, algorithm="maxim
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{(b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3)\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2),x, algorithm="frica
```

```
[Out] integral((d*x + c)/((b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3)*s
qrt(b*x^3 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2),x, algorithm="giac"

[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

3.66 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=590

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7bd-4ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{a+bx^3}(7bd-4ag)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2e\sqrt{a+bx^3}}{3b}+\frac{2fx\sqrt{a+bx^3}}{5b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*f*x*Sqrt[a + b*x^3])/(5*b) + (2*
g*x^2*Sqrt[a + b*x^3])/(7*b) + (2*(7*b*d - 4*a*g)*Sqrt[a + b*x^3]
)/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt
[2 - Sqrt[3]]*a^(1/3)*(7*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b
^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1
/3)*(5*b*c - 2*a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rubi [A] time = 1.00036, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7bd-4ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{a+bx^3}(7bd-4ag)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2e\sqrt{a+bx^3}}{3b}+\frac{2fx\sqrt{a+bx^3}}{5b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*f*x*Sqrt[a + b*x^3])/(5*b) + (2*
g*x^2*Sqrt[a + b*x^3])/(7*b) + (2*(7*b*d - 4*a*g)*Sqrt[a + b*x^3]
)/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt
[2 - Sqrt[3]]*a^(1/3)*(7*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b
^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1
/3)*(5*b*c - 2*a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

<u>**Rubi in Sympy [A]**</u> time = 136.716, size = 527, normalized size = 0.89

$$\frac{4\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(ag-\frac{7bd}{4}\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{7b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2e\sqrt{a+bx^3}}{3b}+\frac{2fx\sqrt{a+bx^3}}{5b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5\sqrt[3]{a}\left(-\sqrt{3}+1\right)(4ag-7bd)+\sqrt[3]{b}(14af-35bc)\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+}\right.\right.}{105b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{8\sqrt{a+bx^3}\left(ag-\frac{7bd}{4}\right)}{7b^{\frac{5}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] 4*3**(1/4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/
3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3)
+ 2)*(a**(1/3) + b**(1/3)*x)*(a*g - 7*b*d/4)*elliptic_e(asin((-a*
*(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**
(1/3)*x)), -7 - 4*sqrt(3))/(7*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) +
b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*
x**3)) + 2*e*sqrt(a + b*x**3)/(3*b) + 2*f*x*sqrt(a + b*x**3)/(5*b
) + 2*g*x**2*sqrt(a + b*x**3)/(7*b) - 2*3**(3/4)*sqrt((a**(2/3) -
 a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b
**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-5*a**(
1/3)*(-sqrt(3) + 1)*(4*a*g - 7*b*d) + b**(1/3)*(14*a*f - 35*b*c))
*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3
)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(105*b**(5/3)*sqr
t(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(
1/3)*x)**2)*sqrt(a + b*x**3)) - 8*sqrt(a + b*x**3)*(a*g - 7*b*d/4
)/(7*b**(5/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x))
```

Mathematica [C] time = 2.10678, size = 357, normalized size = 0.61

$$-\frac{2 i 3^{3/4} \sqrt[3]{a} \sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}} \sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1} F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\left(35b\left(\sqrt[3]{a}d+\sqrt[3]{-b}c\right)-2a\left(10\sqrt[3]{a}\cdots\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] -(2*(-b)^(2/3)*(a + b*x^3)*(35*e + 3*x*(7*f + 5*g*x)) - 30*(-1)^(
2/3)*3^(1/4)*a^(2/3)*(7*b*d - 4*a*g)*Sqrt[((-1)^(5/6)*(-a^(1/3) +
 (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^
(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(
1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + (2*I)*3^(3/4)*a^(1/3)*(3
5*b*((-b)^(1/3)*c + a^(1/3)*d) - 2*a*(7*(-b)^(1/3)*f + 10*a^(1/3)
*g))*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1
+ ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[Ar
cSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^
(1/3)])/(105*(-b)^(5/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.01, size = 1491, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*
(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)))+2/3*e*(b*x^3+a)^(1/2)/b+f*(2/5/b*x*(b*x^
3+a)^(1/2)+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)))+g*(2/7/b*x^2*(b*x^3+a)^(1/2)+8/21*I*a/b^2*3^(1/2
)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
```

```
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 4.32436, size = 187, normalized size = 0.32

$$e\left(\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for}\ b = 0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases}\right) + \frac{cx\,\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\,\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\,\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\,\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*
b), True)) + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((
1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3
)) + f*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar
(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + g*x**5*gamma(5/3)*hyper((1/2,
5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

3.67 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=594

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\Big(\sqrt[3]{b}(2af+bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4a}{3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(bd-4ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}(bd-4ag)}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}-\frac{2e\sqrt{a+bx^3}}{3ab}$$

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3
]) - (2*e*Sqrt[a + b*x^3])/(3*a*b) - (2*(b*d - 4*a*g)*Sqrt[a + b*
x^3])/(3*a*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2
 - Sqrt[3]]*(b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)
*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*(b*c + 2*a*f) + (1 - Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.871258, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\Big(\sqrt[3]{b}(2af+bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4a}{3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(bd-4ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}(bd-4ag)}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}-\frac{2e\sqrt{a+bx^3}}{3ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3
]) - (2*e*Sqrt[a + b*x^3])/(3*a*b) - (2*(b*d - 4*a*g)*Sqrt[a + b*
```

```
x^3])/(3*a*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2
 - Sqrt[3]]*(b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)
*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*(b*c + 2*a*f) + (1 - Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 98.2651, size = 524, normalized size = 0.88

$$-\frac{2e\sqrt{a+bx^3}}{3ab}-\frac{2x\left(af-bc-bex^2+x\left(ag-bd\right)\right)}{3ab\sqrt{a+bx^3}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}\left(-\sqrt{3}+1\right)\left(4ag-bd\right)+\sqrt[3]{b}\left(2af+bc\right)\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\right.}{9ab^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{a+bx^3}\left(4ag-bd\right)}{3ab^{\frac{5}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(4ag-bd\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3a^{\frac{2}{3}}b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] -2*e*sqrt(a + b*x**3)/(3*a*b) - 2*x*(a*f - b*c - b*e*x**2 + x*(a*
g - b*d))/(3*a*b*sqrt(a + b*x**3)) + 2*3**(3/4)*sqrt((a**(2/3) -
a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b*
*(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-a**(1/3
)*(-sqrt(3) + 1)*(4*a*g - b*d) + b**(1/3)*(2*a*f + b*c))*elliptic
_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sq
rt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(9*a*b**(5/3)*sqrt(a**(1/3
)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**
2)*sqrt(a + b*x**3)) + 2*sqrt(a + b*x**3)*(4*a*g - b*d)/(3*a*b**(
5/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) - 3**(1/4)*sqrt((a**(
2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3
)) + b**(1/3)*x)**2)*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(
4*a*g - b*d)*elliptic_e(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)
*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(3*a*
*(2/3)*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(
1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 2.03153, size = 354, normalized size = 0.6

$$-\frac{2i3^{3/4}\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)\left(4a^{4/3}g-\sqrt[3]{a}bd+2a\sqrt[3]{-b}f+\sqrt[3]{-}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] -(6*(-b)^(2/3)*(b*x*(c + d*x) - a*(e + x*(f + g*x))) + 6*(-1)^(2/
3)*3^(1/4)*a^(2/3)*(b*d - 4*a*g)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b
)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3
)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)
*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + (2*I)*3^(3/4)*a^(1/3)*((-b)^
(1/3)*b*c - a^(1/3)*b*d + 2*a*(-b)^(1/3)*f + 4*a^(4/3)*g)*Sqrt[((
-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3
)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(
-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(9*a
*(-b)^(5/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.009, size = 1547, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] c*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(2/3/a*x^2/((x^3+a/b)*b)^(1/2
)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))))-2/3*e/b/(b*x^3+a)^(1/2)+f*(-2/3/b*x/((x^
3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)))+g*(-2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2),x, algorithm="maxima
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2),x, algorithm="fricas
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Sympy [A] time = 42.0473, size = 189, normalized size = 0.32

$$e\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for}\ b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\,\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\,\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{3}{2} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\,\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{3}{2} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\,\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3
/2)), True)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hype
r((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma
(5/3)) + f*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + g*x**5*gamma(5/3)*hyper((
3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/
3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

3.68 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=628

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4ag+5bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}(4ag+5bd)}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2(3ae-x(x(4ag+5bd)+2af+7bc))}{27a^2b\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2af+7bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+\right.}{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}}$$

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3
/2)) - (2*(5*b*d + 4*a*g)*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*(3*a*e - x*(7*b*c + 2*a*f + (
5*b*d + 4*a*g)*x)))/(27*a^2*b*Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3
]]*(5*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(5
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(
7*b*c + 2*a*f) + (1 - Sqrt[3])*a^(1/3)*(5*b*d + 4*a*g))*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.02861, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4ag+5bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}(4ag+5bd)}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2(3ae-x(x(4ag+5bd)+2af+7bc))}{27a^2b\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2af+7bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+\right.}{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3
/2)) - (2*(5*b*d + 4*a*g)*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*(3*a*e - x*(7*b*c + 2*a*f + (
5*b*d + 4*a*g)*x)))/(27*a^2*b*Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3
]]*(5*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(5
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(
7*b*c + 2*a*f) + (1 - Sqrt[3])*a^(1/3)*(5*b*d + 4*a*g))*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 107.722, size = 564, normalized size = 0.9

$$-\frac{2x\left(af-bc-bex^2+x\left(ag-bd\right)\right)}{9ab\left(a+bx^3\right)^{\frac{3}{2}}}-\frac{4\left(\frac{3ae}{2}-\frac{x(2af+7bc+x(4ag+5bd))}{2}\right)}{27a^2b\sqrt{a+bx^3}}$$

$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{a}\left(-\sqrt{3}+1\right)(4ag+5bd)+\sqrt[3]{b}\left(2af+7bc\right)\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\right)}{81a^2b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{8\sqrt{a+bx^3}\left(ag+\frac{5bd}{4}\right)}{27a^2b^{\frac{5}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

$$+\frac{4\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(ag+\frac{5bd}{4}\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{27a^{\frac{5}{3}}b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(5/2),x)
```

```
[Out] -2*x*(a*f - b*c - b*e*x**2 + x*(a*g - b*d))/(9*a*b*(a + b*x**3)**
(3/2)) - 4*(3*a*e/2 - x*(2*a*f + 7*b*c + x*(4*a*g + 5*b*d))/2)/(2
7*a**2*b*sqrt(a + b*x**3)) + 2*3**(3/4)*sqrt((a**(2/3) - a**(1/3)
*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x
)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(a**(1/3)*(-sqrt(
3) + 1)*(4*a*g + 5*b*d) + b**(1/3)*(2*a*f + 7*b*c))*elliptic_f(as
in((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)
) + b**(1/3)*x)), -7 - 4*sqrt(3))/(81*a**2*b**(5/3)*sqrt(a**(1/3)
*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2
)*sqrt(a + b*x**3)) - 8*sqrt(a + b*x**3)*(a*g + 5*b*d/4)/(27*a**2
*b**(5/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 4*3**(1/4)*sqr
t((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 +
 sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/
3)*x)*(a*g + 5*b*d/4)*elliptic_e(asin((-a**(1/3)*(-1 + sqrt(3)) +
 b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(
3))/(27*a**(5/3)*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 1.41387, size = 329, normalized size = 0.52

$$-\frac{2\left(-3(-b)^{2/3}\left(3a(a(e+x(f+gx))-bx(c+dx))-x\left(a+bx^3\right)(2af+4agx+7bc+5bdx)\right)+i3^{3/4}\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(-3*(-b)^(2/3)*(-(x*(7*b*c + 2*a*f + 5*b*d*x + 4*a*g*x)*(a +
b*x^3)) + 3*a*(-(b*x*(c + d*x)) + a*(e + x*(f + g*x)))) + I*3^(3/
4)*a^(1/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*S
qrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*(a + b
*x^3)*((-1)^(1/6)*Sqrt[3]*a^(1/3)*(5*b*d + 4*a*g)*EllipticE[ArcSi
n[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/
3)] + (7*(-b)^(1/3)*b*c - 5*a^(1/3)*b*d + 2*a*(-b)^(1/3)*f - 4*a^
(4/3)*g)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(
1/3)]/3^(1/4)], (-1)^(1/3)])))/(81*a^2*(-b)^(5/3)*(a + b*x^3)^(3/
2))
```

Maple [B] time = 0.009, size = 1673, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x)
```

```
[Out] c*(2/9/a*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+14/27/a^2*x/((x^3+a/b)
*b)^(1/2)-14/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)))+d*(2/9/a*x^2/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+10/
27/a^2*x^2/((x^3+a/b)*b)^(1/2)+10/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/
3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))-2/9*e/b
/(b*x^3+a)^(3/2)+f*(-2/9*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+4/27/b
/a*x/((x^3+a/b)*b)^(1/2)-4/81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g*(-2/9*x^2/b^3*(b*x^3+a)^(1/2)/(x
^3+a/b)^2+8/27/b/a*x^2/((x^3+a/b)*b)^(1/2)+8/81*I/b^2/a*3^(1/2)*(
```

```
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2),x, algorithm="maxima
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx^4 + fx^3 + ex^2 + dx + c}{(b^2x^6 + 2\,abx^3 + a^2)\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2),x, algorithm="fricas
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b^2*x^6 + 2*a*b*x^3
+ a^2)*sqrt(b*x^3 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

3.69 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx$

Optimal. Leaf size=676

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4ag+11bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\,3^{3/4}a^{8/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}(4ag+11bd)}{81a^3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{405a^3b\sqrt{a+bx^3}}$$

$$-\frac{2(9ae-x(x(4ag+11bd)+2af+13bc))}{135a^2b\,(a+bx^3)^{3/2}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(2af+13bc)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4a\right.}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\,(a+bx^3)^{5/2}}$$

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(
5/2)) + (2*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(405*a^
3*b*Sqrt[a + b*x^3]) - (2*(11*b*d + 4*a*g)*Sqrt[a + b*x^3])/(81*a
^3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*(9*a*e - x*(
13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(135*a^2*b*(a + b*x^3)^(3/
2)) + (Sqrt[2 - Sqrt[3]]*(11*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(27*3^(3/4)*a^(8/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt
[2 + Sqrt[3]]*(7*b^(1/3)*(13*b*c + 2*a*f) + 5*(1 - Sqrt[3])*a^(1/
3)*(11*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(5
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.29215, antiderivative size = 676, normalized size of antiderivative = 1., number of

steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4ag+11bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}a^{8/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}(4ag+11bd)}{81a^3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{405a^3b\sqrt{a+bx^3}}$$

$$-\frac{2(9ae-x(x(4ag+11bd)+2af+13bc))}{135a^2b\left(a+bx^3\right)^{3/2}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(2af+13bc)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4a\right.}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(
5/2)) + (2*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(405*a^
3*b*Sqrt[a + b*x^3]) - (2*(11*b*d + 4*a*g)*Sqrt[a + b*x^3])/(81*a
^3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*(9*a*e - x*(
13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(135*a^2*b*(a + b*x^3)^(3/
2)) + (Sqrt[2 - Sqrt[3]]*(11*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(27*3^(3/4)*a^(8/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt
[2 + Sqrt[3]]*(7*b^(1/3)*(13*b*c + 2*a*f) + 5*(1 - Sqrt[3])*a^(1/
3)*(11*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(5
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 135.762, size = 612, normalized size = 0.91

$$-\frac{2x\left(af-bc-bex^2+x\left(ag-bd\right)\right)}{15ab\left(a+bx^3\right)^{\frac{5}{2}}}-\frac{4\left(\frac{9ae}{2}-\frac{x(2af+13bc+x(4ag+11bd))}{2}\right)}{135a^2b\left(a+bx^3\right)^{\frac{3}{2}}}+\frac{8x\left(\frac{7af}{2}+\frac{91bc}{4}+x\left(5ag+\frac{55bd}{4}\right)\right)}{405a^3b\sqrt{a+bx^3}}$$

$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{a}\left(-\sqrt{3}+1\right)(4ag+11bd)+\sqrt[3]{b}\left(14af+91bc\right)\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\cdots}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\cdots}\right.\right.}{1215a^3b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}\,(4ag+11bd)}{81a^3b^{\frac{5}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(4ag+11bd)\,E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{81a^{\frac{8}{3}}b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(7/2),x)
```

```
[Out] -2*x*(a*f - b*c - b*e*x**2 + x*(a*g - b*d))/(15*a*b*(a + b*x**3)*
*(5/2)) - 4*(9*a*e/2 - x*(2*a*f + 13*b*c + x*(4*a*g + 11*b*d))/2)
/(135*a**2*b*(a + b*x**3)**(3/2)) + 8*x*(7*a*f/2 + 91*b*c/4 + x*(
5*a*g + 55*b*d/4))/(405*a**3*b*sqrt(a + b*x**3)) + 2*3**(3/4)*sqr
t((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 +
 sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3
)*x)*(5*a**(1/3)*(-sqrt(3) + 1)*(4*a*g + 11*b*d) + b**(1/3)*(14*a
*f + 91*b*c))*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3
)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(121
5*a**3*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(
1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 2*sqrt(a + b*x
**3)*(4*a*g + 11*b*d)/(81*a**3*b**(5/3)*(a**(1/3)*(1 + sqrt(3)) +
 b**(1/3)*x)) + 3**(1/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b
**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqr
t(3) + 2)*(a**(1/3) + b**(1/3)*x)*(4*a*g + 11*b*d)*elliptic_e(asi
n((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3))
 + b**(1/3)*x)), -7 - 4*sqrt(3))/(81*a**(8/3)*b**(5/3)*sqrt(a**(1
/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)
**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 1.48168, size = 366, normalized size = 0.54

$$-\frac{2\left(-3(-b)^{2/3}\left(27a^2(a(e+x(f+gx))-bx(c+dx))-3ax\left(a+bx^3\right)(2af+4agx+13bc+11bdx)-x\left(a+bx^3\right)^2(14af+2\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (-2*(-3*(-b)^(2/3)*(-3*a*x*(13*b*c + 2*a*f + 11*b*d*x + 4*a*g*x)*
(a + b*x^3) - x*(91*b*c + 14*a*f + 55*b*d*x + 20*a*g*x)*(a + b*x^
3)^2 + 27*a^2*(-(b*x*(c + d*x)) + a*(e + x*(f + g*x)))) + I*3^(3/
```

```
4)*a^(1/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*S
qrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*(a + b
*x^3)^2*(5*(-1)^(1/6)*Sqrt[3]*a^(1/3)*(11*b*d + 4*a*g)*EllipticE[
ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1
)^(1/3)] + (91*(-b)^(1/3)*b*c - 55*a^(1/3)*b*d + 14*a*(-b)^(1/3)*
f - 20*a^(4/3)*g)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/
3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])))/(1215*a^3*(-b)^(5/3)*(a +
 b*x^3)^(5/2))
```

Maple [B] time = 0.008, size = 1793, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x)
```

```
[Out] c*(2/15/a*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^3+26/135/a^2*x/b^2*(b*x
^3+a)^(1/2)/(x^3+a/b)^2+182/405/a^3*x/((x^3+a/b)*b)^(1/2)-182/121
5*I/a^3*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))+d*(2/15/a*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^3+22/135/a^2*x^2/b
^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+22/81/a^3*x^2/((x^3+a/b)*b)^(1/2)+
22/243*I/a^3*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/
b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))))-2/15*e/b/(b*x^3+a)^(5/2)+f*(-2/15*x/b
^4*(b*x^3+a)^(1/2)/(x^3+a/b)^3+4/135/a*x/b^3*(b*x^3+a)^(1/2)/(x^3
+a/b)^2+28/405/b/a^2*x/((x^3+a/b)*b)^(1/2)-28/1215*I/b^2/a^2*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g*(-2/15*x^
2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^3+8/135/a*x^2/b^3*(b*x^3+a)^(1/2)
/(x^3+a/b)^2+8/81/b/a^2*x^2/((x^3+a/b)*b)^(1/2)+8/243*I/b^2/a^2*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2),x, algorithm="maxima
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx^4 + fx^3 + ex^2 + dx + c}{(b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3)\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2),x, algorithm="fricas
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b^3*x^9 + 3*a*b^2*x^
6 + 3*a^2*b*x^3 + a^3)*sqrt(b*x^3 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

3.70 $\int \frac{(a+bx)^2}{c+dx^3}\,dx$

Optimal. Leaf size=186

$$\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}-\frac{a\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

```
[Out] -((a*(2*b*c^(1/3) + a*d^(1/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(2/3))) - (a*(2*b*c^(1/3) - a
*d^(1/3))*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(2/3)) + (a*(2*b
*c^(1/3) - a*d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x
^2])/(6*c^(2/3)*d^(2/3)) + (b^2*Log[c + d*x^3])/(3*d)
```

Rubi [A] time = 0.331619, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}-\frac{a\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^2/(c + d*x^3),x]
```


```
[Out] -((a*(2*b*c^(1/3) + a*d^(1/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(2/3))) - (a*(2*b*c^(1/3) - a
*d^(1/3))*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(2/3)) + (a*(2*b
*c^(1/3) - a*d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x
^2])/(6*c^(2/3)*d^(2/3)) + (b^2*Log[c + d*x^3])/(3*d)
```

Rubi in Sympy [A] time = 42.178, size = 175, normalized size = 0.94

$$\frac{a\left(a\sqrt[3]{d}-2b\sqrt[3]{c}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{\frac{2}{3}}d^{\frac{2}{3}}}-\frac{a\left(a\sqrt[3]{d}-2b\sqrt[3]{c}\right)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}d^{\frac{2}{3}}}-\frac{\sqrt{3}a\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{\frac{2}{3}}d^{\frac{2}{3}}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**2/(d*x**3+c),x)
```

```
[Out] a*(a*d**(1/3) - 2*b*c**(1/3))*log(c**(1/3) + d**(1/3)*x)/(3*c**(2
/3)*d**(2/3)) - a*(a*d**(1/3) - 2*b*c**(1/3))*log(c**(2/3) - c**(
1/3)*d**(1/3)*x + d**(2/3)*x**2)/(6*c**(2/3)*d**(2/3)) - sqrt(3)*
a*(a*d**(1/3) + 2*b*c**(1/3))*atan(sqrt(3)*(c**(1/3)/3 - 2*d**(1/
3)*x/3)/c**(1/3))/(3*c**(2/3)*d**(2/3)) + b**2*log(c + d*x**3)/(3
*d)
```

Mathematica [A] time = 0.165063, size = 200, normalized size = 1.08

$$-\frac{\left(a^2\sqrt[3]{c}\sqrt[3]{d}-2abc^{2/3}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6cd^{2/3}}+\frac{\left(a^2\sqrt[3]{c}\sqrt[3]{d}-2abc^{2/3}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3cd^{2/3}}+\frac{\left(a^2\sqrt[3]{c}\sqrt[3]{d}+2abc^{2/3}\right)\tan^{-1}\left(\frac{2\sqrt[3]{d}x-\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}cd^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] ((2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3))*ArcTan[(-c^(1/3) + 2*d^(1/
3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c*d^(2/3)) + ((-2*a*b*c^(2/3)
+ a^2*c^(1/3)*d^(1/3))*Log[c^(1/3) + d^(1/3)*x])/(3*c*d^(2/3)) -
((-2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2])/(6*c*d^(2/3)) + (b^2*Log[c + d*x^3])/(3*d)
```

Maple [A] time = 0.004, size = 211, normalized size = 1.1

$$\frac{a^2}{3\,d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{a^2}{6\,d}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{a^2\sqrt{3}}{3\,d}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{2\,ab}{3\,d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}+\frac{ab}{3\,d}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}+\frac{2\,\sqrt{3}ab}{3\,d}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}+\frac{b^2\ln\left(dx^3+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^2/(d*x^3+c),x)
```

```
[Out] 1/3*a^2/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6*a^2/d/(c/d)^(2/3)*ln(
x^2-x*(c/d)^(1/3)+(c/d)^(2/3))+1/3*a^2/d/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))-2/3*a*b/d/(c/d)^(1/3)*ln(x+(c
/d)^(1/3))+1/3*a*b/d/(c/d)^(1/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3)
)+2/3*a*b*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)
*x-1))+1/3*b^2*ln(d*x^3+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^2/(d*x^3 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^2/(d*x^3 + c),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.98675, size = 156, normalized size = 0.84

$$\mathrm{RootSum}\left(27t^3c^2d^3 - 27t^2b^2c^2d^2 + t\left(18a^3bcd^2 + 9b^4c^2d\right) - a^6d^2 + 2a^3b^3cd - b^6c^2, \left(t \mapsto t\log\left(x + \frac{18t^2bc^2d^2 + 3ta^3cd^2 -}{a^5d^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**2/(d*x**3+c),x)
```

```
[Out] RootSum(27*_t**3*c**2*d**3 - 27*_t**2*b**2*c**2*d**2 + _t*(18*a**
3*b*c*d**2 + 9*b**4*c**2*d) - a**6*d**2 + 2*a**3*b**3*c*d - b**6*
c**2, Lambda(_t, _t*log(x + (18*_t**2*b*c**2*d**2 + 3*_t*a**3*c*d
**2 - 12*_t*b**3*c**2*d + 7*a**3*b**2*c*d + 2*b**5*c**2)/(a**5*d*
*2 + 8*a**2*b**3*c*d))))
```

GIAC/XCAS [A] time = 0.21605, size = 262, normalized size = 1.41

$$\frac{b^2\ln\left(\left|dx^3+c\right|\right)}{3\,d} - \frac{\left(2\,abd\left(-\frac{c}{d}\right)^{\frac{1}{3}} + a^2d\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd} + \frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}a^2d - 2\left(-cd^2\right)^{\frac{2}{3}}ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2} + \frac{\left(\left(-cd^2\right)^{\frac{1}{3}}a^2cd^3 + 2\left(-cd^2\right)^{\frac{2}{3}}abcd^2\right)\ln\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^2/(d*x^3 + c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*ln(abs(d*x^3 + c))/d - 1/3*(2*a*b*d*(-c/d)^(1/3) + a^2*d)
*(-c/d)^(1/3)*ln(abs(x - (-c/d)^(1/3)))/(c*d) + 1/3*sqrt(3)*((-c*
d^2)^(1/3)*a^2*d - 2*(-c*d^2)^(2/3)*a*b)*arctan(1/3*sqrt(3)*(2*x
+ (-c/d)^(1/3))/(-c/d)^(1/3))/(c*d^2) + 1/6*((-c*d^2)^(1/3)*a^2*c
*d^3 + 2*(-c*d^2)^(2/3)*a*b*c*d^2)*ln(x^2 + x*(-c/d)^(1/3) + (-c/
d)^(2/3))/(c^2*d^4)
```

3.71 $\int \frac{(a+bx)^3}{c+dx^3}\,dx$

Optimal. Leaf size=222

$$\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{\left(a^3(-d)-3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}+\frac{ab^2\log\left(c+dx^3\right)}{d}+\frac{b^3x}{d}$$

```
[Out] (b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^
(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)
) - ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1
/3)*x])/(3*c^(2/3)*d^(4/3)) + ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) -
 a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3
)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d
```

Rubi [A] time = 0.574891, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471

$$\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{\left(a^3(-d)-3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}+\frac{ab^2\log\left(c+dx^3\right)}{d}+\frac{b^3x}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^
(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)
) - ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1
/3)*x])/(3*c^(2/3)*d^(4/3)) + ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) -
 a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3
)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ab^2\log\left(c+dx^3\right)}{d}+\frac{\int b^3\,dx}{d}+\frac{\left(a^3d-3a^2b\sqrt[3]{c}d^{\frac{2}{3}}-b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{\frac{2}{3}}d^{\frac{4}{3}}}-\frac{\left(a^3d-3a^2b\sqrt[3]{c}d^{\frac{2}{3}}-b^3c\right)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}d^{\frac{4}{3}}}-\frac{\sqrt{3}\left(a^3d+3a^2b\sqrt[3]{c}d^{\frac{2}{3}}-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{\frac{2}{3}}d^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**3/(d*x**3+c),x)
```

```
[Out] a*b**2*log(c + d*x**3)/d + Integral(b**3, x)/d + (a**3*d - 3*a**2
*b*c**(1/3)*d**(2/3) - b**3*c)*log(c**(1/3) + d**(1/3)*x)/(3*c**(
2/3)*d**(4/3)) - (a**3*d - 3*a**2*b*c**(1/3)*d**(2/3) - b**3*c)*l
og(c**(2/3) - c**(1/3)*d**(1/3)*x + d**(2/3)*x**2)/(6*c**(2/3)*d*
*(4/3)) - sqrt(3)*(a**3*d + 3*a**2*b*c**(1/3)*d**(2/3) - b**3*c)*
atan(sqrt(3)*(c**(1/3)/3 - 2*d**(1/3)*x/3)/c**(1/3))/(3*c**(2/3)*
d**(4/3))
```

Mathematica [A] time = 0.331785, size = 214, normalized size = 0.96

$$\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-2\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)+2\sqrt{3}\left(a^3(-d)\right.}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (6*b^3*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b^3*c - 3*a^2*b*c^(1/3)*d^(
2/3) - a^3*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] - 2*(b^
3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x] +
 (b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2] + 6*a*b^2*c^(2/3)*d^(1/3)*Log[c + d*x^3]
)/(6*c^(2/3)*d^(4/3))
```

Maple [A] time = 0.006, size = 325, normalized size = 1.5

$$\begin{aligned}&\frac{b^3x}{d}+\frac{a^3}{3\,d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{b^3c}{3\,d^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{a^3}{6\,d}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}\\&+\frac{b^3c}{6\,d^2}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}a^3}{3\,d}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}\\&-\frac{\sqrt{3}b^3c}{3\,d^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{a^2b}{d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}\\&+\frac{a^2b}{2\,d}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}+\frac{a^2b\sqrt{3}}{d}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}+\frac{ab^2\ln\left(dx^3+c\right)}{d}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^3/(d*x^3+c),x)
```

```
[Out] b^3*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^3-1/3/d^2/(c/d)^(2/
3)*ln(x+(c/d)^(1/3))*b^3*c-1/6/d/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)
+(c/d)^(2/3))*a^3+1/6/d^2/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^
(2/3))*b^3*c+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d
)^(1/3)*x-1))*a^3-1/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(c/d)^(1/3)*x-1))*b^3*c-1/d*a^2*b/(c/d)^(1/3)*ln(x+(c/d)^(1/3)
)+1/2/d*a^2*b/(c/d)^(1/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))+1/d*a
^2*b*3^(1/2)/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))+
a*b^2*ln(d*x^3+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^3/(d*x^3 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^3/(d*x^3 + c),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 4.73957, size = 245, normalized size = 1.1

$$\frac{b^3x}{d} + \operatorname{RootSum}\left(27t^3c^2d^4 - 81t^2ab^2c^2d^3 + t\left(27a^5bcd^3 + 54a^2b^4c^2d^2\right) - a^9d^3 + 3a^6b^3cd^2 - 3a^3b^6c^2d + b^9c^3, \left(t \mapsto t\log\left(x + \frac{27t\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**3/(d*x**3+c),x)
```

```
[Out] b**3*x/d + RootSum(27*_t**3*c**2*d**4 - 81*_t**2*a*b**2*c**2*d**3
 + _t*(27*a**5*b*c*d**3 + 54*a**2*b**4*c**2*d**2) - a**9*d**3 + 3
*a**6*b**3*c*d**2 - 3*a**3*b**6*c**2*d + b**9*c**3, Lambda(_t, _t
*log(x + (27*_t**2*a**2*b*c**2*d**3 + 3*_t*a**6*c*d**3 - 60*_t*a*
*3*b**3*c**2*d**2 + 3*_t*b**6*c**3*d + 15*a**7*b**2*c*d**2 + 15*a
**4*b**5*c**2*d - 3*a*b**8*c**3)/(a**9*d**3 + 24*a**6*b**3*c*d**2
 + 3*a**3*b**6*c**2*d - b**9*c**3))))
```

GIAC/XCAS [A] time = 0.216398, size = 327, normalized size = 1.47

$$\frac{b^3x}{d} + \frac{ab^2\ln\left(\left|dx^3+c\right|\right)}{d} - \frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}b^3c - \left(-cd^2\right)^{\frac{1}{3}}a^3d + 3\left(-cd^2\right)^{\frac{2}{3}}a^2b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3cd^2} - \frac{\left(\left(-cd^2\right)^{\frac{1}{3}}b^3c - \left(-cd^2\right)^{\frac{1}{3}}a^3d - 3\left(-cd^2\right)^{\frac{2}{3}}a^2b\right)\ln\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6cd^2} - \frac{\left(3a^2bd^3\left(-\frac{c}{d}\right)^{\frac{1}{3}} - b^3cd^2 + a^3d^3\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^3/(d*x^3 + c),x, algorithm="giac")
```

```
[Out] b^3*x/d + a*b^2*ln(abs(d*x^3 + c))/d - 1/3*sqrt(3)*((-c*d^2)^(1/3
)*b^3*c - (-c*d^2)^(1/3)*a^3*d + 3*(-c*d^2)^(2/3)*a^2*b)*arctan(1
/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c*d^2) - 1/6*((-c*
d^2)^(1/3)*b^3*c - (-c*d^2)^(1/3)*a^3*d - 3*(-c*d^2)^(2/3)*a^2*b)
*ln(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c*d^2) - 1/3*(3*a^2*b*d
^3*(-c/d)^(1/3) - b^3*c*d^2 + a^3*d^3)*(-c/d)^(1/3)*ln(abs(x - (-
c/d)^(1/3)))/(c*d^3)
```

3.72 $\int \frac{(a+bx)^4}{c+dx^3}\,dx$

Optimal. Leaf size=282

$$\frac{2a^2b^2\log\left(c+dx^3\right)}{d}-\frac{\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)-\sqrt[3]{d}\left(4ab^3c-a^4d\right)\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{5/3}}+\frac{\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)-\sqrt[3]{d}\left(4ab^3c-a^4d\right)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{5/3}}+\frac{\left(a^4\left(-d^{4/3}\right)-4a^3b\sqrt[3]{c}d+4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}}+\frac{4ab^3x}{d}+\frac{b^4x^2}{2d}$$

```
[Out] (4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^(4/3) + 4*a*b^3*c*d^(1/
3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(c^(1/3) - 2*d^(1/3)
*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(5/3)) + ((b*c^(1/3)*(
b^3*c - 4*a^3*d) - d^(1/3)*(4*a*b^3*c - a^4*d))*Log[c^(1/3) + d^(
1/3)*x])/(3*c^(2/3)*d^(5/3)) - ((b*c^(1/3)*(b^3*c - 4*a^3*d) - d^
(1/3)*(4*a*b^3*c - a^4*d))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2])/(6*c^(2/3)*d^(5/3)) + (2*a^2*b^2*Log[c + d*x^3])/d
```

Rubi [A] time = 0.796534, antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471

$$\frac{2a^2b^2\log\left(c+dx^3\right)}{d}+\frac{\left(a^4(-d)-\frac{b\sqrt[3]{c}\left(b^3c-4a^3d\right)}{\sqrt[3]{d}}+4ab^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}+\frac{\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)-\sqrt[3]{d}\left(4ab^3c-a^4d\right)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{5/3}}+\frac{\left(a^4\left(-d^{4/3}\right)-4a^3b\sqrt[3]{c}d+4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}}+\frac{4ab^3x}{d}+\frac{b^4x^2}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^(4/3) + 4*a*b^3*c*d^(1/
3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(c^(1/3) - 2*d^(1/3)
*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(5/3)) + ((b*c^(1/3)*(
b^3*c - 4*a^3*d) - d^(1/3)*(4*a*b^3*c - a^4*d))*Log[c^(1/3) + d^(
1/3)*x])/(3*c^(2/3)*d^(5/3)) + ((4*a*b^3*c - a^4*d - (b*c^(1/3)*(
b^3*c - 4*a^3*d))/d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (2*a^2*b^2*Log[c + d*x^3])/d
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2a^2b^2\log\left(c+dx^3\right)}{d}+\frac{4ab^3x}{d}+\frac{b^4\int x\,dx}{d}-\frac{\sqrt{3}\left(a\sqrt[3]{d}\left(a^3d-4b^3c\right)+b\sqrt[3]{c}\left(4a^3d-b^3c\right)\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{\frac{2}{3}}d^{\frac{5}{3}}}+\frac{\left(a^4d^{\frac{4}{3}}-4a^3b\sqrt[3]{c}d-4ab^3c\sqrt[3]{d}+b^4c^{\frac{4}{3}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{\frac{2}{3}}d^{\frac{5}{3}}}-\frac{\left(a^4d^{\frac{4}{3}}-4a^3b\sqrt[3]{c}d-4ab^3c\sqrt[3]{d}+b^4c^{\frac{4}{3}}\right)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}d^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**4/(d*x**3+c),x)
```

```
[Out] 2*a**2*b**2*log(c + d*x**3)/d + 4*a*b**3*x/d + b**4*Integral(x, x
)/d - sqrt(3)*(a*d**(1/3)*(a**3*d - 4*b**3*c) + b*c**(1/3)*(4*a**
3*d - b**3*c))*atan(sqrt(3)*(c**(1/3)/3 - 2*d**(1/3)*x/3)/c**(1/3
))/(3*c**(2/3)*d**(5/3)) + (a**4*d**(4/3) - 4*a**3*b*c**(1/3)*d -
 4*a*b**3*c*d**(1/3) + b**4*c**(4/3))*log(c**(1/3) + d**(1/3)*x)/
(3*c**(2/3)*d**(5/3)) - (a**4*d**(4/3) - 4*a**3*b*c**(1/3)*d - 4*
a*b**3*c*d**(1/3) + b**4*c**(4/3))*log(c**(2/3) - c**(1/3)*d**(1/
3)*x + d**(2/3)*x**2)/(6*c**(2/3)*d**(5/3))
```

Mathematica [A] time = 0.49217, size = 277, normalized size = 0.98

$$\frac{12a^2b^2d^{2/3}\log\left(c+dx^3\right)-\frac{\left(a^4d^{4/3}-4a^3b\sqrt[3]{c}d-4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{2/3}}+\frac{2\left(a^4d^{4/3}-4a^3b\sqrt[3]{c}d-4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\log\left(\sqrt[3]{\right.}}{c^{2/3}}}{6d^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (24*a*b^3*d^(2/3)*x + 3*b^4*d^(2/3)*x^2 + (2*Sqrt[3]*(b^4*c^(4/3)
 + 4*a*b^3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(1
 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) + (2*(b^4*c^(4/3) - 4
*a*b^3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d + a^4*d^(4/3))*Log[c^(1/3) +
 d^(1/3)*x])/c^(2/3) - ((b^4*c^(4/3) - 4*a*b^3*c*d^(1/3) - 4*a^3*
b*c^(1/3)*d + a^4*d^(4/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2])/c^(2/3) + 12*a^2*b^2*d^(2/3)*Log[c + d*x^3])/(6*d^(5/3)
)
```

Maple [A] time = 0.006, size = 446, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^4/(d*x^3+c),x)
```

```
[Out] 1/2*b^4*x^2/d+4*a*b^3*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^4
-4/3/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a*b^3*c-1/6/d/(c/d)^(2/3)*
ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*a^4+2/3/d^2/(c/d)^(2/3)*ln(x^2-
x*(c/d)^(1/3)+(c/d)^(2/3))*a*b^3*c+1/3/d/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^4-4/3/d^2/(c/d)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a*b^3*c-4/3/d/(c/d)^(1
/3)*ln(x+(c/d)^(1/3))*a^3*b+1/3/d^2/(c/d)^(1/3)*ln(x+(c/d)^(1/3))
*b^4*c+2/3/d/(c/d)^(1/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*a^3*b-
1/6/d^2/(c/d)^(1/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*b^4*c+4/3/d
*3^(1/2)/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^3*
b-1/3/d^2*3^(1/2)/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x
-1))*b^4*c+2*a^2*b^2*ln(d*x^3+c)/d
```

Maple [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^4/(d*x^3 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^4/(d*x^3 + c),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 8.68548, size = 325, normalized size = 1.15

$$\frac{4ab^3x}{d}+\frac{b^4x^2}{2d}$$
$$+\mathrm{RootSum}\left(27t^3c^2d^5-162t^2a^2b^2c^2d^4+t\left(36a^7bcd^4+171a^4b^4c^2d^3+36ab^7c^3d^2\right)-a^{12}d^4+4a^9b^3cd^3-6a^6b^6c^2d^2+4a^3b^9c^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**4/(d*x**3+c),x)
```

```
[Out] 4*a*b**3*x/d + b**4*x**2/(2*d) + RootSum(27*_t**3*c**2*d**5 - 162
*_t**2*a**2*b**2*c**2*d**4 + _t*(36*a**7*b*c*d**4 + 171*a**4*b**4
*c**2*d**3 + 36*a*b**7*c**3*d**2) - a**12*d**4 + 4*a**9*b**3*c*d*
*3 - 6*a**6*b**6*c**2*d**2 + 4*a**3*b**9*c**3*d - b**12*c**4, Lam
bda(_t, _t*log(x + (36*_t**2*a**3*b*c**2*d**4 - 9*_t**2*b**4*c**3
*d**3 + 3*_t*a**8*c*d**4 - 168*_t*a**5*b**3*c**2*d**3 + 84*_t*a**
2*b**6*c**3*d**2 + 26*a**10*b**2*c*d**3 + 48*a**7*b**5*c**2*d**2
- 66*a**4*b**8*c**3*d - 8*a*b**11*c**4)/(a**12*d**4 + 52*a**9*b**
3*c*d**3 - 52*a**3*b**9*c**3*d - b**12*c**4))))
```

GIAC/XCAS [A] time = 0.215461, size = 427, normalized size = 1.51

$$\frac{2\,a^2b^2\ln\left(\left|dx^3+c\right|\right)}{d}+\frac{b^4dx^2+8\,ab^3dx}{2\,d^2}$$
$$-\frac{\sqrt{3}\left(4\,\left(-cd^2\right)^{\frac{1}{3}}ab^3cd-\left(-cd^2\right)^{\frac{1}{3}}a^4d^2-\left(-cd^2\right)^{\frac{2}{3}}b^4c+4\,\left(-cd^2\right)^{\frac{2}{3}}a^3bd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^3}$$
$$-\frac{\left(4\,\left(-cd^2\right)^{\frac{1}{3}}ab^3cd-\left(-cd^2\right)^{\frac{1}{3}}a^4d^2+\left(-cd^2\right)^{\frac{2}{3}}b^4c-4\,\left(-cd^2\right)^{\frac{2}{3}}a^3bd\right)\ln\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^3}$$
$$+\frac{\left(b^4cd^4\left(-\frac{c}{d}\right)^{\frac{1}{3}}-4\,a^3bd^5\left(-\frac{c}{d}\right)^{\frac{1}{3}}+4\,ab^3cd^4-a^4d^5\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)^4/(d*x^3 + c),x, algorithm="giac")
```

```
[Out] 2*a^2*b^2*ln(abs(d*x^3 + c))/d + 1/2*(b^4*d*x^2 + 8*a*b^3*d*x)/d^
2 - 1/3*sqrt(3)*(4*(-c*d^2)^(1/3)*a*b^3*c*d - (-c*d^2)^(1/3)*a^4*
d^2 - (-c*d^2)^(2/3)*b^4*c + 4*(-c*d^2)^(2/3)*a^3*b*d)*arctan(1/3
```

```
*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c*d^3) - 1/6*(4*(-c*
d^2)^(1/3)*a*b^3*c*d - (-c*d^2)^(1/3)*a^4*d^2 + (-c*d^2)^(2/3)*b^
4*c - 4*(-c*d^2)^(2/3)*a^3*b*d)*ln(x^2 + x*(-c/d)^(1/3) + (-c/d)^
(2/3))/(c*d^3) + 1/3*(b^4*c*d^4*(-c/d)^(1/3) - 4*a^3*b*d^5*(-c/d)
^(1/3) + 4*a*b^3*c*d^4 - a^4*d^5)*(-c/d)^(1/3)*ln(abs(x - (-c/d)^
(1/3)))/(c*d^5)
```

3.73 $\int \frac{(a+bx+cx^2)^2}{d+ex^3} dx$

Optimal. Leaf size=272

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(\sqrt[3]{e}\left(2bcd-a^2e\right)-\sqrt[3]{d}\left(c^2d-2abe\right)\right)}{6d^{2/3}e^{5/3}} - \frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}\left(2bcd-a^2e\right)-\sqrt[3]{d}\left(c^2d-2abe\right)\right)}{3d^{2/3}e^{5/3}} + \frac{(2ac+b^2)\log\left(d+ex^3\right)}{3e} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-ae\left(a\sqrt[3]{e}+2b\sqrt[3]{d}\right)+2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{\sqrt{3}d^{2/3}e^{5/3}} + \frac{2bcx}{e} + \frac{c^2x^2}{2e}$$

```
[Out] (2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^(4/3) + 2*b*c*d*e^(1/3) -
 a*(2*b*d^(1/3) + a*e^(1/3))*e)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(S
qrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(5/3)) - ((e^(1/3)*(2*b*c*d
- a^2*e) - d^(1/3)*(c^2*d - 2*a*b*e))*Log[d^(1/3) + e^(1/3)*x])/(
3*d^(2/3)*e^(5/3)) + ((e^(1/3)*(2*b*c*d - a^2*e) - d^(1/3)*(c^2*d
 - 2*a*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d
^(2/3)*e^(5/3)) + ((b^2 + 2*a*c)*Log[d + e*x^3])/(3*e)
```

Rubi [A] time = 0.889187, antiderivative size = 270, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^2(-e)-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}+2bcd\right)}{6d^{2/3}e^{4/3}} - \frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}\left(2bcd-a^2e\right)-\sqrt[3]{d}\left(c^2d-2abe\right)\right)}{3d^{2/3}e^{5/3}} + \frac{(2ac+b^2)\log\left(d+ex^3\right)}{3e} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-ae\left(a\sqrt[3]{e}+2b\sqrt[3]{d}\right)+2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{\sqrt{3}d^{2/3}e^{5/3}} + \frac{2bcx}{e} + \frac{c^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] (2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^(4/3) + 2*b*c*d*e^(1/3) -
 a*(2*b*d^(1/3) + a*e^(1/3))*e)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(S
qrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(5/3)) - ((e^(1/3)*(2*b*c*d
- a^2*e) - d^(1/3)*(c^2*d - 2*a*b*e))*Log[d^(1/3) + e^(1/3)*x])/(
3*d^(2/3)*e^(5/3)) + ((2*b*c*d - a^2*e - (d^(1/3)*(c^2*d - 2*a*b*
e))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d
^(2/3)*e^(4/3)) + ((b^2 + 2*a*c)*Log[d + e*x^3])/(3*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2bcx}{e} + \frac{c^2\int x\,dx}{e} + \frac{(2ac+b^2)\log\left(d+ex^3\right)}{3e} + \frac{\left(a^2e^{\frac{4}{3}}-2ab\sqrt[3]{d}e-2bcd\sqrt[3]{e}+c^2d^{\frac{4}{3}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{\frac{2}{3}}e^{\frac{5}{3}}} - \frac{\left(a^2e^{\frac{4}{3}}-2ab\sqrt[3]{d}e-2bcd\sqrt[3]{e}+c^2d^{\frac{4}{3}}\right)\log\left(d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2\right)}{6d^{\frac{2}{3}}e^{\frac{5}{3}}} - \frac{\sqrt{3}\left(a^2e^{\frac{4}{3}}+2ab\sqrt[3]{d}e-2bcd\sqrt[3]{e}-c^2d^{\frac{4}{3}}\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{d}}{3}-\frac{2\sqrt[3]{e}x}{3}\right)}{\sqrt[3]{d}}\right)}{3d^{\frac{2}{3}}e^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**2/(e*x**3+d),x)
```

```
[Out] 2*b*c*x/e + c**2*Integral(x, x)/e + (2*a*c + b**2)*log(d + e*x**3
)/(3*e) + (a**2*e**(4/3) - 2*a*b*d**(1/3)*e - 2*b*c*d*e**(1/3) +
c**2*d**(4/3))*log(d**(1/3) + e**(1/3)*x)/(3*d**(2/3)*e**(5/3)) -
 (a**2*e**(4/3) - 2*a*b*d**(1/3)*e - 2*b*c*d*e**(1/3) + c**2*d**(
4/3))*log(d**(2/3) - d**(1/3)*e**(1/3)*x + e**(2/3)*x**2)/(6*d**(
2/3)*e**(5/3)) - sqrt(3)*(a**2*e**(4/3) + 2*a*b*d**(1/3)*e - 2*b*
c*d*e**(1/3) - c**2*d**(4/3))*atan(sqrt(3)*(d**(1/3)/3 - 2*e**(1/
3)*x/3)/d**(1/3))/(3*d**(2/3)*e**(5/3))
```

Mathematica [A] time = 0.799703, size = 269, normalized size = 0.99

$$\frac{2e^{2/3}\left(2ac+b^2\right)\log\left(d+ex^3\right)-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae\left(a\sqrt[3]{e}-2b\sqrt[3]{d}\right)-2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{d^{2/3}}+\frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae\left(a\sqrt[3]{e}-2b\sqrt[3]{d}\right)-2bc\right.}{d^{2/3}}}{6e^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] (12*b*c*e^(2/3)*x + 3*c^2*e^(2/3)*x^2 + (2*Sqrt[3]*(c*d^(2/3) - a
*e^(2/3))*(c*d^(2/3) + 2*b*d^(1/3)*e^(1/3) + a*e^(2/3))*ArcTan[(1
 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (2*(c^2*d^(4/3) - 2
*b*c*d*e^(1/3) + a*(-2*b*d^(1/3) + a*e^(1/3))*e)*Log[d^(1/3) + e^
(1/3)*x])/d^(2/3) - ((c^2*d^(4/3) - 2*b*c*d*e^(1/3) + a*(-2*b*d^(
1/3) + a*e^(1/3))*e)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^
2])/d^(2/3) + 2*(b^2 + 2*a*c)*e^(2/3)*Log[d + e*x^3])/(6*e^(5/3))
```

Maple [B] time = 0.007, size = 444, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^2/(e*x^3+d),x)
```

```
[Out] 1/2*c^2*x^2/e+2*b*c*x/e+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^2-2
/3/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*b*c*d-1/6/e/(d/e)^(2/3)*ln(x
^2-x*(d/e)^(1/3)+(d/e)^(2/3))*a^2+1/3/e^2/(d/e)^(2/3)*ln(x^2-x*(d
/e)^(1/3)+(d/e)^(2/3))*b*c*d+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2-2/3/e^2/(d/e)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b*c*d-2/3/e/(d/e)^(1/3)*ln(x
+(d/e)^(1/3))*a*b+1/3/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*c^2*d+1/3
/e/(d/e)^(1/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*a*b-1/6/e^2/(d/e
)^(1/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*c^2*d+2/3/e*3^(1/2)/(d/
e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b-1/3/e^2*3^(1
/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c^2*d+2/3
/e*ln(e*x^3+d)*a*c+1/3/e*ln(e*x^3+d)*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^2/(e*x^3 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^2/(e*x^3 + d),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 15.6229, size = 546, normalized size = 2.01

$$\frac{2bcx}{e}+\frac{c^2x^2}{2e}$$
$$+\mathrm{RootSum}\left(27t^3d^2e^5+t^2\left(-54acd^2e^4-27b^2d^2e^4\right)+t\left(18a^3bde^4+27a^2c^2d^2e^3+9b^4d^2e^3+18bc^3d^3e^2\right)-a^6e^4-6a^4bcde^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**2/(e*x**3+d),x)
```

```
[Out] 2*b*c*x/e + c**2*x**2/(2*e) + RootSum(27*_t**3*d**2*e**5 + _t**2*
(-54*a*c*d**2*e**4 - 27*b**2*d**2*e**4) + _t*(18*a**3*b*d*e**4 +
27*a**2*c**2*d**2*e**3 + 9*b**4*d**2*e**3 + 18*b*c**3*d**3*e**2)
- a**6*e**4 - 6*a**4*b*c*d*e**3 + 2*a**3*b**3*d*e**3 - 2*a**3*c**
3*d**2*e**2 - 9*a**2*b**2*c**2*d**2*e**2 + 6*a*b**4*c*d**2*e**2 -
 6*a*b*c**4*d**3*e - b**6*d**2*e**2 + 2*b**3*c**3*d**3*e - c**6*d
**4, Lambda(_t, _t*log(x + (18*_t**2*a*b*d**2*e**4 - 9*_t**2*c**2
*d**3*e**3 + 3*_t*a**4*d*e**4 - 36*_t*a**2*b*c*d**2*e**3 - 12*_t*
a*b**3*d**2*e**3 + 12*_t*a*c**3*d**3*e**2 + 18*_t*b**2*c**2*d**3*
e**2 - 2*a**5*c*d*e**3 + 7*a**4*b**2*d*e**3 + 8*a**3*b*c**2*d**2*
e**2 - 4*a**2*b**3*c*d**2*e**2 - 2*a**2*c**4*d**3*e + 2*a*b**5*d*
*2*e**2 + 4*a*b**2*c**3*d**3*e - 5*b**4*c**2*d**3*e - 4*b*c**5*d*
*4)/(a**6*e**4 - 6*a**4*b*c*d*e**3 + 8*a**3*b**3*d*e**3 + 6*a*b*c
**4*d**3*e - 8*b**3*c**3*d**3*e - c**6*d**4)))
```

GIAC/XCAS [A] time = 0.218029, size = 383, normalized size = 1.41

$$\frac{1}{3}\left(b^2+2\,ac\right)e^{(-1)}\ln\left(\left|x^3e+d\right|\right)$$
$$-\frac{\sqrt{3}\left(2\left(-de^2\right)^{\frac{1}{3}}bcde-\left(-de^2\right)^{\frac{2}{3}}c^2d+2\left(-de^2\right)^{\frac{2}{3}}abe-\left(-de^2\right)^{\frac{1}{3}}a^2e^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-3)}}{3\,d}$$
$$+\frac{\left(\left(-de^{(-1)}\right)^{\frac{1}{3}}c^2de^4+2\,bcde^4-2\left(-de^{(-1)}\right)^{\frac{1}{3}}abe^5-a^2e^5\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-5)}\ln\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3\,d}$$
$$+\frac{1}{2}\left(c^2x^2e+4\,bcxe\right)e^{(-2)}$$
$$-\frac{\left(2\left(-de^2\right)^{\frac{1}{3}}bcde+\left(-de^2\right)^{\frac{2}{3}}c^2d-2\left(-de^2\right)^{\frac{2}{3}}abe-\left(-de^2\right)^{\frac{1}{3}}a^2e^2\right)e^{(-3)}\ln\left(x^2+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^2/(e*x^3 + d),x, algorithm="giac")
```

```
[Out] 1/3*(b^2 + 2*a*c)*e^(-1)*ln(abs(x^3*e + d)) - 1/3*sqrt(3)*(2*(-d*
e^2)^(1/3)*b*c*d*e - (-d*e^2)^(2/3)*c^2*d + 2*(-d*e^2)^(2/3)*a*b*
e - (-d*e^2)^(1/3)*a^2*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))
^(1/3))/(-d*e^(-1))^(1/3))*e^(-3)/d + 1/3*((-d*e^(-1))^(1/3)*c^2*
d*e^4 + 2*b*c*d*e^4 - 2*(-d*e^(-1))^(1/3)*a*b*e^5 - a^2*e^5)*(-d*
e^(-1))^(1/3)*e^(-5)*ln(abs(x - (-d*e^(-1))^(1/3)))/d + 1/2*(c^2*
x^2*e + 4*b*c*x*e)*e^(-2) - 1/6*(2*(-d*e^2)^(1/3)*b*c*d*e + (-d*e
^2)^(2/3)*c^2*d - 2*(-d*e^2)^(2/3)*a*b*e - (-d*e^2)^(1/3)*a^2*e^2
)*e^(-3)*ln(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/d
```

3.74 $\int \frac{(a+bx+cx^2)^3}{d+ex^3} dx$

Optimal. Leaf size=416

$$-\frac{\log(d+ex^3)\left(a^2(-c)e-ab^2e+bc^2d\right)}{e^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(-e\left(b^3d-a^3e\right)+3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-6abcde+c^3d^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(-e\left(b^3d-a^3e\right)+3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-6abcde+c^3d^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(a^3e^2+3a^2b\sqrt[3]{d}e^{5/3}-6abcde-3ac^2d^{4/3}e^{2/3}-b^3de-3b^2cd^{4/3}e^{2/3}+c^3d^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}-\frac{x\left(-6abce+b^3(-e)+c^3d\right)}{e^2}+\frac{3cx^2\left(ac+b^2\right)}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}$$

```
[Out] -(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2
*e) + (b*c^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/
3)*e^(2/3) - 3*a*c^2*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*
a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(
Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c^3*d^2 - 6*a*b*
c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d + a*c^2*d
- a^2*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) - ((c^3
*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c
*d + a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3
)*x^2])/(6*d^(2/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[
d + e*x^3])/e^2
```

Rubi [A] time = 1.31076, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{\log(d+ex^3)\left(a^2(-c)e-ab^2e+bc^2d\right)}{e^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(-e\left(b^3d-a^3e\right)+3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-6abcde+c^3d^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(-e\left(b^3d-a^3e\right)+3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-6abcde+c^3d^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(a^3e^2+3a^2b\sqrt[3]{d}e^{5/3}-6abcde-3ac^2d^{4/3}e^{2/3}-b^3de-3b^2cd^{4/3}e^{2/3}+c^3d^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}-\frac{x\left(-6abce+b^3(-e)+c^3d\right)}{e^2}+\frac{3cx^2\left(ac+b^2\right)}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```


```
[Out] -(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2
*e) + (b*c^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/
3)*e^(2/3) - 3*a*c^2*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*
a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(
Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c^3*d^2 - 6*a*b*
c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d + a*c^2*d
- a^2*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) - ((c^3
*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c
*d + a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3
)*x^2])/(6*d^(2/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[
d + e*x^3])/e^2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**3/(e*x**3+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.866991, size = 439, normalized size = 1.06

$$12\sqrt[3]{e}\log\left(d+ex^3\right)\left(a^2ce+ab^2e-bc^2d\right)-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(e\left(a^3e+3a^2b\sqrt[3]{d}e^{2/3}-b^3d\right)-3c\left(2abde+b^2d^{4/3}e^{2/3}\right)-3ac^2d^{4/3}e^{2/3}+c^3d^2\right)}{d^{2/3}}-\cdots$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c^3*d) + b^3*e + 6*a*b*c*e)*x + 18*c*(b^2 + a*c)*e
^(4/3)*x^2 + 12*b*c^2*e^(4/3)*x^3 + 3*c^3*e^(4/3)*x^4 - (4*Sqrt[3
]*(c^3*d^2 - 3*a*c^2*d^(4/3)*e^(2/3) + e*(-(b^3*d) + 3*a^2*b*d^(1
/3)*e^(2/3) + a^3*e) - 3*c*(b^2*d^(4/3)*e^(2/3) + 2*a*b*d*e))*Arc
Tan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (4*(c^3*d^2 +
 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4/3)*e^(2/3) - b^3*d*e - 6*
a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Log[d^(1/3) + e^(1
/3)*x])/d^(2/3) - (2*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2
*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3
) + a^3*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2
/3) + 12*e^(1/3)*(-(b*c^2*d) + a*b^2*e + a^2*c*e)*Log[d + e*x^3])
/(12*e^(7/3))
```

Maple [B] time = 0.008, size = 837, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^3/(e*x^3+d),x)
```

```
[Out] -2/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)
)*a*b*c*d-1/3/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)
^(1/3)*x-1))*b^3*d+1/3/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(d/e)^(1/3)*x-1))*c^3*d^2+1/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))
*a*c^2*d+1/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*b^2*c*d-1/2/e^2/(d/e
)^(1/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*a*c^2*d-1/2/e^2/(d/e)^(
1/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*b^2*c*d+1/e*3^(1/2)/(d/e)^
(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2*b+1/3/e/(d/e)^(
2/3)*ln(x+(d/e)^(1/3))*a^3+1/2/e/(d/e)^(1/3)*ln(x^2-x*(d/e)^(1/3)
+(d/e)^(2/3))*a^2*b-1/e^2*ln(e*x^3+d)*b*c^2*d-1/6/e^3/(d/e)^(2/3)
*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*c^3*d^2+1/3/e/(d/e)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^3+1/6/e^2/(d/e)^(2/
3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*b^3*d-2/e^2/(d/e)^(2/3)*ln(x
+(d/e)^(1/3))*a*b*c*d-1/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2
)*(2/(d/e)^(1/3)*x-1))*a*c^2*d+1/3/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/
3))*c^3*d^2+6/e*a*b*c*x-1/6/e/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d
/e)^(2/3))*a^3+1/e*ln(e*x^3+d)*a^2*c+1/e*ln(e*x^3+d)*a*b^2+1/4*c^
3*x^4/e+b*c^2*x^3/e+1/e*b^3*x+3/2/e*x^2*a*c^2+3/2/e*b^2*c*x^2-1/e
^2*c^3*d*x-1/3/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*b^3*d-1/e/(d/e)^
```

```
(1/3)*ln(x+(d/e)^(1/3))*a^2*b-1/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/
3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b^2*c*d+1/e^2/(d/e)^(2/3)*ln(x^2-x
*(d/e)^(1/3)+(d/e)^(2/3))*a*b*c*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^3/(e*x^3 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^3/(e*x^3 + d),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 124.454, size = 1314, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**3/(e*x**3+d),x)
```

```
[Out] b*c**2*x**3/e + c**3*x**4/(4*e) + RootSum(27*_t**3*d**2*e**7 + _t
**2*(-81*a**2*c*d**2*e**6 - 81*a*b**2*d**2*e**6 + 81*b*c**2*d**3*
e**5) + _t*(27*a**5*b*d*e**6 + 54*a**4*c**2*d**2*e**5 - 27*a**3*b
**2*c*d**2*e**5 + 54*a**2*b**4*d**2*e**5 + 27*a**2*b*c**3*d**3*e*
*4 + 27*a*b**3*c**2*d**3*e**4 - 27*a*c**5*d**4*e**3 + 27*b**5*c*d
**3*e**4 + 54*b**2*c**4*d**4*e**3) - a**9*e**6 - 9*a**7*b*c*d*e**
5 + 3*a**6*b**3*d*e**5 - 3*a**6*c**3*d**2*e**4 - 27*a**5*b**2*c**
2*d**2*e**4 + 18*a**4*b**4*c*d**2*e**4 - 18*a**4*b*c**4*d**3*e**3
 - 3*a**3*b**6*d**2*e**4 - 21*a**3*b**3*c**3*d**3*e**3 - 3*a**3*c
**6*d**4*e**2 + 27*a**2*b**5*c**2*d**3*e**3 - 27*a**2*b**2*c**5*d
**4*e**2 - 9*a*b**7*c*d**3*e**3 + 18*a*b**4*c**4*d**4*e**2 - 9*a*
b*c**7*d**5*e + b**9*d**3*e**3 - 3*b**6*c**3*d**4*e**2 + 3*b**3*c
**6*d**5*e - c**9*d**6, Lambda(_t, _t*log(x + (27*_t**2*a**2*b*d*
*2*e**6 - 27*_t**2*a*c**2*d**3*e**5 - 27*_t**2*b**2*c*d**3*e**5 +
 3*_t*a**6*d*e**6 - 90*_t*a**4*b*c*d**2*e**5 - 60*_t*a**3*b**3*d*
*2*e**5 + 60*_t*a**3*c**3*d**3*e**4 + 270*_t*a**2*b**2*c**2*d**3*
e**4 + 90*_t*a*b**4*c*d**3*e**4 - 90*_t*a*b*c**4*d**4*e**3 + 3*_t
*b**6*d**3*e**4 - 60*_t*b**3*c**3*d**4*e**3 + 3*_t*c**6*d**5*e**2
 - 3*a**8*c*d*e**5 + 15*a**7*b**2*d*e**5 + 30*a**6*b*c**2*d**2*e*
*4 - 48*a**5*b**3*c*d**2*e**4 - 15*a**5*c**4*d**3*e**3 + 15*a**4*
b**5*d**2*e**4 - 15*a**4*b**2*c**3*d**3*e**3 - 15*a**3*b**4*c**2*
d**3*e**3 - 48*a**3*b*c**5*d**4*e**2 - 30*a**2*b**6*c*d**3*e**3 +
 15*a**2*b**3*c**4*d**4*e**2 + 15*a**2*c**7*d**5*e - 3*a*b**8*d**
3*e**3 - 48*a*b**5*c**3*d**4*e**2 - 30*a*b**2*c**6*d**5*e - 15*b*
*7*c**2*d**4*e**2 - 15*b**4*c**5*d**5*e + 3*b*c**8*d**6)/(a**9*e*
*6 - 18*a**7*b*c*d*e**5 + 24*a**6*b**3*d*e**5 + 3*a**6*c**3*d**2*
e**4 + 27*a**5*b**2*c**2*d**2*e**4 - 45*a**4*b**4*c*d**2*e**4 + 4
5*a**4*b*c**4*d**3*e**3 + 3*a**3*b**6*d**2*e**4 - 60*a**3*b**3*c*
```

```
*3*d**3*e**3 - 24*a**3*c**6*d**4*e**2 - 27*a**2*b**5*c**2*d**3*e*
*3 + 27*a**2*b**2*c**5*d**4*e**2 - 18*a*b**7*c*d**3*e**3 - 45*a*b
**4*c**4*d**4*e**2 - 18*a*b*c**7*d**5*e - b**9*d**3*e**3 - 24*b**
6*c**3*d**4*e**2 - 3*b**3*c**6*d**5*e + c**9*d**6)))) + x**2*(3*a
*c**2 + 3*b**2*c)/(2*e) + x*(6*a*b*c*e + b**3*e - c**3*d)/e**2
```

GIAC/XCAS [A] time = 0.224708, size = 648, normalized size = 1.56

$$-\left(bc^2d - ab^2e - a^2ce\right)e^{(-2)}\ln\left(\left|x^3e + d\right|\right)$$
$$+\frac{\sqrt{3}\left(\left(-de^2\right)^{\frac{1}{3}}c^3d^2 - \left(-de^2\right)^{\frac{1}{3}}b^3de - 6\left(-de^2\right)^{\frac{1}{3}}abcde + 3\left(-de^2\right)^{\frac{2}{3}}b^2cd + 3\left(-de^2\right)^{\frac{2}{3}}ac^2d - 3\left(-de^2\right)^{\frac{2}{3}}a^2be + \left(-de^2\right)^{\frac{1}{3}}a\right.}{3d}$$
$$-\frac{\left(c^3d^2e^7 - 3\left(-de^{(-1)}\right)^{\frac{1}{3}}b^2cde^8 - 3\left(-de^{(-1)}\right)^{\frac{1}{3}}ac^2de^8 - b^3de^8 - 6\,abcde^8 + 3\left(-de^{(-1)}\right)^{\frac{1}{3}}a^2be^9 + a^3e^9\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-9)}\mathrm{l}}{3d}$$
$$+\frac{1}{4}\left(c^3x^4e^3 + 4\,bc^2x^3e^3 + 6\,b^2cx^2e^3 + 6\,ac^2x^2e^3 - 4\,c^3dxe^2 + 4\,b^3xe^3 + 24\,abcxe^3\right)e^{(-4)}$$
$$+\frac{\left(\left(-de^2\right)^{\frac{1}{3}}c^3d^2 - \left(-de^2\right)^{\frac{1}{3}}b^3de - 6\left(-de^2\right)^{\frac{1}{3}}abcde - 3\left(-de^2\right)^{\frac{2}{3}}b^2cd - 3\left(-de^2\right)^{\frac{2}{3}}ac^2d + 3\left(-de^2\right)^{\frac{2}{3}}a^2be + \left(-de^2\right)^{\frac{1}{3}}a^3e^2\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^3/(e*x^3 + d),x, algorithm="giac")
```

```
[Out] -(b*c^2*d - a*b^2*e - a^2*c*e)*e^(-2)*ln(abs(x^3*e + d)) + 1/3*sq
rt(3)*((-d*e^2)^(1/3)*c^3*d^2 - (-d*e^2)^(1/3)*b^3*d*e - 6*(-d*e^
2)^(1/3)*a*b*c*d*e + 3*(-d*e^2)^(2/3)*b^2*c*d + 3*(-d*e^2)^(2/3)*
a*c^2*d - 3*(-d*e^2)^(2/3)*a^2*b*e + (-d*e^2)^(1/3)*a^3*e^2)*arct
an(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-3
)/d - 1/3*(c^3*d^2*e^7 - 3*(-d*e^(-1))^(1/3)*b^2*c*d*e^8 - 3*(-d*
e^(-1))^(1/3)*a*c^2*d*e^8 - b^3*d*e^8 - 6*a*b*c*d*e^8 + 3*(-d*e^(
-1))^(1/3)*a^2*b*e^9 + a^3*e^9)*(-d*e^(-1))^(1/3)*e^(-9)*ln(abs(x
 - (-d*e^(-1))^(1/3)))/d + 1/4*(c^3*x^4*e^3 + 4*b*c^2*x^3*e^3 + 6
*b^2*c*x^2*e^3 + 6*a*c^2*x^2*e^3 - 4*c^3*d*x*e^2 + 4*b^3*x*e^3 +
24*a*b*c*x*e^3)*e^(-4) + 1/6*((-d*e^2)^(1/3)*c^3*d^2 - (-d*e^2)^(
1/3)*b^3*d*e - 6*(-d*e^2)^(1/3)*a*b*c*d*e - 3*(-d*e^2)^(2/3)*b^2*
c*d - 3*(-d*e^2)^(2/3)*a*c^2*d + 3*(-d*e^2)^(2/3)*a^2*b*e + (-d*e
^2)^(1/3)*a^3*e^2)*e^(-3)*ln(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-
1))^(2/3))/d
```

3.75 $\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$

Optimal. Leaf size=645

$$-\frac{x^2\left(-6a^2c^2e-12ab^2ce+b^4(-e)+4bc^3d\right)}{2e^2}-\frac{2x\left(-6a^2bce-2ab^3e+2ac^3d+3b^2c^2d\right)}{e^2}+\frac{\log\left(d+ex^3\right)\left(-4ce\left(b^3d-a^3e\right)+6a^2b^2e^2-12abc^2de+c^4d^2\right)}{3e^3}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-e\left(a^3(-e)-3a^2b\sqrt[3]{d}e^{2/3}+3ab^2d^{2/3}\sqrt[3]{e}+b^3d\right)+6c^2\left(bd^{5/3}\sqrt[3]{e}-ad^{4/3}e^{2/3}\right)-12abcde\cdots}{\sqrt{3}d^{2/3}e^{8/3}}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(\sqrt[3]{e}\left(a^4e^2-12a^2bcde-4ab^3de+4ac^3d^2+6b^2c^2d^2\right)+\sqrt[3]{d}\left(-4b\left(a^3e^2+c^3d^2\right)+6a^2c^2de+1\cdots\right.\right.}{6d^{2/3}e^{8/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}\left(a^4e^2-12a^2bcde-4ab^3de+4ac^3d^2+6b^2c^2d^2\right)+\sqrt[3]{d}\left(-4b\left(a^3e^2+c^3d^2\right)+6a^2c^2de+12ab^2cde+b^4\cdots\right.\right.}{3d^{2/3}e^{8/3}}-\frac{cx^3\left(-12abce-4b^3e+c^3d\right)}{3e^2}+\frac{c^2x^4\left(2ac+3b^2\right)}{2e}+\frac{4bc^3x^5}{5e}+\frac{c^4x^6}{6e}$$

```
[Out] (-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 -
((4*b*c^3*d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) -
(c*(c^3*d - 4*b^3*e - 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*
a*c)*x^4)/(2*e) + (4*b*c^3*x^5)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^(
1/3) + a*e^(1/3))*(4*c^3*d^2 + 6*c^2*(b*d^(5/3)*e^(1/3) - a*d^(4/
3)*e^(2/3)) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b^2*d^(2/3)*e^(1/3) -
 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)
/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(8/3)) + ((e^(1/3)*(6*b^2
*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2)
+ d^(1/3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^
2 + a^3*e^2)))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(8/3)) - ((
e^(1/3)*(6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d
*e + a^4*e^2) + d^(1/3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e
 - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^
(2/3)*x^2])/(6*d^(2/3)*e^(8/3)) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*
a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3])/(3*e^3)
```

Rubi [A] time = 2.20409, antiderivative size = 643, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{x^2\left(-6a^2c^2e-12ab^2ce+b^4(-e)+4bc^3d\right)}{2e^2}-\frac{2x\left(-6a^2bce-2ab^3e+2ac^3d+3b^2c^2d\right)}{e^2}+\frac{\log\left(d+ex^3\right)\left(-4ce\left(b^3d-a^3e\right)+6a^2b^2e^2-12abc^2de+c^4d^2\right)}{3e^3}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-e\left(a^3(-e)-3a^2b\sqrt[3]{d}e^{2/3}+3ab^2d^{2/3}\sqrt[3]{e}+b^3d\right)+6c^2\left(bd^{5/3}\sqrt[3]{e}-ad^{4/3}e^{2/3}\right)-12abcde\cdots}{\sqrt{3}d^{2/3}e^{8/3}}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^4e^2-12a^2bcde+\frac{\sqrt[3]{d}\left(-4b\left(a^3e^2+c^3d^2\right)+6a^2c^2de+12ab^2cde+b^4de\right)}{\sqrt[3]{e}}-4ab^3de+4ac^3d^2+6b^2c^2d^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}\left(a^4e^2-12a^2bcde-4ab^3de+4ac^3d^2+6b^2c^2d^2\right)+\sqrt[3]{d}\left(-4b\left(a^3e^2+c^3d^2\right)+6a^2c^2de+12ab^2cde+b^4\cdots\right.\right.}{3d^{2/3}e^{8/3}}-\frac{cx^3\left(-12abce-4b^3e+c^3d\right)}{3e^2}+\frac{c^2x^4\left(2ac+3b^2\right)}{2e}+\frac{4bc^3x^5}{5e}+\frac{c^4x^6}{6e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 -
((4*b*c^3*d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) -
(c*(c^3*d - 4*b^3*e - 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*
```

```
a*c)*x^4)/(2*e) + (4*b*c^3*x^5)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^(
1/3) + a*e^(1/3))*(4*c^3*d^2 + 6*c^2*(b*d^(5/3)*e^(1/3) - a*d^(4/
3)*e^(2/3)) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b^2*d^(2/3)*e^(1/3) -
 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)
/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(8/3)) + ((e^(1/3)*(6*b^2
*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2)
+ d^(1/3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^
2 + a^3*e^2)))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(8/3)) - ((
6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*
e^2 + (d^(1/3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c
^3*d^2 + a^3*e^2)))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^
(2/3)*x^2])/(6*d^(2/3)*e^(7/3)) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*
a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3])/(3*e^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**4/(e*x**3+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.752682, size = 678, normalized size = 1.05

$$\frac{15e^{2/3}x^2\left(6a^2c^2e+12ab^2ce+b^4e-4bc^3d\right)+60e^{2/3}x\left(6a^2bce+2ab^3e-2ac^3d-3b^2c^2d\right)+\frac{10\log\left(d+ex^3\right)\left(4ce\left(a^3e-b^3d\right)+6a^2b^2e^2\cdots\right.}{\sqrt[3]{e}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (60*e^(2/3)*(-3*b^2*c^2*d - 2*a*c^3*d + 2*a*b^3*e + 6*a^2*b*c*e)*
x + 15*e^(2/3)*(-4*b*c^3*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*
x^2 + 10*c*e^(2/3)*(-(c^3*d) + 4*b^3*e + 12*a*b*c*e)*x^3 + 15*c^2
*(3*b^2 + 2*a*c)*e^(5/3)*x^4 + 24*b*c^3*e^(5/3)*x^5 + 5*c^4*e^(5/
3)*x^6 + (10*Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*(-4*c^3*d^2 + c^2*(-
6*b*d^(5/3)*e^(1/3) + 6*a*d^(4/3)*e^(2/3)) + 12*a*b*c*d*e + e*(b^
3*d + 3*a*b^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))
*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (10*(4*a*
c^3*d^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d
*e^(4/3) + a^4*e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e
) - 4*b*(c^3*d^(7/3) + 3*a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[
d^(1/3) + e^(1/3)*x])/d^(2/3) - (5*(4*a*c^3*d^2*e^(1/3) + b^4*d^(
4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4/3) + a^4*e^(7/3) +
6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^(7/3) + 3*
a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(2/3) + (10*(c^4*d^2 - 12*a*b*c^2*d*e + 6*a^
2*b^2*e^2 + 4*c*e*(-(b^3*d) + a^3*e))*Log[d + e*x^3])/e^(1/3))/(3
0*e^(8/3))
```

Maple [B] time = 0.01, size = 1339, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^4/(e*x^3+d),x)
```

```
[Out] -4/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)
)*a^2*b*c*d-4/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)
^(1/3)*x-1))*a*b^2*c*d-4/3/e^3/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*b*c^
3*d^2-1/e^2/(d/e)^(1/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*a^2*c^2
*d+2/3/e^3/(d/e)^(1/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*b*c^3*d^
2+4/3/e*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1
))*a^3*b-1/3/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^
(1/3)*x-1))*b^4*d-4/3/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a*b^3*d+4
/3/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a*c^3*d^2+2/e^2/(d/e)^(1/3)*
ln(x+(d/e)^(1/3))*a^2*c^2*d-2/3/e^3/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1
/3)+(d/e)^(2/3))*a*c^3*d^2-1/e^3/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)
+(d/e)^(2/3))*b^2*c^2*d^2+2/3/e^2/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3
)+(d/e)^(2/3))*a*b^3*d-4/e^2*ln(e*x^3+d)*a*b*c^2*d+2/e^3/(d/e)^(2
/3)*ln(x+(d/e)^(1/3))*b^2*c^2*d^2+2/3/e/(d/e)^(1/3)*ln(x^2-x*(d/e
)^(1/3)+(d/e)^(2/3))*a^3*b+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(d/e)^(1/3)*x-1))*a^4+1/e*x^4*a*c^3+4/e*a*b^3*x+3/2/e*x
^4*b^2*c^2+4/3/e*x^3*b^3*c-4/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*a^
3*b-1/6/e^2/(d/e)^(1/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*b^4*d-4
/3/e^2*ln(e*x^3+d)*b^3*c*d+1/3/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*
b^4*d-2/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)
*x-1))*a^2*c^2*d+2/e*ln(e*x^3+d)*a^2*b^2+1/3/e^3*ln(e*x^3+d)*c^4*
d^2+3/e*x^2*a^2*c^2-1/6/e/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^
(2/3))*a^4+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^4+4/3/e*ln(e*x^3
+d)*a^3*c-2/e^2*x^2*b*c^3*d+4/3/e^3*3^(1/2)/(d/e)^(1/3)*arctan(1/
3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b*c^3*d^2-1/3/e^2*x^3*c^4*d+1/6*c^
4*x^6/e-4/e^2*a*c^3*d*x-6/e^2*x*b^2*c^2*d+1/2/e*x^2*b^4+4/e^2/(d/
e)^(1/3)*ln(x+(d/e)^(1/3))*a*b^2*c*d-2/e^2/(d/e)^(1/3)*ln(x^2-x*(
d/e)^(1/3)+(d/e)^(2/3))*a*b^2*c*d+2/e^3/(d/e)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b^2*c^2*d^2+4/3/e^3/(d/e)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*c^3*d^2+6/e*x
^2*a*b^2*c+4/e*x^3*a*b*c^2-4/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^
2*b*c*d+2/e^2/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*a^2*b
*c*d-4/3/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3
)*x-1))*a*b^3*d+12/e*a^2*b*c*x+4/5*b*c^3*x^5/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^4/(e*x^3 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^4/(e*x^3 + d),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**4/(e*x**3+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.221811, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^4/(e*x^3 + d),x, algorithm="giac")
```

```
[Out] Done
```

3.76 $\int \frac{2x^2+x^4}{1+x^3}\,dx$

Optimal. Leaf size=43

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x^2/2 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x] + Log[1 -
x + x^2]/2
```

Rubi [A] time = 0.125172, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x] + Log[1 -
x + x^2]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\log(x+1)+\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**4+2*x**2)/(x**3+1),x)
```

```
[Out] log(x + 1) + log(x**2 - x + 1)/2 - sqrt(3)*atan(sqrt(3)*(2*x/3 -
1/3))/3 + Integral(x, x)
```

Mathematica [A] time = 0.024239, size = 54, normalized size = 1.26

$$\frac{1}{6}\left(4\log\left(x^3+1\right)+3x^2-\log\left(x^2-x+1\right)+2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] (3*x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Lo
g[1 - x + x^2] + 4*Log[1 + x^3])/6
```

Maple [A] time = 0.007, size = 38, normalized size = 0.9

$$\frac{x^2}{2}+\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+2*x^2)/(x^3+1),x)
```

```
[Out] 1/2*x^2+1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+l
n(1+x)
```

Maxima [A] time = 1.52392, size = 50, normalized size = 1.16

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right) + \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2)/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2
 - x + 1) + log(x + 1)
```

Fricas [A] time = 0.225107, size = 65, normalized size = 1.51

$$\frac{1}{6}\sqrt{3}\left(\sqrt{3}x^2 + \sqrt{3}\log\left(x^2-x+1\right) + 2\sqrt{3}\log\left(x+1\right) - 2\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2)/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(sqrt(3)*x^2 + sqrt(3)*log(x^2 - x + 1) + 2*sqrt(3)*l
og(x + 1) - 2*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.153301, size = 44, normalized size = 1.02

$$\frac{x^2}{2} + \log\left(x+1\right) + \frac{\log\left(x^2-x+1\right)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+2*x**2)/(x**3+1),x)
```

```
[Out] x**2/2 + log(x + 1) + log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3
)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.212748, size = 51, normalized size = 1.19

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\ln\left(x^2-x+1\right) + \ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2)/(x^3 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*ln(x^2
- x + 1) + ln(abs(x + 1))
```

3.77 $\int \frac{2x^2+x^4}{1-x^3} dx$

Optimal. Leaf size=46

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -x^2/2 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x] - Log[1 +
 x + x^2]/2
```

Rubi [A] time = 0.125808, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x^2 + x^4)/(1 - x^3),x]
```

```
[Out] -x^2/2 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x] - Log[1 +
 x + x^2]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\log(-x+1)-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}-\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**4+2*x**2)/(-x**3+1),x)
```

```
[Out] -log(-x + 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(sqrt(3)*(2*x/3
+ 1/3))/3 - Integral(x, x)
```

Mathematica [A] time = 0.0233239, size = 54, normalized size = 1.17

$$\frac{1}{6}\left(-4\log\left(1-x^3\right)-3x^2+\log\left(x^2+x+1\right)-2\log(1-x)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x^2 + x^4)/(1 - x^3),x]
```

```
[Out] (-3*x^2 - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + Lo
g[1 + x + x^2] - 4*Log[1 - x^3])/6
```

Maple [A] time = 0.008, size = 38, normalized size = 0.8

$$-\frac{x^2}{2}-\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+2*x^2)/(-x^3+1),x)
```

```
[Out] -1/2*x^2-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-
ln(-1+x)
```

Maxima [A] time = 1.51471, size = 50, normalized size = 1.09

$$-\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) - \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 + 2*x^2)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^
2 + x + 1) - log(x - 1)
```

Fricas [A] time = 0.244687, size = 62, normalized size = 1.35

$$-\frac{1}{6}\sqrt{3}\left(\sqrt{3}x^2 + \sqrt{3}\log\left(x^2+x+1\right) + 2\sqrt{3}\log\left(x-1\right) + 2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 + 2*x^2)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*(sqrt(3)*x^2 + sqrt(3)*log(x^2 + x + 1) + 2*sqrt(3)*
log(x - 1) + 2*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.152768, size = 46, normalized size = 1.

$$-\frac{x^2}{2} - \log\left(x-1\right) - \frac{\log\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+2*x**2)/(-x**3+1),x)
```

```
[Out] -x**2/2 - log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(
3)*x/3 + sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.211878, size = 51, normalized size = 1.11

$$-\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\ln\left(x^2+x+1\right) - \ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 + 2*x^2)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*ln(x^2
 + x + 1) - ln(abs(x - 1))
```

3.78 $\int \frac{1-x+4x^3}{1+x^3}\,dx$

Optimal. Leaf size=44

$$\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 +
Log[1 - x + x^2]/3
```

Rubi [A] time = 0.0752904, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] 4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 +
Log[1 - x + x^2]/3
```

Rubi in Sympy [A] time = 15.5135, size = 44, normalized size = 1.

$$4x-\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{3}-\frac{4\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**3-x+1)/(x**3+1),x)
```

```
[Out] 4*x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - 4*sqrt(3)*atan(sqrt(
3)*(2*x/3 - 1/3))/3
```

Mathematica [A] time = 0.0145698, size = 44, normalized size = 1.

$$\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)-\frac{4\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] 4*x - (4*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 +
 Log[1 - x + x^2]/3
```

Maple [A] time = 0.008, size = 38, normalized size = 0.9

$$4\,x+\frac{\ln\left(x^2-x+1\right)}{3}-\frac{4\sqrt{3}}{3}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)-\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^3-x+1)/(x^3+1),x)
```

```
[Out] 4*x+1/3*ln(x^2-x+1)-4/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-2/3*l
n(1+x)
```

Maxima [A] time = 1.52395, size = 50, normalized size = 1.14

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^3 - x + 1)/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 -
x + 1) - 2/3*log(x + 1)
```

Fricas [A] time = 0.233204, size = 63, normalized size = 1.43

$$\frac{1}{9}\sqrt{3}\left(12\sqrt{3}x+\sqrt{3}\log\left(x^2-x+1\right)-2\sqrt{3}\log\left(x+1\right)-12\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^3 - x + 1)/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*(12*sqrt(3)*x + sqrt(3)*log(x^2 - x + 1) - 2*sqrt(3)*
log(x + 1) - 12*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.15081, size = 48, normalized size = 1.09

$$4x-\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{3}-\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**3-x+1)/(x**3+1),x)
```

```
[Out] 4*x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - 4*sqrt(3)*atan(2*sqr
t(3)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.212904, size = 51, normalized size = 1.16

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\ln\left(x^2-x+1\right)-\frac{2}{3}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^3 - x + 1)/(x^3 + 1),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*ln(x^2 - x
 + 1) - 2/3*ln(abs(x + 1))
```

3.79 $\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=230

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[
(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[A
rcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqr
t[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.126238, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[
(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[A
rcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqr
t[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 11.6607, size = 206, normalized size = 0.9

$$\frac{2\sqrt{x^3+1}}{x+1+\sqrt{3}}-\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}+\frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(x**3 + 1)/(x + 1 + sqrt(3)) - 3**(1/4)*sqrt((x**2 - x + 1)
/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_e(asin
((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x +
 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1)) + 4*3**(1/4)*sqrt((x**2
 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(x + 1)*ellipti
c_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(s
qrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.187205, size = 127, normalized size = 0.55

$$\frac{\sqrt[4]{3}\sqrt{-\sqrt[6]{-1}\left(x+(-1)^{2/3}\right)}\sqrt{(-1)^{2/3}x^2+\sqrt[3]{-1}x+1}\left(-2E\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}(x+1)}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+\sqrt[6]{-1}\left(\sqrt{3}+(2-i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)}}{\sqrt[4]{}}\right.\right.\right.}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (3^(1/4)*Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]*Sqrt[1 + (-1)^(1/3)
*x + (-1)^(2/3)*x^2]*(-2*EllipticE[ArcSin[Sqrt[-((-1)^(5/6)*(1 +
x))]/3^(1/4)], (-1)^(1/3)] + (-1)^(1/6)*((2 - I) + Sqrt[3])*Ellip
ticF[ArcSin[Sqrt[-((-1)^(5/6)*(1 + x))]/3^(1/4)], (-1)^(1/3)]))/S
qrt[1 + x^3]
```

Maple [B] time = 0.02, size = 407, normalized size = 1.8

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+2\,\frac{\sqrt{3}\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\left(\left(-3/2-i/2\sqrt{3}\right)\mathit{EllipticE}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+}{-3/2-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))+2*3^(1/2)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+
1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Elliptic
F(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*(
(-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(
1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Sympy [A] time = 1.98338, size = 92, normalized size = 0.4

$$\frac{x^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\,x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{5}{3}\right)}+\frac{x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\,x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\,x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(
3*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_p
olar(I*pi))/(3*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2
), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.80 $\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}} \, dx$

Optimal. Leaf size=257

$$-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin
[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sq
rt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[
ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sq
rt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.150188, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin
[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sq
rt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[
ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sq
rt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 14.6655, size = 206, normalized size = 0.8

$$-\frac{2\sqrt{-x^3+1}}{-x+1+\sqrt{3}}+\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,E\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -2*sqrt(-x**3 + 1)/(-x + 1 + sqrt(3)) + 3**(1/4)*sqrt((x**2 + x +
 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_e
(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sq
rt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1)) - 4*3**(1/4)*
sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(-x
+ 1)*elliptic_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 -
 4*sqrt(3))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1)
)
```

Mathematica [C] time = 0.123504, size = 112, normalized size = 0.44

$$\frac{2\sqrt[4]{3}\sqrt{(-1)^{5/6}(x-1)}\sqrt{x^2+x+1}\left(iF\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+(-1)^{2/3}E\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*3^(1/4)*Sqrt[(-1)^(5/6)*(-1 + x)]*Sqrt[1 + x + x^2]*((-1)^(2/3
)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)]
+ I*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)
]))/Sqrt[1 - x^3]
```

Maple [A] time = 0.028, size = 368, normalized size = 1.4

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

$$-2i\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\sqrt{3}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3+}}$$

$$+\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)EllipticE\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2*I*(I*(x+1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1
/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/
2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1
/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3/2+1/2*I*3^(1
/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(
I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(
1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Sympy [A] time = 2.47376, size = 97, normalized size = 0.38

$$-\frac{x^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\,\Gamma\left(\frac{5}{3}\right)}+\frac{x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi)
)/(3*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*ex
p_polar(2*I*pi))/(3*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3
, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.81 $\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=144

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((
1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.0777194, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((
1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 6.76168, size = 109, normalized size = 0.76

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,E\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] 2*sqrt(x**3 - 1)/(-x - sqrt(3) + 1) - 3**(1/4)*sqrt((x**2 + x + 1
)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_e(as
in((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sqrt(
(x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.11397, size = 110, normalized size = 0.76

$$\frac{2\sqrt[4]{3}\sqrt{(-1)^{5/6}(x-1)}\sqrt{x^2+x+1}\left(iF\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+(-1)^{2/3}E\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*3^(1/4)*Sqrt[(-1)^(5/6)*(-1 + x)]*Sqrt[1 + x + x^2]*((-1)^(2/3
)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)]
+ I*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)
]))/Sqrt[-1 + x^3]
```

Maple [B] time = 0.015, size = 407, normalized size = 2.8

$$2\,{\frac {-3/2-i/2\sqrt {3}}{\sqrt {{x}^{3}-1}}\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2-i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2+i/2\sqrt {3}}{3/2+i/2\sqrt {3}}}}{\it EllipticF} \left(\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}},\sqrt {{\frac {3/2+i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}} \right) }$$
$$+2\,{\frac {\sqrt {3} \left(-3/2-i/2\sqrt {3} \right) }{\sqrt {{x}^{3}-1}}\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2-i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2+i/2\sqrt {3}}{3/2+i/2\sqrt {3}}}}{\it EllipticF} \left(\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}},\sqrt {{\frac {3/2+i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}} \right) }$$
$$-2\,{\frac {-3/2-i/2\sqrt {3}}{\sqrt {{x}^{3}-1}}\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2-i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2+i/2\sqrt {3}}{3/2+i/2\sqrt {3}}}} \left(\left(3/2-i/2\sqrt {3} \right) {\it EllipticE} \left(\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}},\sqrt {{\frac {3/2}{3/2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2)
)/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/
2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))
^(1/2))+2*3^(1/2)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2
)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Ellipt
icF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2))-2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*
I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/
2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2
)*((3/2-1/2*I*3^(1/2))*EllipticE(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1
/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*
3^(1/2))*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1),x, algorithm="fricas")
```

```
[Out] integral(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Sympy [A] time = 2.40001, size = 82, normalized size = 0.57

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
- sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma
(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(
4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.82 $\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSi
n[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-(
(1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.0747656, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSi
n[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-(
(1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 6.53129, size = 114, normalized size = 0.84

$$-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -2*sqrt(-x**3 - 1)/(x - sqrt(3) + 1) + 3**(1/4)*sqrt((x**2 - x +
1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_e(asi
n((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sqrt((-x
 - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.200061, size = 147, normalized size = 1.09

$$\frac{(1-i)\sqrt[6]{-1}\sqrt[4]{3}\sqrt{-(-1)^{5/6}+ix}\sqrt{-\sqrt[3]{-1}x^2-(-1)^{2/3}x+1}\left(-\left(1+\sqrt{3}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\sqrt[6]{-1}\left(x+(-1)^{2/3}\right)}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+(1+i)\sqrt[6]{-1}E\left(\sin\cdots\right.\right.}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] ((1 - I)*(-1)^(1/6)*3^(1/4)*Sqrt[-(-1)^(5/6) + I*x]*Sqrt[1 - (-1)
^(2/3)*x - (-1)^(1/3)*x^2]*((1 + I)*(-1)^(1/6)*EllipticE[ArcSin[S
qrt[-((-1)^(1/6)*((-1)^(2/3) + x))]/3^(1/4)], (-1)^(1/3)] - (1 +
Sqrt[3])*EllipticF[ArcSin[Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]/3^
(1/4)], (-1)^(1/3)]))/Sqrt[-1 - x^3]
```

Maple [B] time = 0.015, size = 370, normalized size = 2.7

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^{3}}}$$
$$-2i\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^{3}-1}}$$
$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}\left(\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathit{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2*I*(I*(x-1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3
/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I
*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*Elli
pticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1
/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^{3}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x+\sqrt{3}+1}{\sqrt{-x^{3}-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Sympy [A] time = 2.02625, size = 99, normalized size = 0.73

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi)
)/(3*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,
), x**3*exp_polar(I*pi))/(3*gamma(4/3)) - I*x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.83 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=468

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*a
^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)
```

Rubi [A] time = 0.285541, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[
```

```
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*a
^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)
```

Rubi in Sympy [A] time = 25.0419, size = 410, normalized size = 0.88

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3
)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3) +
 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 + sqrt
(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4
*sqrt(3))/(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/
3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 4*3**(1/4)
*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1
/3) + b**(1/3)*x)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**
(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/
(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sq
rt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 2*sqrt(a + b*x**3)/(
b**(1/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x))
```

Mathematica [C] time = 0.501834, size = 225, normalized size = 0.48

$$\frac{2ia^{2/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(\left(\left(3+\sqrt{3}\right)\sqrt[3]{-b}+\sqrt{3}\sqrt[3]{b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-3\sqrt[6]{-1}\sqrt[3]{\ldots}\right.}{3^{3/4}(-b)^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] ((2*I)*a^(2/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3
)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*(-
```

```
3*(-1)^(1/6)*b^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^
(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + ((3 + Sqrt[3])*(-b)^(1/
3) + Sqrt[3]*b^(1/3))*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)
^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(3/4)*(-b)^(2/3)*Sq
rt[a + b*x^3])
```

Maple [B] time = 0.119, size = 1003, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))-2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a),x, algorithm="maxi
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a),x, algorithm="frica
```

```
[Out] integral((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.63644, size = 122, normalized size = 0.26

$$\frac{\sqrt[3]{b}x^{2}\,\Gamma\left(\frac{2}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{x\,\Gamma\left(\frac{1}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\,\Gamma\left(\frac{1}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2
), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sq
rt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*a**(1/6)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a),x, algorithm="giac
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

3.84 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=481

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*
a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) -
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3
])
```

Rubi [A] time = 0.309419, antiderivative size = 481, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt
```

```
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*
a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) -
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3
])
```

Rubi in Sympy [A] time = 28.0204, size = 413, normalized size = 0.86

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(-\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\sqrt{a-bx^{3}}}+\frac{4\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\sqrt{a-bx^{3}}}-\frac{2\sqrt{a-bx^{3}}}{\sqrt[3]{b}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3
)*x**2)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(-sqrt(3) +
 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin(-(-a**(1/3)*(-1 + sqr
t(3)) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)), -7 -
4*sqrt(3))/(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1
/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3)) + 4*3**(1/4
)*a**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/
(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(
1/3) - b**(1/3)*x)*elliptic_f(asin((a**(1/3)*(-1 + sqrt(3)) + b**
(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)), -7 - 4*sqrt(3))/
(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(1 + sq
rt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3)) - 2*sqrt(a - b*x**3)/(
b**(1/3)*(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x))
```

Mathematica [C] time = 0.312613, size = 182, normalized size = 0.38

$$\frac{2\sqrt[4]{3}a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^{2}}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(iF\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+(-1)^{2/3}E\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-\right.\right.}{\sqrt[3]{b}\sqrt{a-bx^{3}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (2*3^(1/4)*a^(2/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + b^(1/3)*x))/a^(1/
3)]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*((-1)^(
```

```
2/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^(1/3)*x)/a^(1/3)]/3
^(1/4)], (-1)^(1/3)] + I*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*b
^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(b^(1/3)*Sqrt[a - b*x^
3])
```

Maple [B] time = 0.135, size = 949, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] 2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x
-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)
^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^
(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1
/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(
-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I*a
^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1
/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*
(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I
*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2
)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2
)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-
3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1
/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^
2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/
2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/
2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a),x, algorithm="ma
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a),x, algorithm="fr
```

```
[Out] integral(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x
)
```

Sympy [A] time = 4.03459, size = 128, normalized size = 0.27

$$-\frac{\sqrt[3]{b}x^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_po
lar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3,
1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
 + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_pola
r(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a),x, algorithm="gi
```

```
[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a),
x)
```

3.85 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=271

$$\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^
(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.160202, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028

$$\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```


```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^
(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi in Sympy [A] time = 13.7682, size = 224, normalized size = 0.83

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}-\frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3
)*x**2)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) +
 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin((a**(1/3)*(1 + sqrt(3
)) - b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)), -7 + 4
*sqrt(3))/(b**(1/3)*sqrt(-a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1
/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a + b*x**3)) - 2*sqrt(
-a + b*x**3)/(b**(1/3)*(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x))
```

Mathematica [C] time = 0.598886, size = 257, normalized size = 0.95

$$\frac{2\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{-b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{-b}x\left(\sqrt[3]{-a}+\sqrt[3]{-b}x\right)}{(-a)^{2/3}}+1}\left(i\left(\left(3+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{-b}-\sqrt{3}\sqrt[3]{-a}\sqrt[3]{b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right.\right.\right.}{3^{3/4}(-b)^{2/3}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*(-b)^(1/3)*x))/a
)]*Sqrt[1 + ((-b)^(1/3)*x*((-a)^(1/3) + (-b)^(1/3)*x))/(-a)^(2/3)
]*(3*(-1)^(2/3)*(-a)^(1/3)*b^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5
/6) - (I*(-b)^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] + I*((3
+ Sqrt[3])*a^(1/3)*(-b)^(1/3) - Sqrt[3]*(-a)^(1/3)*b^(1/3))*Ellip
ticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/(-a)^(1/3)]/3^(1/
4)], (-1)^(1/3)]))/(3^(3/4)*(-b)^(2/3)*Sqrt[-a + b*x^3])
```

Maple [B] time = 0.06, size = 952, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] 2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x
-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)
^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^
(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1
/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/
3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-
3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I*a^
(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/
3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(
I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(
x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a
*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*
(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^
(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)
^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),
```

```
(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3} + 1\right)}{\sqrt{b x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a),x, algorithm="max
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3} + 1\right)}{\sqrt{b x^{3} - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a),x, algorithm="fri
```

```
[Out] integral(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 4.04426, size = 112, normalized size = 0.41

$$\frac{i \sqrt[3]{b} x^{2} \Gamma\left(\frac{2}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| {\frac{b x^{3}}{a}} \right)}}{3 \sqrt{a} \Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3} i x \Gamma\left(\frac{1}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3}}{a}} \right)}}{3 \sqrt[6]{a} \Gamma\left(\frac{4}{3}\right)} - \frac{i x \Gamma\left(\frac{1}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3}}{a}} \right)}}{3 \sqrt[6]{a} \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3
*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (
4/3,), b*x**3/a)/(3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hyper((
1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/6)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3} + 1\right)}{\sqrt{b x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a),x, algorithm="gia
```

```
[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x
)
```

3.86 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b
^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.147579, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b
^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi in Sympy [A] time = 13.138, size = 224, normalized size = 0.84

$$\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] 3**(1/4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)
*x**2)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) +
 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((a**(1/3)*(1 + sqrt(3
)) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)), -7 + 4
*sqrt(3))/(b**(1/3)*sqrt(-a**(1/3)*(a**(1/3) + b**(1/3)*x)/(-a**(
1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a - b*x**3)) + 2*sqrt
(-a - b*x**3)/(b**(1/3)*(a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x))
```

Mathematica [C] time = 0.51842, size = 227, normalized size = 0.85

$$\frac{2i\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{-a}+\sqrt[3]{b}x\right)}{(-a)^{2/3}}+1}\left(\left(\sqrt{3}\sqrt[3]{-a}+\left(3+\sqrt{3}\right)\sqrt[3]{a}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-3\sqrt[6]{-1}\right.}{3^{3/4}\sqrt[3]{b}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] ((2*I)*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*b^(1/3)*x))/
a)]*Sqrt[1 + (b^(1/3)*x*((-a)^(1/3) + b^(1/3)*x))/(-a)^(2/3)]*(-3
*(-1)^(1/6)*(-a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^(
1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] + (Sqrt[3]*(-a)^(1/3) +
 (3 + Sqrt[3])*a^(1/3))*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^
(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(3/4)*b^(1/3)*Sqr
t[-a - b*x^3])
```

Maple [B] time = 0.046, size = 1012, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))-2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
```

```
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} + 1\right)}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a),x, algorithm="max
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} + 1\right)}{\sqrt{-bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a),x, algorithm="fri
```

```
[Out] integral((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

Sympy [A] time = 3.9529, size = 129, normalized size = 0.48

$$-\frac{i\sqrt[3]{b}x^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)} - \frac{ix\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hy
per((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gam
ma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} + 1\right)}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a),x, algorithm="giac
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x
)
```

3.87 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=520

$$\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (2*(b/a)^(1/3)*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqr
t[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/
a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Rubi [A] time = 0.452089, antiderivative size = 520, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(b/a)^(1/3)*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqr
t[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/
a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Rubi in Sympy [A] time = 32.3627, size = 450, normalized size = 0.87

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$
$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(-\sqrt{3}+1\right)+\sqrt[3]{b}\left(1+\sqrt{3}\right)\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\right.}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*(b/a)**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sq
rt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/
3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3
)*x)), -7 - 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/
3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
 + 2*(b/a)**(1/3)*sqrt(a + b*x**3)/(b**(2/3)*(a**(1/3)*(1 + sqrt(
3)) + b**(1/3)*x)) + 2*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sq
rt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-a**(1/3)*(b/a)**(1/3)*(
-sqrt(3) + 1) + b**(1/3)*(1 + sqrt(3)))*elliptic_f(asin((-a**(1/3
)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)
*x)), -7 - 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1
/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)
)
```

Mathematica [C] time = 0.462142, size = 243, normalized size = 0.47

$$\frac{2i\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(\left(\sqrt{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}+\left(3+\sqrt{3}\right)\sqrt[3]{-b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-3\sqrt[6]{-}\right.}{3^{3/4}(-b)^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] ((2*I)*a^(1/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3
)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*(-
3*(-1)^(1/6)*a^(1/3)*(b/a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6
) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + ((3 + Sqrt[
3])*(-b)^(1/3) + Sqrt[3]*a^(1/3)*(b/a)^(1/3))*EllipticF[ArcSin[Sq
rt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])
)/(3^(3/4)*(-b)^(2/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.056, size = 1004, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
-2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b/
a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-
a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 1.32564, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

3.88 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=533

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[Out] (-2*(b/a)^(1/3)*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sq
rt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b
/a)^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*
(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqr
t[a - b*x^3])
```

Rubi [A] time = 0.397507, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(1/3)*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sq
rt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b
/a)^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*
(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqr
t[a - b*x^3])
```

Rubi in Sympy [A] time = 35.8613, size = 450, normalized size = 0.84

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(-\sqrt{3}+1\right)+\sqrt[3]{b}\left(1+\sqrt{3}\right)\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\cdots\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*(b/a)**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sq
rt(-sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin((a**(1/3
)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)
*x)), -7 - 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3
)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3))
- 2*(b/a)**(1/3)*sqrt(a - b*x**3)/(b**(2/3)*(a**(1/3)*(1 + sqrt(3
)) - b**(1/3)*x)) + 2*3**(3/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)
*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqr
t(sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*(-a**(1/3)*(b/a)**(1/3)*(-
sqrt(3) + 1) + b**(1/3)*(1 + sqrt(3)))*elliptic_f(asin((a**(1/3)*
(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x
)), -7 - 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3
)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3))
```

Mathematica [C] time = 0.389734, size = 232, normalized size = 0.44

$$\frac{2\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(i\left(\left(3+\sqrt{3}\right)\sqrt[3]{b}-\sqrt{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+3(-1)^{2/3}\sqrt[3]{a}\cdots\right)}{3^{3/4}b^{2/3}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (2*a^(1/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + b^(1/3)*x))/a^(1/3)]*Sqrt
[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*(3*(-1)^(2/3)*a
^(1/3)*(b/a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^(1/3)
*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + I*((3 + Sqrt[3])*b^(1/3) - S
qrt[3]*a^(1/3)*(b/a)^(1/3))*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (
I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(3/4)*b^(2/3)*Sq
rt[a - b*x^3])
```

Maple [B] time = 0.056, size = 950, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*
b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))
^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I/b*(a*b^2)
^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*
3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))
^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(
1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2)^
(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^
(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(
1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2
)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1
/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^
2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-b x^{3} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Sympy [A] time = 1.69432, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

3.89 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=256

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{bx^3-a}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}\sqrt{bx^3-a}}$$

```
[Out] (2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)
) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a
)^(1/3)*x + (b/a)^(2/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*Ell
ipticE[ArcSin[(1 + Sqrt[3] - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^
(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[-((1 - (b/a)^(1/3)*
x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.179784, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{bx^3-a}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```


```
[Out] (2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)
) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a
)^(1/3)*x + (b/a)^(2/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*Ell
ipticE[ArcSin[(1 + Sqrt[3] - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^
(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[-((1 - (b/a)^(1/3)*
x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi in Sympy [A] time = 35.6471, size = 449, normalized size = 1.75

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$
$$-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a+bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$
$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\left(1+\sqrt{3}\right)\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}+\sqrt[3]{b}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*(b/a)**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*s
qrt(sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin((a**(1/3
)*(1 + sqrt(3)) - b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b**(1/3
)*x)), -7 + 4*sqrt(3))/(b**(2/3)*sqrt(-a**(1/3)*(a**(1/3) - b**(1
/3)*x)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a + b*x**
3)) - 2*(b/a)**(1/3)*sqrt(-a + b*x**3)/(b**(2/3)*(a**(1/3)*(-1 +
sqrt(3)) + b**(1/3)*x)) - 2*3**(3/4)*sqrt((a**(2/3) + a**(1/3)*b*
*(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)*
*2)*(1 + sqrt(3))*sqrt(-sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*(-a*
*(1/3)*(b/a)**(1/3) + b**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sq
rt(3)) - b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)), -7
 + 4*sqrt(3))/(3*b**(2/3)*sqrt(-a**(1/3)*(a**(1/3) - b**(1/3)*x)/
(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a + b*x**3))
```

Mathematica [C] time = 0.559792, size = 267, normalized size = 1.04

$$\frac{2\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{-b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{-b}x\left(\sqrt[3]{-a}+\sqrt[3]{-b}x\right)}{(-a)^{2/3}}+1}\left(i\left(-\sqrt{3}\sqrt[3]{-a}\sqrt[3]{\frac{b}{a}}+\sqrt{3}\sqrt[3]{-b}+3\sqrt[3]{-b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right.\right.\right.}{3^{3/4}(-b)^{2/3}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*(-b)^(1/3)*x))/a
)]*Sqrt[1 + ((-b)^(1/3)*x*((-a)^(1/3) + (-b)^(1/3)*x))/(-a)^(2/3)
]*(3*(-1)^(2/3)*(-a)^(1/3)*(b/a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1
)^(5/6) - (I*(-b)^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] + I*
(3*(-b)^(1/3) + Sqrt[3]*(-b)^(1/3) - Sqrt[3]*(-a)^(1/3)*(b/a)^(1/
3))*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/(-a)^(1/
3)]/3^(1/4)], (-1)^(1/3)]))/(3^(3/4)*(-b)^(2/3)*Sqrt[-a + b*x^3])
```

Maple [B] time = 0.029, size = 953, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*
b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))
^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/
2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I/b*(a*b^2)^
(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^
(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/
2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2)^(1
/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1
/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2
)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(
1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/
b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a),x, algorithm="fricas")
```

```
[Out] integral(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 1.72006, size = 0, normalized size = 0.

NaN

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a),x, algorithm="giac")
```

```
[Out] integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

3.90 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=251

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}\sqrt{-a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}$$

```
[Out] (-2*(b/a)^(2/3)*Sqrt[-a - b*x^3])/(b*(1 - Sqrt[3] + (b/a)^(1/3)*x
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/
a)^(1/3)*x + (b/a)^(2/3)*x^2)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2]*El
lipticE[ArcSin[(1 + Sqrt[3] + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)
^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[-((1 + (b/a)^(1/3)
*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.148912, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}\sqrt{-a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(2/3)*Sqrt[-a - b*x^3])/(b*(1 - Sqrt[3] + (b/a)^(1/3)*x
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/
a)^(1/3)*x + (b/a)^(2/3)*x^2)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2]*El
lipticE[ArcSin[(1 + Sqrt[3] + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)
^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[-((1 + (b/a)^(1/3)
*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi in Sympy [A] time = 33.9891, size = 449, normalized size = 1.79

$$\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$+\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}$$
$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\left(1+\sqrt{3}\right)\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}+\sqrt[3]{b}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-a)**(1/2),x)
```

```
[Out] 3**(1/4)*a**(1/3)*(b/a)**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)
*x + b**(2/3)*x**2)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*s
qrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((a**(1/3
)*(1 + sqrt(3)) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3
)*x)), -7 + 4*sqrt(3))/(b**(2/3)*sqrt(-a**(1/3)*(a**(1/3) + b**(1
/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a - b*x*
*3)) + 2*(b/a)**(1/3)*sqrt(-a - b*x**3)/(b**(2/3)*(a**(1/3)*(-1 +
 sqrt(3)) - b**(1/3)*x)) + 2*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b
**(1/3)*x + b**(2/3)*x**2)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x
)**2)*(1 + sqrt(3))*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-
a**(1/3)*(b/a)**(1/3) + b**(1/3))*elliptic_f(asin((a**(1/3)*(1 +
sqrt(3)) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)),
-7 + 4*sqrt(3))/(3*b**(2/3)*sqrt(-a**(1/3)*(a**(1/3) + b**(1/3)*x
)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a - b*x**3))
```

Mathematica [C] time = 0.510876, size = 245, normalized size = 0.98

$$\frac{2i\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{-a}+\sqrt[3]{b}x\right)}{(-a)^{2/3}}+1}\left(\left(\sqrt{3}\sqrt[3]{-a}\sqrt[3]{\frac{b}{a}}+\left(3+\sqrt{3}\right)\sqrt[3]{b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-3\right.}{3^{3/4}b^{2/3}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] ((2*I)*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*b^(1/3)*x))/
a)]*Sqrt[1 + (b^(1/3)*x*((-a)^(1/3) + b^(1/3)*x))/(-a)^(2/3)]*(-3
*(-1)^(1/6)*(-a)^(1/3)*(b/a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5
/6) - (I*b^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] + ((3 + Sqr
t[3])*b^(1/3) + Sqrt[3]*(-a)^(1/3)*(b/a)^(1/3))*EllipticF[ArcSin[
Sqrt[-(-1)^(5/6) - (I*b^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)
]))/(3^(3/4)*b^(2/3)*Sqrt[-a - b*x^3])
```

Maple [B] time = 0.019, size = 1013, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)-2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(
b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a),x, algorithm="fricas")
```

```
[Out] integral((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Sympy [A] time = 1.45204, size = 0, normalized size = 0.

NaN

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a),x, algorithm="giac")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

3.91 $\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[
(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.054056, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[
(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 6.45859, size = 110, normalized size = 0.87

$$\frac{2\sqrt{x^3+1}}{x+1+\sqrt{3}}-\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(x**3 + 1)/(x + 1 + sqrt(3)) - 3**(1/4)*sqrt((x**2 - x + 1)
/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_e(asin
((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x +
 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.201512, size = 127, normalized size = 1.

$$\frac{\sqrt[4]{3}\sqrt{-\sqrt[6]{-1}\left(x+(-1)^{2/3}\right)}\sqrt{(-1)^{2/3}x^2+\sqrt[3]{-1}x+1}\left(-2E\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}(x+1)}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+\sqrt[6]{-1}\left(\sqrt{3}+(-2-i)\right)F\left(\sin^{-1}\left(\sqrt{-(\cdots}\right.\right.\right.}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (3^(1/4)*Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]*Sqrt[1 + (-1)^(1/3)
*x + (-1)^(2/3)*x^2]*(-2*EllipticE[ArcSin[Sqrt[-((-1)^(5/6)*(1 +
x))]/3^(1/4)], (-1)^(1/3)] + (-1)^(1/6)*((-2 - I) + Sqrt[3])*Elli
pticF[ArcSin[Sqrt[-((-1)^(5/6)*(1 + x))]/3^(1/4)], (-1)^(1/3)]))/
Sqrt[1 + x^3]
```

Maple [B] time = 0.021, size = 407, normalized size = 3.2

$$-2\,\frac{\sqrt{3}\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\left(\left(-3/2-i/2\sqrt{3}\right)\mathit{EllipticE}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+}{-3/2-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] -2*3^(1/2)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*
((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*
3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Ellipti
cF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-
1/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^
(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*
((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*
((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)
,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^
(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Sympy [A] time = 1.9902, size = 92, normalized size = 0.72

$$\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(
3*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x*
*3*exp_polar(I*pi))/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2
), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^{3} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.92 $\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin
[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.0677622, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin
[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 8.30459, size = 110, normalized size = 0.77

$$-\frac{2\sqrt{-x^3+1}}{-x+1+\sqrt{3}}+\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,E\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -2*sqrt(-x**3 + 1)/(-x + 1 + sqrt(3)) + 3**(1/4)*sqrt((x**2 + x +
 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_e
(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sq
rt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.125475, size = 112, normalized size = 0.79

$$\frac{2\sqrt[4]{3}\sqrt{(-1)^{5/6}(x-1)}\sqrt{x^2+x+1}\left((-1)^{2/3}E\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-iF\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*3^(1/4)*Sqrt[(-1)^(5/6)*(-1 + x)]*Sqrt[1 + x + x^2]*((-1)^(2/3
)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)]
- I*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)
]))/Sqrt[1 - x^3]
```

Maple [B] time = 0.019, size = 368, normalized size = 2.6

$$2i\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3+}}$$
$$+\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}\left(\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)EllipticE\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{-}\right.\right.$$
$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x-3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/
2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3/2
+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/
3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2)))-2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2
*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(
1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Sympy [A] time = 2.50213, size = 97, normalized size = 0.68

$$-\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi)
)/(3*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,),
 x**3*exp_polar(2*I*pi))/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3
, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^{3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.93 $\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=264

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((
1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2
- Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])
/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.126456, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((
1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2
- Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])
/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 13.0806, size = 202, normalized size = 0.77

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,E\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] 2*sqrt(x**3 - 1)/(-x - sqrt(3) + 1) - 3**(1/4)*sqrt((x**2 + x + 1
)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_e(as
in((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sqrt(
(x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1)) + 4*3**(1/4)*sqrt(
(x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1)
*elliptic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*s
qrt(3))/(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.0994565, size = 110, normalized size = 0.42

$$\frac{2\sqrt[4]{3}\sqrt{(-1)^{5/6}(x-1)}\sqrt{x^2+x+1}\left((-1)^{2/3}E\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-iF\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*3^(1/4)*Sqrt[(-1)^(5/6)*(-1 + x)]*Sqrt[1 + x + x^2]*((-1)^(2/3
)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)]
- I*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)
]))/Sqrt[-1 + x^3]
```

Maple [A] time = 0.014, size = 407, normalized size = 1.5

$$-2\,\frac{\sqrt{3}\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$
$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\left(\left(3/2-i/2\sqrt{3}\right)\mathit{EllipticE}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\right.\right.$$
$$+2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x-3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -2*3^(1/2)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/
2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*
I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-
1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2))-2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+
1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2
-1/2*I*3^(1/2))*EllipticE(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3
/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2)
)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2)))+2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(
-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1
/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1),x, algorithm="fricas")
```

```
[Out] integral(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Sympy [A] time = 2.50298, size = 82, normalized size = 0.31

$$\frac{ix^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\,\Gamma\left(\frac{5}{3}\right)}-\frac{ix\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
- I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) +
 sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(
4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.94 $\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=247

$$-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSi
n[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-(
(1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]]
)/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.12889, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSi
n[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-(
(1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]]
)/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 13.1312, size = 211, normalized size = 0.85

$$-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -2*sqrt(-x**3 - 1)/(x - sqrt(3) + 1) + 3**(1/4)*sqrt((x**2 - x +
1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_e(asi
n((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sqrt((-x
 - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1)) - 4*3**(1/4)*sqrt((x
**2 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*ell
iptic_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3)
)/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.20911, size = 147, normalized size = 0.6

$$\frac{(1+i)\sqrt[6]{-1}\sqrt[4]{3}\sqrt{-(-1)^{5/6}+ix}\sqrt{-\sqrt[3]{-1}x^2-(-1)^{2/3}x+1}\left(-\left(\sqrt{3}-1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\sqrt[6]{-1}\left(x+(-1)^{2/3}\right)}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+(1-i)\sqrt[6]{-1}E\left(\sin\right.\right.}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] ((1 + I)*(-1)^(1/6)*3^(1/4)*Sqrt[-(-1)^(5/6) + I*x]*Sqrt[1 - (-1)
^(2/3)*x - (-1)^(1/3)*x^2]*((1 - I)*(-1)^(1/6)*EllipticE[ArcSin[S
qrt[-((-1)^(1/6)*((-1)^(2/3) + x))]/3^(1/4)], (-1)^(1/3)] - (-1 +
 Sqrt[3])*EllipticF[ArcSin[Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]/3
^(1/4)], (-1)^(1/3)]))/Sqrt[-1 - x^3]
```

Maple [A] time = 0.016, size = 370, normalized size = 1.5

$$2i\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3-1}}$$
$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathit{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i}{\frac{3}{2}+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x-3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] 2*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
,(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-
I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*Ellip
ticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/
2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1),x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Sympy [A] time = 2.09817, size = 97, normalized size = 0.39

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi)
)/(3*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*
exp_polar(I*pi))/(3*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.95 $\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=126

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

```
[Out] (-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.063252, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 6.26944, size = 110, normalized size = 0.87

$$-\frac{2\sqrt{x^3+1}}{x+1+\sqrt{3}}+\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1-x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] -2*sqrt(x**3 + 1)/(x + 1 + sqrt(3)) + 3**(1/4)*sqrt((x**2 - x + 1
)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_e(asi
n((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x
+ 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.222668, size = 129, normalized size = 1.02

$$\frac{\sqrt[4]{3}\sqrt{-\sqrt[6]{-1}\left(x+(-1)^{2/3}\right)}\sqrt{(-1)^{2/3}x^2+\sqrt[3]{-1}x+1}\left(2E\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}(x+1)}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+\sqrt[6]{-1}\left(-\sqrt{3}+(2+i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)}}{}\right.\right.\right.}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

```
[Out] (3^(1/4)*Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]*Sqrt[1 + (-1)^(1/3)
*x + (-1)^(2/3)*x^2]*(2*EllipticE[ArcSin[Sqrt[-((-1)^(5/6)*(1 + x
))]/3^(1/4)], (-1)^(1/3)] + (-1)^(1/6)*((2 + I) - Sqrt[3])*Ellipt
icF[ArcSin[Sqrt[-((-1)^(5/6)*(1 + x))]/3^(1/4)], (-1)^(1/3)]))/Sq
rt[1 + x^3]
```

Maple [B] time = 0.018, size = 407, normalized size = 3.2

$$2\,{\frac {\sqrt {3} \left(3/2-i/2\sqrt {3} \right) }{\sqrt {{x}^{3}+1}}\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2-i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2+i/2\sqrt {3}}{-3/2+i/2\sqrt {3}}}}{\it EllipticF} \left(\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}},\sqrt {{\frac {-3/2+i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}} \right) }-2\,{\frac {3/2-i/2\sqrt {3}}{\sqrt {{x}^{3}+1}}\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2-i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2+i/2\sqrt {3}}{-3/2+i/2\sqrt {3}}}}{\it EllipticF} \left(\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}},\sqrt {{\frac {-3/2+i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}} \right) }-2\,{\frac {3/2-i/2\sqrt {3}}{\sqrt {{x}^{3}+1}}\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2-i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2+i/2\sqrt {3}}{-3/2+i/2\sqrt {3}}}} \left(\left(-3/2-i/2\sqrt {3} \right) {\it EllipticE} \left(\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}},\sqrt {{\frac {-3/2}{-3/2}}} \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1-x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*3^(1/2)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3
^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x
)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+
1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Elliptic
F(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*(
(-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(
1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Sympy [A] time = 2.34903, size = 92, normalized size = 0.73

$$-\frac{x^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{5}{3}\right)}-\frac{x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1-x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/
(3*gamma(5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_
polar(I*pi))/(3*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/
2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.96 $\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=143

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[
(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.0673292, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[
(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 7.01661, size = 110, normalized size = 0.77

$$\frac{2\sqrt{-x^3+1}}{-x+1+\sqrt{3}}-\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,E\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(-x**3 + 1)/(-x + 1 + sqrt(3)) - 3**(1/4)*sqrt((x**2 + x +
1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_e(
asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqr
t((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.121531, size = 112, normalized size = 0.78

$$-\frac{2\sqrt[4]{3}\sqrt{(-1)^{5/6}(x-1)}\sqrt{x^2+x+1}\left((-1)^{2/3}E\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-iF\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

```
[Out] (-2*3^(1/4)*Sqrt[(-1)^(5/6)*(-1 + x)]*Sqrt[1 + x + x^2]*((-1)^(2/
3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)]
 - I*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3
)]))/Sqrt[1 - x^3]
```

Maple [B] time = 0.013, size = 368, normalized size = 2.6

$$-2i\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3+}}$$
$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)EllipticE\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\right.\right.$$
$$+\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*(x+1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1
/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3/
2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1
/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2)))+2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/
2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^
(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Sympy [A] time = 2.08967, size = 97, normalized size = 0.68

$$\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))
/(3*gamma(5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp
_polar(2*I*pi))/(3*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3,
 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^{3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.97 $\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=263

$$-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSi
n[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]]
)/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.131254, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```


```
[Out] (-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSi
n[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]]
)/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 12.246, size = 202, normalized size = 0.77

$$-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,E\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -2*sqrt(x**3 - 1)/(-x - sqrt(3) + 1) + 3**(1/4)*sqrt((x**2 + x +
1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_e(a
sin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sqrt
((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1)) - 4*3**(1/4)*sqrt
((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1
)*elliptic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*
sqrt(3))/(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.105284, size = 110, normalized size = 0.42

$$-\frac{2\sqrt[4]{3}\sqrt{(-1)^{5/6}(x-1)}\sqrt{x^2+x+1}\left((-1)^{2/3}E\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-iF\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*3^(1/4)*Sqrt[(-1)^(5/6)*(-1 + x)]*Sqrt[1 + x + x^2]*((-1)^(2/
3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)]
 - I*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3
)]))/Sqrt[-1 + x^3]
```

Maple [A] time = 0.014, size = 407, normalized size = 1.6

$$2\,{\frac{\sqrt{3}\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}}\sqrt{{\frac{-1+x}{-3/2-i/2\sqrt{3}}}}\sqrt{{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}}\sqrt{{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}}{\it EllipticF}\left(\sqrt{{\frac{-1+x}{-3/2-i/2\sqrt{3}}}},\sqrt{{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}}\right)$$
$$+2\,{\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}}\sqrt{{\frac{-1+x}{-3/2-i/2\sqrt{3}}}}\sqrt{{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}}\sqrt{{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}}\left(\left(3/2-i/2\sqrt{3}\right){\it EllipticE}\left(\sqrt{{\frac{-1+x}{-3/2-i/2\sqrt{3}}}},\sqrt{{\frac{3/2}{3/2}}}\right.\right.$$
$$-2\,{\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}}\sqrt{{\frac{-1+x}{-3/2-i/2\sqrt{3}}}}\sqrt{{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}}\sqrt{{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}}{\it EllipticF}\left(\sqrt{{\frac{-1+x}{-3/2-i/2\sqrt{3}}}},\sqrt{{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*3^(1/2)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2
)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I
*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1
+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2))+2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2
)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-
1/2*I*3^(1/2))*EllipticE(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2))
*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2)))-2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-
3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Sympy [A] time = 2.11365, size = 82, normalized size = 0.31

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
 - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamm
a(4/3)) + I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma
(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.98 $\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=248

$$\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((
1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2
- Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])
/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.127578, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

```
[Out] (2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((
1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2
- Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])
/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 13.0755, size = 211, normalized size = 0.85

$$\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1-x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] 2*sqrt(-x**3 - 1)/(x - sqrt(3) + 1) - 3**(1/4)*sqrt((x**2 - x + 1
)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_e(asin
((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sqrt((-x
- 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1)) + 4*3**(1/4)*sqrt((x*
*2 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elli
ptic_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))
/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.199515, size = 146, normalized size = 0.59

$$\frac{(1+i)\sqrt[6]{-1}\sqrt[4]{3}\sqrt{-(-1)^{5/6}+ix}\sqrt{-\sqrt[3]{-1}x^2-(-1)^{2/3}x+1}\left(\left(\sqrt{3}-1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\sqrt[6]{-1}(x+(-1)^{2/3})}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-(1-i)\sqrt[6]{-1}E\left(\sin^{-1}\right.\right.}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

```
[Out] ((1 + I)*(-1)^(1/6)*3^(1/4)*Sqrt[-(-1)^(5/6) + I*x]*Sqrt[1 - (-1)
^(2/3)*x - (-1)^(1/3)*x^2]*((-1 + I)*(-1)^(1/6)*EllipticE[ArcSin[
Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]/3^(1/4)], (-1)^(1/3)] + (-1
+ Sqrt[3])*EllipticF[ArcSin[Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]/
3^(1/4)], (-1)^(1/3)]))/Sqrt[-1 - x^3]
```

Maple [A] time = 0.015, size = 370, normalized size = 1.5

$$-2\,i\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3-1}}$$
$$+\frac{2\,i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$+\frac{2\,i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathit{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i}{\frac{3}{2}+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1-x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*(x-1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3
/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I
*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*Elli
pticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1
/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1),x, algorithm="fricas")
```

```
[Out] integral(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Sympy [A] time = 2.44392, size = 97, normalized size = 0.39

$$\frac{ix^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}ix\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}+\frac{ix\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1-x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))
/(3*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,)
, x**3*exp_polar(I*pi))/(3*gamma(4/3)) + I*x*gamma(1/3)*hyper((1/
3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.99 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=256

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.126241, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```



```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 12.0684, size = 221, normalized size = 0.86

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3
)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3) +
 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 + sqrt
(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4
*sqrt(3))/(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/
3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 2*sqrt(a +
 b*x**3)/(b**(1/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x))
```

Mathematica [C] time = 0.473726, size = 225, normalized size = 0.88

$$\frac{2ia^{2/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(\left(\left(\sqrt{3}-3\right)\sqrt[3]{-b}+\sqrt{3}\sqrt[3]{b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-3\sqrt[6]{-1}\cdots\right.}{3^{3/4}(-b)^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] ((2*I)*a^(2/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3
)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*(-
3*(-1)^(1/6)*b^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^
(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + ((-3 + Sqrt[3])*(-b)^(1
/3) + Sqrt[3]*b^(1/3))*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b
)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(3/4)*(-b)^(2/3)*S
qrt[a + b*x^3])
```

Maple [B] time = 0.059, size = 1003, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] 2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/
3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
```

```
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a),x, algorithm="maxi
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a),x, algorithm="fric
```

```
[Out] integral((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.75701, size = 122, normalized size = 0.48

$$\frac{\sqrt[3]{b}x^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)} + \frac{x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((
1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/
3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*a**(1/6)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a),x, algorithm="giac

[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

3.100 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=263

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.115506, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```


```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi in Sympy [A] time = 13.5502, size = 226, normalized size = 0.86

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(-\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3
)*x**2)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(-sqrt(3) +
 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin(-(-a**(1/3)*(-1 + sqr
t(3)) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)), -7 -
4*sqrt(3))/(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1
/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3)) - 2*sqrt(a
- b*x**3)/(b**(1/3)*(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x))
```

Mathematica [C] time = 0.326896, size = 182, normalized size = 0.69

$$\frac{2\sqrt[4]{3}a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left((-1)^{2/3}E\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-iF\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-\right.\right.\right.}{\sqrt[3]{b}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (2*3^(1/4)*a^(2/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + b^(1/3)*x))/a^(1/
3)]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*((-1)^(
2/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^(1/3)*x)/a^(1/3)]/3
^(1/4)], (-1)^(1/3)] - I*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*b
^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(b^(1/3)*Sqrt[a - b*x^
3])
```

Maple [B] time = 0.062, size = 949, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b
^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^
(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/
2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*
3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/
3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(
I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3
))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))
*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^
(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*
b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))
^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
```

```
^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a),x, algorithm="ma
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{-bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a),x, algorithm="fr
```

```
[Out] integral(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x
)
```

Sympy [A] time = 4.11752, size = 128, normalized size = 0.49

$$-\frac{\sqrt[3]{b}x^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)} + \frac{x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_po
lar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hype
r((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gam
ma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_pola
r(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a),x, algorithm="gia
```

```
[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a),
x)
```

3.101 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=497

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^
(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
 + b*x^3])
```

Rubi [A] time = 0.310935, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt
```

```
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^
(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
 + b*x^3])
```

Rubi in Sympy [A] time = 28.9467, size = 413, normalized size = 0.83

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}-\frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3
)*x**2)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) +
 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin((a**(1/3)*(1 + sqrt(3
)) - b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)), -7 + 4
*sqrt(3))/(b**(1/3)*sqrt(-a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1
/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a + b*x**3)) + 4*3**(1
/4)*a**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2
)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3) + 2)*(
a**(1/3) - b**(1/3)*x)*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) -
b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)), -7 + 4*sqrt
(3))/(b**(1/3)*sqrt(-a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(
-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a + b*x**3)) - 2*sqrt(-a +
b*x**3)/(b**(1/3)*(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x))
```

Mathematica [C] time = 0.662983, size = 257, normalized size = 0.52

$$\frac{2\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{-b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{-b}x\left(\sqrt[3]{-a}+\sqrt[3]{-b}x\right)}{(-a)^{2/3}}+1}\left(i\left(\left(\sqrt{3}-3\right)\sqrt[3]{a}\sqrt[3]{-b}-\sqrt{3}\sqrt[3]{-a}\sqrt[3]{b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\cdots\right.\right.}{3^{3/4}(-b)^{2/3}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*(-b)^(1/3)*x))/a
)]*Sqrt[1 + ((-b)^(1/3)*x*((-a)^(1/3) + (-b)^(1/3)*x))/(-a)^(2/3)
```

```
]*(3*(-1)^(2/3)*(-a)^(1/3)*b^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5
/6) - (I*(-b)^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] + I*((-3
 + Sqrt[3])*a^(1/3)*(-b)^(1/3) - Sqrt[3]*(-a)^(1/3)*b^(1/3))*Elli
pticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/(-a)^(1/3)]/3^(1
/4)], (-1)^(1/3)]))/(3^(3/4)*(-b)^(2/3)*Sqrt[-a + b*x^3])
```

Maple [B] time = 0.02, size = 952, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b
^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^
(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2
)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/
2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3
^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3
))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I
*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))
^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^(1
/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^
2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(
1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a),x, algorithm="max
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a),x, algorithm="fri
```

```
[Out] integral(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 4.32418, size = 112, normalized size = 0.23

$$\frac{i\sqrt[3]{b}x^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{ix\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3
*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b
*x**3/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((
1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/6)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a),x, algorithm="gia
```

```
[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x
)
```

3.102 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=488

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b
^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt
[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-
a - b*x^3])
```

Rubi [A] time = 0.293479, antiderivative size = 488, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqr
```

```
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b
^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt
[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-
a - b*x^3])
```

Rubi in Sympy [A] time = 27.6699, size = 415, normalized size = 0.85

$$\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-a-bx^{3}}} - \frac{4\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-a-bx^{3}}} + \frac{2\sqrt{-a-bx^{3}}}{\sqrt[3]{b}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] 3**(1/4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)
*x**2)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) +
 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((a**(1/3)*(1 + sqrt(3
)) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)), -7 + 4
*sqrt(3))/(b**(1/3)*sqrt(-a**(1/3)*(a**(1/3) + b**(1/3)*x)/(-a**(
1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a - b*x**3)) - 4*3**(
1/4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**
2)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3) + 2)
*(a**(1/3) + b**(1/3)*x)*elliptic_f(asin((a**(1/3)*(1 + sqrt(3))
+ b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)), -7 + 4*sq
rt(3))/(b**(1/3)*sqrt(-a**(1/3)*(a**(1/3) + b**(1/3)*x)/(-a**(1/3
)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a - b*x**3)) + 2*sqrt(-a
 - b*x**3)/(b**(1/3)*(a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x))
```

Mathematica [C] time = 0.517527, size = 227, normalized size = 0.47

$$\frac{2i\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{-a}+\sqrt[3]{b}x\right)}{(-a)^{2/3}}+1}\left(\left(\sqrt{3}\sqrt[3]{-a}+\left(\sqrt{3}-3\right)\sqrt[3]{a}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-3\sqrt[6]{-1}}{3^{3/4}\sqrt[3]{b}\sqrt{-a-bx^{3}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] ((2*I)*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*b^(1/3)*x))/
a)]*Sqrt[1 + (b^(1/3)*x*((-a)^(1/3) + b^(1/3)*x))/(-a)^(2/3)]*(-3
```

```
*(-1)^(1/6)*(-a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^(
1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] + (Sqrt[3]*(-a)^(1/3) +
 (-3 + Sqrt[3])*a^(1/3))*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*b
^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(3/4)*b^(1/3)*Sq
rt[-a - b*x^3])
```

Maple [B] time = 0.027, size = 1012, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] 2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2
/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^
(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a),x, algorithm="max
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{-bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a),x, algorithm="fri
```

```
[Out] integral((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

Sympy [A] time = 4.06708, size = 128, normalized size = 0.26

$$-\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3
, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
 + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a),x, algorithm="gia
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x
)
```

3.103 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=241

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)
^(1/3)*x + (b/a)^(2/3)*x^2)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Elli
pticE[ArcSin[(1 - Sqrt[3] + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(
1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[(1 + (b/a)^(1/3)*x)/
(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.154738, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)
^(1/3)*x + (b/a)^(2/3)*x^2)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Elli
pticE[ArcSin[(1 - Sqrt[3] + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(
1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[(1 + (b/a)^(1/3)*x)/
(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 32.7379, size = 444, normalized size = 1.84

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\left(-\sqrt{3}+1\right)\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}+\sqrt[3]{b}\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*(b/a)**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sq
rt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/
3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3
)*x)), -7 - 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/
3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
 + 2*(b/a)**(1/3)*sqrt(a + b*x**3)/(b**(2/3)*(a**(1/3)*(1 + sqrt(
3)) + b**(1/3)*x)) + 2*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*(-
sqrt(3) + 1)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-a**(1/3)
*(b/a)**(1/3) + b**(1/3))*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3
)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*s
qrt(3))/(3*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/
3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 0.468901, size = 243, normalized size = 1.01

$$\frac{2i\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(\left(\sqrt{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}+\left(\sqrt{3}-3\right)\sqrt[3]{-b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-3\sqrt[6]{-}\right.}{3^{3/4}(-b)^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] ((2*I)*a^(1/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3
)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*(-
3*(-1)^(1/6)*a^(1/3)*(b/a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6
) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + ((-3 + Sqrt
[3])*(-b)^(1/3) + Sqrt[3]*a^(1/3)*(b/a)^(1/3))*EllipticF[ArcSin[S
qrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)]
))/(3^(3/4)*(-b)^(2/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.048, size = 1004, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] 2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*3^(1
/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a
)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 1.38769, size = 0, normalized size = 0.

NaN

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

3.104 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)}$$

```
[Out] (-2*(b/a)^(2/3)*Sqrt[a - b*x^3])/(b*(1 + Sqrt[3] - (b/a)^(1/3)*x)
) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a
)^(1/3)*x + (b/a)^(2/3)*x^2)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Ell
ipticE[ArcSin[(1 - Sqrt[3] - (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^
(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[(1 - (b/a)^(1/3)*x)
/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.152683, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```


```
[Out] (-2*(b/a)^(2/3)*Sqrt[a - b*x^3])/(b*(1 + Sqrt[3] - (b/a)^(1/3)*x)
) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a
)^(1/3)*x + (b/a)^(2/3)*x^2)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Ell
ipticE[ArcSin[(1 - Sqrt[3] - (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^
(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[(1 - (b/a)^(1/3)*x)
/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi in Sympy [A] time = 35.7004, size = 444, normalized size = 1.79

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\sqrt{a-bx^{3}}}$$
$$-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^{3}}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}$$
$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\left(-\sqrt{3}+1\right)\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}+\sqrt[3]{b}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\sqrt{a-bx^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*(b/a)**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sq
rt(-sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin((a**(1/3
)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)
*x)), -7 - 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3
)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3))
- 2*(b/a)**(1/3)*sqrt(a - b*x**3)/(b**(2/3)*(a**(1/3)*(1 + sqrt(3
)) - b**(1/3)*x)) + 2*3**(3/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)
*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*(-s
qrt(3) + 1)*sqrt(sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*(-a**(1/3)*
(b/a)**(1/3) + b**(1/3))*elliptic_f(asin((a**(1/3)*(-1 + sqrt(3))
 + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)), -7 - 4*sqr
t(3))/(3*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)
*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3))
```

Mathematica [C] time = 0.397025, size = 232, normalized size = 0.94

$$\frac{2\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^{2}}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(i\left(\left(\sqrt{3}-3\right)\sqrt[3]{b}-\sqrt{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+3(-1)^{2/3}\sqrt[3]{a}\cdots\right.}{3^{3/4}b^{2/3}\sqrt{a-bx^{3}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (2*a^(1/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + b^(1/3)*x))/a^(1/3)]*Sqrt
[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*(3*(-1)^(2/3)*a
^(1/3)*(b/a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^(1/3)
*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + I*((-3 + Sqrt[3])*b^(1/3) -
Sqrt[3]*a^(1/3)*(b/a)^(1/3))*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) -
(I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(3/4)*b^(2/3)*S
qrt[a - b*x^3])
```

Maple [B] time = 0.05, size = 950, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3
))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I
*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(
x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a
*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1
/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3)
)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*
(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(
a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))
^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*3^(1
/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3)
)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*
(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(
a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x
+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*
b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Sympy [A] time = 1.75645, size = 0, normalized size = 0.

NaN

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

3.105 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=549

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[Out] (2*(b/a)^(1/3)*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*
x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) - (
2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3
)*(b/a)^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^
(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.495688, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(b/a)^(1/3)*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*
x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) - (
2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3
)*(b/a)^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^
(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3])
```

Rubi in Sympy [A] time = 37.3886, size = 454, normalized size = 0.83

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-a+bx^{3}}}-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a+bx^{3}}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(1+\sqrt{3}\right)-\sqrt[3]{b}\left(-\sqrt{3}+1\right)\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+\right.}{3b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-a+bx^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*(b/a)**(1/3)*sqrt((a**(2/3) + a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*s
qrt(sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin((a**(1/3
)*(1 + sqrt(3)) - b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b**(1/3
)*x)), -7 + 4*sqrt(3))/(b**(2/3)*sqrt(-a**(1/3)*(a**(1/3) - b**(1
/3)*x)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a + b*x**
3)) - 2*(b/a)**(1/3)*sqrt(-a + b*x**3)/(b**(2/3)*(a**(1/3)*(-1 +
sqrt(3)) + b**(1/3)*x)) + 2*3**(3/4)*sqrt((a**(2/3) + a**(1/3)*b*
*(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)*
*2)*sqrt(-sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*(a**(1/3)*(b/a)**(
1/3)*(1 + sqrt(3)) - b**(1/3)*(-sqrt(3) + 1))*elliptic_f(asin((a*
*(1/3)*(1 + sqrt(3)) - b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b*
*(1/3)*x)), -7 + 4*sqrt(3))/(3*b**(2/3)*sqrt(-a**(1/3)*(a**(1/3)
- b**(1/3)*x)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a
+ b*x**3))
```

Mathematica [C] time = 0.570496, size = 267, normalized size = 0.49

$$\frac{2\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{-b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{-b}x\left(\sqrt[3]{-a}+\sqrt[3]{-b}x\right)}{(-a)^{2/3}}+1}\left(i\left(-\sqrt{3}\sqrt[3]{-a}\sqrt[3]{\frac{b}{a}}+\sqrt{3}\sqrt[3]{-b}-3\sqrt[3]{-b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right.\right.\right.}{3^{3/4}(-b)^{2/3}\sqrt{bx^{3}-a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*(-b)^(1/3)*x))/a
)]*Sqrt[1 + ((-b)^(1/3)*x*((-a)^(1/3) + (-b)^(1/3)*x))/(-a)^(2/3)
]*(3*(-1)^(2/3)*(-a)^(1/3)*(b/a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1
)^(5/6) - (I*(-b)^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] + I*
(-3*(-b)^(1/3) + Sqrt[3]*(-b)^(1/3) - Sqrt[3]*(-a)^(1/3)*(b/a)^(1
/3))*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/(-a)^(1
/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(3/4)*(-b)^(2/3)*Sqrt[-a + b*x^3]
)
```

Maple [B] time = 0.02, size = 953, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] -2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3
))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I
*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x
+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*
b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/
2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))
/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(
x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a
*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(
1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1
/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*3^(1/2
)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b
^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/
(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x
+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*
b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/
2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{bx^3 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a),x, algorithm="fricas")
```

```
[Out] integral(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 1.7375, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a),x, algorithm="giac")
```

```
[Out] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

3.106 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=540

$$\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (-2*(b/a)^(1/3)*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) +
(2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/
3)*(b/a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.388655, antiderivative size = 540, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(1/3)*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) +
(2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/
3)*(b/a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2)]*Sqrt[-a - b*x^3])
```

Rubi in Sympy [A] time = 35.4923, size = 456, normalized size = 0.84

$$\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$+\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}$$

$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(1+\sqrt{3}\right)-\sqrt[3]{b}\left(-\sqrt{3}+1\right)\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7\cdots\right.}{3b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-a)**(1/2),x)
```

```
[Out] 3**(1/4)*a**(1/3)*(b/a)**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)
*x + b**(2/3)*x**2)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*s
qrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((a**(1/3
)*(1 + sqrt(3)) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3
)*x)), -7 + 4*sqrt(3))/(b**(2/3)*sqrt(-a**(1/3)*(a**(1/3) + b**(1
/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a - b*x*
*3)) + 2*(b/a)**(1/3)*sqrt(-a - b*x**3)/(b**(2/3)*(a**(1/3)*(-1 +
 sqrt(3)) - b**(1/3)*x)) - 2*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b
**(1/3)*x + b**(2/3)*x**2)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x
)**2)*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(a**(1/3)*(b/a)*
*(1/3)*(1 + sqrt(3)) - b**(1/3)*(-sqrt(3) + 1))*elliptic_f(asin((
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) +
b**(1/3)*x)), -7 + 4*sqrt(3))/(3*b**(2/3)*sqrt(-a**(1/3)*(a**(1/3
) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(
-a - b*x**3))
```

Mathematica [C] time = 0.484302, size = 245, normalized size = 0.45

$$\frac{2i\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{-a}+\sqrt[3]{b}x\right)}{(-a)^{2/3}}+1}\left(\left(\sqrt{3}\sqrt[3]{-a}\sqrt[3]{\frac{b}{a}}+\left(\sqrt{3}-3\right)\sqrt[3]{b}\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-3\cdots\right.}{3^{3/4}b^{2/3}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] ((2*I)*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*b^(1/3)*x))/
a)]*Sqrt[1 + (b^(1/3)*x*((-a)^(1/3) + b^(1/3)*x))/(-a)^(2/3)]*(-3
*(-1)^(1/6)*(-a)^(1/3)*(b/a)^(1/3)*EllipticE[ArcSin[Sqrt[-(-1)^(5
/6) - (I*b^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] + ((-3 + Sq
rt[3])*b^(1/3) + Sqrt[3]*(-a)^(1/3)*(b/a)^(1/3))*EllipticF[ArcSin
[Sqrt[-(-1)^(5/6) - (I*b^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3
)]))/(3^(3/4)*b^(2/3)*Sqrt[-a - b*x^3])
```

Maple [B] time = 0.028, size = 1013, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] 2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*3^(
1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b
/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*
(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^{3}-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a),x, algorithm="fricas")
```

```
[Out] integral((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Sympy [A] time = 1.50767, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a),x, algorithm="giac")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

3.107 $\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=490

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.38411, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
```

```
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 28.4946, size = 430, normalized size = 0.88

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}+\frac{2d\sqrt{a+bx^{3}}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3)
 + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 + sq
rt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 -
 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 2*d*sqrt
(a + b*x**3)/(b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 2
*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1
/3) + b**(1/3)*x)*(-a**(1/3)*d*(-sqrt(3) + 1) + b**(1/3)*c)*ellip
tic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 +
 sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/
3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)*
*2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 0.329792, size = 221, normalized size = 0.45

$$-\frac{2\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^{2}}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left((-1)^{2/3}\sqrt{3}\sqrt[3]{a}dE\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-i\left(\sqrt[3]{a}d+\sqrt[3]{-b}c\right)F\cdots\right)}{\sqrt[4]{3}(-b)^{2/3}\sqrt{a+bx^{3}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a^(1/3)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*
Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*((-1)
```

```
^(2/3)*Sqrt[3]*a^(1/3)*d*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(
-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] - I*((-b)^(1/3)*c + a
^(1/3)*d)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^
(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(1/4)*(-b)^(2/3)*Sqrt[a + b*x^3
])
```

Maple [A] time = 0.006, size = 720, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*
(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.16881, size = 78, normalized size = 0.16

$$\frac{cx\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5
/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

3.108 $\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=503

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[Out] (2*d*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*S
qrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*c + (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2
/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.438419, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[a - b*x^3],x]
```


```
[Out] (2*d*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*S
qrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b
```

```
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*c + (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2
/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi in Sympy [A] time = 30.4166, size = 430, normalized size = 0.85

$$\frac{\sqrt[4]{3}\sqrt[3]{a}d\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{2d\sqrt{a-bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-b*x**3+a)**(1/2),x)
```

```
[Out] 3**(1/4)*a**(1/3)*d*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/
3)*x**2)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(-sqrt(3)
+ 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin((a**(1/3)*(-1 + sqrt
(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)), -7 - 4
*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/
3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3)) + 2*d*sqrt(a
 - b*x**3)/(b**(2/3)*(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)) + 2*3
**(3/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a*
*(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3
) - b**(1/3)*x)*(a**(1/3)*d*(-sqrt(3) + 1) + b**(1/3)*c)*elliptic
_f(asin((a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqr
t(3)) - b**(1/3)*x)), -7 - 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/3)*(
a**(1/3) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*
sqrt(a - b*x**3))
```

Mathematica [C] time = 0.249839, size = 208, normalized size = 0.41

$$-\frac{2\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left((-1)^{2/3}\sqrt{3}a^{2/3}dE\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-i\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\cdots\right.\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*Sqrt[((-1)^(5/6)*(-a^(1/3) + b^(1/3)*x))/a^(1/3)]*Sqrt[1 + (b
^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*((-1)^(2/3)*Sqrt[3]*a^
```

```
(2/3)*d*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^(1/3)*x)/a^(1/3)
]/3^(1/4)], (-1)^(1/3)] - I*a^(1/3)*(b^(1/3)*c + a^(1/3)*d)*Ellip
ticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (
-1)^(1/3)]))/(3^(1/4)*b^(2/3)*Sqrt[a - b*x^3])
```

Maple [A] time = 0.006, size = 681, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-b*x^3+a)^(1/2),x)
```

```
[Out] 2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(
a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)
))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))
*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*d*3^(
1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3
))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I
*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3)
)^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*
3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{-bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(-b*x^3 + a), x)
```

Sympy [A] time = 2.25458, size = 82, normalized size = 0.16

$$\frac{cx\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{2i\pi}}{a}\right)}{3\sqrt{a}\left(\frac{4}{3}\right)}+\frac{dx^{2}\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{2i\pi}}{a}\right)}{3\sqrt{a}\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-b*x**3+a)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)
/a)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3),
(5/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx+c}{\sqrt{-bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

3.109 $\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=515

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2d\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[Out] (-2*d*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)
*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)
*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]
)/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2 - Sqrt[3]]
*(b^(1/3)*c + (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]])/(3^
(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3
])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.451936, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2d\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (-2*d*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)
*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)
*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
```

```
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]
)/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2 - Sqrt[3]]
*(b^(1/3)*c + (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt
[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]])/(3^
(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3
])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi in Sympy [A] time = 31.1736, size = 434, normalized size = 0.84

$$\frac{\sqrt[4]{3}\sqrt[3]{a}d\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$
$$+\frac{2d\sqrt{-a+bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$
$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(\sqrt[3]{a}d\left(1+\sqrt{3}\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**3-a)**(1/2),x)
```

```
[Out] 3**(1/4)*a**(1/3)*d*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/
3)*x**2)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3)
+ 2)*(a**(1/3) - b**(1/3)*x)*elliptic_e(asin((a**(1/3)*(1 + sqrt(
3)) - b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)), -7 +
4*sqrt(3))/(b**(2/3)*sqrt(-a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(
1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a + b*x**3)) + 2*d*sq
rt(-a + b*x**3)/(b**(2/3)*(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x))
 - 2*3**(3/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**
2)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3) + 2)*
(a**(1/3) - b**(1/3)*x)*(a**(1/3)*d*(1 + sqrt(3)) + b**(1/3)*c)*e
lliptic_f(asin((a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)/(-a**(1/3)*(
-1 + sqrt(3)) - b**(1/3)*x)), -7 + 4*sqrt(3))/(3*b**(2/3)*sqrt(-a
**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/
3)*x)**2)*sqrt(-a + b*x**3))
```

Mathematica [C] time = 0.41676, size = 236, normalized size = 0.46

$$-\frac{2\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{-b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{-b}x\left(\sqrt[3]{-a}+\sqrt[3]{-b}x\right)}{(-a)^{2/3}}+1}\left((-1)^{2/3}\sqrt{3}\sqrt[3]{-a}dE\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-i\left(\sqrt[3]{-a}d\right.\right.}{\sqrt[4]{3}(-b)^{2/3}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (-2*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*(-b)^(1/3)*x))/
a)]*Sqrt[1 + ((-b)^(1/3)*x*((-a)^(1/3) + (-b)^(1/3)*x))/(-a)^(2/3
```

```
)]*((-1)^(2/3)*Sqrt[3]*(-a)^(1/3)*d*EllipticE[ArcSin[Sqrt[-(-1)^(
5/6) - (I*(-b)^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] - I*((-
b)^(1/3)*c + (-a)^(1/3)*d)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I
*(-b)^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(1/4)*(-b)^
(2/3)*Sqrt[-a + b*x^3])
```

Maple [A] time = 0.007, size = 683, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^3-a)^(1/2),x)
```

```
[Out] 2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(
a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)
))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))
*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(
1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*d*3^(1
/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3)
)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*
(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(
a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^
(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^3 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^3 - a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^3 - a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 2.34541, size = 73, normalized size = 0.14

$$-\frac{icx\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*
gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**
3/a)/(3*sqrt(a)*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^3 - a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

3.110 $\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=508

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$-\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (-2*d*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]]
)/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2 - Sqrt[3]]
*(b^(1/3)*c - (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.416953, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$-\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*d*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
```

```
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]]
)/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2 - Sqrt[3]]
*(b^(1/3)*c - (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi in Sympy [A] time = 30.4718, size = 435, normalized size = 0.86

$$\frac{\sqrt[4]{3}\sqrt[3]{a}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2d\sqrt{-a-bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}d\left(1+\sqrt{3}\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-b*x**3-a)**(1/2),x)
```

```
[Out] 3**(1/4)*a**(1/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/
3)*x**2)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3)
 + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((a**(1/3)*(1 + sqrt
(3)) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)), -7 +
 4*sqrt(3))/(b**(2/3)*sqrt(-a**(1/3)*(a**(1/3) + b**(1/3)*x)/(-a*
*(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-a - b*x**3)) + 2*d*
sqrt(-a - b*x**3)/(b**(2/3)*(a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x
)) + 2*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x
**2)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3) +
2)*(a**(1/3) + b**(1/3)*x)*(-a**(1/3)*d*(1 + sqrt(3)) + b**(1/3)*
c)*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)/(-a**(1/
3)*(-1 + sqrt(3)) + b**(1/3)*x)), -7 + 4*sqrt(3))/(3*b**(2/3)*sqr
t(-a**(1/3)*(a**(1/3) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b
**(1/3)*x)**2)*sqrt(-a - b*x**3))
```

Mathematica [C] time = 0.3837, size = 223, normalized size = 0.44

$$-\frac{2\sqrt[3]{-a}\sqrt{-\frac{(-1)^{5/6}\left((-a)^{2/3}\sqrt[3]{b}x+a\right)}{a}}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{-a}+\sqrt[3]{b}x\right)}{(-a)^{2/3}}+1}\left((-1)^{2/3}\sqrt{3}\sqrt[3]{-a}dE\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{b}x}{\sqrt[3]{-a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-i\left(\sqrt[3]{-a}d+\sqrt[3]{b}c\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*(-a)^(1/3)*Sqrt[-(((-1)^(5/6)*(a + (-a)^(2/3)*b^(1/3)*x))/a)]
*Sqrt[1 + (b^(1/3)*x*((-a)^(1/3) + b^(1/3)*x))/(-a)^(2/3)]*((-1)^
```

```
(2/3)*Sqrt[3]*(-a)^(1/3)*d*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I
*b^(1/3)*x)/(-a)^(1/3)]/3^(1/4)], (-1)^(1/3)] - I*(b^(1/3)*c + (-
a)^(1/3)*d)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*b^(1/3)*x)/(-a
)^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(3^(1/4)*b^(2/3)*Sqrt[-a - b*x^3
])
```

Maple [A] time = 0.006, size = 726, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/
b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^3 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^3 - a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{-bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^3 - a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(-b*x^3 - a), x)
```

Sympy [A] time = 2.24707, size = 83, normalized size = 0.16

$$-\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(a)*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3
), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^3 - a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

3.111 $\int \frac{c+dx}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=246

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2d\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*d*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]
]*d*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[Arc
Sin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]
]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 +
 x^3])
```

Rubi [A] time = 0.208461, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2d\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*d*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]
]*d*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[Arc
Sin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]
]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 +
 x^3])
```

Rubi in Sympy [A] time = 12.4651, size = 221, normalized size = 0.9

$$\frac{2d\sqrt{x^3+1}}{x+1+\sqrt{3}}-\frac{\sqrt[4]{3}d\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\left(c-d+\sqrt{3}d\right)F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] 2*d*sqrt(x**3 + 1)/(x + 1 + sqrt(3)) - 3**(1/4)*d*sqrt((x**2 - x
+ 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_e(
asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt(
(x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1)) + 2*3**(3/4)*sqrt((
x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(x + 1)*(c
- d + sqrt(3)*d)*elliptic_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(
3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(
x**3 + 1))
```

Mathematica [A]   time = 0.176978, size = 136, normalized size = 0.55

$$-\frac{2\sqrt{-\sqrt[6]{-1}\left(x+(-1)^{2/3}\right)}\sqrt{(-1)^{2/3}x^2+\sqrt[3]{-1}x+1}\left(\sqrt[6]{-1}\sqrt{3}\left((-1)^{2/3}d-c\right)F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}(x+1)}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+3dE\left(\sin^{-1}\left(\frac{\sqrt{-(}}{}\right.\right.\right.}{3^{3/4}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x)/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]*Sqrt[1 + (-1)^(1/3)*x +
(-1)^(2/3)*x^2]*(3*d*EllipticE[ArcSin[Sqrt[-((-1)^(5/6)*(1 + x))]
/3^(1/4)], (-1)^(1/3)] + (-1)^(1/6)*Sqrt[3]*(-c + (-1)^(2/3)*d)*E
llipticF[ArcSin[Sqrt[-((-1)^(5/6)*(1 + x))]/3^(1/4)], (-1)^(1/3)]
))/(3^(3/4)*Sqrt[1 + x^3])
```

Maple [A]   time = 0.007, size = 291, normalized size = 1.2

$$2\,\frac{c\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+2\,\frac{d\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\left(\left(-3/2-i/2\sqrt{3}\right)\mathit{EllipticE}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3}{-3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2*c*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2)
)/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2))+2*d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2
)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*
I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I
*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*Ellip
ticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{x^3 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(x^3 + 1), x)
```

Sympy [A] time = 1.73082, size = 61, normalized size = 0.25

$$\frac{cx\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3
*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*e
xp_polar(I*pi))/(3*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{x^3 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)
```

3.112 $\int \frac{c+dx}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=271

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{2d\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (2*d*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]
]*d*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[Arc
Sin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[
(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]
]*(c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x
)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 -
4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x
^3])
```

Rubi [A] time = 0.221322, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{2d\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*d*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]
]*d*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[Arc
Sin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[
(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]
]*(c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x
)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 -
4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x
^3])
```

Rubi in Sympy [A] time = 14.9134, size = 221, normalized size = 0.82

$$\frac{2d\sqrt{-x^3+1}}{-x+1+\sqrt{3}}-\frac{\sqrt[4]{3}d\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,E\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\left(c-\sqrt{3}d+d\right)F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] 2*d*sqrt(-x**3 + 1)/(-x + 1 + sqrt(3)) - 3**(1/4)*d*sqrt((x**2 +
x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*ellipti
c_e(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/
(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1)) - 2*3**(3/
4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(
-x + 1)*(c - sqrt(3)*d + d)*elliptic_f(asin((-x - sqrt(3) + 1)/(-
x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((-x + 1)/(-x + 1 + sqr
t(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.155923, size = 121, normalized size = 0.45

$$\frac{2i\sqrt{(-1)^{5/6}(x-1)}\sqrt{x^2+x+1}\left(\sqrt{3}(c+d)F\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-3\sqrt[6]{-1}dE\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\right)}{3^{3/4}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x)/Sqrt[1 - x^3],x]
```

```
[Out] ((2*I)*Sqrt[(-1)^(5/6)*(-1 + x)]*Sqrt[1 + x + x^2]*(-3*(-1)^(1/6)
*d*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)]
 + Sqrt[3]*(c + d)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/
4)], (-1)^(1/3)]))/(3^(3/4)*Sqrt[1 - x^3])
```

Maple [A] time = 0.006, size = 267, normalized size = 1.

$$-\frac{2i}{3}c\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

$$-\frac{2i}{3}d\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathit{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*c*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/
(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*d*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+
1)^(1/2)*((-3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2
))+EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),
(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx + c}{\sqrt{-x^3 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(-x^3 + 1), x)
```

Sympy [A] time = 1.79453, size = 65, normalized size = 0.24

$$\frac{cx\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/
(3*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3
*exp_polar(2*I*pi))/(3*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-x^3 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

3.113 $\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=275

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2d\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (-2*d*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[
3]]*d*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[A
rcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqr
t[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - S
qrt[3]]*(c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[
3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
 -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3])
```

Rubi [A] time = 0.208419, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2d\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*d*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[
3]]*d*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[A
rcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqr
t[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - S
qrt[3]]*(c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[
3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
 -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3])
```

Rubi in Sympy [A] time = 13.1052, size = 218, normalized size = 0.79

$$-\frac{2d\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{\sqrt[4]{3}d\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,E\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\left(c+d+\sqrt{3}d\right)F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -2*d*sqrt(x**3 - 1)/(-x - sqrt(3) + 1) + 3**(1/4)*d*sqrt((x**2 +
x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic
_e(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(
sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1)) - 2*3**(3/4)*
sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x
 + 1)*(c + d + sqrt(3)*d)*elliptic_f(asin((-x + 1 + sqrt(3))/(-x
- sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((x - 1)/(-x - sqrt(3) +
1)**2)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.133674, size = 119, normalized size = 0.43

$$\frac{2i\sqrt{(-1)^{5/6}(x-1)}\sqrt{x^2+x+1}\left(\sqrt{3}(c+d)F\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)-3\sqrt[6]{-1}dE\left(\sin^{-1}\left(\frac{\sqrt{-ix-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\right)}{3^{3/4}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x)/Sqrt[-1 + x^3],x]
```

```
[Out] ((2*I)*Sqrt[(-1)^(5/6)*(-1 + x)]*Sqrt[1 + x + x^2]*(-3*(-1)^(1/6)
*d*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/4)], (-1)^(1/3)]
 + Sqrt[3]*(c + d)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - I*x]/3^(1/
4)], (-1)^(1/3)]))/(3^(3/4)*Sqrt[-1 + x^3])
```

Maple [A] time = 0.006, size = 291, normalized size = 1.1

$$2\,\frac{c\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$
$$+2\,\frac{d\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\left(\left(3/2-i/2\sqrt{3}\right)\mathit{EllipticE}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(x^3-1)^(1/2),x)
```

```
[Out] 2*c*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+
1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/
2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-
3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))+2*d*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^
(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1
/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*
I*3^(1/2))*EllipticE(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2))*Ell
ipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(x^3 - 1),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx+c}{\sqrt{x^3-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(x^3 - 1),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(x^3 - 1), x)
```

Sympy [A] time = 1.8178, size = 56, normalized size = 0.2

$$-\frac{icx\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\,\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
- I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/
3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx+c}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(x^3 - 1),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)
```

3.114 $\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=261

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} - \frac{2d\sqrt{-x^3-1}}{x-\sqrt{3}+1} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (-2*d*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[
3]]*d*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[A
rcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqr
t[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - S
qrt[3]]*(c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)
], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]
*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.200216, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} - \frac{2d\sqrt{-x^3-1}}{x-\sqrt{3}+1} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*d*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[
3]]*d*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[A
rcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqr
t[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - S
qrt[3]]*(c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)
], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]
*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 12.9329, size = 226, normalized size = 0.87

$$-\frac{2d\sqrt{-x^3-1}}{x-\sqrt{3}+1} + \frac{\sqrt[4]{3}d\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\left(c-d\left(1+\sqrt{3}\right)\right)(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] -2*d*sqrt(-x**3 - 1)/(x - sqrt(3) + 1) + 3**(1/4)*d*sqrt((x**2 -
x + 1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_e
(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sqrt
((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1)) + 2*3**(3/4)*sqr
t((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(c - d*
(1 + sqrt(3)))*(x + 1)*elliptic_f(asin((x + 1 + sqrt(3))/(x - sqr
t(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2
)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.181432, size = 152, normalized size = 0.58

$$\frac{2\sqrt[6]{-1}\sqrt{-(-1)^{5/6}+ix}\sqrt{-\sqrt[3]{-1}x^2-(-1)^{2/3}x+1}\left(\sqrt{3}\left((-1)^{2/3}c-d\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\sqrt[6]{-1}\left(x+(-1)^{2/3}\right)}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+3\sqrt[6]{-1}dE\left(\sin^{-1}\left(\cdots\right.\right.\right.}{3^{3/4}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x)/Sqrt[-1 - x^3],x]
```

```
[Out] (2*(-1)^(1/6)*Sqrt[-(-1)^(5/6) + I*x]*Sqrt[1 - (-1)^(2/3)*x - (-1
)^(1/3)*x^2]*(3*(-1)^(1/6)*d*EllipticE[ArcSin[Sqrt[-((-1)^(1/6)*(
(-1)^(2/3) + x))]/3^(1/4)], (-1)^(1/3)] + Sqrt[3]*((-1)^(2/3)*c -
 d)*EllipticF[ArcSin[Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]/3^(1/4)
], (-1)^(1/3)]))/(3^(3/4)*Sqrt[-1 - x^3])
```

Maple [A] time = 0.007, size = 269, normalized size = 1.

$$-\frac{2i}{3}c\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3\cdots}}$$
$$-\frac{2i}{3}d\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)EllipticE\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{\cdots}{\frac{3}{2}\cdots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*c*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*d*3^(1
/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1
/2)*((3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-Elli
pticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1
/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx+c}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-x^3 - 1),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx + c}{\sqrt{-x^3 - 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-x^3 - 1),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(-x^3 - 1), x)
```

Sympy [A] time = 1.75819, size = 66, normalized size = 0.25

$$-\frac{icx\,\Gamma\left(\frac{1}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))
/(3*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x
**3*exp_polar(I*pi))/(3*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-x^3 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-x^3 - 1),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

3.115 $\int \frac{c+dx}{a-bx^4}\,dx$

Optimal. Leaf size=87

$$\frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] (c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x
^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.15086, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$\frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x
^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])
```

Rubi in Sympy [A] time = 21.5933, size = 82, normalized size = 0.94

$$\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}\sqrt[4]{b}}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-b*x**4+a),x)
```

```
[Out] d*atanh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*sqrt(b)) + c*atan(b**(1/
4)*x/a**(1/4))/(2*a**(3/4)*b**(1/4)) + c*atanh(b**(1/4)*x/a**(1/4
))/(2*a**(3/4)*b**(1/4))
```

Mathematica [A] time = 0.0632107, size = 134, normalized size = 1.54

$$\frac{-\left(\sqrt[4]{a}d+\sqrt[4]{b}c\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)+\sqrt[4]{b}c\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)+2\sqrt[4]{b}c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\sqrt[4]{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)-\sqrt[4]{a}d\log\left(\sqrt[4]{a}+\sqrt[4]{b}\right.}{4a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (2*b^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(1/4)*c + a^(1/4)*d
)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*c*Log[a^(1/4) + b^(1/4)*x] -
 a^(1/4)*d*Log[a^(1/4) + b^(1/4)*x] + a^(1/4)*d*Log[Sqrt[a] + Sqr
t[b]*x^2])/(4*a^(3/4)*Sqrt[b])
```

Maple [A] time = 0.008, size = 101, normalized size = 1.2

$$\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{d}{4}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*c*(a/
b)^(1/4)/a*arctan(x/(a/b)^(1/4))-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*
b)^(1/2))/(-a-x^2*(a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x + c)/(b*x^4 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x + c)/(b*x^4 - a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.36486, size = 126, normalized size = 1.45

$$-\operatorname{RootSum}\left(256t^4a^3b^2-32t^2a^2bd^2-16tabc^2d+ad^4-bc^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2+16t^2a^2bc^2d+8ta^2d^4-4tabc^4}{4acd^4+bc^5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c
**2*d + a*d**4 - b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b
*d**2 + 16*_t**2*a**2*b*c**2*d + 8*_t*a**2*d**4 - 4*_t*a*b*c**4 +
 5*a*c**2*d**3)/(4*a*c*d**4 + b*c**5))))
```

GIAC/XCAS [A] time = 0.216631, size = 304, normalized size = 3.49

$$\frac{\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\ln\left(x^{2}+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab}-\frac{\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\ln\left(x^{2}-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}bd+\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^{2}}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}bd+\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x + c)/(b*x^4 - a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/(a*b) - 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*ln(x^2 - sqrt(2)*x
*(-a/b)^(1/4) + sqrt(-a/b))/(a*b) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*
b)*b*d + (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*
b)*b*d + (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)
```

3.116 $\int \frac{c+dx}{a+bx^4}\, dx$

Optimal. Leaf size=219

$$-\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) +
(c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^
(1/4)) - (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rubi [A] time = 0.397686, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6

$$-\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) +
(c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^
(1/4)) - (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rubi in Sympy [A] time = 58.5732, size = 207, normalized size = 0.95

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{b}}+\frac{\sqrt{2}c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{b}}-\frac{\sqrt{2}c\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{b}}+\frac{\sqrt{2}c\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**4+a),x)
```

```
[Out] d*atan(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*sqrt(b)) - sqrt(2)*c*log(
-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(8*a**(3/4
)*b**(1/4)) + sqrt(2)*c*log(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a)
 + sqrt(b)*x**2)/(8*a**(3/4)*b**(1/4)) - sqrt(2)*c*atan(1 - sqrt(
2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(1/4)) + sqrt(2)*c*atan(1
+ sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(1/4))
```

Mathematica [A] time = 0.140262, size = 184, normalized size = 0.84

$$\frac{-2\left(2\sqrt[4]{a}d+\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{b}c-2\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)+\sqrt{2}\sqrt[4]{b}c\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right.\right.}{8a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(1/4)*c + 2*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)] + 2*(Sqrt[2]*b^(1/4)*c - 2*a^(1/4)*d)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*c*(-Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/4)*Sqrt[b])
```

Maple [A] time = 0.009, size = 151, normalized size = 0.7

$$\frac{c\sqrt{2}}{8\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{d}{2}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*c*(a/b)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*c*(a/b)^(1/4)/a*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/2*d/(a*b)^(1/2)*arctan(x^2
*(b/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.35383, size = 124, normalized size = 0.57

$$\operatorname{RootSum}\left(256t^4a^3b^2 + 32t^2a^2bd^2 - 16tabc^2d + ad^4 + bc^4, \left(t \mapsto t\log\left(x + \frac{-128t^3a^3bd^2 - 16t^2a^2bc^2d - 8ta^2d^4 - 4tabc^4 + }{4acd^4 - bc^5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c*
*2*d + a*d**4 + b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*
d**2 - 16*_t**2*a**2*b*c**2*d - 8*_t*a**2*d**4 - 4*_t*a*b*c**4 +
5*a*c**2*d**3)/(4*a*c*d**4 - b*c**5))))
```

GIAC/XCAS [A] time = 0.216012, size = 288, normalized size = 1.32

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}} c\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}} c\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}bd - \left(ab^3\right)^{\frac{1}{4}} bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}bd - \left(ab^3\right)^{\frac{1}{4}} bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*c*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt
(a/b))/(a*b) - 1/8*sqrt(2)*(a*b^3)^(1/4)*c*ln(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d
- (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4
))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d - (a
*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a*b^2)
```

3.117 $\int \frac{c+dx}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=110

$$\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\,(a-bx^4)}$$

```
[Out] (x*(c + d*x))/(4*a*(a - b*x^4)) + (3*c*ArcTan[(b^(1/4)*x)/a^(1/4)
])/(8*a^(7/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^
(7/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sq
rt[b])
```

Rubi [A] time = 0.18865, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\,(a-bx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a - b*x^4)) + (3*c*ArcTan[(b^(1/4)*x)/a^(1/4)
])/(8*a^(7/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^
(7/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sq
rt[b])
```

Rubi in Sympy [A] time = 32.9588, size = 102, normalized size = 0.93

$$\frac{x\,(c+dx)}{4a\,(a-bx^4)}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}+\frac{3c\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}\sqrt[4]{b}}+\frac{3c\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] x*(c + d*x)/(4*a*(a - b*x**4)) + d*atanh(sqrt(b)*x**2/sqrt(a))/(4
*a**(3/2)*sqrt(b)) + 3*c*atan(b**(1/4)*x/a**(1/4))/(8*a**(7/4)*b*
*(1/4)) + 3*c*atanh(b**(1/4)*x/a**(1/4))/(8*a**(7/4)*b**(1/4))
```

Mathematica [A] time = 0.343325, size = 168, normalized size = 1.53

$$\frac{\frac{4ax(c+dx)}{a-bx^4}-\frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c+2\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c-2\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{6\sqrt[4]{a}c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{2\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{16a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + d*x))/(a - b*x^4) + (6*a^(1/4)*c*ArcTan[(b^(1/4)*x)/
a^(1/4)])/b^(1/4) - ((3*a^(1/4)*b^(1/4)*c + 2*Sqrt[a]*d)*Log[a^(1
```

```
/4) - b^(1/4)*x])/Sqrt[b] + ((3*a^(1/4)*b^(1/4)*c - 2*Sqrt[a]*d)*
Log[a^(1/4) + b^(1/4)*x])/Sqrt[b] + (2*Sqrt[a]*d*Log[Sqrt[a] + Sq
rt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Maple [A] time = 0.007, size = 142, normalized size = 1.3

$$-\frac{cx}{4\,a\,(bx^4-a)}+\frac{3\,c}{16\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3\,c}{8\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{dx^2}{4\,a\,(bx^4-a)}-\frac{d}{8\,a}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] -1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x
-(a/b)^(1/4)))+3/8*c/a^2*(a/b)^(1/4)*arctan(x/(a/b)^(1/4))-1/4*d*
x^2/a/(b*x^4-a)-1/8*d/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x
^2*(a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 - a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 - a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.41347, size = 155, normalized size = 1.41

$$\operatorname{RootSum}\left(65536t^4a^7b^2-2048t^2a^4bd^2+1152ta^2bc^2d+16ad^4-81bc^4,\left(t\mapsto t\log\left(x+\frac{32768t^3a^6bd^2+4608t^2a^4bc^2d-51}{192acd^4+}\right.\right.\right.$$
$$-\frac{cx+dx^2}{-4a^2+4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**2 - 2048*_t**2*a**4*b*d**2 + 1152*_t*
a**2*b*c**2*d + 16*a*d**4 - 81*b*c**4, Lambda(_t, _t*log(x + (327
```

```
68*_t**3*a**6*b*d**2 + 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**
4 + 1296*_t*a**2*b*c**4 + 360*a*c**2*d**3)/(192*a*c*d**4 + 243*b*
c**5)))) - (c*x + d*x**2)/(-4*a**2 + 4*a*b*x**4)
```

GIAC/XCAS [A] time = 0.221915, size = 343, normalized size = 3.12

$$\frac{3\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\ln\left(x^{2}+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^{2}b}-\frac{3\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\ln\left(x^{2}-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^{2}b}-\frac{dx^{2}+cx}{4\,(bx^{4}-a)a}-\frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-ab}bd-3\,\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^{2}b^{2}}-\frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-ab}bd-3\,\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 - a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + s
qrt(-a/b))/(a^2*b) - 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*ln(x^2 - sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b) - 1/4*(d*x^2 + c*x)/((b*x
^4 - a)*a) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b*d - 3*(-a*b^3)^
(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/(a^2*b^2) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b*d - 3*(-
a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))
/(-a/b)^(1/4))/(a^2*b^2)
```

3.118 $\int \frac{c+dx}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=241

$$-\frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\left(a+bx^4\right)}$$

```
[Out] (x*(c + d*x))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]
])/(4*a^(3/2)*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*c*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^
(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rubi [A] time = 0.440223, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]
])/(4*a^(3/2)*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*c*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^
(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rubi in Sympy [A] time = 74.7733, size = 231, normalized size = 0.96

$$\frac{x\left(c+dx\right)}{4a\left(a+bx^4\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}-\frac{3\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}\sqrt[4]{b}}+\frac{3\sqrt{2}c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}\sqrt[4]{b}}-\frac{3\sqrt{2}c\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\sqrt[4]{b}}+\frac{3\sqrt{2}c\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] x*(c + d*x)/(4*a*(a + b*x**4)) + d*atan(sqrt(b)*x**2/sqrt(a))/(4*
a**(3/2)*sqrt(b)) - 3*sqrt(2)*c*log(-sqrt(2)*a**(1/4)*b**(1/4)*x
+ sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(1/4)) + 3*sqrt(2)*c*lo
g(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7
/4)*b**(1/4)) - 3*sqrt(2)*c*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))
/(16*a**(7/4)*b**(1/4)) + 3*sqrt(2)*c*atan(1 + sqrt(2)*b**(1/4)*x
```

```
/a**(1/4))/(16*a**(7/4)*b**(1/4))
```

Mathematica [A] time = 0.396176, size = 224, normalized size = 0.93

$$\frac{\frac{8a^{3/4}x(c+dx)}{a+bx^4}-\frac{2\left(4\sqrt[4]{a}d+3\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}}+\frac{2\left(3\sqrt{2}\sqrt[4]{b}c-4\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{b}}-\frac{3\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{b}}+3\sqrt{2}c\log(}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(c + d*x))/(a + b*x^4) - (2*(3*Sqrt[2]*b^(1/4)*c +
4*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] + (
2*(3*Sqrt[2]*b^(1/4)*c - 4*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/Sqrt[b] - (3*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (3*Sqrt[2]*c*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(32*a^(7/4))
```

Maple [A] time = 0.007, size = 188, normalized size = 0.8

$$\frac{cx}{4\,a\left(bx^4+a\right)}+\frac{3\,c\sqrt{2}}{32\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{3\,c\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\,c\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{dx^2}{4\,a\left(bx^4+a\right)}+\frac{d}{4\,a}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] 1/4*c*x/a/(b*x^4+a)+3/32*c/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^
(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2)))+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+
1)+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
+1/4*d*x^2/a/(b*x^4+a)+1/4*d/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.41194, size = 155, normalized size = 0.64

$$\mathrm{RootSum}\left(65536t^4a^7b^2 + 2048t^2a^4bd^2 - 1152ta^2bc^2d + 16ad^4 + 81bc^4, \left(t \mapsto t\log\left(x + \frac{-32768t^3a^6bd^2 - 4608t^2a^4bc^2d - 5}{192acd^4 -}\right.\right.\right.$$
$$+ \frac{cx + dx^2}{4a^2 + 4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**2 + 2048*_t**2*a**4*b*d**2 - 1152*_t*
a**2*b*c**2*d + 16*a*d**4 + 81*b*c**4, Lambda(_t, _t*log(x + (-32
768*_t**3*a**6*b*d**2 - 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d*
*4 - 1296*_t*a**2*b*c**4 + 360*a*c**2*d**3)/(192*a*c*d**4 - 243*b
*c**5)))) + (c*x + d*x**2)/(4*a**2 + 4*a*b*x**4)
```

GIAC/XCAS [A] time = 0.217511, size = 321, normalized size = 1.33

$$\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}} c\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b} - \frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}} c\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b} + \frac{dx^2 + cx}{4\left(bx^4 + a\right)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}bd + 3\left(ab^3\right)^{\frac{1}{4}} bc\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}bd + 3\left(ab^3\right)^{\frac{1}{4}} bc\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*b^3)^(1/4)*c*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b) - 3/32*sqrt(2)*(a*b^3)^(1/4)*c*ln(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a^2*b) + 1/4*(d*x^2 + c*x)/((b*x^4 + a
)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)*b*
c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)
*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^2*b^2)
```

3.119 $\int \frac{c+dx}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=136

$$\frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(7c+6dx)}{32a^2\,(a-bx^4)}+\frac{x(c+dx)}{8a\,(a-bx^4)^2}$$

```
[Out] (x*(c + d*x))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a
- b*x^4)) + (21*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/
4)) + (21*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) +
 (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.234074, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(7c+6dx)}{32a^2\,(a-bx^4)}+\frac{x(c+dx)}{8a\,(a-bx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a
- b*x^4)) + (21*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/
4)) + (21*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) +
 (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 42.2882, size = 128, normalized size = 0.94

$$\frac{x\,(c+dx)}{8a\,(a-bx^4)^2}+\frac{x\,(7c+6dx)}{32a^2\,(a-bx^4)}+\frac{3d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}+\frac{21c\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}\sqrt[4]{b}}+\frac{21c\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] x*(c + d*x)/(8*a*(a - b*x**4)**2) + x*(7*c + 6*d*x)/(32*a**2*(a -
 b*x**4)) + 3*d*atanh(sqrt(b)*x**2/sqrt(a))/(16*a**(5/2)*sqrt(b))
 + 21*c*atan(b**(1/4)*x/a**(1/4))/(64*a**(11/4)*b**(1/4)) + 21*c*
atanh(b**(1/4)*x/a**(1/4))/(64*a**(11/4)*b**(1/4))
```

Mathematica [A] time = 0.338919, size = 193, normalized size = 1.42

$$\frac{\frac{16a^2x(c+dx)}{(a-bx^4)^2}+\frac{4ax(7c+6dx)}{a-bx^4}-\frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c+4\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c-4\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{42\sqrt[4]{a}c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{12\sqrt{a}d\log(\cdots}{\sqrt{b}\cdots}}{128a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + d*x))/(a - b*x^4)^2 + (4*a*x*(7*c + 6*d*x))/(a -
b*x^4) + (42*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(1/4) - (3*
```

```
(7*a^(1/4)*b^(1/4)*c + 4*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqr
t[b] + (3*(7*a^(1/4)*b^(1/4)*c - 4*Sqrt[a]*d)*Log[a^(1/4) + b^(1/
4)*x])/Sqrt[b] + (12*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b
])/(128*a^3)
```

Maple [A] time = 0.01, size = 180, normalized size = 1.3

$$\frac{cx}{8\,a\,(bx^4-a)^2}-\frac{7\,cx}{32\,a^2\,(bx^4-a)}+\frac{21\,c}{128\,a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{21\,c}{64\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{dx^2}{8\,a\,(bx^4-a)^2}-\frac{3\,dx^2}{16\,a^2\,(bx^4-a)}-\frac{3\,d}{32\,a^2}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(a/b)^(
1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+21/64*c/a^3*(a/b)^(1/4)*
arctan(x/(a/b)^(1/4))+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x
^4-a)-3/32*d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b
)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x + c)/(b*x^4 - a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x + c)/(b*x^4 - a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 4.34004, size = 194, normalized size = 1.43

$$-\operatorname{RootSum}\left(268435456t^4a^{11}b^2-4718592t^2a^6bd^2-2709504ta^3bc^2d+20736ad^4-194481bc^4,\left(t\mapsto t\log\left(x+\frac{-67108864\ldots}{}\right.\right.\right.-\frac{-11acx-10adx^2+7bcx^5+6bdx^6}{32a^4-64a^3bx^4+32a^2b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**2 - 4718592*_t**2*a**6*b*d**2 -
 2709504*_t*a**3*b*c**2*d + 20736*a*d**4 - 194481*b*c**4, Lambda(
_t, _t*log(x + (-67108864*_t**3*a**9*b*d**2 + 9633792*_t**2*a**6*
b*c**2*d + 589824*_t*a**4*d**4 - 2765952*_t*a**3*b*c**4 + 423360*
a*c**2*d**3)/(193536*a*c*d**4 + 453789*b*c**5)))) - (-11*a*c*x -
10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 - 64*a**3*b*x**4
+ 32*a**2*b**2*x**8)
```

GIAC/XCAS [A] time = 0.221663, size = 367, normalized size = 2.7

$$\frac{21\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b}-\frac{21\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b}+\frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd+7\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2}+\frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd+7\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2}-\frac{6\,bdx^6+7\,bcx^5-10\,adx^2-11\,acx}{32\left(bx^4-a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x + c)/(b*x^4 - a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) +
 sqrt(-a/b))/(a^3*b) - 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*ln(x^2 - s
qrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sq
rt(2)*sqrt(-a*b)*b*d + 7*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^2) + 3/128*sqrt(
2)*(4*sqrt(2)*sqrt(-a*b)*b*d + 7*(-a*b^3)^(1/4)*b*c)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^2) - 1/3
2*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*
a^2)
```

3.120 $\int \frac{c+dx}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=266

$$-\frac{21c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

```
[Out] (x*(c + d*x))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a
+ b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[
b]) - (21*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*
a^(11/4)*b^(1/4)) + (21*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
)/(64*Sqrt[2]*a^(11/4)*b^(1/4)) - (21*c*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(1/4)) + (
21*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128
*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rubi [A] time = 0.506791, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{21c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a
+ b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[
b]) - (21*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*
a^(11/4)*b^(1/4)) + (21*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
)/(64*Sqrt[2]*a^(11/4)*b^(1/4)) - (21*c*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(1/4)) + (
21*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128
*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rubi in Sympy [A] time = 86.4273, size = 257, normalized size = 0.97

$$\frac{x\left(c+dx\right)}{8a\left(a+bx^4\right)^2}+\frac{x\left(7c+6dx\right)}{32a^2\left(a+bx^4\right)}+\frac{3d\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}-\frac{21\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{256a^{\frac{11}{4}}\sqrt[4]{b}}+\frac{21\sqrt{2}c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{256a^{\frac{11}{4}}\sqrt[4]{b}}-\frac{21\sqrt{2}c\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}\sqrt[4]{b}}+\frac{21\sqrt{2}c\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] x*(c + d*x)/(8*a*(a + b*x**4)**2) + x*(7*c + 6*d*x)/(32*a**2*(a +
 b*x**4)) + 3*d*atan(sqrt(b)*x**2/sqrt(a))/(16*a**(5/2)*sqrt(b))
- 21*sqrt(2)*c*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(
```

```
b)*x**2)/(256*a**(11/4)*b**(1/4)) + 21*sqrt(2)*c*log(sqrt(2)*a**(
1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(256*a**(11/4)*b**(1/4)
) - 21*sqrt(2)*c*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(128*a**(1
1/4)*b**(1/4)) + 21*sqrt(2)*c*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4
))/(128*a**(11/4)*b**(1/4))
```

Mathematica [A] time = 0.410365, size = 249, normalized size = 0.94

$$\frac{\frac{32a^{7/4}x(c+dx)}{(a+bx^4)^2}+\frac{8a^{3/4}x(7c+6dx)}{a+bx^4}-\frac{6\left(8\sqrt[4]{a}d+7\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}}+\frac{6\left(7\sqrt{2}\sqrt[4]{b}c-8\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{b}}-\frac{21\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}\right.}{\sqrt[4]{b}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(c + d*x))/(a + b*x^4)^2 + (8*a^(3/4)*x*(7*c + 6*d
*x))/(a + b*x^4) - (6*(7*Sqrt[2]*b^(1/4)*c + 8*a^(1/4)*d)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] + (6*(7*Sqrt[2]*b^(1/4)
*c - 8*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b
] - (21*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/b^(1/4) + (21*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(256*a^(11/4))
```

Maple [A] time = 0.008, size = 222, normalized size = 0.8

$$\frac{cx}{8\,a\left(bx^4+a\right)^2}+\frac{7\,cx}{32\,a^2\left(bx^4+a\right)}+\frac{21\,c\sqrt{2}}{256\,a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{21\,c\sqrt{2}}{128\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{21\,c\sqrt{2}}{128\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{dx^2}{8\,a\left(bx^4+a\right)^2}+\frac{3\,dx^2}{16\,a^2\left(bx^4+a\right)}+\frac{3\,d}{16\,a^2}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(a/b)^(
1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/8*d*x^2/a/(b*x^4+a)^2+3/16*d/a^
2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 4.26566, size = 192, normalized size = 0.72

$$\operatorname{RootSum}\left(268435456t^4a^{11}b^2 + 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 + 194481bc^4, \left(t \mapsto t\log\left(x + \frac{-67108864t^3a\right.\right.\right.$$
$$+ \frac{11acx + 10adx^2 + 7bcx^5 + 6bdx^6}{32a^4 + 64a^3bx^4 + 32a^2b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**2 + 4718592*_t**2*a**6*b*d**2 -
2709504*_t*a**3*b*c**2*d + 20736*a*d**4 + 194481*b*c**4, Lambda(_
t, _t*log(x + (-67108864*_t**3*a**9*b*d**2 - 9633792*_t**2*a**6*b
*c**2*d - 589824*_t*a**4*d**4 - 2765952*_t*a**3*b*c**4 + 423360*a
*c**2*d**3)/(193536*a*c*d**4 - 453789*b*c**5)))) + (11*a*c*x + 10
*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 + 64*a**3*b*x**4 +
32*a**2*b**2*x**8)
```

GIAC/XCAS [A] time = 0.221637, size = 346, normalized size = 1.3

$$\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}c\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b} - \frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}c\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b}$$
$$+ \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2}$$
$$+ \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} + \frac{6\,bdx^6+7\,bcx^5+10\,adx^2+11\,acx}{32\left(bx^4+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*b^3)^(1/4)*c*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a^3*b) - 21/256*sqrt(2)*(a*b^3)^(1/4)*c*ln(x^2 - sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*
sqrt(a*b)*b*d + 7*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^2) + 3/128*sqrt(2)*(4*sqrt
(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^2) + 1/32*(6*b*d*x^6 +
 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/((b*x^4 + a)^2*a^2)
```

3.121 $\int \frac{c+dx}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=162

$$\frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(c+dx)}{12a\left(a-bx^4\right)^3}$$

```
[Out] (x*(c + d*x))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*
(a - b*x^4)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a - b*x^4)) + (77*
c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (77*c*Arc
Tanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (5*d*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b])
```

Rubi [A] time = 0.292439, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(c+dx)}{12a\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a - b*x^4)^4,x]
```

```
[Out] (x*(c + d*x))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*
(a - b*x^4)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a - b*x^4)) + (77*
c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (77*c*Arc
Tanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (5*d*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 52.3235, size = 151, normalized size = 0.93

$$\frac{x\left(c+dx\right)}{12a\left(a-bx^4\right)^3}+\frac{x\left(11c+10dx\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx\right)}{384a^3\left(a-bx^4\right)}+\frac{5d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}}+\frac{77c\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}\sqrt[4]{b}}+\frac{77c\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] x*(c + d*x)/(12*a*(a - b*x**4)**3) + x*(11*c + 10*d*x)/(96*a**2*(
a - b*x**4)**2) + x*(77*c + 60*d*x)/(384*a**3*(a - b*x**4)) + 5*d
*atanh(sqrt(b)*x**2/sqrt(a))/(32*a**(7/2)*sqrt(b)) + 77*c*atan(b*
*(1/4)*x/a**(1/4))/(256*a**(15/4)*b**(1/4)) + 77*c*atanh(b**(1/4)
*x/a**(1/4))/(256*a**(15/4)*b**(1/4))
```

Mathematica [A] time = 0.431859, size = 217, normalized size = 1.34

$$\frac{\frac{128a^3x(c+dx)}{(a-bx^4)^3}+\frac{16a^2x(11c+10dx)}{(a-bx^4)^2}+\frac{4ax(77c+60dx)}{a-bx^4}-\frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}c+40\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}c-40\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{462\sqrt[4]{a}c}{}}{1536a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + d*x))/(a - b*x^4)^3 + (16*a^2*x*(11*c + 10*d*x))
/(a - b*x^4)^2 + (4*a*x*(77*c + 60*d*x))/(a - b*x^4) + (462*a^(1/
4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(1/4) - (3*(77*a^(1/4)*b^(1/4
)*c + 40*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqrt[b] + (3*(77*a^
(1/4)*b^(1/4)*c - 40*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt[b]
 + (120*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Maple [A] time = 0.022, size = 184, normalized size = 1.1

$$\frac{1}{(bx^4-a)^3}\left(-\frac{5\,b^2dx^{10}}{32\,a^3}-\frac{77\,b^2cx^9}{384\,a^3}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}-\frac{11\,dx^2}{32\,a}-\frac{51\,cx}{128\,a}\right)$$
$$+\frac{77\,c}{512\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{77\,c}{256\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{5\,d}{64}\ln\left(1\left(-a^4+x^2\sqrt{ba^7}\right)\left(-a^4-x^2\sqrt{ba^7}\right)^{-1}\right)\frac{1}{\sqrt{ba^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-5/32*d/a^3*b^2*x^10-77/384*c/a^3*b^2*x^9+5/12/a^2*d*b*x^6+33/64
/a^2*c*b*x^5-11/32*d/a*x^2-51/128/a*c*x)/(b*x^4-a)^3+77/512*c*(a/
b)^(1/4)/a^4*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+77/256*c*(a/b)^(
1/4)/a^4*arctan(x/(a/b)^(1/4))-5/64*d/(b*a^7)^(1/2)*ln((-a^4+x^2*
(b*a^7)^(1/2))/(-a^4-x^2*(b*a^7)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 - a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 - a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 16.2026, size = 231, normalized size = 1.43

$$\operatorname{RootSum}\left(68719476736t^4a^{15}b^2 - 838860800t^2a^8bd^2 + 485703680ta^4bc^2d + 2560000ad^4 - 35153041bc^4, \left(t \mapsto t\log\left(x + \frac{42\cdots}{}\right.\right.\right.$$
$$- \frac{153a^2cx + 132a^2dx^2 - 198abcx^5 - 160abdx^6 + 77b^2cx^9 + 60b^2dx^{10}}{-384a^6 + 1152a^5bx^4 - 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 - 838860800*_t**2*a**8*b*d**
2 + 485703680*_t*a**4*b*c**2*d + 2560000*a*d**4 - 35153041*b*c**4
, Lambda(_t, _t*log(x + (429496729600*_t**3*a**12*b*d**2 + 621700
71040*_t**2*a**8*b*c**2*d - 2621440000*_t*a**5*d**4 + 17998356992
*_t*a**4*b*c**4 + 1897280000*a*c**2*d**3)/(788480000*a*c*d**4 + 2
706784157*b*c**5)))) - (153*a**2*c*x + 132*a**2*d*x**2 - 198*a*b*
c*x**5 - 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*x**10)/(-384
*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3*x*
*12)
```

GIAC/XCAS [A] time = 0.223605, size = 400, normalized size = 2.47

$$\frac{77\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\ln\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024\,a^4b} - \frac{77\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\ln\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024\,a^4b}$$
$$- \frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{-ab}bd - 77\,\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}$$
$$- \frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{-ab}bd - 77\,\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}$$
$$- \frac{60\,b^2dx^{10} + 77\,b^2cx^9 - 160\,abdx^6 - 198\,abcx^5 + 132\,a^2dx^2 + 153\,a^2cx}{384\left(bx^4 - a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 - a)^4,x, algorithm="giac")
```

```
[Out] 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4)
+ sqrt(-a/b))/(a^4*b) - 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*ln(x^2 -
 sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b) - 1/512*sqrt(2)*(40
*sqrt(2)*sqrt(-a*b)*b*d - 77*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^2) - 1/512*s
qrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d - 77*(-a*b^3)^(1/4)*b*c)*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^2)
 - 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*
c*x^5 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

3.122 $\int \frac{c+dx}{(a+bx^4)^4} \, dx$

Optimal. Leaf size=291

$$-\frac{77c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{77c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{77c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{77c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(77c+60dx)}{384a^3(a+bx^4)}+\frac{x(11c+10dx)}{96a^2(a+bx^4)^2}+\frac{x(c+dx)}{12a(a+bx^4)^3}$$

```
[Out] (x*(c + d*x))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*
(a + b*x^4)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a + b*x^4)) + (5*d
*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (77*c*ArcT
an[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4
)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*
a^(15/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15
/4)*b^(1/4))
```

Rubi [A] time = 0.580991, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{77c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{77c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{77c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{77c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(77c+60dx)}{384a^3(a+bx^4)}+\frac{x(11c+10dx)}{96a^2(a+bx^4)^2}+\frac{x(c+dx)}{12a(a+bx^4)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] (x*(c + d*x))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*
(a + b*x^4)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a + b*x^4)) + (5*d
*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (77*c*ArcT
an[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4
)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*
a^(15/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15
/4)*b^(1/4))
```

Rubi in Sympy [A] time = 98.4271, size = 280, normalized size = 0.96

$$\frac{x\left(c+dx\right)}{12a\left(a+bx^4\right)^3}+\frac{x\left(11c+10dx\right)}{96a^2\left(a+bx^4\right)^2}+\frac{x\left(77c+60dx\right)}{384a^3\left(a+bx^4\right)}+\frac{5d\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}}-\frac{77\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{1024a^{\frac{15}{4}}\sqrt[4]{b}}+\frac{77\sqrt{2}c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{1024a^{\frac{15}{4}}\sqrt[4]{b}}-\frac{77\sqrt{2}c\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{15}{4}}\sqrt[4]{b}}+\frac{77\sqrt{2}c\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{15}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] x*(c + d*x)/(12*a*(a + b*x**4)**3) + x*(11*c + 10*d*x)/(96*a**2*(
a + b*x**4)**2) + x*(77*c + 60*d*x)/(384*a**3*(a + b*x**4)) + 5*d
*atan(sqrt(b)*x**2/sqrt(a))/(32*a**(7/2)*sqrt(b)) - 77*sqrt(2)*c*
log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(1024*
a**(15/4)*b**(1/4)) + 77*sqrt(2)*c*log(sqrt(2)*a**(1/4)*b**(1/4)*
x + sqrt(a) + sqrt(b)*x**2)/(1024*a**(15/4)*b**(1/4)) - 77*sqrt(2
)*c*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(512*a**(15/4)*b**(1/4)
) + 77*sqrt(2)*c*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(512*a**(1
5/4)*b**(1/4))
```

Mathematica [A] time = 0.510721, size = 274, normalized size = 0.94

$$\frac{\frac{256a^{11/4}x(c+dx)}{(a+bx^4)^3}+\frac{32a^{7/4}x(11c+10dx)}{(a+bx^4)^2}+\frac{8a^{3/4}x(77c+60dx)}{a+bx^4}-\frac{6\left(80\sqrt[4]{a}d+77\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}}+\frac{6\left(77\sqrt{2}\sqrt[4]{b}c-80\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{b}}-}{3072a^{15/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^(11/4)*x*(c + d*x))/(a + b*x^4)^3 + (32*a^(7/4)*x*(11*c +
 10*d*x))/(a + b*x^4)^2 + (8*a^(3/4)*x*(77*c + 60*d*x))/(a + b*x^
4) - (6*(77*Sqrt[2]*b^(1/4)*c + 80*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/Sqrt[b] + (6*(77*Sqrt[2]*b^(1/4)*c - 80*a^(
1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (231*S
qrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
b^(1/4) + (231*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/b^(1/4))/(3072*a^(15/4))
```

Maple [A] time = 0.024, size = 224, normalized size = 0.8

$$\begin{aligned}&\frac{1}{(bx^4+a)^3}\left(\frac{5\,b^2dx^{10}}{32\,a^3}+\frac{77\,b^2cx^9}{384\,a^3}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}+\frac{11\,dx^2}{32\,a}+\frac{51\,cx}{128\,a}\right)\\&+\frac{77\,c\sqrt{2}}{1024\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\\&+\frac{77\,c\sqrt{2}}{512\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\\&+\frac{77\,c\sqrt{2}}{512\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{5\,d}{32}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ba^7}}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+5/12/a^2*d*b*x^6+33/64/
a^2*c*b*x^5+11/32*d/a*x^2+51/128/a*c*x)/(b*x^4+a)^3+77/1024*c*(a/
b)^(1/4)/a^4*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(
x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+77/512*c*(a/b)^(1/4)/a^4*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+77/512*c*(a/b)^(1/4)/a^4*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+5/32*d/(b*a^7)^(1/2)*arct
an(x^2*(b/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 16.1953, size = 231, normalized size = 0.79

$$\operatorname{RootSum}\left(68719476736t^4a^{15}b^2 + 838860800t^2a^8bd^2 - 485703680ta^4bc^2d + 2560000ad^4 + 35153041bc^4, \left(t \mapsto t\log\left(x + \frac{-4\cdots}{}\right)\right)\right)$$
$$+ \frac{153a^2cx + 132a^2dx^2 + 198abcx^5 + 160abdx^6 + 77b^2cx^9 + 60b^2dx^{10}}{384a^6 + 1152a^5bx^4 + 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 + 838860800*_t**2*a**8*b*d**
2 - 485703680*_t*a**4*b*c**2*d + 2560000*a*d**4 + 35153041*b*c**4
, Lambda(_t, _t*log(x + (-429496729600*_t**3*a**12*b*d**2 - 62170
071040*_t**2*a**8*b*c**2*d - 2621440000*_t*a**5*d**4 - 1799835699
2*_t*a**4*b*c**4 + 1897280000*a*c**2*d**3)/(788480000*a*c*d**4 -
2706784157*b*c**5)))) + (153*a**2*c*x + 132*a**2*d*x**2 + 198*a*b
*c*x**5 + 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*x**10)/(384
*a**6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*x*
*12)
```

GIAC/XCAS [A] time = 0.219318, size = 378, normalized size = 1.3

$$\frac{77\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\operatorname{cln}\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b} - \frac{77\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\operatorname{cln}\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b}$$
$$+ \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}bd + 77\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}$$
$$+ \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}bd + 77\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}$$
$$+ \frac{60\,b^2dx^{10} + 77\,b^2cx^9 + 160\,abdx^6 + 198\,abcx^5 + 132\,a^2dx^2 + 153\,a^2cx}{384\left(bx^4 + a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(b*x^4 + a)^4,x, algorithm="giac")
```

```
[Out] 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(a^4*b) - 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*ln(x^2 - sqr
t(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(
2)*sqrt(a*b)*b*d + 77*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^2) + 1/512*sqrt(2)*(40
*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^2) + 1/384*(60*b
^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*a^
2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

3.123 $\int \frac{c+dx}{1-x^4}\, dx$

Optimal. Leaf size=24

$$\frac{1}{2}c\tan^{-1}(x)+\frac{1}{2}c\tanh^{-1}(x)+\frac{1}{2}d\tanh^{-1}\left(x^2\right)$$

```
[Out] (c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2
```

Rubi [A] time = 0.0450354, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{2}c\tan^{-1}(x)+\frac{1}{2}c\tanh^{-1}(x)+\frac{1}{2}d\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(1 - x^4),x]

[Out] (c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2
```

Rubi in Sympy [A] time = 7.25757, size = 20, normalized size = 0.83

$$\frac{c\operatorname{atan}(x)}{2}+\frac{c\operatorname{atanh}(x)}{2}+\frac{d\operatorname{atanh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-x**4+1),x)

[Out] c*atan(x)/2 + c*atanh(x)/2 + d*atanh(x**2)/2
```

Mathematica [A] time = 0.0262536, size = 42, normalized size = 1.75

$$\frac{1}{4}\left(-(c+d)\log(1-x)+c\log(x+1)+2c\tan^{-1}(x)+d\log\left(x^2+1\right)-d\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(1 - x^4),x]

[Out] (2*c*ArcTan[x] - (c + d)*Log[1 - x] + c*Log[1 + x] - d*Log[1 + x]
 + d*Log[1 + x^2])/4
```

Maple [B] time = 0.007, size = 44, normalized size = 1.8

$$-\frac{\ln(-1+x)c}{4}-\frac{\ln(-1+x)d}{4}+\frac{\ln(1+x)c}{4}-\frac{\ln(1+x)d}{4}+\frac{d\ln\left(x^2+1\right)}{4}+\frac{c\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-x^4+1),x)

[Out] -1/4*ln(-1+x)*c-1/4*ln(-1+x)*d+1/4*ln(1+x)*c-1/4*ln(1+x)*d+1/4*d*
ln(x^2+1)+1/2*c*arctan(x)
```

Maxima [A] time = 1.51955, size = 47, normalized size = 1.96

$$\frac{1}{2}\,c\arctan\left(x\right)+\frac{1}{4}\,d\log\left(x^{2}+1\right)+\frac{1}{4}\left(c-d\right)\log\left(x+1\right)-\frac{1}{4}\left(c+d\right)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x + c)/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1
/4*(c + d)*log(x - 1)
```

Fricas [A] time = 0.227966, size = 47, normalized size = 1.96

$$\frac{1}{2}\,c\arctan\left(x\right)+\frac{1}{4}\,d\log\left(x^{2}+1\right)+\frac{1}{4}\left(c-d\right)\log\left(x+1\right)-\frac{1}{4}\left(c+d\right)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x + c)/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1
/4*(c + d)*log(x - 1)
```

Sympy [A] time = 0.771547, size = 313, normalized size = 13.04

$$\frac{\left(c-d\right)\log\left(x+\frac{c^{4}(c-d)+5c^{2}d^{3}+c^{2}d(c-d)^{2}-2d^{4}(c-d)+2d^{2}(c-d)^{3}}{c^{5}+4cd^{4}}\right)}{4}-\frac{\left(c+d\right)\log\left(x+\frac{-c^{4}(c+d)+5c^{2}d^{3}+c^{2}d(c+d)^{2}+2d^{4}(c+d)-2d^{2}(c+d)^{3}}{c^{5}+4cd^{4}}\right)}{4}-\left(-\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^{4}\left(-\frac{ic}{4}-\frac{d}{4}\right)+5c^{2}d^{3}+16c^{2}d\left(-\frac{ic}{4}-\frac{d}{4}\right)^{2}+8d^{4}\left(-\frac{ic}{4}-\frac{d}{4}\right)-128d^{2}\left(-\frac{ic}{4}-\frac{d}{4}\right)^{3}}{c^{5}+4cd^{4}}\right)-\left(\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^{4}\left(\frac{ic}{4}-\frac{d}{4}\right)+5c^{2}d^{3}+16c^{2}d\left(\frac{ic}{4}-\frac{d}{4}\right)^{2}+8d^{4}\left(\frac{ic}{4}-\frac{d}{4}\right)-128d^{2}\left(\frac{ic}{4}-\frac{d}{4}\right)^{3}}{c^{5}+4cd^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-x**4+1),x)
```

```
[Out] (c - d)*log(x + (c**4*(c - d) + 5*c**2*d**3 + c**2*d*(c - d)**2 -
 2*d**4*(c - d) + 2*d**2*(c - d)**3)/(c**5 + 4*c*d**4))/4 - (c +
d)*log(x + (-c**4*(c + d) + 5*c**2*d**3 + c**2*d*(c + d)**2 + 2*d
**4*(c + d) - 2*d**2*(c + d)**3)/(c**5 + 4*c*d**4))/4 - (-I*c/4 -
 d/4)*log(x + (-4*c**4*(-I*c/4 - d/4) + 5*c**2*d**3 + 16*c**2*d*(
-I*c/4 - d/4)**2 + 8*d**4*(-I*c/4 - d/4) - 128*d**2*(-I*c/4 - d/4
)**3)/(c**5 + 4*c*d**4)) - (I*c/4 - d/4)*log(x + (-4*c**4*(I*c/4
- d/4) + 5*c**2*d**3 + 16*c**2*d*(I*c/4 - d/4)**2 + 8*d**4*(I*c/4
 - d/4) - 128*d**2*(I*c/4 - d/4)**3)/(c**5 + 4*c*d**4))
```

GIAC/XCAS [A] time = 0.208833, size = 50, normalized size = 2.08

$$\frac{1}{2}\,c\arctan\left(x\right)+\frac{1}{4}\,d\ln\left(x^{2}+1\right)+\frac{1}{4}\left(c-d\right)\ln\left(|x+1|\right)-\frac{1}{4}\left(c+d\right)\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x + c)/(x^4 - 1),x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*ln(x^2 + 1) + 1/4*(c - d)*ln(abs(x + 1))
- 1/4*(c + d)*ln(abs(x - 1))
```

3.124 $\int \frac{c+dx}{1+x^4}\,dx$

Optimal. Leaf size=98

$$-\frac{c\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{c\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{c\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{c\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}+\frac{1}{2}d\tan^{-1}\left(x^2\right)$$

```
[Out] (d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*Ar
cTan[1 + Sqrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(
4*Sqrt[2]) + (c*Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Rubi [A] time = 0.151425, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692

$$-\frac{c\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{c\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{c\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{c\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}+\frac{1}{2}d\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/(1 + x^4),x]
```

```
[Out] (d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*Ar
cTan[1 + Sqrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(
4*Sqrt[2]) + (c*Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 19.0504, size = 88, normalized size = 0.9

$$-\frac{\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}c\,\text{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}c\,\text{atan}\left(\sqrt{2}x+1\right)}{4}+\frac{d\,\text{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(x**4+1),x)
```

```
[Out] -sqrt(2)*c*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*c*log(x**2 + sqr
t(2)*x + 1)/8 + sqrt(2)*c*atan(sqrt(2)*x - 1)/4 + sqrt(2)*c*atan(
sqrt(2)*x + 1)/4 + d*atan(x**2)/2
```

Mathematica [C] time = 0.119185, size = 99, normalized size = 1.01

$$\frac{1}{4}\left(-\left(\sqrt[4]{-1}c+id\right)\log\left(\sqrt[4]{-1}-x\right)+\left(-(-1)^{3/4}c+id\right)\log\left((-1)^{3/4}-x\right)+\left(\sqrt[4]{-1}c-id\right)\log\left(x+\sqrt[4]{-1}\right)+\left((-1)^{3/4}c+id\right)\log\left(x+(-1)^{3/4}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/(1 + x^4),x]
```

```
[Out] (-(((-1)^(1/4)*c + I*d)*Log[(-1)^(1/4) - x]) + (-((-1)^(3/4)*c) +
 I*d)*Log[(-1)^(3/4) - x] + ((-1)^(1/4)*c - I*d)*Log[(-1)^(1/4) +
 x] + ((-1)^(3/4)*c + I*d)*Log[(-1)^(3/4) + x])/4
```

Maple [A] time = 0.005, size = 68, normalized size = 0.7

$$\frac{c\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{c\arctan\left(x\sqrt{2}-1\right)\sqrt{2}}{4}+\frac{c\sqrt{2}}{8}\ln\left(\frac{1+x^{2}+x\sqrt{2}}{1+x^{2}-x\sqrt{2}}\right)+\frac{d\arctan\left(x^{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(x^4+1),x)
```

```
[Out] 1/4*c*arctan(1+x*2^(1/2))*2^(1/2)+1/4*c*arctan(x*2^(1/2)-1)*2^(1/
2)+1/8*c*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+1/2*d*ar
ctan(x^2)
```

Maxima [A] time = 1.52263, size = 116, normalized size = 1.18

$$\frac{1}{8}\sqrt{2}c\log\left(x^{2}+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}c\log\left(x^{2}-\sqrt{2}x+1\right)+\frac{1}{4}\left(\sqrt{2}c-2d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\left(\sqrt{2}c+2d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 -
sqrt(2)*x + 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2))) + 1/4*(sqrt(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.744517, size = 83, normalized size = 0.85

$$\operatorname{RootSum}\left(256t^{4}+32t^{2}d^{2}-16tc^{2}d+c^{4}+d^{4},\left(t\mapsto t\log\left(x+\frac{128t^{3}d^{2}+16t^{2}c^{2}d+4tc^{4}+8td^{4}-5c^{2}d^{3}}{c^{5}-4cd^{4}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(x**4+1),x)
```

```
[Out] RootSum(256*_t**4 + 32*_t**2*d**2 - 16*_t*c**2*d + c**4 + d**4, L
ambda(_t, _t*log(x + (128*_t**3*d**2 + 16*_t**2*c**2*d + 4*_t*c**
4 + 8*_t*d**4 - 5*c**2*d**3)/(c**5 - 4*c*d**4))))
```

GIAC/XCAS [A] time = 0.213021, size = 116, normalized size = 1.18

$$\frac{1}{8}\sqrt{2}c\ln\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}c\ln\left(x^2-\sqrt{2}x+1\right)$$
$$+\frac{1}{4}\left(\sqrt{2}c-2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{4}\left(\sqrt{2}c+2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/(x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*c*ln(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*ln(x^2 - sq
rt(2)*x + 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2))) + 1/4*(sqrt(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
))
```

3.125 $\int \frac{c+dx+ex^2}{a-bx^4} \, dx$

Optimal. Leaf size=116

$$\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*
b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*Sqrt[b])
```

Rubi [A] time = 0.21089, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*
b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*Sqrt[b])
```

Rubi in Sympy [A] time = 26.7577, size = 105, normalized size = 0.91

$$\frac{d\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{a}e-\sqrt{b}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] d*atanh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*sqrt(b)) - (sqrt(a)*e -
sqrt(b)*c)*atan(b**(1/4)*x/a**(1/4))/(2*a**(3/4)*b**(3/4)) + (sqr
t(a)*e + sqrt(b)*c)*atanh(b**(1/4)*x/a**(1/4))/(2*a**(3/4)*b**(3/
4))
```

Mathematica [A] time = 0.112758, size = 187, normalized size = 1.61

$$\frac{-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{a}e+\sqrt{b}c\right)+2\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\sqrt{b}c\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)+\sqrt[4]{a}\sqrt[4]{b}d\log\left(\sqrt{a}+\sqrt{b}x^2\right.}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]
*c + a^(1/4)*b^(1/4)*d + Sqrt[a]*e)*Log[a^(1/4) - b^(1/4)*x] + Sq
```

```
rt[b]*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*b^(1/4)*d*Log[a^(1/4)
+ b^(1/4)*x] + Sqrt[a]*e*Log[a^(1/4) + b^(1/4)*x] + a^(1/4)*b^(1/
4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3/4)*b^(3/4))
```

Maple [B] time = 0.003, size = 161, normalized size = 1.4

$$\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{d}{4}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$
$$-\frac{e}{2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e}{4b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*c*(a/
b)^(1/4)/a*arctan(x/(a/b)^(1/4))-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*
b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/2*e/b/(a/b)^(1/4)*arctan(x/(a/b
)^(1/4))+1/4*e/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 10.5564, size = 471, normalized size = 4.06

$$-\operatorname{RootSum}\left(256t^4a^3b^3+t^2\left(-64a^2b^2ce-32a^2b^2d^2\right)+t\left(-16a^2bde^2-16ab^2c^2d\right)-a^2e^4+2abc^2e^2-4abcd^2e+abd^4-b^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 + _t**2*(-64*a**2*b**2*c*e - 32*a**2
*b**2*d**2) + _t*(-16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) - a**2*e*
*4 + 2*a*b*c**2*e**2 - 4*a*b*c*d**2*e + a*b*d**4 - b**2*c**4, Lam
bda(_t, _t*log(x + (-64*_t**3*a**4*b**2*e**3 - 64*_t**3*a**3*b**3
*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 48*_t**2*a**3*b**2*c*d*e**
2 - 32*_t**2*a**3*b**2*d**3*e - 16*_t**2*a**2*b**3*c**3*d + 12*_t
*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 + 16*_t*a**2*b**2*c**3*e*
*2 - 36*_t*a**2*b**2*c**2*d**2*e - 8*_t*a**2*b**2*c*d**4 + 4*_t*a
*b**3*c**5 + 3*a**3*d*e**5 - 5*a**2*b*c*d**3*e**2 + 2*a**2*b*d**5
*e + 5*a*b**2*c**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 + a**2*b*
c**2*e**4 - 8*a**2*b*c*d**2*e**3 + 4*a**2*b*d**4*e**2 - a*b**2*c*
*4*e**2 + 8*a*b**2*c**3*d**2*e - 4*a*b**2*c**2*d**4 - b**3*c**6))
))
```

GIAC/XCAS [A] time = 0.221783, size = 396, normalized size = 3.41

$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - (-a*b^3)^(1/4)*b^2*c - (
-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/
(-a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - (
-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b
^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-
a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*
b^3)
```

3.126 $\int \frac{c+dx+ex^2}{a+bx^4}\,dx$

Optimal. Leaf size=277

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]
*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c
- Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3
/4)*b^(3/4))
```

Rubi [A] time = 0.441168, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a + b*x^4),x]
```


```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]
*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c
- Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3
/4)*b^(3/4))
```

Rubi in Sympy [A] time = 70.4338, size = 258, normalized size = 0.93

$$\frac{d\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{b}c\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{b}c\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] d*atan(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*sqrt(b)) + sqrt(2)*(sqrt(
a)*e - sqrt(b)*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt
```

```
(b) + b*x**2)/(8*a**(3/4)*b**(3/4)) - sqrt(2)*(sqrt(a)*e - sqrt(b
)*c)*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/
(8*a**(3/4)*b**(3/4)) - sqrt(2)*(sqrt(a)*e + sqrt(b)*c)*atan(1 -
sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(3/4)) + sqrt(2)*(sqr
t(a)*e + sqrt(b)*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(
3/4)*b**(3/4))
```

Mathematica [A] time = 0.278707, size = 229, normalized size = 0.83

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e+\sqrt{2}\sqrt{b}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e+\sqrt{2}\sqrt{b}c\right)-\sqrt{2}\left(\sqrt{b}c-\right.}{8a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[b]*c -
 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] - Sqrt[2]*(Sqrt[b]*c - Sqrt[a]*e)*(Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/4)*b^(3/4))
```

Maple [A] time = 0.005, size = 280, normalized size = 1.

$$\frac{c\sqrt{2}}{8\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{d}{2}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}+\frac{e\sqrt{2}}{8\,b}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{e\sqrt{2}}{4\,b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e\sqrt{2}}{4\,b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*c*(a/b)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*c*(a/b)^(1/4)/a*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/2*d/(a*b)^(1/2)*arctan(x^2
*(b/a)^(1/2))+1/8*e/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*
e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*e/b/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 10.4491, size = 466, normalized size = 1.68

$$\operatorname{RootSum}\left(256t^4a^3b^3 + t^2\left(64a^2b^2ce + 32a^2b^2d^2\right) + t\left(16a^2bde^2 - 16ab^2c^2d\right) + a^2e^4 + 2abc^2e^2 - 4abcd^2e + abd^4 + b^2c^4, \left(t \mapsto \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + _t**2*(64*a**2*b**2*c*e + 32*a**2*b
**2*d**2) + _t*(16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) + a**2*e**4
+ 2*a*b*c**2*e**2 - 4*a*b*c*d**2*e + a*b*d**4 + b**2*c**4, Lambda
(_t, _t*log(x + (64*_t**3*a**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**
2*e + 128*_t**3*a**3*b**3*c*d**2 + 48*_t**2*a**3*b**2*c*d*e**2 -
32*_t**2*a**3*b**2*d**3*e + 16*_t**2*a**2*b**3*c**3*d + 12*_t*a**
3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 - 16*_t*a**2*b**2*c**3*e**2 +
 36*_t*a**2*b**2*c**2*d**2*e + 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**
3*c**5 + 3*a**3*d*e**5 + 5*a**2*b*c*d**3*e**2 - 2*a**2*b*d**5*e +
 5*a*b**2*c**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 - a**2*b*c**2
*e**4 + 8*a**2*b*c*d**2*e**3 - 4*a**2*b*d**4*e**2 - a*b**2*c**4*e
**2 + 8*a*b**2*c**3*d**2*e - 4*a*b**2*c**2*d**4 + b**3*c**6))))
```

GIAC/XCAS [A] time = 0.220282, size = 371, normalized size = 1.34

$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d - \left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d - \left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)
^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b
^2*c - (a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b
))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*
ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.127 $\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=146

$$\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}$$

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b
]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)
) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.270704, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b
]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)
) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 41.703, size = 131, normalized size = 0.9

$$\frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}-\frac{\left(\sqrt{a}e-3\sqrt{b}c\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{3}{4}}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] x*(c + d*x + e*x**2)/(4*a*(a - b*x**4)) + d*atanh(sqrt(b)*x**2/sq
rt(a))/(4*a**(3/2)*sqrt(b)) - (sqrt(a)*e - 3*sqrt(b)*c)*atan(b**(
1/4)*x/a**(1/4))/(8*a**(7/4)*b**(3/4)) + (sqrt(a)*e + 3*sqrt(b)*c
)*atanh(b**(1/4)*x/a**(1/4))/(8*a**(7/4)*b**(3/4))
```

Mathematica [A] time = 0.542465, size = 211, normalized size = 1.45

$$\frac{-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c-2\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}-\frac{2\sqrt[4]{a}\left(\sqrt{a}e-3\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{4ax(c+x(d+ex))}{a-bx^4}}{16a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + x*(d + e*x)))/(a - b*x^4) - (2*a^(1/4)*(-3*Sqrt[b]*c
 + Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((3*a^(1/4)*
```

```
Sqrt[b]*c + 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4
)*x])/b^(3/4) + ((3*a^(1/4)*Sqrt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(
3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (2*Sqrt[a]*d*Log[Sqrt
[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Maple [B] time = 0.007, size = 228, normalized size = 1.6

$$-\frac{cx}{4\,a\,(bx^4-a)}+\frac{3\,c}{16\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3\,c}{8\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{dx^2}{4\,a\,(bx^4-a)}-\frac{d}{8\,a}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{ex^3}{4\,a\,(bx^4-a)}$$
$$-\frac{e}{8\,ab}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e}{16\,ab}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] -1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x
-(a/b)^(1/4)))+3/8*c/a^2*(a/b)^(1/4)*arctan(x/(a/b)^(1/4))-1/4*d*
x^2/a/(b*x^4-a)-1/8*d/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x
^2*(a*b)^(1/2)))-1/4*e*x^3/a/(b*x^4-a)-1/8*e/a/b/(a/b)^(1/4)*arct
an(x/(a/b)^(1/4))+1/16*e/a/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a
/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 13.6669, size = 507, normalized size = 3.47

$$\operatorname{RootSum}\left(65536t^4a^7b^3+t^2\left(-3072a^4b^2ce-2048a^4b^2d^2\right)+t\left(128a^3bde^2+1152a^2b^2c^2d\right)-a^2e^4+18abc^2e^2-48abcd^2e+\right.$$
$$-\frac{cx+dx^2+ex^3}{-4a^2+4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048
*a**4*b**2*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d)
 - a**2*e**4 + 18*a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 -
 81*b**2*c**4, Lambda(_t, _t*log(x + (4096*_t**3*a**7*b**2*e**3 +
 36864*_t**3*a**6*b**3*c**2*e - 98304*_t**3*a**6*b**3*c*d**2 + 46
08*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*d**3*e - 13824
*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4*b*d*
*2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d*
*2*e + 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3
*d*e**5 - 120*a**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*
c**4*d*e - 1080*a*b**2*c**3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4
 - 96*a**2*b*c*d**2*e**3 + 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e
**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2*d**4 - 729*b**3*c*
*6)))) - (c*x + d*x**2 + e*x**3)/(-4*a**2 + 4*a*b*x**4)
```

GIAC/XCAS [A] time = 0.220682, size = 440, normalized size = 3.01

$$-\frac{x^3e+dx^2+cx}{4\,(bx^4-a)a}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}b^2d+3\left(-ab^3\right)^{\frac14}b^2c+\left(-ab^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3}$$
$$+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}b^2d+3\left(-ab^3\right)^{\frac14}b^2c+\left(-ab^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3}$$
$$+\frac{\sqrt{2}\left(3\left(-ab^3\right)^{\frac14}b^2c-\left(-ab^3\right)^{\frac34}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$-\frac{\sqrt{2}\left(3\left(-ab^3\right)^{\frac14}b^2c-\left(-ab^3\right)^{\frac34}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 - a)*a) + 1/16*sqrt(2)*(2*sqrt
(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*e)
*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a
^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(
1/4)*b^2*c + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(-a*b^3)^
(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) +
 sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (
-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a
^2*b^3)
```

3.128 $\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)}$$

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) +
 ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a
^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4))
```

Rubi [A] time = 0.567999, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) +
 ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a
^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4))
```

Rubi in Sympy [A] time = 88.0503, size = 287, normalized size = 0.93

$$\frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{a}e-3\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{7}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e-3\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{7}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e+3\sqrt{b}c\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}e+3\sqrt{b}c\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] x*(c + d*x + e*x**2)/(4*a*(a + b*x**4)) + d*atan(sqrt(b)*x**2/sqr
t(a))/(4*a**(3/2)*sqrt(b)) + sqrt(2)*(sqrt(a)*e - 3*sqrt(b)*c)*lo
g(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(32*a*
*(7/4)*b**(3/4)) - sqrt(2)*(sqrt(a)*e - 3*sqrt(b)*c)*log(sqrt(2)*
a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(32*a**(7/4)*b**(
3/4)) - sqrt(2)*(sqrt(a)*e + 3*sqrt(b)*c)*atan(1 - sqrt(2)*b**(1/
4)*x/a**(1/4))/(16*a**(7/4)*b**(3/4)) + sqrt(2)*(sqrt(a)*e + 3*sq
rt(b)*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(3
/4))
```

Mathematica [A] time = 0.81362, size = 305, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e-3\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}+\frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}-\frac{2\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e}{b^{3/4}}}{32a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a*x*(c + x*(d + e*x)))/(a + b*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sq
rt[b]*c + 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b
]*c - 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-3*a^(1/4)*Sqrt[b]*c +
 a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/b^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(32*a^2)
```

Maple [A] time = 0.006, size = 344, normalized size = 1.1

$$\frac{cx}{4\,a\,(bx^4+a)}+\frac{3\,c\sqrt{2}}{32\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{3\,c\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\,c\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{dx^2}{4\,a\,(bx^4+a)}+\frac{d}{4\,a}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}+\frac{ex^3}{4\,a\,(bx^4+a)}$$
$$+\frac{e\sqrt{2}}{32\,ab}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{e\sqrt{2}}{16\,ab}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e\sqrt{2}}{16\,ab}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] 1/4*c*x/a/(b*x^4+a)+3/32*c/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^
(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2)))+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+
```

```
1)+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
+1/4*d*x^2/a/(b*x^4+a)+1/4*d/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2)
)+1/4*e*x^3/a/(b*x^4+a)+1/32*e/a/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a
/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2)))+1/16*e/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x+1)+1/16*e/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 13.705, size = 505, normalized size = 1.64

$$\text{RootSum}\left(65536t^4a^7b^3 + t^2\left(3072a^4b^2ce + 2048a^4b^2d^2\right) + t\left(128a^3bde^2 - 1152a^2b^2c^2d\right) + a^2e^4 + 18abc^2e^2 - 48abcd^2e + 16\right.$$
$$+ \frac{cx + dx^2 + ex^3}{4a^2 + 4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*
a**4*b**2*d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d)
+ a**2*e**4 + 18*a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 +
81*b**2*c**4, Lambda(_t, _t*log(x + (4096*_t**3*a**7*b**2*e**3 -
36864*_t**3*a**6*b**3*c**2*e + 98304*_t**3*a**6*b**3*c*d**2 + 460
8*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*d**3*e + 13824*
_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*b*d**
2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**
2*e + 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*
d*e**5 + 120*a**2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c
**4*d*e - 1080*a*b**2*c**3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4
+ 96*a**2*b*c*d**2*e**3 - 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e*
*2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2*d**4 + 729*b**3*c**
6)))) + (c*x + d*x**2 + e*x**3)/(4*a**2 + 4*a*b*x**4)
```

GIAC/XCAS [A] time = 0.220738, size = 413, normalized size = 1.34

$$\frac{x^3e + dx^2 + cx}{4\,(bx^4 + a)a} + \frac{\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ab}b^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ab}b^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\Big(3\,\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\Big)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\Big(3\,\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\Big)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3
) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2
*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c
- (a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*
ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.129 $\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=179

$$\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}$$

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x + 5*e
*x^2))/(32*a^2*(a - b*x^4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTa
n[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c +
5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4))
+ (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.353912, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x + 5*e
*x^2))/(32*a^2*(a - b*x^4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTa
n[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c +
5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4))
+ (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 53.5629, size = 167, normalized size = 0.93

$$\frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{3d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}-\frac{\left(5\sqrt{a}e-21\sqrt{b}c\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{3}{4}}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] x*(c + d*x + e*x**2)/(8*a*(a - b*x**4)**2) + x*(7*c + 6*d*x + 5*e
*x**2)/(32*a**2*(a - b*x**4)) + 3*d*atanh(sqrt(b)*x**2/sqrt(a))/(
16*a**(5/2)*sqrt(b)) - (5*sqrt(a)*e - 21*sqrt(b)*c)*atan(b**(1/4)
*x/a**(1/4))/(64*a**(11/4)*b**(3/4)) + (5*sqrt(a)*e + 21*sqrt(b)*
c)*atanh(b**(1/4)*x/a**(1/4))/(64*a**(11/4)*b**(3/4))
```

Mathematica [A] time = 0.661725, size = 244, normalized size = 1.36

$$\frac{-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}c-12\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{16a^2x(c+x(d+ex))}{(a-bx^4)^2}+\frac{2\sqrt[4]{a}\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan}{b^{3/4}}}{128a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + x*(d + e*x)))/(a - b*x^4)^2 + (4*a*x*(7*c + x*(6*
d + 5*e*x)))/(a - b*x^4) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e
)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c +
 12*Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^
(3/4) + ((21*a^(1/4)*Sqrt[b]*c - 12*Sqrt[a]*b^(1/4)*d + 5*a^(3/4)
*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*Sqrt[a]*d*Log[Sqrt[a]
 + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [B] time = 0.007, size = 286, normalized size = 1.6

$$\frac{cx}{8\,a\,(bx^4-a)^2}-\frac{7\,cx}{32\,a^2\,(bx^4-a)}+\frac{21\,c}{128\,a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{21\,c}{64\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{dx^2}{8\,a\,(bx^4-a)^2}-\frac{3\,dx^2}{16\,a^2\,(bx^4-a)}$$
$$-\frac{3\,d}{32\,a^2}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}+\frac{ex^3}{8\,a\,(bx^4-a)^2}-\frac{5\,ex^3}{32\,a^2\,(bx^4-a)}$$
$$-\frac{5\,e}{64\,a^2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5\,e}{128\,a^2b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(a/b)^(
1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+21/64*c/a^3*(a/b)^(1/4)*
arctan(x/(a/b)^(1/4))+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x
^4-a)-3/32*d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b
)^(1/2)))+1/8*e*x^3/a/(b*x^4-a)^2-5/32*e/a^2*x^3/(b*x^4-a)-5/64*e
/a^2/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))+5/128*e/a^2/b/(a/b)^(1/4
)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 18.7221, size = 563, normalized size = 3.15

$$-\operatorname{RootSum}\left(268435456t^4a^{11}b^3 + t^2\left(-6881280a^6b^2ce - 4718592a^6b^2d^2\right) + t\left(-153600a^4bde^2 - 2709504a^3b^2c^2d\right) - 625a^2e\right.$$
$$-\frac{-11acx - 10adx^2 - 9aex^3 + 7bcx^5 + 6bdx^6 + 5bex^7}{32a^4 - 64a^3bx^4 + 32a^2b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c
*e - 4718592*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 270950
4*a**3*b**2*c**2*d) - 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480
*a*b*c*d**2*e + 20736*a*b*d**4 - 194481*b**2*c**4, Lambda(_t, _t*
log(x + (-262144000*_t**3*a**10*b**2*e**3 - 4624220160*_t**3*a**9
*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 309657600*_t*
*2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e - 182078
6688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*
_t*a**5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 36578304
0*_t*a**4*b**2*c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 5227
64928*_t*a**3*b**3*c**5 + 112500*a**3*d*e**5 - 4536000*a**2*b*c*d
**3*e**2 + 2488320*a**2*b*d**5*e + 58344300*a*b**2*c**4*d*e - 800
15040*a*b**2*c**3*d**3)/(15625*a**3*e**6 + 275625*a**2*b*c**2*e**
4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a**2*b*d**4*e**2 - 48620
25*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b*
*2*c**2*d**4 - 85766121*b**3*c**6)))) - (-11*a*c*x - 10*a*d*x**2
- 9*a*e*x**3 + 7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 - 6
4*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

GIAC/XCAS [A] time = 0.224729, size = 481, normalized size = 2.69

$$-\frac{5\,bx^7e + 6\,bdx^6 + 7\,bcx^5 - 9\,ax^3e - 10\,adx^2 - 11\,acx}{32\,(bx^4 - a)^2a^2}$$
$$-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d - 21\left(-ab^3\right)^{\frac{1}{4}}b^2c - 5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d - 21\left(-ab^3\right)^{\frac{1}{4}}b^2c - 5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+\frac{\sqrt{2}\left(21\left(-ab^3\right)^{\frac{1}{4}}b^2c - 5\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$-\frac{\sqrt{2}\left(21\left(-ab^3\right)^{\frac{1}{4}}b^2c - 5\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="giac")
```

```
[Out] -1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*x^3*e - 10*a*d*x^2
 - 11*a*c*x)/((b*x^4 - a)^2*a^2) - 1/128*sqrt(2)*(12*sqrt(2)*sqrt
(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^
3) - 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1/
4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(-a*b^3
)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2
*c - 5*(-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-
a/b))/(a^3*b^3)
```

3.130 $\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=341

$$-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}b^{3/4}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)}+\frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2}$$

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x + 5*e
*x^2))/(32*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])
/(16*a^(5/2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((2
1*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt
[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4
)*b^(3/4))
```

Rubi [A] time = 0.692528, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}b^{3/4}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)}+\frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```


```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x + 5*e
*x^2))/(32*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])
/(16*a^(5/2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((2
1*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt
[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4
)*b^(3/4))
```

Rubi in Sympy [A] time = 103.555, size = 326, normalized size = 0.96

$$\frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)}+\frac{3d\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}+\frac{\sqrt{2}\left(5\sqrt{a}e-21\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{256a^{\frac{11}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(5\sqrt{a}e-21\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{256a^{\frac{11}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(5\sqrt{a}e+21\sqrt{b}c\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(5\sqrt{a}e+21\sqrt{b}c\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] x*(c + d*x + e*x**2)/(8*a*(a + b*x**4)**2) + x*(7*c + 6*d*x + 5*e
*x**2)/(32*a**2*(a + b*x**4)) + 3*d*atan(sqrt(b)*x**2/sqrt(a))/(1
6*a**(5/2)*sqrt(b)) + sqrt(2)*(5*sqrt(a)*e - 21*sqrt(b)*c)*log(-s
qrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(256*a**(1
1/4)*b**(3/4)) - sqrt(2)*(5*sqrt(a)*e - 21*sqrt(b)*c)*log(sqrt(2)
*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(256*a**(11/4)*b
**(3/4)) - sqrt(2)*(5*sqrt(a)*e + 21*sqrt(b)*c)*atan(1 - sqrt(2)*
b**(1/4)*x/a**(1/4))/(128*a**(11/4)*b**(3/4)) + sqrt(2)*(5*sqrt(a
)*e + 21*sqrt(b)*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(128*a*
*(11/4)*b**(3/4))
```

Mathematica [A] time = 0.631277, size = 337, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(5a^{3/4}e-21\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}+\frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{b}c-5a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}+\frac{32a^2x(c+x(d+ex))}{\left(a+bx^4\right)^2}-\frac{2\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right.}{\cdots}}{256a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(c + x*(d + e*x)))/(a + b*x^4)^2 + (8*a*x*(7*c + x*(6*
d + 5*e*x)))/(a + b*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*
a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*
a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-21*a^(1/4)*Sqrt[b]*c + 5*a^
(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Maple [A] time = 0.007, size = 396, normalized size = 1.2

$$\frac{cx}{8a(bx^4+a)^2}+\frac{7cx}{32a^2(bx^4+a)} + \frac{21c\sqrt{2}}{256a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{21c\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{21c\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right) + \frac{dx^2}{8a(bx^4+a)^2}+\frac{3dx^2}{16a^2(bx^4+a)}+\frac{3d}{16a^2}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}+\frac{ex^3}{8a(bx^4+a)^2} + \frac{5ex^3}{32a^2(bx^4+a)}+\frac{5e\sqrt{2}}{256a^2b}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{5e\sqrt{2}}{128a^2b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5e\sqrt{2}}{128a^2b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(a/b)^(
1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/8*d*x^2/a/(b*x^4+a)^2+3/16*d/a^
2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/
8*e*x^3/a/(b*x^4+a)^2+5/32*e/a^2*x^3/(b*x^4+a)+5/256*e/a^2/b/(a/b
)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+5/128*e/a^2/b/(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+5/128*e/a^2/b/(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 18.332, size = 558, normalized size = 1.64

$$\mathrm{RootSum}\left(268435456t^4a^{11}b^3 + t^2\left(6881280a^6b^2ce + 4718592a^6b^2d^2\right) + t\left(153600a^4bde^2 - 2709504a^3b^2c^2d\right) + 625a^2e^4 + 22\right.$$
$$+ \frac{11acx + 10adx^2 + 9aex^3 + 7bcx^5 + 6bdx^6 + 5bex^7}{32a^4 + 64a^3bx^4 + 32a^2b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e
 + 4718592*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a
**3*b**2*c**2*d) + 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*
b*c*d**2*e + 20736*a*b*d**4 + 194481*b**2*c**4, Lambda(_t, _t*log
(x + (262144000*_t**3*a**10*b**2*e**3 - 4624220160*_t**3*a**9*b**
3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 309657600*_t**2*a
**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e + 1820786688
*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a
**5*b*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t
*a**4*b**2*c**2*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 52276492
8*_t*a**3*b**3*c**5 + 112500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*
e**2 - 2488320*a**2*b*d**5*e + 58344300*a*b**2*c**4*d*e - 8001504
0*a*b**2*c**3*d**3)/(15625*a**3*e**6 - 275625*a**2*b*c**2*e**4 +
3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b*d**4*e**2 - 4862025*a
*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c
**2*d**4 + 85766121*b**3*c**6)))) + (11*a*c*x + 10*a*d*x**2 + 9*a
*e*x**3 + 7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 + 64*a**
3*b*x**4 + 32*a**2*b**2*x**8)
```

GIAC/XCAS [A] time = 0.224588, size = 454, normalized size = 1.33

$$\frac{5\,bx^7e + 6\,bdx^6 + 7\,bcx^5 + 9\,ax^3e + 10\,adx^2 + 11\,acx}{32\,(bx^4+a)^2a^2} + \frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{ab}b^2d + 21\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{ab}b^2d + 21\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(21\,\left(ab^3\right)^{\frac{1}{4}}b^2c - 5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{256\,a^3b^3} - \frac{\sqrt{2}\left(21\,\left(ab^3\right)^{\frac{1}{4}}b^2c - 5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*x^3*e + 10*a*d*x^2
+ 11*a*c*x)/((b*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(
a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1
/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c
 + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*
c - 5*(a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(
3/4)*e)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3)
```

3.131 $\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=211

$$\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x + 
9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))
/(384*a^3*(a - b*x^4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b
^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*
Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) +
 (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b])
```

Rubi [A] time = 0.441644, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x + 
9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))
/(384*a^3*(a - b*x^4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b
^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*
Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) +
 (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 69.4565, size = 197, normalized size = 0.93

$$\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{5d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}}-\frac{\left(15\sqrt{a}e-77\sqrt{b}c\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{3}{4}}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] x*(c + d*x + e*x**2)/(12*a*(a - b*x**4)**3) + x*(11*c + 10*d*x + 
9*e*x**2)/(96*a**2*(a - b*x**4)**2) + x*(77*c + 60*d*x + 45*e*x**
2)/(384*a**3*(a - b*x**4)) + 5*d*atanh(sqrt(b)*x**2/sqrt(a))/(32*
a**(7/2)*sqrt(b)) - (15*sqrt(a)*e - 77*sqrt(b)*c)*atan(b**(1/4)*x
/a**(1/4))/(256*a**(15/4)*b**(3/4)) + (15*sqrt(a)*e + 77*sqrt(b)*
c)*atanh(b**(1/4)*x/a**(1/4))/(256*a**(15/4)*b**(3/4))
```

Mathematica [A] time = 0.532324, size = 276, normalized size = 1.31

$$\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c+40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c-40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{128a^3x(c+x(d+ex))}{(a-bx^4)^3}+\frac{16a^2x(11c+x(10d+}{(a-bx^4)^2}}{1536a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + x*(d + e*x)))/(a - b*x^4)^3 + (4*a*x*(77*c + 15*
x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*d + 9*e*x
)))/(a - b*x^4)^2 + (6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcT
an[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*
Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/
4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)
*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*Sqrt[a]*d*Log[Sqrt[a
] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Maple [A] time = 0.019, size = 281, normalized size = 1.3

$$\frac{1}{(bx^4-a)^3}\left(-\frac{15\,b^2ex^{11}}{128\,a^3}-\frac{5\,b^2dx^{10}}{32\,a^3}-\frac{77\,b^2cx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}-\frac{113\,ex^3}{384\,a}-\frac{11\,dx^2}{32\,a}-\frac{51\,cx}{128\,a}\right)$$
$$+\frac{77\,c}{512\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{77\,c}{256\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{5\,d}{64}\ln\left(1\left(-a^4+x^2\sqrt{ba^7}\right)\left(-a^4-x^2\sqrt{ba^7}\right)^{-1}\right)\frac{1}{\sqrt{ba^7}}$$
$$-\frac{15\,e}{256\,a^3b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{15\,e}{512\,a^3b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128*e/a^3*b^2*x^11-5/32*d/a^3*b^2*x^10-77/384*c/a^3*b^2*x^9+
21/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5-113/384/a*e*
x^3-11/32*d/a*x^2-51/128/a*c*x)/(b*x^4-a)^3+77/512*c*(a/b)^(1/4)/
a^4*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+77/256*c*(a/b)^(1/4)/a^4*
arctan(x/(a/b)^(1/4))-5/64*d/(b*a^7)^(1/2)*ln((-a^4+x^2*(b*a^7)^(
1/2))/(-a^4-x^2*(b*a^7)^(1/2)))-15/256*e/a^3/b/(a/b)^(1/4)*arctan
(x/(a/b)^(1/4))+15/512*e/a^3/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-
(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 34.2366, size = 612, normalized size = 2.9

$$\mathrm{RootSum}\Big(68719476736t^4a^{15}b^3 + t^2\left(-1211105280a^8b^2ce - 838860800a^8b^2d^2\right) + t\left(18432000a^5bde^2 + 485703680a^4b^2c^2d\right)$$

$$-\frac{153a^2cx + 132a^2dx^2 + 113a^2ex^3 - 198abcx^5 - 160abdx^6 - 126abex^7 + 77b^2cx^9 + 60b^2dx^{10} + 45b^2ex^{11}}{-384a^6 + 1152a^5bx^4 - 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(-1211105280*a**8*b*
*2*c*e - 838860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 +
 485703680*a**4*b**2*c**2*d) - 50625*a**2*e**4 + 2668050*a*b*c**2
*e**2 - 7392000*a*b*c*d**2*e + 2560000*a*b*d**4 - 35153041*b**2*c
**4, Lambda(_t, _t*log(x + (452984832000*_t**3*a**13*b**2*e**3 +
11936653639680*_t**3*a**12*b**3*c**2*e - 33071248179200*_t**3*a**
12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2 - 50331648
0000*_t**2*a**9*b**2*d**3*e - 4787095470080*_t**2*a**8*b**3*c**3*
d - 5987520000*_t*a**6*b*c*e**4 - 8294400000*_t*a**6*b*d**2*e**3
- 210370406400*_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2
*c**2*d**2*e + 201850880000*_t*a**5*b**2*c*d**4 - 1385873488384*_
t*a**4*b**3*c**5 + 91125000*a**3*d*e**5 - 5544000000*a**2*b*c*d**
3*e**2 + 3072000000*a**2*b*d**5*e + 105459123000*a*b**2*c**4*d*e
- 146090560000*a*b**2*c**3*d**3)/(11390625*a**3*e**6 + 300155625*
a**2*b*c**2*e**4 - 3326400000*a**2*b*c*d**2*e**3 + 2304000000*a**
2*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b**2*
c**3*d**2*e - 60712960000*a*b**2*c**2*d**4 - 208422380089*b**3*c*
*6)))) - (153*a**2*c*x + 132*a**2*d*x**2 + 113*a**2*e*x**3 - 198*
a*b*c*x**5 - 160*a*b*d*x**6 - 126*a*b*e*x**7 + 77*b**2*c*x**9 + 6
0*b**2*d*x**10 + 45*b**2*e*x**11)/(-384*a**6 + 1152*a**5*b*x**4 -
 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

GIAC/XCAS [A] time = 0.222059, size = 531, normalized size = 2.52

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d + 77\left(-ab^3\right)^{\frac{1}{4}}b^2c + 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d + 77\left(-ab^3\right)^{\frac{1}{4}}b^2c + 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3} + \frac{\sqrt{2}\left(77\left(-ab^3\right)^{\frac{1}{4}}b^2c - 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024a^4b^3} - \frac{\sqrt{2}\left(77\left(-ab^3\right)^{\frac{1}{4}}b^2c - 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024a^4b^3} - \frac{45b^2x^{11}e + 60b^2dx^{10} + 77b^2cx^9 - 126abx^7e - 160abdx^6 - 198abcx^5 + 113a^2x^3e + 132a^2dx^2 + 153a^2cx}{384\left(bx^4-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^
2*c + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqr
t(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c + 15*(-a*b^3)^(3/4)*e)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*
b^3) + 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 15*(-a*b^3)^(3/4
)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/
1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 15*(-a*b^3)^(3/4)*e)*ln(x
^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/384*(45*b
^2*x^11*e + 60*b^2*d*x^10 + 77*b^2*c*x^9 - 126*a*b*x^7*e - 160*a*
b*d*x^6 - 198*a*b*c*x^5 + 113*a^2*x^3*e + 132*a^2*d*x^2 + 153*a^2
*c*x)/((b*x^4 - a)^3*a^3)
```

3.132 $\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=372

$$-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}a^{15/4}b^{3/4}}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x +
9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))
/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*
a^(7/2)*Sqrt[b]) - ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*S
qrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt
[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/
4)*b^(3/4))
```

Rubi [A] time = 0.803892, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}a^{15/4}b^{3/4}}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```


```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x +
9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))
/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*
a^(7/2)*Sqrt[b]) - ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*S
qrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt
[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/
4)*b^(3/4))
```

Rubi in Sympy [A] time = 122.227, size = 357, normalized size = 0.96

$$\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)}+\frac{5d\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}}+\frac{\sqrt{2}\left(15\sqrt{a}e-77\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(15\sqrt{a}e-77\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(15\sqrt{a}e+77\sqrt{b}c\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{15}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(15\sqrt{a}e+77\sqrt{b}c\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{15}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] x*(c + d*x + e*x**2)/(12*a*(a + b*x**4)**3) + x*(11*c + 10*d*x +
9*e*x**2)/(96*a**2*(a + b*x**4)**2) + x*(77*c + 60*d*x + 45*e*x**
2)/(384*a**3*(a + b*x**4)) + 5*d*atan(sqrt(b)*x**2/sqrt(a))/(32*a
**(7/2)*sqrt(b)) + sqrt(2)*(15*sqrt(a)*e - 77*sqrt(b)*c)*log(-sqr
t(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(1024*a**(15
/4)*b**(3/4)) - sqrt(2)*(15*sqrt(a)*e - 77*sqrt(b)*c)*log(sqrt(2)
*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(1024*a**(15/4)*
b**(3/4)) - sqrt(2)*(15*sqrt(a)*e + 77*sqrt(b)*c)*atan(1 - sqrt(2
)*b**(1/4)*x/a**(1/4))/(512*a**(15/4)*b**(3/4)) + sqrt(2)*(15*sqr
t(a)*e + 77*sqrt(b)*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(512
*a**(15/4)*b**(3/4))
```

Mathematica [A] time = 0.963351, size = 369, normalized size = 0.99

$$\frac{3\sqrt{2}\left(15a^{3/4}e-77\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}+\frac{3\sqrt{2}\left(77\sqrt[4]{a}\sqrt{b}c-15a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}+\frac{256a^3x(c+x(d+ex))}{\left(a+bx^4\right)^3}+\frac{32a^2x(11c+}{(a+b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^3*x*(c + x*(d + e*x)))/(a + b*x^4)^3 + (8*a*x*(77*c + 15*
x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*d + 9*e*x
)))/(a + b*x^4)^2 - (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)
*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/b^(3/4) + (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c - 80*a^(1/4
)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*Sqrt[b]*c + 15*a^(3
/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^
(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^
4)
```

Maple [A] time = 0.022, size = 393, normalized size = 1.1

$$\frac{1}{(bx^4+a)^3}\left(\frac{15\,b^2ex^{11}}{128\,a^3}+\frac{5\,b^2dx^{10}}{32\,a^3}+\frac{77\,b^2cx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}+\frac{113\,ex^3}{384\,a}+\frac{11\,dx^2}{32\,a}+\frac{51\,cx}{128\,a}\right)$$
$$+\frac{77\,c\sqrt{2}}{1024\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{77\,c\sqrt{2}}{512\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{77\,c\sqrt{2}}{512\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{5\,d}{32}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ba^7}}+\frac{15\,e\sqrt{2}}{1024\,a^3b}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{15\,e\sqrt{2}}{512\,a^3b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{15\,e\sqrt{2}}{512\,a^3b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+2
1/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5+113/384/a*e*x
^3+11/32*d/a*x^2+51/128/a*c*x)/(b*x^4+a)^3+77/1024*c*(a/b)^(1/4)/
a^4*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)
^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+77/512*c*(a/b)^(1/4)/a^4*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x+1)+77/512*c*(a/b)^(1/4)/a^4*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1)+5/32*d/(b*a^7)^(1/2)*arctan(x^2*(b
/a)^(1/2))+15/1024*e/a^3/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4
)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))
+15/512*e/a^3/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+
1)+15/512*e/a^3/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 34.1186, size = 610, normalized size = 1.64

$$\mathrm{RootSum}\Big(68719476736t^4a^{15}b^3 + t^2\left(1211105280a^8b^2ce + 838860800a^8b^2d^2\right) + t\left(18432000a^5bde^2 - 485703680a^4b^2c^2d\right) +$$
$$+\frac{153a^2cx + 132a^2dx^2 + 113a^2ex^3 + 198abcx^5 + 160abdx^6 + 126abex^7 + 77b^2cx^9 + 60b^2dx^{10} + 45b^2ex^{11}}{384a^6 + 1152a^5bx^4 + 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(1211105280*a**8*b**
2*c*e + 838860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 -
485703680*a**4*b**2*c**2*d) + 50625*a**2*e**4 + 2668050*a*b*c**2*
e**2 - 7392000*a*b*c*d**2*e + 2560000*a*b*d**4 + 35153041*b**2*c*
*4, Lambda(_t, _t*log(x + (452984832000*_t**3*a**13*b**2*e**3 - 1
1936653639680*_t**3*a**12*b**3*c**2*e + 33071248179200*_t**3*a**1
2*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2 - 503316480
000*_t**2*a**9*b**2*d**3*e + 4787095470080*_t**2*a**8*b**3*c**3*d
 + 5987520000*_t*a**6*b*c*e**4 + 8294400000*_t*a**6*b*d**2*e**3 -
 210370406400*_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*
c**2*d**2*e + 201850880000*_t*a**5*b**2*c*d**4 + 1385873488384*_t
*a**4*b**3*c**5 + 91125000*a**3*d*e**5 + 5544000000*a**2*b*c*d**3
*e**2 - 3072000000*a**2*b*d**5*e + 105459123000*a*b**2*c**4*d*e -
 146090560000*a*b**2*c**3*d**3)/(11390625*a**3*e**6 - 300155625*a
**2*b*c**2*e**4 + 3326400000*a**2*b*c*d**2*e**3 - 2304000000*a**2
*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b**2*c
**3*d**2*e - 60712960000*a*b**2*c**2*d**4 + 208422380089*b**3*c**
6)))) + (153*a**2*c*x + 132*a**2*d*x**2 + 113*a**2*e*x**3 + 198*a
*b*c*x**5 + 160*a*b*d*x**6 + 126*a*b*e*x**7 + 77*b**2*c*x**9 + 60
*b**2*d*x**10 + 45*b**2*e*x**11)/(384*a**6 + 1152*a**5*b*x**4 + 1
152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

GIAC/XCAS [A] time = 0.223793, size = 504, normalized size = 1.35

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d + 77\left(ab^3\right)^{\frac14}b^2c + 15\left(ab^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d + 77\left(ab^3\right)^{\frac14}b^2c + 15\left(ab^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac14}b^2c - 15\left(ab^3\right)^{\frac34}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac14} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$-\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac14}b^2c - 15\left(ab^3\right)^{\frac34}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac14} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$+\frac{45\,b^2x^{11}e + 60\,b^2dx^{10} + 77\,b^2cx^9 + 126\,abx^7e + 160\,abdx^6 + 198\,abcx^5 + 113\,a^2x^3e + 132\,a^2dx^2 + 153\,a^2cx}{384\left(bx^4+a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*
c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b
)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1
024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*ln(x^2
+ sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(
77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(
```

```
a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^2*x^11*e + 60*b^2
*d*x^10 + 77*b^2*c*x^9 + 126*a*b*x^7*e + 160*a*b*d*x^6 + 198*a*b*
c*x^5 + 113*a^2*x^3*e + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)
^3*a^3)
```

3.133 $\int a\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=28

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9
```

Rubi [A] time = 0.0330414, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

Antiderivative was successfully verified.

```
[In]  Int[a*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+a\int e^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a*(f*x**4+e)**2,x)
```

```
[Out] 2*a*e*f*x**5/5 + a*f**2*x**9/9 + a*Integral(e**2, x)
```

Mathematica [A] time = 0.00308368, size = 27, normalized size = 0.96

$$a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[a*(e + f*x^4)^2,x]
```

```
[Out] a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

Maple [A] time = 0.002, size = 24, normalized size = 0.9

$$a\left(\frac{f^2x^9}{9}+\frac{2\,efx^5}{5}+e^2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a*(f*x^4+e)^2,x)
```

```
[Out] a*(1/9*f^2*x^9+2/5*e*f*x^5+e^2*x)
```

Maxima [A] time = 1.36085, size = 34, normalized size = 1.21

$$\frac{1}{45}\left(5 f^{2} x^{9} + 18 e f x^{5} + 45 e^{2} x\right) a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*a,x, algorithm="maxima")
```

```
[Out] 1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a
```

Fricas [A] time = 0.198251, size = 1, normalized size = 0.04

$$\frac{1}{9} x^{9} f^{2} a + \frac{2}{5} x^{5} f e a + x e^{2} a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*a,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*f^2*a + 2/5*x^5*f*e*a + x*e^2*a
```

Sympy [A] time = 0.043121, size = 27, normalized size = 0.96

$$a e^{2} x + \frac{2 a e f x^{5}}{5} + \frac{a f^{2} x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9
```

GIAC/XCAS [A] time = 0.20868, size = 34, normalized size = 1.21

$$\frac{1}{45}\left(5 f^{2} x^{9} + 18 f x^{5} e + 45 x e^{2}\right) a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*a,x, algorithm="giac")
```

```
[Out] 1/45*(5*f^2*x^9 + 18*f*x^5*e + 45*x*e^2)*a
```

3.134 $\int bx\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10
```

Rubi [A] time = 0.0429488, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[b*x*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$be^2\int x\,dx+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(b*x*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*Integral(x, x) + b*e*f*x**6/3 + b*f**2*x**10/10
```

Mathematica [A] time = 0.00245683, size = 32, normalized size = 0.97

$$b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[b*x*(e + f*x^4)^2,x]
```

```
[Out] b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

Maple [A] time = 0.001, size = 27, normalized size = 0.8

$$b\left(\frac{f^2x^{10}}{10}+\frac{efx^6}{3}+\frac{e^2x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(b*x*(f*x^4+e)^2,x)
```

```
[Out] b*(1/10*f^2*x^10+1/3*e*f*x^6+1/2*e^2*x^2)
```

Maxima [A] time = 7.47718, size = 36, normalized size = 1.09

$$\frac{1}{30}\left(3 f^2 x^{10} + 10 e f x^6 + 15 e^2 x^2\right) b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*b*x,x, algorithm="maxima")
```

```
[Out] 1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b
```

Fricas [A] time = 0.195803, size = 1, normalized size = 0.03

$$\frac{1}{10} x^{10} f^2 b + \frac{1}{3} x^6 f e b + \frac{1}{2} x^2 e^2 b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*b*x,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*f^2*b + 1/3*x^6*f*e*b + 1/2*x^2*e^2*b
```

Sympy [A] time = 0.048491, size = 29, normalized size = 0.88

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b*x*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10
```

GIAC/XCAS [A] time = 0.207203, size = 36, normalized size = 1.09

$$\frac{1}{30}\left(3 f^2 x^{10} + 10 f x^6 e + 15 x^2 e^2\right) b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*b*x,x, algorithm="giac")
```

```
[Out] 1/30*(3*f^2*x^10 + 10*f*x^6*e + 15*x^2*e^2)*b
```

3.135 $\int (a+bx)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=60

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^
2*x^9)/9 + (b*f^2*x^10)/10
```

Rubi [A] time = 0.121415, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^
2*x^9)/9 + (b*f^2*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+be^2\int x\,dx+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+e^2\int a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*Integral(x, x) + b*e*f*x*
*6/3 + b*f**2*x**10/10 + e**2*Integral(a, x)
```

Mathematica [A] time = 0.00328463, size = 60, normalized size = 1.

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^
2*x^9)/9 + (b*f^2*x^10)/10
```

Maple [A] time = 0.001, size = 51, normalized size = 0.9

$$ae^2x+\frac{be^2x^2}{2}+\frac{2\,aefx^5}{5}+\frac{befx^6}{3}+\frac{af^2x^9}{9}+\frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(f*x^4+e)^2,x)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1
/10*b*f^2*x^10
```

Maxima [A] time = 6.07744, size = 68, normalized size = 1.13

$$\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{2}\,be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(b*x + a),x, algorithm="maxima")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 +
 1/2*b*e^2*x^2 + a*e^2*x
```

Fricas [A] time = 0.19369, size = 1, normalized size = 0.02

$$\frac{1}{10}x^{10}f^2b+\frac{1}{9}x^9f^2a+\frac{1}{3}x^6feb+\frac{2}{5}x^5fea+\frac{1}{2}x^2e^2b+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(b*x + a),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*f^2*b + 1/9*x^9*f^2*a + 1/3*x^6*f*e*b + 2/5*x^5*f*e*a +
 1/2*x^2*e^2*b + x*e^2*a
```

Sympy [A] time = 0.057771, size = 58, normalized size = 0.97

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f
*x**6/3 + b*f**2*x**10/10
```

GIAC/XCAS [A] time = 0.207954, size = 68, normalized size = 1.13

$$\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(b*x + a),x, algorithm="giac")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*f*x^6*e + 2/5*a*f*x^5*e +
 1/2*b*x^2*e^2 + a*x*e^2
```

3.136 $\int cx^2 \left(e+fx^4\right)^2 dx$

Optimal. Leaf size=33

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Rubi [A] time = 0.0445637, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[c*x^2*(e + f*x^4)^2,x]

[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Rubi in Sympy [A] time = 6.00733, size = 31, normalized size = 0.94

$$\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(c*x**2*(f*x**4+e)**2,x)

[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

Mathematica [A] time = 0.00190006, size = 33, normalized size = 1.

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[c*x^2*(e + f*x^4)^2,x]

[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Maple [A] time = 0., size = 27, normalized size = 0.8

$$c\left(\frac{f^2x^{11}}{11}+\frac{2\,efx^7}{7}+\frac{e^2x^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(c*x^2*(f*x^4+e)^2,x)

[Out] c*(1/11*f^2*x^11+2/7*e*f*x^7+1/3*e^2*x^3)
```

Maxima [A] time = 1.37696, size = 36, normalized size = 1.09

$$\frac{1}{231}\left(21 f^2 x^{11} + 66 e f x^7 + 77 e^2 x^3\right) c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*c*x^2,x, algorithm="maxima")
```

```
[Out] 1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c
```

Fricas [A] time = 0.196707, size = 1, normalized size = 0.03

$$\frac{1}{11}x^{11}f^2c + \frac{2}{7}x^7 fec + \frac{1}{3}x^3e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*c*x^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 2/7*x^7*f*e*c + 1/3*x^3*e^2*c
```

Sympy [A] time = 0.049801, size = 31, normalized size = 0.94

$$\frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x**2*(f*x**4+e)**2,x)
```

```
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

GIAC/XCAS [A] time = 0.210504, size = 36, normalized size = 1.09

$$\frac{1}{231}\left(21 f^2 x^{11} + 66 f x^7 e + 77 x^3 e^2\right) c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*c*x^2,x, algorithm="giac")
```

```
[Out] 1/231*(21*f^2*x^11 + 66*f*x^7*e + 77*x^3*e^2)*c
```

3.137 $\int (a + cx^2) (e + fx^4)^2 \, dx$

Optimal. Leaf size=60

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*
f^2*x^9)/9 + (c*f^2*x^11)/11
```

Rubi [A] time = 0.0737615, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*
f^2*x^9)/9 + (c*f^2*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11} + e^2 \int a \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 +
 c*f**2*x**11/11 + e**2*Integral(a, x)
```

Mathematica [A] time = 0.00401419, size = 60, normalized size = 1.

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*
f^2*x^9)/9 + (c*f^2*x^11)/11
```

Maple [A] time = 0.001, size = 51, normalized size = 0.9

$$ae^2x + \frac{ce^2x^3}{3} + \frac{2\,aefx^5}{5} + \frac{2\,cefx^7}{7} + \frac{af^2x^9}{9} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)*(f*x^4+e)^2,x)
```

```
[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1
/11*c*f^2*x^11
```

Maxima [A] time = 1.36468, size = 68, normalized size = 1.13

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{9}\,af^2x^9 + \frac{2}{7}\,cefx^7 + \frac{2}{5}\,aefx^5 + \frac{1}{3}\,ce^2x^3 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(c*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 +
 1/3*c*e^2*x^3 + a*e^2*x
```

Fricas [A] time = 0.197068, size = 1, normalized size = 0.02

$$\frac{1}{11}x^{11}f^2c + \frac{1}{9}x^9f^2a + \frac{2}{7}x^7fec + \frac{2}{5}x^5fea + \frac{1}{3}x^3e^2c + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(c*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 1/9*x^9*f^2*a + 2/7*x^7*f*e*c + 2/5*x^5*f*e*a +
 1/3*x^3*e^2*c + x*e^2*a
```

Sympy [A] time = 0.059471, size = 60, normalized size = 1.

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e
*f*x**7/7 + c*f**2*x**11/11
```

GIAC/XCAS [A] time = 0.209684, size = 68, normalized size = 1.13

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{9}\,af^2x^9 + \frac{2}{7}\,cfx^7e + \frac{2}{5}\,afx^5e + \frac{1}{3}\,cx^3e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(c*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 2/5*a*f*x^5*e +
 1/3*c*x^3*e^2 + a*x*e^2
```

3.138 $\int (bx + cx^2) (e + fx^4)^2 \, dx$

Optimal. Leaf size=65

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 +
 (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Rubi [A] time = 0.163398, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 +
 (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$be^2 \int x \, dx + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*Integral(x, x) + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x
**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

Mathematica [A] time = 0.00435401, size = 65, normalized size = 1.

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 +
 (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Maple [A] time = 0.001, size = 54, normalized size = 0.8

$$\frac{be^2x^2}{2} + \frac{ce^2x^3}{3} + \frac{befx^6}{3} + \frac{2\,cefx^7}{7} + \frac{bf^2x^{10}}{10} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(f*x^4+e)^2,x)
```

```
[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^
2*x^10+1/11*c*f^2*x^11
```

Maxima [A] time = 1.37063, size = 72, normalized size = 1.11

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{2}{7}\,cefx^7 + \frac{1}{3}\,befx^6 + \frac{1}{3}\,ce^2x^3 + \frac{1}{2}\,be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(c*x^2 + b*x),x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6
 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

Fricas [A] time = 0.195482, size = 1, normalized size = 0.02

$$\frac{1}{11}x^{11}f^2c + \frac{1}{10}x^{10}f^2b + \frac{2}{7}x^7fec + \frac{1}{3}x^6feb + \frac{1}{3}x^3e^2c + \frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(c*x^2 + b*x),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 1/10*x^10*f^2*b + 2/7*x^7*f*e*c + 1/3*x^6*f*e*b
 + 1/3*x^3*e^2*c + 1/2*x^2*e^2*b
```

Sympy [A] time = 0.060234, size = 61, normalized size = 0.94

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 +
2*c*e*f*x**7/7 + c*f**2*x**11/11
```

GIAC/XCAS [A] time = 0.210393, size = 72, normalized size = 1.11

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{2}{7}\,cfx^7e + \frac{1}{3}\,bfx^6e + \frac{1}{3}\,cx^3e^2 + \frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(c*x^2 + b*x),x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*f*x^7*e + 1/3*b*f*x^6*e
 + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2
```

3.139 $\int (a+bx+cx^2) (e+fx^4)^2 \, dx$

Optimal. Leaf size=92

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*
f*x^6)/3 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c
*f^2*x^11)/11
```

Rubi [A] time = 0.104594, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*
f*x^6)/3 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c
*f^2*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+be^2\int x\,dx+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+e^2\int a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*Integral(x, x) + b*e*f*x*
*6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*
x**11/11 + e**2*Integral(a, x)
```

Mathematica [A] time = 0.00591937, size = 92, normalized size = 1.

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*
f*x^6)/3 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c
*f^2*x^11)/11
```

Maple [A] time = 0., size = 77, normalized size = 0.8

$$ae^2x+\frac{be^2x^2}{2}+\frac{ce^2x^3}{3}+\frac{2\,aefx^5}{5}+\frac{befx^6}{3}+\frac{2\,cefx^7}{7}+\frac{af^2x^9}{9}+\frac{bf^2x^{10}}{10}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)*(f*x^4+e)^2,x)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2
/7*c*e*f*x^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

Maxima [A] time = 1.40846, size = 103, normalized size = 1.12

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(c*x^2 + b*x + a),x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7
 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
+ a*e^2*x
```

Fricas [A] time = 0.207, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}f^2c+\frac{1}{10}x^{10}f^2b+\frac{1}{9}x^9f^2a+\frac{2}{7}x^7fec+\frac{1}{3}x^6feb+\frac{2}{5}x^5fea+\frac{1}{3}x^3e^2c+\frac{1}{2}x^2e^2b+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(c*x^2 + b*x + a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 1/10*x^10*f^2*b + 1/9*x^9*f^2*a + 2/7*x^7*f*e*c
 + 1/3*x^6*f*e*b + 2/5*x^5*f*e*a + 1/3*x^3*e^2*c + 1/2*x^2*e^2*b
+ x*e^2*a
```

Sympy [A] time = 0.064328, size = 90, normalized size = 0.98

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f
*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f*
*2*x**11/11
```

GIAC/XCAS [A] time = 0.210448, size = 103, normalized size = 1.12

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cfx^7e+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{3}\,cx^3e^2+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(c*x^2 + b*x + a),x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e
 + 1/3*b*f*x^6*e + 2/5*a*f*x^5*e + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2
+ a*x*e^2
```

3.140 $\int dx^3 \left(e+fx^4\right)^2 \, dx$

Optimal. Leaf size=17

$$\frac{d\left(e+fx^4\right)^3}{12f}$$

```
[Out] (d*(e + f*x^4)^3)/(12*f)
```

Rubi [A] time = 0.0180704, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In]  Int[d*x^3*(e + f*x^4)^2,x]
```

```
[Out] (d*(e + f*x^4)^3)/(12*f)
```

Rubi in Sympy [A] time = 2.56475, size = 12, normalized size = 0.71

$$\frac{d\left(e+fx^4\right)^3}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d*x**3*(f*x**4+e)**2,x)
```

```
[Out] d*(e + f*x**4)**3/(12*f)
```

Mathematica [A] time = 0.00149336, size = 33, normalized size = 1.94

$$\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d*x^3*(e + f*x^4)^2,x]
```

```
[Out] (d*e^2*x^4)/4 + (d*e*f*x^8)/4 + (d*f^2*x^12)/12
```

Maple [A] time = 0.001, size = 27, normalized size = 1.6

$$d\left(\frac{f^2x^{12}}{12}+\frac{efx^8}{4}+\frac{e^2x^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d*x^3*(f*x^4+e)^2,x)
```

```
[Out] d*(1/12*f^2*x^12+1/4*e*f*x^8+1/4*e^2*x^4)
```

Maxima [A] time = 1.41036, size = 20, normalized size = 1.18

$$\frac{\left(fx^4+e\right)^3 d}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*d*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*(f*x^4 + e)^3*d/f
```

Fricas [A] time = 0.203853, size = 1, normalized size = 0.06

$$\frac{1}{12}x^{12}f^2d+\frac{1}{4}x^8fed+\frac{1}{4}x^4e^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*d*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/4*x^8*f*e*d + 1/4*x^4*e^2*d
```

Sympy [A] time = 0.049223, size = 29, normalized size = 1.71

$$\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d*x**3*(f*x**4+e)**2,x)
```

```
[Out] d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

GIAC/XCAS [A] time = 0.209642, size = 22, normalized size = 1.29

$$\frac{\left(fx^4+e\right)^3 d}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*d*x^3,x, algorithm="giac")
```

```
[Out] 1/12*(f*x^4 + e)^3*d/f
```

3.141 $\int (a+dx^3)(e+fx^4)^2\,dx$

Optimal. Leaf size=45

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9 + (d*(e + f*x^4)^3)/(12
*f)
```

Rubi [A] time = 0.0619573, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d(e+fx^4)^3}{12f}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9 + (d*(e + f*x^4)^3)/(12
*f)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+a\int e^2\,dx+\frac{d(e+fx^4)^3}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+a)*(f*x**4+e)**2,x)
```

```
[Out] 2*a*e*f*x**5/5 + a*f**2*x**9/9 + a*Integral(e**2, x) + d*(e + f*x
**4)**3/(12*f)
```

Mathematica [A] time = 0.00411818, size = 60, normalized size = 1.33

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (d*e*f*x^8)/4 + (a*f^
2*x^9)/9 + (d*f^2*x^12)/12
```

Maple [A] time = 0.001, size = 51, normalized size = 1.1

$$\frac{df^2x^{12}}{12}+\frac{af^2x^9}{9}+\frac{defx^8}{4}+\frac{2\,aefx^5}{5}+\frac{de^2x^4}{4}+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+a)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/5*a*e*f*x^5+1/4*d*e
^2*x^4+a*e^2*x
```

Maxima [A] time = 1.4422, size = 68, normalized size = 1.51

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 +
 1/4*d*e^2*x^4 + a*e^2*x
```

Fricas [A] time = 0.2041, size = 1, normalized size = 0.02

$$\frac{1}{12}x^{12}f^2d+\frac{1}{9}x^9f^2a+\frac{1}{4}x^8fed+\frac{2}{5}x^5fea+\frac{1}{4}x^4e^2d+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d + 2/5*x^5*f*e*a +
 1/4*x^4*e^2*d + x*e^2*a
```

Sympy [A] time = 0.05881, size = 58, normalized size = 1.29

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + d*e**2*x**4/4 + d*e*f
*x**8/4 + d*f**2*x**12/12
```

GIAC/XCAS [A] time = 0.209483, size = 68, normalized size = 1.51

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,dfx^8e+\frac{2}{5}\,afx^5e+\frac{1}{4}\,dx^4e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/5*a*f*x^5*e +
 1/4*d*x^4*e^2 + a*x*e^2
```

3.142 $\int (bx+dx^3)(e+fx^4)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^
3)/(12*f)
```

Rubi [A] time = 0.142103, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^
3)/(12*f)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{b\int^{x^2}e^2\,dx}{2}+\frac{d(e+fx^4)^3}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e*f*x**6/3 + b*f**2*x**10/10 + b*Integral(e**2, (x, x**2))/2 +
d*(e + f*x**4)**3/(12*f)
```

Mathematica [A] time = 0.00486726, size = 65, normalized size = 1.3

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4 + (
b*f^2*x^10)/10 + (d*f^2*x^12)/12
```

Maple [A] time = 0.001, size = 54, normalized size = 1.1

$$\frac{df^2x^{12}}{12}+\frac{bf^2x^{10}}{10}+\frac{defx^8}{4}+\frac{befx^6}{3}+\frac{de^2x^4}{4}+\frac{be^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+b*x)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/4*d*e*f*x^8+1/3*b*e*f*x^6+1/4*d
*e^2*x^4+1/2*b*e^2*x^2
```

Maxima [A] time = 1.39247, size = 72, normalized size = 1.44

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + b*x),x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6
 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2
```

Fricas [A] time = 0.201566, size = 1, normalized size = 0.02

$$\frac{1}{12}x^{12}f^2d+\frac{1}{10}x^{10}f^2b+\frac{1}{4}x^8fed+\frac{1}{3}x^6feb+\frac{1}{4}x^4e^2d+\frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + b*x),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/10*x^10*f^2*b + 1/4*x^8*f*e*d + 1/3*x^6*f*e*b
 + 1/4*x^4*e^2*d + 1/2*x^2*e^2*b
```

Sympy [A] time = 0.060153, size = 60, normalized size = 1.2

$$\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 +
d*e*f*x**8/4 + d*f**2*x**12/12
```

GIAC/XCAS [A] time = 0.209465, size = 72, normalized size = 1.44

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,dfx^8e+\frac{1}{3}\,bfx^6e+\frac{1}{4}\,dx^4e^2+\frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + b*x),x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 1/3*b*f*x^6*e
 + 1/4*d*x^4*e^2 + 1/2*b*x^2*e^2
```

3.143 $\int (a + bx + dx^3) (e + fx^4)^2 \, dx$

Optimal. Leaf size=77

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d(e + fx^4)^3}{12f}$$

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^
2*x^9)/9 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rubi [A] time = 0.144073, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d(e + fx^4)^3}{12f}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^
2*x^9)/9 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2aefx^5}{5} + \frac{af^2x^9}{9} + be^2 \int x \, dx + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{d(e + fx^4)^3}{12f} + e^2 \int a \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*Integral(x, x) + b*e*f*x*
*6/3 + b*f**2*x**10/10 + d*(e + f*x**4)**3/(12*f) + e**2*Integral
(a, x)
```

Mathematica [A] time = 0.00555362, size = 92, normalized size = 1.19

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{4}de^2x^4 + \frac{1}{4}defx^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (b*e*
f*x^6)/3 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*f
^2*x^12)/12
```

Maple [A] time = 0.001, size = 77, normalized size = 1.

$$\frac{df^2x^{12}}{12} + \frac{bf^2x^{10}}{10} + \frac{af^2x^9}{9} + \frac{defx^8}{4} + \frac{befx^6}{3} + \frac{2\,aefx^5}{5} + \frac{de^2x^4}{4} + \frac{be^2x^2}{2} + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+b*x+a)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/9*a*f^2*x^9+1/4*d*e*f*x^8+1/3*b
*e*f*x^6+2/5*a*e*f*x^5+1/4*d*e^2*x^4+1/2*b*e^2*x^2+a*e^2*x
```

Maxima [A] time = 1.36915, size = 103, normalized size = 1.34

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + b*x + a),x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8
 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2
+ a*e^2*x
```

Fricas [A] time = 0.204155, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}f^2d+\frac{1}{10}x^{10}f^2b+\frac{1}{9}x^9f^2a+\frac{1}{4}x^8fed+\frac{1}{3}x^6feb+\frac{2}{5}x^5fea+\frac{1}{4}x^4e^2d+\frac{1}{2}x^2e^2b+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + b*x + a),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/10*x^10*f^2*b + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d
 + 1/3*x^6*f*e*b + 2/5*x^5*f*e*a + 1/4*x^4*e^2*d + 1/2*x^2*e^2*b
+ x*e^2*a
```

Sympy [A] time = 0.06577, size = 88, normalized size = 1.14

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f
*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2
*x**12/12
```

GIAC/XCAS [A] time = 0.208341, size = 103, normalized size = 1.34

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,dfx^8e+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{4}\,dx^4e^2+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + b*x + a),x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e
 + 1/3*b*f*x^6*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/2*b*x^2*e^2
+ a*x*e^2
```

3.144 $\int \left(cx^2+dx^3\right) \left(e+fx^4\right)^2 \, dx$

Optimal. Leaf size=65

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 +
 (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Rubi [A] time = 0.1929, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 +
 (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Rubi in Sympy [A] time = 14.3557, size = 44, normalized size = 0.68

$$\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c*x**2)*(f*x**4+e)**2,x)
```

```
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*(e + f*x**4)
**3/(12*f)
```

Mathematica [A] time = 0.00467111, size = 65, normalized size = 1.

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 +
 (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Maple [A] time = 0.002, size = 54, normalized size = 0.8

$$\frac{ce^2x^3}{3}+\frac{de^2x^4}{4}+\frac{2\,cefx^7}{7}+\frac{defx^8}{4}+\frac{cf^2x^{11}}{11}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c*x^2)*(f*x^4+e)^2,x)
```

```
[Out] 1/3*c*e^2*x^3+1/4*d*e^2*x^4+2/7*c*e*f*x^7+1/4*d*e*f*x^8+1/11*c*f^
2*x^11+1/12*d*f^2*x^12
```

Maxima [A] time = 1.38366, size = 72, normalized size = 1.11

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + c*x^2),x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7
 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3
```

Fricas [A] time = 0.20125, size = 1, normalized size = 0.02

$$\frac{1}{12}x^{12}f^2d+\frac{1}{11}x^{11}f^2c+\frac{1}{4}x^8fed+\frac{2}{7}x^7fec+\frac{1}{4}x^4e^2d+\frac{1}{3}x^3e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + c*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/11*x^11*f^2*c + 1/4*x^8*f*e*d + 2/7*x^7*f*e*c
 + 1/4*x^4*e^2*d + 1/3*x^3*e^2*c
```

Sympy [A] time = 0.060133, size = 61, normalized size = 0.94

$$\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c*x**2)*(f*x**4+e)**2,x)
```

```
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4
+ d*e*f*x**8/4 + d*f**2*x**12/12
```

GIAC/XCAS [A] time = 0.210158, size = 72, normalized size = 1.11

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,dfx^8e+\frac{2}{7}\,cfx^7e+\frac{1}{4}\,dx^4e^2+\frac{1}{3}\,cx^3e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + c*x^2),x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*f*x^8*e + 2/7*c*f*x^7*e
 + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2
```

3.145 $\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=77

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*
f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rubi [A] time = 0.116891, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*
f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{d\left(e+fx^4\right)^3}{12f}+e^2\int a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 +
 c*f**2*x**11/11 + d*(e + f*x**4)**3/(12*f) + e**2*Integral(a, x)
```

Mathematica [A] time = 0.00620127, size = 92, normalized size = 1.19

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (2*c*
e*f*x^7)/7 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d
*f^2*x^12)/12
```

Maple [A] time = 0.001, size = 77, normalized size = 1.

$$\frac{df^2x^{12}}{12}+\frac{cf^2x^{11}}{11}+\frac{af^2x^9}{9}+\frac{defx^8}{4}+\frac{2\,cefx^7}{7}+\frac{2\,aefx^5}{5}+\frac{de^2x^4}{4}+\frac{ce^2x^3}{3}+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c*x^2+a)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/7*c
*e*f*x^7+2/5*a*e*f*x^5+1/4*d*e^2*x^4+1/3*c*e^2*x^3+a*e^2*x
```

Maxima [A] time = 1.37371, size = 103, normalized size = 1.34

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{11}\,cf^2x^{11} + \frac{1}{9}\,af^2x^9 + \frac{1}{4}\,defx^8 + \frac{2}{7}\,cefx^7 + \frac{2}{5}\,aefx^5 + \frac{1}{4}\,de^2x^4 + \frac{1}{3}\,ce^2x^3 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + c*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8
 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3
+ a*e^2*x
```

Fricas [A] time = 0.202007, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}f^2d + \frac{1}{11}x^{11}f^2c + \frac{1}{9}x^9f^2a + \frac{1}{4}x^8fed + \frac{2}{7}x^7fec + \frac{2}{5}x^5fea + \frac{1}{4}x^4e^2d + \frac{1}{3}x^3e^2c + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + c*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/11*x^11*f^2*c + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d
 + 2/7*x^7*f*e*c + 2/5*x^5*f*e*a + 1/4*x^4*e^2*d + 1/3*x^3*e^2*c
+ x*e^2*a
```

Sympy [A] time = 0.066001, size = 90, normalized size = 1.17

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e
*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f*
*2*x**12/12
```

GIAC/XCAS [A] time = 0.207852, size = 103, normalized size = 1.34

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{11}\,cf^2x^{11} + \frac{1}{9}\,af^2x^9 + \frac{1}{4}\,dfx^8e + \frac{2}{7}\,cfx^7e + \frac{2}{5}\,afx^5e + \frac{1}{4}\,dx^4e^2 + \frac{1}{3}\,cx^3e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + c*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e
 + 2/7*c*f*x^7*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2
+ a*x*e^2
```

3.146 $\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=97

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (
2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/1
1 + (d*f^2*x^12)/12
```

Rubi [A] time = 0.165651, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (
2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/1
1 + (d*f^2*x^12)/12
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$be^2\int x\,dx+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*Integral(x, x) + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x
**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*(e + f*x**4)**3/(12*
f)
```

Mathematica [A] time = 0.00674172, size = 97, normalized size = 1.

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (
2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/1
1 + (d*f^2*x^12)/12
```

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{be^2x^2}{2}+\frac{ce^2x^3}{3}+\frac{de^2x^4}{4}+\frac{befx^6}{3}+\frac{2\,cefx^7}{7}+\frac{defx^8}{4}+\frac{bf^2x^{10}}{10}+\frac{cf^2x^{11}}{11}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x)
```

```
[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/4*d*e^2*x^4+1/3*b*e*f*x^6+2/7*c*e*f
*x^7+1/4*d*e*f*x^8+1/10*b*f^2*x^10+1/11*c*f^2*x^11+1/12*d*f^2*x^1
2
```

Maxima [A] time = 6.91078, size = 107, normalized size = 1.1

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{1}{4}\,defx^8 + \frac{2}{7}\,cefx^7 + \frac{1}{3}\,befx^6 + \frac{1}{4}\,de^2x^4 + \frac{1}{3}\,ce^2x^3 + \frac{1}{2}\,be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + c*x^2 + b*x),x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x
^8 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^
3 + 1/2*b*e^2*x^2
```

Fricas [A] time = 0.202662, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}f^2d + \frac{1}{11}x^{11}f^2c + \frac{1}{10}x^{10}f^2b + \frac{1}{4}x^8fed + \frac{2}{7}x^7fec + \frac{1}{3}x^6feb + \frac{1}{4}x^4e^2d + \frac{1}{3}x^3e^2c + \frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + c*x^2 + b*x),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/11*x^11*f^2*c + 1/10*x^10*f^2*b + 1/4*x^8*f*e
*d + 2/7*x^7*f*e*c + 1/3*x^6*f*e*b + 1/4*x^4*e^2*d + 1/3*x^3*e^2*
c + 1/2*x^2*e^2*b
```

Sympy [A] time = 0.066996, size = 92, normalized size = 0.95

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 +
2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 +
 d*f**2*x**12/12
```

GIAC/XCAS [A] time = 0.209136, size = 107, normalized size = 1.1

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{1}{4}\,dfx^8e + \frac{2}{7}\,cfx^7e + \frac{1}{3}\,bfx^6e + \frac{1}{4}\,dx^4e^2 + \frac{1}{3}\,cx^3e^2 + \frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e)^2*(d*x^3 + c*x^2 + b*x),x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8
*e + 2/7*c*f*x^7*e + 1/3*b*f*x^6*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^
2 + 1/2*b*x^2*e^2
```

3.147 $\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx$

Optimal. Leaf size=109

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*
d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b
^2*e*x^11)/11 + (f*(a + b*x^4)^3)/(12*b)
```

Rubi [A] time = 0.163476, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*
d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b
^2*e*x^11)/11 + (f*(a + b*x^4)^3)/(12*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2d\int x\,dx+\frac{a^2ex^3}{3}+a^2\int c\,dx+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{f\left(a+bx^4\right)^3}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*d*Integral(x, x) + a**2*e*x**3/3 + a**2*Integral(c, x) + 2*a
*b*c*x**5/5 + a*b*d*x**6/3 + 2*a*b*e*x**7/7 + b**2*c*x**9/9 + b**
2*d*x**10/10 + b**2*e*x**11/11 + f*(a + b*x**4)**3/(12*b)
```

Mathematica [A] time = 0.0119555, size = 124, normalized size = 1.14

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*
c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2
*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Maple [A] time = 0.001, size = 103, normalized size = 0.9

$$\frac{b^2fx^{12}}{12}+\frac{b^2ex^{11}}{11}+\frac{b^2dx^{10}}{10}+\frac{b^2cx^9}{9}+\frac{fabx^8}{4}+\frac{2\,abex^7}{7}+\frac{abdx^6}{3}+\frac{2\,abcx^5}{5}+\frac{fa^2x^4}{4}+\frac{a^2ex^3}{3}+\frac{a^2dx^2}{2}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

```
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4
*f*a*b*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*f*a^2*x^
4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x
```

Maxima [A] time = 5.93498, size = 138, normalized size = 1.27

$$\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(f*x^3 + e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x
^9 + 1/4*a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^
5 + 1/4*a^2*f*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Fricas [A] time = 0.198317, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}fb^2+\frac{1}{11}x^{11}eb^2+\frac{1}{10}x^{10}db^2+\frac{1}{9}x^9cb^2+\frac{1}{4}x^8fba+\frac{2}{7}x^7eba+\frac{1}{3}x^6dba+\frac{2}{5}x^5cba+\frac{1}{4}x^4fa^2+\frac{1}{3}x^3ea^2+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(f*x^3 + e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b
^2 + 1/4*x^8*f*b*a + 2/7*x^7*e*b*a + 1/3*x^6*d*b*a + 2/5*x^5*c*b*
a + 1/4*x^4*f*a^2 + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.074912, size = 121, normalized size = 1.11

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*
c*x**5/5 + a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*
x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

GIAC/XCAS [A] time = 0.215479, size = 142, normalized size = 1.3

$$\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2x^{11}e+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abx^7e+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2x^3e+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(f*x^3 + e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x
^9 + 1/4*a*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^
5 + 1/4*a^2*f*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

3.148 $\int (c + dx + ex^2 + fx^3) (a + bx^4)^3 \, dx$

Optimal. Leaf size=151

$$a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \frac{f(a+bx^4)^4}{16b} + \frac{1}{13}b^3cx^{13} + \frac{1}{14}b^3dx^{14} + \frac{1}{15}b^3ex^{15}$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^
2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x
^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14
 + (b^3*e*x^15)/15 + (f*(a + b*x^4)^4)/(16*b)
```

Rubi [A] time = 0.223836, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \frac{f(a+bx^4)^4}{16b} + \frac{1}{13}b^3cx^{13} + \frac{1}{14}b^3dx^{14} + \frac{1}{15}b^3ex^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^
2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x
^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14
 + (b^3*e*x^15)/15 + (f*(a + b*x^4)^4)/(16*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3d\int x\,dx + \frac{a^3ex^3}{3} + a^3\int c\,dx + \frac{3a^2bcx^5}{5} + \frac{a^2bdx^6}{2} + \frac{3a^2bex^7}{7} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{b^3cx^{13}}{13} + \frac{b^3dx^{14}}{14} + \frac{b^3ex^{15}}{15} + \frac{f(a+bx^4)^4}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*d*Integral(x, x) + a**3*e*x**3/3 + a**3*Integral(c, x) + 3*a
**2*b*c*x**5/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + a*b**2*c*x
**9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + b**3*c*x**13/
13 + b**3*d*x**14/14 + b**3*e*x**15/15 + f*(a + b*x**4)**4/(16*b)
```

Mathematica [A] time = 0.00851507, size = 180, normalized size = 1.19

$$a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{1}{4}a^3fx^4 + \frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{3}{8}a^2bfx^8 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \frac{1}{4}ab^2fx^{12} + \frac{1}{13}b^3cx^{13} + \frac{1}{14}b^3dx^{14} + \frac{1}{15}b^3ex^{15} + \frac{1}{16}b^3fx^{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*
b*c*x^5)/5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8
)/8 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11
 + (a*b^2*f*x^12)/4 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*
x^15)/15 + (b^3*f*x^16)/16
```

Maple [A] time = 0.001, size = 151, normalized size = 1.

$$\frac{b^3fx^{16}}{16}+\frac{b^3ex^{15}}{15}+\frac{b^3dx^{14}}{14}+\frac{b^3cx^{13}}{13}+\frac{ab^2fx^{12}}{4}+\frac{3\,ab^2ex^{11}}{11}+\frac{3\,ab^2dx^{10}}{10}+\frac{ab^2cx^9}{3}+\frac{3\,fa^2bx^8}{8}+\frac{3\,a^2bex^7}{7}+\frac{a^2bdx^6}{2}+\frac{3\,a^2bcx^5}{5}+\frac{a^3fx^4}{4}+\frac{a^3ex^3}{3}+\frac{a^3dx^2}{2}+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/16*b^3*f*x^16+1/15*b^3*e*x^15+1/14*b^3*d*x^14+1/13*b^3*c*x^13+1
/4*a*b^2*f*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x
^9+3/8*f*a^2*b*x^8+3/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^
5+1/4*a^3*f*x^4+1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

Maxima [A] time = 1.36898, size = 203, normalized size = 1.34

$$\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^3*(f*x^3 + e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*
x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 +
 1/3*a*b^2*c*x^9 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*
d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3
*d*x^2 + a^3*c*x
```

Fricas [A] time = 0.197519, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}fb^3+\frac{1}{15}x^{15}eb^3+\frac{1}{14}x^{14}db^3+\frac{1}{13}x^{13}cb^3+\frac{1}{4}x^{12}fb^2a+\frac{3}{11}x^{11}eb^2a+\frac{3}{10}x^{10}db^2a+\frac{1}{3}x^9cb^2a+\frac{3}{8}x^8fba^2+\frac{3}{7}x^7eba^2+\frac{1}{2}x^6dba^2+\frac{3}{5}x^5cba^2+\frac{1}{4}x^4fa^3+\frac{1}{3}x^3ea^3+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^3*(f*x^3 + e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/14*x^14*d*b^3 + 1/13*x^13*c
*b^3 + 1/4*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a +
 1/3*x^9*c*b^2*a + 3/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*
b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*x^4*f*a^3 + 1/3*x^3*e*a^3 + 1/2*x^2
*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.087232, size = 180, normalized size = 1.19

$$a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{a^3fx^4}{4} + \frac{3a^2bcx^5}{5} + \frac{a^2bdx^6}{2} + \frac{3a^2bex^7}{7} + \frac{3a^2bfx^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{ab^2fx^{12}}{4} + \frac{b^3cx^{13}}{13} + \frac{b^3dx^{14}}{14} + \frac{b^3ex^{15}}{15} + \frac{b^3fx^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2
*b*c*x**5/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x*
*8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/1
1 + a*b**2*f*x**12/4 + b**3*c*x**13/13 + b**3*d*x**14/14 + b**3*e
*x**15/15 + b**3*f*x**16/16
```

GIAC/XCAS [A] time = 0.208601, size = 208, normalized size = 1.38

$$\frac{1}{16}\,b^3fx^{16} + \frac{1}{15}\,b^3x^{15}e + \frac{1}{14}\,b^3dx^{14} + \frac{1}{13}\,b^3cx^{13} + \frac{1}{4}\,ab^2fx^{12} + \frac{3}{11}\,ab^2x^{11}e + \frac{3}{10}\,ab^2dx^{10} + \frac{1}{3}\,ab^2cx^9 + \frac{3}{8}\,a^2bfx^8 + \frac{3}{7}\,a^2bx^7e + \frac{1}{2}\,a^2bdx^6 + \frac{3}{5}\,a^2bcx^5 + \frac{1}{4}\,a^3fx^4 + \frac{1}{3}\,a^3x^3e + \frac{1}{2}\,a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^3*(f*x^3 + e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*
x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 +
 1/3*a*b^2*c*x^9 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*
d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3
*d*x^2 + a^3*c*x
```

3.149 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=155

$$\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)}$$

```
[Out] (a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + ((3*Sqrt[b]*c
 - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) +
((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/
4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[
b])
```

Rubi [A] time = 0.299263, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] (a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + ((3*Sqrt[b]*c
 - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) +
((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/
4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[
b])
```

Rubi in Sympy [A] time = 44.3899, size = 138, normalized size = 0.89

$$\frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}-\frac{\left(\sqrt{a}e-3\sqrt{b}c\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{3}{4}}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] (a*f + b*x*(c + d*x + e*x**2))/(4*a*b*(a - b*x**4)) + d*atanh(sqr
t(b)*x**2/sqrt(a))/(4*a**(3/2)*sqrt(b)) - (sqrt(a)*e - 3*sqrt(b)*
c)*atan(b**(1/4)*x/a**(1/4))/(8*a**(7/4)*b**(3/4)) + (sqrt(a)*e +
 3*sqrt(b)*c)*atanh(b**(1/4)*x/a**(1/4))/(8*a**(7/4)*b**(3/4))
```

Mathematica [A] time = 0.288028, size = 220, normalized size = 1.42

$$\frac{-\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d\right)+\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c-2\sqrt{a}\sqrt[4]{b}d\right)+\frac{4a(af+bx(c+x(d+ex}{a-bx^4}}{16a^2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*(a*f + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*a^(1/4)*b^(1
/4)*(-3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - b^(1
/4)*(3*a^(1/4)*Sqrt[b]*c + 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a
^(1/4) - b^(1/4)*x] + b^(1/4)*(3*a^(1/4)*Sqrt[b]*c - 2*Sqrt[a]*b^
(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x] + 2*Sqrt[a]*Sqrt[b]
*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^2*b)
```

Maple [B] time = 0.007, size = 248, normalized size = 1.6

$$-\frac{cx}{4\,a\,(bx^4-a)}+\frac{3\,c}{16\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3\,c}{8\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{dx^2}{4\,a\,(bx^4-a)}-\frac{d}{8\,a}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{ex^3}{4\,a\,(bx^4-a)}$$
$$-\frac{e}{8\,ab}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e}{16\,ab}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{fx^4}{4\,a\,(bx^4-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] -1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x
-(a/b)^(1/4)))+3/8*c/a^2*(a/b)^(1/4)*arctan(x/(a/b)^(1/4))-1/4*d*
x^2/a/(b*x^4-a)-1/8*d/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x
^2*(a*b)^(1/2)))-1/4*e*x^3/a/(b*x^4-a)-1/8*e/a/b/(a/b)^(1/4)*arct
an(x/(a/b)^(1/4))+1/16*e/a/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a
/b)^(1/4)))-1/4*f*x^4/a/(b*x^4-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 30.8461, size = 518, normalized size = 3.34

$$\operatorname{RootSum}\left(65536t^4a^7b^3+t^2\left(-3072a^4b^2ce-2048a^4b^2d^2\right)+t\left(128a^3bde^2+1152a^2b^2c^2d\right)-a^2e^4+18abc^2e^2-48abcd^2e+\right.$$
$$-\frac{af+bcx+bdx^2+bex^3}{-4a^2b+4ab^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048
*a**4*b**2*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d)
 - a**2*e**4 + 18*a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 -
 81*b**2*c**4, Lambda(_t, _t*log(x + (4096*_t**3*a**7*b**2*e**3 +
 36864*_t**3*a**6*b**3*c**2*e - 98304*_t**3*a**6*b**3*c*d**2 + 46
08*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*d**3*e - 13824
*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4*b*d*
*2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d*
*2*e + 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3
*d*e**5 - 120*a**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*
c**4*d*e - 1080*a*b**2*c**3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4
 - 96*a**2*b*c*d**2*e**3 + 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e
**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2*d**4 - 729*b**3*c*
*6)))) - (a*f + b*c*x + b*d*x**2 + b*e*x**3)/(-4*a**2*b + 4*a*b**
2*x**4)
```

GIAC/XCAS [A] time = 0.222741, size = 452, normalized size = 2.92

$$-\frac{bx^3e+bdx^2+bcx+af}{4\,(bx^4-a)ab}+\frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-ab}b^2d+3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-ab}b^2d+3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b^3}-\frac{\sqrt{2}\left(3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*f)/((b*x^4 - a)*a*b) + 1/16*s
qrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a
*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-
a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d
+ 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2
)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*
(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(
1/4)*b^2*c - (-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/(a^2*b^3)
```

3.150 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=188

$$\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}$$

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + (a*f + b*x*(c
+ d*x + e*x^2))/(8*a*b*(a - b*x^4)^2) + ((21*Sqrt[b]*c - 5*Sqrt[a
]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sq
rt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)
*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt
[b])
```

Rubi [A] time = 0.359382, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + (a*f + b*x*(c
+ d*x + e*x^2))/(8*a*b*(a - b*x^4)^2) + ((21*Sqrt[b]*c - 5*Sqrt[a
]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sq
rt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)
*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt
[b])
```

Rubi in Sympy [A] time = 56.9845, size = 173, normalized size = 0.92

$$\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{3d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}-\frac{\left(5\sqrt{a}e-21\sqrt{b}c\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{3}{4}}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] (a*f + b*x*(c + d*x + e*x**2))/(8*a*b*(a - b*x**4)**2) + x*(7*c +
 6*d*x + 5*e*x**2)/(32*a**2*(a - b*x**4)) + 3*d*atanh(sqrt(b)*x**
2/sqrt(a))/(16*a**(5/2)*sqrt(b)) - (5*sqrt(a)*e - 21*sqrt(b)*c)*a
tan(b**(1/4)*x/a**(1/4))/(64*a**(11/4)*b**(3/4)) + (5*sqrt(a)*e +
 21*sqrt(b)*c)*atanh(b**(1/4)*x/a**(1/4))/(64*a**(11/4)*b**(3/4))
```

Mathematica [A] time = 0.517807, size = 253, normalized size = 1.35

$$-\frac{\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}c-12\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{16a^2(af+bx(c+x(d+ex)))}{b(a-bx^4)^2}+\frac{2\sqrt[4]{a}\left(21\sqrt{b}c-5\sqrt{\cdots}\right.}{b^{3\cdots}}}{128a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a*x*(7*c + x*(6*d + 5*e*x)))/(a - b*x^4) + (16*a^2*(a*f + b*x
*(c + x*(d + e*x))))/(b*(a - b*x^4)^2) + (2*a^(1/4)*(21*Sqrt[b]*c
 - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((21*a^(1/
4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) -
b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[b]*c - 12*Sqrt[a]*b^(1/4)
*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*Sqrt[a]
*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [B] time = 0.008, size = 326, normalized size = 1.7

$$\frac{cx}{8\,a\,(bx^4-a)^2}-\frac{7\,cx}{32\,a^2\,(bx^4-a)}+\frac{21\,c}{128\,a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{21\,c}{64\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{dx^2}{8\,a\,(bx^4-a)^2}-\frac{3\,dx^2}{16\,a^2\,(bx^4-a)}$$
$$-\frac{3\,d}{32\,a^2}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$
$$+\frac{ex^3}{8\,a\,(bx^4-a)^2}-\frac{5\,ex^3}{32\,a^2\,(bx^4-a)}-\frac{5\,e}{64\,a^2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{5\,e}{128\,a^2b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{fx^4}{8\,a\,(bx^4-a)^2}-\frac{fx^4}{8\,a^2\,(bx^4-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(a/b)^(
1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+21/64*c/a^3*(a/b)^(1/4)*
arctan(x/(a/b)^(1/4))+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x
^4-a)-3/32*d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b
)^(1/2)))+1/8*e*x^3/a/(b*x^4-a)^2-5/32*e/a^2*x^3/(b*x^4-a)-5/64*e
/a^2/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))+5/128*e/a^2/b/(a/b)^(1/4
)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/8*f*x^4/a/(b*x^4-a)^2-1/8
*f/a^2*x^4/(b*x^4-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 126.867, size = 583, normalized size = 3.1

$$-\operatorname{RootSum}\left(268435456t^4a^{11}b^3+t^2\left(-6881280a^6b^2ce-4718592a^6b^2d^2\right)+t\left(-153600a^4bde^2-2709504a^3b^2c^2d\right)-625a^2e\right.$$
$$-\frac{-4a^2f-11abcx-10abdx^2-9abex^3+7b^2cx^5+6b^2dx^6+5b^2ex^7}{32a^4b-64a^3b^2x^4+32a^2b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c
*e - 4718592*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 270950
4*a**3*b**2*c**2*d) - 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480
*a*b*c*d**2*e + 20736*a*b*d**4 - 194481*b**2*c**4, Lambda(_t, _t*
log(x + (-262144000*_t**3*a**10*b**2*e**3 - 4624220160*_t**3*a**9
*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 309657600*_t*
*2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e - 182078
6688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*
_t*a**5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 36578304
0*_t*a**4*b**2*c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 5227
64928*_t*a**3*b**3*c**5 + 112500*a**3*d*e**5 - 4536000*a**2*b*c*d
**3*e**2 + 2488320*a**2*b*d**5*e + 58344300*a*b**2*c**4*d*e - 800
15040*a*b**2*c**3*d**3)/(15625*a**3*e**6 + 275625*a**2*b*c**2*e**
4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a**2*b*d**4*e**2 - 48620
25*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b*
*2*c**2*d**4 - 85766121*b**3*c**6)))) - (-4*a**2*f - 11*a*b*c*x -
 10*a*b*d*x**2 - 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5
*b**2*e*x**7)/(32*a**4*b - 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)
```

GIAC/XCAS [A] time = 0.225142, size = 505, normalized size = 2.69

$$-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-21\left(-ab^3\right)^{\frac{1}{4}}b^2c-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-21\left(-ab^3\right)^{\frac{1}{4}}b^2c-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}+\frac{\sqrt{2}\left(21\left(-ab^3\right)^{\frac{1}{4}}b^2c-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256a^3b^3}-\frac{\sqrt{2}\left(21\left(-ab^3\right)^{\frac{1}{4}}b^2c-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256a^3b^3}-\frac{5b^2x^7e+6b^2dx^6+7b^2cx^5-9abx^3e-10abdx^2-11abcx-4a^2f}{32\left(bx^4-a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b
^2*c - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) - 1/128*sqrt(2)*(12*sqrt(2)*sqr
t(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b
^3) + 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e
)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/256
*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*ln(x^2 -
sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/32*(5*b^2*x^7*
e + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9*a*b*x^3*e - 10*a*b*d*x^2 - 11*a
*b*c*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

3.151 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=220

$$\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c +
 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4)) + (a*f + b*x*(c + d*x
+ e*x^2))/(12*a*b*(a - b*x^4)^3) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)
*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + ((77*Sqrt[
b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*
b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[
b])
```

Rubi [A] time = 0.436592, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c +
 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4)) + (a*f + b*x*(c + d*x
+ e*x^2))/(12*a*b*(a - b*x^4)^3) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)
*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + ((77*Sqrt[
b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*
b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[
b])
```

Rubi in Sympy [A] time = 72.5857, size = 204, normalized size = 0.93

$$\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{5d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}}-\frac{\left(15\sqrt{a}e-77\sqrt{b}c\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{3}{4}}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] (a*f + b*x*(c + d*x + e*x**2))/(12*a*b*(a - b*x**4)**3) + x*(11*c
 + 10*d*x + 9*e*x**2)/(96*a**2*(a - b*x**4)**2) + x*(77*c + 60*d*
x + 45*e*x**2)/(384*a**3*(a - b*x**4)) + 5*d*atanh(sqrt(b)*x**2/s
qrt(a))/(32*a**(7/2)*sqrt(b)) - (15*sqrt(a)*e - 77*sqrt(b)*c)*ata
n(b**(1/4)*x/a**(1/4))/(256*a**(15/4)*b**(3/4)) + (15*sqrt(a)*e +
 77*sqrt(b)*c)*atanh(b**(1/4)*x/a**(1/4))/(256*a**(15/4)*b**(3/4)
)
```

Mathematica [A] time = 0.62815, size = 286, normalized size = 1.3

$$\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c+40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c-40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}-\frac{128a^3(af+bx(c+x(d+ex)))}{b(bx^4-a)^3}+\frac{16a^2x(11c+}{(a-}}{1536a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*
c + x*(10*d + 9*e*x)))/(a - b*x^4)^2 - (128*a^3*(a*f + b*x*(c + x
*(d + e*x))))/(b*(-a + b*x^4)^3) + (6*a^(1/4)*(77*Sqrt[b]*c - 15*
Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - (3*(77*a^(1/4)*
Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) - b^
(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*Sqrt[a]*b^(1/4)
*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*Sqrt[
a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Maple [A] time = 0.02, size = 287, normalized size = 1.3

$$\frac{1}{(bx^4-a)^3}\left(-\frac{15\,b^2ex^{11}}{128\,a^3}-\frac{5\,b^2dx^{10}}{32\,a^3}-\frac{77\,b^2cx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}-\frac{113\,ex^3}{384\,a}-\frac{11\,dx^2}{32\,a}-\frac{51\,cx}{128\,a}-\frac{f}{12\,b}\right)$$
$$+\frac{77\,c}{512\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{77\,c}{256\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{5\,d}{64}\ln\left(1\left(-a^4+x^2\sqrt{ba^7}\right)\left(-a^4-x^2\sqrt{ba^7}\right)^{-1}\right)\frac{1}{\sqrt{ba^7}}$$
$$-\frac{15\,e}{256\,a^3b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{15\,e}{512\,a^3b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128*e/a^3*b^2*x^11-5/32*d/a^3*b^2*x^10-77/384*c/a^3*b^2*x^9+
21/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5-113/384/a*e*
x^3-11/32*d/a*x^2-51/128/a*c*x-1/12*f/b)/(b*x^4-a)^3+77/512*c*(a/
b)^(1/4)/a^4*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+77/256*c*(a/b)^(
1/4)/a^4*arctan(x/(a/b)^(1/4))-5/64*d/(b*a^7)^(1/2)*ln((-a^4+x^2*
(b*a^7)^(1/2))/(-a^4-x^2*(b*a^7)^(1/2)))-15/256*e/a^3/b/(a/b)^(1/
4)*arctan(x/(a/b)^(1/4))+15/512*e/a^3/b/(a/b)^(1/4)*ln((x+(a/b)^(
1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="fricas")

[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.222513, size = 555, normalized size = 2.52

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d + 77\left(-ab^3\right)^{\frac{1}{4}}b^2c + 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d + 77\left(-ab^3\right)^{\frac{1}{4}}b^2c + 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3} + \frac{\sqrt{2}\left(77\left(-ab^3\right)^{\frac{1}{4}}b^2c - 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024a^4b^3} - \frac{\sqrt{2}\left(77\left(-ab^3\right)^{\frac{1}{4}}b^2c - 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024a^4b^3} - \frac{45b^3x^{11}e + 60b^3dx^{10} + 77b^3cx^9 - 126ab^2x^7e - 160ab^2dx^6 - 198ab^2cx^5 + 113a^2bx^3e + 132a^2bdx^2 + 153a^2bcx + 32a^3}{384\left(bx^4 - a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="giac")

[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^
2*c + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqr
t(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c + 15*(-a*b^3)^(3/4)*e)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*
b^3) + 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 15*(-a*b^3)^(3/4
)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/
1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 15*(-a*b^3)^(3/4)*e)*ln(x
^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/384*(45*b
^3*x^11*e + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 126*a*b^2*x^7*e - 160*
a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2
 + 153*a^2*b*c*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

3.152 $\int \frac{a}{2+3x^4}\,dx$

Optimal. Leaf size=119

$$-\frac{a\log\left(\sqrt{3}x^2-2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{3}x^2+2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}$$

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x]
)/(4*6^(1/4)) - (a*Log[Sqrt[2] - 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2]
)/(8*6^(1/4)) + (a*Log[Sqrt[2] + 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2]
)/(8*6^(1/4))
```

Rubi [A] time = 0.208288, antiderivative size = 101, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[a/(2 + 3*x^4),x]
```

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x]
)/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
+ (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rubi in Sympy [A] time = 18.4003, size = 90, normalized size = 0.76

$$-\frac{6^{\frac{3}{4}}a\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{6^{\frac{3}{4}}a\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{6^{\frac{3}{4}}a\,\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{6^{\frac{3}{4}}a\,\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a/(3*x**4+2),x)
```

```
[Out] -6**(3/4)*a*log(3*x**2 - 6**(3/4)*x + sqrt(6))/48 + 6**(3/4)*a*lo
g(3*x**2 + 6**(3/4)*x + sqrt(6))/48 + 6**(3/4)*a*atan(6**(1/4)*x
- 1)/24 + 6**(3/4)*a*atan(6**(1/4)*x + 1)/24
```

Mathematica [A] time = 0.0558671, size = 78, normalized size = 0.66

$$\frac{a\left(-\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\right)}{8\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a/(2 + 3*x^4),x]
```

```
[Out] (a*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] - Log[2 -
2*6^(1/4)*x + Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))
/(8*6^(1/4))
```

Maple [A] time = 0.004, size = 114, normalized size = 1.

$$\frac{a\sqrt{3}\,\sqrt[4]{6}\,\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+\frac{a\sqrt{3}\,\sqrt[4]{6}\,\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)+\frac{a\sqrt{3}\,\sqrt[4]{6}\,\sqrt{2}}{48}\ln\left(1\left(x^{2}+\frac{\sqrt{3}\,\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^{2}-\frac{\sqrt{3}\,\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^
(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1
/3*6^(1/2)))
```

Maxima [A] time = 1.52661, size = 166, normalized size = 1.39

$$\frac{1}{48}\left(2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(\sqrt{3}x^{2}+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/48*(2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x +
 3^(1/4)*2^(3/4))) + 2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)
*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 3^(3/4)*2^(3/4)*log(sqrt(3)*x
^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 3^(3/4)*2^(3/4)*log(sqrt(3)*x
^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*a
```

Fricas [A] time = 0.236978, size = 366, normalized size = 3.08

$$-\frac{1}{192}\cdot 24^{\frac{3}{4}}\left(4\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}}{24^{\frac{1}{4}}ax+24^{\frac{1}{4}}\sqrt{\frac{1}{6}}a\sqrt{\frac{\sqrt{6}\left(\sqrt{6}a^{2}x^{2}+24^{\frac{1}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+2\sqrt{a^{4}}\right)}{a^{2}}}+\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}}\right)+4\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}\arctan\left(\frac{}{24^{\frac{1}{4}}ax+24^{\frac{1}{4}}\sqrt{\frac{1}{6}}\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] -1/192*24^(3/4)*(4*sqrt(2)*(a^4)^(1/4)*arctan(sqrt(2)*(a^4)^(1/4)
/(24^(1/4)*a*x + 24^(1/4)*sqrt(1/6)*a*sqrt(sqrt(6)*(sqrt(6)*a^2*x
^2 + 24^(1/4)*sqrt(2)*(a^4)^(1/4)*a*x + 2*sqrt(a^4))/a^2) + sqrt(
2)*(a^4)^(1/4))) + 4*sqrt(2)*(a^4)^(1/4)*arctan(sqrt(2)*(a^4)^(1/
4)/(24^(1/4)*a*x + 24^(1/4)*sqrt(1/6)*a*sqrt(sqrt(6)*(sqrt(6)*a^2
*x^2 - 24^(1/4)*sqrt(2)*(a^4)^(1/4)*a*x + 2*sqrt(a^4))/a^2) - sqr
t(2)*(a^4)^(1/4))) - sqrt(2)*(a^4)^(1/4)*log(2*sqrt(6)*a^2*x^2 +
2*24^(1/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(a^4)) + sqrt(2)*(a^4)
^(1/4)*log(2*sqrt(6)*a^2*x^2 - 2*24^(1/4)*sqrt(2)*(a^4)^(1/4)*a*x
 + 4*sqrt(a^4)))
```

Sympy [A] time = 0.783878, size = 88, normalized size = 0.74

$$a\left(-\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a/(3*x**4+2),x)
```

```
[Out] a*(-6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(3/4)*l
og(x**2 + 6**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(3/4)*atan(6**(1/4)*x
 - 1)/12 + 6**(3/4)*atan(6**(1/4)*x + 1)/12)
```

GIAC/XCAS [A] time = 0.216631, size = 131, normalized size = 1.1

$$\frac{1}{48}\left(2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+6^{\frac{3}{4}}\ln\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/48*(2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/
3)^(1/4))) + 2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt
(2)*(2/3)^(1/4))) + 6^(3/4)*ln(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt
(2/3)) - 6^(3/4)*ln(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*a
```

3.153 $\int \frac{bx}{2+3x^4}\,dx$

Optimal. Leaf size=22

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Rubi [A] time = 0.0362842, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x)/(2 + 3*x^4),x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Rubi in Sympy [A] time = 2.87435, size = 19, normalized size = 0.86

$$\frac{\sqrt{6}b\,\mathrm{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(b*x/(3*x**4+2),x)

[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12
```

Mathematica [A] time = 0.0182195, size = 22, normalized size = 1.

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x)/(2 + 3*x^4),x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Maple [A] time = 0.002, size = 16, normalized size = 0.7

$$\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(b*x/(3*x^4+2),x)

[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.51463, size = 20, normalized size = 0.91

$$\frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b*x/(3*x^4 + 2),x, algorithm="maxima")

[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Fricas [A] time = 0.219502, size = 20, normalized size = 0.91

$$\frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b*x/(3*x^4 + 2),x, algorithm="fricas")

[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Sympy [A] time = 0.103153, size = 19, normalized size = 0.86

$$\frac{\sqrt{6}b\operatorname{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b*x/(3*x**4+2),x)

[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12
```

GIAC/XCAS [A] time = 0.213107, size = 20, normalized size = 0.91

$$\frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b*x/(3*x^4 + 2),x, algorithm="giac")

[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

3.154 $\int \frac{a+bx}{2+3x^4} \, dx$

Optimal. Leaf size=141

$$-\frac{a \log \left(\sqrt{3} x^2-2^{3/4} \sqrt[4]{3} x+\sqrt{2}\right)}{8 \sqrt[4]{6}}+\frac{a \log \left(\sqrt{3} x^2+2^{3/4} \sqrt[4]{3} x+\sqrt{2}\right)}{8 \sqrt[4]{6}}-\frac{a \tan^{-1}\left(1-\sqrt[4]{6} x\right)}{4 \sqrt[4]{6}}+\frac{a \tan^{-1}\left(\sqrt[4]{6} x+1\right)}{4 \sqrt[4]{6}}+\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}} x^2\right)}{2 \sqrt{6}}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])
/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqr
t[2] - 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2])/(8*6^(1/4)) + (a*Log[Sqr
t[2] + 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2])/(8*6^(1/4))
```

Rubi [A] time = 0.227079, antiderivative size = 123, normalized size of antiderivative = 0.87, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$-\frac{a \log \left(3 x^2-6^{3/4} x+\sqrt{6}\right)}{8 \sqrt[4]{6}}+\frac{a \log \left(3 x^2+6^{3/4} x+\sqrt{6}\right)}{8 \sqrt[4]{6}}-\frac{a \tan^{-1}\left(1-\sqrt[4]{6} x\right)}{4 \sqrt[4]{6}}+\frac{a \tan^{-1}\left(\sqrt[4]{6} x+1\right)}{4 \sqrt[4]{6}}+\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}} x^2\right)}{2 \sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])
/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqr
t[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)
*x + 3*x^2])/(8*6^(1/4))
```

Rubi in Sympy [A] time = 23.5077, size = 110, normalized size = 0.78

$$-\frac{6^{\frac{3}{4}} a \log \left(3 x^2-6^{\frac{3}{4}} x+\sqrt{6}\right)}{48}+\frac{6^{\frac{3}{4}} a \log \left(3 x^2+6^{\frac{3}{4}} x+\sqrt{6}\right)}{48}+\frac{6^{\frac{3}{4}} a \operatorname{atan}\left(\sqrt[4]{6} x-1\right)}{24}+\frac{6^{\frac{3}{4}} a \operatorname{atan}\left(\sqrt[4]{6} x+1\right)}{24}+\frac{\sqrt{6} b \operatorname{atan}\left(\frac{\sqrt{6} x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)/(3*x**4+2),x)
```

```
[Out] -6**(3/4)*a*log(3*x**2 - 6**(3/4)*x + sqrt(6))/48 + 6**(3/4)*a*lo
g(3*x**2 + 6**(3/4)*x + sqrt(6))/48 + 6**(3/4)*a*atan(6**(1/4)*x
- 1)/24 + 6**(3/4)*a*atan(6**(1/4)*x + 1)/24 + sqrt(6)*b*atan(sqr
t(6)*x**2/2)/12
```

Mathematica [A] time = 0.109898, size = 107, normalized size = 0.76

$$\frac{-2\left(\sqrt[4]{6} a+2 b\right) \tan^{-1}\left(1-\sqrt[4]{6} x\right)+2\left(\sqrt[4]{6} a-2 b\right) \tan^{-1}\left(\sqrt[4]{6} x+1\right)+\sqrt[4]{6} a\left(\log \left(\sqrt{6} x^2+2 \sqrt[4]{6} x+2\right)-\log \left(\sqrt{6} x^2-2 \sqrt[4]{6} x+2\right)\right)}{8 \sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (-2*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*(6^(1/4)*a - 2*b)
*ArcTan[1 + 6^(1/4)*x] + 6^(1/4)*a*(-Log[2 - 2*6^(1/4)*x + Sqrt[6
]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*Sqrt[6])
```

Maple [A] time = 0.004, size = 129, normalized size = 0.9

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)\\+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(1\left(x^{2}+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^{2}-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^{2}\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^
(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1
/3*6^(1/2)))+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.52984, size = 198, normalized size = 1.4

$$\frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^{2}+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^{2}-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt
(2)) - 1/48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x
 + sqrt(2)) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a - 2*sqrt(2)*b)*arct
an(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sq
rt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4
)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.796032, size = 88, normalized size = 0.62

$$\operatorname{RootSum}\left(18432t^4 + 384t^2b^2 - 96ta^2b + 3a^4 + 2b^4, \left(t \mapsto t\log\left(x + \frac{3072t^3b^2 + 192t^2a^2b + 24ta^4 + 32tb^4 - 10a^2b^3}{3a^5 - 8ab^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(18432*_t**4 + 384*_t**2*b**2 - 96*_t*a**2*b + 3*a**4 + 2*
b**4, Lambda(_t, _t*log(x + (3072*_t**3*b**2 + 192*_t**2*a**2*b +
 24*_t*a**4 + 32*_t*b**4 - 10*a**2*b**3)/(3*a**5 - 8*a*b**4))))
```

GIAC/XCAS [A] time = 0.222151, size = 155, normalized size = 1.1

$$\frac{1}{48} \cdot 6^{\frac{3}{4}} a \ln\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}} x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48} \cdot 6^{\frac{3}{4}} a \ln\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}} x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{3}{4}} a - 2\sqrt{6} b\right) \arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}} a + 2\sqrt{6} b\right) \arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/48*6^(3/4)*a*ln(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48
*6^(3/4)*a*ln(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^
(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt
(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqr
t(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)))
```

3.155 $\int \frac{cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=119

$$\frac{c\log\left(\sqrt{3}x^2-2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{3}x^2+2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}$$

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x]
)/(2*6^(3/4)) + (c*Log[Sqrt[2] - 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2]
)/(4*6^(3/4)) - (c*Log[Sqrt[2] + 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2]
)/(4*6^(3/4))
```

Rubi [A] time = 0.173275, antiderivative size = 101, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x]
)/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4))
- (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Rubi in Sympy [A] time = 19.3459, size = 90, normalized size = 0.76

$$\frac{\sqrt[4]{6}c\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{24}-\frac{\sqrt[4]{6}c\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{24}+\frac{\sqrt[4]{6}c\,\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}c\,\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(c*x**2/(3*x**4+2),x)
```

```
[Out] 6**(1/4)*c*log(3*x**2 - 6**(3/4)*x + sqrt(6))/24 - 6**(1/4)*c*log
(3*x**2 + 6**(3/4)*x + sqrt(6))/24 + 6**(1/4)*c*atan(6**(1/4)*x -
 1)/12 + 6**(1/4)*c*atan(6**(1/4)*x + 1)/12
```

Mathematica [A] time = 0.0297655, size = 78, normalized size = 0.66

$$\frac{c\left(\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\right)}{4\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (c*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] + Log[2 -
2*6^(1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))
/(4*6^(3/4))
```

Maple [A] time = 0.004, size = 114, normalized size = 1.

$$\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)$$
$$+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(1\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(c*x^2/(3*x^4+2),x)
```

```
[Out] 1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6
^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+
1/3*6^(1/2)))
```

Maxima [A] time = 1.55481, size = 166, normalized size = 1.39

$$\frac{1}{24}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(c*x^2/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/24*(2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x +
 3^(1/4)*2^(3/4))) + 2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)
*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) - 3^(1/4)*2^(1/4)*log(sqrt(3)*x
^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 3^(1/4)*2^(1/4)*log(sqrt(3)*x
^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*c
```

Fricas [A] time = 0.236057, size = 385, normalized size = 3.24

$$-\frac{1}{432}$$
$$\cdot 54^{\frac{3}{4}}\left(4\sqrt{2}\left(c^4\right)^{\frac{1}{4}}\arctan\left(\frac{27\sqrt{2}\left(c^4\right)^{\frac{3}{4}}}{3\cdot 54^{\frac{3}{4}}c^3x+54^{\frac{3}{4}}\sqrt{\frac{1}{6}}c^3\sqrt{\frac{\sqrt{6}\left(9\sqrt{6}c^3x^2+54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{3}{4}}x+18\sqrt{c^4}c\right)}{c^3}}+27\sqrt{2}\left(c^4\right)^{\frac{3}{4}}}\right)+4\sqrt{2}\left(c^4\right)^{\frac{1}{4}}\arctan\left(\frac{}{3\cdot 54^{\frac{3}{4}}c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(c*x^2/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] -1/432*54^(3/4)*(4*sqrt(2)*(c^4)^(1/4)*arctan(27*sqrt(2)*(c^4)^(3
/4)/(3*54^(3/4)*c^3*x + 54^(3/4)*sqrt(1/6)*c^3*sqrt(sqrt(6)*(9*sq
rt(6)*c^3*x^2 + 54^(3/4)*sqrt(2)*(c^4)^(3/4)*x + 18*sqrt(c^4)*c)/
c^3) + 27*sqrt(2)*(c^4)^(3/4))) + 4*sqrt(2)*(c^4)^(1/4)*arctan(27
*sqrt(2)*(c^4)^(3/4)/(3*54^(3/4)*c^3*x + 54^(3/4)*sqrt(1/6)*c^3*s
qrt(sqrt(6)*(9*sqrt(6)*c^3*x^2 - 54^(3/4)*sqrt(2)*(c^4)^(3/4)*x +
 18*sqrt(c^4)*c)/c^3) - 27*sqrt(2)*(c^4)^(3/4))) + sqrt(2)*(c^4)^
(1/4)*log(9*sqrt(6)*c^3*x^2 + 54^(3/4)*sqrt(2)*(c^4)^(3/4)*x + 18
*sqrt(c^4)*c) - sqrt(2)*(c^4)^(1/4)*log(9*sqrt(6)*c^3*x^2 - 54^(3
/4)*sqrt(2)*(c^4)^(3/4)*x + 18*sqrt(c^4)*c))
```

Sympy [A] time = 0.762353, size = 88, normalized size = 0.74

$$c\left(\frac{\sqrt[4]{6}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}-\frac{\sqrt[4]{6}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x**2/(3*x**4+2),x)
```

```
[Out] c*(6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 - 6**(1/4)*lo
g(x**2 + 6**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(1/4)*atan(6**(1/4)*x
- 1)/12 + 6**(1/4)*atan(6**(1/4)*x + 1)/12)
```

GIAC/XCAS [A] time = 0.224924, size = 131, normalized size = 1.1

$$\frac{1}{24}\left(2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-6^{\frac{1}{4}}\ln\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^2/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/24*(2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/
3)^(1/4))) + 2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt
(2)*(2/3)^(1/4))) - 6^(1/4)*ln(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt
(2/3)) + 6^(1/4)*ln(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*c
```

3.156 $\int \frac{a+cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=141

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\,6^{3/4}}$$

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6
]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c
)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2
*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))
```

Rubi [A] time = 0.233549, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6
]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c
)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2
*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))
```

Rubi in Sympy [A] time = 21.9858, size = 124, normalized size = 0.88

$$\frac{\sqrt[4]{6}\left(-\sqrt{6}a+2c\right)\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}-\frac{\sqrt[4]{6}\left(-\sqrt{6}a+2c\right)\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)/(3*x**4+2),x)
```

```
[Out] 6**(1/4)*(-sqrt(6)*a + 2*c)*log(3*x**2 - 6**(3/4)*x + sqrt(6))/48
 - 6**(1/4)*(-sqrt(6)*a + 2*c)*log(3*x**2 + 6**(3/4)*x + sqrt(6))
/48 + 6**(1/4)*(sqrt(6)*a + 2*c)*atan(6**(1/4)*x - 1)/24 + 6**(1/
4)*(sqrt(6)*a + 2*c)*atan(6**(1/4)*x + 1)/24
```

Mathematica [A] time = 0.0924101, size = 113, normalized size = 0.8

$$-\frac{\left(\sqrt{6}a-2c\right)\left(\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)\right)-2\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\right.}{8\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(1/4)*x +
 Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

Maple [B] time = 0.003, size = 226, normalized size = 1.6

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(1\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right) + \frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(1\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^
(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1
/3*6^(1/2)))+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^
(1/2)*6^(3/4)*x+1)+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1
/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-
1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/
4)*x*2^(1/2)+1/3*6^(1/2)))
```

Maxima [A] time = 1.53162, size = 225, normalized size = 1.6

$$\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c)*arctan(1/6*3^(3/4)*2
^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*3^(1/4)*2^(3/4)*(s
qrt(3)*a + sqrt(2)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3
^(1/4)*2^(3/4))) + 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*l
og(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 1/48*3^(1/4)*2^(3
/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x +
 sqrt(2))
```

Fricas [A] time = 0.409903, size = 4479, normalized size = 31.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*((9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(2*sqrt(6)*a*c -
sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))*log(-6*sqrt(6)*(243*a^10 + 2754
*a^8*c^2 + 4968*a^6*c^4 + 3312*a^4*c^6 + 816*a^2*c^8 + 32*c^10)*x
^2 + 216*(27*a^7*c + 78*a^5*c^3 + 52*a^3*c^5 + 8*a*c^7)*sqrt(9*a^
4 + 12*a^2*c^2 + 4*c^4)*x^2 + 2*sqrt(2)*(9*a^4 + 12*a^2*c^2 + 4*c
^4)^(1/4)*(216^(1/4)*sqrt(6)*(189*a^6*c + 630*a^4*c^3 + 252*a^2*c
^5 + 8*c^7)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*x - 3*216^(1/4)*(81*
a^9 + 1188*a^7*c^2 + 2016*a^5*c^4 + 1008*a^3*c^6 + 112*a*c^8)*x)*
sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*
c^2 + 4*c^4)*a*c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*
a^4 + 12*a^2*c^2 + 4*c^4)*a*c)) - 12*(81*a^8 + 864*a^6*c^2 + 1080
*a^4*c^4 + 384*a^2*c^6 + 16*c^8 - 6*sqrt(6)*(9*a^5*c + 20*a^3*c^3
 + 4*a*c^5)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))*sqrt(9*a^4 + 12*a^2
*c^2 + 4*c^4)) - (9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(2*sqrt(6)*a*
c - sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))*log(-6*sqrt(6)*(243*a^10 +
2754*a^8*c^2 + 4968*a^6*c^4 + 3312*a^4*c^6 + 816*a^2*c^8 + 32*c^1
0)*x^2 + 216*(27*a^7*c + 78*a^5*c^3 + 52*a^3*c^5 + 8*a*c^7)*sqrt(
9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 - 2*sqrt(2)*(9*a^4 + 12*a^2*c^2 +
 4*c^4)^(1/4)*(216^(1/4)*sqrt(6)*(189*a^6*c + 630*a^4*c^3 + 252*a
^2*c^5 + 8*c^7)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*x - 3*216^(1/4)*
(81*a^9 + 1188*a^7*c^2 + 2016*a^5*c^4 + 1008*a^3*c^6 + 112*a*c^8)
*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*
a^2*c^2 + 4*c^4)*a*c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqr
t(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)) - 12*(81*a^8 + 864*a^6*c^2 +
1080*a^4*c^4 + 384*a^2*c^6 + 16*c^8 - 6*sqrt(6)*(9*a^5*c + 20*a^3
*c^3 + 4*a*c^5)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))*sqrt(9*a^4 + 12
*a^2*c^2 + 4*c^4)) + 4*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*sqrt(9*
a^4 - 12*a^2*c^2 + 4*c^4)*arctan(6*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(
1/4)*(sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^3 + 2*a*c^2)
- 2*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*
c^4)*c)/(sqrt(2)*sqrt(1/3)*(216^(1/4)*sqrt(6)*sqrt(9*a^4 + 12*a^2
*c^2 + 4*c^4)*(9*a^4 - 4*c^4) - 12*216^(1/4)*(9*a^5*c - 4*a*c^5))
*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2
*c^2 + 4*c^4)*a*c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9
*a^4 + 12*a^2*c^2 + 4*c^4)*a*c))*sqrt((3*sqrt(6)*(243*a^10 + 2754
*a^8*c^2 + 4968*a^6*c^4 + 3312*a^4*c^6 + 816*a^2*c^8 + 32*c^10)*x
^2 - 108*(27*a^7*c + 78*a^5*c^3 + 52*a^3*c^5 + 8*a*c^7)*sqrt(9*a^
4 + 12*a^2*c^2 + 4*c^4)*x^2 + sqrt(2)*(9*a^4 + 12*a^2*c^2 + 4*c^4
)^(1/4)*(216^(1/4)*sqrt(6)*(189*a^6*c + 630*a^4*c^3 + 252*a^2*c^5
 + 8*c^7)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*x - 3*216^(1/4)*(81*a^
9 + 1188*a^7*c^2 + 2016*a^5*c^4 + 1008*a^3*c^6 + 112*a*c^8)*x)*sq
rt((9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^
2 + 4*c^4)*a*c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^
4 + 12*a^2*c^2 + 4*c^4)*a*c)) + 6*(81*a^8 + 864*a^6*c^2 + 1080*a^
4*c^4 + 384*a^2*c^6 + 16*c^8 - 6*sqrt(6)*(9*a^5*c + 20*a^3*c^3 +
4*a*c^5)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))*sqrt(9*a^4 + 12*a^2*c^
2 + 4*c^4))/(sqrt(6)*(243*a^10 + 2754*a^8*c^2 + 4968*a^6*c^4 + 33
12*a^4*c^6 + 816*a^2*c^8 + 32*c^10) - 36*(27*a^7*c + 78*a^5*c^3 +
 52*a^3*c^5 + 8*a*c^7)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))) + sqrt(
2)*(216^(1/4)*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*(9*a^4 - 4
*c^4)*x - 12*216^(1/4)*(9*a^5*c - 4*a*c^5)*x)*sqrt((9*a^4 + 12*a^
2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/(
9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 +
4*c^4)*a*c)) + 6*(18*a^4*c - 8*c^5 - sqrt(6)*sqrt(9*a^4 + 12*a^2*
c^2 + 4*c^4)*(3*a^3 - 2*a*c^2))*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4
))) + 4*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*sqrt(9*a^4 - 12*a^2*c^
2 + 4*c^4)*arctan(6*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(sqrt(6)*s
qrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^3 + 2*a*c^2) - 2*sqrt(9*a^4
+ 12*a^2*c^2 + 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*c)/(sqrt(2
)*sqrt(1/3)*(216^(1/4)*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*(
9*a^4 - 4*c^4) - 12*216^(1/4)*(9*a^5*c - 4*a*c^5))*sqrt((9*a^4 +
12*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a
*c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c
^2 + 4*c^4)*a*c))*sqrt((3*sqrt(6)*(243*a^10 + 2754*a^8*c^2 + 4968
*a^6*c^4 + 3312*a^4*c^6 + 816*a^2*c^8 + 32*c^10)*x^2 - 108*(27*a^
7*c + 78*a^5*c^3 + 52*a^3*c^5 + 8*a*c^7)*sqrt(9*a^4 + 12*a^2*c^2
+ 4*c^4)*x^2 - sqrt(2)*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(216^(1
/4)*sqrt(6)*(189*a^6*c + 630*a^4*c^3 + 252*a^2*c^5 + 8*c^7)*sqrt(
9*a^4 + 12*a^2*c^2 + 4*c^4)*x - 3*216^(1/4)*(81*a^9 + 1188*a^7*c^
```

```
2 + 2016*a^5*c^4 + 1008*a^3*c^6 + 112*a*c^8)*x)*sqrt((9*a^4 + 12*
a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)
/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2
+ 4*c^4)*a*c)) + 6*(81*a^8 + 864*a^6*c^2 + 1080*a^4*c^4 + 384*a^2
*c^6 + 16*c^8 - 6*sqrt(6)*(9*a^5*c + 20*a^3*c^3 + 4*a*c^5)*sqrt(9
*a^4 + 12*a^2*c^2 + 4*c^4))*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))/(sq
rt(6)*(243*a^10 + 2754*a^8*c^2 + 4968*a^6*c^4 + 3312*a^4*c^6 + 81
6*a^2*c^8 + 32*c^10) - 36*(27*a^7*c + 78*a^5*c^3 + 52*a^3*c^5 + 8
*a*c^7)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))) + sqrt(2)*(216^(1/4)*s
qrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*(9*a^4 - 4*c^4)*x - 12*21
6^(1/4)*(9*a^5*c - 4*a*c^5)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 -
 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/(9*a^4 + 36*a^2*
c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)) -
6*(18*a^4*c - 8*c^5 - sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*(3
*a^3 - 2*a*c^2))*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4))))/((2*216^(1
/4)*sqrt(6)*a*c - 216^(1/4)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))*sqr
t((9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2
 + 4*c^4)*a*c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4
 + 12*a^2*c^2 + 4*c^4)*a*c)))
```

Sympy [A] time = 0.610627, size = 68, normalized size = 0.48

$$\operatorname{RootSum}\left(55296t^4+2304t^2ac+9a^4+12a^2c^2+4c^4,\ \left(t\mapsto t\log\left(x+\frac{-4608t^3c+72ta^3-144tac^2}{9a^4-4c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + 2304*_t**2*a*c + 9*a**4 + 12*a**2*c**2 + 4*
c**4, Lambda(_t, _t*log(x + (-4608*_t**3*c + 72*_t*a**3 - 144*_t*
a*c**2)/(9*a**4 - 4*c**4))))
```

GIAC/XCAS [A] time = 0.225005, size = 177, normalized size = 1.26

$$\begin{aligned}&\frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\&+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\&+\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\ln\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\ln\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*
x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan
(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(
3/4)*a - 2*6^(1/4)*c)*ln(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
 - 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*ln(x^2 - sqrt(2)*(2/3)^(1/4)*x
+ sqrt(2/3))
```

3.157 $\int \frac{bx+cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=141

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(\sqrt{3}x^2-2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{3}x^2+2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])
/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqr
t[2] - 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2])/(4*6^(3/4)) - (c*Log[Sqr
t[2] + 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2])/(4*6^(3/4))
```

Rubi [A] time = 0.240347, antiderivative size = 123, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])
/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqr
t[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)
*x + 3*x^2])/(4*6^(3/4))
```

Rubi in Sympy [A] time = 27.0423, size = 110, normalized size = 0.78

$$\frac{\sqrt{6}b\,\mathrm{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}+\frac{\sqrt[4]{6}c\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{24}-\frac{\sqrt[4]{6}c\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{24}+\frac{\sqrt[4]{6}c\,\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}c\,\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12 + 6**(1/4)*c*log(3*x**2 - 6**(3
/4)*x + sqrt(6))/24 - 6**(1/4)*c*log(3*x**2 + 6**(3/4)*x + sqrt(6
))/24 + 6**(1/4)*c*atan(6**(1/4)*x - 1)/12 + 6**(1/4)*c*atan(6**(
1/4)*x + 1)/12
```

Mathematica [A] time = 0.092438, size = 99, normalized size = 0.7

$$\frac{-2\left(\sqrt[4]{6}b+c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(c-\sqrt[4]{6}b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)+c\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-c\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)}{4\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*(-(6^(1/4)*b) + c)*
ArcTan[1 + 6^(1/4)*x] + c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - c*
Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2])/(4*6^(3/4))
```

Maple [A] time = 0.004, size = 129, normalized size = 0.9

$$\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)$$
$$+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(1\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)/(3*x^4+2),x)
```

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/72*c*3^(1/2)*6^(3/4)*2^(
1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*c*3^(1/2)*6^(3/
4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*3^(1/2
)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/
2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))
```

Maxima [A] time = 1.53638, size = 198, normalized size = 1.4

$$\frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c-2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$
$$+\frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c+2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{1}{24}$$
$$\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c - 2*sqrt(3)*b)*arctan(1/6*3^(3/4)
*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(2)*(3^(1/4)
*2^(3/4)*c + 2*sqrt(3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x
 - 3^(1/4)*2^(3/4))) - 1/24*3^(1/4)*2^(1/4)*c*log(sqrt(3)*x^2 + 3
^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*3^(1/4)*2^(1/4)*c*log(sqrt(3)*
x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.80539, size = 85, normalized size = 0.6

$$\mathrm{RootSum}\left(27648t^4 + 576t^2b^2 + 96tbc^2 + 3b^4 + 2c^4, \left(t \mapsto t\log\left(x + \frac{-1152t^3c^2 + 288t^2b^3 - 36tb^2c^2 + 3b^5 - 3bc^4}{6b^4c - c^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] RootSum(27648*_t**4 + 576*_t**2*b**2 + 96*_t*b*c**2 + 3*b**4 + 2*
c**4, Lambda(_t, _t*log(x + (-1152*_t**3*c**2 + 288*_t**2*b**3 -
36*_t*b**2*c**2 + 3*b**5 - 3*b*c**4)/(6*b**4*c - c**5))))
```

GIAC/XCAS [A] time = 0.223479, size = 154, normalized size = 1.09

$$-\frac{1}{24}\cdot 6^{\frac{1}{4}}c\ln\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24}\cdot 6^{\frac{1}{4}}c\ln\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{12}\left(\sqrt{6}b - 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\left(\sqrt{6}b + 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] -1/24*6^(1/4)*c*ln(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/2
4*6^(1/4)*c*ln(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/12*(s
qrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(
2)*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2
)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)))
```

3.158 $\int \frac{a+bx+cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=163

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan
[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1
/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x
+ 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*
x + 3*x^2])/(8*6^(3/4))
```

Rubi [A] time = 0.28505, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan
[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1
/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x
+ 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*
x + 3*x^2])/(8*6^(3/4))
```

Rubi in Sympy [A] time = 30.0588, size = 144, normalized size = 0.88

$$\frac{\sqrt{6}b\,\mathrm{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}+\frac{\sqrt[4]{6}\left(-\sqrt{6}a+2c\right)\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}-\frac{\sqrt[4]{6}\left(-\sqrt{6}a+2c\right)\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12 + 6**(1/4)*(-sqrt(6)*a + 2*c)*l
og(3*x**2 - 6**(3/4)*x + sqrt(6))/48 - 6**(1/4)*(-sqrt(6)*a + 2*c
)*log(3*x**2 + 6**(3/4)*x + sqrt(6))/48 + 6**(1/4)*(sqrt(6)*a + 2
*c)*atan(6**(1/4)*x - 1)/24 + 6**(1/4)*(sqrt(6)*a + 2*c)*atan(6**
(1/4)*x + 1)/24
```

Mathematica [A] time = 0.165397, size = 129, normalized size = 0.79

$$\frac{-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)-\left(\sqrt{6}a-2c\right)\left(\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)\right.}{8\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*(Sq
rt[6]*a - 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - (Sqrt[6]*a -
 2*c)*(Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x +
 Sqrt[6]*x^2]))/(8*6^(3/4))
```

Maple [B] time = 0.003, size = 241, normalized size = 1.5

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)$$
$$+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(1\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)$$
$$+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)$$
$$+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(1\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^
(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1
/3*6^(1/2)))+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/72*c*3^(1/2
)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*c*
3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1
/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1
/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))
```

Maxima [A] time = 1.53579, size = 252, normalized size = 1.55

$$\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$
$$+\frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a-2\sqrt{3}\sqrt{2}b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$
$$+\frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a+2\sqrt{3}\sqrt{2}b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^
(1/4)*2^(3/4)*x + sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sq
rt(2)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*(3
^(3/4)*2^(3/4)*a - 2*sqrt(3)*sqrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*arc
tan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*(
3^(3/4)*2^(3/4)*a + 2*sqrt(3)*sqrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*ar
ctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 6.00409, size = 292, normalized size = 1.79

$$\operatorname{RootSum}\left(55296t^4 + t^2\left(2304ac + 1152b^2\right) + t\left(-288a^2b + 192bc^2\right) + 9a^4 + 12a^2c^2 - 24ab^2c + 6b^4 + 4c^4, \left(t \mapsto t\log\left(x + \frac{}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + _t**2*(2304*a*c + 1152*b**2) + _t*(-288*a**
2*b + 192*b*c**2) + 9*a**4 + 12*a**2*c**2 - 24*a*b**2*c + 6*b**4
+ 4*c**4, Lambda(_t, _t*log(x + (-13824*_t**3*a**2*c + 27648*_t**
3*a*b**2 + 9216*_t**3*c**3 + 1728*_t**2*a**3*b + 3456*_t**2*a*b*c
**2 - 2304*_t**2*b**3*c + 216*_t*a**5 - 576*_t*a**3*c**2 + 1296*_
t*a**2*b**2*c + 288*_t*a*b**4 + 288*_t*a*c**4 + 288*_t*b**2*c**3
+ 90*a**4*b*c - 90*a**3*b**3 + 60*a*b**3*c**2 - 24*b**5*c + 24*b*
c**5)/(27*a**6 - 18*a**4*c**2 + 144*a**3*b**2*c - 72*a**2*b**4 -
12*a**2*c**4 + 96*a*b**2*c**3 - 48*b**4*c**2 + 8*c**6))))
```

GIAC/XCAS [A] time = 0.227504, size = 193, normalized size = 1.18

$$\begin{aligned}&\frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\&+\frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\&+\frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\ln\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\ln\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(
2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqr
t(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(
2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*ln(x^2 + sqrt(2
)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*ln(
x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.159 $\int \frac{dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[Out] (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.0108337, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x^3)/(2 + 3*x^4),x]

[Out] (d*Log[2 + 3*x^4])/12
```

Rubi in Sympy [A] time = 2.1779, size = 10, normalized size = 0.77

$$\frac{d\log\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d*x**3/(3*x**4+2),x)

[Out] d*log(3*x**4 + 2)/12
```

Mathematica [A] time = 0.00477415, size = 13, normalized size = 1.

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x^3)/(2 + 3*x^4),x]

[Out] (d*Log[2 + 3*x^4])/12
```

Maple [A] time = 0.002, size = 12, normalized size = 0.9

$$\frac{d\ln\left(3\,x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d*x^3/(3*x^4+2),x)

[Out] 1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.396, size = 15, normalized size = 1.15

$$\frac{1}{12} d \log \left(3 x^{4} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d*x^3/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/12*d*log(3*x^4 + 2)
```

Fricas [A] time = 0.217436, size = 15, normalized size = 1.15

$$\frac{1}{12} d \log \left(3 x^{4} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d*x^3/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] 1/12*d*log(3*x^4 + 2)
```

Sympy [A] time = 0.08263, size = 10, normalized size = 0.77

$$\frac{d \log \left(3x^{4} + 2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d*x**3/(3*x**4+2),x)
```

```
[Out] d*log(3*x**4 + 2)/12
```

GIAC/XCAS [A] time = 0.216708, size = 15, normalized size = 1.15

$$\frac{1}{12} d \ln \left(3 x^{4} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d*x^3/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/12*d*ln(3*x^4 + 2)
```

3.160 $\int \frac{a+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=132

$$-\frac{a\log\left(\sqrt{3}x^2-2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{3}x^2+2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x]
)/(4*6^(1/4)) - (a*Log[Sqrt[2] - 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2]
)/(8*6^(1/4)) + (a*Log[Sqrt[2] + 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2]
)/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.208991, antiderivative size = 114, normalized size of antiderivative = 0.86, number of steps used = 12, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x]
)/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
+ (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3
*x^4])/12
```

Rubi in Sympy [A] time = 23.7964, size = 102, normalized size = 0.77

$$-\frac{6^{\frac{3}{4}}a\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{6^{\frac{3}{4}}a\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{6^{\frac{3}{4}}a\,\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{6^{\frac{3}{4}}a\,\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{24}+\frac{d\log\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+a)/(3*x**4+2),x)
```

```
[Out] -6**(3/4)*a*log(3*x**2 - 6**(3/4)*x + sqrt(6))/48 + 6**(3/4)*a*lo
g(3*x**2 + 6**(3/4)*x + sqrt(6))/48 + 6**(3/4)*a*atan(6**(1/4)*x
- 1)/24 + 6**(3/4)*a*atan(6**(1/4)*x + 1)/24 + d*log(3*x**4 + 2)/
12
```

Mathematica [A] time = 0.0610137, size = 108, normalized size = 0.82

$$\frac{1}{48}\left(-6^{3/4}a\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+6^{3/4}a\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\,6^{3/4}a\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\,6^{3/4}a\tan^{-1}\left(\sqrt[4]{6}x+1\right)+4d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(3/4)*a*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*a*ArcTan[1 + 6^(1
/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a
*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [A] time = 0.004, size = 125, normalized size = 1.

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(1\left(x^{2}+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^{2}-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{d\ln\left(3x^{4}+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^
(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1
/3*6^(1/2)))+1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.54513, size = 201, normalized size = 1.52

$$\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3a\right)\log\left(\sqrt{3}x^{2}+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^{2}-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + a)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x +
3^(1/4)*2^(3/4))) + 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(
1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/144*3^(3/4)*2^(3/4)*(2*
3^(1/4)*2^(1/4)*d + 3*a)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sq
rt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d - 3*a)*log(sq
rt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [A] time = 0.246206, size = 389, normalized size = 2.95

$$\frac{1}{576}\cdot 24^{\frac{3}{4}}\left(\left(2\cdot 24^{\frac{1}{4}}d+3\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}\right)\log\left(2\sqrt{6}a^{2}x^{2}+2\cdot 24^{\frac{1}{4}}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+4\sqrt{a^{4}}\right)+\left(2\cdot 24^{\frac{1}{4}}d-3\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}\right)\log\left(2\sqrt{6}a^{2}x^{2}-2\cdot 24\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + a)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] 1/576*24^(3/4)*((2*24^(1/4)*d + 3*sqrt(2)*(a^4)^(1/4))*log(2*sqrt
(6)*a^2*x^2 + 2*24^(1/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(a^4)) +
 (2*24^(1/4)*d - 3*sqrt(2)*(a^4)^(1/4))*log(2*sqrt(6)*a^2*x^2 - 2
*24^(1/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(a^4)) - 12*sqrt(2)*(a^
4)^(1/4)*arctan(sqrt(2)*(a^4)^(1/4)/(24^(1/4)*a*x + 24^(1/4)*sqrt
(1/6)*a*sqrt(sqrt(6)*(sqrt(6)*a^2*x^2 + 24^(1/4)*sqrt(2)*(a^4)^(1
/4)*a*x + 2*sqrt(a^4))/a^2) + sqrt(2)*(a^4)^(1/4))) - 12*sqrt(2)*
(a^4)^(1/4)*arctan(sqrt(2)*(a^4)^(1/4)/(24^(1/4)*a*x + 24^(1/4)*s
qrt(1/6)*a*sqrt(sqrt(6)*(sqrt(6)*a^2*x^2 - 24^(1/4)*sqrt(2)*(a^4)
^(1/4)*a*x + 2*sqrt(a^4))/a^2) - sqrt(2)*(a^4)^(1/4))))
```

Sympy [A] time = 0.503587, size = 51, normalized size = 0.39

$$\operatorname{RootSum}\left(165888t^4 - 55296t^3d + 6912t^2d^2 - 384td^3 + 27a^4 + 8d^4, \left(t \mapsto t\log\left(x + \frac{24t - 2d}{3a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + 6912*_t**2*d**2 - 384*_t*d
**3 + 27*a**4 + 8*d**4, Lambda(_t, _t*log(x + (24*_t - 2*d)/(3*a)
)))
```

GIAC/XCAS [A] time = 0.223524, size = 147, normalized size = 1.11

$$\frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a + 4\,d\right)\ln\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 4\,d\right)\ln\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + a)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3
)^(1/4))) + 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x -
sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a + 4*d)*ln(x^2 + sqrt(2)*(
2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 4*d)*ln(x^2 - sqrt(
2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.161 $\int \frac{bx+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=36

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12} d \log\left(3x^4+2\right)$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) + (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.0744684, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12} d \log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) + (d*Log[2 + 3*x^4])/12
```

Rubi in Sympy [A] time = 9.55335, size = 31, normalized size = 0.86

$$\frac{\sqrt{6} b \operatorname{atan}\left(\frac{\sqrt{6} x^2}{2}\right)}{12} + \frac{d \log\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+b*x)/(3*x**4+2),x)
```

```
[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12 + d*log(3*x**4 + 2)/12
```

Mathematica [C] time = 0.0570111, size = 65, normalized size = 1.81

$$\frac{1}{24}\left(2d+i\sqrt{6}b\right)\log\left(\sqrt{6}-3ix^2\right)+\frac{1}{24}\left(2d-i\sqrt{6}b\right)\log\left(\sqrt{6}+3ix^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] ((I*Sqrt[6]*b + 2*d)*Log[Sqrt[6] - (3*I)*x^2])/24 + (((-I)*Sqrt[6
]*b + 2*d)*Log[Sqrt[6] + (3*I)*x^2])/24
```

Maple [A] time = 0.004, size = 28, normalized size = 0.8

$$\frac{d \ln\left(3x^4+2\right)}{12} + \frac{b\sqrt{6}}{12} \arctan\left(\frac{x^2\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+b*x)/(3*x^4+2),x)
```

```
[Out] 1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.52483, size = 153, normalized size = 4.25

$$-\frac{1}{12}\sqrt{3}\sqrt{2}b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\sqrt{3}\sqrt{2}b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}d\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{12}d\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x +
 3^(1/4)*2^(3/4))) + 1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^
(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/12*d*log(sqrt(3)*x^2 +
 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/12*d*log(sqrt(3)*x^2 - 3^(1/4)*
2^(3/4)*x + sqrt(2))
```

Fricas [A] time = 0.228033, size = 42, normalized size = 1.17

$$\frac{1}{72}\sqrt{6}\left(\sqrt{6}d\log\left(3x^4+2\right)+6b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(6)*(sqrt(6)*d*log(3*x^4 + 2) + 6*b*arctan(1/2*sqrt(6)*x
^2))
```

Sympy [A] time = 0.497149, size = 53, normalized size = 1.47

$$\left(-\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2-\frac{\sqrt{6}i}{3}\right)+\left(\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2+\frac{\sqrt{6}i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+b*x)/(3*x**4+2),x)
```

```
[Out] (-sqrt(6)*I*b/24 + d/12)*log(x**2 - sqrt(6)*I/3) + (sqrt(6)*I*b/2
4 + d/12)*log(x**2 + sqrt(6)*I/3)
```

GIAC/XCAS [A] time = 0.221105, size = 126, normalized size = 3.5

$$-\frac{1}{12}\sqrt{6}b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\sqrt{6}b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}d\ln\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+\frac{1}{12}d\ln\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/
3)^(1/4))) + 1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x -
 sqrt(2)*(2/3)^(1/4))) + 1/12*d*ln(x^2 + sqrt(2)*(2/3)^(1/4)*x +
sqrt(2/3)) + 1/12*d*ln(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.162 $\int \frac{a+bx+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=154

$$-\frac{a \log \left(\sqrt{3}x^2-2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{8\sqrt[4]{6}}+\frac{a \log \left(\sqrt{3}x^2+2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{8\sqrt[4]{6}}-\frac{a \tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a \tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d \log \left(3x^4+2\right)$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])
/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqr
t[2] - 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2])/(8*6^(1/4)) + (a*Log[Sqr
t[2] + 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2])/(8*6^(1/4)) + (d*Log[2 +
 3*x^4])/12
```

Rubi [A] time = 0.262121, antiderivative size = 136, normalized size of antiderivative = 0.88, number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{a \log \left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a \log \left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a \tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a \tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d \log \left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])
/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqr
t[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)
*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi in Sympy [A] time = 31.1556, size = 122, normalized size = 0.79

$$-\frac{6^{\frac{3}{4}} a \log \left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{6^{\frac{3}{4}} a \log \left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{6^{\frac{3}{4}} a \operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{6^{\frac{3}{4}} a \operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{24}+\frac{\sqrt{6} b \operatorname{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}+\frac{d \log \left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+b*x+a)/(3*x**4+2),x)
```

```
[Out] -6**(3/4)*a*log(3*x**2 - 6**(3/4)*x + sqrt(6))/48 + 6**(3/4)*a*lo
g(3*x**2 + 6**(3/4)*x + sqrt(6))/48 + 6**(3/4)*a*atan(6**(1/4)*x
- 1)/24 + 6**(3/4)*a*atan(6**(1/4)*x + 1)/24 + sqrt(6)*b*atan(sqr
t(6)*x**2/2)/12 + d*log(3*x**4 + 2)/12
```

Mathematica [A] time = 0.130159, size = 128, normalized size = 0.83

$$\frac{1}{48}\left(-2\sqrt{6}\left(\sqrt[4]{6}a+2b\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt{6}\left(\sqrt[4]{6}a-2b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)-6^{3/4}a \log \left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+6^{3/4}a \log \left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)+4d \log \left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*Sqrt[6]*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*Sqrt[6]*(
6^(1/4)*a - 2*b)*ArcTan[1 + 6^(1/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1
/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^
2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [A]   time = 0.003, size = 140, normalized size = 0.9

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)$$
$$+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(1\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)$$
$$+\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)+\frac{d\ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((d*x^3+b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^
(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1
/3*6^(1/2)))+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/12*d*ln(3*x
^4+2)
```

Maxima [A]   time = 1.55219, size = 231, normalized size = 1.5

$$\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3\,a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}$$
$$\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$
$$+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$
$$+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^3 + b*x + a)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(sqrt(3)*x^2
 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4
)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
 + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3
/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(3)*(3^(1
/4)*2^(3/4)*a + 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3
)*x - 3^(1/4)*2^(3/4)))
```

Fricas [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x + a)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.2686, size = 199, normalized size = 1.29

$$\operatorname{RootSum}\left(165888t^4 - 55296t^3d + t^2\left(3456b^2 + 6912d^2\right) + t\left(-864a^2b - 576b^2d - 384d^3\right) + 27a^4 + 72a^2bd + 18b^4 + 24b^2d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(3456*b**2 + 6912*d*
*2) + _t*(-864*a**2*b - 576*b**2*d - 384*d**3) + 27*a**4 + 72*a**
2*b*d + 18*b**4 + 24*b**2*d**2 + 8*d**4, Lambda(_t, _t*log(x + (2
7648*_t**3*b**2 + 1728*_t**2*a**2*b - 6912*_t**2*b**2*d + 216*_t*
a**4 - 288*_t*a**2*b*d + 288*_t*b**4 + 576*_t*b**2*d**2 - 18*a**4
*d - 90*a**2*b**3 + 12*a**2*b*d**2 - 24*b**4*d - 16*b**2*d**3)/(2
7*a**5 - 72*a*b**4))))
```

GIAC/XCAS [A] time = 0.225803, size = 169, normalized size = 1.1

$$\begin{aligned}&\frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\&+\frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\&+\frac{1}{48}\left(6^{\frac{3}{4}}a + 4d\right)\ln\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 4d\right)\ln\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x + a)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*
x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan
(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(
3/4)*a + 4*d)*ln(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*
(6^(3/4)*a - 4*d)*ln(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.163 $\int \frac{cx^2+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=132

$$\frac{c\log\left(\sqrt{3}x^2-2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{3}x^2+2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x]
)/(2*6^(3/4)) + (c*Log[Sqrt[2] - 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2]
)/(4*6^(3/4)) - (c*Log[Sqrt[2] + 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2]
)/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.23999, antiderivative size = 114, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x]
)/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4))
- (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3
*x^4])/12
```

Rubi in Sympy [A] time = 28.3706, size = 102, normalized size = 0.77

$$\frac{\sqrt[4]{6}c\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{24}-\frac{\sqrt[4]{6}c\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{24}+\frac{\sqrt[4]{6}c\,\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}c\,\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{12}+\frac{d\log\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c*x**2)/(3*x**4+2),x)
```

```
[Out] 6**(1/4)*c*log(3*x**2 - 6**(3/4)*x + sqrt(6))/24 - 6**(1/4)*c*log
(3*x**2 + 6**(3/4)*x + sqrt(6))/24 + 6**(1/4)*c*atan(6**(1/4)*x -
 1)/12 + 6**(1/4)*c*atan(6**(1/4)*x + 1)/12 + d*log(3*x**4 + 2)/1
2
```

Mathematica [A] time = 0.0488684, size = 108, normalized size = 0.82

$$\frac{1}{24}\left(\sqrt[4]{6}c\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\sqrt[4]{6}c\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\sqrt[4]{6}c\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}c\tan^{-1}\left(\sqrt[4]{6}x+1\right)+2d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*c*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*c*ArcTan[1 + 6^(1
/4)*x] + 6^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(1/4)*c
*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

Maple [A] time = 0.003, size = 125, normalized size = 1.

$$\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(1\left(x^{2}-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^{2}+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{d\ln\left(3\,x^{4}+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((d*x^3+c*x^2)/(3*x^4+2),x)
```

```
[Out] 1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6
^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+
1/3*6^(1/2)))+1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.53629, size = 205, normalized size = 1.55

$$\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}c\right)\log\left(\sqrt{3}x^{2}+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}c\right)\log\left(\sqrt{3}x^{2}-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^3 + c*x^2)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*
x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4
)*2^(3/4)*d + sqrt(3)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sq
rt(2)) + 1/12*3^(1/4)*2^(1/4)*c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqr
t(3)*x + 3^(1/4)*2^(3/4))) + 1/12*3^(1/4)*2^(1/4)*c*arctan(1/6*3^
(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [A] time = 0.2405, size = 440, normalized size = 3.33

$$\frac{1}{1296}\cdot 54^{\frac{3}{4}}\left(\left(2\cdot 54^{\frac{1}{4}}d-3\sqrt{2}\left(c^{4}\right)^{\frac{1}{4}}\right)\log\left(3\sqrt{6}c^{3}x^{2}+54^{\frac{1}{4}}\sqrt{6}\sqrt{2}\left(c^{4}\right)^{\frac{3}{4}}x+6\sqrt{c^{4}}c\right)+\left(2\cdot 54^{\frac{1}{4}}d+3\sqrt{2}\left(c^{4}\right)^{\frac{1}{4}}\right)\log\left(3\sqrt{6}c^{3}x^{2}-54^{\frac{1}{4}}\sqrt{6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^3 + c*x^2)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] 1/1296*54^(3/4)*((2*54^(1/4)*d - 3*sqrt(2)*(c^4)^(1/4))*log(3*sqr
t(6)*c^3*x^2 + 54^(1/4)*sqrt(6)*sqrt(2)*(c^4)^(3/4)*x + 6*sqrt(c^
```

```
4)*c) + (2*54^(1/4)*d + 3*sqrt(2)*(c^4)^(1/4))*log(3*sqrt(6)*c^3*
x^2 - 54^(1/4)*sqrt(6)*sqrt(2)*(c^4)^(3/4)*x + 6*sqrt(c^4)*c) - 1
2*sqrt(2)*(c^4)^(1/4)*arctan(9*sqrt(2)*(c^4)^(3/4)/(3*54^(1/4)*sq
rt(6)*c^3*x + 54^(1/4)*sqrt(6)*sqrt(1/2)*c^3*sqrt(sqrt(6)*(3*sqrt
(6)*c^3*x^2 + 54^(1/4)*sqrt(6)*sqrt(2)*(c^4)^(3/4)*x + 6*sqrt(c^4
)*c)/c^3) + 9*sqrt(2)*(c^4)^(3/4))) - 12*sqrt(2)*(c^4)^(1/4)*arct
an(9*sqrt(2)*(c^4)^(3/4)/(3*54^(1/4)*sqrt(6)*c^3*x + 54^(1/4)*sqr
t(6)*sqrt(1/2)*c^3*sqrt(sqrt(6)*(3*sqrt(6)*c^3*x^2 - 54^(1/4)*sqr
t(6)*sqrt(2)*(c^4)^(3/4)*x + 6*sqrt(c^4)*c)/c^3) - 9*sqrt(2)*(c^4
)^(3/4))))
```

Sympy [A] time = 0.428352, size = 70, normalized size = 0.53

$$\operatorname{RootSum}\left(41472t^4 - 13824t^3d + 1728t^2d^2 - 96td^3 + 3c^4 + 2d^4, \left(t \mapsto t\log\left(x + \frac{3456t^3 - 864t^2d + 72td^2 - 2d^3}{3c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c*x**2)/(3*x**4+2),x)
```

```
[Out] RootSum(41472*_t**4 - 13824*_t**3*d + 1728*_t**2*d**2 - 96*_t*d**
3 + 3*c**4 + 2*d**4, Lambda(_t, _t*log(x + (3456*_t**3 - 864*_t**
2*d + 72*_t*d**2 - 2*d**3)/(3*c**3))))
```

GIAC/XCAS [A] time = 0.223075, size = 147, normalized size = 1.11

$$\frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\left(6^{\frac{1}{4}}c - 2d\right)\ln\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{1}{4}}c + 2d\right)\ln\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3
)^(1/4))) + 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x -
sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4)*c - 2*d)*ln(x^2 + sqrt(2)*(
2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(1/4)*c + 2*d)*ln(x^2 - sqrt(
2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.164 $\int \frac{a+cx^2+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=154

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\,6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6
]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c
)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2
*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*
x^4])/12
```

Rubi [A] time = 0.283733, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\,6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6
]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c
)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2
*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*
x^4])/12
```

Rubi in Sympy [A] time = 30.2491, size = 136, normalized size = 0.88

$$\frac{d\log\left(3x^4+2\right)}{12}+\frac{\sqrt[4]{6}\left(-\sqrt{6}a+2c\right)\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}-\frac{\sqrt[4]{6}\left(-\sqrt{6}a+2c\right)\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c*x**2+a)/(3*x**4+2),x)
```

```
[Out] d*log(3*x**4 + 2)/12 + 6**(1/4)*(-sqrt(6)*a + 2*c)*log(3*x**2 - 6
**(3/4)*x + sqrt(6))/48 - 6**(1/4)*(-sqrt(6)*a + 2*c)*log(3*x**2
+ 6**(3/4)*x + sqrt(6))/48 + 6**(1/4)*(sqrt(6)*a + 2*c)*atan(6**(
1/4)*x - 1)/24 + 6**(1/4)*(sqrt(6)*a + 2*c)*atan(6**(1/4)*x + 1)/
24
```

Mathematica [A] time = 0.275876, size = 148, normalized size = 0.96

$$\frac{1}{48}\left(-\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)+4d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(
Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]*a - 2*c
)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*
Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [B] time = 0.003, size = 237, normalized size = 1.5

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)$$
$$+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(1\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)$$
$$+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)$$
$$+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(1\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{d\ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c*x^2+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^
(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1
/3*6^(1/2)))+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^
(1/2)*6^(3/4)*x+1)+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1
/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-
1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/
4)*x*2^(1/2)+1/3*6^(1/2)))+1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.55211, size = 263, normalized size = 1.71

$$-\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$
$$+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$
$$+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$
$$+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + a)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d -
 3*a)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3
/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(s
qrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/
4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2
*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*
a + 2*3^(3/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x
- 3^(1/4)*2^(3/4)))
```

Fricas [A] time = 0.412381, size = 4869, normalized size = 31.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + a)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*((sqrt(2)*(2*216^(1/4)*sqrt(6)*a*c*d - 216^(1/4)*sqr
t(9*a^4 + 12*a^2*c^2 + 4*c^4)*d)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4
 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/(9*a^4 + 36*a^
2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c))
+ 3*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(2*sqrt(6)*a*c - sqrt(9*a^
4 + 12*a^2*c^2 + 4*c^4)))*log(-6*sqrt(6)*(243*a^10 + 2754*a^8*c^2
 + 4968*a^6*c^4 + 3312*a^4*c^6 + 816*a^2*c^8 + 32*c^10)*x^2 + 216
*(27*a^7*c + 78*a^5*c^3 + 52*a^3*c^5 + 8*a*c^7)*sqrt(9*a^4 + 12*a
^2*c^2 + 4*c^4)*x^2 + 2*sqrt(2)*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4
)*(216^(1/4)*sqrt(6)*(189*a^6*c + 630*a^4*c^3 + 252*a^2*c^5 + 8*c
^7)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*x - 3*216^(1/4)*(81*a^9 + 11
88*a^7*c^2 + 2016*a^5*c^4 + 1008*a^3*c^6 + 112*a*c^8)*x)*sqrt((9*
a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*
c^4)*a*c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12
*a^2*c^2 + 4*c^4)*a*c)) - 12*(81*a^8 + 864*a^6*c^2 + 1080*a^4*c^4
 + 384*a^2*c^6 + 16*c^8 - 6*sqrt(6)*(9*a^5*c + 20*a^3*c^3 + 4*a*c
^5)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))*sqrt(9*a^4 + 12*a^2*c^2 + 4
*c^4)) + (sqrt(2)*(2*216^(1/4)*sqrt(6)*a*c*d - 216^(1/4)*sqrt(9*a
^4 + 12*a^2*c^2 + 4*c^4)*d)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*
sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/(9*a^4 + 36*a^2*c^2
 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)) - 3*(
9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(2*sqrt(6)*a*c - sqrt(9*a^4 + 1
2*a^2*c^2 + 4*c^4)))*log(-6*sqrt(6)*(243*a^10 + 2754*a^8*c^2 + 49
68*a^6*c^4 + 3312*a^4*c^6 + 816*a^2*c^8 + 32*c^10)*x^2 + 216*(27*
a^7*c + 78*a^5*c^3 + 52*a^3*c^5 + 8*a*c^7)*sqrt(9*a^4 + 12*a^2*c^
2 + 4*c^4)*x^2 - 2*sqrt(2)*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(21
6^(1/4)*sqrt(6)*(189*a^6*c + 630*a^4*c^3 + 252*a^2*c^5 + 8*c^7)*s
qrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*x - 3*216^(1/4)*(81*a^9 + 1188*a^
7*c^2 + 2016*a^5*c^4 + 1008*a^3*c^6 + 112*a*c^8)*x)*sqrt((9*a^4 +
 12*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*
a*c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*
c^2 + 4*c^4)*a*c)) - 12*(81*a^8 + 864*a^6*c^2 + 1080*a^4*c^4 + 38
4*a^2*c^6 + 16*c^8 - 6*sqrt(6)*(9*a^5*c + 20*a^3*c^3 + 4*a*c^5)*s
qrt(9*a^4 + 12*a^2*c^2 + 4*c^4))*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)
) + 12*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*sqrt(9*a^4 - 12*a^2*c^2
 + 4*c^4)*arctan(6*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(sqrt(6)*sq
rt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^3 + 2*a*c^2) - 2*sqrt(9*a^4 +
 12*a^2*c^2 + 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*c)/(sqrt(2)
*sqrt(1/3)*(216^(1/4)*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*(9
*a^4 - 4*c^4) - 12*216^(1/4)*(9*a^5*c - 4*a*c^5))*sqrt((9*a^4 + 1
2*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*
c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^
2 + 4*c^4)*a*c))*sqrt((3*sqrt(6)*(243*a^10 + 2754*a^8*c^2 + 4968*
a^6*c^4 + 3312*a^4*c^6 + 816*a^2*c^8 + 32*c^10)*x^2 - 108*(27*a^7
*c + 78*a^5*c^3 + 52*a^3*c^5 + 8*a*c^7)*sqrt(9*a^4 + 12*a^2*c^2 +
 4*c^4)*x^2 + sqrt(2)*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(216^(1/
4)*sqrt(6)*(189*a^6*c + 630*a^4*c^3 + 252*a^2*c^5 + 8*c^7)*sqrt(9
*a^4 + 12*a^2*c^2 + 4*c^4)*x - 3*216^(1/4)*(81*a^9 + 1188*a^7*c^2
 + 2016*a^5*c^4 + 1008*a^3*c^6 + 112*a*c^8)*x)*sqrt((9*a^4 + 12*a
^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/
(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 +
 4*c^4)*a*c)) + 6*(81*a^8 + 864*a^6*c^2 + 1080*a^4*c^4 + 384*a^2*
c^6 + 16*c^8 - 6*sqrt(6)*(9*a^5*c + 20*a^3*c^3 + 4*a*c^5)*sqrt(9*
a^4 + 12*a^2*c^2 + 4*c^4))*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))/(sqr
t(6)*(243*a^10 + 2754*a^8*c^2 + 4968*a^6*c^4 + 3312*a^4*c^6 + 816
*a^2*c^8 + 32*c^10) - 36*(27*a^7*c + 78*a^5*c^3 + 52*a^3*c^5 + 8*
a*c^7)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))) + sqrt(2)*(216^(1/4)*sq
rt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*(9*a^4 - 4*c^4)*x - 12*216
^(1/4)*(9*a^5*c - 4*a*c^5)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 -
2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/(9*a^4 + 36*a^2*c
^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)) + 6
*(18*a^4*c - 8*c^5 - sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*(3*
a^3 - 2*a*c^2))*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4))) + 12*(9*a^4
```

```
+ 12*a^2*c^2 + 4*c^4)^(1/4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*arct
an(6*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(sqrt(6)*sqrt(9*a^4 - 12*
a^2*c^2 + 4*c^4)*(3*a^3 + 2*a*c^2) - 2*sqrt(9*a^4 + 12*a^2*c^2 +
4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*c)/(sqrt(2)*sqrt(1/3)*(21
6^(1/4)*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*(9*a^4 - 4*c^4)
- 12*216^(1/4)*(9*a^5*c - 4*a*c^5))*sqrt((9*a^4 + 12*a^2*c^2 + 4*
c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/(9*a^4 + 36
*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c
))*sqrt((3*sqrt(6)*(243*a^10 + 2754*a^8*c^2 + 4968*a^6*c^4 + 3312
*a^4*c^6 + 816*a^2*c^8 + 32*c^10)*x^2 - 108*(27*a^7*c + 78*a^5*c^
3 + 52*a^3*c^5 + 8*a*c^7)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 -
sqrt(2)*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4)*(216^(1/4)*sqrt(6)*(18
9*a^6*c + 630*a^4*c^3 + 252*a^2*c^5 + 8*c^7)*sqrt(9*a^4 + 12*a^2*
c^2 + 4*c^4)*x - 3*216^(1/4)*(81*a^9 + 1188*a^7*c^2 + 2016*a^5*c^
4 + 1008*a^3*c^6 + 112*a*c^8)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4
 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/(9*a^4 + 36*a^
2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c))
+ 6*(81*a^8 + 864*a^6*c^2 + 1080*a^4*c^4 + 384*a^2*c^6 + 16*c^8 -
 6*sqrt(6)*(9*a^5*c + 20*a^3*c^3 + 4*a*c^5)*sqrt(9*a^4 + 12*a^2*c
^2 + 4*c^4))*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))/(sqrt(6)*(243*a^10
 + 2754*a^8*c^2 + 4968*a^6*c^4 + 3312*a^4*c^6 + 816*a^2*c^8 + 32*
c^10) - 36*(27*a^7*c + 78*a^5*c^3 + 52*a^3*c^5 + 8*a*c^7)*sqrt(9*
a^4 + 12*a^2*c^2 + 4*c^4))) + sqrt(2)*(216^(1/4)*sqrt(6)*sqrt(9*a
^4 + 12*a^2*c^2 + 4*c^4)*(9*a^4 - 4*c^4)*x - 12*216^(1/4)*(9*a^5*
c - 4*a*c^5)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt
(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4
*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)) - 6*(18*a^4*c - 8
*c^5 - sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*(3*a^3 - 2*a*c^2)
)*(9*a^4 + 12*a^2*c^2 + 4*c^4)^(1/4))))/((2*216^(1/4)*sqrt(6)*a*c
 - 216^(1/4)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4))*sqrt((9*a^4 + 12*a
^2*c^2 + 4*c^4 - 2*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 + 4*c^4)*a*c)/
(9*a^4 + 36*a^2*c^2 + 4*c^4 - 4*sqrt(6)*sqrt(9*a^4 + 12*a^2*c^2 +
 4*c^4)*a*c)))
```

Sympy [A] time = 2.01234, size = 148, normalized size = 0.96

$$\operatorname{RootSum}\left(165888t^4 - 55296t^3d + t^2\left(6912ac + 6912d^2\right) + t\left(-1152acd - 384d^3\right) + 27a^4 + 36a^2c^2 + 48acd^2 + 12c^4 + 8d^4, \left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x**3+c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 6912*d**
2) + _t*(-1152*a*c*d - 384*d**3) + 27*a**4 + 36*a**2*c**2 + 48*a*
c*d**2 + 12*c**4 + 8*d**4, Lambda(_t, _t*log(x + (-13824*_t**3*c
+ 3456*_t**2*c*d + 216*_t*a**3 - 432*_t*a*c**2 - 288*_t*c*d**2 -
18*a**3*d + 36*a*c**2*d + 8*c*d**3)/(27*a**4 - 12*c**4))))
```

GIAC/XCAS [A] time = 0.226449, size = 185, normalized size = 1.2

$$\begin{aligned}&\frac{1}{24}\left(6^{\frac{3}{4}}a + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\&+\frac{1}{24}\left(6^{\frac{3}{4}}a + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\&+\frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c + 4d\right)\ln\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)\\&-\frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c - 4d\right)\ln\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + a)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*
x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan
(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(
3/4)*a - 2*6^(1/4)*c + 4*d)*ln(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt
(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c - 4*d)*ln(x^2 - sqrt(2)*(2
/3)^(1/4)*x + sqrt(2/3))
```

3.165 $\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=154

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(\sqrt{3}x^2-2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{4\,6^{3/4}}-\frac{c\log\left(\sqrt{3}x^2+2^{3/4}\sqrt[4]{3}x+\sqrt{2}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\,6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])
/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqr
t[2] - 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2])/(4*6^(3/4)) - (c*Log[Sqr
t[2] + 2^(3/4)*3^(1/4)*x + Sqrt[3]*x^2])/(4*6^(3/4)) + (d*Log[2 +
 3*x^4])/12
```

Rubi [A] time = 0.323716, antiderivative size = 136, normalized size of antiderivative = 0.88, number of steps used = 16, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\,6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])
/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqr
t[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)
*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi in Sympy [A] time = 36.8169, size = 122, normalized size = 0.79

$$\frac{\sqrt{6}b\,\mathrm{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}+\frac{\sqrt[4]{6}c\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{24}-\frac{\sqrt[4]{6}c\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{24}+\frac{\sqrt[4]{6}c\,\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}c\,\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{12}+\frac{d\log\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12 + 6**(1/4)*c*log(3*x**2 - 6**(3
/4)*x + sqrt(6))/24 - 6**(1/4)*c*log(3*x**2 + 6**(3/4)*x + sqrt(6
))/24 + 6**(1/4)*c*atan(6**(1/4)*x - 1)/12 + 6**(1/4)*c*atan(6**(
1/4)*x + 1)/12 + d*log(3*x**4 + 2)/12
```

Mathematica [A] time = 0.126703, size = 125, normalized size = 0.81

$$\frac{1}{24}\left(-2\sqrt[4]{6}\left(\sqrt[4]{6}b+c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}\left(c-\sqrt[4]{6}b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)+\sqrt[4]{6}c\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\sqrt[4]{6}c\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)+2d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(-(
6^(1/4)*b) + c)*ArcTan[1 + 6^(1/4)*x] + 6^(1/4)*c*Log[2 - 2*6^(1/
4)*x + Sqrt[6]*x^2] - 6^(1/4)*c*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2
] + 2*d*Log[2 + 3*x^4])/24
```

Maple [A] time = 0.005, size = 140, normalized size = 0.9

$$\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(1\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{d\ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c*x^2+b*x)/(3*x^4+2),x)
```

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/72*c*3^(1/2)*6^(3/4)*2^(
1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*c*3^(1/2)*6^(3/
4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*3^(1/2
)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/
2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/12*d*ln(3*
x^4+2)
```

Maxima [A] time = 1.61724, size = 235, normalized size = 1.53

$$\frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c-6b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c+6b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c - 6*b)*arctan(1/6*3^(3/4)
*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*sqrt(2)*
(3^(3/4)*2^(3/4)*c + 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x
 - 3^(1/4)*2^(3/4))) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d -
sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/72*
3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3)*c)*log(sqrt(3)*x^2 -
 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.24829, size = 189, normalized size = 1.23

$$\operatorname{RootSum}\left(82944t^4 - 27648t^3d + t^2\left(1728b^2 + 3456d^2\right) + t\left(-288b^2d + 288bc^2 - 192d^3\right) + 9b^4 + 12b^2d^2 - 24bc^2d + 6c^4 + 4a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] RootSum(82944*_t**4 - 27648*_t**3*d + _t**2*(1728*b**2 + 3456*d**
2) + _t*(-288*b**2*d + 288*b*c**2 - 192*d**3) + 9*b**4 + 12*b**2*
d**2 - 24*b*c**2*d + 6*c**4 + 4*d**4, Lambda(_t, _t*log(x + (-345
6*_t**3*c**2 + 864*_t**2*b**3 + 864*_t**2*c**2*d - 144*_t*b**3*d
- 108*_t*b**2*c**2 - 72*_t*c**2*d**2 + 9*b**5 + 6*b**3*d**2 + 9*b
**2*c**2*d - 9*b*c**4 + 2*c**2*d**3)/(18*b**4*c - 3*c**5))))
```

GIAC/XCAS [A] time = 0.225389, size = 167, normalized size = 1.08

$$-\frac{1}{12}\left(\sqrt{6}b - 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\left(\sqrt{6}b + 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\left(6^{\frac{1}{4}}c - 2d\right)\ln\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{1}{4}}c + 2d\right)\ln\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x
 + sqrt(2)*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/
4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4
)*c - 2*d)*ln(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^
(1/4)*c + 2*d)*ln(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.166 $\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan
[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1
/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x
+ 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*
x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.339016, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan
[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1
/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x
+ 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*
x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi in Sympy [A] time = 38.9786, size = 156, normalized size = 0.89

$$\frac{\sqrt{6}b\,\mathrm{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}+\frac{d\log\left(3x^4+2\right)}{12}+\frac{\sqrt[4]{6}\left(-\sqrt{6}a+2c\right)\log\left(3x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}-\frac{\sqrt[4]{6}\left(-\sqrt{6}a+2c\right)\log\left(3x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)}{48}+\frac{\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12 + d*log(3*x**4 + 2)/12 + 6**(1/
4)*(-sqrt(6)*a + 2*c)*log(3*x**2 - 6**(3/4)*x + sqrt(6))/48 - 6**
(1/4)*(-sqrt(6)*a + 2*c)*log(3*x**2 + 6**(3/4)*x + sqrt(6))/48 +
6**(1/4)*(sqrt(6)*a + 2*c)*atan(6**(1/4)*x - 1)/24 + 6**(1/4)*(sq
rt(6)*a + 2*c)*atan(6**(1/4)*x + 1)/24
```

Mathematica [A] time = 0.259358, size = 164, normalized size = 0.93

$$\frac{1}{48}\left(-2\sqrt[4]{6}\tan^{-1}\left(1-\sqrt[4]{6}x\right)\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)+2\sqrt[4]{6}\tan^{-1}\left(\sqrt[4]{6}x+1\right)\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)\right.$$
$$\left.-\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)+4d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x]
 + 2*6^(1/4)*(Sqrt[6]*a - 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x
] - 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2]
+ 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] +
4*d*Log[2 + 3*x^4])/48
```

Maple [A] time = 0.004, size = 252, normalized size = 1.4

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)$$
$$+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(1\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)$$
$$+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}+1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)$$
$$+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(1\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{d\ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)
*x+1)+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6
^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^
(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1
/3*6^(1/2)))+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/72*c*3^(1/2
)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*c*
3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1
/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1
/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))
+1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.5612, size = 279, normalized size = 1.59

$$-\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$
$$+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$
$$+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c-6\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$
$$+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c+6\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x + a)/(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d -
 3*a)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3
/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(s
qrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/
4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c - 6*sqrt(2)*b)*arctan(1/6*3^(3
/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*(3*3^
(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c + 6*sqrt(2)*b)*arctan(1/6*3
^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c*x^2 + b*x + a)/(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 12.9611, size = 580, normalized size = 3.3

$$\operatorname{RootSum}\left(165888t^4 - 55296t^3d + t^2\left(6912ac + 3456b^2 + 6912d^2\right) + t\left(-864a^2b - 1152acd - 576b^2d + 576bc^2 - 384d^3\right) + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 3456*b**
2 + 6912*d**2) + _t*(-864*a**2*b - 1152*a*c*d - 576*b**2*d + 576*
b*c**2 - 384*d**3) + 27*a**4 + 72*a**2*b*d + 36*a**2*c**2 - 72*a*
b**2*c + 48*a*c*d**2 + 18*b**4 + 24*b**2*d**2 - 48*b*c**2*d + 12*
c**4 + 8*d**4, Lambda(_t, _t*log(x + (-41472*_t**3*a**2*c + 82944
*_t**3*a*b**2 + 27648*_t**3*c**3 + 5184*_t**2*a**3*b + 10368*_t**
2*a**2*c*d - 20736*_t**2*a*b**2*d + 10368*_t**2*a*b*c**2 - 6912*_
t**2*b**3*c - 6912*_t**2*c**3*d + 648*_t*a**5 - 864*_t*a**3*b*d -
 1728*_t*a**3*c**2 + 3888*_t*a**2*b**2*c - 864*_t*a**2*c*d**2 + 8
64*_t*a*b**4 + 1728*_t*a*b**2*d**2 - 1728*_t*a*b*c**2*d + 864*_t*
a*c**4 + 1152*_t*b**3*c*d + 864*_t*b**2*c**3 + 576*_t*c**3*d**2 -
 54*a**5*d + 270*a**4*b*c - 270*a**3*b**3 + 36*a**3*b*d**2 + 144*
a**3*c**2*d - 324*a**2*b**2*c*d + 24*a**2*c*d**3 - 72*a*b**4*d +
180*a*b**3*c**2 - 48*a*b**2*d**3 + 72*a*b*c**2*d**2 - 72*a*c**4*d
 - 72*b**5*c - 48*b**3*c*d**2 - 72*b**2*c**3*d + 72*b*c**5 - 16*c
**3*d**3)/(81*a**6 - 54*a**4*c**2 + 432*a**3*b**2*c - 216*a**2*b*
*4 - 36*a**2*c**4 + 288*a*b**2*c**3 - 144*b**4*c**2 + 24*c**6)))
```

GIAC/XCAS [A] time = 0.23005, size = 201, normalized size = 1.14

$$\begin{aligned}
&\frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) \\
&+ \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) \\
&+ \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c + 4d\right)\ln\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) \\
&- \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c - 4d\right)\ln\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x + a)/(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(
2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqr
t(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(
2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c + 4*d)*ln(x^2 +
sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*
c - 4*d)*ln(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.167 $\int \frac{1+x+x^2+x^3}{1-x^4} \, dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -Log[1 - x]
```

Rubi [A] time = 0.0143256, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)/(1 - x^4),x]

[Out] -Log[1 - x]
```

Rubi in Sympy [A] time = 3.65869, size = 5, normalized size = 0.62

$$-\log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+x+1)/(-x**4+1),x)

[Out] -log(-x + 1)
```

Mathematica [A] time = 0.00181174, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]

[Out] -Log[1 - x]
```

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+x+1)/(-x^4+1),x)

[Out] -ln(-1+x)
```

Maxima [A] time = 1.3778, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

Fricas [A] time = 0.214339, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A] time = 0.046167, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

```
[Out] -log(x - 1)
```

GIAC/XCAS [A] time = 0.210105, size = 9, normalized size = 1.12

$$-\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)/(x^4 - 1),x, algorithm="giac")
```

```
[Out] -ln(abs(x - 1))
```

3.168 $\int \frac{1+x+x^2+x^3}{1+x^4}\,dx$

Optimal. Leaf size=53

$$\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}$$

```
[Out] ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2
]*x]/Sqrt[2] + Log[1 + x^4]/4
```

Rubi [A] time = 0.111879, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471

$$\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2
]*x]/Sqrt[2] + Log[1 + x^4]/4
```

Rubi in Sympy [A] time = 16.6124, size = 48, normalized size = 0.91

$$\frac{\log\left(x^4+1\right)}{4}+\frac{\operatorname{atan}\left(x^2\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+x+1)/(x**4+1),x)
```

```
[Out] log(x**4 + 1)/4 + atan(x**2)/2 + sqrt(2)*atan(sqrt(2)*x - 1)/2 +
sqrt(2)*atan(sqrt(2)*x + 1)/2
```

Mathematica [A] time = 0.0642398, size = 50, normalized size = 0.94

$$\frac{1}{4}\left(\log\left(x^4+1\right)-2\left(1+\sqrt{2}\right)\tan^{-1}\left(1-\sqrt{2}x\right)+2\left(\sqrt{2}-1\right)\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] (-2*(1 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(-1 + Sqrt[2])*ArcTan
[1 + Sqrt[2]*x] + Log[1 + x^4])/4
```

Maple [B] time = 0.004, size = 102, normalized size = 1.9

$$\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{2}+\frac{\arctan\left(x\sqrt{2}-1\right)\sqrt{2}}{2}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)$$
$$+\frac{\arctan\left(x^2\right)}{2}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+\frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+x+1)/(x^4+1),x)
```

```
[Out] 1/2*arctan(1+x*2^(1/2))*2^(1/2)+1/2*arctan(x*2^(1/2)-1)*2^(1/2)+1
/8*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+1/2*arctan(x^2
)+1/8*2^(1/2)*ln((1+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))+1/4*ln(x^4+
1)
```

Maxima [A] time = 1.51933, size = 103, normalized size = 1.94

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$
$$+\frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) +
1/4*sqrt(2)*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1
/4*log(x^2 + sqrt(2)*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 0.241714, size = 217, normalized size = 4.09

$$-\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(3\sqrt{2}-4\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(3\sqrt{2}-4\right)}\left(\sqrt{2}+1\right)}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)$$
$$-\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(3\sqrt{2}+4\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(3\sqrt{2}+4\right)}\left(\sqrt{2}-1\right)}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)$$
$$+\frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] -sqrt(1/2)*sqrt(sqrt(2)*(3*sqrt(2) - 4))*arctan(sqrt(1/2)*sqrt(sq
rt(2)*(3*sqrt(2) - 4))*(sqrt(2) + 1)/(sqrt(2)*x + sqrt(2)*sqrt(x^
2 + sqrt(2)*x + 1) + 1)) - sqrt(1/2)*sqrt(sqrt(2)*(3*sqrt(2) + 4)
)*arctan(sqrt(1/2)*sqrt(sqrt(2)*(3*sqrt(2) + 4))*(sqrt(2) - 1)/(s
qrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) - 1)) + 1/4*log(x^2
+ sqrt(2)*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] time = 0.675776, size = 73, normalized size = 1.38

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4}+2\left(\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x-1\right)+2\left(-\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+1)/(x**4+1),x)
```

```
[Out] log(x**2 - sqrt(2)*x + 1)/4 + log(x**2 + sqrt(2)*x + 1)/4 + 2*(1/
4 + sqrt(2)/4)*atan(sqrt(2)*x - 1) + 2*(-1/4 + sqrt(2)/4)*atan(sq
rt(2)*x + 1)
```

GIAC/XCAS [A] time = 0.211986, size = 95, normalized size = 1.79

$$\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{4}\ln\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\ln\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)/(x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*(sqrt
(2) + 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*ln(x^2 + sqrt(
2)*x + 1) + 1/4*ln(x^2 - sqrt(2)*x + 1)
```

3.169 $\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx$

Optimal. Leaf size=124

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] -((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(
3/4)) + ((Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(
3/4)*b^(3/4)) + ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]
) - Log[a - b*x^4]/(4*b)
```

Rubi [A] time = 0.242426, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] -((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(
3/4)) + ((Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(
3/4)*b^(3/4)) + ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]
) - Log[a - b*x^4]/(4*b)
```

Rubi in Sympy [A] time = 30.3221, size = 109, normalized size = 0.88

$$-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+x+1)/(-b*x**4+a),x)
```

```
[Out] -log(a - b*x**4)/(4*b) + atanh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*s
qrt(b)) - (sqrt(a) - sqrt(b))*atan(b**(1/4)*x/a**(1/4))/(2*a**(3/
4)*b**(3/4)) + (sqrt(a) + sqrt(b))*atanh(b**(1/4)*x/a**(1/4))/(2*
a**(3/4)*b**(3/4))
```

Mathematica [A] time = 0.0919052, size = 203, normalized size = 1.64

$$-\frac{\left(a^{3/4}+\sqrt{a}\sqrt[4]{b}+\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{4ab^{3/4}}-\frac{\left(-a^{3/4}+\sqrt{a}\sqrt[4]{b}-\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4ab^{3/4}}+\frac{\left(\sqrt[4]{a}\sqrt{b}-a^{3/4}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}}-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] ((-a^(3/4) + a^(1/4)*Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b
^(3/4)) - ((a^(3/4) + Sqrt[a]*b^(1/4) + a^(1/4)*Sqrt[b])*Log[a^(1
/4) - b^(1/4)*x])/(4*a*b^(3/4)) - ((-a^(3/4) + Sqrt[a]*b^(1/4) -
a^(1/4)*Sqrt[b])*Log[a^(1/4) + b^(1/4)*x])/(4*a*b^(3/4)) + Log[Sq
rt[a] + Sqrt[b]*x^2]/(4*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)
```

Maple [B] time = 0.005, size = 171, normalized size = 1.4

$$\frac{1}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{1}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{1}{4}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{1}{2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{1}{4b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\ln\left(bx^4-a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+x+1)/(-b*x^4+a),x)
```

```
[Out] 1/4*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*(a/b)^(
1/4)/a*arctan(x/(a/b)^(1/4))-1/4/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/
2))/(-a-x^2*(a*b)^(1/2)))-1/2/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))
+1/4/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b*ln(b
*x^4-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)/(b*x^4 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)/(b*x^4 - a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.95533, size = 187, normalized size = 1.51

$$-\operatorname{RootSum}\left(256t^4a^3b^4-256t^3a^3b^3+t^2\left(96a^3b^2-96a^2b^3\right)+t\left(-16a^3b+32a^2b^2-16ab^3\right)+a^3-3a^2b+3ab^2-b^3,\left(t\mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+1)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a*
*3*b**2 - 96*a**2*b**3) + _t*(-16*a**3*b + 32*a**2*b**2 - 16*a*b*
*3) + a**3 - 3*a**2*b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (-
64*_t**3*a**3*b**3 + 48*_t**2*a**3*b**2 + 16*_t**2*a**2*b**3 - 12
*_t*a**3*b + 16*_t*a**2*b**2 - 4*_t*a*b**3 + a**3 - 2*a**2*b + a*
b**2)/(a**2*b - 2*a*b**2 + b**3))))
```

GIAC/XCAS [A] time = 0.217803, size = 392, normalized size = 3.16

$$-\frac{\ln\left(\left|bx^4-a\right|\right)}{4\,b}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\sqrt{2}\sqrt{-ab^3}b+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\sqrt{2}\sqrt{-ab^3}b+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)/(b*x^4 - a),x, algorithm="giac")
```

```
[Out] -1/4*ln(abs(b*x^4 - a))/b + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 - sqr
t(2)*sqrt(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + s
qrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3
)^(1/4)*b^2 + sqrt(2)*sqrt(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8
*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*ln(x^2 + sqrt(2)*x
*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)
*b^2 - (-a*b^3)^(3/4))*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/(a*b^3)
```

3.170 $\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$

Optimal. Leaf size=277

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\log\left(a+bx^4\right)}{4b}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + S
qrt[b])*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*b^(3/4)) + ((Sqrt[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + Log[a
 + b*x^4]/(4*b)
```

Rubi [A] time = 0.474812, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\log\left(a+bx^4\right)}{4b}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```



```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + S
qrt[b])*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*b^(3/4)) + ((Sqrt[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + Log[a
 + b*x^4]/(4*b)
```

Rubi in Sympy [A] time = 73.5883, size = 255, normalized size = 0.92

$$\frac{\log\left(a+bx^4\right)}{4b}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}+\sqrt{b}\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}+\sqrt{b}\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+x+1)/(b*x**4+a),x)
```

```
[Out] log(a + b*x**4)/(4*b) + atan(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*sqr
t(b)) + sqrt(2)*(sqrt(a) - sqrt(b))*log(-sqrt(2)*a**(1/4)*b**(3/4
```

```
)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a**(3/4)*b**(3/4)) - sqrt(2)*(
sqrt(a) - sqrt(b))*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt
(b) + b*x**2)/(8*a**(3/4)*b**(3/4)) - sqrt(2)*(sqrt(a) + sqrt(b))
*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(3/4)) + sq
rt(2)*(sqrt(a) + sqrt(b))*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(
4*a**(3/4)*b**(3/4))
```

Mathematica [A] time = 0.448622, size = 283, normalized size = 1.02

$$\frac{\sqrt{2}\sqrt[4]{b}\left(a^{3/4}-\sqrt[4]{a}\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{b}-a^{3/4}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+2a\log\left(a+\right.}{8al}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*(Sqrt[2]*Sqrt[a] + 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b
])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*(S
qrt[2]*Sqrt[a] - 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(a^(3/4) - a^(1/4)
*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + Sqrt[2]*(-a^(3/4) + a^(1/4)*Sqrt[b])*b^(1/4)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a*Log[a + b*x^4]
)/(8*a*b)
```

Maple [A] time = 0.004, size = 286, normalized size = 1.

$$\begin{aligned}
&\frac{\sqrt{2}}{8\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\\
&+\frac{\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{1}{2}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}\\
&+\frac{\sqrt{2}}{8\,b}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\\
&+\frac{\sqrt{2}}{4\,b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}}{4\,b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\ln\left(bx^4+a\right)}{4\,b}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+x+1)/(b*x^4+a),x)
```

```
[Out] 1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*(a/b)^(1/4)/a*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*(a/b)^(1/4)/a*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/2/(a*b)^(1/2)*arctan(x^2*(b/a)^(
1/2))+1/8/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4/b/(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/b/(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*ln(b*x^4+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.90628, size = 187, normalized size = 0.68

$$\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\left(96a^3b^2 + 96a^2b^3\right) + t\left(-16a^3b - 32a^2b^2 - 16ab^3\right) + a^3 + 3a^2b + 3ab^2 + b^3, \left(t \mapsto t\,\mathrm{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+1)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**
3*b**2 + 96*a**2*b**3) + _t*(-16*a**3*b - 32*a**2*b**2 - 16*a*b**
3) + a**3 + 3*a**2*b + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (64
*_t**3*a**3*b**3 - 48*_t**2*a**3*b**2 + 16*_t**2*a**2*b**3 + 12*_
t*a**3*b + 16*_t*a**2*b**2 + 4*_t*a*b**3 - a**3 - 2*a**2*b - a*b*
*2)/(a**2*b + 2*a*b**2 + b**3))))
```

GIAC/XCAS [A] time = 0.219624, size = 365, normalized size = 1.32

$$\frac{\ln\left(\left|bx^4 + a\right|\right)}{4\,b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 - \sqrt{2}\sqrt{ab^3}b + \left(ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 + \sqrt{2}\sqrt{ab^3}b + \left(ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 - \left(ab^3\right)^{\frac{3}{4}}\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 - \left(ab^3\right)^{\frac{3}{4}}\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*ln(abs(b*x^4 + a))/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 - sqrt(
2)*sqrt(a*b^3)*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)
```

```
*b^2 + sqrt(2)*sqrt(a*b^3)*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((
a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*ln(x^2 + sqrt(2)*x*(a/b)^(1/4)
+ sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a*b^3)^(
3/4))*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.171 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$

Optimal. Leaf size=148

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}$$

```
[Out] -((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*
ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(S
qrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4
*b)
```

Rubi [A] time = 0.435726, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*
ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(S
qrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4
*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{\int g\,dx}{b}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\left(\sqrt{a}\sqrt{b}e+ag+bc\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -f*log(a - b*x**4)/(4*b) - Integral(g, x)/b + d*atanh(sqrt(b)*x**
2/sqrt(a))/(2*sqrt(a)*sqrt(b)) + (-sqrt(a)*sqrt(b)*e + a*g + b*c)
*atan(b**(1/4)*x/a**(1/4))/(2*a**(3/4)*b**(5/4)) + (sqrt(a)*sqrt(
b)*e + a*g + b*c)*atanh(b**(1/4)*x/a**(1/4))/(2*a**(3/4)*b**(5/4)
)
```

Mathematica [A] time = 0.167182, size = 249, normalized size = 1.68

$$\frac{-a^{3/4}\sqrt[4]{b}f\log\left(a-bx^4\right)-4a^{3/4}\sqrt[4]{b}gx-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e+ag+bc\right)+\sqrt[4]{a}b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)-\sqrt[4]{a}b}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*b^(1/4)*g*x + 2*(b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTa
n[(b^(1/4)*x)/a^(1/4)] - (b*c + a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[
b]*e + a*g)*Log[a^(1/4) - b^(1/4)*x] + b*c*Log[a^(1/4) + b^(1/4)*
x] - a^(1/4)*b^(3/4)*d*Log[a^(1/4) + b^(1/4)*x] + Sqrt[a]*Sqrt[b]
*e*Log[a^(1/4) + b^(1/4)*x] + a*g*Log[a^(1/4) + b^(1/4)*x] + a^(1
/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3/4)*b^(1/4)*f*Log[
a - b*x^4])/(4*a^(3/4)*b^(5/4))
```

Maple [B] time = 0.006, size = 244, normalized size = 1.7

$$-\frac{gx}{b}+\frac{g}{2\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{c}{2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{g}{4\,b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{c}{4\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)-\frac{d}{4}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$
$$-\frac{e}{2\,b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e}{4\,b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{f\ln\left(bx^4-a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -g*x/b+1/2/b*(a/b)^(1/4)*arctan(x/(a/b)^(1/4))*g+1/2*c*(a/b)^(1/4
)/a*arctan(x/(a/b)^(1/4))+1/4/b*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x
-(a/b)^(1/4)))*g+1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^
(1/4)))-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(
1/2)))-1/2*e/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))+1/4*e/b/(a/b)^(1
/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b*f*ln(b*x^4-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22272, size = 539, normalized size = 3.64

$$-\frac{gx}{b}-\frac{f\ln\left(\left|bx^4-a\right|\right)}{4\,b}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}ab^4d+\left(-ab^3\right)^{\frac{1}{4}}ab^4c+\left(-ab^3\right)^{\frac{1}{4}}a^2b^3g+\left(-ab^3\right)^{\frac{3}{4}}ab^2e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,a^2b^5}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}ab^4d+\left(-ab^3\right)^{\frac{1}{4}}ab^4c+\left(-ab^3\right)^{\frac{1}{4}}a^2b^3g+\left(-ab^3\right)^{\frac{3}{4}}ab^2e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,a^2b^5}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}ab^4c+\left(-ab^3\right)^{\frac{1}{4}}a^2b^3g-\left(-ab^3\right)^{\frac{3}{4}}ab^2e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,a^2b^5}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}ab^4c+\left(-ab^3\right)^{\frac{1}{4}}a^2b^3g-\left(-ab^3\right)^{\frac{3}{4}}ab^2e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="giac")
```

```
[Out] -g*x/b - 1/4*f*ln(abs(b*x^4 - a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(-
a*b)*a*b^4*d + (-a*b^3)^(1/4)*a*b^4*c + (-a*b^3)^(1/4)*a^2*b^3*g
+ (-a*b^3)^(3/4)*a*b^2*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/(a^2*b^5) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b
)*a*b^4*d + (-a*b^3)^(1/4)*a*b^4*c + (-a*b^3)^(1/4)*a^2*b^3*g + (
-a*b^3)^(3/4)*a*b^2*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(
1/4))/(-a/b)^(1/4))/(a^2*b^5) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*a*b^4
*c + (-a*b^3)^(1/4)*a^2*b^3*g - (-a*b^3)^(3/4)*a*b^2*e)*ln(x^2 +
sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^5) - 1/8*sqrt(2)*((-a
*b^3)^(1/4)*a*b^4*c + (-a*b^3)^(1/4)*a^2*b^3*g - (-a*b^3)^(3/4)*a
*b^2*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^5)
```

3.172 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2} dx$

Optimal. Leaf size=172

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) +
 ((3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(8*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x
^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.34306, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) +
 ((3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(8*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x
^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 63.3251, size = 158, normalized size = 0.92

$$\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{d\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}-\frac{\left(-\sqrt{a}\sqrt{b}e+ag-3bc\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{5}{4}}}-\frac{\left(\sqrt{a}\sqrt{b}e+ag-3bc\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] x*(a*g + b*c + b*d*x + b*e*x**2 + b*f*x**3)/(4*a*b*(a - b*x**4))
+ d*atanh(sqrt(b)*x**2/sqrt(a))/(4*a**(3/2)*sqrt(b)) - (-sqrt(a)*
sqrt(b)*e + a*g - 3*b*c)*atanh(b**(1/4)*x/a**(1/4))/(8*a**(7/4)*b
**(5/4)) - (sqrt(a)*sqrt(b)*e + a*g - 3*b*c)*atan(b**(1/4)*x/a**(
1/4))/(8*a**(7/4)*b**(5/4))
```

Mathematica [A] time = 1.02567, size = 221, normalized size = 1.28

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{a-bx^4}-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e-ag+3bc\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-2\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e\right.}{16a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b
*x^4) - 2*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a
^(1/4)] - (3*b*c + 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)
*Log[a^(1/4) - b^(1/4)*x] + (3*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[a
]*Sqrt[b]*e - a*g)*Log[a^(1/4) + b^(1/4)*x] + 2*a^(1/4)*b^(3/4)*d
*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(5/4))
```

Maple [B] time = 0.013, size = 305, normalized size = 1.8

$$\frac{1}{bx^4-a}\left(-\frac{ex^3}{4a}-\frac{dx^2}{4a}-\frac{(ag+bc)x}{4ab}-\frac{f}{4b}\right)-\frac{g}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{3c}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{g}{16ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3c}{16a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{bd}{8}\ln\left(1\left(-a^2b+x^2\sqrt{a^3b^3}\right)\left(-a^2b-x^2\sqrt{a^3b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^3b^3}}$$
$$-\frac{e}{8ab}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e}{16ab}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4/a*e*x^3-1/4*d/a*x^2-1/4*(a*g+b*c)/a/b*x-1/4*f/b)/(b*x^4-a)-
1/8*(a/b)^(1/4)/a/b*arctan(x/(a/b)^(1/4))*g+3/8*c/a^2*(a/b)^(1/4)
*arctan(x/(a/b)^(1/4))-1/16*(a/b)^(1/4)/a/b*ln((x+(a/b)^(1/4))/(x
-(a/b)^(1/4)))*g+3/16*c/a^2*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/
b)^(1/4)))-1/8*b*d/(a^3*b^3)^(1/2)*ln((-a^2*b+x^2*(a^3*b^3)^(1/2)
)/(-a^2*b-x^2*(a^3*b^3)^(1/2)))-1/8*e/a/b/(a/b)^(1/4)*arctan(x/(a
/b)^(1/4))+1/16*e/a/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.223102, size = 528, normalized size = 3.07

$$-\frac{bx^3e + bdx^2 + bcx + agx + af}{4\,(bx^4 - a)ab} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-ab}b^2d + 3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c - \left(-ab^3\right)^{\frac{1}{4}}abg + \left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-ab}b^2d + 3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c - \left(-ab^3\right)^{\frac{1}{4}}abg + \left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c - \left(-ab^3\right)^{\frac{1}{4}}abg - \left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\left(3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c - \left(-ab^3\right)^{\frac{1}{4}}abg - \left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a)*a*b)
+ 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2
*c - (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(
2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3
)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(-a*b^
3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*ln(x^2
+ sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(
3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)
*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3)
```

3.173 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=221

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(-ag+7bc+6bdx+5bex^2\right)+4af}{32a^2b\left(a-bx^4\right)}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2)
 + (4*a*f + x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a -
 b*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4
)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[
b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4))
 + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.541059, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(-ag+7bc+6bdx+5bex^2\right)+4af}{32a^2b\left(a-bx^4\right)}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2)
 + (4*a*f + x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a -
 b*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4
)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[
b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4))
 + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 96.4441, size = 212, normalized size = 0.96

$$\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af-x\left(ag-7bc-6bdx-5bex^2\right)}{32a^2b\left(a-bx^4\right)}+\frac{3d\,\text{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}-\frac{\left(-5\sqrt{a}\sqrt{b}e+3ag-21bc\right)\text{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{5}{4}}}-\frac{\left(5\sqrt{a}\sqrt{b}e+3ag-21bc\right)\text{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] x*(a*g + b*c + b*d*x + b*e*x**2 + b*f*x**3)/(8*a*b*(a - b*x**4)**
2) + (4*a*f - x*(a*g - 7*b*c - 6*b*d*x - 5*b*e*x**2))/(32*a**2*b*
(a - b*x**4)) + 3*d*atanh(sqrt(b)*x**2/sqrt(a))/(16*a**(5/2)*sqrt
(b)) - (-5*sqrt(a)*sqrt(b)*e + 3*a*g - 21*b*c)*atanh(b**(1/4)*x/a
**(1/4))/(64*a**(11/4)*b**(5/4)) - (5*sqrt(a)*sqrt(b)*e + 3*a*g -
 21*b*c)*atan(b**(1/4)*x/a**(1/4))/(64*a**(11/4)*b**(5/4))
```

Mathematica [A] time = 0.99216, size = 263, normalized size = 1.19

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\left(a^2(4f+3gx)+abx\left(11c+x\left(10d+9ex+gx^3\right)\right)-b^2x^5(7c+x(6d+5ex))\right)}{(a-bx^4)^2}-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(12\sqrt[4]{a}b^{3/4}d+5\sqrt{a}\sqrt{b}e-3ag+21bc\right)+\log}{128}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a^2*(4*f + 3*g*x) - b^2*x^5*(7*c + x*(6*d +
5*e*x)) + a*b*x*(11*c + x*(10*d + 9*e*x + g*x^3))))/(a - b*x^4)^2
 + 2*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^
(1/4)] - (21*b*c + 12*a^(1/4)*b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3
*a*g)*Log[a^(1/4) - b^(1/4)*x] + (21*b*c - 12*a^(1/4)*b^(3/4)*d +
 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) + b^(1/4)*x] + 12*a^(1/
4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^(5/4))
```

Maple [A] time = 0.02, size = 344, normalized size = 1.6

$$
\begin{aligned}
&-\frac{1}{(bx^4-a)^2}\left(\frac{5\,bex^7}{32\,a^2}+\frac{3\,bdx^6}{16\,a^2}-\frac{(ag-7\,bc)\,x^5}{32\,a^2}-\frac{9\,ex^3}{32\,a}-\frac{5\,dx^2}{16\,a}-\frac{(3\,ag+11\,bc)\,x}{32\,ab}-\frac{f}{8\,b}\right)\\
&-\frac{3\,g}{128\,a^2b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{21\,c}{128\,a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\\
&-\frac{3\,g}{64\,a^2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{21\,c}{64\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\\
&-\frac{3\,bd}{32}\ln\left(1\left(-a^3b+x^2\sqrt{a^5b^3}\right)\left(-a^3b-x^2\sqrt{a^5b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^5b^3}}\\
&-\frac{5\,e}{64\,a^2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5\,e}{128\,a^2b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(5/32/a^2*b*e*x^7+3/16/a^2*d*b*x^6-1/32*(a*g-7*b*c)/a^2*x^5-9/32
/a*e*x^3-5/16*d/a*x^2-1/32*(3*a*g+11*b*c)/a/b*x-1/8*f/b)/(b*x^4-a
)^2-3/128*(a/b)^(1/4)/a^2/b*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g
+21/128*c/a^3*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-3/6
4*(a/b)^(1/4)/a^2/b*arctan(x/(a/b)^(1/4))*g+21/64*c/a^3*(a/b)^(1/
4)*arctan(x/(a/b)^(1/4))-3/32*b*d/(a^5*b^3)^(1/2)*ln((-a^3*b+x^2*
(a^5*b^3)^(1/2))/(-a^3*b-x^2*(a^5*b^3)^(1/2)))-5/64*e/a^2/b/(a/b)
^(1/4)*arctan(x/(a/b)^(1/4))+5/128*e/a^2/b/(a/b)^(1/4)*ln((x+(a/b
)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225514, size = 595, normalized size = 2.69

$$-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-21\left(-ab^3\right)^{\frac{1}{4}}b^2c+3\left(-ab^3\right)^{\frac{1}{4}}abg-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-21\left(-ab^3\right)^{\frac{1}{4}}b^2c+3\left(-ab^3\right)^{\frac{1}{4}}abg-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}+\frac{\sqrt{2}\left(21\left(-ab^3\right)^{\frac{1}{4}}b^2c-3\left(-ab^3\right)^{\frac{1}{4}}abg-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256a^3b^3}-\frac{\sqrt{2}\left(21\left(-ab^3\right)^{\frac{1}{4}}b^2c-3\left(-ab^3\right)^{\frac{1}{4}}abg-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256a^3b^3}-\frac{5b^2x^7e+6b^2dx^6+7b^2cx^5-abgx^5-9abx^3e-10abdx^2-11abcx-3a^2gx-4a^2f}{32\left(bx^4-a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b
^2*c + 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sq
rt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) - 1/12
8*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c
+ 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/256*sqr
t(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^
3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^
3) - 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*
b*g - 5*(-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(
-a/b))/(a^3*b^3) - 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5
- a*b*g*x^5 - 9*a*b*x^3*e - 10*a*b*d*x^2 - 11*a*b*c*x - 3*a^2*g*x
 - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

3.174 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=266

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3
) + (x*(7*(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a
- b*x^4)) + (8*a*f + x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(96
*a^2*b*(a - b*x^4)^2) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*
ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c +
15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*
a^(15/4)*b^(5/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7
/2)*Sqrt[b])
```

Rubi [A] time = 0.649152, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```


```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3
) + (x*(7*(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a
- b*x^4)) + (8*a*f + x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(96
*a^2*b*(a - b*x^4)^2) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*
ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c +
15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*
a^(15/4)*b^(5/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7
/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 119.523, size = 255, normalized size = 0.96

$$\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{8af-x\left(ag-11bc-10bdx-9bex^2\right)}{96a^2b\left(a-bx^4\right)^2}-\frac{x\left(7ag-77bc-60bdx-45bex^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{5d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}}-\frac{\left(-15\sqrt{a}\sqrt{b}e+7ag-77bc\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{5}{4}}}-\frac{\left(15\sqrt{a}\sqrt{b}e+7ag-77bc\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] x*(a*g + b*c + b*d*x + b*e*x**2 + b*f*x**3)/(12*a*b*(a - b*x**4)*
*3) + (8*a*f - x*(a*g - 11*b*c - 10*b*d*x - 9*b*e*x**2))/(96*a**2
*b*(a - b*x**4)**2) - x*(7*a*g - 77*b*c - 60*b*d*x - 45*b*e*x**2)
/(384*a**3*b*(a - b*x**4)) + 5*d*atanh(sqrt(b)*x**2/sqrt(a))/(32*
a**(7/2)*sqrt(b)) - (-15*sqrt(a)*sqrt(b)*e + 7*a*g - 77*b*c)*atan
h(b**(1/4)*x/a**(1/4))/(256*a**(15/4)*b**(5/4)) - (15*sqrt(a)*sqr
t(b)*e + 7*a*g - 77*b*c)*atan(b**(1/4)*x/a**(1/4))/(256*a**(15/4)
*b**(5/4))
```

Mathematica [A] time = 0.628692, size = 313, normalized size = 1.18

$$\frac{\frac{128a^{11/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{(a-bx^4)^3}+\frac{16a^{7/4}\sqrt[4]{b}x(-ag+11bc+bx(10d+9ex))}{(a-bx^4)^2}+\frac{4a^{3/4}\sqrt[4]{b}x(-7ag+77bc+15bx(4d+3ex))}{a-bx^4}-3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(40\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*x*(77*b*c - 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a
 - b*x^4) + (16*a^(7/4)*b^(1/4)*x*(11*b*c - a*g + b*x*(10*d + 9*e
*x)))/(a - b*x^4)^2 + (128*a^(11/4)*b^(1/4)*(a*(f + g*x) + b*x*(c
 + x*(d + e*x))))/(a - b*x^4)^3 + 6*(77*b*c - 15*Sqrt[a]*Sqrt[b]*
e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(77*b*c + 40*a^(1/4)*b
^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*Log[a^(1/4) - b^(1/4)*x]
 + 3*(77*b*c - 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e - 7*a*
g)*Log[a^(1/4) + b^(1/4)*x] + 120*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] +
 Sqrt[b]*x^2])/(1536*a^(15/4)*b^(5/4))
```

Maple [A] time = 0.021, size = 384, normalized size = 1.4

$$\frac{1}{(bx^4-a)^3}\left(-\frac{15\,b^2ex^{11}}{128\,a^3}-\frac{5\,b^2dx^{10}}{32\,a^3}+\frac{(7\,ag-77\,bc)\,bx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}+\frac{5\,bdx^6}{12\,a^2}-\frac{(3\,ag-33\,bc)\,x^5}{64\,a^2}-\frac{113\,ex^3}{384\,a}-\frac{11\,dx^2}{32\,a}-\frac{(7\,a}{}\right.$$

$$-\frac{7\,g}{256\,a^3b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{77\,c}{256\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{7\,g}{512\,a^3b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$

$$+\frac{77\,c}{512\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)-\frac{5\,bd}{64}\ln\left(1\left(-a^4b+x^2\sqrt{a^7b^3}\right)\left(-a^4b-x^2\sqrt{a^7b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^7b^3}}$$

$$-\frac{15\,e}{256\,a^3b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{15\,e}{512\,a^3b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128*e/a^3*b^2*x^11-5/32*d/a^3*b^2*x^10+7/384*(a*g-11*b*c)/a^
3*b*x^9+21/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6-3/64/a^2*(a*g-11*b*c)*
x^5-113/384/a*e*x^3-11/32*d/a*x^2-1/128*(7*a*g+51*b*c)/a/b*x-1/12
*f/b)/(b*x^4-a)^3-7/256*(a/b)^(1/4)/a^3/b*arctan(x/(a/b)^(1/4))*g
+77/256*c*(a/b)^(1/4)/a^4*arctan(x/(a/b)^(1/4))-7/512*(a/b)^(1/4)
/a^3/b*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+77/512*c*(a/b)^(1/4)
/a^4*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-5/64*b*d/(a^7*b^3)^(1/2)
*ln((-a^4*b+x^2*(a^7*b^3)^(1/2))/(-a^4*b-x^2*(a^7*b^3)^(1/2)))-15
/256*e/a^3/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))+15/512*e/a^3/b/(a/
b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.221355, size = 662, normalized size = 2.49

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d+77\left(-ab^3\right)^{\frac{1}{4}}b^2c-7\left(-ab^3\right)^{\frac{1}{4}}abg+15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d+77\left(-ab^3\right)^{\frac{1}{4}}b^2c-7\left(-ab^3\right)^{\frac{1}{4}}abg+15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3}+\frac{\sqrt{2}\left(77\left(-ab^3\right)^{\frac{1}{4}}b^2c-7\left(-ab^3\right)^{\frac{1}{4}}abg-15\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024a^4b^3}-\frac{\sqrt{2}\left(77\left(-ab^3\right)^{\frac{1}{4}}b^2c-7\left(-ab^3\right)^{\frac{1}{4}}abg-15\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024a^4b^3}-\frac{45b^3x^{11}e+60b^3dx^{10}+77b^3cx^9-7ab^2gx^9-126ab^2x^7e-160ab^2dx^6-198ab^2cx^5+18a^2bgx^5+113a^2bx^3e+132a^2b}{384\left(bx^4-a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^
2*c - 7*(-a*b^3)^(1/4)*a*b*g + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sq
rt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/51
2*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c
- 7*(-a*b^3)^(1/4)*a*b*g + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/1024*s
qrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a
*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4
*b^3) - 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4
)*a*b*g - 15*(-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/(a^4*b^3) - 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77
*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126*a*b^2*x^7*e - 160*a*b^2*d*x^6 -
198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*
x^2 + 153*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b
)
```

3.175 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$

Optimal. Leaf size=319

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}$$

```
[Out] (g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) -
 ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e
 - a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
 + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a
 + b*x^4])/(4*b)
```

Rubi [A] time = 0.778091, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```


```
[Out] (g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) -
 ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e
 - a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
 + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a
 + b*x^4])/(4*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{f\log\left(a+bx^4\right)}{4b}+\frac{\int g\,dx}{b}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\sqrt{2}\left(-\sqrt{a}\sqrt{b}e+ag-bc\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\left(-\sqrt{a}\sqrt{b}e+ag-bc\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}\sqrt{b}e+ag-bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}\sqrt{b}e+ag-bc\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] f*log(a + b*x**4)/(4*b) + Integral(g, x)/b + d*atan(sqrt(b)*x**2/
sqrt(a))/(2*sqrt(a)*sqrt(b)) + sqrt(2)*(-sqrt(a)*sqrt(b)*e + a*g
- b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(5/4)
) - sqrt(2)*(-sqrt(a)*sqrt(b)*e + a*g - b*c)*atan(1 + sqrt(2)*b**
(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(5/4)) + sqrt(2)*(sqrt(a)*sqrt(b
)*e + a*g - b*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(
b) + b*x**2)/(8*a**(3/4)*b**(5/4)) - sqrt(2)*(sqrt(a)*sqrt(b)*e +
 a*g - b*c)*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b
*x**2)/(8*a**(3/4)*b**(5/4))
```

Mathematica [A] time = 0.768171, size = 311, normalized size = 0.97

$$2a^{3/4}\sqrt[4]{b}f\log\left(a+bx^4\right)+8a^{3/4}\sqrt[4]{b}gx-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e-\sqrt{2}ag+\sqrt{2}bc\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*g*x - 2*(Sqrt[2]*b*c + 2*a^(1/4)*b^(3/4)*d + S
qrt[2]*Sqrt[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)] + 2*(Sqrt[2]*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*S
qrt[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)] + Sqrt[2]*(-(b*c) + Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(b*c - Sqrt[a
]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2] + 2*a^(3/4)*b^(1/4)*f*Log[a + b*x^4])/(8*a^(3/4)*b^(5/4)
)
```

Maple [A] time = 0.007, size = 429, normalized size = 1.3

$$\frac{gx}{b}-\frac{\sqrt{2}g}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{\sqrt{2}g}{8b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{c\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}g}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$
$$+\frac{d}{2}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}+\frac{e\sqrt{2}}{8b}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{e\sqrt{2}}{4b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e\sqrt{2}}{4b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{f\ln\left(bx^4+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] g*x/b-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*g
+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/8/
b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/
(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+1/8*c*(a/b)^(1/4)/a*2^
(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)
*x*2^(1/2)+(a/b)^(1/2)))-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x+1)*g+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x+1)+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e/b/
(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x
^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*e/b/(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*e/b/(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*f*ln(b*x^4+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.222165, size = 459, normalized size = 1.44

$$\frac{gx}{b} + \frac{f\ln\left(\left|bx^4+a\right|\right)}{4\,b} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d + \left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{1}{4}}abg + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d + \left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{1}{4}}abg + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] g*x/b + 1/4*f*ln(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*
b)*b^2*d + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3
/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + (a*b^3)^(1/4)*
b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*(
(a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*ln(x
^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a
*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*ln(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.176 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=341

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) +
 (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*b*c
+ Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*
g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^
(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4)) +
 ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rubi [A] time = 0.667445, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```


```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) +
 (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*b*c
+ Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*
g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^
(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4)) +
 ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rubi in Sympy [A] time = 115.932, size = 325, normalized size = 0.95

$$-\frac{x\left(ag-bc-bdx-bex^2-bfx^3\right)}{4ab\left(a+bx^4\right)}+\frac{d\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}-\frac{\sqrt{2}\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{7}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{7}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] -x*(a*g - b*c - b*d*x - b*e*x**2 - b*f*x**3)/(4*a*b*(a + b*x**4))
 + d*atan(sqrt(b)*x**2/sqrt(a))/(4*a**(3/2)*sqrt(b)) - sqrt(2)*(-
sqrt(a)*sqrt(b)*e + a*g + 3*b*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x
 + sqrt(a)*sqrt(b) + b*x**2)/(32*a**(7/4)*b**(5/4)) + sqrt(2)*(-s
qrt(a)*sqrt(b)*e + a*g + 3*b*c)*log(sqrt(2)*a**(1/4)*b**(3/4)*x +
 sqrt(a)*sqrt(b) + b*x**2)/(32*a**(7/4)*b**(5/4)) - sqrt(2)*(sqrt
(a)*sqrt(b)*e + a*g + 3*b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4)
)/(16*a**(7/4)*b**(5/4)) + sqrt(2)*(sqrt(a)*sqrt(b)*e + a*g + 3*b
*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(5/4))
```

Mathematica [A] time = 0.375136, size = 319, normalized size = 0.94

$$-\frac{8a^{3/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e+\sqrt{2}ag+3\sqrt{2}bc\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a +
b*x^4) - 2*(3*Sqrt[2]*b*c + 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]
*Sqrt[b]*e + Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 + 2*(3*Sqrt[2]*b*c - 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[
b]*e + Sqrt[2]*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqr
t[2]*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c - Sqrt[a]*Sqrt[b]*
e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
(32*a^(7/4)*b^(5/4))
```

Maple [A] time = 0.013, size = 484, normalized size = 1.4

$$\frac{1}{bx^4+a}\left(\frac{ex^3}{4a}+\frac{dx^2}{4a}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}\right)+\frac{\sqrt{2}g}{16ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{3c\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{\sqrt{2}g}{32ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{3c\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{\sqrt{2}g}{16ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3c\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{bd}{4}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{a^3b^3}}+\frac{e\sqrt{2}}{32ab}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e\sqrt{2}}{16ab}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e\sqrt{2}}{16ab}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (1/4/a*e*x^3+1/4*d/a*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1
/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*g+3/1
6*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/32/
b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
)/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+3/32*c/a^2*(a/b)^(1/
4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^
(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x+1)*g+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*b*d/(a^3*b^3)^(1/2)*arctan(x^2*(b/a
)^(1/2))+1/32*e/a/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(
1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/16*e
/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/16*e/a
/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.221529, size = 493, normalized size = 1.45

$$\frac{bx^3e + bdx^2 + bcx - agx - af}{4\,(bx^4 + a)ab} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) +
 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c
+ (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*s
qrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*
b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c
 + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^
2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.177 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{4af-x\left(ag+7bc+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2)
 - (4*a*f - x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a +
 b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b
]) - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5
*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*
e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e
 + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi [A] time = 0.929769, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{4af-x\left(ag+7bc+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```


```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2)
 - (4*a*f - x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a +
 b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b
]) - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5
*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*
e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e
 + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi in Sympy [A] time = 149.134, size = 386, normalized size = 0.98

$$-\frac{x\left(ag-bc-bdx-bex^2-bfx^3\right)}{8ab\left(a+bx^4\right)^2}-\frac{4af-x\left(ag+7bc+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)}+\frac{3d\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}-\frac{\sqrt{2}\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{256a^{\frac{11}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{256a^{\frac{11}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] -x*(a*g - b*c - b*d*x - b*e*x**2 - b*f*x**3)/(8*a*b*(a + b*x**4)*
*2) - (4*a*f - x*(a*g + 7*b*c + 6*b*d*x + 5*b*e*x**2))/(32*a**2*b
*(a + b*x**4)) + 3*d*atan(sqrt(b)*x**2/sqrt(a))/(16*a**(5/2)*sqrt
(b)) - sqrt(2)*(-5*sqrt(a)*sqrt(b)*e + 3*a*g + 21*b*c)*log(-sqrt(
2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(256*a**(11/4)
*b**(5/4)) + sqrt(2)*(-5*sqrt(a)*sqrt(b)*e + 3*a*g + 21*b*c)*log(
sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(256*a**(
11/4)*b**(5/4)) - sqrt(2)*(5*sqrt(a)*sqrt(b)*e + 3*a*g + 21*b*c)*
atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(128*a**(11/4)*b**(5/4)) +
sqrt(2)*(5*sqrt(a)*sqrt(b)*e + 3*a*g + 21*b*c)*atan(1 + sqrt(2)*b
**(1/4)*x/a**(1/4))/(128*a**(11/4)*b**(5/4))
```

Mathematica [A] time = 0.596171, size = 366, normalized size = 0.93

$$-\frac{32a^{7/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{(a+bx^4)^2}+\frac{8a^{3/4}\sqrt[4]{b}x(ag+7bc+bx(6d+5ex))}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(24\sqrt[4]{a}b^{3/4}d+5\sqrt{2}\sqrt{a}\sqrt{b}e+3\sqrt{2}ag+\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x)))/(a + b*x
^4) - (32*a^(7/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/
(a + b*x^4)^2 - 2*(21*Sqrt[2]*b*c + 24*a^(1/4)*b^(3/4)*d + 5*Sqrt
[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b*c - 24*a^(1/4)*b^(3/4)*d + 5*Sqrt
[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt[2]*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)] + Sqrt[2]*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(
21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(5/4))
```

Maple [A] time = 0.018, size = 521, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (5/32/a^2*b*e*x^7+3/16/a^2*d*b*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/
a*e*x^3+5/16*d/a*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)
^2+3/128*(a/b)^(1/4)/a^2/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1
)*g+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
-1)+3/256*(a/b)^(1/4)/a^2/b*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+21/256*c
/a^3*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+3/128*(a/b)^(1/4)/a^2
/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+21/128*c/a^3*(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/16*b*d/(a^5*b^3)^(
1/2)*arctan(x^2*(b/a)^(1/2))+5/256*e/a^2/b/(a/b)^(1/4)*2^(1/2)*ln
((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/
2)+(a/b)^(1/2)))+5/128*e/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x+1)+5/128*e/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225624, size = 562, normalized size = 1.43

$$\frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{ab}b^2d+21\,\left(ab^3\right)^{\frac{1}{4}}b^2c+3\,\left(ab^3\right)^{\frac{1}{4}}abg+5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{ab}b^2d+21\,\left(ab^3\right)^{\frac{1}{4}}b^2c+3\,\left(ab^3\right)^{\frac{1}{4}}abg+5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(21\,\left(ab^3\right)^{\frac{1}{4}}b^2c+3\,\left(ab^3\right)^{\frac{1}{4}}abg-5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}-\frac{\sqrt{2}\left(21\,\left(ab^3\right)^{\frac{1}{4}}b^2c+3\,\left(ab^3\right)^{\frac{1}{4}}abg-5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}+\frac{5\,b^2x^7e+6\,b^2dx^6+7\,b^2cx^5+abgx^5+9\,abx^3e+10\,abdx^2+11\,abcx-3\,a^2gx-4\,a^2f}{32\,(bx^4+a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*
c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(
2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^
3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b
^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*ln(x
^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)
*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4
)*e)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32
*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*x^3
*e + 10*a*b*d*x^2 + 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a
)^2*a^2*b)
```

3.178 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=437

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(7(ag+11bc)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x\left(ag+11bc+10bdx+9bex^2\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3
) + (x*(7*(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a
+ b*x^4)) - (8*a*f - x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(96
*a^2*b*(a + b*x^4)^2) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a
^(7/2)*Sqrt[b]) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4))
 + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 1
5*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 1
5*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))
```

Rubi [A] time = 1.09223, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(7(ag+11bc)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x\left(ag+11bc+10bdx+9bex^2\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```


```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3
) + (x*(7*(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a
+ b*x^4)) - (8*a*f - x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(96
*a^2*b*(a + b*x^4)^2) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a
^(7/2)*Sqrt[b]) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4))
 + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 1
5*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
```

```
)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 1
5*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))
```

Rubi in Sympy [A] time = 177.693, size = 428, normalized size = 0.98

$$-\frac{x\left(ag-bc-bdx-bex^2-bfx^3\right)}{12ab\left(a+bx^4\right)^3}-\frac{8af-x\left(ag+11bc+10bdx+9bex^2\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(7ag+77bc+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)}+\frac{5d\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}}-\frac{\sqrt{2}\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{1024a^{\frac{15}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{1024a^{\frac{15}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{15}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{15}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] -x*(a*g - b*c - b*d*x - b*e*x**2 - b*f*x**3)/(12*a*b*(a + b*x**4)
**3) - (8*a*f - x*(a*g + 11*b*c + 10*b*d*x + 9*b*e*x**2))/(96*a**
2*b*(a + b*x**4)**2) + x*(7*a*g + 77*b*c + 60*b*d*x + 45*b*e*x**2
)/(384*a**3*b*(a + b*x**4)) + 5*d*atan(sqrt(b)*x**2/sqrt(a))/(32*
a**(7/2)*sqrt(b)) - sqrt(2)*(-15*sqrt(a)*sqrt(b)*e + 7*a*g + 77*b
*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/
(1024*a**(15/4)*b**(5/4)) + sqrt(2)*(-15*sqrt(a)*sqrt(b)*e + 7*a*
g + 77*b*c)*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b
*x**2)/(1024*a**(15/4)*b**(5/4)) - sqrt(2)*(15*sqrt(a)*sqrt(b)*e
+ 7*a*g + 77*b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(512*a**(
15/4)*b**(5/4)) + sqrt(2)*(15*sqrt(a)*sqrt(b)*e + 7*a*g + 77*b*c)
*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(512*a**(15/4)*b**(5/4))
```

Mathematica [A] time = 0.694736, size = 411, normalized size = 0.94

$$-\frac{\frac{256a^{11/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{(a+bx^4)^3}+\frac{32a^{7/4}\sqrt[4]{b}x(ag+11bc+bx(10d+9ex))}{(a+bx^4)^2}+\frac{8a^{3/4}\sqrt[4]{b}x(7ag+77bc+15bx(4d+3ex))}{a+bx^4}-6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a
 + b*x^4) + (32*a^(7/4)*b^(1/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e
*x)))/(a + b*x^4)^2 - (256*a^(11/4)*b^(1/4)*(a*(f + g*x) - b*x*(c
 + x*(d + e*x))))/(a + b*x^4)^3 - 6*(77*Sqrt[2]*b*c + 80*a^(1/4)*
b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 7*Sqrt[2]*a*g)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b*c - 80*a^(1/4)
*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 7*Sqrt[2]*a*g)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(77*b*c - 15*Sqrt[a
]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + 3*Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15
```

```
/4)*b^(5/4))
```

Maple [A] time = 0.021, size = 562, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+7/384*(a*g+11*b*c)/a^3
*b*x^9+21/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6+3/64/a^2*(a*g+11*b*c)*x
^5+113/384/a*e*x^3+11/32*d/a*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*
f/b)/(b*x^4+a)^3+7/512*(a/b)^(1/4)/a^3/b*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x-1)*g+77/512*c*(a/b)^(1/4)/a^4*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x-1)+7/1024*(a/b)^(1/4)/a^3/b*2^(1/2)*ln((x^2+(a/b)^
(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2)))*g+77/1024*c*(a/b)^(1/4)/a^4*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^
(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+7/512
*(a/b)^(1/4)/a^3/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+77/5
12*c*(a/b)^(1/4)/a^4*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+5/32
*b*d/(a^7*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/1024*e/a^3/b/(a/b
)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+15/512*e/a^3/b/(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+15/512*e/a^3/b/(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.222268, size = 629, normalized size = 1.44

$$\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d + 77\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\,\left(ab^3\right)^{\frac{1}{4}}abg + 15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d + 77\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\,\left(ab^3\right)^{\frac{1}{4}}abg + 15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(77\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\,\left(ab^3\right)^{\frac{1}{4}}abg - 15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3} - \frac{\sqrt{2}\left(77\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\,\left(ab^3\right)^{\frac{1}{4}}abg - 15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3} + \frac{45\,b^3x^{11}e + 60\,b^3dx^{10} + 77\,b^3cx^9 + 7\,ab^2gx^9 + 126\,ab^2x^7e + 160\,ab^2dx^6 + 198\,ab^2cx^5 + 18\,a^2bgx^5 + 113\,a^2bx^3e + 132\,a^2bd}{384\,(bx^4 + a)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*
c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2
)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt
(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b
^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(
a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*
ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sq
rt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3
)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3)
+ 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g
*x^9 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 18*a
^2*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x -
21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

3.179 $\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{4}(1-x)^4$$

```
[Out] -(1 - x)^4/4
```

Rubi [A] time = 0.012919, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$-\frac{1}{4}(1-x)^4$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

```
[Out] -(1 - x)^4/4
```

Rubi in Sympy [A] time = 4.0795, size = 7, normalized size = 0.64

$$-\frac{(-x+1)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)**3/(x**3+x**2+x+1)**3,x)
```

```
[Out] -(-x + 1)**4/4
```

Mathematica [A] time = 0.00281969, size = 9, normalized size = 0.82

$$-\frac{1}{4}(x-1)^4$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

```
[Out] -(-1 + x)^4/4
```

Maple [A] time = 0.001, size = 8, normalized size = 0.7

$$-\frac{(-1+x)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)^3/(x^3+x^2+x+1)^3,x)
```

```
[Out] -1/4*(-1+x)^4
```

Maxima [A] time = 1.42326, size = 20, normalized size = 1.82

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)^3/(x^3 + x^2 + x + 1)^3,x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Fricas [A] time = 0.207356, size = 20, normalized size = 1.82

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)^3/(x^3 + x^2 + x + 1)^3,x, algorithm="fricas")
```

```
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Sympy [A] time = 0.073051, size = 15, normalized size = 1.36

$$-\frac{x^4}{4} + x^3 - \frac{3x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)**3/(x**3+x**2+x+1)**3,x)
```

```
[Out] -x**4/4 + x**3 - 3*x**2/2 + x
```

GIAC/XCAS [A] time = 0.210623, size = 20, normalized size = 1.82

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)^3/(x^3 + x^2 + x + 1)^3,x, algorithm="giac")
```

```
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

3.180 $\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{3}(1-x)^3$$

```
[Out] -(1 - x)^3/3
```

Rubi [A] time = 0.0144015, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$-\frac{1}{3}(1-x)^3$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]

[Out] -(1 - x)^3/3
```

Rubi in Sympy [A] time = 4.0638, size = 7, normalized size = 0.64

$$-\frac{(-x+1)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)**2/(x**3+x**2+x+1)**2,x)

[Out] -(-x + 1)**3/3
```

Mathematica [A] time = 0.00123161, size = 14, normalized size = 1.27

$$\frac{x^3}{3}-x^2+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]

[Out] x - x^2 + x^3/3
```

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$\frac{(-1+x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)^2/(x^3+x^2+x+1)^2,x)

[Out] 1/3*(-1+x)^3
```

Maxima [A] time = 1.37128, size = 16, normalized size = 1.45

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 1)^2/(x^3 + x^2 + x + 1)^2,x, algorithm="maxima")

[Out] 1/3*x^3 - x^2 + x
```

Fricas [A] time = 0.214174, size = 16, normalized size = 1.45

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 1)^2/(x^3 + x^2 + x + 1)^2,x, algorithm="fricas")

[Out] 1/3*x^3 - x^2 + x
```

Sympy [A] time = 0.065068, size = 8, normalized size = 0.73

$$\frac{x^3}{3} - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)**2/(x**3+x**2+x+1)**2,x)

[Out] x**3/3 - x**2 + x
```

GIAC/XCAS [A] time = 0.209303, size = 16, normalized size = 1.45

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 1)^2/(x^3 + x^2 + x + 1)^2,x, algorithm="giac")

[Out] 1/3*x^3 - x^2 + x
```

3.181 $\int \frac{1-x^4}{1+x+x^2+x^3}\,dx$

Optimal. Leaf size=9

$$x-\frac{x^2}{2}$$

```
[Out] x - x^2/2
```

Rubi [A] time = 0.0123478, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$x-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 + x + x^2 + x^3),x]

[Out] x - x^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x-\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**3+x**2+x+1),x)

[Out] x - Integral(x, x)
```

Mathematica [A] time = 0.0010825, size = 9, normalized size = 1.

$$x-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 + x + x^2 + x^3),x]

[Out] x - x^2/2
```

Maple [A] time = 0.003, size = 8, normalized size = 0.9

$$x-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^3+x^2+x+1),x)

[Out] x-1/2*x^2
```

Maxima [A] time = 1.38845, size = 9, normalized size = 1.

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^3 + x^2 + x + 1),x, algorithm="maxima")

[Out] -1/2*x^2 + x
```

Fricas [A] time = 0.209718, size = 9, normalized size = 1.

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^3 + x^2 + x + 1),x, algorithm="fricas")

[Out] -1/2*x^2 + x
```

Sympy [A] time = 0.043413, size = 5, normalized size = 0.56

$$-\frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**3+x**2+x+1),x)

[Out] -x**2/2 + x
```

GIAC/XCAS [A] time = 0.21047, size = 9, normalized size = 1.

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^3 + x^2 + x + 1),x, algorithm="giac")

[Out] -1/2*x^2 + x
```

3.182 $\int \frac{1+x+x^2+x^3}{1-x^4} \, dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -Log[1 - x]
```

Rubi [A] time = 0.0127494, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)/(1 - x^4),x]

[Out] -Log[1 - x]
```

Rubi in Sympy [A] time = 3.64762, size = 5, normalized size = 0.62

$$-\log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+x+1)/(-x**4+1),x)

[Out] -log(-x + 1)
```

Mathematica [A] time = 0.00169271, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]

[Out] -Log[1 - x]
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+x+1)/(-x^4+1),x)

[Out] -ln(-1+x)
```

Maxima [A] time = 1.36826, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

Fricas [A] time = 0.213344, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A] time = 0.044821, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

```
[Out] -log(x - 1)
```

GIAC/XCAS [A] time = 0.210258, size = 9, normalized size = 1.12

$$-\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)/(x^4 - 1),x, algorithm="giac")
```

```
[Out] -ln(abs(x - 1))
```

3.183 $\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx$

Optimal. Leaf size=7

$$\frac{1}{1-x}$$

```
[Out] (1 - x)^(-1)
```

Rubi [A] time = 0.0133343, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{1}{1-x}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]

[Out] (1 - x)^(-1)
```

Rubi in Sympy [A] time = 4.16705, size = 3, normalized size = 0.43

$$\frac{1}{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+x+1)**2/(-x**4+1)**2,x)

[Out] 1/(-x + 1)
```

Mathematica [A] time = 0.00190998, size = 7, normalized size = 1.

$$-\frac{1}{x-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]

[Out] -(-1 + x)^(-1)
```

Maple [A] time = 0.002, size = 8, normalized size = 1.1

$$-(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+x+1)^2/(-x^4+1)^2,x)

[Out] -1/(-1+x)
```

Maxima [A] time = 6.69503, size = 9, normalized size = 1.29

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)^2/(x^4 - 1)^2,x, algorithm="maxima")
```

```
[Out] -1/(x - 1)
```

Fricas [A] time = 0.214316, size = 9, normalized size = 1.29

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)^2/(x^4 - 1)^2,x, algorithm="fricas")
```

```
[Out] -1/(x - 1)
```

Sympy [A] time = 0.085161, size = 5, normalized size = 0.71

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+1)**2/(-x**4+1)**2,x)
```

```
[Out] -1/(x - 1)
```

GIAC/XCAS [A] time = 0.209931, size = 9, normalized size = 1.29

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)^2/(x^4 - 1)^2,x, algorithm="giac")
```

```
[Out] -1/(x - 1)
```

3.184 $\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2(1-x)^2}$$

```
[Out] 1/(2*(1 - x)^2)
```

Rubi [A] time = 0.0163131, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{1}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]

[Out] 1/(2*(1 - x)^2)
```

Rubi in Sympy [A] time = 4.29654, size = 7, normalized size = 0.64

$$\frac{1}{2\left(-x+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+x+1)**3/(-x**4+1)**3,x)

[Out] 1/(2*(-x + 1)**2)
```

Mathematica [A] time = 0.00271378, size = 9, normalized size = 0.82

$$\frac{1}{2(x-1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]

[Out] 1/(2*(-1 + x)^2)
```

Maple [A] time = 0.003, size = 8, normalized size = 0.7

$$\frac{1}{2\,(-1+x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+x+1)^3/(-x^4+1)^3,x)

[Out] 1/2/(-1+x)^2
```

Maxima [A] time = 5.949, size = 16, normalized size = 1.45

$$\frac{1}{2\,(x^2 - 2\,x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)^3/(x^4 - 1)^3,x, algorithm="maxima")

[Out] 1/2/(x^2 - 2*x + 1)
```

Fricas [A] time = 0.220694, size = 16, normalized size = 1.45

$$\frac{1}{2\,(x^2 - 2\,x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)^3/(x^4 - 1)^3,x, algorithm="fricas")

[Out] 1/2/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.11103, size = 10, normalized size = 0.91

$$\frac{1}{2x^2 - 4x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+1)**3/(-x**4+1)**3,x)

[Out] 1/(2*x**2 - 4*x + 2)
```

GIAC/XCAS [A] time = 0.21115, size = 9, normalized size = 0.82

$$\frac{1}{2\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 + x^2 + x + 1)^3/(x^4 - 1)^3,x, algorithm="giac")

[Out] 1/2/(x - 1)^2
```

3.185 $\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} dx$

Optimal. Leaf size=11

$$\frac{1}{3(1-x)^3}$$

```
[Out] 1/(3*(1 - x)^3)
```

Rubi [A] time = 0.0154507, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{1}{3(1-x)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
```

```
[Out] 1/(3*(1 - x)^3)
```

Rubi in Sympy [A] time = 4.29994, size = 7, normalized size = 0.64

$$\frac{1}{3(-x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+x+1)**4/(-x**4+1)**4,x)
```

```
[Out] 1/(3*(-x + 1)**3)
```

Mathematica [A] time = 0.00227476, size = 9, normalized size = 0.82

$$-\frac{1}{3(x-1)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
```

```
[Out] -1/(3*(-1 + x)^3)
```

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$-\frac{1}{3(-1+x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+x+1)^4/(-x^4+1)^4,x)
```

```
[Out] -1/3/(-1+x)^3
```

Maxima [A] time = 1.39801, size = 23, normalized size = 2.09

$$-\frac{1}{3\left(x^{3}-3x^{2}+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)^4/(x^4 - 1)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Fricas [A] time = 0.21031, size = 23, normalized size = 2.09

$$-\frac{1}{3\left(x^{3}-3x^{2}+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)^4/(x^4 - 1)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [A] time = 0.1231, size = 17, normalized size = 1.55

$$-\frac{1}{3x^{3}-9x^{2}+9x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+1)**4/(-x**4+1)**4,x)
```

```
[Out] -1/(3*x**3 - 9*x**2 + 9*x - 3)
```

GIAC/XCAS [A] time = 0.209831, size = 9, normalized size = 0.82

$$-\frac{1}{3\left(x-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 1)^4/(x^4 - 1)^4,x, algorithm="giac")
```

```
[Out] -1/3/(x - 1)^3
```

3.186 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx$

Optimal. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}$$

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Arc
Tan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*S
qrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4))
 + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2
)) - (f*Log[a - b*x^4])/(4*b)
```

Rubi [A] time = 0.526217, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Arc
Tan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*S
qrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4))
 + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2
)) - (f*Log[a - b*x^4])/(4*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{hx^2}{2b}-\frac{\int g\,dx}{b}+\frac{(ah+bd)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{3}{2}}}+\frac{\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\left(\sqrt{a}\sqrt{b}e+ag+bc\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -f*log(a - b*x**4)/(4*b) - h*x**2/(2*b) - Integral(g, x)/b + (a*h
 + b*d)*atanh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*b**(3/2)) + (-sqrt
(a)*sqrt(b)*e + a*g + b*c)*atan(b**(1/4)*x/a**(1/4))/(2*a**(3/4)*
b**(5/4)) + (sqrt(a)*sqrt(b)*e + a*g + b*c)*atanh(b**(1/4)*x/a**(
1/4))/(2*a**(3/4)*b**(5/4))
```

Mathematica [A] time = 0.551448, size = 256, normalized size = 1.55

$$\frac{-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{5/4}h+\sqrt{a}b^{3/4}e+\sqrt[4]{a}bd+a\sqrt[4]{b}g+b^{5/4}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{5/4}(-h)+\sqrt{a}b^{3/4}e-\sqrt[4]{a}bd+a\sqrt[4]{b}g+b^{5/}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*Sqrt[b]*g*x - 2*a^(3/4)*Sqrt[b]*h*x^2 + 2*b^(1/4)*(b*
c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(5/
4)*c + a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + a^(5/4)*h)
*Log[a^(1/4) - b^(1/4)*x] + (b^(5/4)*c - a^(1/4)*b*d + Sqrt[a]*b^
(3/4)*e + a*b^(1/4)*g - a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] + a^(
1/4)*(b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3/4)*Sqrt[b]*f*L
og[a - b*x^4])/(4*a^(3/4)*b^(3/2))
```

Maple [B] time = 0.009, size = 296, normalized size = 1.8

$$-\frac{hx^2}{2b}-\frac{gx}{b}+\frac{g}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$+\frac{g}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{ah}{4b}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$
$$-\frac{d}{4}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{e}{2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{e}{4b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{f\ln\left(bx^4-a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/2*h*x^2/b-g*x/b+1/2/b*(a/b)^(1/4)*arctan(x/(a/b)^(1/4))*g+1/2*
c*(a/b)^(1/4)/a*arctan(x/(a/b)^(1/4))+1/4/b*(a/b)^(1/4)*ln((x+(a/
b)^(1/4))/(x-(a/b)^(1/4)))*g+1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4
))/(x-(a/b)^(1/4)))-1/4/b/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a
-x^2*(a*b)^(1/2)))*a*h-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/
(-a-x^2*(a*b)^(1/2)))-1/2*e/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))+1
/4*e/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b*f*ln
(b*x^4-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="max
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="fri
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225768, size = 539, normalized size = 3.27

$$-\frac{f\ln\left(\left|bx^4-a\right|\right)}{4\,b}-\frac{bhx^2+2\,bgx}{2\,b^2}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\sqrt{2}\sqrt{-ab}abh+\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{1}{4}}abg+\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\sqrt{2}\sqrt{-ab}abh+\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{1}{4}}abg+\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{1}{4}}abg-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{1}{4}}abg-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="gia
```

```
[Out] -1/4*f*ln(abs(b*x^4 - a))/b - 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/4*s
qrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b*h + (-a
*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3)
 + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b
*h + (-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))
/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b
*g - (-a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*
a*b*g - (-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(
-a/b))/(a*b^3)
```

3.187 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx$

Optimal. Leaf size=188

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}$$

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*
c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*
b^(7/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(
b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(
4*b)
```

Rubi [A] time = 0.64994, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*
c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*
b^(7/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(
b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(
4*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{\int g\,dx}{b}+\frac{(ah+bd)\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{3}{2}}}-\frac{\left(\sqrt{a}\,(ai+be)-\sqrt{b}\,(ag+bc)\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{7}{4}}}+\frac{\left(\sqrt{a}\,(ai+be)+\sqrt{b}\,(ag+bc)\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -f*log(a - b*x**4)/(4*b) - h*x**2/(2*b) - i*x**3/(3*b) - Integral
(g, x)/b + (a*h + b*d)*atanh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*b**
(3/2)) - (sqrt(a)*(a*i + b*e) - sqrt(b)*(a*g + b*c))*atan(b**(1/4
)*x/a**(1/4))/(2*a**(3/4)*b**(7/4)) + (sqrt(a)*(a*i + b*e) + sqrt
(b)*(a*g + b*c))*atanh(b**(1/4)*x/a**(1/4))/(2*a**(3/4)*b**(7/4))
```

Mathematica [A] time = 0.800465, size = 301, normalized size = 1.6

$$-\frac{\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{5/4}\sqrt[4]{b}h+a^{3/2}i+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-a^{5/4}\sqrt[4]{b}h+a^{3/2}i-\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\cdots\right.}{\cdots}}{12b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 + (6*(b^(3/2
)*c - Sqrt[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a
^(1/4)])/a^(3/4) - (3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*
e + a*Sqrt[b]*g + a^(5/4)*b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^
(1/4)*x])/a^(3/4) + (3*(b^(3/2)*c - a^(1/4)*b^(5/4)*d + Sqrt[a]*b
*e + a*Sqrt[b]*g - a^(5/4)*b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) + b
^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sqrt[b]
*x^2])/Sqrt[a] - 3*b^(3/4)*f*Log[a - b*x^4])/(12*b^(7/4))
```

Maple [B] time = 0.009, size = 367, normalized size = 2.

$$-\frac{ix^3}{3b}-\frac{hx^2}{2b}-\frac{gx}{b}+\frac{g}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$+\frac{g}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{ah}{4b}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$
$$-\frac{d}{4}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{ai}{2b^2}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{e}{2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{ai}{4b^2}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{e}{4b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{f\ln\left(bx^4-a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/3*i*x^3/b-1/2*h*x^2/b-g*x/b+1/2/b*(a/b)^(1/4)*arctan(x/(a/b)^(
1/4))*g+1/2*c*(a/b)^(1/4)/a*arctan(x/(a/b)^(1/4))+1/4/b*(a/b)^(1/
4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+1/4*c*(a/b)^(1/4)/a*ln((
x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b/(a*b)^(1/2)*ln((-a+x^2*(a*b
)^(1/2))/(-a-x^2*(a*b)^(1/2)))*a*h-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(
a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/2/b^2/(a/b)^(1/4)*arctan(x/(a
/b)^(1/4))*a*i-1/2*e/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))+1/4/b^2/
(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*a*i+1/4*e/b/(a/b)
^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b*f*ln(b*x^4-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorit
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorit
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.229693, size = 807, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorit
```

```
[Out] 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 - sqrt(2)*(-a*b^3)^(3/4)*ln(x^2
+ sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4) + 1/8*i*(2*sqrt(2)*(-
a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/
b)^(1/4))/b^4 + sqrt(2)*(-a*b^3)^(3/4)*ln(x^2 - sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/b^4) - 1/4*f*ln(abs(b*x^4 - a))/b + 1/4*sqrt(
2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b*h + (-a*b^3
)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1
/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b*h +
 (-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*
b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g -
 (-a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*
g - (-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/(a*b^3) - 1/6*(2*b^2*i*x^3 + 3*b^2*h*x^2 + 6*b^2*g*x)/b^3
```

3.188 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$

Optimal. Leaf size=205

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(aj+bf)\log\left(a-bx^4\right)}{4b^2}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}$$

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - ((b*
e - (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/
4)])/(2*a^(1/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a]
+ a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d
+ a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((b*
f + a*j)*Log[a - b*x^4])/(4*b^2)
```

Rubi [A] time = 0.658252, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.196

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(aj+bf)\log\left(a-bx^4\right)}{4b^2}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),x]
```


```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - ((b*
e - (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/
4)])/(2*a^(1/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a]
+ a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d
+ a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((b*
f + a*j)*Log[a - b*x^4])/(4*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ix^3}{3b}-\frac{\int g\,dx}{b}-\frac{\int^{x^2}h\,dx}{2b}-\frac{\int^{x^2}x\,dx}{2b}-\frac{(a+bf)\log\left(a-bx^4\right)}{4b^2}+\frac{(ah+bd)\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{3}{2}}}-\frac{\left(\sqrt{a}\,(ai+be)-\sqrt{b}\,(ag+bc)\right)\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{7}{4}}}+\frac{\left(\sqrt{a}\,(ai+be)+\sqrt{b}\,(ag+bc)\right)\mathrm{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-
b*x**4+a),x)
```

```
[Out] -i*x**3/(3*b) - Integral(g, x)/b - Integral(h, (x, x**2))/(2*b) -
 Integral(x, (x, x**2))/(2*b) - (a + b*f)*log(a - b*x**4)/(4*b**2
) + (a*h + b*d)*atanh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*b**(3/2))
- (sqrt(a)*(a*i + b*e) - sqrt(b)*(a*g + b*c))*atan(b**(1/4)*x/a**
(1/4))/(2*a**(3/4)*b**(7/4)) + (sqrt(a)*(a*i + b*e) + sqrt(b)*(a*
g + b*c))*atanh(b**(1/4)*x/a**(1/4))/(2*a**(3/4)*b**(7/4))
```

Mathematica [A] time = 1.03089, size = 318, normalized size = 1.55

$$-\frac{\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{5/4}\sqrt[4]{b}h+a^{3/2}i+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-a^{5/4}\sqrt[4]{b}h+a^{3/2}i-\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\cdots\right.}{\cdots}}{12b\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 - 3*b^(3/4)*
j*x^4 + (6*(b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*Ar
cTan[(b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*(b^(3/2)*c + a^(1/4)*b^(5
/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g + a^(5/4)*b^(1/4)*h + a^(3/2)*i
)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4) + (3*(b^(3/2)*c - a^(1/4)*b^(
5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/4)*h + a^(3/2)*
i)*Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h)*Log
[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[a] - (3*(b*f + a*j)*Log[a - b*x^4])
/b^(1/4))/(12*b^(7/4))
```

Maple [B] time = 0.009, size = 393, normalized size = 1.9

$$-\frac{jx^4}{4b}-\frac{ix^3}{3b}-\frac{hx^2}{2b}-\frac{gx}{b}+\frac{g}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{g}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)-\frac{ah}{4b}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$
$$-\frac{d}{4}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{ai}{2b^2}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{e}{2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{ai}{4b^2}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{e}{4b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\ln\left(bx^4-a\right)aj}{4b^2}-\frac{f\ln\left(bx^4-a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/4*j*x^4/b-1/3*i*x^3/b-1/2*h*x^2/b-g*x/b+1/2/b*(a/b)^(1/4)*arct
an(x/(a/b)^(1/4))*g+1/2*c*(a/b)^(1/4)/a*arctan(x/(a/b)^(1/4))+1/4
/b*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+1/4*c*(a/b)^
(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b/(a*b)^(1/2)*ln(
(-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*a*h-1/4*d/(a*b)^(1/2)*
ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/2/b^2/(a/b)^(1/4)
*arctan(x/(a/b)^(1/4))*a*i-1/2*e/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/
4))+1/4/b^2/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*a*i+1
/4*e/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b^2*ln
(b*x^4-a)*a*j-1/4/b*f*ln(b*x^4-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x,
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x,
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.230491, size = 828, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x,
```

```
[Out] 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 - sqrt(2)*(-a*b^3)^(3/4)*ln(x^2
+ sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4) + 1/8*i*(2*sqrt(2)*(-
a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/
b)^(1/4))/b^4 + sqrt(2)*(-a*b^3)^(3/4)*ln(x^2 - sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/b^4) - 1/4*(b*f + a*j)*ln(abs(b*x^4 - a))/b^2
 + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b
*h + (-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))
/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-
a*b)*a*b*h + (-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^
3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b
)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(
1/4)*a*b*g - (-a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3
)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4)
 + sqrt(-a/b))/(a*b^3) - 1/12*(3*b^3*j*x^4 + 4*b^3*i*x^3 + 6*b^3*
h*x^2 + 12*b^3*g*x)/b^4
```

3.189 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$

Optimal. Leaf size=337

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}$$

```
[Out] (g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[
a]])/(2*Sqrt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) +
 ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e
 - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(
4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rubi [A] time = 0.874254, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```


```
[Out] (g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[
a]])/(2*Sqrt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) +
 ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e
 - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(
4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{f\log\left(a+bx^4\right)}{4b}+\frac{hx^2}{2b}+\frac{\int g\,dx}{b}-\frac{(ah-bd)\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{3}{2}}}+\frac{\sqrt{2}\left(-\sqrt{a}\sqrt{b}e+ag-bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\left(-\sqrt{a}\sqrt{b}e+ag-bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}\sqrt{b}e+ag-bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}\sqrt{b}e+ag-bc\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] f*log(a + b*x**4)/(4*b) + h*x**2/(2*b) + Integral(g, x)/b - (a*h
- b*d)*atan(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*b**(3/2)) + sqrt(2)*
(-sqrt(a)*sqrt(b)*e + a*g - b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(
1/4))/(4*a**(3/4)*b**(5/4)) - sqrt(2)*(-sqrt(a)*sqrt(b)*e + a*g -
 b*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(5/4))
 + sqrt(2)*(sqrt(a)*sqrt(b)*e + a*g - b*c)*log(-sqrt(2)*a**(1/4)*
b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a**(3/4)*b**(5/4)) - sq
rt(2)*(sqrt(a)*sqrt(b)*e + a*g - b*c)*log(sqrt(2)*a**(1/4)*b**(3/
4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a**(3/4)*b**(5/4))
```

Mathematica [A] time = 0.742981, size = 342, normalized size = 1.01

$$-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-2a^{5/4}h+\sqrt{2}\sqrt{a}b^{3/4}e+2\sqrt[4]{a}bd-\sqrt{2}a\sqrt[4]{b}g+\sqrt{2}b^{5/4}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(2a^{5/4}h+\sqrt{2}\sqrt{a}b^{3/4}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(5/4)*c + 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*
e - Sqrt[2]*a*b^(1/4)*g - 2*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[2]*Sq
rt[a]*b^(3/4)*e - Sqrt[2]*a*b^(1/4)*g + 2*a^(5/4)*h)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4)*(Sqrt[2]*(-(b*c) + Sqrt[a]*
Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2] + Sqrt[2]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*(2*x*(
2*g + h*x) + f*Log[a + b*x^4]))))/(8*a^(3/4)*b^(3/2))
```

Maple [A] time = 0.007, size = 462, normalized size = 1.4

$$\frac{hx^{2}}{2\,b}+\frac{gx}{b}-\frac{\sqrt{2}g}{4\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{\sqrt{2}g}{8\,b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^{2}+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^{2}-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{c\sqrt{2}}{8\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^{2}+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^{2}-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}g}{4\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{ah}{2\,b}\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}$$
$$+\frac{d}{2}\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}+\frac{e\sqrt{2}}{8\,b}\ln\left(1\left(x^{2}-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^{2}+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{e\sqrt{2}}{4\,b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e\sqrt{2}}{4\,b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{f\ln\left(bx^{4}+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/2*h*x^2/b+g*x/b-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x-1)*g+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)-1/8/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+1/8*c*(a/b
)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2
-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4/b*(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x+1)-1/2/b/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/
2))*a*h+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e/b/(a/b)^(
1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*e/b/(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x-1)+1/4*f*ln(b*x^4+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="fric
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.222625, size = 506, normalized size = 1.5

$$\frac{f\ln\left(\left|bx^4+a\right|\right)}{4\,b}+\frac{bhx^2+2\,bgx}{2\,b^2}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d+\sqrt{2}\sqrt{ab}abh+\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{1}{4}}abg+\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d+\sqrt{2}\sqrt{ab}abh+\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{1}{4}}abg+\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{1}{4}}abg-\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{1}{4}}abg-\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="giac
```

```
[Out] 1/4*f*ln(abs(b*x^4 + a))/b + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/4*sq
rt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3
)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*s
qrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^
3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*
sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4
)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sq
rt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*
e)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.190 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx$

Optimal. Leaf size=384

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}$$

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sq
rt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g)
+ Sqrt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e
 - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
(3/4)*b^(7/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rubi [A] time = 1.23481, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```


```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sq
rt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g)
+ Sqrt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e
 - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
```

```
(3/4)*b^(7/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{f\log\left(a+bx^4\right)}{4b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{\int g\,dx}{b}-\frac{(ah-bd)\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{3}{2}}}$$
$$-\frac{\sqrt{2}\left(\sqrt{a}\,(ai-be)-\sqrt{b}\,(ag-bc)\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{7}{4}}}$$
$$+\frac{\sqrt{2}\left(\sqrt{a}\,(ai-be)-\sqrt{b}\,(ag-bc)\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{7}{4}}}$$
$$+\frac{\sqrt{2}\left(\sqrt{a}\,(ai-be)+\sqrt{b}\,(ag-bc)\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{7}{4}}}$$
$$-\frac{\sqrt{2}\left(\sqrt{a}\,(ai-be)+\sqrt{b}\,(ag-bc)\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] f*log(a + b*x**4)/(4*b) + h*x**2/(2*b) + i*x**3/(3*b) + Integral(
g, x)/b - (a*h - b*d)*atan(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*b**(3
/2)) - sqrt(2)*(sqrt(a)*(a*i - b*e) - sqrt(b)*(a*g - b*c))*log(-s
qrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a**(3/4
)*b**(7/4)) + sqrt(2)*(sqrt(a)*(a*i - b*e) - sqrt(b)*(a*g - b*c))
*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a
**(3/4)*b**(7/4)) + sqrt(2)*(sqrt(a)*(a*i - b*e) + sqrt(b)*(a*g -
 b*c))*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(7/4)
) - sqrt(2)*(sqrt(a)*(a*i - b*e) + sqrt(b)*(a*g - b*c))*atan(1 +
sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(7/4))
```

Mathematica [A] time = 0.587434, size = 427, normalized size = 1.11

$$\frac{6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g-\sqrt{2}b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(2a^{5/4}\sqrt[4]{b}h-\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be-\sqrt{2}\right.}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + (6*(-(Sqrt
[2]*b^(3/2)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt
[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*
c - 2*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e - Sqrt[2]*a*Sqrt[b]
*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]*b
*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]*b*
e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/a^(3/4) + 6*b^(3/4)*f*Log[a + b*x^4])/(24*b^(
7/4))
```

Maple [B] time = 0.007, size = 603, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/3*i*x^3/b+1/2*h*x^2/b+g*x/b-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x-1)*g+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x-1)-1/8/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)
*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*
g+1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4/b*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+1/4*c*(a/b)^(1/4)/a*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/2/b/(a*b)^(1/2)*arctan(x
^2*(b/a)^(1/2))*a*h+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))-1/8
/b^2/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*a*i+1/8*e/b/(a/b)^(1/
4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^
(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4/b^2/(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x-1)*a*i+1/4*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x-1)-1/4/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/b)^(1/4)*x+1)*a*i+1/4*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x+1)+1/4*f*ln(b*x^4+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorith
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorith
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22828, size = 759, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorith
```

```
[Out] -1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(a/b)^(1/4))/(a/b)^(1/4))/b^4 - sqrt(2)*(a*b^3)^(3/4)*ln(x^2 + s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4) - 1/8*i*(2*sqrt(2)*(a*b^3)
^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/b^4 + sqrt(2)*(a*b^3)^(3/4)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/b^4) + 1/4*f*ln(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)
*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c
- (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2
)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c
 - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*ln(x^2 +
sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)
^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*ln(x^2 - sq
rt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/6*(2*b^2*i*x^3 + 3*b
^2*h*x^2 + 6*b^2*g*x)/b^3
```

3.191 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$

Optimal. Leaf size=402

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{(bf-aj)\log\left(a+bx^4\right)}{4b^2}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}$$

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d -
 a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt
[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g
) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b
*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(
b*e - a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*
b^2)
```

Rubi [A] time = 1.2483, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{(bf-aj)\log\left(a+bx^4\right)}{4b^2}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d -
 a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt
[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g
) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b
*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(
```

```
b*e - a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*
b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ix^3}{3b}+\frac{\int g\,dx}{b}+\frac{\int^{x^2} h\,dx}{2b}+\frac{\int^{x^2} x\,dx}{2b}-\frac{(a-bf)\log\left(a+bx^4\right)}{4b^2}-\frac{(ah-bd)\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{3}{2}}}$$
$$-\frac{\sqrt{2}\left(\sqrt{a}\,(ai-be)-\sqrt{b}\,(ag-bc)\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{7}{4}}}$$
$$+\frac{\sqrt{2}\left(\sqrt{a}\,(ai-be)-\sqrt{b}\,(ag-bc)\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{7}{4}}}$$
$$+\frac{\sqrt{2}\left(\sqrt{a}\,(ai-be)+\sqrt{b}\,(ag-bc)\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{7}{4}}}$$
$$-\frac{\sqrt{2}\left(\sqrt{a}\,(ai-be)+\sqrt{b}\,(ag-bc)\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] i*x**3/(3*b) + Integral(g, x)/b + Integral(h, (x, x**2))/(2*b) +
Integral(x, (x, x**2))/(2*b) - (a - b*f)*log(a + b*x**4)/(4*b**2)
 - (a*h - b*d)*atan(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)*b**(3/2)) -
sqrt(2)*(sqrt(a)*(a*i - b*e) - sqrt(b)*(a*g - b*c))*log(-sqrt(2)*
a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a**(3/4)*b**(7
/4)) + sqrt(2)*(sqrt(a)*(a*i - b*e) - sqrt(b)*(a*g - b*c))*log(sq
rt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a**(3/4)
*b**(7/4)) + sqrt(2)*(sqrt(a)*(a*i - b*e) + sqrt(b)*(a*g - b*c))*
atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(7/4)) - sqr
t(2)*(sqrt(a)*(a*i - b*e) + sqrt(b)*(a*g - b*c))*atan(1 + sqrt(2)
*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(7/4))
```

Mathematica [A] time = 0.626573, size = 445, normalized size = 1.11

$$\frac{\frac{6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g-\sqrt{2}b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(2a^{5/4}\sqrt[4]{b}h-\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be-\sqrt{2}\right.}{a^{3/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + 6*b^(3/4)*
j*x^4 + (6*(-(Sqrt[2]*b^(3/2)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*
Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + Sqrt[2]
*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (6
*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e -
 Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[2]*a^(3/2)*i)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]*(b^(
3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3
/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (6*(b*f - a*j)*Lo
g[a + b*x^4])/b^(1/4))/(24*b^(7/4))
```

Maple [B] time = 0.007, size = 627, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/4*j*x^4/b+1/3*i*x^3/b+1/2*h*x^2/b+g*x/b-1/4/b*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+1/4*c*(a/b)^(1/4)/a*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x-1)*g+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x-1)-1/8/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/
4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))
)*g+1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/2/b/(a*b)^(1
/2)*arctan(x^2*(b/a)^(1/2))*a*h+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a
)^(1/2))-1/8/b^2/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2
)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*a*i+1/8*e
/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))
/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4/b^2/(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*a*i+1/4*e/b/(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/b^2/(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1)*a*i+1/4*e/b/(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/4/b^2*ln(b*x^4+a)*a*j+1/4*f*ln(b*
x^4+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, a
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, a
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.227175, size = 780, normalized size = 1.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, a
```

```
[Out] -1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(a/b)^(1/4))/(a/b)^(1/4))/b^4 - sqrt(2)*(a*b^3)^(3/4)*ln(x^2 + s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4) - 1/8*i*(2*sqrt(2)*(a*b^3)
^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/b^4 + sqrt(2)*(a*b^3)^(3/4)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/b^4) + 1/4*(b*f - a*j)*ln(abs(b*x^4 + a))/b^2 + 1/4*sqrt
(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^
(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqr
t(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)
^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sq
rt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*
e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt
(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)
*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/12*(3*b^
3*j*x^4 + 4*b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*g*x)/b^4
```

3.192 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=184

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b
*x^4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^
(1/4)])/(8*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*
ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + ((b*d - a*h)*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.462176, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b
*x^4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^
(1/4)])/(8*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*
ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + ((b*d - a*h)*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))
```

Rubi in Sympy [A] time = 79.1263, size = 167, normalized size = 0.91

$$\frac{x\left(ag+bc+bex^2+bfx^3+x\left(ah+bd\right)\right)}{4ab\left(a-bx^4\right)}-\frac{(ah-bd)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}b^{\frac{3}{2}}}-\frac{\left(-\sqrt{a}\sqrt{b}e+ag-3bc\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{5}{4}}}-\frac{\left(\sqrt{a}\sqrt{b}e+ag-3bc\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] x*(a*g + b*c + b*e*x**2 + b*f*x**3 + x*(a*h + b*d))/(4*a*b*(a - b
*x**4)) - (a*h - b*d)*atanh(sqrt(b)*x**2/sqrt(a))/(4*a**(3/2)*b**
(3/2)) - (-sqrt(a)*sqrt(b)*e + a*g - 3*b*c)*atanh(b**(1/4)*x/a**(
1/4))/(8*a**(7/4)*b**(5/4)) - (sqrt(a)*sqrt(b)*e + a*g - 3*b*c)*a
tan(b**(1/4)*x/a**(1/4))/(8*a**(7/4)*b**(5/4))
```

Mathematica [A] time = 0.302911, size = 257, normalized size = 1.4

$$\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2a^{5/4}h-\sqrt{a}b^{3/4}e-2\sqrt[4]{a}bd+a\sqrt[4]{b}g-3b^{5/4}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(2a^{5/4}h+\sqrt{a}b^{3/4}e-2\sqrt[4]{a}bd-a\sqrt[4]{b}g+3b\right.}{16a^{7/4}b^{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x)))
)/(a - b*x^4) - 2*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcT
an[(b^(1/4)*x)/a^(1/4)] + (-3*b^(5/4)*c - 2*a^(1/4)*b*d - Sqrt[a]
*b^(3/4)*e + a*b^(1/4)*g + 2*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x]
+ (3*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e - a*b^(1/4)*g
+ 2*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4)*(-(b*d) + a*h
)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(3/2))
```

Maple [B] time = 0.013, size = 372, normalized size = 2.

$$\frac{1}{bx^4-a}\left(-\frac{ex^3}{4a}-\frac{(ah+bd)x^2}{4ab}-\frac{(ag+bc)x}{4ab}-\frac{f}{4b}\right)$$
$$-\frac{g}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{3c}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{g}{16ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3c}{16a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{ah}{8}\ln\left(1\left(-a^2b+x^2\sqrt{a^3b^3}\right)\left(-a^2b-x^2\sqrt{a^3b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^3b^3}}$$
$$-\frac{bd}{8}\ln\left(1\left(-a^2b+x^2\sqrt{a^3b^3}\right)\left(-a^2b-x^2\sqrt{a^3b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^3b^3}}$$
$$-\frac{e}{8ab}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e}{16ab}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4/a*e*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4*f/b)/
(b*x^4-a)-1/8*(a/b)^(1/4)/a/b*arctan(x/(a/b)^(1/4))*g+3/8*c/a^2*(
a/b)^(1/4)*arctan(x/(a/b)^(1/4))-1/16*(a/b)^(1/4)/a/b*ln((x+(a/b)
^(1/4))/(x-(a/b)^(1/4)))*g+3/16*c/a^2*(a/b)^(1/4)*ln((x+(a/b)^(1/
4))/(x-(a/b)^(1/4)))+1/8/(a^3*b^3)^(1/2)*ln((-a^2*b+x^2*(a^3*b^3)
^(1/2))/(-a^2*b-x^2*(a^3*b^3)^(1/2)))*a*h-1/8*b*d/(a^3*b^3)^(1/2)
*ln((-a^2*b+x^2*(a^3*b^3)^(1/2))/(-a^2*b-x^2*(a^3*b^3)^(1/2)))-1/
8*e/a/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))+1/16*e/a/b/(a/b)^(1/4)*
ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="fr
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225356, size = 574, normalized size = 3.12

$$-\frac{bx^3e+bdx^2+ahx^2+bcx+agx+af}{4\,(bx^4-a)ab}-\frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-ab}b^2d-2\,\sqrt{2}\sqrt{-ab}abh-3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{1}{4}}abg-\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}-\frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-ab}b^2d-2\,\sqrt{2}\sqrt{-ab}abh-3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{1}{4}}abg-\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{1}{4}}abg-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b^3}-\frac{\sqrt{2}\left(3\,\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{1}{4}}abg-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algorithm="gi
```

```
[Out] -1/4*(b*x^3*e + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/((b*x^4
- a)*a*b) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*
sqrt(-a*b)*a*b*h - 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g
- (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4
))/(-a/b)^(1/4))/(a^2*b^3) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b
^2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h - 3*(-a*b^3)^(1/4)*b^2*c + (-a*
b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(-a
*b^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*ln(x
^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2
)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)
*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3)
```

3.193 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=203

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*
b*(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)
*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt
[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]
])/(4*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.593013, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*
b*(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)
*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt
[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]
])/(4*a^(3/2)*b^(3/2))
```

Rubi in Sympy [A] time = 86.02, size = 194, normalized size = 0.96

$$\frac{x\left(ag+bc+bfx^3+x^2(ai+be)+x(ah+bd)\right)}{4ab\left(a-bx^4\right)}-\frac{(ah-bd)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}b^{\frac{3}{2}}}-\frac{\left(\sqrt{a}(3ai-be)+a\sqrt{b}g-3b^{\frac{3}{2}}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{7}{4}}}+\frac{\left(3a^{\frac{3}{2}}i-\sqrt{a}be-a\sqrt{b}g+3b^{\frac{3}{2}}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] x*(a*g + b*c + b*f*x**3 + x**2*(a*i + b*e) + x*(a*h + b*d))/(4*a*
b*(a - b*x**4)) - (a*h - b*d)*atanh(sqrt(b)*x**2/sqrt(a))/(4*a**(
3/2)*b**(3/2)) - (sqrt(a)*(3*a*i - b*e) + a*sqrt(b)*g - 3*b**(3/2
)*c)*atanh(b**(1/4)*x/a**(1/4))/(8*a**(7/4)*b**(7/4)) + (3*a**(3/
2)*i - sqrt(a)*b*e - a*sqrt(b)*g + 3*b**(3/2)*c)*atan(b**(1/4)*x/
a**(1/4))/(8*a**(7/4)*b**(7/4))
```

Mathematica [A] time = 0.44694, size = 302, normalized size = 1.49

$$\frac{4a^{3/4}b^{3/4}(a(f+x(g+x(h+ix)))+bx(c+x(d+ex)))}{a-bx^4}+\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{a}be+a\sqrt{b}g-3b^{3/2}c\right)+\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + x*(h +
 i*x)))))/(a - b*x^4) + 2*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*
g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-3*b^(3/2)*c - 2*
a^(1/4)*b^(5/4)*d - Sqrt[a]*b*e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)
*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + (3*b^(3/2)*c - 2*a^(
1/4)*b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h
- 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4)*b^(1/4)*(-(b*
d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(7/4))
```

Maple [B] time = 0.013, size = 441, normalized size = 2.2

$$\frac{1}{bx^4-a}\left(-\frac{(ai+be)x^3}{4ab}-\frac{(ah+bd)x^2}{4ab}-\frac{(ag+bc)x}{4ab}-\frac{f}{4b}\right)$$
$$-\frac{g}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{3c}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{g}{16ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3c}{16a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{ah}{8}\ln\left(1\left(-a^2b+x^2\sqrt{a^3b^3}\right)\left(-a^2b-x^2\sqrt{a^3b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^3b^3}}$$
$$-\frac{bd}{8}\ln\left(1\left(-a^2b+x^2\sqrt{a^3b^3}\right)\left(-a^2b-x^2\sqrt{a^3b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^3b^3}}$$
$$+\frac{3i}{8b^2}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{e}{8ab}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{3i}{16b^2}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e}{16ab}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i+b*e)/a/b*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x
-1/4*f/b)/(b*x^4-a)-1/8*(a/b)^(1/4)/a/b*arctan(x/(a/b)^(1/4))*g+3
/8*c/a^2*(a/b)^(1/4)*arctan(x/(a/b)^(1/4))-1/16*(a/b)^(1/4)/a/b*l
n((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+3/16*c/a^2*(a/b)^(1/4)*ln((x
+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/8/(a^3*b^3)^(1/2)*ln((-a^2*b+x^2
*(a^3*b^3)^(1/2))/(-a^2*b-x^2*(a^3*b^3)^(1/2)))*a*h-1/8*b*d/(a^3*
b^3)^(1/2)*ln((-a^2*b+x^2*(a^3*b^3)^(1/2))/(-a^2*b-x^2*(a^3*b^3)^
(1/2)))+3/8/b^2/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))*i-1/8*e/a/b/(a/
b)^(1/4)*arctan(x/(a/b)^(1/4))-3/16/b^2/(a/b)^(1/4)*ln((x+(a/b)^(
1/4))/(x-(a/b)^(1/4)))*i+1/16*e/a/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4)
)/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algori
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algori
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.229687, size = 848, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x, algori
```

```
[Out] -3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) - sqrt(2)*(-a*b^3)^(3/4)*l
n(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 3/32*i*(2
*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^
(1/4))/(-a/b)^(1/4))/(a*b^4) + sqrt(2)*(-a*b^3)^(3/4)*ln(x^2 - sq
rt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 1/4*(a*i*x^3 + b*x^
3*e + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a)*a*b)
 - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)
*a*b*h - 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)
^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^
(1/4))/(a^2*b^3) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*s
qrt(2)*sqrt(-a*b)*a*b*h - 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)
*a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(-a*b^3)^(1/4
)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*ln(x^2 + sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b
^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*ln(x^2
 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3)
```

3.194 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=225

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a-bx^4\right)}{4b^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{4ab\left(a-bx^4\right)}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3
))/(4*a*b*(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a]
- 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e
 + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a
^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)
/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) + (j*Log[a - b*x^4])/(4*b^2)
```

Rubi [A] time = 0.743876, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a-bx^4\right)}{4b^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3
))/(4*a*b*(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a]
- 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e
 + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a
^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)
/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) + (j*Log[a - b*x^4])/(4*b^2)
```

Rubi in Sympy [A] time = 131.579, size = 212, normalized size = 0.94

$$\frac{j\log\left(a-bx^4\right)}{4b^2}+\frac{x\left(ag+bc+x^3\left(aj+bf\right)+x^2\left(ai+be\right)+x\left(ah+bd\right)\right)}{4ab\left(a-bx^4\right)}-\frac{(ah-bd)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}b^{\frac{3}{2}}}-\frac{\left(\sqrt{a}\left(3ai-be\right)+a\sqrt{b}g-3b^{\frac{3}{2}}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{7}{4}}}+\frac{\left(3a^{\frac{3}{2}}i-\sqrt{a}be-a\sqrt{b}g+3b^{\frac{3}{2}}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-
b*x**4+a)**2,x)
```

```
[Out] j*log(a - b*x**4)/(4*b**2) + x*(a*g + b*c + x**3*(a*j + b*f) + x*
*2*(a*i + b*e) + x*(a*h + b*d))/(4*a*b*(a - b*x**4)) - (a*h - b*d
)*atanh(sqrt(b)*x**2/sqrt(a))/(4*a**(3/2)*b**(3/2)) - (sqrt(a)*(3
*a*i - b*e) + a*sqrt(b)*g - 3*b**(3/2)*c)*atanh(b**(1/4)*x/a**(1/
4))/(8*a**(7/4)*b**(7/4)) + (3*a**(3/2)*i - sqrt(a)*b*e - a*sqrt(
b)*g + 3*b**(3/2)*c)*atan(b**(1/4)*x/a**(1/4))/(8*a**(7/4)*b**(7/
4))
```

Mathematica [A] time = 0.459414, size = 338, normalized size = 1.5

$$\frac{\frac{\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{a}be+a\sqrt{b}g-3b^{3/2}c\right)}{a^{7/4}}+\frac{\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h-3a^{3/2}i-2\sqrt[4]{a}b^{5/4}d+\sqrt{a}be-a\sqrt{b}g+3b^{3/2}c\right)}{a^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2,x
```

```
[Out] ((4*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b*(f + x*(g + x*(h + i*x
)))))/(a*(a - b*x^4)) + (2*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e - a
*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(7/4) +
(b^(1/4)*(-3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d - Sqrt[a]*b*e + a*Sq
rt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4
)*x])/a^(7/4) + (b^(1/4)*(3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqr
t[a]*b*e - a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a
^(1/4) + b^(1/4)*x])/a^(7/4) + (2*Sqrt[b]*(b*d - a*h)*Log[Sqrt[a]
 + Sqrt[b]*x^2])/a^(3/2) + 4*j*Log[a - b*x^4])/(16*b^2)
```

Maple [B] time = 0.015, size = 466, normalized size = 2.1

$$\frac{1}{bx^4-a}\left(-\frac{(ai+be)x^3}{4ab}-\frac{(ah+bd)x^2}{4ab}-\frac{(ag+bc)x}{4ab}-\frac{aj+bf}{4b^2}\right)$$
$$-\frac{g}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{3c}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$
$$-\frac{g}{16ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3c}{16a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{ah}{8}\ln\left(1\left(-a^2b+x^2\sqrt{a^3b^3}\right)\left(-a^2b-x^2\sqrt{a^3b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^3b^3}}$$
$$-\frac{bd}{8}\ln\left(1\left(-a^2b+x^2\sqrt{a^3b^3}\right)\left(-a^2b-x^2\sqrt{a^3b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^3b^3}}+\frac{3i}{8b^2}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{e}{8ab}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{3i}{16b^2}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{e}{16ab}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{j\ln\left(ab\left(bx^4-a\right)\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i+b*e)/a/b*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x
-1/4*(a*j+b*f)/b^2)/(b*x^4-a)-1/8*(a/b)^(1/4)/a/b*arctan(x/(a/b)^
(1/4))*g+3/8*c/a^2*(a/b)^(1/4)*arctan(x/(a/b)^(1/4))-1/16*(a/b)^(
1/4)/a/b*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+3/16*c/a^2*(a/b)^(
1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/8/(a^3*b^3)^(1/2)*ln((
-a^2*b+x^2*(a^3*b^3)^(1/2))/(-a^2*b-x^2*(a^3*b^3)^(1/2)))*a*h-1/8
*b*d/(a^3*b^3)^(1/2)*ln((-a^2*b+x^2*(a^3*b^3)^(1/2))/(-a^2*b-x^2*
(a^3*b^3)^(1/2)))+3/8/b^2/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))*i-1/8
*e/a/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))-3/16/b^2/(a/b)^(1/4)*ln(
(x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*i+1/16*e/a/b/(a/b)^(1/4)*ln((x+(
```

```
a/b)^(1/4))/(x-(a/b)^(1/4)))+1/4*j/b^2*ln(a*b*(b*x^4-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.229775, size = 884, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^2,x

[Out] -3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) - sqrt(2)*(-a*b^3)^(3/4)*l
n(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 3/32*i*(2
*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^
(1/4))/(-a/b)^(1/4))/(a*b^4) + sqrt(2)*(-a*b^3)^(3/4)*ln(x^2 - sq
rt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) + 1/4*j*ln(abs(b*x^4
- a))/b^2 - 1/4*((a*i + b*e)*x^3 + (b*d + a*h)*x^2 + (b*c + a*g)*
x + (a*b*f + a^2*j)/b)/((b*x^4 - a)*a*b) - 1/16*sqrt(2)*(2*sqrt(2
)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h - 3*(-a*b^3)^(1/4
)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) - 1/16*
sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h
- 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*
e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
(a^2*b^3) + 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)
```

```
*a*b*g - (-a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt
(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^
3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4
) + sqrt(-a/b))/(a^2*b^3)
```

3.195 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=353

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(ah+bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b
*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b
^(3/2)) - ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt
[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*
Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*
a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*
b^(5/4))
```

Rubi [A] time = 0.779079, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(ah+bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b
*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b
^(3/2)) - ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt
[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*
Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*
a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*
b^(5/4))
```

Rubi in Sympy [A] time = 131.176, size = 333, normalized size = 0.94

$$-\frac{x\left(ag-bc-bex^2-bfx^3+x(ah-bd)\right)}{4ab(a+bx^4)}+\frac{(ah+bd)\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}b^{\frac{3}{2}}}$$
$$-\frac{\sqrt{2}\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{7}{4}}b^{\frac{5}{4}}}$$
$$+\frac{\sqrt{2}\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{7}{4}}b^{\frac{5}{4}}}$$
$$-\frac{\sqrt{2}\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] -x*(a*g - b*c - b*e*x**2 - b*f*x**3 + x*(a*h - b*d))/(4*a*b*(a +
b*x**4)) + (a*h + b*d)*atan(sqrt(b)*x**2/sqrt(a))/(4*a**(3/2)*b**
(3/2)) - sqrt(2)*(-sqrt(a)*sqrt(b)*e + a*g + 3*b*c)*log(-sqrt(2)*
a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(32*a**(7/4)*b**(
5/4)) + sqrt(2)*(-sqrt(a)*sqrt(b)*e + a*g + 3*b*c)*log(sqrt(2)*a*
*(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(32*a**(7/4)*b**(5/
4)) - sqrt(2)*(sqrt(a)*sqrt(b)*e + a*g + 3*b*c)*atan(1 - sqrt(2)*
b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(5/4)) + sqrt(2)*(sqrt(a)*sq
rt(b)*e + a*g + 3*b*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*
a**(7/4)*b**(5/4))
```

Mathematica [A] time = 0.471884, size = 359, normalized size = 1.02

$$-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4a^{5/4}h+\sqrt{2}\sqrt{a}b^{3/4}e+4\sqrt[4]{a}bd+\sqrt{2}a\sqrt[4]{b}g+3\sqrt{2}b^{5/4}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-4a^{5/4}h+\sqrt{2}\sqrt{a}b^{3/}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*
x))))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(5/4)*c + 4*a^(1/4)*b*d + Sqrt
[2]*Sqrt[a]*b^(3/4)*e + Sqrt[2]*a*b^(1/4)*g + 4*a^(5/4)*h)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b^(5/4)*c - 4*a^
(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e + Sqrt[2]*a*b^(1/4)*g - 4*a
^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/
4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*b*c - Sqrt[a]*Sq
rt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(32*a^(7/4)*b^(3/2))
```

Maple [A] time = 0.013, size = 519, normalized size = 1.5

$$\frac{1}{bx^4+a}\left(\frac{ex^3}{4a}-\frac{(ah-bd)\,x^2}{4ab}-\frac{(ag-bc)\,x}{4ab}-\frac{f}{4b}\right)$$
$$+\frac{\sqrt{2}g}{16\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{3\,c\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{\sqrt{2}g}{32\,ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{3\,c\sqrt{2}}{32\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}g}{16\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\,c\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$
$$+\frac{ah}{4}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{a^3b^3}}+\frac{bd}{4}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{a^3b^3}}$$
$$+\frac{e\sqrt{2}}{32\,ab}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{e\sqrt{2}}{16\,ab}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e\sqrt{2}}{16\,ab}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (1/4/a*e*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(
b*x^4+a)+1/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x-1)*g+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x-1)+1/32/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+3/32*c/a^2
*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(
x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/16/b/a*(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+3/16*c/a^2*(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a^3*b^3)^(1/2)*arctan(x^
2*(b/a)^(1/2))*a*h+1/4*b*d/(a^3*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2)
)+1/32*e/a/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a
/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/16*e/a/b/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/16*e/a/b/(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="fr
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.224194, size = 537, normalized size = 1.52

$$\frac{bx^3e + bdx^2 - ahx^2 + bcx - agx - af}{4\,(bx^4 + a)ab} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d + 2\,\sqrt{2}\sqrt{ab}abh + 3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d + 2\,\sqrt{2}\sqrt{ab}abh + 3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="gi
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 - a*h*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 +
 a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sq
rt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*
sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a
*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*
c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b
^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*
(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.196 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=395

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{(ah+bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*
b*(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a
^(3/2)*b^(3/2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i)
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(
7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) - ((
Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)
) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(7/4))
```

Rubi [A] time = 1.16444, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{(ah+bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x]
```


```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*
b*(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a
^(3/2)*b^(3/2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i)
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(
7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) - ((
Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)
```

```
) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(7/4))
```

Rubi in Sympy [A] time = 139.125, size = 364, normalized size = 0.92

$$
\begin{aligned}
&-\frac{x\left(ag-bc-bfx^{3}+x^{2}(ai-be)+x(ah-bd)\right)}{4ab\left(a+bx^{4}\right)}+\frac{(ah+bd)\operatorname{atan}\left(\frac{\sqrt{b}x^{2}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}b^{\frac{3}{2}}}\\
&+\frac{\sqrt{2}\left(\sqrt{a}(3ai+be)-\sqrt{b}(ag+3bc)\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^{2}\right)}{32a^{\frac{7}{4}}b^{\frac{7}{4}}}\\
&-\frac{\sqrt{2}\left(\sqrt{a}(3ai+be)-\sqrt{b}(ag+3bc)\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^{2}\right)}{32a^{\frac{7}{4}}b^{\frac{7}{4}}}\\
&-\frac{\sqrt{2}\left(\sqrt{a}(3ai+be)+\sqrt{b}(ag+3bc)\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{7}{4}}}\\
&+\frac{\sqrt{2}\left(\sqrt{a}(3ai+be)+\sqrt{b}(ag+3bc)\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{7}{4}}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] -x*(a*g - b*c - b*f*x**3 + x**2*(a*i - b*e) + x*(a*h - b*d))/(4*a
*b*(a + b*x**4)) + (a*h + b*d)*atan(sqrt(b)*x**2/sqrt(a))/(4*a**(
3/2)*b**(3/2)) + sqrt(2)*(sqrt(a)*(3*a*i + b*e) - sqrt(b)*(a*g +
3*b*c))*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x*
*2)/(32*a**(7/4)*b**(7/4)) - sqrt(2)*(sqrt(a)*(3*a*i + b*e) - sqr
t(b)*(a*g + 3*b*c))*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqr
t(b) + b*x**2)/(32*a**(7/4)*b**(7/4)) - sqrt(2)*(sqrt(a)*(3*a*i +
 b*e) + sqrt(b)*(a*g + 3*b*c))*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/
4))/(16*a**(7/4)*b**(7/4)) + sqrt(2)*(sqrt(a)*(3*a*i + b*e) + sqr
t(b)*(a*g + 3*b*c))*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**
(7/4)*b**(7/4))
```

Mathematica [A] time = 0.715132, size = 415, normalized size = 1.05

$$
-\frac{8a^{3/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+3
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + x*
(h + i*x)))))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^
(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b
^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)] + 2*(3*Sqrt[2]*b^(3/2)*c - 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*S
qrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g - 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2
]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-
3*b^(3/2)*c + Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b^(3/2)
*c - Sqrt[a]*b*e + a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(7/4))
```

Maple [B] time = 0.016, size = 658, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x
-1/4*f/b)/(b*x^4+a)+1/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)*g+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)+1/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x-1)*g+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x-1)+1/32/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^
(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+3/3
2*c/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4/(a^3*b^3)^(1/2
)*arctan(x^2*(b/a)^(1/2))*a*h+1/4*b*d/(a^3*b^3)^(1/2)*arctan(x^2*
(b/a)^(1/2))+3/32/b^2/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*i+1/
32*e/a/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^
(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+3/16/b^2/(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*i+1/16*e/a/b/(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/16/b^2/(a/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*i+1/16*e/a/b/(a/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algori
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algori
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.23202, size = 795, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algori
```

```
[Out] 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) - sqrt(2)*(a*b^3)^(3/4)*ln(x^2
 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)) + 3/32*i*(2*sqrt(2
)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a
/b)^(1/4))/(a*b^4) + sqrt(2)*(a*b^3)^(3/4)*ln(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a*b^4)) - 1/4*(a*i*x^3 - b*x^3*e - b*d*x^2
 + a*h*x^2 - b*c*x + a*g*x + a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt(2
)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b
^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1
/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*
h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^
2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*
g - (a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a
*b*g - (a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b
))/(a^2*b^3)
```

3.197 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=417

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{(ah+bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a+bx^4\right)}{4b^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{4ab\left(a+bx^4\right)}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3
))/(4*a*b*(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a
]])/(4*a^(3/2)*b^(3/2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e
+ 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/
4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a
^(7/4)*b^(7/4)) + (j*Log[a + b*x^4])/(4*b^2)
```

Rubi [A] time = 1.27795, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{(ah+bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a+bx^4\right)}{4b^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2,x]
```


```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3
))/(4*a*b*(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a
]])/(4*a^(3/2)*b^(3/2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e
+ 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/
4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a
```

```
^(7/4)*b^(7/4)) + (j*Log[a + b*x^4])/(4*b^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2
```

```
[Out] Timed out
```

Mathematica [A] time = 0.708339, size = 460, normalized size = 1.1

$$-\frac{2\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+3\sqrt{2}b^{3/2}c\right)}{a^{7/4}}+\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i-4\sqrt[4]{a}b^{5/4}\right.}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2,x
```

```
[Out] ((8*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x
)))))/(a*(a + b*x^4)) - (2*b^(1/4)*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/
4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5
/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/a^(7/4) + (2*b^(1/4)*(3*Sqrt[2]*b^(3/2)*c - 4*a^(1/4
)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g - 4*a^(5/
4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/a^(7/4) + (Sqrt[2]*b^(1/4)*(-3*b^(3/2)*c + Sqrt[a]*b*
e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/a^(7/4) + (Sqrt[2]*b^(1/4)*(3*b^(3/2)*c - S
qrt[a]*b*e + a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + 8*j*Log[a + b*x^4])/(32*
b^2)
```

Maple [B] time = 0.016, size = 682, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x
+1/4*(a*j-b*f)/b^2)/(b*x^4+a)+1/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)*g+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)+1/32/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a
/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2)))*g+3/32*c/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/16
/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+3/16*c
/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a^3
*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2))*a*h+1/4*b*d/(a^3*b^3)^(1/2)*a
rctan(x^2*(b/a)^(1/2))+3/32/b^2/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)
^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2)))*i+1/32*e/a/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1
/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+3/16/b^
2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*i+1/16*e/a/
```

```
b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/16/b^2/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*i+1/16*e/a/b/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*j/b^2*ln(a*
b*(b*x^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.228776, size = 833, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x

[Out] 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) - sqrt(2)*(a*b^3)^(3/4)*ln(x^2
 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)) + 3/32*i*(2*sqrt(2
)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a
/b)^(1/4))/(a*b^4) + sqrt(2)*(a*b^3)^(3/4)*ln(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a*b^4)) + 1/4*j*ln(abs(b*x^4 + a))/b^2 - 1
/4*((a*i - b*e)*x^3 - (b*d - a*h)*x^2 - (b*c - a*g)*x + (a*b*f -
a^2*j)/b)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b
^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3
)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*s
qrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*
```

```
c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3
*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*ln(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)
*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*
ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.198 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=241

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(2x(3bd-ah)-ag+7bc+5bex^2\right)+4af}{32a^2b\left(a-bx^4\right)}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b
*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + 5*b*e*x^
2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a
*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((21*b*c
 + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64
*a^(11/4)*b^(5/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]
])/(16*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.710137, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(2x(3bd-ah)-ag+7bc+5bex^2\right)+4af}{32a^2b\left(a-bx^4\right)}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```


```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b
*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + 5*b*e*x^
2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a
*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((21*b*c
 + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64
*a^(11/4)*b^(5/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]
])/(16*a^(5/2)*b^(3/2))
```

Rubi in Sympy [A] time = 117.367, size = 226, normalized size = 0.94

$$\frac{x\left(ag+bc+bex^2+bfx^3+x\left(ah+bd\right)\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af-x\left(ag-7bc-5bex^2+2x\left(ah-3bd\right)\right)}{32a^2b\left(a-bx^4\right)}-\frac{\left(ah-3bd\right)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}b^{\frac{3}{2}}}-\frac{\left(-5\sqrt{a}\sqrt{b}e+3ag-21bc\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{5}{4}}}-\frac{\left(5\sqrt{a}\sqrt{b}e+3ag-21bc\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] x*(a*g + b*c + b*e*x**2 + b*f*x**3 + x*(a*h + b*d))/(8*a*b*(a - b
*x**4)**2) + (4*a*f - x*(a*g - 7*b*c - 5*b*e*x**2 + 2*x*(a*h - 3*
b*d)))/(32*a**2*b*(a - b*x**4)) - (a*h - 3*b*d)*atanh(sqrt(b)*x**
2/sqrt(a))/(16*a**(5/2)*b**(3/2)) - (-5*sqrt(a)*sqrt(b)*e + 3*a*g
 - 21*b*c)*atanh(b**(1/4)*x/a**(1/4))/(64*a**(11/4)*b**(5/4)) - (
5*sqrt(a)*sqrt(b)*e + 3*a*g - 21*b*c)*atan(b**(1/4)*x/a**(1/4))/(
64*a**(11/4)*b**(5/4))
```

Mathematica [A] time = 0.554865, size = 309, normalized size = 1.28

$$\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(4a^{5/4}h-5\sqrt{a}b^{3/4}e-12\sqrt[4]{a}bd+3a\sqrt[4]{b}g-21b^{5/4}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(4a^{5/4}h+5\sqrt{a}b^{3/4}e-12\sqrt[4]{a}bd-3a\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) - a*(g + 2*h*x))
)/(a - b*x^4) + (16*a^(7/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f
 + x*(g + h*x))))/(a - b*x^4)^2 + 2*b^(1/4)*(21*b*c - 5*Sqrt[a]*S
qrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-21*b^(5/4)*c -
12*a^(1/4)*b*d - 5*Sqrt[a]*b^(3/4)*e + 3*a*b^(1/4)*g + 4*a^(5/4)*
h)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(5/4)*c - 12*a^(1/4)*b*d + 5*
Sqrt[a]*b^(3/4)*e - 3*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4) + b^
(1/4)*x] - 4*a^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(
128*a^(11/4)*b^(3/2))
```

Maple [B] time = 0.017, size = 418, normalized size = 1.7

$$-\frac{1}{(bx^4-a)^2}\left(\frac{5\,bex^7}{32\,a^2}-\frac{(ah-3\,bd)\,x^6}{16\,a^2}-\frac{(ag-7\,bc)\,x^5}{32\,a^2}-\frac{9\,ex^3}{32\,a}-\frac{(ah+5\,bd)\,x^2}{16\,ab}-\frac{(3\,ag+11\,bc)\,x}{32\,ab}-\frac{f}{8\,b}\right)$$
$$-\frac{3\,g}{64\,a^2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{21\,c}{64\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{3\,g}{128\,a^2b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{21\,c}{128\,a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{ah}{32}\ln\left(1\left(-a^3b+x^2\sqrt{a^5b^3}\right)\left(-a^3b-x^2\sqrt{a^5b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^5b^3}}$$
$$-\frac{3\,bd}{32}\ln\left(1\left(-a^3b+x^2\sqrt{a^5b^3}\right)\left(-a^3b-x^2\sqrt{a^5b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^5b^3}}$$
$$-\frac{5\,e}{64\,a^2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5\,e}{128\,a^2b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(5/32/a^2*b*e*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-7*b*c)/a^2*
x^5-9/32/a*e*x^3-1/16*(a*h+5*b*d)/a/b*x^2-1/32*(3*a*g+11*b*c)/a/b
*x-1/8*f/b)/(b*x^4-a)^2-3/64*(a/b)^(1/4)/a^2/b*arctan(x/(a/b)^(1/
4))*g+21/64*c/a^3*(a/b)^(1/4)*arctan(x/(a/b)^(1/4))-3/128*(a/b)^(
1/4)/a^2/b*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+21/128*c/a^3*(a/
b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/32/(a^5*b^3)^(1/2)
*ln((-a^3*b+x^2*(a^5*b^3)^(1/2))/(-a^3*b-x^2*(a^5*b^3)^(1/2)))*a*
h-3/32*b*d/(a^5*b^3)^(1/2)*ln((-a^3*b+x^2*(a^5*b^3)^(1/2))/(-a^3*
b-x^2*(a^5*b^3)^(1/2)))-5/64*e/a^2/b/(a/b)^(1/4)*arctan(x/(a/b)^(
1/4))+5/128*e/a^2/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="m
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="f
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22894, size = 656, normalized size = 2.72

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-4\sqrt{2}\sqrt{-ab}abh+21\left(-ab^3\right)^{\frac{1}{4}}b^2c-3\left(-ab^3\right)^{\frac{1}{4}}abg+5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-4\sqrt{2}\sqrt{-ab}abh+21\left(-ab^3\right)^{\frac{1}{4}}b^2c-3\left(-ab^3\right)^{\frac{1}{4}}abg+5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(21\left(-ab^3\right)^{\frac{1}{4}}b^2c-3\left(-ab^3\right)^{\frac{1}{4}}abg-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b^3}-\frac{\sqrt{2}\left(21\left(-ab^3\right)^{\frac{1}{4}}b^2c-3\left(-ab^3\right)^{\frac{1}{4}}abg-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b^3}-\frac{5\,b^2x^7e+6\,b^2dx^6-2\,abhx^6+7\,b^2cx^5-abgx^5-9\,abx^3e-10\,abdx^2-2\,a^2hx^2-11\,abcx-3\,a^2gx-4\,a^2f}{32\left(bx^4-a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algorithm="g
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)
*a*b*h + 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a
*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-
a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*
d - 4*sqrt(2)*sqrt(-a*b)*a*b*h + 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*
b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21
*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)
*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/2
56*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*
(-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(
a^3*b^3) - 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 - 2*a*b*h*x^6 + 7*b^2*
c*x^5 - a*b*g*x^5 - 9*a*b*x^3*e - 10*a*b*d*x^2 - 2*a^2*h*x^2 - 11
*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

3.199 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=268

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(2x(3bd-ah)+x^2(5be-3ai)-ag+7bc\right)+4af}{32a^2b\,(a-bx^4)}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{8ab\,(a-bx^4)^2}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*
b*(a - b*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x +
(5*b*e - 3*a*i)*x^2))/(32*a^2*b*(a - b*x^4)) - ((5*b*e - (3*Sqrt[
b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(
64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a]
 - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((
3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.959422, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(2x(3bd-ah)+x^2(5be-3ai)-ag+7bc\right)+4af}{32a^2b\,(a-bx^4)}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{8ab\,(a-bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*
b*(a - b*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x +
(5*b*e - 3*a*i)*x^2))/(32*a^2*b*(a - b*x^4)) - ((5*b*e - (3*Sqrt[
b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(
64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a]
 - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((
3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rubi in Sympy [A] time = 127.972, size = 252, normalized size = 0.94

$$\frac{x\left(ag+bc+bfx^3+x^2\left(ai+be\right)+x\left(ah+bd\right)\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af-x\left(ag-7bc+x^2\left(3ai-5be\right)+2x\left(ah-3bd\right)\right)}{32a^2b\left(a-bx^4\right)}-\frac{\left(ah-3bd\right)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}b^{\frac{3}{2}}}+\frac{\left(\sqrt{a}\left(6a-5be\right)-3a\sqrt{b}g+21b^{\frac{3}{2}}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{7}{4}}}-\frac{\left(\sqrt{a}\left(6a-5be\right)+3a\sqrt{b}g-21b^{\frac{3}{2}}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] x*(a*g + b*c + b*f*x**3 + x**2*(a*i + b*e) + x*(a*h + b*d))/(8*a*
b*(a - b*x**4)**2) + (4*a*f - x*(a*g - 7*b*c + x**2*(3*a*i - 5*b*
e) + 2*x*(a*h - 3*b*d)))/(32*a**2*b*(a - b*x**4)) - (a*h - 3*b*d)
*atanh(sqrt(b)*x**2/sqrt(a))/(16*a**(5/2)*b**(3/2)) + (sqrt(a)*(6
*a - 5*b*e) - 3*a*sqrt(b)*g + 21*b**(3/2)*c)*atan(b**(1/4)*x/a**(
1/4))/(64*a**(11/4)*b**(7/4)) - (sqrt(a)*(6*a - 5*b*e) + 3*a*sqrt
(b)*g - 21*b**(3/2)*c)*atanh(b**(1/4)*x/a**(1/4))/(64*a**(11/4)*b
**(7/4))
```

Mathematica [A] time = 0.673875, size = 359, normalized size = 1.34

$$\frac{16a^{7/4}b^{3/4}(a(f+x(g+x(h+ix)))+bx(c+x(d+ex)))}{(a-bx^4)^2}-\frac{4a^{3/4}b^{3/4}x(a(g+x(2h+3ix))-b(7c+x(6d+5ex)))}{a-bx^4}+\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(4a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-12\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x]
```

```
[Out] ((-4*a^(3/4)*b^(3/4)*x*(-(b*(7*c + x*(6*d + 5*e*x))) + a*(g + x*(
2*h + 3*i*x))))/(a - b*x^4) + (16*a^(7/4)*b^(3/4)*(b*x*(c + x*(d
+ e*x)) + a*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*(21*b^(
3/2)*c - 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(
1/4)*x)/a^(1/4)] + (-21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d - 5*Sqrt
[a]*b*e + 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[
a^(1/4) - b^(1/4)*x] + (21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d + 5*S
qrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*L
og[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*b^(1/4)*(-3*b*d + a*h)*Log[Sq
rt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^(7/4))
```

Maple [B] time = 0.017, size = 501, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(-1/32*(3*a*i-5*b*e)/a^2*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-
7*b*c)/a^2*x^5-1/32*(a*i+9*b*e)/a/b*x^3-1/16*(a*h+5*b*d)/a/b*x^2-
1/32*(3*a*g+11*b*c)/a/b*x-1/8*f/b)/(b*x^4-a)^2-3/64*(a/b)^(1/4)/a
^2/b*arctan(x/(a/b)^(1/4))*g+21/64*c/a^3*(a/b)^(1/4)*arctan(x/(a/
b)^(1/4))-3/128*(a/b)^(1/4)/a^2/b*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))*g+21/128*c/a^3*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)
))+1/32/(a^5*b^3)^(1/2)*ln((-a^3*b+x^2*(a^5*b^3)^(1/2))/(-a^3*b-x
^2*(a^5*b^3)^(1/2)))*a*h-3/32*b*d/(a^5*b^3)^(1/2)*ln((-a^3*b+x^2*
(a^5*b^3)^(1/2))/(-a^3*b-x^2*(a^5*b^3)^(1/2)))+3/64/a/b^2/(a/b)^(
1/4)*arctan(x/(a/b)^(1/4))*i-5/64*e/a^2/b/(a/b)^(1/4)*arctan(x/(a
/b)^(1/4))-3/128/a/b^2/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1
/4)))*i+5/128*e/a^2/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algor
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algor
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.235573, size = 942, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,x, algor
```

```
[Out] -3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(-a*b^3)^(3/4
)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4)) - 3/25
6*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(-a*b^3)^(3/4)*ln(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4)) + 1/128*sqr
t(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a*b*h +
21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/
4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4
))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqr
t(2)*sqrt(-a*b)*a*b*h + 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4
)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(-a*b^3)
^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e)*ln(x^
2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/256*sqrt(2
)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^
(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3)
+ 1/32*(3*a*b*i*x^7 - 5*b^2*x^7*e - 6*b^2*d*x^6 + 2*a*b*h*x^6 - 7
*b^2*c*x^5 + a*b*g*x^5 + a^2*i*x^3 + 9*a*b*x^3*e + 10*a*b*d*x^2 +
 2*a^2*h*x^2 + 11*a*b*c*x + 3*a^2*g*x + 4*a^2*f)/((b*x^4 - a)^2*a
^2*b)
```

3.200 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=285

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4a(bf-aj)}{32a^2b^2(a-bx^4)}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{8ab(a-bx^4)^2}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3
))/(8*a*b*(a - b*x^4)^2) + (4*a*(b*f - a*j) + x*(b*(7*b*c - a*g)
+ 2*b*(3*b*d - a*h)*x + b*(5*b*e - 3*a*i)*x^2))/(32*a^2*b^2*(a -
b*x^4)) - ((5*b*e - (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*Ar
cTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sq
rt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)
])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a]])/(16*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.924048, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4a(bf-aj)}{32a^2b^2(a-bx^4)}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{8ab(a-bx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3,x]
```


```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3
))/(8*a*b*(a - b*x^4)^2) + (4*a*(b*f - a*j) + x*(b*(7*b*c - a*g)
+ 2*b*(3*b*d - a*h)*x + b*(5*b*e - 3*a*i)*x^2))/(32*a^2*b^2*(a -
b*x^4)) - ((5*b*e - (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*Ar
cTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sq
rt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)
])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a]])/(16*a^(5/2)*b^(3/2))
```

Rubi in Sympy [A] time = 141.004, size = 267, normalized size = 0.94

$$\frac{x\left(ag+bc+x^3(aj+bf)+x^2(ai+be)+x(ah+bd)\right)}{8ab(a-bx^4)^2}-\frac{4a(aj-bf)+x\left(bx^2(3ai-5be)+2bx(ah-3bd)+b(ag-7bc)\right)}{32a^2b^2(a-bx^4)}-\frac{(ah-3bd)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}b^{\frac{3}{2}}}+\frac{\left(\sqrt{a}(6a-5be)-3a\sqrt{b}g+21b^{\frac{3}{2}}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{7}{4}}}-\frac{\left(\sqrt{a}(6a-5be)+3a\sqrt{b}g-21b^{\frac{3}{2}}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-
b*x**4+a)**3,x)
```

```
[Out] x*(a*g + b*c + x**3*(a*j + b*f) + x**2*(a*i + b*e) + x*(a*h + b*d
))/(8*a*b*(a - b*x**4)**2) - (4*a*(a*j - b*f) + x*(b*x**2*(3*a*i
- 5*b*e) + 2*b*x*(a*h - 3*b*d) + b*(a*g - 7*b*c)))/(32*a**2*b**2*
(a - b*x**4)) - (a*h - 3*b*d)*atanh(sqrt(b)*x**2/sqrt(a))/(16*a**
(5/2)*b**(3/2)) + (sqrt(a)*(6*a - 5*b*e) - 3*a*sqrt(b)*g + 21*b**
(3/2)*c)*atan(b**(1/4)*x/a**(1/4))/(64*a**(11/4)*b**(7/4)) - (sqr
t(a)*(6*a - 5*b*e) + 3*a*sqrt(b)*g - 21*b**(3/2)*c)*atanh(b**(1/4
)*x/a**(1/4))/(64*a**(11/4)*b**(7/4))
```

Mathematica [A] time = 0.542887, size = 380, normalized size = 1.33

$$\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(4a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-12\sqrt[4]{a}b^{5/4}d-5\sqrt{a}be+3a\sqrt{b}g-21b^{3/2}c\right)+\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(4a^{5/4}\sqrt[4]{b}h-3a^{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3,x
```

```
[Out] ((-4*a^(3/4)*(8*a^2*j - b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g
+ x*(2*h + 3*i*x))))/(a - b*x^4) + (16*a^(7/4)*(a^2*j + b^2*x*(c
+ x*(d + e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^2 +
2*b^(1/4)*(21*b^(3/2)*c - 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/
2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + b^(1/4)*(-21*b^(3/2)*c - 12*a
^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1
/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*(21*b^(3/
2)*c - 12*a^(1/4)*b^(5/4)*d + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a
^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1
/4)*Sqrt[b]*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11
/4)*b^2)
```

Maple [B] time = 0.017, size = 517, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(-1/32*(3*a*i-5*b*e)/a^2*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-
7*b*c)/a^2*x^5-1/4*j*x^4/b-1/32*(a*i+9*b*e)/a/b*x^3-1/16*(a*h+5*b
*d)/a/b*x^2-1/32*(3*a*g+11*b*c)/a/b*x+1/8*(a*j-b*f)/b^2)/(b*x^4-a
)^2-3/64*(a/b)^(1/4)/a^2/b*arctan(x/(a/b)^(1/4))*g+21/64*c/a^3*(a
/b)^(1/4)*arctan(x/(a/b)^(1/4))-3/128*(a/b)^(1/4)/a^2/b*ln((x+(a/
b)^(1/4))/(x-(a/b)^(1/4)))*g+21/128*c/a^3*(a/b)^(1/4)*ln((x+(a/b)
^(1/4))/(x-(a/b)^(1/4)))+1/32/(a^5*b^3)^(1/2)*ln((-a^3*b+x^2*(a^5
*b^3)^(1/2))/(-a^3*b-x^2*(a^5*b^3)^(1/2)))*a*h-3/32*b*d/(a^5*b^3)
^(1/2)*ln((-a^3*b+x^2*(a^5*b^3)^(1/2))/(-a^3*b-x^2*(a^5*b^3)^(1/2
)))+3/64/a/b^2/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))*i-5/64*e/a^2/b/(
a/b)^(1/4)*arctan(x/(a/b)^(1/4))-3/128/a/b^2/(a/b)^(1/4)*ln((x+(a
/b)^(1/4))/(x-(a/b)^(1/4)))*i+5/128*e/a^2/b/(a/b)^(1/4)*ln((x+(a/
b)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.232313, size = 986, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^3,
```

```
[Out] -3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(-a*b^3)^(3/4
)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4)) - 3/25
6*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(-a*b^3)^(3/4)*ln(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4)) + 1/128*sqr
t(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a*b*h +
21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/
4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4
))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqr
t(2)*sqrt(-a*b)*a*b*h + 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4
)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(-a*b^3)
^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e)*ln(x^
2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/256*sqrt(2
)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^
(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3)
+ 1/32*(3*a*b^2*i*x^7 - 5*b^3*x^7*e - 6*b^3*d*x^6 + 2*a*b^2*h*x^6
 - 7*b^3*c*x^5 + a*b^2*g*x^5 + 8*a^2*b*j*x^4 + a^2*b*i*x^3 + 9*a*
b^2*x^3*e + 10*a*b^2*d*x^2 + 2*a^2*b*h*x^2 + 11*a*b^2*c*x + 3*a^2
*b*g*x + 4*a^2*b*f - 4*a^3*j)/((b*x^4 - a)^2*a^2*b^2)
```

3.201 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=413

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{(ah+3bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}-\frac{4af-x\left(2x(ah+3bd)+ag+7bc+5bex^2\right)}{32a^2b\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b
*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + 5*b*e*x^
2))/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/
Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e +
 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(1
1/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - (
(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((
21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi [A] time = 1.12168, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{(ah+3bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}-\frac{4af-x\left(2x(ah+3bd)+ag+7bc+5bex^2\right)}{32a^2b\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```


```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b
*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + 5*b*e*x^
2))/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/
Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e +
 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(1
1/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - (
(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((
```

```
21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi in Sympy [A] time = 170.297, size = 400, normalized size = 0.97

$$-\frac{x\left(ag-bc-bex^2-bfx^3+x\left(ah-bd\right)\right)}{8ab\left(a+bx^4\right)^2}-\frac{4af-x\left(ag+7bc+5bex^2+2x\left(ah+3bd\right)\right)}{32a^2b\left(a+bx^4\right)}+\frac{\left(ah+3bd\right)\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}b^{\frac{3}{2}}}-\frac{\sqrt{2}\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{256a^{\frac{11}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{256a^{\frac{11}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] -x*(a*g - b*c - b*e*x**2 - b*f*x**3 + x*(a*h - b*d))/(8*a*b*(a +
b*x**4)**2) - (4*a*f - x*(a*g + 7*b*c + 5*b*e*x**2 + 2*x*(a*h + 3
*b*d)))/(32*a**2*b*(a + b*x**4)) + (a*h + 3*b*d)*atan(sqrt(b)*x**
2/sqrt(a))/(16*a**(5/2)*b**(3/2)) - sqrt(2)*(-5*sqrt(a)*sqrt(b)*e
 + 3*a*g + 21*b*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqr
t(b) + b*x**2)/(256*a**(11/4)*b**(5/4)) + sqrt(2)*(-5*sqrt(a)*sqr
t(b)*e + 3*a*g + 21*b*c)*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a
)*sqrt(b) + b*x**2)/(256*a**(11/4)*b**(5/4)) - sqrt(2)*(5*sqrt(a)
*sqrt(b)*e + 3*a*g + 21*b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4)
)/(128*a**(11/4)*b**(5/4)) + sqrt(2)*(5*sqrt(a)*sqrt(b)*e + 3*a*g
 + 21*b*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(128*a**(11/4)*b
**(5/4))
```

Mathematica [A] time = 0.673992, size = 411, normalized size = 1.

$$-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8a^{5/4}h+5\sqrt{2}\sqrt{a}b^{3/4}e+24\sqrt[4]{a}bd+3\sqrt{2}a\sqrt[4]{b}g+21\sqrt{2}b^{5/4}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-8a^{5/4}h+5\sqrt{2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) + a*(g + 2*h*x))
)/(a + b*x^4) - (32*a^(7/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a
*(f + x*(g + h*x))))/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(5/4)*c + 24
*a^(1/4)*b*d + 5*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*
g + 8*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*
Sqrt[2]*b^(5/4)*c - 24*a^(1/4)*b*d + 5*Sqrt[2]*Sqrt[a]*b^(3/4)*e
+ 3*Sqrt[2]*a*b^(1/4)*g - 8*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e -
3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + S
qrt[2]*b^(1/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(3/2
))
```

Maple [A] time = 0.019, size = 562, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (5/32/a^2*b*e*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5+9/32/a*e*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/b*
x-1/8*f/b)/(b*x^4+a)^2+3/128*(a/b)^(1/4)/a^2/b*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)*g+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x+1)+3/128*(a/b)^(1/4)/a^2/b*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x-1)*g+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x-1)+3/256*(a/b)^(1/4)/a^2/b*2^(1/2)*ln((x^2+
(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2)))*g+21/256*c/a^3*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)
*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+
1/16/(a^5*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2))*a*h+3/16*b*d/(a^5*b^
3)^(1/2)*arctan(x^2*(b/a)^(1/2))+5/256*e/a^2/b/(a/b)^(1/4)*2^(1/2
)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2)))+5/128*e/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)+5/128*e/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="fr
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.224841, size = 620, normalized size = 1.5

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+4\sqrt{2}\sqrt{ab}abh+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+4\sqrt{2}\sqrt{ab}abh+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}+\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256a^3b^3}-\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256a^3b^3}+\frac{5b^2x^7e+6b^2dx^6+2abhx^6+7b^2cx^5+abgx^5+9abx^3e+10abdx^2-2a^2hx^2+11abcx-3a^2gx-4a^2f}{32\left(bx^4+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="gi
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a
*b*h + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)
^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sq
rt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*
a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*ln(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b
^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*ln(x
^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x
^7*e + 6*b^2*d*x^6 + 2*a*b*h*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*
b*x^3*e + 10*a*b*d*x^2 - 2*a^2*h*x^2 + 11*a*b*c*x - 3*a^2*g*x - 4
*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

3.202 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=463

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{(ah+3bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}-\frac{4af-x\left(2x(ah+3bd)+x^2(3ai+5be)+ag+7bc\right)}{32a^2b\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*
b*(a + b*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x +
(5*b*e + 3*a*i)*x^2))/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*Arc
Tan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7
*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c
 + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a
*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b
]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rubi [A] time = 1.56646, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{(ah+3bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}-\frac{4af-x\left(2x(ah+3bd)+x^2(3ai+5be)+ag+7bc\right)}{32a^2b\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x]
```



```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*
b*(a + b*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x +
(5*b*e + 3*a*i)*x^2))/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*Arc
```

```
Tan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7
*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c
 + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a
*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b
]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.12589, size = 473, normalized size = 1.02

$$-\frac{32a^{7/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{(a+bx^4)^2}+\frac{8a^{3/4}b^{3/4}x(ag+ax(2h+3ix)+7bc+bx(6d+5ex))}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*b^(3/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x) + a*x*(2*h
 + 3*i*x)))/(a + b*x^4) - (32*a^(7/4)*b^(3/4)*(-(b*x*(c + x*(d +
e*x))) + a*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^2 - 2*(21*Sqrt
[2]*b^(3/2)*c + 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*
Sqrt[2]*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b^(3/2)*c
 - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqr
t[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b^(3/2)*c + 5*Sqrt[a]*b
*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(21*b^(3/2)*c - 5*Sqrt[a]*b*e +
 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(7/4))
```

Maple [A] time = 0.019, size = 717, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*
b*c)/a^2*x^5-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/
32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)^2+3/128*(a/b)^(1/4)/a^
2/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*g+21/128*c/a^3*(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/256*(a/b)^(1/4)/a
^2/b*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+21/256*c/a^3*(a/b)^(1/4)*2^(1/2
)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2
```

```
^(1/2)+(a/b)^(1/2)))+3/128*(a/b)^(1/4)/a^2/b*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x+1)*g+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)+1/16/(a^5*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2)
)*a*h+3/16*b*d/(a^5*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2))+3/256/a/b^
2/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/
(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*i+5/256*e/a^2/b/(a/b)^(1
/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)
^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+3/128/a/b^2/(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x-1)*i+5/128*e/a^2/b/(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/128/a/b^2/(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)*i+5/128*e/a^2/b/(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algori
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algori
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.230481, size = 892, normalized size = 1.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algori
```

```
[Out] 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(a*b^3)^(3/4)*ln(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 3/256*i*(2*
sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
```

```
4))/(a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(a*b^3)^(3/4)*ln(x^2 - sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 1/128*sqrt(2)*(12*sqrt
(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4
)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128
*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h
+ 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4
)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/
4)*a*b*g - 5*(a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a
*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(3*a*b*i*x^7 + 5*b^2*x^7*e + 6
*b^2*d*x^6 + 2*a*b*h*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 - a^2*i*x^3 +
9*a*b*x^3*e + 10*a*b*d*x^2 - 2*a^2*h*x^2 + 11*a*b*c*x - 3*a^2*g*x
 - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

3.203 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=480

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{(ah+3bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}-\frac{4a(aj+bf)-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{32a^2b^2\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{8ab\left(a+bx^4\right)^2}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3
))/(8*a*b*(a + b*x^4)^2) - (4*a*(b*f + a*j) - x*(b*(7*b*c + a*g)
+ 2*b*(3*b*d + a*h)*x + b*(5*b*e + 3*a*i)*x^2))/(32*a^2*b^2*(a +
b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/
2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i)
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b
^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/
4)) - ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^
(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*
a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(12
8*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rubi [A] time = 1.55887, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{(ah+3bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}-\frac{4a(aj+bf)-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{32a^2b^2\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3
))/(8*a*b*(a + b*x^4)^2) - (4*a*(b*f + a*j) - x*(b*(7*b*c + a*g)
```

```
+ 2*b*(3*b*d + a*h)*x + b*(5*b*e + 3*a*i)*x^2))/(32*a^2*b^2*(a +
b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/
2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i)
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b
^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/
4)) - ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^
(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*
a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(12
8*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3
```

```
[Out] Timed out
```

Mathematica [A] time = 0.972867, size = 500, normalized size = 1.04

$$-2\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+24\sqrt[4]{a}b^{5/4}d+5\sqrt{2}\sqrt{a}be+3\sqrt{2}a\sqrt{b}g+21\sqrt{2}b^{3/2}c\right)+2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3,x
```

```
[Out] ((8*a^(3/4)*(-8*a^2*j + b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g
+ x*(2*h + 3*i*x))))/(a + b*x^4) + (32*a^(7/4)*(a^2*j + b^2*x*(c
+ x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^2 -
2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*c + 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2
]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 3*S
qrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(
1/4)*(21*Sqrt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqr
t[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2
]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^
(1/4)*(-21*b^(3/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*
i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[
2]*b^(1/4)*(21*b^(3/2)*c - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g - 3*a^(3
/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(2
56*a^(11/4)*b^2)
```

Maple [A] time = 0.016, size = 732, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*
b*c)/a^2*x^5-1/4*j*x^4/b-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d
)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*(a*j+b*f)/b^2)/(b*x^4+a)^
2+3/128*(a/b)^(1/4)/a^2/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)
*g+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+
```

```
1)+3/128*(a/b)^(1/4)/a^2/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1
)*g+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
-1)+3/256*(a/b)^(1/4)/a^2/b*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+21/256*c
/a^3*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/16/(a^5*b^3)^(1/2)*
arctan(x^2*(b/a)^(1/2))*a*h+3/16*b*d/(a^5*b^3)^(1/2)*arctan(x^2*(
b/a)^(1/2))+3/256/a/b^2/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x
*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*i+
5/256*e/a^2/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+3/128/a/b^2/
(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*i+5/128*e/a^2
/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/128/a/b^
2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*i+5/128*e/a
^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x

[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x

[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.233974, size = 936, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x

[Out] 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(a*b^3)^(3/4)*ln(
```

```
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 3/256*i*(2*
sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(a*b^3)^(3/4)*ln(x^2 - sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 1/128*sqrt(2)*(12*sqrt
(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4
)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128
*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h
+ 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4
)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/
4)*a*b*g - 5*(a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a
*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(3*a*b^2*i*x^7 + 5*b^3*x^7*e +
 6*b^3*d*x^6 + 2*a*b^2*h*x^6 + 7*b^3*c*x^5 + a*b^2*g*x^5 - 8*a^2*
b*j*x^4 - a^2*b*i*x^3 + 9*a*b^2*x^3*e + 10*a*b^2*d*x^2 - 2*a^2*b*
h*x^2 + 11*a*b^2*c*x - 3*a^2*b*g*x - 4*a^2*b*f - 4*a^3*j)/((b*x^4
 + a)^2*a^2*b^2)
```

3.204 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=293

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(2x(5bd-ah)-ag+11bc+9bex^2\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a -
b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 45*b*e*x^
2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d
 - a*h)*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b*c - 15*
Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(1
5/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((5*b*d - a*h)*Arc
Tanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rubi [A] time = 0.900172, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(2x(5bd-ah)-ag+11bc+9bex^2\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a -
b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 45*b*e*x^
2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d
 - a*h)*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b*c - 15*
Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(1
5/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((5*b*d - a*h)*Arc
Tanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rubi in Sympy [A] time = 145.162, size = 274, normalized size = 0.94

$$\frac{x\left(ag+bc+bex^2+bfx^3+x\left(ah+bd\right)\right)}{12ab\left(a-bx^4\right)^3}+\frac{8af-x\left(ag-11bc-9bex^2+2x\left(ah-5bd\right)\right)}{96a^2b\left(a-bx^4\right)^2}-\frac{x\left(7ag-77bc-45bex^2+12x\left(ah-5bd\right)\right)}{384a^3b\left(a-bx^4\right)}-\frac{\left(ah-5bd\right)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}b^{\frac{3}{2}}}-\frac{\left(-15\sqrt{a}\sqrt{b}e+7ag-77bc\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{5}{4}}}-\frac{\left(15\sqrt{a}\sqrt{b}e+7ag-77bc\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] x*(a*g + b*c + b*e*x**2 + b*f*x**3 + x*(a*h + b*d))/(12*a*b*(a -
b*x**4)**3) + (8*a*f - x*(a*g - 11*b*c - 9*b*e*x**2 + 2*x*(a*h -
5*b*d)))/(96*a**2*b*(a - b*x**4)**2) - x*(7*a*g - 77*b*c - 45*b*e
*x**2 + 12*x*(a*h - 5*b*d))/(384*a**3*b*(a - b*x**4)) - (a*h - 5*
b*d)*atanh(sqrt(b)*x**2/sqrt(a))/(32*a**(7/2)*b**(3/2)) - (-15*sq
rt(a)*sqrt(b)*e + 7*a*g - 77*b*c)*atanh(b**(1/4)*x/a**(1/4))/(256
*a**(15/4)*b**(5/4)) - (15*sqrt(a)*sqrt(b)*e + 7*a*g - 77*b*c)*at
an(b**(1/4)*x/a**(1/4))/(256*a**(15/4)*b**(5/4))
```

Mathematica [A] time = 0.764228, size = 360, normalized size = 1.23

$$-3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(-8a^{5/4}h+15\sqrt{a}b^{3/4}e+40\sqrt[4]{a}bd-7a\sqrt[4]{b}g+77b^{5/4}c\right)+3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(8a^{5/4}h+15\sqrt{a}b^{3/4}e-40\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(77*b*c - 7*a*g + 60*b*d*x - 12*a*h*x + 45*
b*e*x^2))/(a - b*x^4) + (16*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d
 + 9*e*x) - a*(g + 2*h*x)))/(a - b*x^4)^2 + (128*a^(11/4)*Sqrt[b]
*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a - b*x^4)^3 + 6
*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*
x)/a^(1/4)] - 3*(77*b^(5/4)*c + 40*a^(1/4)*b*d + 15*Sqrt[a]*b^(3/
4)*e - 7*a*b^(1/4)*g - 8*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + 3*
(77*b^(5/4)*c - 40*a^(1/4)*b*d + 15*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/
4)*g + 8*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 24*a^(1/4)*(-5*b*d
 + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(1536*a^(15/4)*b^(3/2))
```

Maple [A] time = 0.021, size = 463, normalized size = 1.6

$$\frac{1}{(bx^4-a)^3}\left(-\frac{15\,b^2ex^{11}}{128\,a^3}+\frac{(ah-5\,bd)\,bx^{10}}{32\,a^3}+\frac{(7\,ag-77\,bc)\,bx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}-\frac{(ah-5\,bd)\,x^6}{12\,a^2}-\frac{(3\,ag-33\,bc)\,x^5}{64\,a^2}-\frac{113\,ex^3}{384\,a}-\right.$$
$$-\frac{7\,g}{256\,a^3b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{77\,c}{256\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{7\,g}{512\,a^3b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{77\,c}{512\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{ah}{64}\ln\left(1\left(-a^4b+x^2\sqrt{a^7b^3}\right)\left(-a^4b-x^2\sqrt{a^7b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^7b^3}}$$
$$-\frac{5\,bd}{64}\ln\left(1\left(-a^4b+x^2\sqrt{a^7b^3}\right)\left(-a^4b-x^2\sqrt{a^7b^3}\right)^{-1}\right)\frac{1}{\sqrt{a^7b^3}}$$
$$-\frac{15\,e}{256\,a^3b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{15\,e}{512\,a^3b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128*e/a^3*b^2*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(a*g-11
*b*c)/a^3*b*x^9+21/64/a^2*b*e*x^7-1/12/a^2*(a*h-5*b*d)*x^6-3/64/a
^2*(a*g-11*b*c)*x^5-113/384/a*e*x^3-1/32*(a*h+11*b*d)/a/b*x^2-1/1
28*(7*a*g+51*b*c)/a/b*x-1/12*f/b)/(b*x^4-a)^3-7/256*(a/b)^(1/4)/a
^3/b*arctan(x/(a/b)^(1/4))*g+77/256*c*(a/b)^(1/4)/a^4*arctan(x/(a
/b)^(1/4))-7/512*(a/b)^(1/4)/a^3/b*ln((x+(a/b)^(1/4))/(x-(a/b)^(1
/4)))*g+77/512*c*(a/b)^(1/4)/a^4*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4
```

```
)))+1/64/(a^7*b^3)^(1/2)*ln((-a^4*b+x^2*(a^7*b^3)^(1/2))/(-a^4*b-
x^2*(a^7*b^3)^(1/2)))*a*h-5/64*b*d/(a^7*b^3)^(1/2)*ln((-a^4*b+x^2
*(a^7*b^3)^(1/2))/(-a^4*b-x^2*(a^7*b^3)^(1/2)))-15/256*e/a^3/b/(a
/b)^(1/4)*arctan(x/(a/b)^(1/4))+15/512*e/a^3/b/(a/b)^(1/4)*ln((x+
(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="fr
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22365, size = 738, normalized size = 2.52

$$-\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^{2}d-8\sqrt{2}\sqrt{-ab}abh-77\left(-ab^{3}\right)^{\frac{1}{4}}b^{2}c+7\left(-ab^{3}\right)^{\frac{1}{4}}abg-15\left(-ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^{4}b^{3}}-\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^{2}d-8\sqrt{2}\sqrt{-ab}abh-77\left(-ab^{3}\right)^{\frac{1}{4}}b^{2}c+7\left(-ab^{3}\right)^{\frac{1}{4}}abg-15\left(-ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^{4}b^{3}}+\frac{\sqrt{2}\left(77\left(-ab^{3}\right)^{\frac{1}{4}}b^{2}c-7\left(-ab^{3}\right)^{\frac{1}{4}}abg-15\left(-ab^{3}\right)^{\frac{3}{4}}e\right)\ln\left(x^{2}+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024a^{4}b^{3}}-\frac{\sqrt{2}\left(77\left(-ab^{3}\right)^{\frac{1}{4}}b^{2}c-7\left(-ab^{3}\right)^{\frac{1}{4}}abg-15\left(-ab^{3}\right)^{\frac{3}{4}}e\right)\ln\left(x^{2}-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024a^{4}b^{3}}-\frac{45b^{3}x^{11}e+60b^{3}dx^{10}-12ab^{2}hx^{10}+77b^{3}cx^{9}-7ab^{2}gx^{9}-126ab^{2}x^{7}e-160ab^{2}dx^{6}+32a^{2}bhx^{6}-198ab^{2}cx^{5}+18a^{2}bg}{384\left(bx^{4}-a\right)^{3}a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algorithm="gi
```

```
[Out] -1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b
)*a*b*h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(
-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/
(-a/b)^(1/4))/(a^4*b^3) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^
2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-
a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)
*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^
(3/4)*e)*ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3)
- 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*
g - 15*(-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-
a/b))/(a^4*b^3) - 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 - 12*a*b^2
*h*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126*a*b^2*x^7*e - 160*a*
b^2*d*x^6 + 32*a^2*b*h*x^6 - 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 1
13*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 12*a^3*h*x^2 + 153*a^2*b*c*x +
 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

3.205 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=331

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai)\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(2x(5bd-ah)+3x^2(3be-ai)-ag+11bc\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a
*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 1
5*(3*b*e - a*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*
c - a*g + 2*(5*b*d - a*h)*x + 3*(3*b*e - a*i)*x^2))/(96*a^2*b*(a
- b*x^4)^2) + (((7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*(3*b*e - a
*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b
*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)
*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rubi [A] time = 1.27834, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai)\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(2x(5bd-ah)+3x^2(3be-ai)-ag+11bc\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x]
```


```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a
*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 1
5*(3*b*e - a*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*
c - a*g + 2*(5*b*d - a*h)*x + 3*(3*b*e - a*i)*x^2))/(96*a^2*b*(a
- b*x^4)^2) + (((7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*(3*b*e - a
*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b
*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)
*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rubi in Sympy [A] time = 155.188, size = 304, normalized size = 0.92

$$\frac{x\left(ag+bc+bfx^3+x^2\left(ai+be\right)+x\left(ah+bd\right)\right)}{12ab\left(a-bx^4\right)^3}+\frac{8af-x\left(ag-11bc+3x^2\left(ai-3be\right)+2x\left(ah-5bd\right)\right)}{96a^2b\left(a-bx^4\right)^2}-\frac{x\left(7ag-77bc+15x^2\left(2a-3be\right)+12x\left(ah-5bd\right)\right)}{384a^3b\left(a-bx^4\right)}-\frac{\left(ah-5bd\right)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}b^{\frac{3}{2}}}+\frac{\left(\sqrt{a}\left(10a-15be\right)-7a\sqrt{b}g+77b^{\frac{3}{2}}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{7}{4}}}-\frac{\left(\sqrt{a}\left(10a-15be\right)+7a\sqrt{b}g-77b^{\frac{3}{2}}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] x*(a*g + b*c + b*f*x**3 + x**2*(a*i + b*e) + x*(a*h + b*d))/(12*a
*b*(a - b*x**4)**3) + (8*a*f - x*(a*g - 11*b*c + 3*x**2*(a*i - 3*
b*e) + 2*x*(a*h - 5*b*d)))/(96*a**2*b*(a - b*x**4)**2) - x*(7*a*g
 - 77*b*c + 15*x**2*(2*a - 3*b*e) + 12*x*(a*h - 5*b*d))/(384*a**3
*b*(a - b*x**4)) - (a*h - 5*b*d)*atanh(sqrt(b)*x**2/sqrt(a))/(32*
a**(7/2)*b**(3/2)) + (sqrt(a)*(10*a - 15*b*e) - 7*a*sqrt(b)*g + 7
7*b**(3/2)*c)*atan(b**(1/4)*x/a**(1/4))/(256*a**(15/4)*b**(7/4))
- (sqrt(a)*(10*a - 15*b*e) + 7*a*sqrt(b)*g - 77*b**(3/2)*c)*atanh
(b**(1/4)*x/a**(1/4))/(256*a**(15/4)*b**(7/4))
```

Mathematica [A] time = 0.742444, size = 422, normalized size = 1.27

$$3\sqrt[4]{a}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(8a^{5/4}\sqrt[4]{b}h+5a^{3/2}i-40\sqrt[4]{a}b^{5/4}d-15\sqrt{a}be+7a\sqrt{b}g-77b^{3/2}c\right)-3\sqrt[4]{a}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-8a^{5/4}\sqrt[4]{b}h\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x]
```

```
[Out] ((-4*a*b^(3/4)*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*
(4*h + 5*i*x)))/(a - b*x^4) - (16*a^2*b^(3/4)*x*(-(b*(11*c + x*(1
0*d + 9*e*x))) + a*(g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a
^3*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + x*(h + i*x)))))
/(a - b*x^4)^3 + 6*a^(1/4)*(77*b^(3/2)*c - 15*Sqrt[a]*b*e - 7*a*S
qrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + 3*a^(1/4)*(
-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*Sqrt[
b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x
] - 3*a^(1/4)*(-77*b^(3/2)*c + 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*
b*e + 7*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1
/4) + b^(1/4)*x] - 24*Sqrt[a]*b^(1/4)*(-5*b*d + a*h)*Log[Sqrt[a]
+ Sqrt[b]*x^2])/(1536*a^4*b^(7/4))
```

Maple [A] time = 0.02, size = 551, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i-3*b*e)/a^3*b*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(
a*g-11*b*c)/a^3*b*x^9-7/64/a^2*(a*i-3*b*e)*x^7-1/12/a^2*(a*h-5*b*
d)*x^6-3/64/a^2*(a*g-11*b*c)*x^5-1/384*(5*a*i+113*b*e)/a/b*x^3-1/
32*(a*h+11*b*d)/a/b*x^2-1/128*(7*a*g+51*b*c)/a/b*x-1/12*f/b)/(b*x
^4-a)^3-7/256*(a/b)^(1/4)/a^3/b*arctan(x/(a/b)^(1/4))*g+77/256*c*
(a/b)^(1/4)/a^4*arctan(x/(a/b)^(1/4))-7/512*(a/b)^(1/4)/a^3/b*ln(
(x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+77/512*c*(a/b)^(1/4)/a^4*ln((x
+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/64/(a^7*b^3)^(1/2)*ln((-a^4*b+x^
2*(a^7*b^3)^(1/2))/(-a^4*b-x^2*(a^7*b^3)^(1/2)))*a*h-5/64*b*d/(a^
7*b^3)^(1/2)*ln((-a^4*b+x^2*(a^7*b^3)^(1/2))/(-a^4*b-x^2*(a^7*b^3
)^(1/2)))+5/256/a^2/b^2/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))*i-15/25
6*e/a^3/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))-5/512/a^2/b^2/(a/b)^(
1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*i+15/512*e/a^3/b/(a/b)^(
1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algori
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algori
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.224947, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x, algori
```

```
[Out] Done
```

3.206 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=349

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai)\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+4a(2bf-aj)}{96a^2b^2\left(a-bx^4\right)^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3
))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*
h)*x + 15*(3*b*e - a*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (4*a*(2*b
*f - a*j) + x*(b*(11*b*c - a*g) + 2*b*(5*b*d - a*h)*x + 3*b*(3*b*
e - a*i)*x^2))/(96*a^2*b^2*(a - b*x^4)^2) + (((7*Sqrt[b]*(11*b*c
- a*g))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(
256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqr
t[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)
) + ((5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^
(3/2))
```

Rubi [A] time = 1.24729, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai)\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+4a(2bf-aj)}{96a^2b^2\left(a-bx^4\right)^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4,x]
```


```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3
))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*
h)*x + 15*(3*b*e - a*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (4*a*(2*b
*f - a*j) + x*(b*(11*b*c - a*g) + 2*b*(5*b*d - a*h)*x + 3*b*(3*b*
e - a*i)*x^2))/(96*a^2*b^2*(a - b*x^4)^2) + (((7*Sqrt[b]*(11*b*c
- a*g))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(
256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqr
t[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)
) + ((5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^
(3/2))
```

Rubi in Sympy [A] time = 168.366, size = 321, normalized size = 0.92

$$\frac{x\left(ag+bc+x^3(aj+bf)+x^2(ai+be)+x(ah+bd)\right)}{12ab(a-bx^4)^3} - \frac{4a(aj-2bf)+x\left(3bx^2(ai-3be)+2bx(ah-5bd)+b(ag-11bc)\right)}{96a^2b^2(a-bx^4)^2} - \frac{x\left(7ag-77bc+15x^2(2a-3be)+12x(ah-5bd)\right)}{384a^3b(a-bx^4)} - \frac{(ah-5bd)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}b^{\frac{3}{2}}} + \frac{\left(\sqrt{a}(10a-15be)-7a\sqrt{b}g+77b^{\frac{3}{2}}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{7}{4}}} - \frac{\left(\sqrt{a}(10a-15be)+7a\sqrt{b}g-77b^{\frac{3}{2}}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{\frac{15}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-
b*x**4+a)**4,x)
```

```
[Out] x*(a*g + b*c + x**3*(a*j + b*f) + x**2*(a*i + b*e) + x*(a*h + b*d
))/(12*a*b*(a - b*x**4)**3) - (4*a*(a*j - 2*b*f) + x*(3*b*x**2*(a
*i - 3*b*e) + 2*b*x*(a*h - 5*b*d) + b*(a*g - 11*b*c)))/(96*a**2*b
**2*(a - b*x**4)**2) - x*(7*a*g - 77*b*c + 15*x**2*(2*a - 3*b*e)
+ 12*x*(a*h - 5*b*d))/(384*a**3*b*(a - b*x**4)) - (a*h - 5*b*d)*a
tanh(sqrt(b)*x**2/sqrt(a))/(32*a**(7/2)*b**(3/2)) + (sqrt(a)*(10*
a - 15*b*e) - 7*a*sqrt(b)*g + 77*b**(3/2)*c)*atan(b**(1/4)*x/a**(
1/4))/(256*a**(15/4)*b**(7/4)) - (sqrt(a)*(10*a - 15*b*e) + 7*a*s
qrt(b)*g - 77*b**(3/2)*c)*atanh(b**(1/4)*x/a**(1/4))/(256*a**(15/
4)*b**(7/4))
```

Mathematica [A] time = 0.743704, size = 439, normalized size = 1.26

$$3\sqrt[4]{a}\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(8a^{5/4}\sqrt[4]{b}h+5a^{3/2}i-40\sqrt[4]{a}b^{5/4}d-15\sqrt{a}be+7a\sqrt{b}g-77b^{3/2}c\right)+3\sqrt[4]{a}\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(8a^{5/4}\sqrt[4]{\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4,x
```

```
[Out] ((-4*a*b*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h +
 5*i*x)))/(a - b*x^4) - (16*a^2*(12*a^2*j - b^2*x*(11*c + x*(10*d
 + 9*e*x)) + a*b*x*(g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a
^3*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b*(f + x*(g + x*(h + i*x)
))))/(a - b*x^4)^3 + 6*a^(1/4)*b^(1/4)*(77*b^(3/2)*c - 15*Sqrt[a]
*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] +
 3*a^(1/4)*b^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 15*Sqr
t[a]*b*e + 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log
[a^(1/4) - b^(1/4)*x] + 3*a^(1/4)*b^(1/4)*(77*b^(3/2)*c - 40*a^(1
/4)*b^(5/4)*d + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4
)*h - 5*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 24*Sqrt[a]*Sqrt[b]*
(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(1536*a^4*b^2)
```

Maple [A] time = 0.02, size = 567, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i-3*b*e)/a^3*b*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(
a*g-11*b*c)/a^3*b*x^9-7/64/a^2*(a*i-3*b*e)*x^7-1/12/a^2*(a*h-5*b*
d)*x^6-3/64/a^2*(a*g-11*b*c)*x^5-1/8*j*x^4/b-1/384*(5*a*i+113*b*e
)/a/b*x^3-1/32*(a*h+11*b*d)/a/b*x^2-1/128*(7*a*g+51*b*c)/a/b*x+1/
24*(a*j-2*b*f)/b^2)/(b*x^4-a)^3-7/256*(a/b)^(1/4)/a^3/b*arctan(x/
(a/b)^(1/4))*g+77/256*c*(a/b)^(1/4)/a^4*arctan(x/(a/b)^(1/4))-7/5
12*(a/b)^(1/4)/a^3/b*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+77/512
*c*(a/b)^(1/4)/a^4*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/64/(a^7*
b^3)^(1/2)*ln((-a^4*b+x^2*(a^7*b^3)^(1/2))/(-a^4*b-x^2*(a^7*b^3)^
(1/2)))*a*h-5/64*b*d/(a^7*b^3)^(1/2)*ln((-a^4*b+x^2*(a^7*b^3)^(1/
2))/(-a^4*b-x^2*(a^7*b^3)^(1/2)))+5/256/a^2/b^2/(a/b)^(1/4)*arcta
n(x/(a/b)^(1/4))*i-15/256*e/a^3/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4
))-5/512/a^2/b^2/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*
i+15/512*e/a^3/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225829, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a)^4,x
```

```
[Out] Done
```

3.207 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4} dx$

Optimal. Leaf size=462

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{(ah+5bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(ag+11bc)+12x(ah+5bd)+45bex^2\right)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x\left(2x(ah+5bd)+ag+11bc+9bex^2\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a +
b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 45*b*e*x^
2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d
 + a*h)*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)
*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((77*b*c +
 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqr
t[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqr
t[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqr
t[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqr
t[2]*a^(15/4)*b^(5/4))
```

Rubi [A] time = 1.32749, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{(ah+5bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(ag+11bc)+12x(ah+5bd)+45bex^2\right)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x\left(2x(ah+5bd)+ag+11bc+9bex^2\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a +
b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 45*b*e*x^
2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d
 + a*h)*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)
*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((77*b*c +
 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqr
t[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqr
t[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)
```

```
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqr
t[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqr
t[2]*a^(15/4)*b^(5/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.866516, size = 461, normalized size = 1.

$$-6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(16a^{5/4}h+15\sqrt{2}\sqrt{a}b^{3/4}e+80\sqrt[4]{a}bd+7\sqrt{2}a\sqrt[4]{b}g+77\sqrt{2}b^{5/4}c\right)+6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-16a^{5/4}h+\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(77*b*c + 7*a*g + 60*b*d*x + 12*a*h*x + 45*
b*e*x^2))/(a + b*x^4) + (32*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d
 + 9*e*x) + a*(g + 2*h*x)))/(a + b*x^4)^2 - (256*a^(11/4)*Sqrt[b]
*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(a + b*x^4)^3
- 6*(77*Sqrt[2]*b^(5/4)*c + 80*a^(1/4)*b*d + 15*Sqrt[2]*Sqrt[a]*b
^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g + 16*a^(5/4)*h)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(5/4)*c - 80*a^(1/4)*b*
d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g - 16*a^(
5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*b^(1/
4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*b^(1/4)*(77*b*c - 15
*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(3/2))
```

Maple [A] time = 0.02, size = 608, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128*e/a^3*b^2*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*
b*c)/a^3*b*x^9+21/64/a^2*b*e*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^
2*(a*g+11*b*c)*x^5+113/384/a*e*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/12
8*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+7/512*(a/b)^(1/4)/a^
3/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+77/512*c*(a/b)^(1/4
)/a^4*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+7/512*(a/b)^(1/4)/a
^3/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*g+77/512*c*(a/b)^(1/
4)/a^4*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+7/1024*(a/b)^(1/4)
/a^3/b*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a
/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+77/1024*c*(a/b)^(1/4)/a^4*2^(
1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*
x*2^(1/2)+(a/b)^(1/2)))+1/32/(a^7*b^3)^(1/2)*arctan(x^2*(b/a)^(1/
2))*a*h+5/32*b*d/(a^7*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/1024*
```

```
e/a^3/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+15/512*e/a^3/b/(a/
b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+15/512*e/a^3/b/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="ma

[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="fr

[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.225241, size = 703, normalized size = 1.52

$$\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d + 8\,\sqrt{2}\sqrt{ab}abh + 77\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\,\left(ab^3\right)^{\frac{1}{4}}abg + 15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d + 8\,\sqrt{2}\sqrt{ab}abh + 77\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\,\left(ab^3\right)^{\frac{1}{4}}abg + 15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(77\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\,\left(ab^3\right)^{\frac{1}{4}}abg - 15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3} - \frac{\sqrt{2}\left(77\,\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\,\left(ab^3\right)^{\frac{1}{4}}abg - 15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3} + \frac{45\,b^3x^{11}e + 60\,b^3dx^{10} + 12\,ab^2hx^{10} + 77\,b^3cx^9 + 7\,ab^2gx^9 + 126\,ab^2x^7e + 160\,ab^2dx^6 + 32\,a^2bhx^6 + 198\,ab^2cx^5 + 18\,a^2bgx}{384\left(bx^4 + a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="gi
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a
*b*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3
)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*s
qrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)
*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1
/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*ln(x^2 +
sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77
*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e
)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(
45*b^3*x^11*e + 60*b^3*d*x^10 + 12*a*b^2*h*x^10 + 77*b^3*c*x^9 +
7*a*b^2*g*x^9 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 32*a^2*b*h*x^
6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*
b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/(
(b*x^4 + a)^3*a^3*b)
```

3.208 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=516

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{(ah+5bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be)\right)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x\left(2x(ah+5bd)+3x^2(ai+3be)+ag+11bc\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a
*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 1
5*(3*b*e + a*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*
c + a*g + 2*(5*b*d + a*h)*x + 3*(3*b*e + a*i)*x^2))/(96*a^2*b*(a
+ b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a
^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e +
 a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(1
5/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a
*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/
4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i
))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*S
qrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]
*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4))
```

Rubi [A] time = 1.90911, antiderivative size = 516, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{(ah+5bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be)\right)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x\left(2x(ah+5bd)+3x^2(ai+3be)+ag+11bc\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a
*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 1
5*(3*b*e + a*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*
c + a*g + 2*(5*b*d + a*h)*x + 3*(3*b*e + a*i)*x^2))/(96*a^2*b*(a
+ b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a
^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e +
 a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(1
5/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a
*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/
4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i
))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*S
qrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]
*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.38262, size = 530, normalized size = 1.03

$$-\frac{256a^{11/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{(a+bx^4)^3}+\frac{32a^{7/4}b^{3/4}x(ag+ax(2h+3ix)+11bc+bx(10d+9ex))}{(a+bx^4)^2}+\frac{8a^{3/4}b^{3/4}x(7ag+3ax(4h+5ix)+77bc+15bx(4d+}{a+bx^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x]
```

```
[Out] ((32*a^(7/4)*b^(3/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x) + a*x*(
2*h + 3*i*x)))/(a + b*x^4)^2 + (8*a^(3/4)*b^(3/4)*x*(77*b*c + 7*a
*g + 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x)))/(a + b*x^4) - (
256*a^(11/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + x*
(h + i*x)))))/(a + b*x^4)^3 - 6*(77*Sqrt[2]*b^(3/2)*c + 80*a^(1/4
)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g + 16
*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(3/2)*c - 80*a^(1/4)*b^(5/4)*d
 + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/4)*b^
(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] + 3*Sqrt[2]*(-77*b^(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g
 + 5*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2] + 3*Sqrt[2]*(77*b^(3/2)*c - 15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g
- 5*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(3072*a^(15/4)*b^(7/4))
```

Maple [A] time = 0.023, size = 768, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(
a*g+11*b*c)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*
d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5-1/384*(5*a*i-113*b*e)/a/b*x^3-1/
32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x
^4+a)^3+7/512*(a/b)^(1/4)/a^3/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x+1)*g+77/512*c*(a/b)^(1/4)/a^4*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+7/512*(a/b)^(1/4)/a^3/b*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)*g+77/512*c*(a/b)^(1/4)/a^4*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x-1)+7/1024*(a/b)^(1/4)/a^3/b*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*
2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*g+7
7/1024*c*(a/b)^(1/4)/a^4*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a
/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/32/(a^7*b^3
)^(1/2)*arctan(x^2*(b/a)^(1/2))*a*h+5/32*b*d/(a^7*b^3)^(1/2)*arct
an(x^2*(b/a)^(1/2))+5/1024/a^2/b^2/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a
/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2)))*i+15/1024*e/a^3/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4
)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))
+5/512/a^2/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)*i+15/512*e/a^3/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x+1)+5/512/a^2/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x-1)*i+15/512*e/a^3/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algori
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algori
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.23049, size = 992, normalized size = 1.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algori
```

```
[Out] 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(a*b^3)^(3/4)*ln
(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 5/1024*i*(
2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^3*b^4) + sqrt(2)*(a*b^3)^(3/4)*ln(x^2 - sqr
t(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 1/512*sqrt(2)*(40*sq
rt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1
/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/
512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b
*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^
(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3
)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c
 + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(15*a*b^2*i*x^11 + 45
*b^3*x^11*e + 60*b^3*d*x^10 + 12*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*
a*b^2*g*x^9 + 42*a^2*b*i*x^7 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6
+ 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 - 5*a^3*i*x^3
 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c
*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

3.209 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4} dx$

Optimal. Leaf size=534

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{(ah+5bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be)\right)}{384a^3b\left(a+bx^4\right)}-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{96a^2b^2\left(a+bx^4\right)^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3
))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*
h)*x + 15*(3*b*e + a*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (4*a*(2*b
*f + a*j) - x*(b*(11*b*c + a*g) + 2*b*(5*b*d + a*h)*x + 3*b*(3*b*
e + a*i)*x^2))/(96*a^2*b^2*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan
[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b
*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c
 + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c +
 a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt
[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4
))
```

Rubi [A] time = 1.86061, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{(ah+5bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be)\right)}{384a^3b\left(a+bx^4\right)}-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{96a^2b^2\left(a+bx^4\right)^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3
))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*
h)*x + 15*(3*b*e + a*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (4*a*(2*b
*f + a*j) - x*(b*(11*b*c + a*g) + 2*b*(5*b*d + a*h)*x + 3*b*(3*b*
e + a*i)*x^2))/(96*a^2*b^2*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan
[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b
*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c
 + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c +
 a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt
[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4
))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4
```

```
[Out] Timed out
```

Mathematica [A] time = 1.12429, size = 555, normalized size = 1.04

$$\frac{-6\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(16a^{5/4}\sqrt[4]{b}h+5\sqrt{2}a^{3/2}i+80\sqrt[4]{a}b^{5/4}d+15\sqrt{2}\sqrt{a}be+7\sqrt{2}a\sqrt{b}g+77\sqrt{2}b^{3/2}c\right)+6\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4,x
```

```
[Out] ((8*a^(3/4)*b*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x) + 3*a*x*(4
*h + 5*i*x)))/(a + b*x^4) - (32*a^(7/4)*(12*a^2*j - b^2*x*(11*c +
 x*(10*d + 9*e*x)) - a*b*x*(g + x*(2*h + 3*i*x))))/(a + b*x^4)^2
+ (256*a^(11/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g
+ x*(h + i*x)))))/(a + b*x^4)^3 - 6*b^(1/4)*(77*Sqrt[2]*b^(3/2)*c
 + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sq
rt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*b^(1/4)*(77*Sqrt[2]*b^(3/2)*c -
80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[
b]*g - 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*b^(1/4)*(-77*b^(3/2)*c + 15
*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*b^(1/4)*(77*b^(3/2)
*c - 15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^2)
```

Maple [A] time = 0.02, size = 784, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(
a*g+11*b*c)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*
d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5-1/8*j*x^4/b-1/384*(5*a*i-113*b*e
)/a/b*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/
24*(a*j+2*b*f)/b^2)/(b*x^4+a)^3+7/512*(a/b)^(1/4)/a^3/b*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+77/512*c*(a/b)^(1/4)/a^4*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+7/512*(a/b)^(1/4)/a^3/b*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x-1)*g+77/512*c*(a/b)^(1/4)/a^4*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+7/1024*(a/b)^(1/4)/a^3/b*2^(1/2
)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2)))*g+77/1024*c*(a/b)^(1/4)/a^4*2^(1/2)*ln((x^2+
(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2)))+1/32/(a^7*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2))*a*h+5/32*
b*d/(a^7*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2))+5/1024/a^2/b^2/(a/b)^
(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/
b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*i+15/1024*e/a^3/b/(a/b)^(1/4)*2^
(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)
*x*2^(1/2)+(a/b)^(1/2)))+5/512/a^2/b^2/(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x+1)*i+15/512*e/a^3/b/(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x+1)+5/512/a^2/b^2/(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1)*i+15/512*e/a^3/b/(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.228247, size = 1035, normalized size = 1.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((j*x^7 + i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x
```

```
[Out] 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(a*b^3)^(3/4)*ln
(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 5/1024*i*(
2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^3*b^4) + sqrt(2)*(a*b^3)^(3/4)*ln(x^2 - sqr
t(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 1/512*sqrt(2)*(40*sq
rt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1
/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/
512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b
*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^
(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3
)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c
 + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(15*a*b^3*i*x^11 + 45
*b^4*x^11*e + 60*b^4*d*x^10 + 12*a*b^3*h*x^10 + 77*b^4*c*x^9 + 7*
a*b^3*g*x^9 + 42*a^2*b^2*i*x^7 + 126*a*b^3*x^7*e + 160*a*b^3*d*x^
6 + 32*a^2*b^2*h*x^6 + 198*a*b^3*c*x^5 + 18*a^2*b^2*g*x^5 - 48*a^
3*b*j*x^4 - 5*a^3*b*i*x^3 + 113*a^2*b^2*x^3*e + 132*a^2*b^2*d*x^2
 - 12*a^3*b*h*x^2 + 153*a^2*b^2*c*x - 21*a^3*b*g*x - 32*a^3*b*f -
 16*a^4*j)/((b*x^4 + a)^3*a^3*b^2)
```

3.210 $\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*S
qrt[a + b*x^4])
```

Rubi [A] time = 0.153843, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*S
qrt[a + b*x^4])
```

Rubi in Sympy [A] time = 13.3842, size = 109, normalized size = 0.9

$$\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] d*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(2*sqrt(b)) + c*sqrt((a +
b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*ell
iptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*b**(1/4)*sq
rt(a + b*x**4))
```

Mathematica [C] time = 0.265937, size = 107, normalized size = 0.88

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{ic\sqrt{\frac{bx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/Sqrt[a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (I*c*Sqr
t[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x]
, -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*x^4])
```

Maple [C] time = 0.006, size = 96, normalized size = 0.8

$$c\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{d}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+1/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx+c}{\sqrt{bx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 2.81163, size = 61, normalized size = 0.5

$$\frac{d\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{cx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper(
(1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/
4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

3.211 $\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*
c*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])
/(b^(1/4)*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.137483, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*
c*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])
/(b^(1/4)*Sqrt[a - b*x^4])
```

Rubi in Sympy [A] time = 14.9437, size = 78, normalized size = 0.9

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-b*x**4+a)**(1/2),x)
```

```
[Out] a**(1/4)*c*sqrt(1 - b*x**4/a)*elliptic_f(asin(b**(1/4)*x/a**(1/4)
), -1)/(b**(1/4)*sqrt(a - b*x**4)) + d*atan(sqrt(b)*x**2/sqrt(a -
 b*x**4))/(2*sqrt(b))
```

Mathematica [C] time = 0.269044, size = 106, normalized size = 1.22

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}-\frac{ic\sqrt{1-\frac{bx^4}{a}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/Sqrt[a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) - (I*c*Sqrt
[1 - (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[a])]*x],
-1])/(Sqrt[-(Sqrt[b]/Sqrt[a])]*Sqrt[a - b*x^4])
```

Maple [A] time = 0.006, size = 90, normalized size = 1.

$$c\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}+\frac{d}{2}\arctan\left(x^2\sqrt{b}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-b*x^4+a)^(1/2),x)
```

```
[Out] c/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1
/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^
(1/2))^(1/2),I)+1/2*d*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx+c}{\sqrt{-bx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(-b*x^4 + a), x)
```

Sympy [A] time = 2.99458, size = 95, normalized size = 1.09

$$d\left(\begin{cases}-\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for } \left|\frac{bx^4}{a}\right|>1\\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise}\end{cases}\right)+\frac{cx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-b*x**4+a)**(1/2),x)
```

```
[Out] d*Piecewise((-I*acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x*
*4/a) > 1), (asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) + c*x
*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)
/(4*sqrt(a)*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

3.212 $\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{bx^4-a}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4
)*c*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1
])/(b^(1/4)*Sqrt[-a + b*x^4])
```

Rubi [A] time = 0.137626, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{bx^4-a}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[-a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4
)*c*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1
])/(b^(1/4)*Sqrt[-a + b*x^4])
```

Rubi in Sympy [A] time = 15.5759, size = 78, normalized size = 0.88

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{-a+bx^4}}+\frac{d\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(b*x**4-a)**(1/2),x)
```

```
[Out] a**(1/4)*c*sqrt(1 - b*x**4/a)*elliptic_f(asin(b**(1/4)*x/a**(1/4)
), -1)/(b**(1/4)*sqrt(-a + b*x**4)) + d*atanh(sqrt(b)*x**2/sqrt(-
a + b*x**4))/(2*sqrt(b))
```

Mathematica [C] time = 0.246353, size = 108, normalized size = 1.21

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}-\frac{ic\sqrt{1-\frac{bx^4}{a}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/Sqrt[-a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) - (I*c*Sq
rt[1 - (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[a])]*x]
, -1])/(Sqrt[-(Sqrt[b]/Sqrt[a])]*Sqrt[-a + b*x^4])
```

Maple [A] time = 0.023, size = 95, normalized size = 1.1

$$c\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{-1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4-a}}+\frac{d}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4-a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(b*x^4-a)^(1/2),x)
```

```
[Out] c/(-1/a^(1/2)*b^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)*(1-b^(
1/2)*x^2/a^(1/2))^(1/2)/(b*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*b
^(1/2))^(1/2),I)+1/2*d*ln(b^(1/2)*x^2+(b*x^4-a)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^4 - a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx+c}{\sqrt{bx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^4 - a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(b*x^4 - a), x)
```

Sympy [A] time = 3.01345, size = 90, normalized size = 1.01

$$d\left(\begin{cases}\frac{\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for}\ \left|\frac{bx^4}{a}\right|>1\\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise}\end{cases}\right)-\frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(b*x**4-a)**(1/2),x)
```

```
[Out] d*Piecewise((acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/
a) > 1), (-I*asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) - I*c
*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4/a)/(4*sqrt(a)*gamm
a(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(b*x^4 - a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

3.213 $\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx$

Optimal. Leaf size=127

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*S
qrt[-a - b*x^4])
```

Rubi [A] time = 0.156468, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)/Sqrt[-a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*S
qrt[-a - b*x^4])
```

Rubi in Sympy [A] time = 14.8655, size = 112, normalized size = 0.88

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}+\frac{c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)/(-b*x**4-a)**(1/2),x)
```

```
[Out] d*atan(sqrt(b)*x**2/sqrt(-a - b*x**4))/(2*sqrt(b)) + c*sqrt((a +
b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*ell
iptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*b**(1/4)*sq
rt(-a - b*x**4))
```

Mathematica [C] time = 0.268889, size = 113, normalized size = 0.89

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}-\frac{ic\sqrt{\frac{bx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{-a-bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)/Sqrt[-a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) - (I*c*Sqr
t[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x]
, -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[-a - b*x^4])
```

Maple [C] time = 0.023, size = 101, normalized size = 0.8

$$c\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,EllipticF\left(x\sqrt{-i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4-a}}+\frac{d}{2}\arctan\left(x^2\sqrt{b}\frac{1}{\sqrt{-bx^4-a}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)/(-b*x^4-a)^(1/2),x)
```

```
[Out] c/(-I/a^(1/2)*b^(1/2))^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(-b*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1
/2)*b^(1/2))^(1/2),I)+1/2*d*arctan(x^2*b^(1/2)/(-b*x^4-a)^(1/2))/
b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-bx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^4 - a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx+c}{\sqrt{-bx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^4 - a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(-b*x^4 - a), x)
```

Sympy [A] time = 2.92057, size = 66, normalized size = 0.52

$$-\frac{id\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}-\frac{icx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)/(-b*x**4-a)**(1/2),x)
```

```
[Out] -I*d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - I*c*x*gamma(1/4)*h
yper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gam
ma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^4 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)/sqrt(-b*x^4 - a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

3.214 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=257

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[Out] (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b
*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.312084, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```



```
[Out] (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b
*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 30.6001, size = 233, normalized size = 0.91

$$-\frac{\sqrt[4]{a}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}e+\sqrt{b}c\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] -a**(1/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(
a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(
b**(3/4)*sqrt(a + b*x**4)) + d*atanh(sqrt(b)*x**2/sqrt(a + b*x**4
))/(2*sqrt(b)) + e*x*sqrt(a + b*x**4)/(sqrt(b)*(sqrt(a) + sqrt(b)
*x**2)) + sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a)
 + sqrt(b)*x**2)*(sqrt(a)*e + sqrt(b)*c)*elliptic_f(2*atan(b**(1/
4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*b**(3/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.349199, size = 201, normalized size = 0.78

$$\frac{-2\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}e+i\sqrt{b}c\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+2\sqrt{a}e\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\right.\right.}{2\sqrt{b}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]] + 2*Sqrt[a]*e*Sqrt[1 + (b*x^4)/a]*EllipticE[I*
ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 2*(I*Sqrt[b]*c + Sqrt
[a]*e)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/S
qrt[a]]*x], -1])/(2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[b]*Sqrt[a + b*
x^4])
```

Maple [C] time = 0.006, size = 193, normalized size = 0.8

$$c\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{d}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}$$
$$+ie\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+1/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+
I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1
/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + dx + c}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 3.36443, size = 102, normalized size = 0.4

$$\frac{d \operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\,\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{1}{2} \\ \frac{5}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\,\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper(
(1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/
4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

3.215 $\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=14

$$\frac{gx}{\sqrt{a+bx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^4]
```

Rubi [A] time = 0.0117671, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{gx}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (g*x)/Sqrt[a + b*x^4]
```

Rubi in Sympy [A] time = 6.02304, size = 12, normalized size = 0.86

$$\frac{gx}{\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*g*x**4+a*g)/(b*x**4+a)**(3/2),x)

[Out] g*x/sqrt(a + b*x**4)
```

Mathematica [A] time = 0.0249728, size = 14, normalized size = 1.

$$\frac{gx}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (g*x)/Sqrt[a + b*x^4]
```

Maple [A] time = 0.006, size = 13, normalized size = 0.9

$$gx\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x)

[Out] g*x/(b*x^4+a)^(1/2)
```

Maxima [A] time = 1.55461, size = 16, normalized size = 1.14

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*g*x^4 - a*g)/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

Fricas [A] time = 0.216265, size = 16, normalized size = 1.14

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*g*x^4 - a*g)/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

Sympy [A] time = 13.5294, size = 80, normalized size = 5.71

$$\frac{gx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*g*x**4+a*g)/(b*x**4+a)**(3/2),x)
```

```
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2),
(9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

GIAC/XCAS [A] time = 0.214519, size = 16, normalized size = 1.14

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*g*x^4 - a*g)/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

3.216 $\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

```
[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.0498754, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 12.11, size = 22, normalized size = 0.76

$$\frac{x(2ag+ex)}{2a\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*g*x**4+a*g+e*x)/(b*x**4+a)**(3/2),x)

[Out] x*(2*a*g + e*x)/(2*a*sqrt(a + b*x**4))
```

Mathematica [A] time = 0.0414228, size = 29, normalized size = 1.

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

Maple [A] time = 0.007, size = 24, normalized size = 0.8

$$\frac{x(2\,ag+ex)}{2\,a}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*x*(2*a*g+e*x)/(b*x^4+a)^(1/2)/a
```

Maxima [A] time = 1.45333, size = 34, normalized size = 1.17

$$\frac{2\,agx + ex^2}{2\,\sqrt{bx^4 + a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*g*x^4 - a*g - e*x)/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*g*x + e*x^2)/(sqrt(b*x^4 + a)*a)
```

Fricas [A] time = 0.217029, size = 46, normalized size = 1.59

$$\frac{\sqrt{bx^4 + a}\left(2\,agx + ex^2\right)}{2\,(abx^4 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*g*x^4 - a*g - e*x)/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*g*x + e*x^2)/(a*b*x^4 + a^2)
```

Sympy [A] time = 19.287, size = 104, normalized size = 3.59

$$\frac{gx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

```
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2),
(9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**
2/(2*a**(3/2)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [A] time = 0.214844, size = 43, normalized size = 1.48

$$-\frac{x\left(\frac{2\,g}{a^2b^4} + \frac{xe}{a^3b^4}\right)}{64\,\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*g*x^4 - a*g - e*x)/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*x*(2*g/(a^2*b^4) + x*e/(a^3*b^4))/sqrt(b*x^4 + a)
```

3.217 $\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

[Out] -(f - 2*b*g*x)/(2*b*Sqrt[a + b*x^4])

Rubi [A] time = 0.0541837, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] -(f - 2*b*g*x)/(2*b*Sqrt[a + b*x^4])

Rubi in Sympy [A] time = 14.0076, size = 22, normalized size = 0.88

$$-\frac{-2bgx+f}{2b\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((-b*g*x**4+f*x**3+a*g)/(b*x**4+a)**(3/2),x)

[Out] -(-2*b*g*x + f)/(2*b*sqrt(a + b*x**4))

Mathematica [A] time = 0.0436706, size = 27, normalized size = 1.08

$$\frac{2bgx-f}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (-f + 2*b*g*x)/(2*b*Sqrt[a + b*x^4])

Maple [A] time = 0.005, size = 24, normalized size = 1.

$$\frac{2\,bgx-f}{2\,b}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x)

```
[Out] 1/2*(2*b*g*x-f)/b/(b*x^4+a)^(1/2)
```

Maxima [A] time = 1.45987, size = 31, normalized size = 1.24

$$\frac{2\,bgx - f}{2\,\sqrt{bx^4 + a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*g*x^4 - f*x^3 - a*g)/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*g*x - f)/(sqrt(b*x^4 + a)*b)
```

Fricas [A] time = 0.218192, size = 45, normalized size = 1.8

$$\frac{\sqrt{bx^4 + a}(2\,bgx - f)}{2\,(b^2x^4 + ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*g*x^4 - f*x^3 - a*g)/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*b*g*x - f)/(b^2*x^4 + a*b)
```

Sympy [A] time = 20.0076, size = 109, normalized size = 4.36

$$f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\: b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{gx\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\,\Gamma\left(\frac{5}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*g*x**4+f*x**3+a*g)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3
/2)), True)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyp
er((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamm
a(9/4))
```

GIAC/XCAS [A] time = 0.216623, size = 30, normalized size = 1.2

$$\frac{2\,gx - \frac{f}{b}}{2\,\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*g*x^4 - f*x^3 - a*g)/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*g*x - f/b)/sqrt(b*x^4 + a)
```

3.218 $\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{-2abgx+af-bex^2}{2ab\sqrt{a+bx^4}}$$

```
[Out] -(a*f - 2*a*b*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.0612339, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032

$$-\frac{-2abgx+af-bex^2}{2ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] -(a*f - 2*a*b*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 16.536, size = 37, normalized size = 0.97

$$-\frac{-4abgx+2af-2bex^2}{4ab\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*g*x**4+f*x**3+a*g+e*x)/(b*x**4+a)**(3/2),x)

[Out] -(-4*a*b*g*x + 2*a*f - 2*b*e*x**2)/(4*a*b*sqrt(a + b*x**4))
```

Mathematica [A] time = 0.0553107, size = 38, normalized size = 1.

$$\frac{2abgx-af+bex^2}{2ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (-(a*f) + 2*a*b*g*x + b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])
```

Maple [A] time = 0.005, size = 35, normalized size = 0.9

$$\frac{2\,abgx+bex^2-af}{2\,ab}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*(2*a*b*g*x+b*e*x^2-a*f)/a/b/(b*x^4+a)^(1/2)
```

Maxima [A] time = 7.14106, size = 59, normalized size = 1.55

$$\frac{\sqrt{bx^4+a}\left(2\,abgx+bex^2-af\right)}{2\,(ab^2x^4+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*g*x^4 - f*x^3 - a*g - e*x)/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*
b)
```

Fricas [A] time = 0.222882, size = 59, normalized size = 1.55

$$\frac{\sqrt{bx^4+a}\left(2\,abgx+bex^2-af\right)}{2\,(ab^2x^4+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*g*x^4 - f*x^3 - a*g - e*x)/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*
b)
```

Sympy [A] time = 26.051, size = 133, normalized size = 3.5

$$f\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{gx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{bgx^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{9}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*g*x**4+f*x**3+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3
/2)), True)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyp
er((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamm
a(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [A] time = 0.219069, size = 42, normalized size = 1.11

$$\frac{\left(2\,g+\frac{xe}{a}\right)x-\frac{f}{b}}{2\,\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*g*x^4 - f*x^3 - a*g - e*x)/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*g + x*e/a)*x - f/b)/sqrt(b*x^4 + a)
```

3.219 $\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{x^4+1}}$$

```
[Out] -(x/Sqrt[1 + x^4])
```

Rubi [A] time = 0.00872242, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(1 + x^4)^(3/2),x]

[Out] -(x/Sqrt[1 + x^4])
```

Rubi in Sympy [A] time = 2.94961, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**4-1)/(x**4+1)**(3/2),x)

[Out] -x/sqrt(x**4 + 1)
```

Mathematica [A] time = 0.0143957, size = 12, normalized size = 1.

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(1 + x^4)^(3/2),x]

[Out] -(x/Sqrt[1 + x^4])
```

Maple [A] time = 0.007, size = 11, normalized size = 0.9

$$-x\frac{1}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^4+1)^(3/2),x)

[Out] -x/(x^4+1)^(1/2)
```

Maxima [A] time = 5.83016, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 1)/(x^4 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(x^4 + 1)
```

Fricas [A] time = 0.215414, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 1)/(x^4 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] -x/sqrt(x^4 + 1)
```

Sympy [A] time = 3.7857, size = 58, normalized size = 4.83

$$\frac{x^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\,\Gamma\left(\frac{9}{4}\right)}-\frac{x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-1)/(x**4+1)**(3/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(
4*gamma(9/4)) - x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_p
olar(I*pi))/(4*gamma(5/4))
```

GIAC/XCAS [A] time = 0.210607, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 1)/(x^4 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] -x/sqrt(x^4 + 1)
```

3.220 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=385

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}}-9ai+15be\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{(2bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{x\sqrt{a+bx^4}(5be-3ai)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5be-3ai)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b}$$

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*
Sqrt[a + b*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e
- 3*a*i)*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) +
 ((2*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2
)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(15*b*e + (
5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.946276, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}}-9ai+15be\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{(2bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{x\sqrt{a+bx^4}(5be-3ai)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5be-3ai)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4],x]
```


```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*
Sqrt[a + b*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e
- 3*a*i)*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) +
 ((2*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2
)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(15*b*e + (
5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 125.926, size = 335, normalized size = 0.87

$$\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)(3ai-5be)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{7}{4}}\sqrt{a+bx^4}}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{ix^3\sqrt{a+bx^4}}{5b}+\frac{\sqrt{a+bx^4}\left(2f+hx^2\right)}{4b}-\frac{x\sqrt{a+bx^4}(3ai-5be)}{5b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{(ah-2bd)\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{\frac{3}{2}}}-\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}(9ai-15be)+\sqrt{b}(5ag-15bc)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{30\sqrt[4]{a}b^{\frac{7}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] a**(1/4)*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a)
+ sqrt(b)*x**2)*(3*a*i - 5*b*e)*elliptic_e(2*atan(b**(1/4)*x/a**(
1/4)), 1/2)/(5*b**(7/4)*sqrt(a + b*x**4)) + g*x*sqrt(a + b*x**4)/
(3*b) + i*x**3*sqrt(a + b*x**4)/(5*b) + sqrt(a + b*x**4)*(2*f + h
*x**2)/(4*b) - x*sqrt(a + b*x**4)*(3*a*i - 5*b*e)/(5*b**(3/2)*(sq
rt(a) + sqrt(b)*x**2)) - (a*h - 2*b*d)*atanh(sqrt(b)*x**2/sqrt(a
+ b*x**4))/(4*b**(3/2)) - sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x*
*2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*(9*a*i - 15*b*e) + sqrt
(b)*(5*a*g - 15*b*c))*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2
)/(30*a**(1/4)*b**(7/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 1.03916, size = 275, normalized size = 0.71

$$\frac{4\sqrt{\frac{bx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)\left(9a^{3/2}i-15\sqrt{a}be+5ia\sqrt{b}g-15ib^{3/2}c\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(15\sqrt{a+bx^4}(2bd-ah)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.\right.}{60b^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[b]*(a + b*x^4)*(30*f + x*(20*g +
 3*x*(5*h + 4*i*x))) + 15*(2*b*d - a*h)*Sqrt[a + b*x^4]*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) - 12*Sqrt[a]*(-5*b*e + 3*a*i)*Sqrt
[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x],
 -1] + 4*((-15*I)*b^(3/2)*c - 15*Sqrt[a]*b*e + (5*I)*a*Sqrt[b]*g
+ 9*a^(3/2)*i)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sq
rt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^(3/2)*Sq
rt[a + b*x^4])
```

Maple [C] time = 0.01, size = 516, normalized size = 1.3

$$
\begin{aligned}
&c\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{d}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}\\
&+ie\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}\\
&+\frac{f}{2b}\sqrt{bx^4+a}+\frac{gx}{3b}\sqrt{bx^4+a}\\
&-\frac{ag}{3b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{hx^2}{4b}\sqrt{bx^4+a}-\frac{ah}{4}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{ix^3}{5b}\sqrt{bx^4+a}\\
&-\frac{3i}{5}ia^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{3i}{5}ia^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+1/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+
I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1
/2))^(1/2),I))+1/2*f*(b*x^4+a)^(1/2)/b+1/3*g*x*(b*x^4+a)^(1/2)/b-
1/3*g*a/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)+1/4*h*x^2*(b*x^4+a)^(1/2)/b-1/4*h*a/b^
(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))+1/5*i*x^3*(b*x^4+a)^(1/2)/b
-3/5*I*i*a^(3/2)/b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/5*I*i*a^(3/2)/b^(3/2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ix^6 + hx^5 + gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a),x, algo
```

```
[Out] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(
b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ix^6 + hx^5 + gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a),x, algo
```

```
[Out] integral((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b
*x^4 + a), x)
```

Sympy [A] time = 8.9426, size = 260, normalized size = 0.68

$$\frac{\sqrt{a}hx^2\sqrt{1+\frac{bx^4}{a}}}{4b}-\frac{ah\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{cx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{gx^5\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{ix^7\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*h*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*h*asinh(sqrt(b)*x**2/
sqrt(a))/(4*b**(3/2)) + f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)),
 (sqrt(a + b*x**4)/(2*b), True)) + d*asinh(sqrt(b)*x**2/sqrt(a))/
(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper(
(1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/
4)) + g*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + i*x**7*gamma(7/4)*hyper((1/2,
 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ix^6+hx^5+gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a),x, algo
```

```
[Out] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(
b*x^4 + a), x)
```

3.221 $\int \frac{1+x}{1+x^5}\,dx$

Optimal. Leaf size=109

$$-\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-x-(-1)^{4/5}\right)+\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left(x+(-1)^{2/5}\right)-\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(x-(-1)^{3/5}\right)$$

```
[Out] -((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^(1/5) - x])/5 + ((-1)^(4/5
)*(1 - (-1)^(4/5))*Log[-(-1)^(4/5) - x])/5 + ((-1)^(2/5)*(1 - (-1
)^(2/5))*Log[(-1)^(2/5) + x])/5 - ((-1)^(3/5)*(1 + (-1)^(3/5))*Lo
g[-(-1)^(3/5) + x])/5
```

Rubi [F] time = 0.048902, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{1+x}{1+x^5},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(1 + x)/(1 + x^5),x]

[Out] Defer[Int][(1 - x + x^2 - x^3 + x^4)^(-1), x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4-x^3+x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(x**5+1),x)

[Out] Integral(1/(x**4 - x**3 + x**2 - x + 1), x)
```

Mathematica [C] time = 0.0173895, size = 51, normalized size = 0.47

$$\text{RootSum}\left[\#1^4-\#1^3+\#1^2-\#1+1\&,\frac{\log(x-\#1)}{4\#1^3-3\#1^2+2\#1-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/(1 + x^5),x]

[Out] RootSum[1 - #1 + #1^2 - #1^3 + #1^4 & , Log[x - #1]/(-1 + 2*#1 -
3*#1^2 + 4*#1^3) & ]
```

Maple [B] time = 0.033, size = 173, normalized size = 1.6

$$-\frac{\sqrt{5}\ln\left(-x\sqrt{5}+2x^{2}-x+2\right)}{10}+\frac{1}{\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{-\sqrt{5}+4x-1}{\sqrt{10-2\sqrt{5}}}\right)+\frac{\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{-\sqrt{5}+4x-1}{\sqrt{10-2\sqrt{5}}}\right)+\frac{\sqrt{5}\ln\left(x\sqrt{5}+2x^{2}-x+2\right)}{10}+\frac{1}{\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{\sqrt{5}+4x-1}{\sqrt{10+2\sqrt{5}}}\right)-\frac{\sqrt{5}}{5\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{\sqrt{5}+4x-1}{\sqrt{10+2\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(x^5+1),x)
```

```
[Out] -1/10*5^(1/2)*ln(-x*5^(1/2)+2*x^2-x+2)+1/(10-2*5^(1/2))^(1/2)*arc
tan((-5^(1/2)+4*x-1)/(10-2*5^(1/2))^(1/2))+1/5/(10-2*5^(1/2))^(1/
2)*arctan((-5^(1/2)+4*x-1)/(10-2*5^(1/2))^(1/2))*5^(1/2)+1/10*5^(
1/2)*ln(x*5^(1/2)+2*x^2-x+2)+1/(10+2*5^(1/2))^(1/2)*arctan((5^(1/
2)+4*x-1)/(10+2*5^(1/2))^(1/2))-1/5/(10+2*5^(1/2))^(1/2)*arctan((
5^(1/2)+4*x-1)/(10+2*5^(1/2))^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{x^5+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^5 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(x^5 + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^5 + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.798626, size = 36, normalized size = 0.33

$$\operatorname{RootSum}\left(125t^{4}-5t+1,\left(t\mapsto t\log\left(\frac{375t^{3}}{11}+\frac{100t^{2}}{11}+\frac{45t}{11}+x-\frac{14}{11}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(x**5+1),x)
```

```
[Out] RootSum(125*_t**4 - 5*_t + 1, Lambda(_t, _t*log(375*_t**3/11 + 10
0*_t**2/11 + 45*_t/11 + x - 14/11)))
```

GIAC/XCAS [A] time = 0.220089, size = 136, normalized size = 1.25

$$\frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{1}{10}\sqrt{5}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{10}\sqrt{5}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^5 + 1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5
) + 10)) + 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x - sqrt(5) - 1)/sqr
t(-2*sqrt(5) + 10)) - 1/10*sqrt(5)*ln(x^2 - 1/2*x*(sqrt(5) + 1) +
 1) + 1/10*sqrt(5)*ln(x^2 + 1/2*x*(sqrt(5) - 1) + 1)
```

3.222 $\int \frac{1-x}{1-x^5}\,dx$

Optimal. Leaf size=109

$$-\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-x-(-1)^{3/5}\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(x+\sqrt[5]{-1}\right)-\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4}\right.$$

```
[Out] -((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^(2/5) - x])/5 + ((-1)^(3/5
)*(1 + (-1)^(3/5))*Log[-(-1)^(3/5) - x])/5 + ((-1)^(1/5)*(1 + (-1
)^(1/5))*Log[(-1)^(1/5) + x])/5 - ((-1)^(4/5)*(1 - (-1)^(4/5))*Lo
g[-(-1)^(4/5) + x])/5
```

Rubi [F] time = 0.0528554, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{1-x}{1-x^5},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(1 - x)/(1 - x^5),x]

[Out] -Defer[Int][(-1 - x - x^2 - x^3 - x^4)^(-1), x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4+x^3+x^2+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)/(-x**5+1),x)

[Out] Integral(1/(x**4 + x**3 + x**2 + x + 1), x)
```

Mathematica [C] time = 0.0174208, size = 47, normalized size = 0.43

$$\text{RootSum}\left[\#1^4+\#1^3+\#1^2+\#1+1\&,\frac{\log(x-\#1)}{4\#1^3+3\#1^2+2\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x)/(1 - x^5),x]

[Out] RootSum[1 + #1 + #1^2 + #1^3 + #1^4 & , Log[x - #1]/(1 + 2*#1 + 3
*#1^2 + 4*#1^3) & ]
```

Maple [B] time = 0.024, size = 169, normalized size = 1.6

$$-\frac{\sqrt{5}\,\ln\left(-x\sqrt{5}+2\,x^{2}+x+2\right)}{10}+\frac{1}{\sqrt{10+2\,\sqrt{5}}}\arctan\left(\frac{1+4\,x-\sqrt{5}}{\sqrt{10+2\,\sqrt{5}}}\right)-\frac{\sqrt{5}}{5\,\sqrt{10+2\,\sqrt{5}}}\arctan\left(\frac{1+4\,x-\sqrt{5}}{\sqrt{10+2\,\sqrt{5}}}\right)+\frac{\sqrt{5}\,\ln\left(x\sqrt{5}+2\,x^{2}+x+2\right)}{10}+\frac{1}{\sqrt{10-2\,\sqrt{5}}}\arctan\left(\frac{1+4\,x+\sqrt{5}}{\sqrt{10-2\,\sqrt{5}}}\right)+\frac{\sqrt{5}}{5\,\sqrt{10-2\,\sqrt{5}}}\arctan\left(\frac{1+4\,x+\sqrt{5}}{\sqrt{10-2\,\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)/(-x^5+1),x)
```

```
[Out] -1/10*5^(1/2)*ln(-x*5^(1/2)+2*x^2+x+2)+1/(10+2*5^(1/2))^(1/2)*arc
tan((1+4*x-5^(1/2))/(10+2*5^(1/2))^(1/2))-1/5/(10+2*5^(1/2))^(1/2
)*arctan((1+4*x-5^(1/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)+1/10*5^(1/
2)*ln(x*5^(1/2)+2*x^2+x+2)+1/(10-2*5^(1/2))^(1/2)*arctan((1+4*x+5
^(1/2))/(10-2*5^(1/2))^(1/2))+1/5/(10-2*5^(1/2))^(1/2)*arctan((1+
4*x+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{x^{5}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^5 - 1),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(x^5 - 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^5 - 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.789137, size = 36, normalized size = 0.33

$$\operatorname{RootSum}\left(125t^{4}+5t+1,\left(t\mapsto t\log\left(\frac{375t^{3}}{11}-\frac{100t^{2}}{11}+\frac{45t}{11}+x+\frac{14}{11}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)/(-x**5+1),x)
```

```
[Out] RootSum(125*_t**4 + 5*_t + 1, Lambda(_t, _t*log(375*_t**3/11 - 10
0*_t**2/11 + 45*_t/11 + x + 14/11)))
```

GIAC/XCAS [A] time = 0.220766, size = 136, normalized size = 1.25

$$\frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{1}{10}\sqrt{5}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{10}\sqrt{5}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^5 - 1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5
) + 10)) + 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x + sqrt(5) + 1)/sqr
t(-2*sqrt(5) + 10)) + 1/10*sqrt(5)*ln(x^2 + 1/2*x*(sqrt(5) + 1) +
 1) - 1/10*sqrt(5)*ln(x^2 - 1/2*x*(sqrt(5) - 1) + 1)
```

3.223 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=208

$$\frac{x^{12}\left(a^2f-abe+b^2d\right)}{12b^3}-\frac{a^3\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^7}+\frac{a^2x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6}-\frac{ax^6\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5}+\frac{x^9\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{9b^4}+\frac{x^{15}(be-af)}{15b^2}+\frac{fx^{18}}{18b}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^6) - (a*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^6)/(6*b^5) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^9)/(9*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(1
2*b^3) + ((b*e - a*f)*x^15)/(15*b^2) + (f*x^18)/(18*b) - (a^3*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rubi [A] time = 0.608796, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x^{12}\left(a^2f-abe+b^2d\right)}{12b^3}-\frac{a^3\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^7}+\frac{a^2x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6}-\frac{ax^6\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5}+\frac{x^9\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{9b^4}+\frac{x^{15}(be-af)}{15b^2}+\frac{fx^{18}}{18b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^6) - (a*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^6)/(6*b^5) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^9)/(9*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(1
2*b^3) + ((b*e - a*f)*x^15)/(15*b^2) + (f*x^18)/(18*b) - (a^3*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^7}+\frac{a\left(a^3f-a^2be+ab^2d-b^3c\right)\int^{x^3}x\,dx}{3b^5}+\frac{fx^{18}}{18b}-\frac{x^{15}\left(af-be\right)}{15b^2}+\frac{x^{12}\left(a^2f-abe+b^2d\right)}{12b^3}-\frac{x^9\left(a^3f-a^2be+ab^2d-b^3c\right)}{9b^4}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\int^{x^3}a^2\,dx}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b
**7) + a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*Integral(x, (x,
x**3))/(3*b**5) + f*x**18/(18*b) - x**15*(a*f - b*e)/(15*b**2) +
x**12*(a**2*f - a*b*e + b**2*d)/(12*b**3) - x**9*(a**3*f - a**2*b
*e + a*b**2*d - b**3*c)/(9*b**4) - (a**3*f - a**2*b*e + a*b**2*d
- b**3*c)*Integral(a**2, (x, x**3))/(3*b**6)
```

Mathematica [A] time = 0.163741, size = 187, normalized size = 0.9

$$\frac{60a^3\log\left(a+bx^3\right)\left(a^3f-a^2be+ab^2d-b^3c\right)+bx^3\left(-60a^5f+30a^4b\left(2e+fx^3\right)-10a^3b^2\left(6d+3ex^3+2fx^6\right)+5a^2b^3\left(12\right.\right.}{180b^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-60*a^5*f + 30*a^4*b*(2*e + f*x^3) - 10*a^3*b^2*(6*d + 3*
e*x^3 + 2*f*x^6) + 5*a^2*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9)
 + b^5*x^6*(20*c + 15*d*x^3 + 12*e*x^6 + 10*f*x^9) - a*b^4*x^3*(3
0*c + 20*d*x^3 + 15*e*x^6 + 12*f*x^9)) + 60*a^3*(-(b^3*c) + a*b^2
*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(180*b^7)
```

Maple [A] time = 0.007, size = 266, normalized size = 1.3

$$\frac{fx^{18}}{18\,b}-\frac{x^{15}af}{15\,b^2}+\frac{x^{15}e}{15\,b}+\frac{x^{12}a^2f}{12\,b^3}-\frac{x^{12}ae}{12\,b^2}+\frac{x^{12}d}{12\,b}-\frac{x^9a^3f}{9\,b^4}+\frac{x^9a^2e}{9\,b^3}-\frac{x^9ad}{9\,b^2}+\frac{x^9c}{9\,b}+\frac{x^6a^4f}{6\,b^5}-\frac{a^3ex^6}{6\,b^4}+\frac{a^2dx^6}{6\,b^3}-\frac{acx^6}{6\,b^2}-\frac{a^5fx^3}{3\,b^6}+\frac{a^4ex^3}{3\,b^5}-\frac{a^3dx^3}{3\,b^4}+\frac{a^2cx^3}{3\,b^3}+\frac{a^6\ln\left(bx^3+a\right)f}{3\,b^7}-\frac{a^5\ln\left(bx^3+a\right)e}{3\,b^6}+\frac{a^4\ln\left(bx^3+a\right)d}{3\,b^5}-\frac{a^3\ln\left(bx^3+a\right)c}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/18*f*x^18/b-1/15/b^2*x^15*a*f+1/15/b*x^15*e+1/12/b^3*x^12*a^2*f
-1/12/b^2*x^12*a*e+1/12/b*x^12*d-1/9/b^4*x^9*a^3*f+1/9/b^3*x^9*a^
2*e-1/9/b^2*x^9*a*d+1/9/b*x^9*c+1/6/b^5*x^6*a^4*f-1/6/b^4*x^6*a^3
*e+1/6/b^3*x^6*a^2*d-1/6/b^2*x^6*a*c-1/3/b^6*a^5*f*x^3+1/3/b^5*a^
4*e*x^3-1/3/b^4*a^3*d*x^3+1/3/b^3*a^2*c*x^3+1/3*a^6/b^7*ln(b*x^3+
a)*f-1/3*a^5/b^6*ln(b*x^3+a)*e+1/3*a^4/b^5*ln(b*x^3+a)*d-1/3*a^3/
b^4*ln(b*x^3+a)*c
```

Maxima [A] time = 1.40123, size = 282, normalized size = 1.36

$$\frac{10\,b^5fx^{18}+12\left(b^5e-ab^4f\right)x^{15}+15\left(b^5d-ab^4e+a^2b^3f\right)x^{12}+20\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^9-30\left(ab^4c-a^2b^3d+a\right.}{180\,b^6}$$
$$-\frac{\left(a^3b^3c-a^4b^2d+a^5be-a^6f\right)\log\left(bx^3+a\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^11/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/180*(10*b^5*f*x^18 + 12*(b^5*e - a*b^4*f)*x^15 + 15*(b^5*d - a*
b^4*e + a^2*b^3*f)*x^12 + 20*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b
^2*f)*x^9 - 30*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^6 +
60*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^3)/b^6 - 1/3*(a^3*
b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log(b*x^3 + a)/b^7
```

Fricas [A] time = 0.232497, size = 284, normalized size = 1.37

$$\frac{10\,b^6fx^{18}+12\left(b^6e-ab^5f\right)x^{15}+15\left(b^6d-ab^5e+a^2b^4f\right)x^{12}+20\left(b^6c-ab^5d+a^2b^4e-a^3b^3f\right)x^9-30\left(ab^5c-a^2b^4d+a\right.}{180\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^11/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/180*(10*b^6*f*x^18 + 12*(b^6*e - a*b^5*f)*x^15 + 15*(b^6*d - a*
b^5*e + a^2*b^4*f)*x^12 + 20*(b^6*c - a*b^5*d + a^2*b^4*e - a^3*b
^3*f)*x^9 - 30*(a*b^5*c - a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*x^6
+ 60*(a^2*b^4*c - a^3*b^3*d + a^4*b^2*e - a^5*b*f)*x^3 - 60*(a^3*
b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log(b*x^3 + a))/b^7
```

Sympy [A] time = 2.42227, size = 192, normalized size = 0.92

$$\frac{a^3\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^7}+\frac{fx^{18}}{18b}-\frac{x^{15}\left(af-be\right)}{15b^2}+\frac{x^{12}\left(a^2f-abe+b^2d\right)}{12b^3}-\frac{x^9\left(a^3f-a^2be+ab^2d-b^3c\right)}{9b^4}+\frac{x^6\left(a^4f-a^3be+a^2b^2d-ab^3c\right)}{6b^5}-\frac{x^3\left(a^5f-a^4be+a^3b^2d-a^2b^3c\right)}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b
**7) + f*x**18/(18*b) - x**15*(a*f - b*e)/(15*b**2) + x**12*(a**2
*f - a*b*e + b**2*d)/(12*b**3) - x**9*(a**3*f - a**2*b*e + a*b**2
*d - b**3*c)/(9*b**4) + x**6*(a**4*f - a**3*b*e + a**2*b**2*d - a
*b**3*c)/(6*b**5) - x**3*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*
b**3*c)/(3*b**6)
```

GIAC/XCAS [A] time = 0.212789, size = 332, normalized size = 1.6

$$\frac{10\,b^5fx^{18}-12\,ab^4fx^{15}+12\,b^5x^{15}e+15\,b^5dx^{12}+15\,a^2b^3fx^{12}-15\,ab^4x^{12}e+20\,b^5cx^9-20\,ab^4dx^9-20\,a^3b^2fx^9+20\,a^2b^3}{180\,b^6}-\frac{\left(a^3b^3c-a^4b^2d-a^6f+a^5be\right)\ln\left(\left|bx^3+a\right|\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^11/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/180*(10*b^5*f*x^18 - 12*a*b^4*f*x^15 + 12*b^5*x^15*e + 15*b^5*d
*x^12 + 15*a^2*b^3*f*x^12 - 15*a*b^4*x^12*e + 20*b^5*c*x^9 - 20*a
*b^4*d*x^9 - 20*a^3*b^2*f*x^9 + 20*a^2*b^3*x^9*e - 30*a*b^4*c*x^6
 + 30*a^2*b^3*d*x^6 + 30*a^4*b*f*x^6 - 30*a^3*b^2*x^6*e + 60*a^2*
b^3*c*x^3 - 60*a^3*b^2*d*x^3 - 60*a^5*f*x^3 + 60*a^4*b*x^3*e)/b^6
 - 1/3*(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*ln(abs(b*x^3 + a
))/b^7
```

3.224 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=170

$$\frac{x^9\left(a^2f-abe+b^2d\right)}{9b^3}+\frac{a^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6}-\frac{ax^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5}+\frac{x^6\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4}+\frac{x^{12}(be-af)}{12b^2}+\frac{fx^{15}}{15b}$$

```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*x^6)/(6*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^9)/(9*b^3) + ((b*e - a*f)*x^12)/(12*b^2) + (f*x^15)/(15*b) +
(a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rubi [A] time = 0.472533, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^9\left(a^2f-abe+b^2d\right)}{9b^3}+\frac{a^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6}-\frac{ax^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5}+\frac{x^6\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4}+\frac{x^{12}(be-af)}{12b^2}+\frac{fx^{15}}{15b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*x^6)/(6*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^9)/(9*b^3) + ((b*e - a*f)*x^12)/(12*b^2) + (f*x^15)/(15*b) +
(a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^6}+\frac{fx^{15}}{15b}-\frac{x^{12}\left(af-be\right)}{12b^2}+\frac{x^9\left(a^2f-abe+b^2d\right)}{9b^3}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\int^{x^3}x\,dx}{3b^4}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\int^{x^3}a\,dx}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] -a**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*
b**6) + f*x**15/(15*b) - x**12*(a*f - b*e)/(12*b**2) + x**9*(a**2
*f - a*b*e + b**2*d)/(9*b**3) - (a**3*f - a**2*b*e + a*b**2*d - b
**3*c)*Integral(x, (x, x**3))/(3*b**4) + (a**3*f - a**2*b*e + a*b
**2*d - b**3*c)*Integral(a, (x, x**3))/(3*b**5)
```

Mathematica [A] time = 0.137552, size = 154, normalized size = 0.91

$$\frac{bx^3\left(60a^4f-30a^3b\left(2e+fx^3\right)+10a^2b^2\left(6d+3ex^3+2fx^6\right)-5ab^3\left(12c+6dx^3+4ex^6+3fx^9\right)+b^4x^3\left(30c+20dx^3+15e\right.\right.}{180b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(60*a^4*f - 30*a^3*b*(2*e + f*x^3) + 10*a^2*b^2*(6*d + 3*e
*x^3 + 2*f*x^6) - 5*a*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) +
b^4*x^3*(30*c + 20*d*x^3 + 15*e*x^6 + 12*f*x^9)) - 60*a^2*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(180*b^6)
```

Maple [A] time = 0.006, size = 218, normalized size = 1.3

$$\frac{fx^{15}}{15\,b}-\frac{x^{12}af}{12\,b^2}+\frac{x^{12}e}{12\,b}+\frac{x^9a^2f}{9\,b^3}-\frac{x^9ae}{9\,b^2}+\frac{x^9d}{9\,b}-\frac{a^3fx^6}{6\,b^4}+\frac{a^2ex^6}{6\,b^3}-\frac{adx^6}{6\,b^2}+\frac{x^6c}{6\,b}+\frac{a^4fx^3}{3\,b^5}-\frac{a^3ex^3}{3\,b^4}+\frac{a^2dx^3}{3\,b^3}-\frac{acx^3}{3\,b^2}-\frac{a^5\ln\left(bx^3+a\right)f}{3\,b^6}+\frac{a^4\ln\left(bx^3+a\right)e}{3\,b^5}-\frac{a^3\ln\left(bx^3+a\right)d}{3\,b^4}+\frac{a^2\ln\left(bx^3+a\right)c}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/15*f*x^15/b-1/12/b^2*x^12*a*f+1/12/b*x^12*e+1/9/b^3*x^9*a^2*f-1
/9/b^2*x^9*a*e+1/9/b*x^9*d-1/6/b^4*x^6*a^3*f+1/6/b^3*x^6*a^2*e-1/
6/b^2*x^6*a*d+1/6/b*x^6*c+1/3/b^5*a^4*f*x^3-1/3/b^4*a^3*e*x^3+1/3
/b^3*a^2*d*x^3-1/3/b^2*a*c*x^3-1/3*a^5/b^6*ln(b*x^3+a)*f+1/3*a^4/
b^5*ln(b*x^3+a)*e-1/3*a^3/b^4*ln(b*x^3+a)*d+1/3*a^2/b^3*ln(b*x^3+
a)*c
```

Maxima [A] time = 1.45629, size = 228, normalized size = 1.34

$$\frac{12\,b^4fx^{15}+15\left(b^4e-ab^3f\right)x^{12}+20\left(b^4d-ab^3e+a^2b^2f\right)x^9+30\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^6-60\left(ab^3c-a^2b^2d+a^3\right.}{180\,b^5}+\frac{\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)\log\left(bx^3+a\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^8/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/180*(12*b^4*f*x^15 + 15*(b^4*e - a*b^3*f)*x^12 + 20*(b^4*d - a*
b^3*e + a^2*b^2*f)*x^9 + 30*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*
f)*x^6 - 60*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3)/b^5 + 1/
3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*log(b*x^3 + a)/b^6
```

Fricas [A] time = 0.227734, size = 230, normalized size = 1.35

$$\frac{12\,b^5fx^{15}+15\left(b^5e-ab^4f\right)x^{12}+20\left(b^5d-ab^4e+a^2b^3f\right)x^9+30\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^6-60\left(ab^4c-a^2b^3d+a^3\right.}{180\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^8/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^5*f*x^15 + 15*(b^5*e - a*b^4*f)*x^12 + 20*(b^5*d - a*
b^4*e + a^2*b^3*f)*x^9 + 30*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^
2*f)*x^6 - 60*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 6
0*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*log(b*x^3 + a))/b^6
```

Sympy [A] time = 2.32156, size = 155, normalized size = 0.91

$$-\frac{a^2\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^6}+\frac{fx^{15}}{15b}-\frac{x^{12}\left(af-be\right)}{12b^2}+\frac{x^9\left(a^2f-abe+b^2d\right)}{9b^3}-\frac{x^6\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^4}+\frac{x^3\left(a^4f-a^3be+a^2b^2d-ab^3c\right)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] -a**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*
b**6) + f*x**15/(15*b) - x**12*(a*f - b*e)/(12*b**2) + x**9*(a**2
*f - a*b*e + b**2*d)/(9*b**3) - x**6*(a**3*f - a**2*b*e + a*b**2*
d - b**3*c)/(6*b**4) + x**3*(a**4*f - a**3*b*e + a**2*b**2*d - a*
b**3*c)/(3*b**5)
```

GIAC/XCAS [A]   time = 0.214389, size = 266, normalized size = 1.56

$$\frac{12\, b^4 f x^{15} - 15\, a b^3 f x^{12} + 15\, b^4 x^{12} e + 20\, b^4 d x^9 + 20\, a^2 b^2 f x^9 - 20\, a b^3 x^9 e + 30\, b^4 c x^6 - 30\, a b^3 d x^6 - 30\, a^3 b f x^6 + 30\, a^2 b^2 x^6 e}{180\, b^5} + \frac{\left(a^2 b^3 c - a^3 b^2 d - a^5 f + a^4 b e\right) \ln\left(\left|b x^3 + a\right|\right)}{3\, b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^8/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/180*(12*b^4*f*x^15 - 15*a*b^3*f*x^12 + 15*b^4*x^12*e + 20*b^4*d
*x^9 + 20*a^2*b^2*f*x^9 - 20*a*b^3*x^9*e + 30*b^4*c*x^6 - 30*a*b^
3*d*x^6 - 30*a^3*b*f*x^6 + 30*a^2*b^2*x^6*e - 60*a*b^3*c*x^3 + 60
*a^2*b^2*d*x^3 + 60*a^4*f*x^3 - 60*a^3*b*x^3*e)/b^5 + 1/3*(a^2*b^
3*c - a^3*b^2*d - a^5*f + a^4*b*e)*ln(abs(b*x^3 + a))/b^6
```

3.225 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=132

$$\frac{x^6\left(a^2f-abe+b^2d\right)}{6b^3}-\frac{a\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5}+\frac{x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{12}}{12b}$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b
*e + a^2*f)*x^6)/(6*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^12)/(
12*b) - (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3
*b^5)
```

Rubi [A] time = 0.347144, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x^6\left(a^2f-abe+b^2d\right)}{6b^3}-\frac{a\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5}+\frac{x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{12}}{12b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b
*e + a^2*f)*x^6)/(6*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^12)/(
12*b) - (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3
*b^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^5}-\left(\frac{a^3f}{3}-\frac{a^2be}{3}+\frac{ab^2d}{3}-\frac{b^3c}{3}\right)\int^{x^3}\frac{1}{b^4}\,dx+\frac{fx^{12}}{12b}-\frac{x^9\left(af-be\right)}{9b^2}+\frac{\left(a^2f-abe+b^2d\right)\int^{x^3}x\,dx}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**5
) - (a**3*f/3 - a**2*b*e/3 + a*b**2*d/3 - b**3*c/3)*Integral(b**(
-4), (x, x**3)) + f*x**12/(12*b) - x**9*(a*f - b*e)/(9*b**2) + (a
**2*f - a*b*e + b**2*d)*Integral(x, (x, x**3))/(3*b**3)
```

Mathematica [A] time = 0.10138, size = 119, normalized size = 0.9

$$\frac{12a\log\left(a+bx^3\right)\left(a^3f-a^2be+ab^2d-b^3c\right)+bx^3\left(-12a^3f+6a^2b\left(2e+fx^3\right)-2ab^2\left(6d+3ex^3+2fx^6\right)+b^3\left(12c+6dx^3\right.\right.}{36b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-12*a^3*f + 6*a^2*b*(2*e + f*x^3) - 2*a*b^2*(6*d + 3*e*x^
3 + 2*f*x^6) + b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9)) + 12*a*(
-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(36*b^5)
```

Maple [A] time = 0.005, size = 170, normalized size = 1.3

$$\frac{f x^{12}}{12 b}-\frac{x^{9} a f}{9 b^{2}}+\frac{x^{9} e}{9 b}+\frac{a^{2} f x^{6}}{6 b^{3}}-\frac{a e x^{6}}{6 b^{2}}+\frac{d x^{6}}{6 b}-\frac{a^{3} f x^{3}}{3 b^{4}}+\frac{a^{2} e x^{3}}{3 b^{3}}-\frac{a d x^{3}}{3 b^{2}}+\frac{c x^{3}}{3 b}+\frac{a^{4} \ln \left(b x^{3}+a\right) f}{3 b^{5}}-\frac{a^{3} \ln \left(b x^{3}+a\right) e}{3 b^{4}}+\frac{a^{2} \ln \left(b x^{3}+a\right) d}{3 b^{3}}-\frac{a \ln \left(b x^{3}+a\right) c}{3 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/12*f*x^12/b-1/9/b^2*x^9*a*f+1/9/b*x^9*e+1/6/b^3*x^6*a^2*f-1/6/b
^2*x^6*a*e+1/6/b*x^6*d-1/3/b^4*a^3*f*x^3+1/3/b^3*a^2*e*x^3-1/3/b^
2*a*d*x^3+1/3/b*c*x^3+1/3*a^4/b^5*ln(b*x^3+a)*f-1/3*a^3/b^4*ln(b*
x^3+a)*e+1/3*a^2/b^3*ln(b*x^3+a)*d-1/3*a/b^2*ln(b*x^3+a)*c
```

Maxima [A] time = 1.41639, size = 174, normalized size = 1.32

$$\frac{3 b^{3} f x^{12}+4 \left(b^{3} e-a b^{2} f\right) x^{9}+6 \left(b^{3} d-a b^{2} e+a^{2} b f\right) x^{6}+12 \left(b^{3} c-a b^{2} d+a^{2} b e-a^{3} f\right) x^{3}}{36 b^{4}}-\frac{\left(a b^{3} c-a^{2} b^{2} d+a^{3} b e-a^{4} f\right) \log \left(b x^{3}+a\right)}{3 b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^5/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/36*(3*b^3*f*x^12 + 4*(b^3*e - a*b^2*f)*x^9 + 6*(b^3*d - a*b^2*e
 + a^2*b*f)*x^6 + 12*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^4
 - 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a)/b^5
```

Fricas [A] time = 0.225766, size = 176, normalized size = 1.33

$$\frac{3 b^{4} f x^{12}+4 \left(b^{4} e-a b^{3} f\right) x^{9}+6 \left(b^{4} d-a b^{3} e+a^{2} b^{2} f\right) x^{6}+12 \left(b^{4} c-a b^{3} d+a^{2} b^{2} e-a^{3} b f\right) x^{3}-12 \left(a b^{3} c-a^{2} b^{2} d+a^{3} b e-\right.}{36 b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^5/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^4*f*x^12 + 4*(b^4*e - a*b^3*f)*x^9 + 6*(b^4*d - a*b^3*e
 + a^2*b^2*f)*x^6 + 12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^
3 - 12*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a))/b^
5
```

Sympy [A] time = 2.23357, size = 117, normalized size = 0.89

$$\frac{a \left(a^{3} f-a^{2} b e+a b^{2} d-b^{3} c\right) \log \left(a+b x^{3}\right)}{3 b^{5}}+\frac{f x^{12}}{12 b}-\frac{x^{9} \left(a f-b e\right)}{9 b^{2}}+\frac{x^{6} \left(a^{2} f-a b e+b^{2} d\right)}{6 b^{3}}-\frac{x^{3} \left(a^{3} f-a^{2} b e+a b^{2} d-b^{3} c\right)}{3 b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**5
) + f*x**12/(12*b) - x**9*(a*f - b*e)/(9*b**2) + x**6*(a**2*f - a
*b*e + b**2*d)/(6*b**3) - x**3*(a**3*f - a**2*b*e + a*b**2*d - b*
*3*c)/(3*b**4)
```

GIAC/XCAS [A] time = 0.213717, size = 200, normalized size = 1.52

$$\frac{3\,b^3fx^{12}-4\,ab^2fx^9+4\,b^3x^9e+6\,b^3dx^6+6\,a^2bfx^6-6\,ab^2x^6e+12\,b^3cx^3-12\,ab^2dx^3-12\,a^3fx^3+12\,a^2bx^3e}{36\,b^4}-\frac{\left(ab^3c-a^2b^2d-a^4f+a^3be\right)\ln\left(\left|bx^3+a\right|\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^5/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/36*(3*b^3*f*x^12 - 4*a*b^2*f*x^9 + 4*b^3*x^9*e + 6*b^3*d*x^6 +
6*a^2*b*f*x^6 - 6*a*b^2*x^6*e + 12*b^3*c*x^3 - 12*a*b^2*d*x^3 - 1
2*a^3*f*x^3 + 12*a^2*b*x^3*e)/b^4 - 1/3*(a*b^3*c - a^2*b^2*d - a^
4*f + a^3*b*e)*ln(abs(b*x^3 + a))/b^5
```

3.226 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=96

$$\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{x^6(be-af)}{6b^2}+\frac{fx^9}{9b}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^6)/(6*b^2)
 + (f*x^9)/(9*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b
*x^3])/(3*b^4)
```

Rubi [A] time = 0.278955, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{x^6(be-af)}{6b^2}+\frac{fx^9}{9b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^6)/(6*b^2)
 + (f*x^9)/(9*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b
*x^3])/(3*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\left(\frac{a^2f}{3}-\frac{abe}{3}+\frac{b^2d}{3}\right)\int^{x^3}\frac{1}{b^3}\,dx+\frac{fx^9}{9b}-\frac{(af-be)\int^{x^3}x\,dx}{3b^2}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] (a**2*f/3 - a*b*e/3 + b**2*d/3)*Integral(b**(-3), (x, x**3)) + f*
x**9/(9*b) - (a*f - b*e)*Integral(x, (x, x**3))/(3*b**2) - (a**3*
f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**4)
```

Mathematica [A] time = 0.0757291, size = 88, normalized size = 0.92

$$\frac{bx^3\left(6a^2f-3ab\left(2e+fx^3\right)+b^2\left(6d+3ex^3+2fx^6\right)\right)+6\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{18b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(6*a^2*f - 3*a*b*(2*e + f*x^3) + b^2*(6*d + 3*e*x^3 + 2*f*
x^6)) + 6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(18
*b^4)
```

Maple [A] time = 0.004, size = 124, normalized size = 1.3

$$\frac{fx^9}{9\,b}-\frac{x^6af}{6\,b^2}+\frac{ex^6}{6\,b}+\frac{a^2fx^3}{3\,b^3}-\frac{aex^3}{3\,b^2}+\frac{dx^3}{3\,b}-\frac{\ln\left(bx^3+a\right)a^3f}{3\,b^4}+\frac{\ln\left(bx^3+a\right)a^2e}{3\,b^3}-\frac{\ln\left(bx^3+a\right)ad}{3\,b^2}+\frac{c\ln\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/9*f*x^9/b-1/6/b^2*x^6*a*f+1/6/b*x^6*e+1/3/b^3*a^2*f*x^3-1/3/b^2
*a*e*x^3+1/3/b*d*x^3-1/3/b^4*ln(b*x^3+a)*a^3*f+1/3/b^3*ln(b*x^3+a
)*a^2*e-1/3/b^2*ln(b*x^3+a)*a*d+1/3*c*ln(b*x^3+a)/b
```

Maxima [A] time = 1.3992, size = 123, normalized size = 1.28

$$\frac{2\,b^2fx^9+3\,(b^2e-abf)\,x^6+6\,(b^2d-abe+a^2f)\,x^3}{18\,b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,\log\left(bx^3+a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^2/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/18*(2*b^2*f*x^9 + 3*(b^2*e - a*b*f)*x^6 + 6*(b^2*d - a*b*e + a^
2*f)*x^3)/b^3 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3
 + a)/b^4
```

Fricas [A] time = 0.212525, size = 124, normalized size = 1.29

$$\frac{2\,b^3fx^9+3\,(b^3e-ab^2f)\,x^6+6\,(b^3d-ab^2e+a^2bf)\,x^3+6\,(b^3c-ab^2d+a^2be-a^3f)\,\log\left(bx^3+a\right)}{18\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^2/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^3*f*x^9 + 3*(b^3*e - a*b^2*f)*x^6 + 6*(b^3*d - a*b^2*e
+ a^2*b*f)*x^3 + 6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3
+ a))/b^4
```

Sympy [A] time = 2.05452, size = 83, normalized size = 0.86

$$\frac{fx^9}{9b}-\frac{x^6\,(af-be)}{6b^2}+\frac{x^3\,(a^2f-abe+b^2d)}{3b^3}-\frac{(a^3f-a^2be+ab^2d-b^3c)\log\left(a+bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] f*x**9/(9*b) - x**6*(a*f - b*e)/(6*b**2) + x**3*(a**2*f - a*b*e +
 b**2*d)/(3*b**3) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a
 + b*x**3)/(3*b**4)
```

GIAC/XCAS [A] time = 0.213935, size = 136, normalized size = 1.42

$$\frac{2\,b^2fx^9-3\,abfx^6+3\,b^2x^6e+6\,b^2dx^3+6\,a^2fx^3-6\,abx^3e}{18\,b^3}+\frac{(b^3c-ab^2d-a^3f+a^2be)\ln\left(\left|bx^3+a\right|\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^2/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/18*(2*b^2*f*x^9 - 3*a*b*f*x^6 + 3*b^2*x^6*e + 6*b^2*d*x^3 + 6*a
^2*f*x^3 - 6*a*b*x^3*e)/b^3 + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*
b*e)*ln(abs(b*x^3 + a))/b^4
```

3.227 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx$

Optimal. Leaf size=80

$$-\frac{\log \left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3}+\frac{x^3(be-af)}{3b^2}+\frac{c\log(x)}{a}+\frac{fx^6}{6b}$$

```
[Out] ((b*e - a*f)*x^3)/(3*b^2) + (f*x^6)/(6*b) + (c*Log[x])/a - ((b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a*b^3)
```

Rubi [A] time = 0.221115, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{\log \left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3}+\frac{x^3(be-af)}{3b^2}+\frac{c\log(x)}{a}+\frac{fx^6}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

```
[Out] ((b*e - a*f)*x^3)/(3*b^2) + (f*x^6)/(6*b) + (c*Log[x])/a - ((b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{af}{3}-\frac{be}{3}\right)\int^{x^3}\frac{1}{b^2}\,dx+\frac{f\int^{x^3}x\,dx}{3b}+\frac{c\log\left(x^3\right)}{3a}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a),x)
```

```
[Out] -(a*f/3 - b*e/3)*Integral(b**(-2), (x, x**3)) + f*Integral(x, (x,
 x**3))/(3*b) + c*log(x**3)/(3*a) + (a**3*f - a**2*b*e + a*b**2*d
 - b**3*c)*log(a + b*x**3)/(3*a*b**3)
```

Mathematica [A] time = 0.0552851, size = 75, normalized size = 0.94

$$\frac{-2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+abx^3\left(-2af+2be+bfx^3\right)+6b^3c\log(x)}{6ab^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

```
[Out] (a*b*x^3*(2*b*e - 2*a*f + b*f*x^3) + 6*b^3*c*Log[x] - 2*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(6*a*b^3)
```

Maple [A] time = 0.009, size = 97, normalized size = 1.2

$$\frac{fx^6}{6\,b}-\frac{ax^3f}{3\,b^2}+\frac{ex^3}{3\,b}+\frac{c\ln(x)}{a}+\frac{a^2\ln\left(bx^3+a\right)f}{3\,b^3}-\frac{ae\ln\left(bx^3+a\right)}{3\,b^2}+\frac{d\ln\left(bx^3+a\right)}{3\,b}-\frac{c\ln\left(bx^3+a\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x)
```

```
[Out] 1/6*f*x^6/b-1/3/b^2*x^3*a*f+1/3*e*x^3/b+c*ln(x)/a+1/3*a^2/b^3*ln(
b*x^3+a)*f-1/3*a*e*ln(b*x^3+a)/b^2+1/3*d*ln(b*x^3+a)/b-1/3*c*ln(b
*x^3+a)/a
```

Maxima [A] time = 7.32697, size = 104, normalized size = 1.3

$$\frac{c\log\left(x^{3}\right)}{3\,a}+\frac{bfx^{6}+2\,(be-af)x^{3}}{6\,b^{2}}-\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(bx^{3}+a\right)}{3\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x),x, algorithm="maxima")
```

```
[Out] 1/3*c*log(x^3)/a + 1/6*(b*f*x^6 + 2*(b*e - a*f)*x^3)/b^2 - 1/3*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/(a*b^3)
```

Fricas [A] time = 0.246527, size = 108, normalized size = 1.35

$$\frac{ab^{2}fx^{6}+6\,b^{3}c\log\left(x\right)+2\,\left(ab^{2}e-a^{2}bf\right)x^{3}-2\,\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(bx^{3}+a\right)}{6\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x),x, algorithm="fricas")
```

```
[Out] 1/6*(a*b^2*f*x^6 + 6*b^3*c*log(x) + 2*(a*b^2*e - a^2*b*f)*x^3 - 2
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/(a*b^3)
```

Sympy [A] time = 9.6601, size = 68, normalized size = 0.85

$$\frac{fx^{6}}{6b}-\frac{x^{3}\left(af-be\right)}{3b^{2}}+\frac{c\log\left(x\right)}{a}+\frac{\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\log\left(\frac{a}{b}+x^{3}\right)}{3ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a),x)
```

```
[Out] f*x**6/(6*b) - x**3*(a*f - b*e)/(3*b**2) + c*log(x)/a + (a**3*f -
 a**2*b*e + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a*b**3)
```

GIAC/XCAS [A] time = 0.209952, size = 107, normalized size = 1.34

$$\frac{c\ln\left(|x|\right)}{a}+\frac{bfx^{6}-2\,afx^{3}+2\,bx^{3}e}{6\,b^{2}}-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\ln\left(\left|bx^{3}+a\right|\right)}{3\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x),x, algorithm="giac")
```

```
[Out] c*ln(abs(x))/a + 1/6*(b*f*x^6 - 2*a*f*x^3 + 2*b*x^3*e)/b^2 - 1/3*
(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*ln(abs(b*x^3 + a))/(a*b^3)
```

3.228 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=81

$$-\frac{\log(x)(bc-ad)}{a^2}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2}-\frac{c}{3ax^3}+\frac{fx^3}{3b}$$

```
[Out] -c/(3*a*x^3) + (f*x^3)/(3*b) - ((b*c - a*d)*Log[x])/a^2 + ((b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^2*b^2)
```

Rubi [A] time = 0.210764, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{\log(x)(bc-ad)}{a^2}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2}-\frac{c}{3ax^3}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

```
[Out] -c/(3*a*x^3) + (f*x^3)/(3*b) - ((b*c - a*d)*Log[x])/a^2 + ((b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^2*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^3} f\,dx}{3b}-\frac{c}{3ax^3}+\frac{(ad-bc)\log\left(x^3\right)}{3a^2}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a),x)
```

```
[Out] Integral(f, (x, x**3))/(3*b) - c/(3*a*x**3) + (a*d - b*c)*log(x**
3)/(3*a**2) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x
**3)/(3*a**2*b**2)
```

Mathematica [A] time = 0.0753243, size = 77, normalized size = 0.95

$$\frac{1}{3}\left(\frac{3\log(x)(ad-bc)}{a^2}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^2b^2}-\frac{c}{ax^3}+\frac{fx^3}{b}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

```
[Out] (-(c/(a*x^3)) + (f*x^3)/b + (3*(-(b*c) + a*d)*Log[x])/a^2 + ((b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(a^2*b^2))/3
```

Maple [A] time = 0.013, size = 94, normalized size = 1.2

$$\frac{fx^3}{3\,b}-\frac{c}{3\,ax^3}+\frac{d\ln(x)}{a}-\frac{\ln(x)\,bc}{a^2}-\frac{a\ln\left(bx^3+a\right)f}{3\,b^2}+\frac{e\ln\left(bx^3+a\right)}{3\,b}-\frac{d\ln\left(bx^3+a\right)}{3\,a}+\frac{b\ln\left(bx^3+a\right)c}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x)
```

```
[Out] 1/3*f*x^3/b-1/3*c/a/x^3+d*ln(x)/a-1/a^2*ln(x)*b*c-1/3*a/b^2*ln(b*
x^3+a)*f+1/3*e*ln(b*x^3+a)/b-1/3*d*ln(b*x^3+a)/a+1/3/a^2*b*ln(b*x
^3+a)*c
```

Maxima [A] time = 1.42186, size = 104, normalized size = 1.28

$$\frac{fx^3}{3\,b} - \frac{(bc-ad)\log\left(x^3\right)}{3\,a^2} + \frac{\left(b^3c - ab^2d + a^2be - a^3f\right)\log\left(bx^3+a\right)}{3\,a^2b^2} - \frac{c}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^4),x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b - 1/3*(b*c - a*d)*log(x^3)/a^2 + 1/3*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*log(b*x^3 + a)/(a^2*b^2) - 1/3*c/(a*x^3)
```

Fricas [A] time = 0.239169, size = 115, normalized size = 1.42

$$\frac{a^2bfx^6 + \left(b^3c - ab^2d + a^2be - a^3f\right)x^3\log\left(bx^3+a\right) - 3\left(b^3c - ab^2d\right)x^3\log(x) - ab^2c}{3\,a^2b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*b*f*x^6 + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3*log(b*
x^3 + a) - 3*(b^3*c - a*b^2*d)*x^3*log(x) - a*b^2*c)/(a^2*b^2*x^3
)
```

Sympy [A] time = 23.0633, size = 70, normalized size = 0.86

$$\frac{fx^3}{3b} - \frac{c}{3ax^3} + \frac{(ad-bc)\log(x)}{a^2} - \frac{\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(\frac{a}{b} + x^3\right)}{3a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a),x)
```

```
[Out] f*x**3/(3*b) - c/(3*a*x**3) + (a*d - b*c)*log(x)/a**2 - (a**3*f -
 a**2*b*e + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a**2*b**2)
```

GIAC/XCAS [A] time = 0.213959, size = 128, normalized size = 1.58

$$\frac{fx^3}{3\,b} - \frac{(bc-ad)\ln(|x|)}{a^2} + \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\ln\left(\left|bx^3+a\right|\right)}{3\,a^2b^2} + \frac{bcx^3 - adx^3 - ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^4),x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b - (b*c - a*d)*ln(abs(x))/a^2 + 1/3*(b^3*c - a*b^2*d -
 a^3*f + a^2*b*e)*ln(abs(b*x^3 + a))/(a^2*b^2) + 1/3*(b*c*x^3 - a
*d*x^3 - a*c)/(a^2*x^3)
```

3.229 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx$

Optimal. Leaf size=95

$$\frac{bc-ad}{3a^2x^3}+\frac{\log(x)\left(a^2e-abd+b^2c\right)}{a^3}-\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b}-\frac{c}{6ax^6}$$

```
[Out] -c/(6*a*x^6) + (b*c - a*d)/(3*a^2*x^3) + ((b^2*c - a*b*d + a^2*e)
*Log[x])/a^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3
])/(3*a^3*b)
```

Rubi [A] time = 0.243478, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{bc-ad}{3a^2x^3}+\frac{\log(x)\left(a^2e-abd+b^2c\right)}{a^3}-\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b}-\frac{c}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

```
[Out] -c/(6*a*x^6) + (b*c - a*d)/(3*a^2*x^3) + ((b^2*c - a*b*d + a^2*e)
*Log[x])/a^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3
])/(3*a^3*b)
```

Rubi in Sympy [A] time = 44.4511, size = 87, normalized size = 0.92

$$-\frac{c}{6ax^6}-\frac{ad-bc}{3a^2x^3}+\frac{\left(a^2e-abd+b^2c\right)\log\left(x^3\right)}{3a^3}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a),x)
```

```
[Out] -c/(6*a*x**6) - (a*d - b*c)/(3*a**2*x**3) + (a**2*e - a*b*d + b**
2*c)*log(x**3)/(3*a**3) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*log(a + b*x**3)/(3*a**3*b)
```

Mathematica [A] time = 0.138683, size = 88, normalized size = 0.93

$$\frac{6\log(x)\left(a^2e-abd+b^2c\right)+\log\left(a+bx^3\right)\left(\frac{2a^3f}{b}-2a^2e+2abd-2b^2c\right)-\frac{a\left(ac+2adx^3-2bcx^3\right)}{x^6}}{6a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

```
[Out] (-((a*(a*c - 2*b*c*x^3 + 2*a*d*x^3))/x^6) + 6*(b^2*c - a*b*d + a^
2*e)*Log[x] + (-2*b^2*c + 2*a*b*d - 2*a^2*e + (2*a^3*f)/b)*Log[a
+ b*x^3])/(6*a^3)
```

Maple [A] time = 0.011, size = 116, normalized size = 1.2

$$-\frac{c}{6\,ax^6}-\frac{d}{3\,ax^3}+\frac{bc}{3\,a^2x^3}+\frac{e\ln(x)}{a}-\frac{\ln(x)\,bd}{a^2}+\frac{\ln(x)\,b^2c}{a^3}+\frac{\ln\left(bx^3+a\right)f}{3\,b}-\frac{e\ln\left(bx^3+a\right)}{3\,a}+\frac{b\ln\left(bx^3+a\right)d}{3\,a^2}-\frac{b^2\ln\left(bx^3+a\right)c}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x)
```

```
[Out] -1/6*c/a/x^6-1/3/a/x^3*d+1/3/a^2/x^3*b*c+e*ln(x)/a-1/a^2*ln(x)*b*
d+1/a^3*ln(x)*b^2*c+1/3/b*ln(b*x^3+a)*f-1/3*e*ln(b*x^3+a)/a+1/3/a
^2*b*ln(b*x^3+a)*d-1/3/a^3*b^2*ln(b*x^3+a)*c
```

Maxima [A] time = 1.38527, size = 126, normalized size = 1.33

$$\frac{(b^2c - abd + a^2e)\log(x^3)}{3\,a^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)\log(bx^3 + a)}{3\,a^3b} + \frac{2\,(bc - ad)x^3 - ac}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^7),x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c - a*b*d + a^2*e)*log(x^3)/a^3 - 1/3*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*log(b*x^3 + a)/(a^3*b) + 1/6*(2*(b*c - a*d)*x^3
 - a*c)/(a^2*x^6)
```

Fricas [A] time = 0.248078, size = 136, normalized size = 1.43

$$-\frac{2\left(b^3c - ab^2d + a^2be - a^3f\right)x^6\log\left(bx^3 + a\right) - 6\left(b^3c - ab^2d + a^2be\right)x^6\log\left(x\right) + a^2bc - 2\left(ab^2c - a^2bd\right)x^3}{6\,a^3bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^7),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6*log(b*x^3 + a) -
6*(b^3*c - a*b^2*d + a^2*b*e)*x^6*log(x) + a^2*b*c - 2*(a*b^2*c -
 a^2*b*d)*x^3)/(a^3*b*x^6)
```

Sympy [A] time = 92.4216, size = 85, normalized size = 0.89

$$-\frac{ac + x^3\left(2ad - 2bc\right)}{6a^2x^6} + \frac{\left(a^2e - abd + b^2c\right)\log\left(x\right)}{a^3} + \frac{\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(\frac{a}{b} + x^3\right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a),x)
```

```
[Out] -(a*c + x**3*(2*a*d - 2*b*c))/(6*a**2*x**6) + (a**2*e - a*b*d + b
**2*c)*log(x)/a**3 + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(
a/b + x**3)/(3*a**3*b)
```

GIAC/XCAS [A] time = 0.215596, size = 170, normalized size = 1.79

$$\frac{\left(b^2c - abd + a^2e\right)\ln\left(|x|\right)}{a^3} - \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\ln\left(\left|bx^3 + a\right|\right)}{3\,a^3b} - \frac{3\,b^2cx^6 - 3\,abdx^6 + 3\,a^2x^6e - 2\,abcx^3 + 2\,a^2dx^3 + a^2c}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^7),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d + a^2*e)*ln(abs(x))/a^3 - 1/3*(b^3*c - a*b^2*d - a
^3*f + a^2*b*e)*ln(abs(b*x^3 + a))/(a^3*b) - 1/6*(3*b^2*c*x^6 - 3
*a*b*d*x^6 + 3*a^2*x^6*e - 2*a*b*c*x^3 + 2*a^2*d*x^3 + a^2*c)/(a^
3*x^6)
```

3.230 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx$

Optimal. Leaf size=128

$$\frac{bc-ad}{6a^2x^6}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4}-\frac{\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^4}-\frac{c}{9ax^9}$$

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/
(3*a^3*x^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^4 +
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Rubi [A] time = 0.307128, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{bc-ad}{6a^2x^6}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4}-\frac{\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^4}-\frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/
(3*a^3*x^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^4 +
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Rubi in Sympy [A] time = 50.1633, size = 117, normalized size = 0.91

$$-\frac{c}{9ax^9}-\frac{ad-bc}{6a^2x^6}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(x^3\right)}{3a^4}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a),x)
```

```
[Out] -c/(9*a*x**9) - (a*d - b*c)/(6*a**2*x**6) - (a**2*e - a*b*d + b**
2*c)/(3*a**3*x**3) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(
x**3)/(3*a**4) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a +
b*x**3)/(3*a**4)
```

Mathematica [A] time = 0.129909, size = 128, normalized size = 1.

$$\frac{bc-ad}{6a^2x^6}+\frac{a^2(-e)+abd-b^2c}{3a^3x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4}+\frac{\log(x)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^4}-\frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) + (-(b^2*c) + a*b*d - a^2*
e)/(3*a^3*x^3) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[x])/
a^4 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4
)
```

Maple [A] time = 0.012, size = 161, normalized size = 1.3

$$-\frac{c}{9\,a x^9}-\frac{d}{6\,a x^6}+\frac{bc}{6\,a^2x^6}-\frac{e}{3\,a x^3}+\frac{bd}{3\,a^2x^3}-\frac{b^2c}{3\,a^3x^3}+\frac{\ln(x)\,f}{a}-\frac{\ln(x)\,be}{a^2}+\frac{\ln(x)\,b^2d}{a^3}-\frac{\ln(x)\,b^3c}{a^4}-\frac{\ln\left(bx^3+a\right)\,f}{3\,a}+\frac{\ln\left(bx^3+a\right)\,be}{3\,a^2}-\frac{\ln\left(bx^3+a\right)\,b^2d}{3\,a^3}+\frac{\ln\left(bx^3+a\right)\,b^3c}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x)
```

```
[Out] -1/9*c/a/x^9-1/6/a/x^6*d+1/6/a^2/x^6*b*c-1/3/a/x^3*e+1/3/a^2/x^3*
b*d-1/3/a^3/x^3*b^2*c+1/a*ln(x)*f-1/a^2*ln(x)*b*e+1/a^3*ln(x)*b^2
*d-1/a^4*ln(x)*b^3*c-1/3/a*ln(b*x^3+a)*f+1/3/a^2*ln(b*x^3+a)*b*e-
1/3/a^3*ln(b*x^3+a)*b^2*d+1/3/a^4*ln(b*x^3+a)*b^3*c
```

Maxima [A] time = 1.43352, size = 169, normalized size = 1.32

$$\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,a^4}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(x^3\right)}{3\,a^4}-\frac{6\left(b^2c-abd+a^2e\right)x^6-3\left(abc-a^2d\right)x^3+2\,a^2c}{18\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^10),x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/a^4 - 1/3*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x^3)/a^4 - 1/18*(6*(b^2*c
 - a*b*d + a^2*e)*x^6 - 3*(a*b*c - a^2*d)*x^3 + 2*a^2*c)/(a^3*x^9
)
```

Fricas [A] time = 0.237382, size = 171, normalized size = 1.34

$$\frac{6\left(b^3c-ab^2d+a^2be-a^3f\right)x^9\log\left(bx^3+a\right)-18\left(b^3c-ab^2d+a^2be-a^3f\right)x^9\log(x)-6\left(ab^2c-a^2bd+a^3e\right)x^6-2\,a^3c+}{18\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^10),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(b*x^3 + a) -
18*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(x) - 6*(a*b^2*c -
a^2*b*d + a^3*e)*x^6 - 2*a^3*c + 3*(a^2*b*c - a^3*d)*x^3)/(a^4*x^
9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.214182, size = 248, normalized size = 1.94

$$-\frac{\left(b^{3} c - a b^{2} d - a^{3} f + a^{2} b e\right) \ln\left(|x|\right)}{a^{4}} + \frac{\left(b^{4} c - a b^{3} d - a^{3} b f + a^{2} b^{2} e\right) \ln\left(\left|b x^{3} + a\right|\right)}{3 \, a^{4} b} + \frac{11 \, b^{3} c x^{9} - 11 \, a b^{2} d x^{9} - 11 \, a^{3} f x^{9} + 11 \, a^{2} b x^{9} e - 6 \, a b^{2} c x^{6} + 6 \, a^{2} b d x^{6} - 6 \, a^{3} x^{6} e + 3 \, a^{2} b c x^{3} - 3 \, a^{3} d x^{3} - 2 \, a^{3} c}{18 \, a^{4} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^10),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*ln(abs(x))/a^4 + 1/3*(b^4*c
- a*b^3*d - a^3*b*f + a^2*b^2*e)*ln(abs(b*x^3 + a))/(a^4*b) + 1/1
8*(11*b^3*c*x^9 - 11*a*b^2*d*x^9 - 11*a^3*f*x^9 + 11*a^2*b*x^9*e
- 6*a*b^2*c*x^6 + 6*a^2*b*d*x^6 - 6*a^3*x^6*e + 3*a^2*b*c*x^3 - 3
*a^3*d*x^3 - 2*a^3*c)/(a^4*x^9)
```

3.231 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx$

Optimal. Leaf size=164

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{6a^3x^6}-\frac{b\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5}+\frac{b\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{12ax^{12}}$$

```
[Out] -c/(12*a*x^12) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e
)/(6*a^3*x^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) +
 (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^5 - (b*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^5)
```

Rubi [A] time = 0.358205, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{6a^3x^6}-\frac{b\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5}+\frac{b\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] -c/(12*a*x^12) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e
)/(6*a^3*x^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) +
 (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^5 - (b*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^5)
```

Rubi in Sympy [A] time = 56.3961, size = 153, normalized size = 0.93

$$-\frac{c}{12ax^{12}}-\frac{ad-bc}{9a^2x^9}-\frac{a^2e-abd+b^2c}{6a^3x^6}-\frac{a^3f-a^2be+ab^2d-b^3c}{3a^4x^3}-\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(x^3\right)}{3a^5}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a),x)
```

```
[Out] -c/(12*a*x**12) - (a*d - b*c)/(9*a**2*x**9) - (a**2*e - a*b*d + b
**2*c)/(6*a**3*x**6) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3
*a**4*x**3) - b*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(x**3)
/(3*a**5) + b*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x
**3)/(3*a**5)
```

Mathematica [A] time = 0.131631, size = 164, normalized size = 1.

$$\frac{-a^4\left(3c+4dx^3+6ex^6+12fx^9\right)+2a^3bx^3\left(2c+3dx^3+6ex^6\right)-6a^2b^2x^6\left(c+2dx^3\right)+36bx^{12}\log(x)\left(a^3(-f)+a^2be-ab^2d\right.}{36a^5x^{12}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] (12*a*b^3*c*x^9 - 6*a^2*b^2*x^6*(c + 2*d*x^3) + 2*a^3*b*x^3*(2*c
+ 3*d*x^3 + 6*e*x^6) - a^4*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) +
 36*b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^12*Log[x] - 12*b*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x^12*Log[a + b*x^3])/(36*a^5*x^12
)
```

Maple [A] time = 0.014, size = 210, normalized size = 1.3

$$-\frac{c}{12\,a x^{12}}-\frac{d}{9\,a x^{9}}+\frac{bc}{9\,a^{2}x^{9}}-\frac{e}{6\,a x^{6}}+\frac{bd}{6\,a^{2}x^{6}}-\frac{b^{2}c}{6\,a^{3}x^{6}}-\frac{f}{3\,a x^{3}}+\frac{be}{3\,a^{2}x^{3}}-\frac{b^{2}d}{3\,a^{3}x^{3}}+\frac{b^{3}c}{3\,a^{4}x^{3}}-\frac{b\ln(x)f}{a^{2}}+\frac{b^{2}\ln(x)e}{a^{3}}-\frac{b^{3}\ln(x)d}{a^{4}}+\frac{b^{4}\ln(x)c}{a^{5}}+\frac{b\ln\left(bx^{3}+a\right)f}{3\,a^{2}}-\frac{b^{2}\ln\left(bx^{3}+a\right)e}{3\,a^{3}}+\frac{b^{3}\ln\left(bx^{3}+a\right)d}{3\,a^{4}}-\frac{b^{4}\ln\left(bx^{3}+a\right)c}{3\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x)
```

```
[Out] -1/12*c/a/x^12-1/9/a/x^9*d+1/9/a^2/x^9*b*c-1/6/a/x^6*e+1/6/a^2/x^
6*b*d-1/6/a^3/x^6*b^2*c-1/3/a/x^3*f+1/3/a^2/x^3*b*e-1/3/a^3/x^3*b
^2*d+1/3/a^4/x^3*b^3*c-1/a^2*b*ln(x)*f+1/a^3*b^2*ln(x)*e-1/a^4*b^
3*ln(x)*d+1/a^5*b^4*ln(x)*c+1/3*b/a^2*ln(b*x^3+a)*f-1/3*b^2/a^3*l
n(b*x^3+a)*e+1/3*b^3/a^4*ln(b*x^3+a)*d-1/3*b^4/a^5*ln(b*x^3+a)*c
```

Maxima [A] time = 1.48564, size = 224, normalized size = 1.37

$$-\frac{\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)\log\left(bx^{3}+a\right)}{3\,a^{5}}+\frac{\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)\log\left(x^{3}\right)}{3\,a^{5}}+\frac{12\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{9}-6\left(ab^{2}c-a^{2}bd+a^{3}e\right)x^{6}-3\,a^{3}c+4\left(a^{2}bc-a^{3}d\right)x^{3}}{36\,a^{4}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^13),x, algorithm="maxima")
```

```
[Out] -1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(b*x^3 + a)/a^5 +
 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x^3)/a^5 + 1/36*
(12*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 6*(a*b^2*c - a^2*b*
d + a^3*e)*x^6 - 3*a^3*c + 4*(a^2*b*c - a^3*d)*x^3)/(a^4*x^12)
```

Fricas [A] time = 0.260637, size = 227, normalized size = 1.38

$$-\frac{12\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{12}\log\left(bx^{3}+a\right)-36\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{12}\log\left(x\right)-12\left(ab^{3}c-a^{2}b^{2}d+a^{3}be\right.}{36\,a^{5}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^13),x, algorithm="fricas")
```

```
[Out] -1/36*(12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(b*x^3
+ a) - 36*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(x) - 1
2*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 6*(a^2*b^2*c - a^
3*b*d + a^4*e)*x^6 + 3*a^4*c - 4*(a^3*b*c - a^4*d)*x^3)/(a^5*x^12
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.214204, size = 317, normalized size = 1.93

$$\frac{\left(b^{4}c-ab^{3}d-a^{3}bf+a^{2}b^{2}e\right)\ln\left(|x|\right)}{a^{5}}-\frac{\left(b^{5}c-ab^{4}d-a^{3}b^{2}f+a^{2}b^{3}e\right)\ln\left(\left|bx^{3}+a\right|\right)}{3\,a^{5}b}-\frac{25\,b^{4}cx^{12}-25\,ab^{3}dx^{12}-25\,a^{3}bfx^{12}+25\,a^{2}b^{2}x^{12}e-12\,ab^{3}cx^{9}+12\,a^{2}b^{2}dx^{9}+12\,a^{4}fx^{9}-12\,a^{3}bx^{9}e+6\,a^{2}b^{2}cx^{6}-6\,a^{3}bc}{36\,a^{5}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^13),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*ln(abs(x))/a^5 - 1/3*(b^5
*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*ln(abs(b*x^3 + a))/(a^5*b)
- 1/36*(25*b^4*c*x^12 - 25*a*b^3*d*x^12 - 25*a^3*b*f*x^12 + 25*a^
2*b^2*x^12*e - 12*a*b^3*c*x^9 + 12*a^2*b^2*d*x^9 + 12*a^4*f*x^9 -
 12*a^3*b*x^9*e + 6*a^2*b^2*c*x^6 - 6*a^3*b*d*x^6 + 6*a^4*x^6*e -
 4*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4*c)/(a^5*x^12)
```

3.232 $\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx$

Optimal. Leaf size=205

$$\frac{bc-ad}{12a^2x^{12}}-\frac{a^2e-abd+b^2c}{9a^3x^9}+\frac{b^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6}-\frac{b^2\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^4x^6}-\frac{c}{15ax^{15}}$$

```
[Out] -c/(15*a*x^15) + (b*c - a*d)/(12*a^2*x^12) - (b^2*c - a*b*d + a^2
*e)/(9*a^3*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*x^6)
 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*x^3) - (b^2*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^6 + (b^2*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rubi [A] time = 0.44397, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{bc-ad}{12a^2x^{12}}-\frac{a^2e-abd+b^2c}{9a^3x^9}+\frac{b^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6}-\frac{b^2\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^4x^6}-\frac{c}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -c/(15*a*x^15) + (b*c - a*d)/(12*a^2*x^12) - (b^2*c - a*b*d + a^2
*e)/(9*a^3*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*x^6)
 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*x^3) - (b^2*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^6 + (b^2*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rubi in Sympy [A] time = 67.1214, size = 190, normalized size = 0.93

$$-\frac{c}{15ax^{15}}-\frac{ad-bc}{12a^2x^{12}}-\frac{a^2e-abd+b^2c}{9a^3x^9}-\frac{a^3f-a^2be+ab^2d-b^3c}{6a^4x^6}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^5x^3}+\frac{b^2\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(x^3\right)}{3a^6}-\frac{b^2\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**16/(b*x**3+a),x)
```

```
[Out] -c/(15*a*x**15) - (a*d - b*c)/(12*a**2*x**12) - (a**2*e - a*b*d +
 b**2*c)/(9*a**3*x**9) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/
(6*a**4*x**6) + b*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**5
*x**3) + b**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(x**3)/(
3*a**6) - b**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*
x**3)/(3*a**6)
```

Mathematica [A] time = 0.407158, size = 194, normalized size = 0.95

$$-\frac{-60b^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+180b^2\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+\frac{a\left(a^4\left(12c+15dx^3+20ex^6+30f\right.\right.}{}}{180a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -((a*(60*b^4*c*x^12 - 30*a*b^3*x^9*(c + 2*d*x^3) + 10*a^2*b^2*x^6
*(2*c + 3*d*x^3 + 6*e*x^6) - 5*a^3*b*x^3*(3*c + 4*d*x^3 + 6*e*x^6
 + 12*f*x^9) + a^4*(12*c + 15*d*x^3 + 20*e*x^6 + 30*f*x^9)))/x^15
 + 180*b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x] - 60*b^2*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(180*a^6)
```

Maple [A] time = 0.013, size = 260, normalized size = 1.3

$$
\begin{aligned}
&-\frac{c}{15\,a x^{15}}-\frac{d}{12\,a x^{12}}+\frac{bc}{12\,a^2x^{12}}-\frac{e}{9\,a x^9}+\frac{bd}{9\,a^2x^9}-\frac{b^2c}{9\,a^3x^9}-\frac{f}{6\,a x^6}+\frac{be}{6\,a^2x^6}-\frac{b^2d}{6\,a^3x^6}\\
&+\frac{b^3c}{6\,a^4x^6}+\frac{b^2\ln(x)\,f}{a^3}-\frac{b^3\ln(x)\,e}{a^4}+\frac{b^4\ln(x)\,d}{a^5}-\frac{b^5\ln(x)\,c}{a^6}+\frac{bf}{3\,a^2x^3}-\frac{b^2e}{3\,a^3x^3}+\frac{b^3d}{3\,a^4x^3}\\
&-\frac{b^4c}{3\,a^5x^3}-\frac{b^2\ln\left(bx^3+a\right)f}{3\,a^3}+\frac{b^3\ln\left(bx^3+a\right)e}{3\,a^4}-\frac{b^4\ln\left(bx^3+a\right)d}{3\,a^5}+\frac{b^5\ln\left(bx^3+a\right)c}{3\,a^6}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x)
```

```
[Out] -1/15*c/a/x^15-1/12/a/x^12*d+1/12/a^2/x^12*b*c-1/9/a/x^9*e+1/9/a^
2/x^9*b*d-1/9/a^3/x^9*b^2*c-1/6/a/x^6*f+1/6/a^2/x^6*b*e-1/6/a^3/x
^6*b^2*d+1/6/a^4/x^6*b^3*c+1/a^3*b^2*ln(x)*f-1/a^4*b^3*ln(x)*e+1/
a^5*b^4*ln(x)*d-1/a^6*b^5*ln(x)*c+1/3/a^2*b/x^3*f-1/3/a^3*b^2/x^3
*e+1/3/a^4*b^3/x^3*d-1/3/a^5*b^4/x^3*c-1/3*b^2/a^3*ln(b*x^3+a)*f+
1/3*b^3/a^4*ln(b*x^3+a)*e-1/3*b^4/a^5*ln(b*x^3+a)*d+1/3*b^5/a^6*l
n(b*x^3+a)*c
```

Maxima [A] time = 1.4418, size = 281, normalized size = 1.37

$$
\frac{\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)\log\left(bx^3+a\right)}{3\,a^6}-\frac{\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)\log\left(x^3\right)}{3\,a^6}
-\frac{60\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^{12}-30\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x^9+20\left(a^2b^2c-a^3bd+a^4e\right)x^6+12\,a^4c-15\left(a^3bc\right.}{180\,a^5x^{15}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^16),x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(b*x^3 + a)/a^6
- 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(x^3)/a^6 - 1/
180*(60*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 30*(a*b^3*
c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 20*(a^2*b^2*c - a^3*b*d +
a^4*e)*x^6 + 12*a^4*c - 15*(a^3*b*c - a^4*d)*x^3)/(a^5*x^15)
```

Fricas [A] time = 0.283746, size = 284, normalized size = 1.39

$$
\frac{60\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{15}\log\left(bx^3+a\right)-180\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{15}\log\left(x\right)-60\left(ab^4c-a^2b^3d+a^3b^2\right.}{180\,a^6x^{15}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^16),x, algorithm="fricas")
```

```
[Out] 1/180*(60*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(b*x^
3 + a) - 180*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(x
```

```
) - 60*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 30*(a^2
*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 20*(a^3*b^2*c - a^4*b
*d + a^5*e)*x^6 - 12*a^5*c + 15*(a^4*b*c - a^5*d)*x^3)/(a^6*x^15)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**16/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.215204, size = 387, normalized size = 1.89

$$-\frac{\left(b^{5}c-ab^{4}d-a^{3}b^{2}f+a^{2}b^{3}e\right)\ln\left(|x|\right)}{a^{6}}+\frac{\left(b^{6}c-ab^{5}d-a^{3}b^{3}f+a^{2}b^{4}e\right)\ln\left(\left|bx^{3}+a\right|\right)}{3\,a^{6}b}+\frac{137\,b^{5}cx^{15}-137\,ab^{4}dx^{15}-137\,a^{3}b^{2}fx^{15}+137\,a^{2}b^{3}x^{15}e-60\,ab^{4}cx^{12}+60\,a^{2}b^{3}dx^{12}+60\,a^{4}bfx^{12}-60\,a^{3}b^{2}x^{12}e+30\,a^{2}b^{3}}{180\,a^{6}x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^16),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*ln(abs(x))/a^6 + 1/3*(
b^6*c - a*b^5*d - a^3*b^3*f + a^2*b^4*e)*ln(abs(b*x^3 + a))/(a^6*
b) + 1/180*(137*b^5*c*x^15 - 137*a*b^4*d*x^15 - 137*a^3*b^2*f*x^1
5 + 137*a^2*b^3*x^15*e - 60*a*b^4*c*x^12 + 60*a^2*b^3*d*x^12 + 60
*a^4*b*f*x^12 - 60*a^3*b^2*x^12*e + 30*a^2*b^3*c*x^9 - 30*a^3*b^2
*d*x^9 - 30*a^5*f*x^9 + 30*a^4*b*x^9*e - 20*a^3*b^2*c*x^6 + 20*a^
4*b*d*x^6 - 20*a^5*x^6*e + 15*a^4*b*c*x^3 - 15*a^5*d*x^3 - 12*a^5
*c)/(a^6*x^15)
```

3.233 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=348

$$\frac{x^{10}\left(a^2f-abe+b^2d\right)}{10b^3}+\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}-\frac{ax^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^5}+\frac{x^7\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{7b^4}+\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^{19/3}}-\frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^{19/3}}+\frac{a^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}b^{19/3}}+\frac{x^{13}(be-af)}{13b^2}+\frac{fx^{16}}{16b}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3)
 + ((b*e - a*f)*x^13)/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) - (a^(7/3)*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) + (a^(7/
3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(6*b^(19/3))
```

Rubi [A] time = 0.774286, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{x^{10}\left(a^2f-abe+b^2d\right)}{10b^3}+\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}-\frac{ax^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^5}+\frac{x^7\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{7b^4}+\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^{19/3}}-\frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^{19/3}}+\frac{a^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}b^{19/3}}+\frac{x^{13}(be-af)}{13b^2}+\frac{fx^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3)
 + ((b*e - a*f)*x^13)/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) - (a^(7/3)*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) + (a^(7/
3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(6*b^(19/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{\frac{7}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{19}{3}}}-\frac{a^{\frac{7}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{19}{3}}}-\frac{\sqrt{3}a^{\frac{7}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{19}{3}}}+\frac{ax^4\left(a^3f-a^2be+ab^2d-b^3c\right)}{4b^5}+\frac{fx^{16}}{16b}-\frac{x^{13}(af-be)}{13b^2}+\frac{x^{10}\left(a^2f-abe+b^2d\right)}{10b^3}-\frac{x^7\left(a^3f-a^2be+ab^2d-b^3c\right)}{7b^4}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\int a^2\,dx}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a**(7/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) + b
**(1/3)*x)/(3*b**(19/3)) - a**(7/3)*(a**3*f - a**2*b*e + a*b**2*d
 - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6
*b**(19/3)) - sqrt(3)*a**(7/3)*(a**3*f - a**2*b*e + a*b**2*d - b*
*3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*b**
(19/3)) + a*x**4*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(4*b**5)
 + f*x**16/(16*b) - x**13*(a*f - b*e)/(13*b**2) + x**10*(a**2*f -
 a*b*e + b**2*d)/(10*b**3) - x**7*(a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)/(7*b**4) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*Integ
ral(a**2, x)/b**6
```

Mathematica [A] time = 0.144861, size = 351, normalized size = 1.01

$$\frac{x^{10}\left(a^2f-abe+b^2d\right)}{10b^3}-\frac{a^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^6}+\frac{ax^4\left(a^3f-a^2be+ab^2d-b^3c\right)}{4b^5}+\frac{x^7\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{7b^4}-\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^{19/3}}+\frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^{19/3}}+\frac{a^{7/3}\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}b^{19/3}}+\frac{x^{13}(be-af)}{13b^2}+\frac{fx^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)
/(10*b^3) + ((b*e - a*f)*x^13)/(13*b^2) + (f*x^16)/(16*b) + (a^(7
/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(-a^(1/3) + 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) + (a^(7/3)*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^
(19/3)) - (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(19/3))
```

Maple [A] time = 0.008, size = 592, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/13/b*x^13*e+1/10/b*x^10*d+1/7/b*x^7*c-1/13/b^2*x^13*a*f+1/10/b^
3*x^10*a^2*f-1/10/b^2*x^10*a*e-1/7/b^4*x^7*a^3*f+1/7/b^3*x^7*a^2*
e-1/7/b^2*x^7*a*d-1/3*a^3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/6
*a^6/b^7/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/16*f*x
^16/b+1/3*a^4/b^5/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*d-1/3*a^3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*c+1/3*a^6/b^7/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3*a^5/b^6/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/4/b^5*x^4*a^4*f-1/4/b
^4*x^4*a^3*e+1/4/b^3*x^4*a^2*d-1/4/b^2*x^4*a*c-1/b^6*a^5*f*x+1/b^
5*a^4*e*x-1/b^4*a^3*d*x+1/b^3*a^2*c*x+1/6*a^5/b^6/(a/b)^(2/3)*ln(
x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/3*a^6/b^7/(a/b)^(2/3)*ln(x+(a/
b)^(1/3))*f-1/3*a^5/b^6/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1/3*a^4/b
^5/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-1/6*a^4/b^5/(a/b)^(2/3)*ln(x^2
-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/6*a^3/b^4/(a/b)^(2/3)*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^9/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.223335, size = 481, normalized size = 1.38

$$\sqrt{3}\left(3640\sqrt{3}\left(a^{2}b^{3}c - a^{3}b^{2}d + a^{4}be - a^{5}f\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2} - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 7280\sqrt{3}\left(a^{2}b^{3}c - a^{3}b^{2}d + a^{4}be - a^{5}f\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^9/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/65520*sqrt(3)*(3640*sqrt(3)*(a^2*b^3*c - a^3*b^2*d + a^4*b*e -
a^5*f)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 7280*
sqrt(3)*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*log
(x + (a/b)^(1/3)) + 21840*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*
f)*(a/b)^(1/3)*arctan(-1/3*(2*sqrt(3)*x - sqrt(3)*(a/b)^(1/3))/(a
/b)^(1/3)) + 3*sqrt(3)*(455*b^5*f*x^16 + 560*(b^5*e - a*b^4*f)*x^
13 + 728*(b^5*d - a*b^4*e + a^2*b^3*f)*x^10 + 1040*(b^5*c - a*b^4
*d + a^2*b^3*e - a^3*b^2*f)*x^7 - 1820*(a*b^4*c - a^2*b^3*d + a^3
*b^2*e - a^4*b*f)*x^4 + 7280*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a
^5*f)*x))/b^6
```

Sympy [A] time = 3.69756, size = 450, normalized size = 1.29

$$\mathrm{RootSum}\left(27t^3b^{19} - a^{16}f^3 + 3a^{15}bef^2 - 3a^{14}b^2df^2 - 3a^{14}b^2e^2f + 3a^{13}b^3cf^2 + 6a^{13}b^3def + a^{13}b^3e^3 - 6a^{12}b^4cef - 3a^{12}b^4\right.$$
$$+ \frac{fx^{16}}{16b} - \frac{x^{13}(af - be)}{13b^2} + \frac{x^{10}(a^2f - abe + b^2d)}{10b^3} - \frac{x^7(a^3f - a^2be + ab^2d - b^3c)}{7b^4}$$
$$+ \frac{x^4(a^4f - a^3be + a^2b^2d - ab^3c)}{4b^5} - \frac{x(a^5f - a^4be + a^3b^2d - a^2b^3c)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**19 - a**16*f**3 + 3*a**15*b*e*f**2 - 3*a**14*
b**2*d*f**2 - 3*a**14*b**2*e**2*f + 3*a**13*b**3*c*f**2 + 6*a**13
*b**3*d*e*f + a**13*b**3*e**3 - 6*a**12*b**4*c*e*f - 3*a**12*b**4
*d**2*f - 3*a**12*b**4*d*e**2 + 6*a**11*b**5*c*d*f + 3*a**11*b**5
*c*e**2 + 3*a**11*b**5*d**2*e - 3*a**10*b**6*c**2*f - 6*a**10*b**
6*c*d*e - a**10*b**6*d**3 + 3*a**9*b**7*c**2*e + 3*a**9*b**7*c*d*
*2 - 3*a**8*b**8*c**2*d + a**7*b**9*c**3, Lambda(_t, _t*log(3*_t*
b**6/(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x))) + f*x
**16/(16*b) - x**13*(a*f - b*e)/(13*b**2) + x**10*(a**2*f - a*b*e
 + b**2*d)/(10*b**3) - x**7*(a**3*f - a**2*b*e + a*b**2*d - b**3*
c)/(7*b**4) + x**4*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(
4*b**5) - x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/b**6
```

GIAC/XCAS [A] time = 0.217692, size = 613, normalized size = 1.76

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2b^3c - \left(-ab^2\right)^{\frac{1}{3}}a^3b^2d - \left(-ab^2\right)^{\frac{1}{3}}a^5f + \left(-ab^2\right)^{\frac{1}{3}}a^4be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7}$$
$$-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}a^2b^3c - \left(-ab^2\right)^{\frac{1}{3}}a^3b^2d - \left(-ab^2\right)^{\frac{1}{3}}a^5f + \left(-ab^2\right)^{\frac{1}{3}}a^4be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^7}$$
$$+\frac{\left(a^3b^{13}c - a^4b^{12}d - a^6b^{10}f + a^5b^{11}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{16}}$$
$$+\frac{455b^{15}fx^{16} - 560ab^{14}fx^{13} + 560b^{15}x^{13}e + 728b^{15}dx^{10} + 728a^2b^{13}fx^{10} - 728ab^{14}x^{10}e + 1040b^{15}cx^7 - 1040ab^{14}dx^7 - 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^9/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3*b^2*d
 - (-a*b^2)^(1/3)*a^5*f + (-a*b^2)^(1/3)*a^4*b*e)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^7 - 1/6*((-a*b^2)^(1/3)*
a^2*b^3*c - (-a*b^2)^(1/3)*a^3*b^2*d - (-a*b^2)^(1/3)*a^5*f + (-a
*b^2)^(1/3)*a^4*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7
+ 1/3*(a^3*b^13*c - a^4*b^12*d - a^6*b^10*f + a^5*b^11*e)*(-a/b)^
(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^16) + 1/7280*(455*b^15*f*x^1
6 - 560*a*b^14*f*x^13 + 560*b^15*x^13*e + 728*b^15*d*x^10 + 728*a
^2*b^13*f*x^10 - 728*a*b^14*x^10*e + 1040*b^15*c*x^7 - 1040*a*b^1
4*d*x^7 - 1040*a^3*b^12*f*x^7 + 1040*a^2*b^13*x^7*e - 1820*a*b^14
*c*x^4 + 1820*a^2*b^13*d*x^4 + 1820*a^4*b^11*f*x^4 - 1820*a^3*b^1
2*x^4*e + 7280*a^2*b^13*c*x - 7280*a^3*b^12*d*x - 7280*a^5*b^10*f
*x + 7280*a^4*b^11*x*e)/b^16
```

3.234 $\int \frac{x^7 (c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

Optimal. Leaf size=316

$$\frac{x^8 (a^2 f - abe + b^2 d)}{8b^3} - \frac{ax^2 (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{2b^5} + \frac{x^5 (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{5b^4} + \frac{a^{5/3} \log \left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{6b^{17/3}} - \frac{a^{5/3} \log \left(\sqrt[3]{a} + \sqrt[3]{b}x\right) (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{3b^{17/3}} - \frac{a^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right) (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{\sqrt{3}b^{17/3}} + \frac{x^{11}(be - af)}{11b^2} + \frac{fx^{14}}{14b}$$

```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^5) + ((b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^8)/(8*b^3) + ((b*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) -
(a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(17/3)) - (a^(5/3)*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(1
7/3)) + (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

Rubi [A] time = 0.697341, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{x^8 (a^2 f - abe + b^2 d)}{8b^3} - \frac{ax^2 (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{2b^5} + \frac{x^5 (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{5b^4} + \frac{a^{5/3} \log \left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{6b^{17/3}} - \frac{a^{5/3} \log \left(\sqrt[3]{a} + \sqrt[3]{b}x\right) (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{3b^{17/3}} - \frac{a^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right) (a^3(-f) + a^2 be - ab^2 d + b^3 c)}{\sqrt{3}b^{17/3}} + \frac{x^{11}(be - af)}{11b^2} + \frac{fx^{14}}{14b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^5) + ((b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^8)/(8*b^3) + ((b*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) -
(a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(17/3)) - (a^(5/3)*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(1
7/3)) + (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{\frac{5}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{17}{3}}}-\frac{a^{\frac{5}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{17}{3}}}+\frac{\sqrt{3}a^{\frac{5}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{17}{3}}}+\frac{a\left(a^3f-a^2be+ab^2d-b^3c\right)\int x\,dx}{b^5}+\frac{fx^{14}}{14b}-\frac{x^{11}(af-be)}{11b^2}+\frac{x^8\left(a^2f-abe+b^2d\right)}{8b^3}-\frac{x^5\left(a^3f-a^2be+ab^2d-b^3c\right)}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a**(5/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) + b
**(1/3)*x)/(3*b**(17/3)) - a**(5/3)*(a**3*f - a**2*b*e + a*b**2*d
 - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6
*b**(17/3)) + sqrt(3)*a**(5/3)*(a**3*f - a**2*b*e + a*b**2*d - b*
*3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*b**
(17/3)) + a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*Integral(x, x
)/b**5 + f*x**14/(14*b) - x**11*(a*f - b*e)/(11*b**2) + x**8*(a**
2*f - a*b*e + b**2*d)/(8*b**3) - x**5*(a**3*f - a**2*b*e + a*b**2
*d - b**3*c)/(5*b**4)
```

Mathematica [A] time = 0.182511, size = 311, normalized size = 0.98

$$\frac{x^8\left(a^2f-abe+b^2d\right)}{8b^3}+\frac{ax^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{2b^5}+\frac{x^5\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{5b^4}-\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^{17/3}}+\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^{17/3}}+\frac{a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}b^{17/3}}+\frac{x^{11}(be-af)}{11b^2}+\frac{fx^{14}}{14b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(2*b^5) + ((b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2
*f)*x^8)/(8*b^3) + ((b*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b)
+ (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(17/3)) + (a^(5/3)*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^
(17/3)) - (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

Maple [B] time = 0.006, size = 554, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] -1/3*a^3/b^4*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))*d+1/5/b*x^5*c+1/11/b*x^11*e+1/14*f*x^14/b-1/5/b^4*x^5*f*a
^3+1/5/b^3*x^5*a^2*e-1/5/b^2*x^5*a*d+1/2/b^5*x^2*a^4*f-1/2/b^4*x^
2*a^3*e+1/2/b^3*x^2*a^2*d-1/2/b^2*x^2*a*c+1/3*a^5/b^6/(a/b)^(1/3)
*ln(x+(a/b)^(1/3))*f-1/3*a^4/b^5/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-
1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c-1/6*a^5/b^6/(a/b)^(1/
3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/6*a^4/b^5/(a/b)^(1/3)*ln
(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e-1/6*a^3/b^4/(a/b)^(1/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-x*(a/
b)^(1/3)+(a/b)^(2/3))*c-1/11/b^2*x^11*a*f+1/8/b^3*x^8*a^2*f-1/8/b
^2*x^8*a*e+1/8/b*x^8*d+1/3*a^3/b^4/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*
d+1/3*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))*c+1/3*a^4/b^5*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))*e-1/3*a^5/b^6*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^7/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221211, size = 462, normalized size = 1.46

$$\frac{\sqrt{3}\left(1540\,\sqrt{3}\left(ab^3c - a^2b^2d + a^3be - a^4f\right)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2 - bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}} + a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right) - 3080\,\sqrt{3}\left(ab^3c - a^2b^2d + a^3be - a^4f\right)\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^7/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/27720*sqrt(3)*(1540*sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^
4*f)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2
)^(1/3)) - 3080*sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(
a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)) - 9240*(a*b^3*c - a^2
*b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*
a*x - sqrt(3)*b*(a^2/b^2)^(2/3))/(b*(a^2/b^2)^(2/3))) + 3*sqrt(3)
*(220*b^4*f*x^14 + 280*(b^4*e - a*b^3*f)*x^11 + 385*(b^4*d - a*b^
3*e + a^2*b^2*f)*x^8 + 616*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f
)*x^5 - 1540*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^2))/b^5
```

Sympy [A] time = 2.93698, size = 496, normalized size = 1.57

$$\operatorname{RootSum}\left(27t^3b^{17} - a^{14}f^3 + 3a^{13}bef^2 - 3a^{12}b^2df^2 - 3a^{12}b^2e^2f + 3a^{11}b^3cf^2 + 6a^{11}b^3def + a^{11}b^3e^3 - 6a^{10}b^4cef - 3a^{10}b^4\right.$$

$$+\frac{fx^{14}}{14b} - \frac{x^{11}\left(af - be\right)}{11b^2} + \frac{x^8\left(a^2f - abe + b^2d\right)}{8b^3} - \frac{x^5\left(a^3f - a^2be + ab^2d - b^3c\right)}{5b^4} + \frac{x^2\left(a^4f - a^3be + a^2b^2d - ab^3c\right)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**17 - a**14*f**3 + 3*a**13*b*e*f**2 - 3*a**12*
b**2*d*f**2 - 3*a**12*b**2*e**2*f + 3*a**11*b**3*c*f**2 + 6*a**11
*b**3*d*e*f + a**11*b**3*e**3 - 6*a**10*b**4*c*e*f - 3*a**10*b**4
*d**2*f - 3*a**10*b**4*d*e**2 + 6*a**9*b**5*c*d*f + 3*a**9*b**5*c
*e**2 + 3*a**9*b**5*d**2*e - 3*a**8*b**6*c**2*f - 6*a**8*b**6*c*d
*e - a**8*b**6*d**3 + 3*a**7*b**7*c**2*e + 3*a**7*b**7*c*d**2 - 3
*a**6*b**8*c**2*d + a**5*b**9*c**3, Lambda(_t, _t*log(9*_t**2*b**
11/(a**9*f**2 - 2*a**8*b*e*f + 2*a**7*b**2*d*f + a**7*b**2*e**2 -
 2*a**6*b**3*c*f - 2*a**6*b**3*d*e + 2*a**5*b**4*c*e + a**5*b**4*
d**2 - 2*a**4*b**5*c*d + a**3*b**6*c**2) + x))) + f*x**14/(14*b)
- x**11*(a*f - b*e)/(11*b**2) + x**8*(a**2*f - a*b*e + b**2*d)/(8
*b**3) - x**5*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(5*b**4) +
x**2*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(2*b**5)
```

GIAC/XCAS [A] time = 0.219766, size = 595, normalized size = 1.88

$$-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{2}{3}}ab^{3}c-\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b^{2}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{4}f+\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^{7}}+\frac{\left(\left(-ab^{2}\right)^{\frac{2}{3}}ab^{3}c-\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b^{2}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{4}f+\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^{7}}-\frac{\left(a^{2}b^{12}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}b^{11}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{5}b^{9}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{4}b^{10}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{14}}+\frac{220b^{13}fx^{14}-280ab^{12}fx^{11}+280b^{13}x^{11}e+385b^{13}dx^{8}+385a^{2}b^{11}fx^{8}-385ab^{12}x^{8}e+616b^{13}cx^{5}-616ab^{12}dx^{5}-616a^{3}}{3080b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^7/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2*d -
 (-a*b^2)^(2/3)*a^4*f + (-a*b^2)^(2/3)*a^3*b*e)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/6*((-a*b^2)^(2/3)*a*
b^3*c - (-a*b^2)^(2/3)*a^2*b^2*d - (-a*b^2)^(2/3)*a^4*f + (-a*b^2
)^(2/3)*a^3*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 - 1/
3*(a^2*b^12*c*(-a/b)^(1/3) - a^3*b^11*d*(-a/b)^(1/3) - a^5*b^9*f*
(-a/b)^(1/3) + a^4*b^10*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (
-a/b)^(1/3)))/(a*b^14) + 1/3080*(220*b^13*f*x^14 - 280*a*b^12*f*x
^11 + 280*b^13*x^11*e + 385*b^13*d*x^8 + 385*a^2*b^11*f*x^8 - 385
*a*b^12*x^8*e + 616*b^13*c*x^5 - 616*a*b^12*d*x^5 - 616*a^3*b^10*
f*x^5 + 616*a^2*b^11*x^5*e - 1540*a*b^12*c*x^2 + 1540*a^2*b^11*d*
x^2 + 1540*a^4*b^9*f*x^2 - 1540*a^3*b^10*x^2*e)/b^14
```

3.235 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=312

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}-\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{x^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^4}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^{16/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^{16/3}}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}b^{16/3}}+\frac{x^{10}(be-af)}{10b^2}+\frac{fx^{13}}{13b}$$

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^
7)/(7*b^3) + ((b*e - a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) - (a^(
4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(16/3)) + (a^(4/3)*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(16/3)
) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Rubi [A] time = 0.663587, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}-\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{x^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^4}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^{16/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^{16/3}}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}b^{16/3}}+\frac{x^{10}(be-af)}{10b^2}+\frac{fx^{13}}{13b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^
7)/(7*b^3) + ((b*e - a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) - (a^(
4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(16/3)) + (a^(4/3)*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(16/3)
) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{\frac{4}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{16}{3}}}+\frac{a^{\frac{4}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{16}{3}}}+\frac{\sqrt{3}a^{\frac{4}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{16}{3}}}+\frac{fx^{13}}{13b}-\frac{x^{10}(af-be)}{10b^2}+\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}-\frac{x^4\left(a^3f-a^2be+ab^2d-b^3c\right)}{4b^4}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\int a\,dx}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] -a**(4/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) +
b**(1/3)*x)/(3*b**(16/3)) + a**(4/3)*(a**3*f - a**2*b*e + a*b**2*
d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
6*b**(16/3)) + sqrt(3)*a**(4/3)*(a**3*f - a**2*b*e + a*b**2*d - b
**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*b*
*(16/3)) + f*x**13/(13*b) - x**10*(a*f - b*e)/(10*b**2) + x**7*(a
**2*f - a*b*e + b**2*d)/(7*b**3) - x**4*(a**3*f - a**2*b*e + a*b*
*2*d - b**3*c)/(4*b**4) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*Integral(a, x)/b**5
```

Mathematica [A] time = 0.171048, size = 306, normalized size = 0.98

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}+\frac{ax\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^5}+\frac{x^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^4}+\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^{16/3}}-\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^{16/3}}+\frac{a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}b^{16/3}}+\frac{x^{10}(be-af)}{10b^2}+\frac{fx^{13}}{13b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^5 + ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^
7)/(7*b^3) + ((b*e - a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) + (a^(
4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(16/3)) - (a^(4/3)*(-(b^3*c) +
 a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(16/3)
) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Maple [B] time = 0.007, size = 544, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] -1/3*a^3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))*d-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3
))*c+1/6*a^3/b^4/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+
1/13*f*x^13/b+1/4/b^3*x^4*a^2*e-1/4/b^2*x^4*a*d+1/b^5*a^4*f*x-1/b
^4*a^3*e*x+1/b^3*a^2*d*x-1/b^2*a*c*x-1/10/b^2*x^10*a*f+1/7/b^3*x^
7*a^2*f-1/7/b^2*x^7*a*e-1/4/b^4*x^4*a^3*f-1/3*a^5/b^6/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))*f+1/10/b*x^10*e+1/3*a^4/b^5/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*e-1/3*a^3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+1/3*a^2
/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c+1/6*a^5/b^6/(a/b)^(2/3)*ln(x
^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f-1/6*a^4/b^5/(a/b)^(2/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))*e-1/3*a^5/b^6/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+1/3*a^4/b^5/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3*a^2/b^3/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/4/b*x^4*c
+1/7/b*x^7*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^6/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219294, size = 429, normalized size = 1.38

$$\frac{\sqrt{3}\left(910\,\sqrt{3}\left(ab^3c - a^2b^2d + a^3be - a^4f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 1820\,\sqrt{3}\left(ab^3c - a^2b^2d + a^3be - a^4f\right)\left(-\frac{a}{b}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^6/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/16380*sqrt(3)*(910*sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4
*f)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 1820*
sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*log(
x - (-a/b)^(1/3)) + 5460*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*
(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*x + sqrt(3)*(-a/b)^(1/3))/(-a/
b)^(1/3)) + 3*sqrt(3)*(140*b^4*f*x^13 + 182*(b^4*e - a*b^3*f)*x^1
0 + 260*(b^4*d - a*b^3*e + a^2*b^2*f)*x^7 + 455*(b^4*c - a*b^3*d
+ a^2*b^2*e - a^3*b*f)*x^4 - 1820*(a*b^3*c - a^2*b^2*d + a^3*b*e
- a^4*f)*x))/b^5
```

Sympy [A] time = 3.67061, size = 411, normalized size = 1.32

$$\operatorname{RootSum}\left(27t^3b^{16} + a^{13}f^3 - 3a^{12}bef^2 + 3a^{11}b^2df^2 + 3a^{11}b^2e^2f - 3a^{10}b^3cf^2 - 6a^{10}b^3def - a^{10}b^3e^3 + 6a^9b^4cef + 3a^9b^4d^2\right.$$
$$+ \frac{fx^{13}}{13b} - \frac{x^{10}\left(af - be\right)}{10b^2} + \frac{x^7\left(a^2f - abe + b^2d\right)}{7b^3}$$
$$- \frac{x^4\left(a^3f - a^2be + ab^2d - b^3c\right)}{4b^4} + \frac{x\left(a^4f - a^3be + a^2b^2d - ab^3c\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**16 + a**13*f**3 - 3*a**12*b*e*f**2 + 3*a**11*
b**2*d*f**2 + 3*a**11*b**2*e**2*f - 3*a**10*b**3*c*f**2 - 6*a**10
*b**3*d*e*f - a**10*b**3*e**3 + 6*a**9*b**4*c*e*f + 3*a**9*b**4*d
**2*f + 3*a**9*b**4*d*e**2 - 6*a**8*b**5*c*d*f - 3*a**8*b**5*c*e*
*2 - 3*a**8*b**5*d**2*e + 3*a**7*b**6*c**2*f + 6*a**7*b**6*c*d*e
+ a**7*b**6*d**3 - 3*a**6*b**7*c**2*e - 3*a**6*b**7*c*d**2 + 3*a*
*5*b**8*c**2*d - a**4*b**9*c**3, Lambda(_t, _t*log(-3*_t*b**5/(a*
*4*f - a**3*b*e + a**2*b**2*d - a*b**3*c) + x))) + f*x**13/(13*b)
 - x**10*(a*f - b*e)/(10*b**2) + x**7*(a**2*f - a*b*e + b**2*d)/(
7*b**3) - x**4*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(4*b**4) +
 x*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/b**5
```

GIAC/XCAS [A] time = 0.215241, size = 541, normalized size = 1.73

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c - \left(-ab^2\right)^{\frac{1}{3}}a^2b^2d - \left(-ab^2\right)^{\frac{1}{3}}a^4f + \left(-ab^2\right)^{\frac{1}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^6} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c - \left(-ab^2\right)^{\frac{1}{3}}a^2b^2d - \left(-ab^2\right)^{\frac{1}{3}}a^4f + \left(-ab^2\right)^{\frac{1}{3}}a^3be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^6} - \frac{\left(a^2b^{11}c - a^3b^{10}d - a^5b^8f + a^4b^9e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{13}} + \frac{140b^{12}fx^{13} - 182ab^{11}fx^{10} + 182b^{12}x^{10}e + 260b^{12}dx^7 + 260a^2b^{10}fx^7 - 260ab^{11}x^7e + 455b^{12}cx^4 - 455ab^{11}dx^4 - 455a^3}{1820b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^6/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*d -
(-a*b^2)^(1/3)*a^4*f + (-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 + 1/6*((-a*b^2)^(1/3)*a*b
^3*c - (-a*b^2)^(1/3)*a^2*b^2*d - (-a*b^2)^(1/3)*a^4*f + (-a*b^2)
^(1/3)*a^3*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^6 - 1/3
*(a^2*b^11*c - a^3*b^10*d - a^5*b^8*f + a^4*b^9*e)*(-a/b)^(1/3)*l
n(abs(x - (-a/b)^(1/3)))/(a*b^13) + 1/1820*(140*b^12*f*x^13 - 182
*a*b^11*f*x^10 + 182*b^12*x^10*e + 260*b^12*d*x^7 + 260*a^2*b^10*
f*x^7 - 260*a*b^11*x^7*e + 455*b^12*c*x^4 - 455*a*b^11*d*x^4 - 45
5*a^3*b^9*f*x^4 + 455*a^2*b^10*x^4*e - 1820*a*b^11*c*x + 1820*a^2
*b^10*d*x + 1820*a^4*b^8*f*x - 1820*a^3*b^9*x*e)/b^13
```

3.236 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=279

$$\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2b^4}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^{14/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^{14/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}b^{14/3}}+\frac{x^8(be-af)}{8b^2}+\frac{fx^{11}}{11b}$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^4) + ((b^2*d - a*b
*e + a^2*f)*x^5)/(5*b^3) + ((b*e - a*f)*x^8)/(8*b^2) + (f*x^11)/(
11*b) + (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(14/3)) + (a^(2
/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/(3*b^(14/3)) - (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(14/3))
```

Rubi [A] time = 0.61526, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2b^4}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^{14/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^{14/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}b^{14/3}}+\frac{x^8(be-af)}{8b^2}+\frac{fx^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^4) + ((b^2*d - a*b
*e + a^2*f)*x^5)/(5*b^3) + ((b*e - a*f)*x^8)/(8*b^2) + (f*x^11)/(
11*b) + (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(14/3)) + (a^(2
/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/(3*b^(14/3)) - (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(14/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{\frac{2}{3}}\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{14}{3}}}+\frac{a^{\frac{2}{3}}\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}\right)}{6b^{\frac{14}{3}}}-\frac{\sqrt{3}a^{\frac{2}{3}}\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{14}{3}}}+\frac{fx^{11}}{11b}-\frac{x^{8}\left(af-be\right)}{8b^{2}}+\frac{x^{5}\left(a^{2}f-abe+b^{2}d\right)}{5b^{3}}-\frac{\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\int x\,dx}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] -a**(2/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) +
b**(1/3)*x)/(3*b**(14/3)) + a**(2/3)*(a**3*f - a**2*b*e + a*b**2*
d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
6*b**(14/3)) - sqrt(3)*a**(2/3)*(a**3*f - a**2*b*e + a*b**2*d - b
**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*b*
*(14/3)) + f*x**11/(11*b) - x**8*(a*f - b*e)/(8*b**2) + x**5*(a**
2*f - a*b*e + b**2*d)/(5*b**3) - (a**3*f - a**2*b*e + a*b**2*d -
b**3*c)*Integral(x, x)/b**4
```

Mathematica [A] time = 0.165219, size = 266, normalized size = 0.95

$$\frac{264b^{5/3}x^{5}\left(a^{2}f-abe+b^{2}d\right)+660b^{2/3}x^{2}\left(a^{3}(-f)+a^{2}be-ab^{2}d+b^{3}c\right)-440a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (660*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 + 264*b^(5/3
)*(b^2*d - a*b*e + a^2*f)*x^5 + 165*b^(8/3)*(b*e - a*f)*x^8 + 120
*b^(11/3)*f*x^11 - 440*Sqrt[3]*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*
b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^
(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3
)*x] + 220*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*b^(14/3))
```

Maple [B] time = 0.006, size = 502, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/11*f*x^11/b-1/8/b^2*x^8*a*f+1/8/b*x^8*e+1/5/b^3*x^5*a^2*f-1/5/b
^2*x^5*a*e+1/5/b*x^5*d-1/2/b^4*x^2*a^3*f+1/2/b^3*x^2*a^2*e-1/2/b^
2*x^2*a*d+1/2/b*x^2*c-1/3*a^4/b^5/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f
+1/3*a^3/b^4/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-1/3*a^2/b^3/(a/b)^(1
```

```
/3)*ln(x+(a/b)^(1/3))*d+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c
+1/6*a^4/b^5/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f-1/6*
a^3/b^4/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/6*a^2/b
^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-1/6*a/b^2/(a/b
)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+1/3*a^4/b^5*3^(1/2)/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3*a^3/b^4*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3
*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*d-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^4/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.220689, size = 410, normalized size = 1.47

$$\frac{\sqrt{3}\left(220\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-440\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^4/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/3960*sqrt(3)*(220*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(
-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(
1/3)) - 440*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2
)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3)) - 1320*(b^3*c - a*b^2*d + a
^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*a*x - sqr
t(3)*b*(-a^2/b^2)^(2/3))/(b*(-a^2/b^2)^(2/3))) + 3*sqrt(3)*(40*b^
3*f*x^11 + 55*(b^3*e - a*b^2*f)*x^8 + 88*(b^3*d - a*b^2*e + a^2*b
*f)*x^5 + 220*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2))/b^4
```

Sympy [A] time = 2.84197, size = 459, normalized size = 1.65

$$\operatorname{RootSum}\left(27t^3b^{14}+a^{11}f^3-3a^{10}bef^2+3a^9b^2df^2+3a^9b^2e^2f-3a^8b^3cf^2-6a^8b^3def-a^8b^3e^3+6a^7b^4cef+3a^7b^4d^2f+\right.$$

$$+\frac{fx^{11}}{11b}-\frac{x^8\left(af-be\right)}{8b^2}+\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}-\frac{x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**14 + a**11*f**3 - 3*a**10*b*e*f**2 + 3*a**9*b
**2*d*f**2 + 3*a**9*b**2*e**2*f - 3*a**8*b**3*c*f**2 - 6*a**8*b**
3*d*e*f - a**8*b**3*e**3 + 6*a**7*b**4*c*e*f + 3*a**7*b**4*d**2*f
 + 3*a**7*b**4*d*e**2 - 6*a**6*b**5*c*d*f - 3*a**6*b**5*c*e**2 -
3*a**6*b**5*d**2*e + 3*a**5*b**6*c**2*f + 6*a**5*b**6*c*d*e + a**
5*b**6*d**3 - 3*a**4*b**7*c**2*e - 3*a**4*b**7*c*d**2 + 3*a**3*b*
*8*c**2*d - a**2*b**9*c**3, Lambda(_t, _t*log(9*_t**2*b**9/(a**7*
f**2 - 2*a**6*b*e*f + 2*a**5*b**2*d*f + a**5*b**2*e**2 - 2*a**4*b
```

```
**3*c*f - 2*a**4*b**3*d*e + 2*a**3*b**4*c*e + a**3*b**4*d**2 - 2*
a**2*b**5*c*d + a*b**6*c**2) + x))) + f*x**11/(11*b) - x**8*(a*f
- b*e)/(8*b**2) + x**5*(a**2*f - a*b*e + b**2*d)/(5*b**3) - x**2*
(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*b**4)
```

GIAC/XCAS [A] time = 0.219166, size = 521, normalized size = 1.87

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6} - \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6} + \frac{\left(ab^{10}c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b^9d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^4b^7f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^3b^8\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{11}} + \frac{40\,b^{10}fx^{11} - 55\,ab^9fx^8 + 55\,b^{10}x^8e + 88\,b^{10}dx^5 + 88\,a^2b^8fx^5 - 88\,ab^9x^5e + 220\,b^{10}cx^2 - 220\,ab^9dx^2 - 220\,a^3b^7fx^2 + 220}{440\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^4/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*
b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 - 1/6*((-a*b^2)^(2/3)*b^3*c -
 (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a
^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^6 + 1/3*(a*b^10
*c*(-a/b)^(1/3) - a^2*b^9*d*(-a/b)^(1/3) - a^4*b^7*f*(-a/b)^(1/3)
 + a^3*b^8*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))
/(a*b^11) + 1/440*(40*b^10*f*x^11 - 55*a*b^9*f*x^8 + 55*b^10*x^8*
e + 88*b^10*d*x^5 + 88*a^2*b^8*f*x^5 - 88*a*b^9*x^5*e + 220*b^10*
c*x^2 - 220*a*b^9*d*x^2 - 220*a^3*b^7*f*x^2 + 220*a^2*b^8*x^2*e)/
b^11
```

3.237 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=274

$$\frac{x^4(a^2f-abe+b^2d)}{4b^3}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^{13/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}b^{13/3}}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^{13/3}}+\frac{x^7(be-af)}{7b^2}+\frac{fx^{10}}{10b}$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a
^2*f)*x^4)/(4*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^10)/(10*b)
+ (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(13/3)) - (a^(1/3)*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^
(13/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3))
```

Rubi [A] time = 0.592991, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{x^4(a^2f-abe+b^2d)}{4b^3}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^{13/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}b^{13/3}}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^{13/3}}+\frac{x^7(be-af)}{7b^2}+\frac{fx^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a
^2*f)*x^4)/(4*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^10)/(10*b)
+ (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(13/3)) - (a^(1/3)*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^
(13/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[3]{a}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{13}{3}}}-\frac{\sqrt[3]{a}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{13}{3}}}-\frac{\sqrt{3}\sqrt[3]{a}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{13}{3}}}-\left(a^3f-a^2be+ab^2d-b^3c\right)\int\frac{1}{b^4}\,dx+\frac{fx^{10}}{10b}-\frac{x^7(af-be)}{7b^2}+\frac{x^4\left(a^2f-abe+b^2d\right)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a**(1/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) + b
**(1/3)*x)/(3*b**(13/3)) - a**(1/3)*(a**3*f - a**2*b*e + a*b**2*d
 - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6
*b**(13/3)) - sqrt(3)*a**(1/3)*(a**3*f - a**2*b*e + a*b**2*d - b*
*3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*b**
(13/3)) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*Integral(b**(-4
), x) + f*x**10/(10*b) - x**7*(a*f - b*e)/(7*b**2) + x**4*(a**2*f
 - a*b*e + b**2*d)/(4*b**3)
```

Mathematica [A] time = 0.170181, size = 264, normalized size = 0.96

$$105b^{4/3}x^4\left(a^2f-abe+b^2d\right)+420\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+140\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x + 105*b^(4/3)*
(b^2*d - a*b*e + a^2*f)*x^4 + 60*b^(7/3)*(b*e - a*f)*x^7 + 42*b^(
10/3)*f*x^10 - 140*Sqrt[3]*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3
)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x]
 - 70*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*b^(13/3))
```

Maple [B] time = 0.005, size = 492, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/10*f*x^10/b-1/7/b^2*x^7*a*f+1/7/b*x^7*e+1/4/b^3*x^4*a^2*f-1/4/b
^2*x^4*a*e+1/4/b*x^4*d-1/b^4*a^3*f*x+1/b^3*a^2*e*x-1/b^2*a*d*x+c*
x/b+1/3*a^4/b^5/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1/3*a^3/b^4/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))*e+1/3*a^2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/
3))*d-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/6*a^4/b^5/(a/b)
^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/6*a^3/b^4/(a/b)^(2/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e-1/6*a^2/b^3/(a/b)^(2/3)*ln(
x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-x*(
a/b)^(1/3)+(a/b)^(2/3))*c+1/3*a^4/b^5/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3*a^3/b^4/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3*a^2/b^3/(a/b)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3*a/b^2/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^3/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2231, size = 355, normalized size = 1.3

$$\frac{\sqrt{3}\left(70\,\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-140\,\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^3/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/1260*sqrt(3)*(70*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a
/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*sqrt(3)*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)
) + 420*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*arctan(-1
/3*(2*sqrt(3)*x - sqrt(3)*(a/b)^(1/3))/(a/b)^(1/3)) + 3*sqrt(3)*(
14*b^3*f*x^10 + 20*(b^3*e - a*b^2*f)*x^7 + 35*(b^3*d - a*b^2*e +
a^2*b*f)*x^4 + 140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x))/b^4
```

Sympy [A] time = 3.44902, size = 371, normalized size = 1.35

$$\operatorname{RootSum}\left(27t^3b^{13}-a^{10}f^3+3a^9bef^2-3a^8b^2df^2-3a^8b^2e^2f+3a^7b^3cf^2+6a^7b^3def+a^7b^3e^3-6a^6b^4cef-3a^6b^4d^2f-\right.$$
$$+\frac{fx^{10}}{10b}-\frac{x^7\left(af-be\right)}{7b^2}+\frac{x^4\left(a^2f-abe+b^2d\right)}{4b^3}-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**13 - a**10*f**3 + 3*a**9*b*e*f**2 - 3*a**8*b*
*2*d*f**2 - 3*a**8*b**2*e**2*f + 3*a**7*b**3*c*f**2 + 6*a**7*b**3
*d*e*f + a**7*b**3*e**3 - 6*a**6*b**4*c*e*f - 3*a**6*b**4*d**2*f
- 3*a**6*b**4*d*e**2 + 6*a**5*b**5*c*d*f + 3*a**5*b**5*c*e**2 + 3
*a**5*b**5*d**2*e - 3*a**4*b**6*c**2*f - 6*a**4*b**6*c*d*e - a**4
*b**6*d**3 + 3*a**3*b**7*c**2*e + 3*a**3*b**7*c*d**2 - 3*a**2*b**
8*c**2*d + a*b**9*c**3, Lambda(_t, _t*log(3*_t*b**4/(a**3*f - a**
2*b*e + a*b**2*d - b**3*c) + x))) + f*x**10/(10*b) - x**7*(a*f -
b*e)/(7*b**2) + x**4*(a**2*f - a*b*e + b**2*d)/(4*b**3) - x*(a**3
*f - a**2*b*e + a*b**2*d - b**3*c)/b**4
```

GIAC/XCAS [A] time = 0.21903, size = 467, normalized size = 1.7

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d-\left(-ab^2\right)^{\frac{1}{3}}a^3f+\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5}$$
$$-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d-\left(-ab^2\right)^{\frac{1}{3}}a^3f+\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5}$$
$$+\frac{\left(ab^9c-a^2b^8d-a^4b^6f+a^3b^7e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{10}}$$
$$+\frac{14\,b^9fx^{10}-20\,ab^8fx^7+20\,b^9x^7e+35\,b^9dx^4+35\,a^2b^7fx^4-35\,ab^8x^4e+140\,b^9cx-140\,ab^8dx-140\,a^3b^6fx+140\,a^2b^7x}{140\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^3/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a
*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/6*((-a*b^2)^(1/3)*b^3*c
- (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*
a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/3*(a*b^9
*c - a^2*b^8*d - a^4*b^6*f + a^3*b^7*e)*(-a/b)^(1/3)*ln(abs(x - (
-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*f*x^10 - 20*a*b^8*f*x^7 +
20*b^9*x^7*e + 35*b^9*d*x^4 + 35*a^2*b^7*f*x^4 - 35*a*b^8*x^4*e +
 140*b^9*c*x - 140*a*b^8*d*x - 140*a^3*b^6*f*x + 140*a^2*b^7*x*e)
/b^10
```

3.238 $\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=245

$$\frac{x^2\left(a^2f-abe+b^2d\right)}{2b^3}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3\sqrt[3]{a}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}\sqrt[3]{a}b^{11/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6\sqrt[3]{a}b^{11/3}}+\frac{x^5(be-af)}{5b^2}+\frac{fx^8}{8b}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^3) + ((b*e - a*f)*x^5)/(5*b^2)
 + (f*x^8)/(8*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(11/
3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(1/3)*b^(11/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(11/3
))
```

Rubi [A] time = 0.480416, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{x^2\left(a^2f-abe+b^2d\right)}{2b^3}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3\sqrt[3]{a}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}\sqrt[3]{a}b^{11/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6\sqrt[3]{a}b^{11/3}}+\frac{x^5(be-af)}{5b^2}+\frac{fx^8}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^3) + ((b*e - a*f)*x^5)/(5*b^2)
 + (f*x^8)/(8*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(11/
3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(1/3)*b^(11/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(11/3
))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{fx^8}{8b}-\frac{x^5\left(af-be\right)}{5b^2}+\frac{\left(a^2f-abe+b^2d\right)\int x\,dx}{b^3}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{\frac{11}{3}}}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6\sqrt[3]{a}b^{\frac{11}{3}}}+\frac{\sqrt{3}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}b^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] f*x**8/(8*b) - x**5*(a*f - b*e)/(5*b**2) + (a**2*f - a*b*e + b**2
*d)*Integral(x, x)/b**3 + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*log(a**(1/3) + b**(1/3)*x)/(3*a**(1/3)*b**(11/3)) - (a**3*f - a*
*2*b*e + a*b**2*d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x +
b**(2/3)*x**2)/(6*a**(1/3)*b**(11/3)) + sqrt(3)*(a**3*f - a**2*b*
e + a*b**2*d - b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)
/a**(1/3))/(3*a**(1/3)*b**(11/3))
```

Mathematica [A] time = 0.2958, size = 231, normalized size = 0.94

$$\frac{60b^{2/3}x^2\left(a^2f-abe+b^2d\right)+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt[3]{a}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt[3]{a}}+\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{}}{120b^{11/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (60*b^(2/3)*(b^2*d - a*b*e + a^2*f)*x^2 + 24*b^(5/3)*(b*e - a*f)*
x^5 + 15*b^(8/3)*f*x^8 + (40*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*
e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) +
 (40*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)
*x])/a^(1/3) + (20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(120*b^(11/3))
```

Maple [B] time = 0.005, size = 450, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/8*f*x^8/b-1/5/b^2*x^5*a*f+1/5/b*x^5*e+1/2/b^3*x^2*a^2*f-1/2/b^2
*x^2*a*e+1/2*d*x^2/b+1/3/b^4/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*a^3*f-
1/3/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*a^2*e+1/3/b^2/(a/b)^(1/3)*l
n(x+(a/b)^(1/3))*a*d-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c-1/6/b^
4/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^3*f+1/6/b^3/(a/
b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^2*e-1/6/b^2/(a/b)^(1
/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a*d+1/6/b/(a/b)^(1/3)*ln(x^
2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-1/3/b^4*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^3*f+1/3/b^3*3^(1/2)/(a/b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*e-1/3/b^2*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*d+1/3/b*3^(
1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.222158, size = 313, normalized size = 1.28

$$\frac{\sqrt{3}\left(20\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\left(ab^2\right)^{\frac{1}{3}}bx^2+ab-\left(ab^2\right)^{\frac{2}{3}}x\right)-40\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(ab+\left(ab^2\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/360*sqrt(3)*(20*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log
((a*b^2)^(1/3)*b*x^2 + a*b - (a*b^2)^(2/3)*x) - 40*sqrt(3)*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*log(a*b + (a*b^2)^(2/3)*x) + 120*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(-1/3*(sqrt(3)*a*b - 2*sq
rt(3)*(a*b^2)^(2/3)*x)/(a*b)) + 3*sqrt(3)*(5*b^2*f*x^8 + 8*(b^2*e
 - a*b*f)*x^5 + 20*(b^2*d - a*b*e + a^2*f)*x^2)*(a*b^2)^(1/3))/((
a*b^2)^(1/3)*b^3)
```

Sympy [A] time = 2.89422, size = 422, normalized size = 1.72

$$\operatorname{RootSum}\left(27t^3ab^{11}-a^9f^3+3a^8bef^2-3a^7b^2df^2-3a^7b^2e^2f+3a^6b^3cf^2+6a^6b^3def+a^6b^3e^3-6a^5b^4cef-3a^5b^4d^2f-\right.$$

$$+\frac{fx^8}{8b}-\frac{x^5\left(af-be\right)}{5b^2}+\frac{x^2\left(a^2f-abe+b^2d\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**11 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b
**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**
3*d*e*f + a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f
 - 3*a**5*b**4*d*e**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 +
3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f - 6*a**3*b**6*c*d*e - a**
3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d**2 - 3*a*b**8*
c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a*b**7/(a**6*f**2 -
 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*
f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5
*c*d + b**6*c**2) + x))) + f*x**8/(8*b) - x**5*(a*f - b*e)/(5*b**
2) + x**2*(a**2*f - a*b*e + b**2*d)/(2*b**3)
```

GIAC/XCAS [A] time = 0.219204, size = 466, normalized size = 1.9

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\left(-ab^2\right)^{\frac{2}{3}}a^3f+\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^5}+\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\left(-ab^2\right)^{\frac{2}{3}}a^3f+\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^5}-\frac{\left(b^8c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^7d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^6\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^8}+\frac{5b^7fx^8-8ab^6fx^5+8b^7x^5e+20b^7dx^2+20a^2b^5fx^2-20ab^6x^2e}{40b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a
*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^5) + 1/6*((-a*b^2)^(2/3)*b^
3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2
/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5) - 1/
3*(b^8*c*(-a/b)^(1/3) - a*b^7*d*(-a/b)^(1/3) - a^3*b^5*f*(-a/b)^(
1/3) + a^2*b^6*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/
3)))/(a*b^8) + 1/40*(5*b^7*f*x^8 - 8*a*b^6*f*x^5 + 8*b^7*x^5*e +
20*b^7*d*x^2 + 20*a^2*b^5*f*x^2 - 20*a*b^6*x^2*e)/b^8
```

3.239 $\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3} dx$

Optimal. Leaf size=240

$$\frac{x\left(a^2f-abe+b^2d\right)}{b^3}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{2/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{2/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{x^4(be-af)}{4b^2}+\frac{fx^7}{7b}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*
x^7)/(7*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) +
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3
*a^(2/3)*b^(10/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

Rubi [A] time = 0.342271, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{x\left(a^2f-abe+b^2d\right)}{b^3}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{2/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{2/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{x^4(be-af)}{4b^2}+\frac{fx^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```


```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*
x^7)/(7*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) +
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3
*a^(2/3)*b^(10/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\left(a^2f-abe+b^2d\right)\int\frac{1}{b^3}\,dx+\frac{fx^7}{7b}-\frac{x^4\left(af-be\right)}{4b^2}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{10}{3}}}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{10}{3}}}+\frac{\sqrt{3}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] (a**2*f - a*b*e + b**2*d)*Integral(b**(-3), x) + f*x**7/(7*b) - x
**4*(a*f - b*e)/(4*b**2) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c
)*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(10/3)) + (a**3*f - a
**2*b*e + a*b**2*d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x +
 b**(2/3)*x**2)/(6*a**(2/3)*b**(10/3)) + sqrt(3)*(a**3*f - a**2*b
*e + a*b**2*d - b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3
)/a**(1/3))/(3*a**(2/3)*b**(10/3))
```

Mathematica [A] time = 0.297413, size = 229, normalized size = 0.95

$$\frac{84\sqrt[3]{b}x\left(a^2f-abe+b^2d\right)+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{2/3}}+\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots\right.}{}}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*(b^2*d - a*b*e + a^2*f)*x + 21*b^(4/3)*(b*e - a*f)*x^
4 + 12*b^(7/3)*f*x^7 + (28*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (
28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
a^(2/3) + (14*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(84*b^(10/3))
```

Maple [B] time = 0.004, size = 442, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/7*f*x^7/b-1/4/b^2*x^4*a*f+1/4/b*x^4*e+1/b^3*a^2*f*x-1/b^2*a*e*x
+d*x/b-1/3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^3*f+1/3/b^3/(a/b)^
(2/3)*ln(x+(a/b)^(1/3))*a^2*e-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*a*d+1/3*c/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/b^4/(a/b)^(2/3)*
ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^3*f-1/6/b^3/(a/b)^(2/3)*ln(x^
2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^2*e+1/6/b^2/(a/b)^(2/3)*ln(x^2-x*(
a/b)^(1/3)+(a/b)^(2/3))*a*d-1/6*c/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))-1/3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))*a^3*f+1/3/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*e-1/3/b^2/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*d+1/3*c/b/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219365, size = 312, normalized size = 1.3

$$\frac{\sqrt{3}\left(14\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)\log\left(\left(-a^2b\right)^{\frac{2}{3}}x^2 + \left(-a^2b\right)^{\frac{1}{3}}ax + a^2\right) - 28\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)\log\left(\left(-a^2b\right)^{\frac{1}{3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/252*sqrt(3)*(14*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log
((-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*a*x + a^2) - 28*sqrt(3)*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*log((-a^2*b)^(1/3)*x - a) + 84*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*(2*sqrt(3)*(-a^2*b)^
(1/3)*x + sqrt(3)*a)/a) + 3*sqrt(3)*(4*b^2*f*x^7 + 7*(b^2*e - a*b
*f)*x^4 + 28*(b^2*d - a*b*e + a^2*f)*x)*(-a^2*b)^(1/3))/((-a^2*b)
^(1/3)*b^3)
```

Sympy [A] time = 3.60088, size = 340, normalized size = 1.42

$$\operatorname{RootSum}\left(27t^3a^2b^{10} + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f + \right.$$
$$+ \frac{fx^7}{7b} - \frac{x^4\left(af - be\right)}{4b^2} + \frac{x\left(a^2f - abe + b^2d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**10 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**
7*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*
b**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**
2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2
 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e +
a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b*
*8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*f - a
**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**7/(7*b) - x**4*(a*f -
b*e)/(4*b**2) + x*(a**2*f - a*b*e + b**2*d)/b**3
```

GIAC/XCAS [A] time = 0.216909, size = 414, normalized size = 1.72

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^4}$$
$$+ \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^4}$$
$$- \frac{\left(b^7c - ab^6d - a^3b^4f + a^2b^5e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^7}$$
$$+ \frac{4b^6fx^7 - 7ab^5fx^4 + 7b^6x^4e + 28b^6dx + 28a^2b^4fx - 28ab^5xe}{28b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*
b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) + 1/6*((-a*b^2)^(1/3)*b^3
*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/
3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) - 1/3
*(b^7*c - a*b^6*d - a^3*b^4*f + a^2*b^5*e)*(-a/b)^(1/3)*ln(abs(x
- (-a/b)^(1/3)))/(a*b^7) + 1/28*(4*b^6*f*x^7 - 7*a*b^5*f*x^4 + 7*
b^6*x^4*e + 28*b^6*d*x + 28*a^2*b^4*f*x - 28*a*b^5*x*e)/b^7
```

3.240 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=227

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{4/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{4/3}b^{8/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{4/3}b^{8/3}}+\frac{x^2(be-af)}{2b^2}-\frac{c}{ax}+\frac{fx^5}{5b}$$

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(8/3)) + ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(8/3)) - ((
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*a^(4/3)*b^(8/3))
```

Rubi [A] time = 0.412511, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{4/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{4/3}b^{8/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{4/3}b^{8/3}}+\frac{x^2(be-af)}{2b^2}-\frac{c}{ax}+\frac{fx^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(8/3)) + ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(8/3)) - ((
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*a^(4/3)*b^(8/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{fx^5}{5b}-\frac{(af-be)\int x\,dx}{b^2}-\frac{c}{ax}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{4}{3}}b^{\frac{8}{3}}}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{4}{3}}b^{\frac{8}{3}}}-\frac{\sqrt{3}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{4}{3}}b^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a),x)
```

```
[Out] f*x**5/(5*b) - (a*f - b*e)*Integral(x, x)/b**2 - c/(a*x) - (a**3*
f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) + b**(1/3)*x)/(3*a
**(4/3)*b**(8/3)) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a
**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(4/3)*b**(8/
3)) - sqrt(3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(3
)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(4/3)*b**(8/3))
```

Mathematica [A] time = 0.264038, size = 224, normalized size = 0.99

$$\frac{15a^{4/3}b^{2/3}x^3(be-af)+6a^{4/3}b^{5/3}fx^6+10x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+10\sqrt{3}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-\right.}{30a^{4/3}b^{8/3}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-30*a^(1/3)*b^(8/3)*c + 15*a^(4/3)*b^(2/3)*(b*e - a*f)*x^3 + 6*a
^(4/3)*b^(5/3)*f*x^6 + 10*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*x*Log[a^(1/3) + b^(1/3)*x] - 5*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(30*a^(4/3)*b^(8/3)*x)
```

Maple [B] time = 0.009, size = 419, normalized size = 1.9

$$\frac{fx^5}{5\,b}-\frac{ax^2f}{2\,b^2}+\frac{ex^2}{2\,b}-\frac{a^2f}{3\,b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{ae}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{d}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{c}{3\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{a^2f}{6\,b^3}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{ae}{6\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{d}{6\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c}{6\,a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{a^2\sqrt{3}f}{3\,b^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{a\sqrt{3}e}{3\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{d\sqrt{3}}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c\sqrt{3}}{3\,a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x)
```

```
[Out] 1/5*f*x^5/b-1/2/b^2*x^2*a*f+1/2*e*x^2/b-1/3*a^2/b^3/(a/b)^(1/3)*l
n(x+(a/b)^(1/3))*f+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-1/3*
d/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))*c+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*
f-1/6*a/b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/6*d
/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1/6/a/(a/b)^(1/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+1/3*a^2/b^3*3^(1/2)/(a/b)^(
```

```
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3*a/b^2*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3*d*3^(1/
2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/a*3^
(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-c/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242164, size = 312, normalized size = 1.37

$$\frac{\sqrt{3}\left(5\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)x\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2-ab+\left(-ab^2\right)^{\frac{2}{3}}x\right)-10\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)x\log\left(ab+\left(\right.\right.\right.}{90\,(-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*sqrt(3)*(5*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*log
((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 10*sqrt(3)*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x*log(a*b + (-a*b^2)^(2/3)*x) + 3
0*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*arctan(-1/3*(sqrt(3)*a*b
- 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)) + 3*sqrt(3)*(2*a*b*f*x^6 + 5
*(a*b*e - a^2*f)*x^3 - 10*b^2*c)*(-a*b^2)^(1/3))/((-a*b^2)^(1/3)*
a*b^2*x)
```

Sympy [A] time = 5.1265, size = 406, normalized size = 1.79

$$\mathrm{RootSum}\left(27t^3a^4b^8+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4cef+3a^5b^4d^2f+\right.$$
$$+\frac{fx^5}{5b}-\frac{x^2\left(af-be\right)}{2b^2}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**4*b**8 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7
*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b
**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2
*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2
- 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e + a
**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b**
8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**3*b**5/(a**6*f
**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b*
*3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a
*b**5*c*d + b**6*c**2) + x))) + f*x**5/(5*b) - x**2*(a*f - b*e)/(
2*b**2) - c/(a*x)
```

GIAC/XCAS [A] time = 0.218532, size = 428, normalized size = 1.89

$$-\frac{c}{ax} + \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2b^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^4} + \frac{2\,b^4fx^5 - 5\,ab^3fx^2 + 5\,b^4x^2e}{10\,b^5} - \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^2),x, algorithm="giac")
```

```
[Out] -c/(a*x) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f
*(-a/b)^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a
/b)^(1/3)))/(a^2*b^2) + 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b
^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4)
 + 1/10*(2*b^4*f*x^5 - 5*a*b^3*f*x^2 + 5*b^4*x^2*e)/b^5 - 1/6*((-
a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*
f + (-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a^2*b^4)
```

3.241 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=224

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{5/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{5/3}b^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{5/3}b^{7/3}}+\frac{x(be-af)}{b^2}-\frac{c}{2ax^2}+\frac{fx^4}{4b}$$

```
[Out] -c/(2*a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(7/3)) + ((b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))
```

Rubi [A] time = 0.379286, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{5/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{5/3}b^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{5/3}b^{7/3}}+\frac{x(be-af)}{b^2}-\frac{c}{2ax^2}+\frac{fx^4}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

```
[Out] -c/(2*a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(7/3)) + ((b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-(af-be)\int\frac{1}{b^2}\,dx+\frac{fx^4}{4b}-\frac{c}{2ax^2}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{5}{3}}b^{\frac{7}{3}}}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{5}{3}}b^{\frac{7}{3}}}-\frac{\sqrt{3}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{5}{3}}b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a),x)
```

```
[Out] -(a*f - b*e)*Integral(b**(-2), x) + f*x**4/(4*b) - c/(2*a*x**2) +
 (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) + b**(1/3)*
x)/(3*a**(5/3)*b**(7/3)) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c
)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(5/3)
*b**(7/3)) - sqrt(3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan
(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(5/3)*b**(
7/3))
```

Mathematica [A]   time = 0.203563, size = 218, normalized size = 0.97

$$\frac{1}{12}\left(\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{5/3}b^{7/3}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{5/3}b^{7/3}}+\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{5/3}b^{7/3}}+\frac{12x(be-af)}{b^2}-\frac{6c}{ax^2}+\frac{3fx^4}{b}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-6*c)/(a*x^2) + (12*(b*e - a*f)*x)/b^2 + (3*f*x^4)/b + (4*Sqrt[
3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/(a^(5/3)*b^(7/3)) + (4*(-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*b^(7/3)) + (2*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(a^(5/3)*b^(7/3)))/12
```

Maple [B]   time = 0.007, size = 414, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x)
```

```
[Out] 1/4*f*x^4/b-1/b^2*a*f*x+e*x/b+1/3*a^2/b^3/(a/b)^(2/3)*ln(x+(a/b)^
(1/3))*f-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1/3/b/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))*d-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/6
*a^2/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/6*a/b^
2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e-1/6/b/(a/b)^(2/
3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/6/a/(a/b)^(2/3)*ln(x^2-x
*(a/b)^(1/3)+(a/b)^(2/3))*c+1/3*a^2/b^3/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3*a/b^2/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3/b/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3/a/(a/b)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/2*c/a/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244861, size = 304, normalized size = 1.36

$$\frac{\sqrt{3}\Big(2\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^2\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2 - \left(a^2b\right)^{\frac{1}{3}}ax + a^2\right) - 4\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^2\log\left(\left(a^2b\right)^{\frac{1}{3}}x\right.}{36\,\left(a^2b\right)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*(2*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2*l
og((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 4*sqrt(3)*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x^2*log((a^2*b)^(1/3)*x + a) - 12*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2*arctan(1/3*(2*sqrt(3)*(a^
2*b)^(1/3)*x - sqrt(3)*a)/a) + 3*sqrt(3)*(a*b*f*x^6 + 4*(a*b*e -
a^2*f)*x^3 - 2*b^2*c)*(a^2*b)^(1/3))/((a^2*b)^(1/3)*a*b^2*x^2)
```

Sympy [A] time = 6.09596, size = 326, normalized size = 1.46

$$\mathrm{RootSum}\Big(27t^3a^5b^7 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2f -$$
$$+ \frac{fx^4}{4b} - \frac{x\,(af - be)}{b^2} - \frac{c}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**5*b**7 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7
*b**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b
**3*d*e*f + a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2
*f - 3*a**5*b**4*d*e**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2
+ 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f - 6*a**3*b**6*c*d*e - a
**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d**2 - 3*a*b**
8*c**2*d + b**9*c**3, Lambda(_t, _t*log(3*_t*a**2*b**2/(a**3*f -
a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**4/(4*b) - x*(a*f - b*
e)/b**2 - c/(2*a*x**2)
```

GIAC/XCAS [A] time = 0.21652, size = 378, normalized size = 1.69

$$\frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2b^2} - \frac{c}{2\,ax^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^3} + \frac{b^3fx^4 - 4\,ab^2fx + 4\,b^3xe}{4\,b^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^3),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*ln(abs(x - (
-a/b)^(1/3)))/(a^2*b^2) - 1/2*c/(a*x^2) - 1/3*sqrt(3)*((-a*b^2)^(
1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*
b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b
)^(1/3))/(a^2*b^3) + 1/4*(b^3*f*x^4 - 4*a*b^2*f*x + 4*b^3*x*e)/b^
4 - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)
^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a^2*b^3)
```

3.242 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx$

Optimal. Leaf size=227

$$\frac{bc-ad}{a^2x}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{7/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{7/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{7/3}b^{5/3}}-\frac{c}{4ax^4}+\frac{fx^2}{2b}$$

```
[Out] -c/(4*a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))
```

Rubi [A] time = 0.428986, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{bc-ad}{a^2x}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{7/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{7/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{7/3}b^{5/3}}-\frac{c}{4ax^4}+\frac{fx^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```


```
[Out] -c/(4*a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{f\int x\,dx}{b}-\frac{c}{4ax^4}-\frac{ad-bc}{a^2x}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{7}{3}}b^{\frac{5}{3}}}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{7}{3}}b^{\frac{5}{3}}}+\frac{\sqrt{3}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{7}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a),x)
```

```
[Out] f*Integral(x, x)/b - c/(4*a*x**4) - (a*d - b*c)/(a**2*x) + (a**3*
f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) + b**(1/3)*x)/(3*a
**(7/3)*b**(5/3)) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a
**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(7/3)*b**(5/
3)) + sqrt(3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(3
)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(7/3)*b**(5/3))
```

Mathematica [A] time = 0.198025, size = 220, normalized size = 0.97

$$\frac{1}{12}\left(\frac{12(bc-ad)}{a^2x}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{7/3}b^{5/3}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{7/3}b^{5/3}}+\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{7/3}b^{5/3}}-\frac{3c}{ax^4}+\frac{6fx^2}{b}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*c)/(a*x^4) + (12*(b*c - a*d))/(a^2*x) + (6*f*x^2)/b + (4*Sqr
t[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]])/(a^(7/3)*b^(5/3)) + (4*(-(b^3*c) + a*b^2*
d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(7/3)*b^(5/3))
+ (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(5/3)))/12
```

Maple [B] time = 0.01, size = 412, normalized size = 1.8

$$\frac{fx^2}{2b}-\frac{c}{4ax^4}-\frac{d}{ax}+\frac{bc}{a^2x}+\frac{af}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{e}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d}{3a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{bc}{3a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{af}{6b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{e}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{d}{6a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{bc}{6a^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{a\sqrt{3}f}{3b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\sqrt{3}e}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{d\sqrt{3}}{3a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{b\sqrt{3}c}{3a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x)
```

```
[Out] 1/2*f*x^2/b-1/4*c/a/x^4-d/a/x+1/a^2/x*b*c+1/3/b^2*a/(a/b)^(1/3)*l
n(x+(a/b)^(1/3))*f-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e+1/3/a/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))*d-1/3*b/a^2/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))*c-1/6/b^2*a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+
1/6/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e-1/6/a/(a/b)
^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/6*b/a^2/(a/b)^(1/3)*
ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-1/3/b^2*a*3^(1/2)/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+1/3/b*3^(1/2)/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/3/a*3^(1/2)/(a/b)
^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/3*b/a^2*3^(1/2
)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.245771, size = 312, normalized size = 1.37

$$\frac{\sqrt{3}\left(2\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)x^4\log\left(\left(ab^2\right)^{\frac{1}{3}}bx^2+ab-\left(ab^2\right)^{\frac{2}{3}}x\right)-4\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)x^4\log\left(ab+\left(ab\right.\right.\right.}{36\left(ab^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^5),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*(2*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^4*l
og((a*b^2)^(1/3)*b*x^2 + a*b - (a*b^2)^(2/3)*x) - 4*sqrt(3)*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x^4*log(a*b + (a*b^2)^(2/3)*x) + 1
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^4*arctan(-1/3*(sqrt(3)*a*
b - 2*sqrt(3)*(a*b^2)^(2/3)*x)/(a*b)) + 3*sqrt(3)*(2*a^2*f*x^6 +
4*(b^2*c - a*b*d)*x^3 - a*b*c)*(a*b^2)^(1/3))/((a*b^2)^(1/3)*a^2*
b*x^4)
```

Sympy [A] time = 16.6246, size = 411, normalized size = 1.81

$$\operatorname{RootSum}\left(27t^3a^7b^5 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2f - \right.$$
$$+ \frac{fx^2}{2b} - \frac{ac + x^3(4ad - 4bc)}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7*b**5 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7
*b**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b
**3*d*e*f + a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2
*f - 3*a**5*b**4*d*e**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2
+ 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f - 6*a**3*b**6*c*d*e - a
**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d**2 - 3*a*b**
8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**5*b**3/(a**6*f
**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b*
*3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a
*b**5*c*d + b**6*c**2) + x))) + f*x**2/(2*b) - (a*c + x**3*(4*a*d
 - 4*b*c))/(4*a**2*x**4)
```

GIAC/XCAS [A] time = 0.219325, size = 417, normalized size = 1.84

$$\frac{fx^2}{2\,b} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}$$
$$- \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b^3}$$
$$+ \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3b^3}$$
$$+ \frac{4\,bcx^3 - 4\,adx^3 - ac}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^5),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^
3*f*(-a/b)^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x -
(-a/b)^(1/3)))/(a^3*b) - 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*
b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*
e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3
) + 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)
^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a^3*b^3) + 1/4*(4*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^2*
x^4)
```

3.243 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx$

Optimal. Leaf size=225

$$\frac{bc-ad}{2a^2x^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{8/3}b^{4/3}}-\frac{c}{5ax^5}+\frac{fx}{b}$$

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Rubi [A] time = 0.381287, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{bc-ad}{2a^2x^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{8/3}b^{4/3}}-\frac{c}{5ax^5}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```


```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int f\,dx}{b}-\frac{c}{5ax^5}-\frac{ad-bc}{2a^2x^2}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{8}{3}}b^{\frac{4}{3}}}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{8}{3}}b^{\frac{4}{3}}}+\frac{\sqrt{3}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{8}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a),x)
```

```
[Out] Integral(f, x)/b - c/(5*a*x**5) - (a*d - b*c)/(2*a**2*x**2) - (a*
*3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) + b**(1/3)*x)/(
3*a**(8/3)*b**(4/3)) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*lo
g(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(8/3)*b**
(4/3)) + sqrt(3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqr
t(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(8/3)*b**(4/3)
)
```

Mathematica [A] time = 0.153976, size = 220, normalized size = 0.98

$$\frac{bc-ad}{2a^2x^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}a^{8/3}b^{4/3}}-\frac{c}{5ax^5}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b + ((-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/(Sqrt[3]*a^(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*b^(4/3)) + ((-(b^3*c)
+ a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Maple [B] time = 0.012, size = 410, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x)
```

```
[Out] f*x/b-1/5*c/a/x^5-1/2*d/a/x^2+1/2/a^2/x^2*b*c-1/3/b^2*a/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))*f+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e-1/3/
a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+1/3*b/a^2/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))*c+1/6/b^2*a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3)
)*f-1/6/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/6/a/(
a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-1/6*b/a^2/(a/b)^(2
/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-1/3/b^2*a/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+1/3/b/(a/b)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/3/a/(a/b)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/3*b/a^2/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23851, size = 313, normalized size = 1.39

$$\frac{\sqrt{3}\left(5\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^5\log\left(\left(-a^2b\right)^{\frac{2}{3}}x^2 + \left(-a^2b\right)^{\frac{1}{3}}ax + a^2\right) - 10\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^5\log\left(\left(-a^2\right.\right.}{90\,(-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^6),x, algorithm="fricas")
```

```
[Out] 1/90*sqrt(3)*(5*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5*l
og((-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*a*x + a^2) - 10*sqrt(3)*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5*log((-a^2*b)^(1/3)*x - a) +
 30*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5*arctan(1/3*(2*sqrt(3)
*(-a^2*b)^(1/3)*x + sqrt(3)*a)/a) + 3*sqrt(3)*(10*a^2*f*x^6 + 5*(
b^2*c - a*b*d)*x^3 - 2*a*b*c)*(-a^2*b)^(1/3))/((-a^2*b)^(1/3)*a^2
*b*x^5)
```

Sympy [A] time = 20.5395, size = 328, normalized size = 1.46

$$\operatorname{RootSum}\left(27t^3a^8b^4 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f + \right.$$
$$+ \frac{fx}{b} - \frac{2ac + x^3\,(5ad - 5bc)}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8*b**4 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7
*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b
**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2
*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2
- 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e + a
**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b**
8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a**3*b/(a**3*f - a*
*2*b*e + a*b**2*d - b**3*c) + x))) + f*x/b - (2*a*c + x**3*(5*a*d
 - 5*b*c))/(10*a**2*x**5)
```

GIAC/XCAS [A] time = 0.217755, size = 370, normalized size = 1.64

$$\frac{fx}{b} - \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}$$
$$+ \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b^2}$$
$$+ \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3b^2}$$
$$+ \frac{5\,bcx^3 - 5\,adx^3 - 2\,ac}{10\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^6),x, algorithm="giac")
```

```
[Out] f*x/b - 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*ln(a
bs(x - (-a/b)^(1/3)))/(a^3*b) + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c
 - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)
*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(
a^3*b^2) + 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (
-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/(a^3*b^2) + 1/10*(5*b*c*x^3 - 5*a*d*x^3 - 2*
a*c)/(a^2*x^5)
```

3.244 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$

Optimal. Leaf size=242

$$\frac{bc-ad}{4a^2x^4}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{10/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{10/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{10/3}b^{2/3}}-\frac{c}{7ax^7}$$

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/
(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(10/3)*b^(2/3)) + ((b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^
(10/3)*b^(2/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/3)*b^(2/3))
```

Rubi [A] time = 0.426618, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{bc-ad}{4a^2x^4}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{10/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{10/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{10/3}b^{2/3}}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/
(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(10/3)*b^(2/3)) + ((b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^
(10/3)*b^(2/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/3)*b^(2/3))
```

Rubi in Sympy [A] time = 63.9127, size = 224, normalized size = 0.93

$$-\frac{c}{7ax^7}-\frac{ad-bc}{4a^2x^4}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{10}{3}}b^{\frac{2}{3}}}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{10}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{10}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a),x)
```

```
[Out] -c/(7*a*x**7) - (a*d - b*c)/(4*a**2*x**4) - (a**2*e - a*b*d + b**
2*c)/(a**3*x) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1
/3) + b**(1/3)*x)/(3*a**(10/3)*b**(2/3)) + (a**3*f - a**2*b*e + a
*b**2*d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x
**2)/(6*a**(10/3)*b**(2/3)) - sqrt(3)*(a**3*f - a**2*b*e + a*b**2
*d - b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))
/(3*a**(10/3)*b**(2/3))
```

Mathematica [A] time = 0.207139, size = 231, normalized size = 0.95

$$\frac{\frac{21a^{4/3}(bc-ad)}{x^4} - \frac{12a^{7/3}c}{x^7} - \frac{84\sqrt[3]{a}(a^2e-abd+b^2c)}{x} + \frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{2/3}} + \frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{2/3}}}{84a^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*c)/x^7 + (21*a^(4/3)*(b*c - a*d))/x^4 - (84*a^(1/3)
*(b^2*c - a*b*d + a^2*e))/x + (28*Sqrt[3]*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3)
 + (28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/b^(2/3) + (14*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(84*a^(10/3))
```

Maple [B] time = 0.01, size = 440, normalized size = 1.8

$$
\begin{aligned}
&-\frac{c}{7\,ax^7}-\frac{d}{4\,ax^4}+\frac{bc}{4\,x^4a^2}-\frac{e}{ax}+\frac{bd}{a^2x}-\frac{b^2c}{a^3x}-\frac{f}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\
&+\frac{e}{3\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{bd}{3\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{b^2c}{3\,a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\
&+\frac{f}{6\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{e}{6\,a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\
&+\frac{bd}{6\,a^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{b^2c}{6\,a^3}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\
&+\frac{\sqrt{3}f}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{\sqrt{3}e}{3\,a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\
&+\frac{\sqrt{3}bd}{3\,a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{\sqrt{3}b^2c}{3\,a^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x)
```

```
[Out] -1/7*c/a/x^7-1/4/a/x^4*d+1/4/a^2/x^4*b*c-e/a/x+1/a^2/x*b*d-1/a^3/
x*b^2*c-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+1/3/a/(a/b)^(1/3)*l
```

```
n(x+(a/b)^(1/3))*e-1/3/a^2*b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d+1/3/
a^3*b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/6/b/(a/b)^(1/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))*f-1/6/a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/
3)+(a/b)^(2/3))*e+1/6/a^2*b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b
)^(2/3))*d-1/6/a^3*b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))*c+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))*f-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))*e+1/3/a^2*3^(1/2)*b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*d-1/3/a^3*3^(1/2)*b^2/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238885, size = 333, normalized size = 1.38

$$\frac{\sqrt{3}\left(14\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^7\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2 - ab + \left(-ab^2\right)^{\frac{2}{3}}x\right) - 28\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^7\log\left(ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^8),x, algorithm="fricas")
```

```
[Out] 1/252*sqrt(3)*(14*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7
*log((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 28*sqrt(3)*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*log(a*b + (-a*b^2)^(2/3)*
x) + 84*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*arctan(-1/3*(sqrt
(3)*a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(28*(b^2
*c - a*b*d + a^2*e)*x^6 - 7*(a*b*c - a^2*d)*x^3 + 4*a^2*c)*(-a*b^
2)^(1/3))/((-a*b^2)^(1/3)*a^3*x^7)
```

Sympy [A] time = 62.5502, size = 432, normalized size = 1.79

$$\operatorname{RootSum}\left(27t^3a^{10}b^2 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f +\right.$$

$$- \frac{4a^2c + x^6\left(28a^2e - 28abd + 28b^2c\right) + x^3\left(7a^2d - 7abc\right)}{28a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10*b**2 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**
7*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*
b**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**
2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2
 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e +
a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b*
*8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**7*b/(a**6*f**
2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3
*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b
```

```
**5*c*d + b**6*c**2) + x))) - (4*a**2*c + x**6*(28*a**2*e - 28*a*
b*d + 28*b**2*c) + x**3*(7*a**2*d - 7*a*b*c))/(28*a**3*x**7)
```

GIAC/XCAS [A] time = 0.221134, size = 444, normalized size = 1.83

$$\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^4} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^4b^2} - \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^4b^2} - \frac{28\,b^2cx^6 - 28\,abdx^6 + 28\,a^2x^6e - 7\,abcx^3 + 7\,a^2dx^3 + 4\,a^2c}{28\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^8),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/
3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))
/a^4 + 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d
 - (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2) - 1/6*((-a*b^2)^
(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a
*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4
*b^2) - 1/28*(28*b^2*c*x^6 - 28*a*b*d*x^6 + 28*a^2*x^6*e - 7*a*b*
c*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^3*x^7)
```

3.245 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$

Optimal. Leaf size=244

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{2a^3x^2}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{11/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{11/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{11/3}\sqrt[3]{b}}-\frac{c}{8ax^8}$$

```
[Out] -c/(8*a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/
(2*a^3*x^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) -
 ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(
3*a^(11/3)*b^(1/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(11/3)*b^(1/3))
```

Rubi [A] time = 0.410485, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{2a^3x^2}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{11/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{11/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{11/3}\sqrt[3]{b}}-\frac{c}{8ax^8}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] -c/(8*a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/
(2*a^3*x^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) -
 ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(
3*a^(11/3)*b^(1/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(11/3)*b^(1/3))
```

Rubi in Sympy [A] time = 65.8415, size = 228, normalized size = 0.93

$$-\frac{c}{8ax^8}-\frac{ad-bc}{5a^2x^5}-\frac{a^2e-abd+b^2c}{2a^3x^2}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{11}{3}}\sqrt[3]{b}}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{11}{3}}\sqrt[3]{b}}-\frac{\sqrt{3}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{11}{3}}\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a),x)
```

```
[Out] -c/(8*a*x**8) - (a*d - b*c)/(5*a**2*x**5) - (a**2*e - a*b*d + b**
2*c)/(2*a**3*x**2) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(
a**(1/3) + b**(1/3)*x)/(3*a**(11/3)*b**(1/3)) - (a**3*f - a**2*b*
e + a*b**2*d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(6*a**(11/3)*b**(1/3)) - sqrt(3)*(a**3*f - a**2*b*e + a
*b**2*d - b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(
1/3))/(3*a**(11/3)*b**(1/3))
```

Mathematica [A] time = 0.207986, size = 231, normalized size = 0.95

$$\frac{\frac{24a^{5/3}(bc-ad)}{x^5}-\frac{15a^{8/3}c}{x^8}-\frac{60a^{2/3}(a^2e-abd+b^2c)}{x^2}+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3f-a^2be+ab^2d-b^3c)}{\sqrt[3]{b}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt[3]{b}}+}{120a^{11/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] ((-15*a^(8/3)*c)/x^8 + (24*a^(5/3)*(b*c - a*d))/x^5 - (60*a^(2/3)
*(b^2*c - a*b*d + a^2*e))/x^2 + (40*Sqrt[3]*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/
3) + (40*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/b^(1/3) + (20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(120*a^(11/3))
```

Maple [B] time = 0.01, size = 441, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x)
```

```
[Out] -1/8*c/a/x^8-1/5/a/x^5*d+1/5/a^2/x^5*b*c-1/2/a/x^2*e+1/2/a^2/x^2*
b*d-1/2/a^3/x^2*b^2*c+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1/3/a
/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1/3/a^2*b/(a/b)^(2/3)*ln(x+(a/b)
^(1/3))*d-1/3/a^3*b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/6/b/(a/b)
^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/6/a/(a/b)^(2/3)*ln(x
^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e-1/6/a^2*b/(a/b)^(2/3)*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))*d+1/6/a^3*b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(
1/3)+(a/b)^(2/3))*c+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))*f-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))*e+1/3/a^2*b/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3/a^3*b^2/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^9),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221883, size = 319, normalized size = 1.31

$$\frac{\sqrt{3}\left(20\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)x^8\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)-40\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)x^8\log\left(\left(a^2b\right)^{\frac{1}{3}}\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^9),x, algorithm="fricas")
```

```
[Out] 1/360*sqrt(3)*(20*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^8
*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 40*sqrt(3)*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^8*log((a^2*b)^(1/3)*x + a) -
120*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^8*arctan(1/3*(2*sqrt(3)
*(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*(20*(b^2*c - a*b*d +
 a^2*e)*x^6 - 8*(a*b*c - a^2*d)*x^3 + 5*a^2*c)*(a^2*b)^(1/3))/((a
^2*b)^(1/3)*a^3*x^8)
```

Sympy [A] time = 106.427, size = 348, normalized size = 1.43

$$\operatorname{RootSum}\left(27t^3a^{11}b-a^9f^3+3a^8bef^2-3a^7b^2df^2-3a^7b^2e^2f+3a^6b^3cf^2+6a^6b^3def+a^6b^3e^3-6a^5b^4cef-3a^5b^4d^2f-\right.$$
$$-\frac{5a^2c+x^6\left(20a^2e-20abd+20b^2c\right)+x^3\left(8a^2d-8abc\right)}{40a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**11*b - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b
**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**
3*d*e*f + a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f
 - 3*a**5*b**4*d*e**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 +
3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f - 6*a**3*b**6*c*d*e - a**
3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d**2 - 3*a*b**8*
c**2*d + b**9*c**3, Lambda(_t, _t*log(3*_t*a**4/(a**3*f - a**2*b*
e + a*b**2*d - b**3*c) + x))) - (5*a**2*c + x**6*(20*a**2*e - 20*
a*b*d + 20*b**2*c) + x**3*(8*a**2*d - 8*a*b*c))/(40*a**3*x**8)
```

GIAC/XCAS [A] time = 0.217408, size = 401, normalized size = 1.64

$$\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4}$$
$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d-\left(-ab^2\right)^{\frac{1}{3}}a^3f+\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4b}$$
$$-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d-\left(-ab^2\right)^{\frac{1}{3}}a^3f+\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4b}$$
$$-\frac{20b^2cx^6-20abdx^6+20a^2x^6e-8abcx^3+8a^2dx^3+5a^2c}{40a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^9),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*ln(abs(x - (
-a/b)^(1/3)))/a^4 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^
(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*ar
ctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/6
*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*
a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^
(2/3))/(a^4*b) - 1/40*(20*b^2*c*x^6 - 20*a*b*d*x^6 + 20*a^2*x^6*e
 - 8*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^3*x^8)
```

3.246 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$

Optimal. Leaf size=277

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{4a^3x^4}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{13/3}} -\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{13/3}} +\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{13/3}} +\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{10ax^{10}}$$

```
[Out] -c/(10*a*x^10) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e
)/(4*a^3*x^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^
(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(13/3)) - (b^(1/3)*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(13/3
)) + (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13/3))
```

Rubi [A] time = 0.501596, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{4a^3x^4}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{13/3}} -\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{13/3}} +\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{13/3}} +\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```


```
[Out] -c/(10*a*x^10) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e
)/(4*a^3*x^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^
(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(13/3)) - (b^(1/3)*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(13/3
)) + (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13/3))
```

Rubi in Sympy [A] time = 78.6698, size = 257, normalized size = 0.93

$$-\frac{c}{10ax^{10}}-\frac{ad-bc}{7a^2x^7}-\frac{a^2e-abd+b^2c}{4a^3x^4}-\frac{a^3f-a^2be+ab^2d-b^3c}{a^4x}+\frac{\sqrt[3]{b}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{13}{3}}}-\frac{\sqrt[3]{b}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{13}{3}}}+\frac{\sqrt{3}\sqrt[3]{b}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{13}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a),x)
```

```
[Out] -c/(10*a*x**10) - (a*d - b*c)/(7*a**2*x**7) - (a**2*e - a*b*d + b
**2*c)/(4*a**3*x**4) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a
**4*x) + b**(1/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**
(1/3) + b**(1/3)*x)/(3*a**(13/3)) - b**(1/3)*(a**3*f - a**2*b*e +
 a*b**2*d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)
*x**2)/(6*a**(13/3)) + sqrt(3)*b**(1/3)*(a**3*f - a**2*b*e + a*b*
*2*d - b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3
))/(3*a**(13/3))
```

Mathematica [A] time = 0.205059, size = 266, normalized size = 0.96

$$\frac{\frac{60a^{7/3}(bc-ad)}{x^7}-\frac{42a^{10/3}c}{x^{10}}-\frac{105a^{4/3}\left(a^2e-abd+b^2c\right)}{x^4}+\frac{420\sqrt[3]{a}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{x}+140\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

```
[Out] ((-42*a^(10/3)*c)/x^10 + (60*a^(7/3)*(b*c - a*d))/x^7 - (105*a^(4
/3)*(b^2*c - a*b*d + a^2*e))/x^4 + (420*a^(1/3)*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f))/x - 140*Sqrt[3]*b^(1/3)*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*
b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1
/3)*x] + 70*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*a^(13/3))
```

Maple [B] time = 0.013, size = 491, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x)
```

```
[Out] -1/10*c/a/x^10-1/7/a/x^7*d+1/7/a^2/x^7*b*c-1/4/a/x^4*e+1/4/a^2/x^
4*b*d-1/4/a^3/x^4*b^2*c-1/a/x*f+1/a^2/x*b*e-1/a^3/x*b^2*d+1/a^4/x
*b^3*c+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-1/3*b/a^2/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))*e+1/3*b^2/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d
```

```
-1/3*b^3/a^4/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c-1/6/a/(a/b)^(1/3)*ln
(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/6*b/a^2/(a/b)^(1/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))*e-1/6*b^2/a^3/(a/b)^(1/3)*ln(x^2-x*(a/b)
^(1/3)+(a/b)^(2/3))*d+1/6*b^3/a^4/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3
)+(a/b)^(2/3))*c-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))*f+1/3*b/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*e-1/3*b^2/a^3*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/3*b^3/a^4*3^(1/2)/(a/b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^11),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.226273, size = 389, normalized size = 1.4

$$\frac{\sqrt{3}\left(70\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^{10}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2 - ax\left(\frac{b}{a}\right)^{\frac{2}{3}} + a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 140\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^{10}\left(\frac{b}{a}\right)^{\frac{1}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^11),x, algorithm="fricas")
```

```
[Out] 1/1260*sqrt(3)*(70*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^
10*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140
*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*log
(b*x + a*(b/a)^(2/3)) - 420*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x
^10*(b/a)^(1/3)*arctan(-1/3*(2*sqrt(3)*b*x - sqrt(3)*a*(b/a)^(2/3
))/(a*(b/a)^(2/3))) + 3*sqrt(3)*(140*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*x^9 - 35*(a*b^2*c - a^2*b*d + a^3*e)*x^6 - 14*a^3*c + 20*
(a^2*b*c - a^3*d)*x^3))/(a^4*x^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.220606, size = 508, normalized size = 1.83

$$-\frac{\left(b^{4}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{3}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}b^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{5}}-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{5}b}+\frac{\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{5}b}+\frac{140\,b^{3}cx^{9}-140\,ab^{2}dx^{9}-140\,a^{3}fx^{9}+140\,a^{2}bx^{9}e-35\,ab^{2}cx^{6}+35\,a^{2}bdx^{6}-35\,a^{3}x^{6}e+20\,a^{2}bcx^{3}-20\,a^{3}dx^{3}-14\,a^{3}c}{140\,a^{4}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^11),x, algorithm="giac")
```

```
[Out] -1/3*(b^4*c*(-a/b)^(1/3) - a*b^3*d*(-a/b)^(1/3) - a^3*b*f*(-a/b)^
(1/3) + a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1
/3)))/a^5 - 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*
b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) + 1/6*((-a*b^
2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f +
(-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(
a^5*b) + 1/140*(140*b^3*c*x^9 - 140*a*b^2*d*x^9 - 140*a^3*f*x^9 +
 140*a^2*b*x^9*e - 35*a*b^2*c*x^6 + 35*a^2*b*d*x^6 - 35*a^3*x^6*e
 + 20*a^2*b*c*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^4*x^10)
```

3.247 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$

Optimal. Leaf size=280

$$\frac{bc-ad}{8a^2x^8}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{14/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{14/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{14/3}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{2a^4x^2}-\frac{c}{11ax^{11}}$$

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e
)/(5*a^3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) -
 (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(14/3)) + (b^(2/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(
14/3)) - (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(14/3))
```

Rubi [A] time = 0.472228, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$\frac{bc-ad}{8a^2x^8}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{14/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{14/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{14/3}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{2a^4x^2}-\frac{c}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e
)/(5*a^3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) -
 (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(14/3)) + (b^(2/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(
14/3)) - (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(14/3))
```

Rubi in Sympy [A] time = 84.0703, size = 260, normalized size = 0.93

$$-\frac{c}{11ax^{11}}-\frac{ad-bc}{8a^2x^8}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{a^3f-a^2be+ab^2d-b^3c}{2a^4x^2}-\frac{b^{\frac{2}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{14}{3}}}+\frac{b^{\frac{2}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{14}{3}}}+\frac{\sqrt{3}b^{\frac{2}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{14}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a),x)
```

```
[Out] -c/(11*a*x**11) - (a*d - b*c)/(8*a**2*x**8) - (a**2*e - a*b*d + b
**2*c)/(5*a**3*x**5) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2
*a**4*x**2) - b**(2/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*lo
g(a**(1/3) + b**(1/3)*x)/(3*a**(14/3)) + b**(2/3)*(a**3*f - a**2*
b*e + a*b**2*d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**
(2/3)*x**2)/(6*a**(14/3)) + sqrt(3)*b**(2/3)*(a**3*f - a**2*b*e +
 a*b**2*d - b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a*
*(1/3))/(3*a**(14/3))
```

Mathematica [A] time = 0.215224, size = 266, normalized size = 0.95

$$\frac{165a^{8/3}(bc-ad)}{x^8} - \frac{120a^{11/3}c}{x^{11}} - \frac{264a^{5/3}\left(a^2e-abd+b^2c\right)}{x^5} + 440b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right) - 440\sqrt{3}b^{2/3}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] ((-120*a^(11/3)*c)/x^11 + (165*a^(8/3)*(b*c - a*d))/x^8 - (264*a^
(5/3)*(b^2*c - a*b*d + a^2*e))/x^5 + (660*a^(2/3)*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f))/x^2 - 440*Sqrt[3]*b^(2/3)*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] +
440*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(
1/3)*x] + 220*b^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*a^(14/3))
```

Maple [B] time = 0.011, size = 493, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x)
```

```
[Out] -1/11*c/a/x^11-1/8/a/x^8*d+1/8/a^2/x^8*b*c-1/5/a/x^5*e+1/5/a^2/x^
5*b*d-1/5/a^3/x^5*b^2*c-1/2/a/x^2*f+1/2/a^2/x^2*b*e-1/2/a^3/x^2*b
^2*d+1/2/a^4/x^2*b^3*c-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+1/3*
b/a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e-1/3*b^2/a^3/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))*d+1/3*b^3/a^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c+1/6/
a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f-1/6*b/a^2/(a/b)
^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/6*b^2/a^3/(a/b)^(2/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-1/6*b^3/a^4/(a/b)^(2/3)*ln(
x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+1/3*b/a^2/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/3*b^2/a^3/(a/b)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/3*b^3/a^4/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^12),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.223798, size = 427, normalized size = 1.52

$$\frac{\sqrt{3}\left(220\,\sqrt{3}\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{11}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)-440\,\sqrt{3}\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^12),x, algorithm="fricas")
```

```
[Out] 1/3960*sqrt(3)*(220*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x
^11*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(
-b^2/a^2)^(2/3)) - 440*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*x^11*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 1320*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*b*x + sqrt(3)*a*(-b^2/a^2)^(1/3))/(a*(-b^2/a^2)^(1/3))
) + 3*sqrt(3)*(220*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 88*(
a*b^2*c - a^2*b*d + a^3*e)*x^6 - 40*a^3*c + 55*(a^2*b*c - a^3*d)*
x^3))/(a^4*x^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217835, size = 456, normalized size = 1.63

$$\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{5}}-\frac{\left(b^{4}c-ab^{3}d-a^{3}bf+a^{2}b^{2}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{5}}+\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{5}}+\frac{220\,b^{3}cx^{9}-220\,ab^{2}dx^{9}-220\,a^{3}fx^{9}+220\,a^{2}bx^{9}e-88\,ab^{2}cx^{6}+88\,a^{2}bdx^{6}-88\,a^{3}x^{6}e+55\,a^{2}bcx^{3}-55\,a^{3}dx^{3}-40\,a^{3}c}{440\,a^{4}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^12),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*
b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^5 - 1/3*(b^4*c - a*b^3*d - a^3*
b*f + a^2*b^2*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/a^5 + 1/6
```

```
*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*
a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^
(2/3))/a^5 + 1/440*(220*b^3*c*x^9 - 220*a*b^2*d*x^9 - 220*a^3*f*x
^9 + 220*a^2*b*x^9*e - 88*a*b^2*c*x^6 + 88*a^2*b*d*x^6 - 88*a^3*x
^6*e + 55*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^4*x^11)
```

3.248 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$

Optimal. Leaf size=313

$$\frac{bc-ad}{10a^2x^{10}}-\frac{a^2e-abd+b^2c}{7a^3x^7}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{16/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{16/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{16/3}}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{4a^4x^4}-\frac{c}{13ax^{13}}$$

```
[Out] -c/(13*a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2
*e)/(7*a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4)
 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^5*x) + (b^(4/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(16/3)) + (b^(4/3)*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(16/3)) - (b^(
4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Rubi [A] time = 0.559978, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{bc-ad}{10a^2x^{10}}-\frac{a^2e-abd+b^2c}{7a^3x^7}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{16/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{16/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{16/3}}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{4a^4x^4}-\frac{c}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -c/(13*a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2
*e)/(7*a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4)
 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^5*x) + (b^(4/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(16/3)) + (b^(4/3)*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(16/3)) - (b^(
4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Rubi in Sympy [A] time = 101.679, size = 291, normalized size = 0.93

$$-\frac{c}{13ax^{13}}-\frac{ad-bc}{10a^2x^{10}}-\frac{a^2e-abd+b^2c}{7a^3x^7}-\frac{a^3f-a^2be+ab^2d-b^3c}{4a^4x^4}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5x}-\frac{b^{\frac{4}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{16}{3}}}+\frac{b^{\frac{4}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{16}{3}}}-\frac{\sqrt{3}b^{\frac{4}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{16}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a),x)
```

```
[Out] -c/(13*a*x**13) - (a*d - b*c)/(10*a**2*x**10) - (a**2*e - a*b*d +
 b**2*c)/(7*a**3*x**7) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/
(4*a**4*x**4) + b*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**5*x
) - b**(4/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3)
 + b**(1/3)*x)/(3*a**(16/3)) + b**(4/3)*(a**3*f - a**2*b*e + a*b*
*2*d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2
)/(6*a**(16/3)) - sqrt(3)*b**(4/3)*(a**3*f - a**2*b*e + a*b**2*d
- b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3
*a**(16/3))
```

Mathematica [A] time = 0.182712, size = 308, normalized size = 0.98

$$\frac{bc-ad}{10a^2x^{10}}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6a^{16/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{16/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{16/3}}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{4a^4x^4}-\frac{c}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -c/(13*a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2
*e)/(7*a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4)
 + (b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(4/3)*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/(Sqrt[3]*a^(16/3)) + (b^(4/3)*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(16/3)) + (b^(4/3
)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Maple [B] time = 0.012, size = 546, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x)
```

```
[Out] 1/3*b/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*f-1/3*b^2/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*e+1/3*b^3/a^4*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))*d-1/3*b^4/a^5*3^(1/2)/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/3*b/a^2/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))*f+1/3*b^2/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-1/3*b^3/a^
4/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d+1/3*b^4/a^5/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))*c+1/6*b/a^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))*f-1/6*b^2/a^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e
+1/6*b^3/a^4/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-1/6*
b^4/a^5/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-1/13*c/a/
x^13+1/10/a^2/x^10*b*c+1/7/a^2/x^7*b*d-1/7/a^3/x^7*b^2*c+1/4/a^2/
x^4*b*e-1/4/a^3/x^4*b^2*d+1/4/a^4/x^4*b^3*c+1/a^2*b/x*f-1/a^3*b^2
/x*e+1/a^4*b^3/x*d-1/a^5*b^4/x*c-1/4/a/x^4*f-1/10/a/x^10*d-1/7/a/
x^7*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^14),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218643, size = 464, normalized size = 1.48

$$\frac{\sqrt{3}\left(910\sqrt{3}\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)-1820\sqrt{3}\left(b^4c-ab^3d+a^2b^2e-a^3b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^14),x, algorithm="fricas")
```

```
[Out] 1/16380*sqrt(3)*(910*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b
*f)*x^13*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/
3)) - 1820*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(
-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) - 5460*(b^4*c - a*b^3*d + a
^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*arctan(-1/3*(2*sqrt(3)*b*x
- sqrt(3)*a*(-b/a)^(2/3))/(a*(-b/a)^(2/3))) - 3*sqrt(3)*(1820*(b^
4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 455*(a*b^3*c - a^2*b^
2*d + a^3*b*e - a^4*f)*x^9 + 260*(a^2*b^2*c - a^3*b*d + a^4*e)*x^
6 + 140*a^4*c - 182*(a^3*b*c - a^4*d)*x^3))/(a^5*x^13)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a),x)
```

[Out] Timed out

GIAC/XCAS [A] time = 0.220513, size = 566, normalized size = 1.81

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\left(-ab^2\right)^{\frac{2}{3}}a^3f+\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6}+\frac{\left(b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^6}-\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\left(-ab^2\right)^{\frac{2}{3}}a^3f+\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6}-\frac{1820\,b^4cx^{12}-1820\,ab^3dx^{12}-1820\,a^3bfx^{12}+1820\,a^2b^2x^{12}e-455\,ab^3cx^9+455\,a^2b^2dx^9+455\,a^4fx^9-455\,a^3bx^9e+260}{1820\,a^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^14),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*
b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^6 + 1/3*(b^5*c*(-a/b)^(1/3) - a
*b^4*d*(-a/b)^(1/3) - a^3*b^2*f*(-a/b)^(1/3) + a^2*b^3*(-a/b)^(1/
3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/a^6 - 1/6*((-a*b^2)^
(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a
*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6
- 1/1820*(1820*b^4*c*x^12 - 1820*a*b^3*d*x^12 - 1820*a^3*b*f*x^12
 + 1820*a^2*b^2*x^12*e - 455*a*b^3*c*x^9 + 455*a^2*b^2*d*x^9 + 45
5*a^4*f*x^9 - 455*a^3*b*x^9*e + 260*a^2*b^2*c*x^6 - 260*a^3*b*d*x
^6 + 260*a^4*x^6*e - 182*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)
/(a^5*x^13)
```

3.249 $\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$

Optimal. Leaf size=315

$$\frac{bc-ad}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{17/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{17/3}}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{17/3}}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^5x^2}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{14ax^{14}}$$

```
[Out] -c/(14*a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2
*e)/(8*a^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5)
 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^(5/3)
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(17/3)) - (b^(5/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(17/3)) +
(b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Rubi [A] time = 0.514963, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$\frac{bc-ad}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{17/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{17/3}}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{17/3}}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^5x^2}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] -c/(14*a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2
*e)/(8*a^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5)
 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^(5/3)
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(17/3)) - (b^(5/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(17/3)) +
(b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Rubi in Sympy [A] time = 113.806, size = 294, normalized size = 0.93

$$-\frac{c}{14ax^{14}}-\frac{ad-bc}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}-\frac{a^3f-a^2be+ab^2d-b^3c}{5a^4x^5}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^5x^2}+\frac{b^{\frac{5}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{17}{3}}}-\frac{b^{\frac{5}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{17}{3}}}-\frac{\sqrt{3}b^{\frac{5}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{17}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**15/(b*x**3+a),x)
```

```
[Out] -c/(14*a*x**14) - (a*d - b*c)/(11*a**2*x**11) - (a**2*e - a*b*d +
 b**2*c)/(8*a**3*x**8) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/
(5*a**4*x**5) + b*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a**5
*x**2) + b**(5/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**
(1/3) + b**(1/3)*x)/(3*a**(17/3)) - b**(5/3)*(a**3*f - a**2*b*e +
 a*b**2*d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)
*x**2)/(6*a**(17/3)) - sqrt(3)*b**(5/3)*(a**3*f - a**2*b*e + a*b*
*2*d - b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3
))/(3*a**(17/3))
```

Mathematica [A] time = 0.188541, size = 311, normalized size = 0.99

$$\frac{bc-ad}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{17/3}}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^{17/3}}+\frac{b^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{17/3}}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^5x^2}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] -c/(14*a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2
*e)/(8*a^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5)
 + (b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(2*a^5*x^2) + (b^(5
/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(17/3)) + (b^(5/3)*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(17/3)) +
(b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Maple [B] time = 0.012, size = 548, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x)
```

```
[Out] 1/3*b/a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*f-1/3*b^2/a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*e+1/3*b^3/a^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))*d-1/3*b^4/a^5/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/3*b/a^2/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))*f-1/3*b^2/a^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1/3*b^3/a^
4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-1/3*b^4/a^5/(a/b)^(2/3)*ln(x+(a
/b)^(1/3))*c-1/6*b/a^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))*f+1/6*b^2/a^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e
-1/6*b^3/a^4/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/5/
a^2/x^5*b*e-1/5/a^3/x^5*b^2*d+1/5/a^4/x^5*b^3*c+1/2/a^2*b/x^2*f-1
/2/a^3*b^2/x^2*e+1/2/a^4*b^3/x^2*d-1/2/a^5*b^4/x^2*c+1/11/a^2/x^1
1*b*c+1/8/a^2/x^8*b*d-1/8/a^3/x^8*b^2*c+1/6*b^4/a^5/(a/b)^(2/3)*l
n(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-1/8/a/x^8*e-1/5/a/x^5*f-1/11/a
/x^11*d-1/14*c/a/x^14
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^15),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.223861, size = 481, normalized size = 1.53

$$\sqrt{3}\Bigg(1540\,\sqrt{3}\big(b^4c-ab^3d+a^2b^2e-a^3bf\big)x^{14}\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-3080\,\sqrt{3}\big(b^4c-ab^3d+a^2b^2e-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^15),x, algorithm="fricas")
```

```
[Out] 1/27720*sqrt(3)*(1540*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*
b*f)*x^14*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a
^2*(b^2/a^2)^(2/3)) - 3080*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e -
 a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 924
0*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*ar
ctan(-1/3*(2*sqrt(3)*b*x - sqrt(3)*a*(b^2/a^2)^(1/3))/(a*(b^2/a^2
)^(1/3))) - 3*sqrt(3)*(1540*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*
f)*x^12 - 616*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 385*(
a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 220*a^4*c - 280*(a^3*b*c - a^4
*d)*x^3))/(a^5*x^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**15/(b*x**3+a),x)
```

[Out] Timed out

GIAC/XCAS [A] time = 0.217412, size = 531, normalized size = 1.69

$$-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{2}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{6}}+\frac{\left(b^{5}c-ab^{4}d-a^{3}b^{2}f+a^{2}b^{3}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{6}}-\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{2}e\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{6}}-\frac{1540\,b^{4}cx^{12}-1540\,ab^{3}dx^{12}-1540\,a^{3}bfx^{12}+1540\,a^{2}b^{2}x^{12}e-616\,ab^{3}cx^{9}+616\,a^{2}b^{2}dx^{9}+616\,a^{4}fx^{9}-616\,a^{3}bx^{9}e+385}{3080\,a^{5}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^15),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^4*c - (-a*b^2)^(1/3)*a*b^3*d - (-a
*b^2)^(1/3)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^6 + 1/3*(b^5*c - a*b^4*d -
 a^3*b^2*f + a^2*b^3*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/a^
6 - 1/6*((-a*b^2)^(1/3)*b^4*c - (-a*b^2)^(1/3)*a*b^3*d - (-a*b^2)
^(1/3)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*ln(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/a^6 - 1/3080*(1540*b^4*c*x^12 - 1540*a*b^3*d*x^
12 - 1540*a^3*b*f*x^12 + 1540*a^2*b^2*x^12*e - 616*a*b^3*c*x^9 +
616*a^2*b^2*d*x^9 + 616*a^4*f*x^9 - 616*a^3*b*x^9*e + 385*a^2*b^2
*c*x^6 - 385*a^3*b*d*x^6 + 385*a^4*x^6*e - 280*a^3*b*c*x^3 + 280*
a^4*d*x^3 + 220*a^4*c)/(a^5*x^14)
```

3.250 $\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx$

Optimal. Leaf size=351

$$\frac{bc-ad}{13a^2x^{13}}-\frac{a^2e-abd+b^2c}{10a^3x^{10}}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{19/3}}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{19/3}}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{19/3}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^5x^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{7a^4x^7}-\frac{c}{16ax^{16}}$$

```
[Out] -c/(16*a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2
*e)/(10*a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^
7) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^6*x) - (b^(7/3)*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(19/3)) - (b^(7/3)*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (b^(7/3)*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*a^(19/3))
```

Rubi [A] time = 0.599039, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{bc-ad}{13a^2x^{13}}-\frac{a^2e-abd+b^2c}{10a^3x^{10}}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{19/3}}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{19/3}}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{19/3}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^5x^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{7a^4x^7}-\frac{c}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```


```
[Out] -c/(16*a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2
*e)/(10*a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^
7) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^6*x) - (b^(7/3)*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(19/3)) - (b^(7/3)*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (b^(7/3)*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*a^(19/3))
```

Rubi in Sympy [A] time = 155.236, size = 326, normalized size = 0.93

$$-\frac{c}{16ax^{16}}-\frac{ad-bc}{13a^2x^{13}}-\frac{a^2e-abd+b^2c}{10a^3x^{10}}-\frac{a^3f-a^2be+ab^2d-b^3c}{7a^4x^7}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^5x^4}-\frac{b^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^6x}+\frac{b^{\frac{7}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{19}{3}}}-\frac{b^{\frac{7}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{19}{3}}}+\frac{\sqrt{3}b^{\frac{7}{3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{19}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**17/(b*x**3+a),x)
```

```
[Out] -c/(16*a*x**16) - (a*d - b*c)/(13*a**2*x**13) - (a**2*e - a*b*d +
 b**2*c)/(10*a**3*x**10) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c
)/(7*a**4*x**7) + b*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(4*a*
*5*x**4) - b**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**6*x)
+ b**(7/3)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**(1/3) +
 b**(1/3)*x)/(3*a**(19/3)) - b**(7/3)*(a**3*f - a**2*b*e + a*b**2
*d - b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/
(6*a**(19/3)) + sqrt(3)*b**(7/3)*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a
**(19/3))
```

Mathematica [A] time = 0.203014, size = 346, normalized size = 0.99

$$\frac{bc-ad}{13a^2x^{13}}-\frac{a^2e-abd+b^2c}{10a^3x^{10}}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{19/3}}+\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^{19/3}}+\frac{b^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}a^{19/3}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^5x^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{7a^4x^7}-\frac{c}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

```
[Out] -c/(16*a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2
*e)/(10*a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^
7) + (b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(4*a^5*x^4) + (b^
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^6*x) + (b^(7/3)*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/(Sqrt[3]*a^(19/3)) + (b^(7/3)*(-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (b^(7/3)
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*a^(19/3))
```

Maple [A] time = 0.014, size = 600, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x)
```

```
[Out] 1/3*b^2/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-1/3*b^4/a^5*3^(1/2)/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/3*b^5/a^6*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/3
*b^2/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*f+1/3*b^3/a^4*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*e-1/7/a/x^7*f-1/13/a/x^13*d-1/10/a/x^10*e+1/3*b^4/a^
5/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-1/3*b^5/a^6/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))*c-1/6*b^2/a^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(
2/3))*f+1/6*b^3/a^4/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
*e-1/6*b^4/a^5/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/
6*b^5/a^6/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+1/a^6*b
^5/x*c+1/4/a^2*b/x^4*f-1/4/a^3*b^2/x^4*e+1/4/a^4*b^3/x^4*d-1/4/a^
5*b^4/x^4*c+1/13/a^2/x^13*b*c+1/10/a^2/x^10*b*d-1/10/a^3/x^10*b^2
*c+1/7/a^2/x^7*b*e-1/7/a^3/x^7*b^2*d+1/7/a^4/x^7*b^3*c-1/a^3*b^2/
x*f+1/a^4*b^3/x*e-1/a^5*b^4/x*d-1/3*b^3/a^4/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))*e-1/16*c/a/x^16
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^17),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219368, size = 514, normalized size = 1.46

$$\frac{\sqrt{3}\left(3640\sqrt{3}\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2 - ax\left(\frac{b}{a}\right)^{\frac{2}{3}} + a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 7280\sqrt{3}\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^17),x, algorithm="fricas")
```

```
[Out] 1/65520*sqrt(3)*(3640*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*
b^2*f)*x^16*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/
3)) - 7280*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16
*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) - 21840*(b^5*c - a*b^4*d +
a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^(1/3)*arctan(-1/3*(2*sqrt(3)*b*
x - sqrt(3)*a*(b/a)^(2/3))/(a*(b/a)^(2/3))) + 3*sqrt(3)*(7280*(b^
5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15 - 1820*(a*b^4*c - a^2
*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 1040*(a^2*b^3*c - a^3*b^2*d
+ a^4*b*e - a^5*f)*x^9 - 728*(a^3*b^2*c - a^4*b*d + a^5*e)*x^6 -
455*a^5*c + 560*(a^4*b*c - a^5*d)*x^3))/(a^6*x^16)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**17/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218915, size = 640, normalized size = 1.82

$$-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{2}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b^{2}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^{7}}-\frac{\left(b^{6}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{5}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}b^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}b^{4}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^{7}}+\frac{\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{2}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b^{2}e\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{7}}+\frac{7280\,b^{5}cx^{15}-7280\,ab^{4}dx^{15}-7280\,a^{3}b^{2}fx^{15}+7280\,a^{2}b^{3}x^{15}e-1820\,ab^{4}cx^{12}+1820\,a^{2}b^{3}dx^{12}+1820\,a^{4}bfx^{12}-1820\,a^{3}b^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)*x^17),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d - (-a
*b^2)^(2/3)*a^3*b*f + (-a*b^2)^(2/3)*a^2*b^2*e)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^7 - 1/3*(b^6*c*(-a/b)^(1/3
) - a*b^5*d*(-a/b)^(1/3) - a^3*b^3*f*(-a/b)^(1/3) + a^2*b^4*(-a/b
)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/a^7 + 1/6*((-a*
b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d - (-a*b^2)^(2/3)*a^3*b*
f + (-a*b^2)^(2/3)*a^2*b^2*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/a^7 + 1/7280*(7280*b^5*c*x^15 - 7280*a*b^4*d*x^15 - 7280*a^3
*b^2*f*x^15 + 7280*a^2*b^3*x^15*e - 1820*a*b^4*c*x^12 + 1820*a^2*
b^3*d*x^12 + 1820*a^4*b*f*x^12 - 1820*a^3*b^2*x^12*e + 1040*a^2*b
^3*c*x^9 - 1040*a^3*b^2*d*x^9 - 1040*a^5*f*x^9 + 1040*a^4*b*x^9*e
 - 728*a^3*b^2*c*x^6 + 728*a^4*b*d*x^6 - 728*a^5*x^6*e + 560*a^4*
b*c*x^3 - 560*a^5*d*x^3 - 455*a^5*c)/(a^6*x^16)
```

3.251 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=220

$$\frac{x^9(3a^2f-2abe+b^2d)}{9b^4}+\frac{a^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^7(a+bx^3)}+\frac{a^2\log(a+bx^3)(-6a^3f+5a^2be-4ab^2d+3b^3c)}{3b^7}-\frac{ax^3(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6}+\frac{x^6(-4a^3f+3a^2be-2ab^2d+b^3c)}{6b^5}+\frac{x^{12}(be-2af)}{12b^3}+\frac{fx^{15}}{15b^2}$$

```
[Out] -(a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/(3*b^6) + ((
b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(6*b^5) + ((b^2*d -
 2*a*b*e + 3*a^2*f)*x^9)/(9*b^4) + ((b*e - 2*a*f)*x^12)/(12*b^3)
+ (f*x^15)/(15*b^2) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(
3*b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^
3*f)*Log[a + b*x^3])/(3*b^7)
```

Rubi [A] time = 0.697603, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^9(3a^2f-2abe+b^2d)}{9b^4}+\frac{a^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^7(a+bx^3)}+\frac{a^2\log(a+bx^3)(-6a^3f+5a^2be-4ab^2d+3b^3c)}{3b^7}-\frac{ax^3(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6}+\frac{x^6(-4a^3f+3a^2be-2ab^2d+b^3c)}{6b^5}+\frac{x^{12}(be-2af)}{12b^3}+\frac{fx^{15}}{15b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -(a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/(3*b^6) + ((
b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(6*b^5) + ((b^2*d -
 2*a*b*e + 3*a^2*f)*x^9)/(9*b^4) + ((b*e - 2*a*f)*x^12)/(12*b^3)
+ (f*x^15)/(15*b^2) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(
3*b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^
3*f)*Log[a + b*x^3])/(3*b^7)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3(a^3f-a^2be+ab^2d-b^3c)}{3b^7(a+bx^3)}-\frac{a^2(6a^3f-5a^2be+4ab^2d-3b^3c)\log(a+bx^3)}{3b^7}+\frac{fx^{15}}{15b^2}-\frac{x^{12}(2af-be)}{12b^3}+\frac{x^9(3a^2f-2abe+b^2d)}{9b^4}-\frac{(4a^3f-3a^2be+2ab^2d-b^3c)\int^{x^3}x\,dx}{3b^5}+\frac{(5a^3f-4a^2be+3ab^2d-2b^3c)\int^{x^3}a\,dx}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -a**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*b**7*(a + b*x**3
)) - a**2*(6*a**3*f - 5*a**2*b*e + 4*a*b**2*d - 3*b**3*c)*log(a +
 b*x**3)/(3*b**7) + f*x**15/(15*b**2) - x**12*(2*a*f - b*e)/(12*b
**3) + x**9*(3*a**2*f - 2*a*b*e + b**2*d)/(9*b**4) - (4*a**3*f -
3*a**2*b*e + 2*a*b**2*d - b**3*c)*Integral(x, (x, x**3))/(3*b**5)
 + (5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)*Integral(a, (x
```

```
, x**3))/(3*b**6)
```

Mathematica [A] time = 0.361985, size = 205, normalized size = 0.93

$$\frac{20b^3x^9\left(3a^2f-2abe+b^2d\right)+30b^2x^6\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)+60abx^3\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)-\frac{60a^3(a^3f-}{}}{180b^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (60*a*b*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x^3 + 30*b^2
*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6 + 20*b^3*(b^2*d -
2*a*b*e + 3*a^2*f)*x^9 + 15*b^4*(b*e - 2*a*f)*x^12 + 12*b^5*f*x^1
5 - (60*a^3*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) +
 60*a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f)*Log[a + b*x^3
])/(180*b^7)
```

Maple [A] time = 0.018, size = 288, normalized size = 1.3

$$\frac{fx^{15}}{15\,b^2}-\frac{x^{12}af}{6\,b^3}+\frac{x^{12}e}{12\,b^2}+\frac{x^9a^2f}{3\,b^4}-\frac{2\,x^9ae}{9\,b^3}+\frac{x^9d}{9\,b^2}-\frac{2\,a^3fx^6}{3\,b^5}+\frac{a^2ex^6}{2\,b^4}-\frac{adx^6}{3\,b^3}+\frac{x^6c}{6\,b^2}+\frac{5\,a^4fx^3}{3\,b^6}-\frac{4\,a^3ex^3}{3\,b^5}+\frac{a^2dx^3}{b^4}-\frac{2\,acx^3}{3\,b^3}-2\,\frac{a^5\ln\left(bx^3+a\right)f}{b^7}+\frac{5\,a^4\ln\left(bx^3+a\right)e}{3\,b^6}-\frac{4\,a^3\ln\left(bx^3+a\right)d}{3\,b^5}+\frac{a^2\ln\left(bx^3+a\right)c}{b^4}-\frac{a^6f}{3\,b^7\left(bx^3+a\right)}+\frac{a^5e}{3\,b^6\left(bx^3+a\right)}-\frac{a^4d}{3\,b^5\left(bx^3+a\right)}+\frac{a^3c}{3\,b^4\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/15*f*x^15/b^2-1/6/b^3*x^12*a*f+1/12/b^2*x^12*e+1/3/b^4*x^9*a^2*
f-2/9/b^3*x^9*a*e+1/9/b^2*x^9*d-2/3/b^5*x^6*a^3*f+1/2/b^4*x^6*a^2
*e-1/3/b^3*x^6*a*d+1/6/b^2*x^6*c+5/3/b^6*a^4*f*x^3-4/3/b^5*a^3*e*
x^3+1/b^4*a^2*d*x^3-2/3/b^3*a*c*x^3-2*a^5/b^7*ln(b*x^3+a)*f+5/3*a
^4/b^6*ln(b*x^3+a)*e-4/3*a^3/b^5*ln(b*x^3+a)*d+a^2/b^4*ln(b*x^3+a
)*c-1/3*a^6/b^7/(b*x^3+a)*f+1/3*a^5/b^6/(b*x^3+a)*e-1/3*a^4/b^5/(
b*x^3+a)*d+1/3*a^3/b^4/(b*x^3+a)*c
```

Maxima [A] time = 1.38471, size = 300, normalized size = 1.36

$$\frac{a^3b^3c-a^4b^2d+a^5be-a^6f}{3\left(b^8x^3+ab^7\right)}+\frac{12\,b^4fx^{15}+15\left(b^4e-2\,ab^3f\right)x^{12}+20\left(b^4d-2\,ab^3e+3\,a^2b^2f\right)x^9+30\left(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf\right)x^6-60\left(2\,ab^3c-\right.}{180\,b^6}+\frac{\left(3\,a^2b^3c-4\,a^3b^2d+5\,a^4be-6\,a^5f\right)\log\left(bx^3+a\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^11/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)/(b^8*x^3 + a*b^7) +
 1/180*(12*b^4*f*x^15 + 15*(b^4*e - 2*a*b^3*f)*x^12 + 20*(b^4*d -
 2*a*b^3*e + 3*a^2*b^2*f)*x^9 + 30*(b^4*c - 2*a*b^3*d + 3*a^2*b^2
*e - 4*a^3*b*f)*x^6 - 60*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5
*a^4*f)*x^3)/b^6 + 1/3*(3*a^2*b^3*c - 4*a^3*b^2*d + 5*a^4*b*e - 6
*a^5*f)*log(b*x^3 + a)/b^7
```

Fricas [A] time = 0.203985, size = 409, normalized size = 1.86

$$\frac{12 b^6 f x^{18} + 3\left(5 b^6 e - 6 a b^5 f\right)x^{15} + 5\left(4 b^6 d - 5 a b^5 e + 6 a^2 b^4 f\right)x^{12} + 10\left(3 b^6 c - 4 a b^5 d + 5 a^2 b^4 e - 6 a^3 b^3 f\right)x^9 + 60 a^3 b^3 c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^11/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^6*f*x^18 + 3*(5*b^6*e - 6*a*b^5*f)*x^15 + 5*(4*b^6*d
- 5*a*b^5*e + 6*a^2*b^4*f)*x^12 + 10*(3*b^6*c - 4*a*b^5*d + 5*a^2
*b^4*e - 6*a^3*b^3*f)*x^9 + 60*a^3*b^3*c - 60*a^4*b^2*d + 60*a^5*
b*e - 60*a^6*f - 30*(3*a*b^5*c - 4*a^2*b^4*d + 5*a^3*b^3*e - 6*a^
4*b^2*f)*x^6 - 60*(2*a^2*b^4*c - 3*a^3*b^3*d + 4*a^4*b^2*e - 5*a^
5*b*f)*x^3 + 60*(3*a^3*b^3*c - 4*a^4*b^2*d + 5*a^5*b*e - 6*a^6*f
+ (3*a^2*b^4*c - 4*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)*log(
b*x^3 + a))/(b^8*x^3 + a*b^7)
```

Sympy [A] time = 19.4944, size = 224, normalized size = 1.02

$$-\frac{a^2\left(6a^3f - 5a^2be + 4ab^2d - 3b^3c\right)\log\left(a + bx^3\right)}{3b^7} - \frac{a^6f - a^5be + a^4b^2d - a^3b^3c}{3ab^7 + 3b^8x^3} + \frac{fx^{15}}{15b^2} - \frac{x^{12}\left(2af - be\right)}{12b^3} + \frac{x^9\left(3a^2f - 2abe + b^2d\right)}{9b^4} - \frac{x^6\left(4a^3f - 3a^2be + 2ab^2d - b^3c\right)}{6b^5} + \frac{x^3\left(5a^4f - 4a^3be + 3a^2b^2d - 2ab^3c\right)}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -a**2*(6*a**3*f - 5*a**2*b*e + 4*a*b**2*d - 3*b**3*c)*log(a + b*x
**3)/(3*b**7) - (a**6*f - a**5*b*e + a**4*b**2*d - a**3*b**3*c)/(
3*a*b**7 + 3*b**8*x**3) + f*x**15/(15*b**2) - x**12*(2*a*f - b*e)
/(12*b**3) + x**9*(3*a**2*f - 2*a*b*e + b**2*d)/(9*b**4) - x**6*(
4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(6*b**5) + x**3*(5*a
**4*f - 4*a**3*b*e + 3*a**2*b**2*d - 2*a*b**3*c)/(3*b**6)
```

GIAC/XCAS [A] time = 0.215197, size = 405, normalized size = 1.84

$$\frac{\left(3 a^2 b^3 c - 4 a^3 b^2 d - 6 a^5 f + 5 a^4 b e\right)\ln\left(\left|b x^3 + a\right|\right)}{3 b^7} - \frac{3 a^2 b^4 c x^3 - 4 a^3 b^3 d x^3 - 6 a^5 b f x^3 + 5 a^4 b^2 x^3 e + 2 a^3 b^3 c - 3 a^4 b^2 d - 5 a^6 f + 4 a^5 b e}{3\left(b x^3 + a\right)b^7} + \frac{12 b^8 f x^{15} - 30 a b^7 f x^{12} + 15 b^8 x^{12} e + 20 b^8 d x^9 + 60 a^2 b^6 f x^9 - 40 a b^7 x^9 e + 30 b^8 c x^6 - 60 a b^7 d x^6 - 120 a^3 b^5 f x^6 + 90 a^2 b^6}{180 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^11/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*b^3*c - 4*a^3*b^2*d - 6*a^5*f + 5*a^4*b*e)*ln(abs(b*x^
3 + a))/b^7 - 1/3*(3*a^2*b^4*c*x^3 - 4*a^3*b^3*d*x^3 - 6*a^5*b*f*
x^3 + 5*a^4*b^2*x^3*e + 2*a^3*b^3*c - 3*a^4*b^2*d - 5*a^6*f + 4*a
^5*b*e)/((b*x^3 + a)*b^7) + 1/180*(12*b^8*f*x^15 - 30*a*b^7*f*x^1
2 + 15*b^8*x^12*e + 20*b^8*d*x^9 + 60*a^2*b^6*f*x^9 - 40*a*b^7*x^
9*e + 30*b^8*c*x^6 - 60*a*b^7*d*x^6 - 120*a^3*b^5*f*x^6 + 90*a^2*
b^6*x^6*e - 120*a*b^7*c*x^3 + 180*a^2*b^6*d*x^3 + 300*a^4*b^4*f*x
^3 - 240*a^3*b^5*x^3*e)/b^10
```

3.252 $\int \frac{x^8\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=180

$$\frac{x^6\left(3a^2f-2abe+b^2d\right)}{6b^4}-\frac{a^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{a\log\left(a+bx^3\right)\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{3b^6}+\frac{x^3\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}+\frac{x^9(be-2af)}{9b^3}+\frac{fx^{12}}{12b^2}$$

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d
 - 2*a*b*e + 3*a^2*f)*x^6)/(6*b^4) + ((b*e - 2*a*f)*x^9)/(9*b^3)
+ (f*x^12)/(12*b^2) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(
3*b^6*(a + b*x^3)) - (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*
f)*Log[a + b*x^3])/(3*b^6)
```

Rubi [A] time = 0.531902, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^6\left(3a^2f-2abe+b^2d\right)}{6b^4}-\frac{a^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{a\log\left(a+bx^3\right)\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{3b^6}+\frac{x^3\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}+\frac{x^9(be-2af)}{9b^3}+\frac{fx^{12}}{12b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d
 - 2*a*b*e + 3*a^2*f)*x^6)/(6*b^4) + ((b*e - 2*a*f)*x^9)/(9*b^3)
+ (f*x^12)/(12*b^2) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(
3*b^6*(a + b*x^3)) - (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*
f)*Log[a + b*x^3])/(3*b^6)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^6\left(a+bx^3\right)}+\frac{a\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)\log\left(a+bx^3\right)}{3b^6}-\left(\frac{4a^3f}{3}-a^2be+\frac{2ab^2d}{3}-\frac{b^3c}{3}\right)\int^{x^3}\frac{1}{b^5}\,dx+\frac{fx^{12}}{12b^2}-\frac{x^9\left(2af-be\right)}{9b^3}+\frac{\left(3a^2f-2abe+b^2d\right)\int^{x^3}x\,dx}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] a**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*b**6*(a + b*x**3)
) + a*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)*log(a + b*x
**3)/(3*b**6) - (4*a**3*f/3 - a**2*b*e + 2*a*b**2*d/3 - b**3*c/3)
*Integral(b**(-5), (x, x**3)) + f*x**12/(12*b**2) - x**9*(2*a*f -
 b*e)/(9*b**3) + (3*a**2*f - 2*a*b*e + b**2*d)*Integral(x, (x, x*
*3))/(3*b**4)
```

Mathematica [A] time = 0.236282, size = 167, normalized size = 0.93

$$\frac{6b^2x^6\left(3a^2f-2abe+b^2d\right)+12bx^3\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)+\frac{12a^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+12a\log\left(a+bx^3\right)\left(5a^3f-\right.}{36b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (12*b*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3 + 6*b^2*(b^2*
d - 2*a*b*e + 3*a^2*f)*x^6 + 4*b^3*(b*e - 2*a*f)*x^9 + 3*b^4*f*x^
12 + (12*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)
+ 12*a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*Log[a + b*x^3
])/(36*b^6)
```

Maple [A] time = 0.019, size = 240, normalized size = 1.3

$$\frac{fx^{12}}{12\,b^2}-\frac{2\,x^9af}{9\,b^3}+\frac{x^9e}{9\,b^2}+\frac{a^2fx^6}{2\,b^4}-\frac{aex^6}{3\,b^3}+\frac{dx^6}{6\,b^2}-\frac{4\,a^3fx^3}{3\,b^5}+\frac{a^2ex^3}{b^4}-\frac{2\,adx^3}{3\,b^3}+\frac{cx^3}{3\,b^2}+\frac{5\,a^4\ln\left(bx^3+a\right)f}{3\,b^6}-\frac{4\,a^3\ln\left(bx^3+a\right)e}{3\,b^5}+\frac{a^2\ln\left(bx^3+a\right)d}{b^4}-\frac{2\,a\ln\left(bx^3+a\right)c}{3\,b^3}+\frac{a^5f}{3\,b^6\left(bx^3+a\right)}-\frac{a^4e}{3\,b^5\left(bx^3+a\right)}+\frac{a^3d}{3\,b^4\left(bx^3+a\right)}-\frac{a^2c}{3\,b^3\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/12*f*x^12/b^2-2/9/b^3*x^9*a*f+1/9/b^2*x^9*e+1/2/b^4*x^6*a^2*f-1
/3/b^3*x^6*a*e+1/6/b^2*x^6*d-4/3/b^5*a^3*f*x^3+1/b^4*a^2*e*x^3-2/
3/b^3*a*d*x^3+1/3/b^2*c*x^3+5/3*a^4/b^6*ln(b*x^3+a)*f-4/3*a^3/b^5
*ln(b*x^3+a)*e+a^2/b^4*ln(b*x^3+a)*d-2/3*a/b^3*ln(b*x^3+a)*c+1/3*
a^5/b^6/(b*x^3+a)*f-1/3*a^4/b^5/(b*x^3+a)*e+1/3*a^3/b^4/(b*x^3+a)
*d-1/3*a^2/b^3/(b*x^3+a)*c
```

Maxima [A] time = 1.40935, size = 243, normalized size = 1.35

$$-\frac{a^2b^3c-a^3b^2d+a^4be-a^5f}{3\left(b^7x^3+ab^6\right)}+\frac{3\,b^3fx^{12}+4\left(b^3e-2\,ab^2f\right)x^9+6\left(b^3d-2\,ab^2e+3\,a^2bf\right)x^6+12\left(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f\right)x^3}{36\,b^5}-\frac{\left(2\,ab^3c-3\,a^2b^2d+4\,a^3be-5\,a^4f\right)\log\left(bx^3+a\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^8/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)/(b^7*x^3 + a*b^6)
+ 1/36*(3*b^3*f*x^12 + 4*(b^3*e - 2*a*b^2*f)*x^9 + 6*(b^3*d - 2*a
*b^2*e + 3*a^2*b*f)*x^6 + 12*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a
^3*f)*x^3)/b^5 - 1/3*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4
*f)*log(b*x^3 + a)/b^6
```

Fricas [A] time = 0.204973, size = 347, normalized size = 1.93

$$3\,b^5fx^{15}+\left(4\,b^5e-5\,ab^4f\right)x^{12}+2\left(3\,b^5d-4\,ab^4e+5\,a^2b^3f\right)x^9+6\left(2\,b^5c-3\,ab^4d+4\,a^2b^3e-5\,a^3b^2f\right)x^6-12\,a^2b^3c+12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^8/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*f*x^15 + (4*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 4*
a*b^4*e + 5*a^2*b^3*f)*x^9 + 6*(2*b^5*c - 3*a*b^4*d + 4*a^2*b^3*e
 - 5*a^3*b^2*f)*x^6 - 12*a^2*b^3*c + 12*a^3*b^2*d - 12*a^4*b*e +
12*a^5*f + 12*(a*b^4*c - 2*a^2*b^3*d + 3*a^3*b^2*e - 4*a^4*b*f)*x
^3 - 12*(2*a^2*b^3*c - 3*a^3*b^2*d + 4*a^4*b*e - 5*a^5*f + (2*a*b
^4*c - 3*a^2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)*log(b*x^3 + a)
)/(b^7*x^3 + a*b^6)
```

Sympy [A] time = 19.1881, size = 180, normalized size = 1.

$$\frac{a\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)\log\left(a+bx^3\right)}{3b^6}+\frac{a^5f-a^4be+a^3b^2d-a^2b^3c}{3ab^6+3b^7x^3}+\frac{fx^{12}}{12b^2}-\frac{x^9\left(2af-be\right)}{9b^3}+\frac{x^6\left(3a^2f-2abe+b^2d\right)}{6b^4}-\frac{x^3\left(4a^3f-3a^2be+2ab^2d-b^3c\right)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] a*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)*log(a + b*x**3)
/(3*b**6) + (a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*
b**6 + 3*b**7*x**3) + f*x**12/(12*b**2) - x**9*(2*a*f - b*e)/(9*b
**3) + x**6*(3*a**2*f - 2*a*b*e + b**2*d)/(6*b**4) - x**3*(4*a**3
*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(3*b**5)
```

GIAC/XCAS [A] time = 0.215384, size = 335, normalized size = 1.86

$$-\frac{\left(2\,ab^3c-3\,a^2b^2d-5\,a^4f+4\,a^3be\right)\ln\left(\left|bx^3+a\right|\right)}{3\,b^6}+\frac{2\,ab^4cx^3-3\,a^2b^3dx^3-5\,a^4bfx^3+4\,a^3b^2x^3e+a^2b^3c-2\,a^3b^2d-4\,a^5f+3\,a^4be}{3\,(bx^3+a)b^6}+\frac{3\,b^6fx^{12}-8\,ab^5fx^9+4\,b^6x^9e+6\,b^6dx^6+18\,a^2b^4fx^6-12\,ab^5x^6e+12\,b^6cx^3-24\,ab^5dx^3-48\,a^3b^3fx^3+36\,a^2b^4x^3e}{36\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^8/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*a*b^3*c - 3*a^2*b^2*d - 5*a^4*f + 4*a^3*b*e)*ln(abs(b*x^3
 + a))/b^6 + 1/3*(2*a*b^4*c*x^3 - 3*a^2*b^3*d*x^3 - 5*a^4*b*f*x^3
 + 4*a^3*b^2*x^3*e + a^2*b^3*c - 2*a^3*b^2*d - 4*a^5*f + 3*a^4*b*
e)/((b*x^3 + a)*b^6) + 1/36*(3*b^6*f*x^12 - 8*a*b^5*f*x^9 + 4*b^6
*x^9*e + 6*b^6*d*x^6 + 18*a^2*b^4*f*x^6 - 12*a*b^5*x^6*e + 12*b^6
*c*x^3 - 24*a*b^5*d*x^3 - 48*a^3*b^3*f*x^3 + 36*a^2*b^4*x^3*e)/b^
8
```

3.253 $\int \frac{x^5\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=140

$$\frac{x^3\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{a\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}+\frac{x^6(be-2af)}{6b^3}+\frac{fx^9}{9b^2}$$

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^6)/(
6*b^3) + (f*x^9)/(9*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/(3*b^5*(a + b*x^3)) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f
)*Log[a + b*x^3])/(3*b^5)
```

Rubi [A] time = 0.369875, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^3\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{a\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}+\frac{x^6(be-2af)}{6b^3}+\frac{fx^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^6)/(
6*b^3) + (f*x^9)/(9*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/(3*b^5*(a + b*x^3)) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f
)*Log[a + b*x^3])/(3*b^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^5\left(a+bx^3\right)}+\left(a^2f-\frac{2abe}{3}+\frac{b^2d}{3}\right)\int^{x^3}\frac{1}{b^4}\,dx+\frac{fx^9}{9b^2}-\frac{(2af-be)\int^{x^3}x\,dx}{3b^3}-\frac{\left(4a^3f-3a^2be+2ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*b**5*(a + b*x**3))
+ (a**2*f - 2*a*b*e/3 + b**2*d/3)*Integral(b**(-4), (x, x**3)) +
f*x**9/(9*b**2) - (2*a*f - b*e)*Integral(x, (x, x**3))/(3*b**3) -
 (4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)*log(a + b*x**3)/(3
*b**5)
```

Mathematica [A] time = 0.180594, size = 129, normalized size = 0.92

$$\frac{6bx^3\left(3a^2f-2abe+b^2d\right)+\frac{6a\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a+bx^3}+6\log\left(a+bx^3\right)\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)+3b^2x^6(be-2af)+}{18b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (6*b*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3 + 3*b^2*(b*e - 2*a*f)*x^6 +
2*b^3*f*x^9 + (6*a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^
3) + 6*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/
(18*b^5)
```

Maple [A] time = 0.016, size = 192, normalized size = 1.4

$$\frac{fx^9}{9\,b^2}-\frac{x^6af}{3\,b^3}+\frac{ex^6}{6\,b^2}+\frac{a^2fx^3}{b^4}-\frac{2\,aex^3}{3\,b^3}+\frac{dx^3}{3\,b^2}-\frac{4\,\ln\left(bx^3+a\right)a^3f}{3\,b^5}+\frac{\ln\left(bx^3+a\right)a^2e}{b^4}-\frac{2\,\ln\left(bx^3+a\right)ad}{3\,b^3}+\frac{\ln\left(bx^3+a\right)c}{3\,b^2}-\frac{a^4f}{3\,b^5\left(bx^3+a\right)}+\frac{a^3e}{3\,b^4\left(bx^3+a\right)}-\frac{a^2d}{3\,b^3\left(bx^3+a\right)}+\frac{ac}{3\,b^2\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/9*f*x^9/b^2-1/3/b^3*x^6*a*f+1/6/b^2*x^6*e+1/b^4*a^2*f*x^3-2/3/b
^3*a*e*x^3+1/3/b^2*d*x^3-4/3/b^5*ln(b*x^3+a)*a^3*f+1/b^4*ln(b*x^3
+a)*a^2*e-2/3/b^3*ln(b*x^3+a)*a*d+1/3/b^2*ln(b*x^3+a)*c-1/3/b^5*a
^4/(b*x^3+a)*f+1/3/b^4*a^3/(b*x^3+a)*e-1/3/b^3*a^2/(b*x^3+a)*d+1/
3/b^2*a/(b*x^3+a)*c
```

Maxima [A] time = 1.37511, size = 186, normalized size = 1.33

$$\frac{ab^3c-a^2b^2d+a^3be-a^4f}{3\left(b^6x^3+ab^5\right)}+\frac{2\,b^2fx^9+3\left(b^2e-2\,abf\right)x^6+6\left(b^2d-2\,abe+3\,a^2f\right)x^3}{18\,b^4}+\frac{\left(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f\right)\log\left(bx^3+a\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^5/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)/(b^6*x^3 + a*b^5) + 1
/18*(2*b^2*f*x^9 + 3*(b^2*e - 2*a*b*f)*x^6 + 6*(b^2*d - 2*a*b*e +
 3*a^2*f)*x^3)/b^4 + 1/3*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f
)*log(b*x^3 + a)/b^5
```

Fricas [A] time = 0.232821, size = 273, normalized size = 1.95

$$\frac{2\,b^4fx^{12}+\left(3\,b^4e-4\,ab^3f\right)x^9+3\left(2\,b^4d-3\,ab^3e+4\,a^2b^2f\right)x^6+6\,ab^3c-6\,a^2b^2d+6\,a^3be-6\,a^4f+6\left(ab^3d-2\,a^2b^2e+3\right.}{18\left(b^6x^3+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^5/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^4*f*x^12 + (3*b^4*e - 4*a*b^3*f)*x^9 + 3*(2*b^4*d - 3*a
*b^3*e + 4*a^2*b^2*f)*x^6 + 6*a*b^3*c - 6*a^2*b^2*d + 6*a^3*b*e -
 6*a^4*f + 6*(a*b^3*d - 2*a^2*b^2*e + 3*a^3*b*f)*x^3 + 6*(a*b^3*c
 - 2*a^2*b^2*d + 3*a^3*b*e - 4*a^4*f + (b^4*c - 2*a*b^3*d + 3*a^2
*b^2*e - 4*a^3*b*f)*x^3)*log(b*x^3 + a))/(b^6*x^3 + a*b^5)
```

Sympy [A] time = 18.4291, size = 138, normalized size = 0.99

$$-\frac{a^4f - a^3be + a^2b^2d - ab^3c}{3ab^5 + 3b^6x^3} + \frac{fx^9}{9b^2} - \frac{x^6\left(2af - be\right)}{6b^3} + \frac{x^3\left(3a^2f - 2abe + b^2d\right)}{3b^4} - \frac{\left(4a^3f - 3a^2be + 2ab^2d - b^3c\right)\log\left(a + bx^3\right)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(3*a*b**5 + 3*b**6*
x**3) + f*x**9/(9*b**2) - x**6*(2*a*f - b*e)/(6*b**3) + x**3*(3*a
**2*f - 2*a*b*e + b**2*d)/(3*b**4) - (4*a**3*f - 3*a**2*b*e + 2*a
*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**5)
```

GIAC/XCAS [A] time = 0.216618, size = 293, normalized size = 2.09

$$\frac{\frac{\left(bx^3+a\right)^3\left(2f-\frac{3\left(4abf-b^2e\right)}{\left(bx^3+a\right)b}+\frac{6\left(b^4d+6a^2b^2f-3ab^3e\right)}{\left(bx^3+a\right)^2b^2}\right)}{b^4}-\frac{6\left(b^3c-2ab^2d-4a^3f+3a^2be\right)\ln\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2\left|b\right|}\right)}{b^4}+\frac{6\left(\frac{ab^6c}{bx^3+a}-\frac{a^2b^5d}{bx^3+a}-\frac{a^4b^3f}{bx^3+a}+\frac{a^3b^4e}{bx^3+a}\right)}{b^7}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^5/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] 1/18*((b*x^3 + a)^3*(2*f - 3*(4*a*b*f - b^2*e)/((b*x^3 + a)*b) +
6*(b^4*d + 6*a^2*b^2*f - 3*a*b^3*e)/((b*x^3 + a)^2*b^2))/b^4 - 6*
(b^3*c - 2*a*b^2*d - 4*a^3*f + 3*a^2*b*e)*ln(abs(b*x^3 + a)/((b*x
^3 + a)^2*abs(b)))/b^4 + 6*(a*b^6*c/(b*x^3 + a) - a^2*b^5*d/(b*x^
3 + a) - a^4*b^3*f/(b*x^3 + a) + a^3*b^4*e/(b*x^3 + a))/b^7)/b
```

3.254 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=103

$$\frac{\log\left(a+bx^3\right)\left(3a^2f-2abe+b^2d\right)}{3b^4}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3b^4\left(a+bx^3\right)}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^6}{6b^2}$$

```
[Out] ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^6)/(6*b^2) - (b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)/(3*b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^
2*f)*Log[a + b*x^3])/(3*b^4)
```

Rubi [A] time = 0.299313, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{\log\left(a+bx^3\right)\left(3a^2f-2abe+b^2d\right)}{3b^4}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3b^4\left(a+bx^3\right)}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^6)/(6*b^2) - (b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)/(3*b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^
2*f)*Log[a + b*x^3])/(3*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{2af}{3}-\frac{be}{3}\right)\int^{x^3}\frac{1}{b^3}\,dx+\frac{f\int^{x^3}x\,dx}{3b^2}+\frac{\left(3a^2f-2abe+b^2d\right)\log\left(a+bx^3\right)}{3b^4}+\frac{a^3f-a^2be+ab^2d-b^3c}{3b^4\left(a+bx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -(2*a*f/3 - b*e/3)*Integral(b**(-3), (x, x**3)) + f*Integral(x, (
x, x**3))/(3*b**2) + (3*a**2*f - 2*a*b*e + b**2*d)*log(a + b*x**3
)/(3*b**4) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*b**4*(a +
 b*x**3))
```

Mathematica [A] time = 0.106698, size = 93, normalized size = 0.9

$$\frac{2\log\left(a+bx^3\right)\left(3a^2f-2abe+b^2d\right)+\frac{2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+2bx^3(be-2af)+b^2fx^6}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (2*b*(b*e - 2*a*f)*x^3 + b^2*f*x^6 + (2*(-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f))/(a + b*x^3) + 2*(b^2*d - 2*a*b*e + 3*a^2*f)*Log[a
+ b*x^3])/(6*b^4)
```

Maple [A] time = 0.016, size = 142, normalized size = 1.4

$$\frac{fx^{6}}{6\,b^{2}}-\frac{2\,ax^{3}f}{3\,b^{3}}+\frac{x^{3}e}{3\,b^{2}}+\frac{\ln\left(bx^{3}+a\right)a^{2}f}{b^{4}}-\frac{2\,\ln\left(bx^{3}+a\right)ae}{3\,b^{3}}+\frac{\ln\left(bx^{3}+a\right)d}{3\,b^{2}}+\frac{a^{3}f}{3\,b^{4}\left(bx^{3}+a\right)}-\frac{a^{2}e}{3\,b^{3}\left(bx^{3}+a\right)}+\frac{ad}{3\,b^{2}\left(bx^{3}+a\right)}-\frac{c}{3\,b\left(bx^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/6*f*x^6/b^2-2/3/b^3*x^3*a*f+1/3/b^2*x^3*e+1/b^4*ln(b*x^3+a)*a^2
*f-2/3/b^3*ln(b*x^3+a)*a*e+1/3/b^2*ln(b*x^3+a)*d+1/3/b^4/(b*x^3+a
)*a^3*f-1/3/b^3/(b*x^3+a)*a^2*e+1/3/b^2/(b*x^3+a)*a*d-1/3/b/(b*x^
3+a)*c
```

Maxima [A] time = 9.60462, size = 132, normalized size = 1.28

$$-\frac{b^{3}c-ab^{2}d+a^{2}be-a^{3}f}{3\left(b^{5}x^{3}+ab^{4}\right)}+\frac{bfx^{6}+2\left(be-2\,af\right)x^{3}}{6\,b^{3}}+\frac{\left(b^{2}d-2\,abe+3\,a^{2}f\right)\log\left(bx^{3}+a\right)}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^2/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(b^5*x^3 + a*b^4) + 1/6*
(b*f*x^6 + 2*(b*e - 2*a*f)*x^3)/b^3 + 1/3*(b^2*d - 2*a*b*e + 3*a^
2*f)*log(b*x^3 + a)/b^4
```

Fricas [A] time = 0.215871, size = 193, normalized size = 1.87

$$\frac{b^{3}fx^{9}+\left(2\,b^{3}e-3\,ab^{2}f\right)x^{6}-2\,b^{3}c+2\,ab^{2}d-2\,a^{2}be+2\,a^{3}f+2\left(ab^{2}e-2\,a^{2}bf\right)x^{3}+2\left(ab^{2}d-2\,a^{2}be+3\,a^{3}f+\left(b^{3}d-2\right.\right.}{6\left(b^{5}x^{3}+ab^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^2/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*f*x^9 + (2*b^3*e - 3*a*b^2*f)*x^6 - 2*b^3*c + 2*a*b^2*d
- 2*a^2*b*e + 2*a^3*f + 2*(a*b^2*e - 2*a^2*b*f)*x^3 + 2*(a*b^2*d
- 2*a^2*b*e + 3*a^3*f + (b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)*log(
b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [A] time = 16.5737, size = 97, normalized size = 0.94

$$\frac{a^{3}f-a^{2}be+ab^{2}d-b^{3}c}{3ab^{4}+3b^{5}x^{3}}+\frac{fx^{6}}{6b^{2}}-\frac{x^{3}\left(2af-be\right)}{3b^{3}}+\frac{\left(3a^{2}f-2abe+b^{2}d\right)\log\left(a+bx^{3}\right)}{3b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3)
+ f*x**6/(6*b**2) - x**3*(2*a*f - b*e)/(3*b**3) + (3*a**2*f - 2*a
*b*e + b**2*d)*log(a + b*x**3)/(3*b**4)
```

GIAC/XCAS [A] time = 0.21559, size = 278, normalized size = 2.7

$$-\frac{1}{6} f\left(\frac{\left(bx^3+a\right)^2\left(\frac{6a}{bx^3+a}-1\right)}{b^4}+\frac{6a^2\ln\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^4}-\frac{2a^3}{(bx^3+a)b^4}\right)+\frac{1}{3}\left(\frac{2a\ln\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^3}+\frac{bx^3+a}{b^3}-\frac{a^2}{(bx^3+a)b^3}\right)e-\frac{d\left(\frac{\ln\left(\frac{|bx^3+a|}{\left(bx^3+a\right)^2|b|}\right)}{b}-\frac{a}{(bx^3+a)b}\right)}{3b}-\frac{c}{3\left(bx^3+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^2/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/6*f*((b*x^3 + a)^2*(6*a/(b*x^3 + a) - 1)/b^4 + 6*a^2*ln(abs(b*
x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^4 - 2*a^3/((b*x^3 + a)*b^4)) +
 1/3*(2*a*ln(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^3 + (b*x^3
+ a)/b^3 - a^2/((b*x^3 + a)*b^3))*e - 1/3*d*(ln(abs(b*x^3 + a)/((
b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b - 1/3*c/((b*x^3 +
a)*b)
```

3.255 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=100

$$\frac{c\log(x)}{a^2}-\frac{\log\left(a+bx^3\right)\left(2a^3f-a^2be+b^3c\right)}{3a^2b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3ab^3\left(a+bx^3\right)}+\frac{fx^3}{3b^2}$$

```
[Out] (f*x^3)/(3*b^2) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a*b^3*(a
 + b*x^3)) + (c*Log[x])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a
+ b*x^3])/(3*a^2*b^3)
```

Rubi [A] time = 0.24865, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{c\log(x)}{a^2}-\frac{\log\left(a+bx^3\right)\left(2a^3f-a^2be+b^3c\right)}{3a^2b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3ab^3\left(a+bx^3\right)}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] (f*x^3)/(3*b^2) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a*b^3*(a
 + b*x^3)) + (c*Log[x])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a
+ b*x^3])/(3*a^2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^3} f\,dx}{3b^2}-\frac{a^3f-a^2be+ab^2d-b^3c}{3ab^3\left(a+bx^3\right)}+\frac{c\log\left(x^3\right)}{3a^2}-\frac{\left(2a^3f-a^2be+b^3c\right)\log\left(a+bx^3\right)}{3a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Integral(f, (x, x**3))/(3*b**2) - (a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)/(3*a*b**3*(a + b*x**3)) + c*log(x**3)/(3*a**2) - (2*a**3
*f - a**2*b*e + b**3*c)*log(a + b*x**3)/(3*a**2*b**3)
```

Mathematica [A] time = 0.228831, size = 95, normalized size = 0.95

$$\frac{\frac{\log\left(a+bx^3\right)\left(-2a^3f+a^2be-b^3c\right)+\frac{a\left(a^3(-f)+a^2b\left(e+fx^3\right)+ab^2\left(fx^6-d\right)+b^3c\right)}{a+bx^3}}{b^3}+3c\log(x)}{3a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] (3*c*Log[x] + ((a*(b^3*c - a^3*f + a^2*b*(e + f*x^3) + a*b^2*(-d
+ f*x^6)))/(a + b*x^3) + (-(b^3*c) + a^2*b*e - 2*a^3*f)*Log[a + b
*x^3])/b^3)/(3*a^2)
```

Maple [A] time = 0.023, size = 125, normalized size = 1.3

$$\frac{fx^{3}}{3\,b^{2}}+\frac{c\ln\left(x\right)}{a^{2}}-\frac{2\,a\ln\left(bx^{3}+a\right)f}{3\,b^{3}}+\frac{\ln\left(bx^{3}+a\right)e}{3\,b^{2}}-\frac{c\ln\left(bx^{3}+a\right)}{3\,a^{2}}-\frac{a^{2}f}{3\,b^{3}\left(bx^{3}+a\right)}+\frac{ae}{3\,b^{2}\left(bx^{3}+a\right)}-\frac{d}{3\,b\left(bx^{3}+a\right)}+\frac{c}{3\,a\left(bx^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x)
```

```
[Out] 1/3*f*x^3/b^2+c*ln(x)/a^2-2/3*a/b^3*ln(b*x^3+a)*f+1/3/b^2*ln(b*x^
3+a)*e-1/3*c*ln(b*x^3+a)/a^2-1/3*a^2/b^3/(b*x^3+a)*f+1/3*a/b^2/(b
*x^3+a)*e-1/3/b/(b*x^3+a)*d+1/3/a/(b*x^3+a)*c
```

Maxima [A] time = 1.38728, size = 135, normalized size = 1.35

$$\frac{fx^{3}}{3\,b^{2}}+\frac{b^{3}c-ab^{2}d+a^{2}be-a^{3}f}{3\,\left(ab^{4}x^{3}+a^{2}b^{3}\right)}+\frac{c\log\left(x^{3}\right)}{3\,a^{2}}-\frac{\left(b^{3}c-a^{2}be+2\,a^{3}f\right)\log\left(bx^{3}+a\right)}{3\,a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x),x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a*b^4*x^
3 + a^2*b^3) + 1/3*c*log(x^3)/a^2 - 1/3*(b^3*c - a^2*b*e + 2*a^3*
f)*log(b*x^3 + a)/(a^2*b^3)
```

Fricas [A] time = 0.243147, size = 196, normalized size = 1.96

$$\frac{a^{2}b^{2}fx^{6}+a^{3}bfx^{3}+ab^{3}c-a^{2}b^{2}d+a^{3}be-a^{4}f-\left(ab^{3}c-a^{3}be+2\,a^{4}f+\left(b^{4}c-a^{2}b^{2}e+2\,a^{3}bf\right)x^{3}\right)\log\left(bx^{3}+a\right)+3\,\left(b^{4}c\right.}{3\,\left(a^{2}b^{4}x^{3}+a^{3}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*b^2*f*x^6 + a^3*b*f*x^3 + a*b^3*c - a^2*b^2*d + a^3*b*e
- a^4*f - (a*b^3*c - a^3*b*e + 2*a^4*f + (b^4*c - a^2*b^2*e + 2*a
^3*b*f)*x^3)*log(b*x^3 + a) + 3*(b^4*c*x^3 + a*b^3*c)*log(x))/(a^
2*b^4*x^3 + a^3*b^3)
```

Sympy [A] time = 54.8426, size = 95, normalized size = 0.95

$$-\frac{a^{3}f-a^{2}be+ab^{2}d-b^{3}c}{3a^{2}b^{3}+3ab^{4}x^{3}}+\frac{fx^{3}}{3b^{2}}+\frac{c\log\left(x\right)}{a^{2}}-\frac{\left(2a^{3}f-a^{2}be+b^{3}c\right)\log\left(\frac{a}{b}+x^{3}\right)}{3a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**2,x)
```

```
[Out] -(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**2*b**3 + 3*a*b**4*
x**3) + f*x**3/(3*b**2) + c*log(x)/a**2 - (2*a**3*f - a**2*b*e +
b**3*c)*log(a/b + x**3)/(3*a**2*b**3)
```

GIAC/XCAS [A] time = 0.213549, size = 169, normalized size = 1.69

$$\frac{fx^3}{3\,b^2} + \frac{c\ln\left(|x|\right)}{a^2} - \frac{\left(b^3c + 2\,a^3f - a^2be\right)\ln\left(\left|bx^3 + a\right|\right)}{3\,a^2b^3} + \frac{b^4cx^3 + 2\,a^3bfx^3 - a^2b^2x^3e + 2\,ab^3c - a^2b^2d + a^4f}{3\,(bx^3 + a)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x),x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b^2 + c*ln(abs(x))/a^2 - 1/3*(b^3*c + 2*a^3*f - a^2*b*e
)*ln(abs(b*x^3 + a))/(a^2*b^3) + 1/3*(b^4*c*x^3 + 2*a^3*b*f*x^3 -
 a^2*b^2*x^3*e + 2*a*b^3*c - a^2*b^2*d + a^4*f)/((b*x^3 + a)*a^2*
b^3)
```

3.256 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^2}\,dx$

Optimal. Leaf size=109

$$\frac{\log(a+bx^3)\left(a^3f-ab^2d+2b^3c\right)}{3a^3b^2}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{3a^2x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^2b^2(a+bx^3)}$$

```
[Out] -c/(3*a^2*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^2*b^2*(
a + b*x^3)) - ((2*b*c - a*d)*Log[x])/a^3 + ((2*b^3*c - a*b^2*d +
a^3*f)*Log[a + b*x^3])/(3*a^3*b^2)
```

Rubi [A] time = 0.277588, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{\log(a+bx^3)\left(a^3f-ab^2d+2b^3c\right)}{3a^3b^2}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{3a^2x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^2b^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -c/(3*a^2*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^2*b^2*(
a + b*x^3)) - ((2*b*c - a*d)*Log[x])/a^3 + ((2*b^3*c - a*b^2*d +
a^3*f)*Log[a + b*x^3])/(3*a^3*b^2)
```

Rubi in Sympy [A] time = 48.4794, size = 102, normalized size = 0.94

$$-\frac{c}{3a^2x^3}+\frac{a^3f-a^2be+ab^2d-b^3c}{3a^2b^2(a+bx^3)}+\frac{(ad-2bc)\log(x^3)}{3a^3}+\frac{\left(a^3f-ab^2d+2b^3c\right)\log(a+bx^3)}{3a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] -c/(3*a**2*x**3) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**
2*b**2*(a + b*x**3)) + (a*d - 2*b*c)*log(x**3)/(3*a**3) + (a**3*f
 - a*b**2*d + 2*b**3*c)*log(a + b*x**3)/(3*a**3*b**2)
```

Mathematica [A] time = 0.136676, size = 97, normalized size = 0.89

$$\frac{\frac{\log(a+bx^3)\left(a^3f-ab^2d+2b^3c\right)}{b^2}+\frac{a\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^2(a+bx^3)}+3\log(x)(ad-2bc)-\frac{ac}{x^3}}{3a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*c)/x^3) + (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(
a + b*x^3)) + 3*(-2*b*c + a*d)*Log[x] + ((2*b^3*c - a*b^2*d + a^3
*f)*Log[a + b*x^3])/b^2)/(3*a^3)
```

Maple [A] time = 0.021, size = 132, normalized size = 1.2

$$-\frac{c}{3\,a^2x^3}+\frac{d\ln(x)}{a^2}-2\,\frac{bc\ln(x)}{a^3}+\frac{f\ln\left(bx^3+a\right)}{3\,b^2}-\frac{d\ln\left(bx^3+a\right)}{3\,a^2}+\frac{2\,bc\ln\left(bx^3+a\right)}{3\,a^3}+\frac{af}{3\,b^2\,(bx^3+a)}-\frac{e}{3\,b\,(bx^3+a)}+\frac{d}{3\,a\,(bx^3+a)}-\frac{bc}{3\,a^2\,(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/3*c/a^2/x^3+d*ln(x)/a^2-2*b*c*ln(x)/a^3+1/3*f*ln(b*x^3+a)/b^2-
1/3*d*ln(b*x^3+a)/a^2+2/3*b*c*ln(b*x^3+a)/a^3+1/3*a/b^2/(b*x^3+a)
*f-1/3/b/(b*x^3+a)*e+1/3/a/(b*x^3+a)*d-1/3/a^2*b/(b*x^3+a)*c
```

Maxima [A] time = 1.38097, size = 157, normalized size = 1.44

$$-\frac{ab^2c+\left(2\,b^3c-ab^2d+a^2be-a^3f\right)x^3}{3\,(a^2b^3x^6+a^3b^2x^3)}-\frac{(2\,bc-ad)\log\left(x^3\right)}{3\,a^3}+\frac{\left(2\,b^3c-ab^2d+a^3f\right)\log\left(bx^3+a\right)}{3\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] -1/3*(a*b^2*c + (2*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(a^2*b
^3*x^6 + a^3*b^2*x^3) - 1/3*(2*b*c - a*d)*log(x^3)/a^3 + 1/3*(2*b
^3*c - a*b^2*d + a^3*f)*log(b*x^3 + a)/(a^3*b^2)
```

Fricas [A] time = 0.232838, size = 232, normalized size = 2.13

$$-\frac{a^2b^2c+\left(2\,ab^3c-a^2b^2d+a^3be-a^4f\right)x^3-\left(\left(2\,b^4c-ab^3d+a^3bf\right)x^6+\left(2\,ab^3c-a^2b^2d+a^4f\right)x^3\right)\log\left(bx^3+a\right)+3\left(\left(2\,b\right.\right.}{3\,(a^3b^3x^6+a^4b^2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*b^2*c + (2*a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3 -
 ((2*b^4*c - a*b^3*d + a^3*b*f)*x^6 + (2*a*b^3*c - a^2*b^2*d + a^
4*f)*x^3)*log(b*x^3 + a) + 3*((2*b^4*c - a*b^3*d)*x^6 + (2*a*b^3*
c - a^2*b^2*d)*x^3)*log(x))/(a^3*b^3*x^6 + a^4*b^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.215592, size = 177, normalized size = 1.62

$$-\frac{(2\,bc-ad)\ln\left(|x|\right)}{a^{3}}+\frac{\left(2\,b^{3}c-ab^{2}d+a^{3}f\right)\ln\left(\left|bx^{3}+a\right|\right)}{3\,a^{3}b^{2}}-\frac{a^{2}bfx^{6}+4\,b^{3}cx^{3}-2\,ab^{2}dx^{3}-a^{3}fx^{3}+2\,a^{2}bx^{3}e+2\,ab^{2}c}{6\,(bx^{6}+ax^{3})a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] -(2*b*c - a*d)*ln(abs(x))/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3*f)*l
n(abs(b*x^3 + a))/(a^3*b^2) - 1/6*(a^2*b*f*x^6 + 4*b^3*c*x^3 - 2*
a*b^2*d*x^3 - a^3*f*x^3 + 2*a^2*b*x^3*e + 2*a*b^2*c)/((b*x^6 + a*
x^3)*a^2*b^2)
```

3.257 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2}\,dx$

Optimal. Leaf size=130

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{6a^2x^6}-\frac{\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)}{3a^4}+\frac{\log(x)\left(a^2e-2abd+3b^2c\right)}{a^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^3b\left(a+bx^3\right)}$$

```
[Out] -c/(6*a^2*x^6) + (2*b*c - a*d)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a
^2*b*e - a^3*f)/(3*a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2
*e)*Log[x])/a^4 - ((3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(3
*a^4)
```

Rubi [A] time = 0.315857, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{6a^2x^6}-\frac{\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)}{3a^4}+\frac{\log(x)\left(a^2e-2abd+3b^2c\right)}{a^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -c/(6*a^2*x^6) + (2*b*c - a*d)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a
^2*b*e - a^3*f)/(3*a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2
*e)*Log[x])/a^4 - ((3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(3
*a^4)
```

Rubi in Sympy [A] time = 51.0906, size = 124, normalized size = 0.95

$$-\frac{c}{6a^2x^6}-\frac{ad-2bc}{3a^3x^3}-\frac{a^3f-a^2be+ab^2d-b^3c}{3a^3b\left(a+bx^3\right)}+\frac{\left(a^2e-2abd+3b^2c\right)\log\left(x^3\right)}{3a^4}-\frac{\left(a^2e-2abd+3b^2c\right)\log\left(a+bx^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**2,x)
```

```
[Out] -c/(6*a**2*x**6) - (a*d - 2*b*c)/(3*a**3*x**3) - (a**3*f - a**2*b
*e + a*b**2*d - b**3*c)/(3*a**3*b*(a + b*x**3)) + (a**2*e - 2*a*b
*d + 3*b**2*c)*log(x**3)/(3*a**4) - (a**2*e - 2*a*b*d + 3*b**2*c)
*log(a + b*x**3)/(3*a**4)
```

Mathematica [A] time = 0.218767, size = 118, normalized size = 0.91

$$-\frac{2\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)-6\log(x)\left(a^2e-2abd+3b^2c\right)+\frac{a^2c}{x^6}+\frac{2a\left(a^3f-a^2be+ab^2d-b^3c\right)}{b\left(a+bx^3\right)}+\frac{2a(ad-2bc)}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -((a^2*c)/x^6 + (2*a*(-2*b*c + a*d))/x^3 + (2*a*(-(b^3*c) + a*b^2
*d - a^2*b*e + a^3*f))/(b*(a + b*x^3)) - 6*(3*b^2*c - 2*a*b*d + a
^2*e)*Log[x] + 2*(3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(6*a
^4)
```

Maple [A] time = 0.023, size = 167, normalized size = 1.3

$$-\frac{c}{6\,a^2x^6}-\frac{d}{3\,a^2x^3}+\frac{2\,bc}{3\,a^3x^3}+\frac{e\ln(x)}{a^2}-2\,\frac{\ln(x)\,bd}{a^3}+3\,\frac{\ln(x)\,b^2c}{a^4}-\frac{e\ln\left(bx^3+a\right)}{3\,a^2}+\frac{2\,\ln\left(bx^3+a\right)bd}{3\,a^3}-\frac{\ln\left(bx^3+a\right)b^2c}{a^4}-\frac{f}{3\,b\,(bx^3+a)}+\frac{e}{3\,a\,(bx^3+a)}-\frac{bd}{3\,a^2\,(bx^3+a)}+\frac{b^2c}{3\,a^3\,(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x)
```

```
[Out] -1/6*c/a^2/x^6-1/3/a^2/x^3*d+2/3/a^3/x^3*b*c+e*ln(x)/a^2-2/a^3*ln
(x)*b*d+3/a^4*ln(x)*b^2*c-1/3*e*ln(b*x^3+a)/a^2+2/3/a^3*ln(b*x^3+
a)*b*d-1/a^4*ln(b*x^3+a)*b^2*c-1/3/b/(b*x^3+a)*f+1/3/a/(b*x^3+a)*
e-1/3/a^2*b/(b*x^3+a)*d+1/3/a^3*b^2/(b*x^3+a)*c
```

Maxima [A] time = 1.3754, size = 186, normalized size = 1.43

$$\frac{2\left(3\,b^3c-2\,ab^2d+a^2be-a^3f\right)x^6-a^2bc+\left(3\,ab^2c-2\,a^2bd\right)x^3}{6\left(a^3b^2x^9+a^4bx^6\right)}-\frac{\left(3\,b^2c-2\,abd+a^2e\right)\log\left(bx^3+a\right)}{3\,a^4}+\frac{\left(3\,b^2c-2\,abd+a^2e\right)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^7),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*b^3*c - 2*a*b^2*d + a^2*b*e - a^3*f)*x^6 - a^2*b*c + (3
*a*b^2*c - 2*a^2*b*d)*x^3)/(a^3*b^2*x^9 + a^4*b*x^6) - 1/3*(3*b^2
*c - 2*a*b*d + a^2*e)*log(b*x^3 + a)/a^4 + 1/3*(3*b^2*c - 2*a*b*d
 + a^2*e)*log(x^3)/a^4
```

Fricas [A] time = 0.228617, size = 281, normalized size = 2.16

$$\frac{2\left(3\,ab^3c-2\,a^2b^2d+a^3be-a^4f\right)x^6-a^3bc+\left(3\,a^2b^2c-2\,a^3bd\right)x^3-2\left(\left(3\,b^4c-2\,ab^3d+a^2b^2e\right)x^9+\left(3\,ab^3c-2\,a^2b^2d+a\right.\right.}{6\left(a^4b^2x^9+a^5bx^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^7),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e - a^4*f)*x^6 - a^3*b*c
+ (3*a^2*b^2*c - 2*a^3*b*d)*x^3 - 2*((3*b^4*c - 2*a*b^3*d + a^2*b
^2*e)*x^9 + (3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(b*x^3 +
a) + 6*((3*b^4*c - 2*a*b^3*d + a^2*b^2*e)*x^9 + (3*a*b^3*c - 2*a^
2*b^2*d + a^3*b*e)*x^6)*log(x))/(a^4*b^2*x^9 + a^5*b*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.214631, size = 271, normalized size = 2.08

$$\frac{\left(3\,b^2c-2\,abd+a^2e\right)\ln\left(|x|\right)}{a^4}-\frac{\left(3\,b^3c-2\,ab^2d+a^2be\right)\ln\left(\left|bx^3+a\right|\right)}{3\,a^4b}+\frac{3\,b^4cx^3-2\,ab^3dx^3+a^2b^2x^3e+4\,ab^3c-3\,a^2b^2d-a^4f+2\,a^3be}{3\,(bx^3+a)a^4b}-\frac{9\,b^2cx^6-6\,abdx^6+3\,a^2x^6e-4\,abcx^3+2\,a^2dx^3+a^2c}{6\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^7),x, algorithm="giac")
```

```
[Out] (3*b^2*c - 2*a*b*d + a^2*e)*ln(abs(x))/a^4 - 1/3*(3*b^3*c - 2*a*b
^2*d + a^2*b*e)*ln(abs(b*x^3 + a))/(a^4*b) + 1/3*(3*b^4*c*x^3 - 2
*a*b^3*d*x^3 + a^2*b^2*x^3*e + 4*a*b^3*c - 3*a^2*b^2*d - a^4*f +
2*a^3*b*e)/((b*x^3 + a)*a^4*b) - 1/6*(9*b^2*c*x^6 - 6*a*b*d*x^6 +
 3*a^2*x^6*e - 4*a*b*c*x^3 + 2*a^2*d*x^3 + a^2*c)/(a^4*x^6)
```

3.258 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2} dx$

Optimal. Leaf size=175

$$\frac{2bc-ad}{6a^3x^6}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{3a^5} -\frac{\log(x)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{a^5}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4\left(a+bx^3\right)}$$

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(6*a^3*x^6) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(3*a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*
(a + b*x^3)) - ((4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[x])
/a^5 + ((4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[a + b*x^3])
/(3*a^5)
```

Rubi [A] time = 0.426039, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{2bc-ad}{6a^3x^6}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{3a^5} -\frac{\log(x)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{a^5}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(6*a^3*x^6) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(3*a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*
(a + b*x^3)) - ((4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[x])
/a^5 + ((4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[a + b*x^3])
/(3*a^5)
```

Rubi in Sympy [A] time = 61.8512, size = 170, normalized size = 0.97

$$-\frac{c}{9a^2x^9}-\frac{ad-2bc}{6a^3x^6}+\frac{a^3f-a^2be+ab^2d-b^3c}{3a^4\left(a+bx^3\right)}-\frac{a^2e-2abd+3b^2c}{3a^4x^3} +\frac{\left(a^3f-2a^2be+3ab^2d-4b^3c\right)\log\left(x^3\right)}{3a^5}-\frac{\left(a^3f-2a^2be+3ab^2d-4b^3c\right)\log\left(a+bx^3\right)}{3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**2,x)
```

```
[Out] -c/(9*a**2*x**9) - (a*d - 2*b*c)/(6*a**3*x**6) + (a**3*f - a**2*b
*e + a*b**2*d - b**3*c)/(3*a**4*(a + b*x**3)) - (a**2*e - 2*a*b*d
 + 3*b**2*c)/(3*a**4*x**3) + (a**3*f - 2*a**2*b*e + 3*a*b**2*d -
4*b**3*c)*log(x**3)/(3*a**5) - (a**3*f - 2*a**2*b*e + 3*a*b**2*d
- 4*b**3*c)*log(a + b*x**3)/(3*a**5)
```

Mathematica [A] time = 0.197316, size = 160, normalized size = 0.91

$$\frac{-\frac{2a^3c}{x^9}-\frac{6a\left(a^2e-2abd+3b^2c\right)}{x^3}-\frac{3a^2(ad-2bc)}{x^6}+\frac{6a\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+6\log\left(a+bx^3\right)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)+18\log}{18a^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-2*b*c + a*d))/x^6 - (6*a*(3*b^2*c - 2*
a*b*d + a^2*e))/x^3 + (6*a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
)/(a + b*x^3) + 18*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f)*Log
[x] + 6*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[a + b*x^3])
/(18*a^5)
```

Maple [A] time = 0.034, size = 229, normalized size = 1.3

$$-\frac{c}{9\,a^{2}x^{9}}-\frac{d}{6\,a^{2}x^{6}}+\frac{bc}{3\,a^{3}x^{6}}-\frac{e}{3\,a^{2}x^{3}}+\frac{2\,bd}{3\,a^{3}x^{3}}-\frac{b^{2}c}{a^{4}x^{3}}+\frac{\ln (x)\,f}{a^{2}}-2\,\frac{\ln (x)\,be}{a^{3}}+3\,\frac{\ln (x)\,b^{2}d}{a^{4}}-4\,\frac{\ln (x)\,b^{3}c}{a^{5}}-\frac{\ln \left(bx^{3}+a\right)f}{3\,a^{2}}+\frac{2\,b\ln \left(bx^{3}+a\right)e}{3\,a^{3}}-\frac{b^{2}\ln \left(bx^{3}+a\right)d}{a^{4}}+\frac{4\,b^{3}\ln \left(bx^{3}+a\right)c}{3\,a^{5}}+\frac{f}{3\,a\left(bx^{3}+a\right)}-\frac{be}{3\,a^{2}\left(bx^{3}+a\right)}+\frac{b^{2}d}{3\,a^{3}\left(bx^{3}+a\right)}-\frac{b^{3}c}{3\,a^{4}\left(bx^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x)
```

```
[Out] -1/9*c/a^2/x^9-1/6/a^2/x^6*d+1/3/a^3/x^6*b*c-1/3/a^2/x^3*e+2/3/a^
3/x^3*b*d-1/a^4/x^3*b^2*c+1/a^2*ln(x)*f-2/a^3*ln(x)*b*e+3/a^4*ln(
x)*b^2*d-4/a^5*ln(x)*b^3*c-1/3/a^2*ln(b*x^3+a)*f+2/3*b/a^3*ln(b*x
^3+a)*e-b^2/a^4*ln(b*x^3+a)*d+4/3*b^3/a^5*ln(b*x^3+a)*c+1/3/a/(b*
x^3+a)*f-1/3*b/a^2/(b*x^3+a)*e+1/3*b^2/a^3/(b*x^3+a)*d-1/3*b^3/a^
4/(b*x^3+a)*c
```

Maxima [A] time = 1.38733, size = 244, normalized size = 1.39

$$-\frac{6\left(4\,b^{3}c-3\,ab^{2}d+2\,a^{2}be-a^{3}f\right)x^{9}+3\left(4\,ab^{2}c-3\,a^{2}bd+2\,a^{3}e\right)x^{6}+2\,a^{3}c-\left(4\,a^{2}bc-3\,a^{3}d\right)x^{3}}{18\left(a^{4}bx^{12}+a^{5}x^{9}\right)}+\frac{\left(4\,b^{3}c-3\,ab^{2}d+2\,a^{2}be-a^{3}f\right)\log \left(bx^{3}+a\right)}{3\,a^{5}}-\frac{\left(4\,b^{3}c-3\,ab^{2}d+2\,a^{2}be-a^{3}f\right)\log \left(x^{3}\right)}{3\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^10),x, algorithm="maxima")
```

```
[Out] -1/18*(6*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*x^9 + 3*(4*a*b
^2*c - 3*a^2*b*d + 2*a^3*e)*x^6 + 2*a^3*c - (4*a^2*b*c - 3*a^3*d)
*x^3)/(a^4*b*x^12 + a^5*x^9) + 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b
*e - a^3*f)*log(b*x^3 + a)/a^5 - 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2
*b*e - a^3*f)*log(x^3)/a^5
```

Fricas [A] time = 0.243035, size = 352, normalized size = 2.01

$$-\frac{6\left(4\,ab^{3}c-3\,a^{2}b^{2}d+2\,a^{3}be-a^{4}f\right)x^{9}+3\left(4\,a^{2}b^{2}c-3\,a^{3}bd+2\,a^{4}e\right)x^{6}+2\,a^{4}c-\left(4\,a^{3}bc-3\,a^{4}d\right)x^{3}-6\left(\left(4\,b^{4}c-3\,ab^{3}d+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^10),x, algorithm="fricas")
```

```
[Out] -1/18*(6*(4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9 + 3*(4
*a^2*b^2*c - 3*a^3*b*d + 2*a^4*e)*x^6 + 2*a^4*c - (4*a^3*b*c - 3*
a^4*d)*x^3 - 6*((4*b^4*c - 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^1
2 + (4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9)*log(b*x^3
```

```
+ a) + 18*((4*b^4*c - 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^12 + (
4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9)*log(x))/(a^5*b*
x^12 + a^6*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.215789, size = 371, normalized size = 2.12

$$-\frac{\left(4\,b^3c-3\,ab^2d-a^3f+2\,a^2be\right)\ln\left(|x|\right)}{a^5}+\frac{\left(4\,b^4c-3\,ab^3d-a^3bf+2\,a^2b^2e\right)\ln\left(\left|bx^3+a\right|\right)}{3\,a^5b}-\frac{4\,b^4cx^3-3\,ab^3dx^3-a^3bfx^3+2\,a^2b^2x^3e+5\,ab^3c-4\,a^2b^2d-2\,a^4f+3\,a^3be}{3\,(bx^3+a)a^5}+\frac{44\,b^3cx^9-33\,ab^2dx^9-11\,a^3fx^9+22\,a^2bx^9e-18\,ab^2cx^6+12\,a^2bdx^6-6\,a^3x^6e+6\,a^2bcx^3-3\,a^3dx^3-2\,a^3c}{18\,a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^10),x, algorithm="giac")
```

```
[Out] -(4*b^3*c - 3*a*b^2*d - a^3*f + 2*a^2*b*e)*ln(abs(x))/a^5 + 1/3*(
4*b^4*c - 3*a*b^3*d - a^3*b*f + 2*a^2*b^2*e)*ln(abs(b*x^3 + a))/(
a^5*b) - 1/3*(4*b^4*c*x^3 - 3*a*b^3*d*x^3 - a^3*b*f*x^3 + 2*a^2*b
^2*x^3*e + 5*a*b^3*c - 4*a^2*b^2*d - 2*a^4*f + 3*a^3*b*e)/((b*x^3
 + a)*a^5) + 1/18*(44*b^3*c*x^9 - 33*a*b^2*d*x^9 - 11*a^3*f*x^9 +
 22*a^2*b*x^9*e - 18*a*b^2*c*x^6 + 12*a^2*b*d*x^6 - 6*a^3*x^6*e +
 6*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^5*x^9)
```

3.259 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx$

Optimal. Leaf size=214

$$\frac{2bc-ad}{9a^3x^9}-\frac{c}{12a^2x^{12}}-\frac{a^2e-2abd+3b^2c}{6a^4x^6}-\frac{b\log\left(a+bx^3\right)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{3a^6}+\frac{b\log(x)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{a^6}+\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

```
[Out] -c/(12*a^2*x^12) + (2*b*c - a*d)/(9*a^3*x^9) - (3*b^2*c - 2*a*b*d
 + a^2*e)/(6*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)
/(3*a^5*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*(a
+ b*x^3)) + (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x]
)/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[a + b*
x^3])/(3*a^6)
```

Rubi [A] time = 0.529954, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{2bc-ad}{9a^3x^9}-\frac{c}{12a^2x^{12}}-\frac{a^2e-2abd+3b^2c}{6a^4x^6}-\frac{b\log\left(a+bx^3\right)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{3a^6}+\frac{b\log(x)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{a^6}+\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] -c/(12*a^2*x^12) + (2*b*c - a*d)/(9*a^3*x^9) - (3*b^2*c - 2*a*b*d
 + a^2*e)/(6*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)
/(3*a^5*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*(a
+ b*x^3)) + (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x]
)/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[a + b*
x^3])/(3*a^6)
```

Rubi in Sympy [A] time = 72.9148, size = 216, normalized size = 1.01

$$-\frac{c}{12a^2x^{12}}-\frac{ad-2bc}{9a^3x^9}-\frac{a^2e-2abd+3b^2c}{6a^4x^6}-\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^5\left(a+bx^3\right)}-\frac{a^3f-2a^2be+3ab^2d-4b^3c}{3a^5x^3}-\frac{b\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)\log\left(x^3\right)}{3a^6}+\frac{b\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)\log\left(a+bx^3\right)}{3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**2,x)
```

```
[Out] -c/(12*a**2*x**12) - (a*d - 2*b*c)/(9*a**3*x**9) - (a**2*e - 2*a*
b*d + 3*b**2*c)/(6*a**4*x**6) - b*(a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)/(3*a**5*(a + b*x**3)) - (a**3*f - 2*a**2*b*e + 3*a*b**2*
d - 4*b**3*c)/(3*a**5*x**3) - b*(2*a**3*f - 3*a**2*b*e + 4*a*b**2
*d - 5*b**3*c)*log(x**3)/(3*a**6) + b*(2*a**3*f - 3*a**2*b*e + 4*
a*b**2*d - 5*b**3*c)*log(a + b*x**3)/(3*a**6)
```

Mathematica [A] time = 0.461654, size = 198, normalized size = 0.93

$$-\frac{\frac{3a^4c}{x^{12}}+\frac{4a^3(ad-2bc)}{x^9}+\frac{6a^2(a^2e-2abd+3b^2c)}{x^6}+\frac{12ab(a^3f-a^2be+ab^2d-b^3c)}{a+bx^3}+\frac{12a(a^3f-2a^2be+3ab^2d-4b^3c)}{x^3}+12b\log\left(a+bx^3\right)\left(-2a^3f+\right.}{36a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] -((3*a^4*c)/x^12 + (4*a^3*(-2*b*c + a*d))/x^9 + (6*a^2*(3*b^2*c -
 2*a*b*d + a^2*e))/x^6 + (12*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e
+ a^3*f))/x^3 + (12*a*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(
a + b*x^3) - 36*b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log
[x] + 12*b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[a + b*
x^3])/(36*a^6)
```

Maple [A] time = 0.029, size = 282, normalized size = 1.3

$$-\frac{c}{12\,a^2x^{12}}-\frac{d}{9\,a^2x^9}+\frac{2\,bc}{9\,a^3x^9}-\frac{e}{6\,a^2x^6}+\frac{bd}{3\,a^3x^6}-\frac{b^2c}{2\,a^4x^6}-\frac{f}{3\,a^2x^3}+\frac{2\,be}{3\,a^3x^3}-\frac{b^2d}{a^4x^3}+\frac{4\,b^3c}{3\,a^5x^3}-2\,\frac{b\ln(x)\,f}{a^3}+3\,\frac{b^2\ln(x)\,e}{a^4}-4\,\frac{b^3\ln(x)\,d}{a^5}+5\,\frac{b^4\ln(x)\,c}{a^6}+\frac{2\,b\ln\left(bx^3+a\right)\,f}{3\,a^3}-\frac{b^2\ln\left(bx^3+a\right)\,e}{a^4}+\frac{4\,b^3\ln\left(bx^3+a\right)\,d}{3\,a^5}-\frac{5\,b^4\ln\left(bx^3+a\right)\,c}{3\,a^6}-\frac{fb}{3\,a^2\left(bx^3+a\right)}+\frac{eb^2}{3\,a^3\left(bx^3+a\right)}-\frac{b^3d}{3\,a^4\left(bx^3+a\right)}+\frac{b^4c}{3\,a^5\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x)
```

```
[Out] -1/12*c/a^2/x^12-1/9/a^2/x^9*d+2/9/a^3/x^9*b*c-1/6/a^2/x^6*e+1/3/
a^3/x^6*b*d-1/2/a^4/x^6*b^2*c-1/3/a^2/x^3*f+2/3/a^3/x^3*b*e-1/a^4
/x^3*b^2*d+4/3/a^5/x^3*b^3*c-2*b/a^3*ln(x)*f+3*b^2/a^4*ln(x)*e-4*
b^3/a^5*ln(x)*d+5*b^4/a^6*ln(x)*c+2/3*b/a^3*ln(b*x^3+a)*f-b^2/a^4
*ln(b*x^3+a)*e+4/3*b^3/a^5*ln(b*x^3+a)*d-5/3*b^4/a^6*ln(b*x^3+a)*
c-1/3*b/a^2/(b*x^3+a)*f+1/3*b^2/a^3/(b*x^3+a)*e-1/3*b^3/a^4/(b*x^
3+a)*d+1/3*b^4/a^5/(b*x^3+a)*c
```

Maxima [A] time = 1.39506, size = 305, normalized size = 1.43

$$\frac{12\left(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf\right)x^{12}+6\left(5\,ab^3c-4\,a^2b^2d+3\,a^3be-2\,a^4f\right)x^9-2\left(5\,a^2b^2c-4\,a^3bd+3\,a^4e\right)x^6-3\,a^4}{36\left(a^5bx^{15}+a^6x^{12}\right)}-\frac{\left(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf\right)\log\left(bx^3+a\right)}{3\,a^6}+\frac{\left(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf\right)\log\left(x^3\right)}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^13),x, algorithm="maxima")
```

```
[Out] 1/36*(12*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*x^12 + 6
*(5*a*b^3*c - 4*a^2*b^2*d + 3*a^3*b*e - 2*a^4*f)*x^9 - 2*(5*a^2*b
^2*c - 4*a^3*b*d + 3*a^4*e)*x^6 - 3*a^4*c + (5*a^3*b*c - 4*a^4*d)
*x^3)/(a^5*b*x^15 + a^6*x^12) - 1/3*(5*b^4*c - 4*a*b^3*d + 3*a^2*
b^2*e - 2*a^3*b*f)*log(b*x^3 + a)/a^6 + 1/3*(5*b^4*c - 4*a*b^3*d
+ 3*a^2*b^2*e - 2*a^3*b*f)*log(x^3)/a^6
```

Fricas [A] time = 0.277506, size = 419, normalized size = 1.96

$$\frac{12\left(5\,ab^{4}c-4\,a^{2}b^{3}d+3\,a^{3}b^{2}e-2\,a^{4}bf\right)x^{12}+6\left(5\,a^{2}b^{3}c-4\,a^{3}b^{2}d+3\,a^{4}be-2\,a^{5}f\right)x^{9}-2\left(5\,a^{3}b^{2}c-4\,a^{4}bd+3\,a^{5}e\right)x^{6}-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^13),x, algorithm="fricas")
```

```
[Out] 1/36*(12*(5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^4*b*f)*x^12
 + 6*(5*a^2*b^3*c - 4*a^3*b^2*d + 3*a^4*b*e - 2*a^5*f)*x^9 - 2*(5
*a^3*b^2*c - 4*a^4*b*d + 3*a^5*e)*x^6 - 3*a^5*c + (5*a^4*b*c - 4*
a^5*d)*x^3 - 12*((5*b^5*c - 4*a*b^4*d + 3*a^2*b^3*e - 2*a^3*b^2*f
)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^4*b*f)*x^12
)*log(b*x^3 + a) + 36*((5*b^5*c - 4*a*b^4*d + 3*a^2*b^3*e - 2*a^3
*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^4*b*f
)*x^12)*log(x))/(a^6*b*x^15 + a^7*x^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.220128, size = 447, normalized size = 2.09

$$\frac{\left(5\,b^{4}c-4\,ab^{3}d-2\,a^{3}bf+3\,a^{2}b^{2}e\right)\ln\left(|x|\right)}{a^{6}}-\frac{\left(5\,b^{5}c-4\,ab^{4}d-2\,a^{3}b^{2}f+3\,a^{2}b^{3}e\right)\ln\left(\left|bx^{3}+a\right|\right)}{3\,a^{6}b}+\frac{5\,b^{5}cx^{3}-4\,ab^{4}dx^{3}-2\,a^{3}b^{2}fx^{3}+3\,a^{2}b^{3}x^{3}e+6\,ab^{4}c-5\,a^{2}b^{3}d-3\,a^{4}bf+4\,a^{3}b^{2}e}{3\,(bx^{3}+a)a^{6}}-\frac{125\,b^{4}cx^{12}-100\,ab^{3}dx^{12}-50\,a^{3}bfx^{12}+75\,a^{2}b^{2}x^{12}e-48\,ab^{3}cx^{9}+36\,a^{2}b^{2}dx^{9}+12\,a^{4}fx^{9}-24\,a^{3}bx^{9}e+18\,a^{2}b^{2}cx^{6}-12}{36\,a^{6}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^13),x, algorithm="giac")
```

```
[Out] (5*b^4*c - 4*a*b^3*d - 2*a^3*b*f + 3*a^2*b^2*e)*ln(abs(x))/a^6 -
1/3*(5*b^5*c - 4*a*b^4*d - 2*a^3*b^2*f + 3*a^2*b^3*e)*ln(abs(b*x^
3 + a))/(a^6*b) + 1/3*(5*b^5*c*x^3 - 4*a*b^4*d*x^3 - 2*a^3*b^2*f*
x^3 + 3*a^2*b^3*x^3*e + 6*a*b^4*c - 5*a^2*b^3*d - 3*a^4*b*f + 4*a
^3*b^2*e)/((b*x^3 + a)*a^6) - 1/36*(125*b^4*c*x^12 - 100*a*b^3*d*
x^12 - 50*a^3*b*f*x^12 + 75*a^2*b^2*x^12*e - 48*a*b^3*c*x^9 + 36*
a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 24*a^3*b*x^9*e + 18*a^2*b^2*c*x^6
- 12*a^3*b*d*x^6 + 6*a^4*x^6*e - 8*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*
a^4*c)/(a^6*x^12)
```

3.260 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=369

$$\frac{x^7\left(3a^2f-2abe+b^2d\right)}{7b^4}-\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{ax\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{b^6}+\frac{x^4\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{4b^5}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{18b^{19/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{9b^{19/3}}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{3\sqrt{3}b^{19/3}}+\frac{x^{10}(be-2af)}{10b^3}+\frac{fx^{13}}{13b^2}$$

```
[Out] -((a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x)/b^6) + ((b^3*
c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a
*b*e + 3*a^2*f)*x^7)/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f
*x^13)/(13*b^2) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*
b^6*(a + b*x^3)) - (a^(4/3)*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e -
16*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*S
qrt[3]*b^(19/3)) + (a^(4/3)*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e -
16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) - (a^(4/3)*(7*b^
3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(18*b^(19/3))
```

Rubi [A] time = 0.974218, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{x^7\left(3a^2f-2abe+b^2d\right)}{7b^4}-\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{ax\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{b^6}+\frac{x^4\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{4b^5}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{18b^{19/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{9b^{19/3}}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{3\sqrt{3}b^{19/3}}+\frac{x^{10}(be-2af)}{10b^3}+\frac{fx^{13}}{13b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -((a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x)/b^6) + ((b^3*
c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a
*b*e + 3*a^2*f)*x^7)/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f
*x^13)/(13*b^2) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*
b^6*(a + b*x^3)) - (a^(4/3)*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e -
16*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*S
qrt[3]*b^(19/3)) + (a^(4/3)*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e -
16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) - (a^(4/3)*(7*b^
3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(18*b^(19/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.879941, size = 364, normalized size = 0.99

$$\frac{x^7\left(3a^2f-2abe+b^2d\right)}{7b^4}+\frac{a^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^6\left(a+bx^3\right)}+\frac{ax\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)}{b^6}+\frac{x^4\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{4b^5}+\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(16a^3f-13a^2be+10ab^2d-7b^3c\right)}{18b^{19/3}}-\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(16a^3f-13a^2be+10ab^2d-7b^3c\right)}{9b^{19/3}}+\frac{a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(16a^3f-13a^2be+10ab^2d-7b^3c\right)}{3\sqrt{3}b^{19/3}}+\frac{x^{10}(be-2af)}{10b^3}+\frac{fx^{13}}{13b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c
- 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b
*e + 3*a^2*f)*x^7)/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x
^13)/(13*b^2) + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(3
*b^6*(a + b*x^3)) + (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^2*b*e
+ 16*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(3*Sqrt[
3]*b^(19/3)) - (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^2*b*e + 16*
a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) + (a^(4/3)*(-7*b^3*
c + 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(18*b^(19/3))
```

Maple [A] time = 0.015, size = 622, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/10/b^2*x^10*e+1/7/b^2*x^7*d+1/4/b^2*x^4*c-16/9*a^5/b^7*f/(a/b)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+13/9*a^4/b^
6*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1
0/9*a^3/b^5*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))+7/9*a^2/b^4*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))+1/13*f*x^13/b^2-13/18*a^4/b^6*e/(a/b)^(2/3)*ln(
x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-10/9*a^3/b^5*d/(a/b)^(2/3)*ln(x+(a
/b)^(1/3))+5/9*a^3/b^5*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(
2/3))+7/9*a^2/b^4*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-7/18*a^2/b^4*c/
(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*a^5/b^6*x/(b*x^
```

```
3+a)*f-1/3*a^4/b^5*x/(b*x^3+a)*e-1/5/b^3*x^10*a*f+8/9*a^5/b^7*f/(
a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+13/9*a^4/b^6*e/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))+1/3*a^3/b^4*x/(b*x^3+a)*d-1/3*a^2/b^3*x/
(b*x^3+a)*c-16/9*a^5/b^7*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+3/4/b^4*
x^4*a^2*e-1/2/b^3*x^4*a*d+5/b^6*a^4*f*x-4/b^5*a^3*e*x+3/b^4*a^2*d
*x-2/b^3*a*c*x+3/7/b^4*x^7*a^2*f-2/7/b^3*x^7*a*e-1/b^5*x^4*a^3*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^9/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.225067, size = 678, normalized size = 1.84

$$\frac{\sqrt{3}\left(910\,\sqrt{3}\left(7\,a^{2}b^{3}c-10\,a^{3}b^{2}d+13\,a^{4}be-16\,a^{5}f+\left(7\,ab^{4}c-10\,a^{2}b^{3}d+13\,a^{3}b^{2}e-16\,a^{4}bf\right)x^{3}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^9/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/49140*sqrt(3)*(910*sqrt(3)*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4
*b*e - 16*a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a
^4*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
) - 1820*sqrt(3)*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^
5*f + (7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)
*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) + 5460*(7*a^2*b^3*c - 10*a^3*
b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 13*a^
3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*x +
 sqrt(3)*(-a/b)^(1/3))/(-a/b)^(1/3)) + 3*sqrt(3)*(420*b^5*f*x^16
+ 42*(13*b^5*e - 16*a*b^4*f)*x^13 + 78*(10*b^5*d - 13*a*b^4*e + 1
6*a^2*b^3*f)*x^10 + 195*(7*b^5*c - 10*a*b^4*d + 13*a^2*b^3*e - 16
*a^3*b^2*f)*x^7 - 1365*(7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e -
 16*a^4*b*f)*x^4 - 1820*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e
- 16*a^5*f)*x))/(b^7*x^3 + a*b^6)
```

Sympy [A] time = 18.4096, size = 490, normalized size = 1.33

$$\frac{x\left(a^{5}f-a^{4}be+a^{3}b^{2}d-a^{2}b^{3}c\right)}{3ab^{6}+3b^{7}x^{3}}$$
$$+\operatorname{RootSum}\left(729t^{3}b^{19}+4096a^{13}f^{3}-9984a^{12}bef^{2}+7680a^{11}b^{2}df^{2}+8112a^{11}b^{2}e^{2}f-5376a^{10}b^{3}cf^{2}-12480a^{10}b^{3}def-219\right.$$
$$+\frac{fx^{13}}{13b^{2}}-\frac{x^{10}\left(2af-be\right)}{10b^{3}}+\frac{x^{7}\left(3a^{2}f-2abe+b^{2}d\right)}{7b^{4}}$$
$$-\frac{x^{4}\left(4a^{3}f-3a^{2}be+2ab^{2}d-b^{3}c\right)}{4b^{5}}+\frac{x\left(5a^{4}f-4a^{3}be+3a^{2}b^{2}d-2ab^{3}c\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 + 3*b
**7*x**3) + RootSum(729*_t**3*b**19 + 4096*a**13*f**3 - 9984*a**1
```

```
2*b*e*f**2 + 7680*a**11*b**2*d*f**2 + 8112*a**11*b**2*e**2*f - 53
76*a**10*b**3*c*f**2 - 12480*a**10*b**3*d*e*f - 2197*a**10*b**3*e
**3 + 8736*a**9*b**4*c*e*f + 4800*a**9*b**4*d**2*f + 5070*a**9*b*
*4*d*e**2 - 6720*a**8*b**5*c*d*f - 3549*a**8*b**5*c*e**2 - 3900*a
**8*b**5*d**2*e + 2352*a**7*b**6*c**2*f + 5460*a**7*b**6*c*d*e +
1000*a**7*b**6*d**3 - 1911*a**6*b**7*c**2*e - 2100*a**6*b**7*c*d*
*2 + 1470*a**5*b**8*c**2*d - 343*a**4*b**9*c**3, Lambda(_t, _t*lo
g(-9*_t*b**6/(16*a**4*f - 13*a**3*b*e + 10*a**2*b**2*d - 7*a*b**3
*c) + x))) + f*x**13/(13*b**2) - x**10*(2*a*f - b*e)/(10*b**3) +
x**7*(3*a**2*f - 2*a*b*e + b**2*d)/(7*b**4) - x**4*(4*a**3*f - 3*
a**2*b*e + 2*a*b**2*d - b**3*c)/(4*b**5) + x*(5*a**4*f - 4*a**3*b
*e + 3*a**2*b**2*d - 2*a*b**3*c)/b**6
```

GIAC/XCAS [A] time = 0.216323, size = 609, normalized size = 1.65

$$\frac{\sqrt{3}\left(7\left(-ab^2\right)^{\frac{1}{3}}ab^3c-10\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d-16\left(-ab^2\right)^{\frac{1}{3}}a^4f+13\left(-ab^2\right)^{\frac{1}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^7}-\frac{\left(7a^2b^3c-10a^3b^2d-16a^5f+13a^4be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab^6}+\frac{\left(7\left(-ab^2\right)^{\frac{1}{3}}ab^3c-10\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d-16\left(-ab^2\right)^{\frac{1}{3}}a^4f+13\left(-ab^2\right)^{\frac{1}{3}}a^3be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^7}-\frac{a^2b^3cx-a^3b^2dx-a^5fx+a^4bxe}{3\left(bx^3+a\right)b^6}+\frac{140b^{24}fx^{13}-364ab^{23}fx^{10}+182b^{24}x^{10}e+260b^{24}dx^7+780a^2b^{22}fx^7-520ab^{23}x^7e+455b^{24}cx^4-910ab^{23}dx^4-1820a}{1820b^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^9/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2
*d - 16*(-a*b^2)^(1/3)*a^4*f + 13*(-a*b^2)^(1/3)*a^3*b*e)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^7 - 1/9*(7*a^2*b
^3*c - 10*a^3*b^2*d - 16*a^5*f + 13*a^4*b*e)*(-a/b)^(1/3)*ln(abs(
x - (-a/b)^(1/3)))/(a*b^6) + 1/18*(7*(-a*b^2)^(1/3)*a*b^3*c - 10*
(-a*b^2)^(1/3)*a^2*b^2*d - 16*(-a*b^2)^(1/3)*a^4*f + 13*(-a*b^2)^
(1/3)*a^3*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 - 1/3*
(a^2*b^3*c*x - a^3*b^2*d*x - a^5*f*x + a^4*b*x*e)/((b*x^3 + a)*b^
6) + 1/1820*(140*b^24*f*x^13 - 364*a*b^23*f*x^10 + 182*b^24*x^10*
e + 260*b^24*d*x^7 + 780*a^2*b^22*f*x^7 - 520*a*b^23*x^7*e + 455*
b^24*c*x^4 - 910*a*b^23*d*x^4 - 1820*a^3*b^21*f*x^4 + 1365*a^2*b^
22*x^4*e - 3640*a*b^23*c*x + 5460*a^2*b^22*d*x + 9100*a^4*b^20*f*
x - 7280*a^3*b^21*x*e)/b^26
```

3.261 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=335

$$\frac{x^5(3a^2f-2abe+b^2d)}{5b^4}+\frac{x^2(-4a^3f+3a^2be-2ab^2d+b^3c)}{2b^5}+\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5(a+bx^3)}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{18b^{17/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{9b^{17/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{3\sqrt{3}b^{17/3}}+\frac{x^8(be-2af)}{8b^3}+\frac{fx^{11}}{11b^2}$$

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/(2*b^5) + ((b^2*d
 - 2*a*b*e + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^8)/(8*b^3)
+ (f*x^11)/(11*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)
/(3*b^5*(a + b*x^3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e
 - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
3*Sqrt[3]*b^(17/3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e
- 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(17/3)) - (a^(2/3)*(5*
b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(18*b^(17/3))
```

Rubi [A] time = 1.42045, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x^5(3a^2f-2abe+b^2d)}{5b^4}+\frac{x^2(-4a^3f+3a^2be-2ab^2d+b^3c)}{2b^5}+\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5(a+bx^3)}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{18b^{17/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{9b^{17/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{3\sqrt{3}b^{17/3}}+\frac{x^8(be-2af)}{8b^3}+\frac{fx^{11}}{11b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/(2*b^5) + ((b^2*d
 - 2*a*b*e + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^8)/(8*b^3)
+ (f*x^11)/(11*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)
/(3*b^5*(a + b*x^3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e
 - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
3*Sqrt[3]*b^(17/3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e
- 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(17/3)) - (a^(2/3)*(5*
b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(18*b^(17/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.325497, size = 319, normalized size = 0.95

$$792b^{5/3}x^5\left(3a^2f-2abe+b^2d\right)+1980b^{2/3}x^2\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)+\frac{1320ab^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a+bx^3}-440a^{2/3}\mathrm{l}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1980*b^(2/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2 + 792
*b^(5/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^5 + 495*b^(8/3)*(b*e - 2*a
*f)*x^8 + 360*b^(11/3)*f*x^11 + (1320*a*b^(2/3)*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*x^2)/(a + b*x^3) - 440*Sqrt[3]*a^(2/3)*(-5*b^3
*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-5*b^3*c + 8*a*b^2*d - 11*a^2*b
*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-5*b^3*c +
 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(3960*b^(17/3))
```

Maple [B] time = 0.017, size = 584, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 14/9*a^4/b^6*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-11/9*a^3/b^5*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))+1/8/b^2*x^8*e+1/5/b^2*x^5*d+1/2/b^2*x^2*c-1/4
/b^3*x^8*a*f+3/5/b^4*x^5*a^2*f-2/5/b^3*x^5*a*e-2/b^5*x^2*a^3*f+1/
11*f*x^11/b^2-1/3*a^4/b^5*x^2/(b*x^3+a)*f+1/3*a^3/b^4*x^2/(b*x^3+
a)*e-5/18*a/b^3*c/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-5
/9*a/b^3*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))+8/9*a^2/b^4*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))+3/2/b^4*x^2*a^2*e-1/b^3*x^2*a*d-1/3*a^2/b^3*x^2/(b
*x^3+a)*d+1/3*a/b^2*x^2/(b*x^3+a)*c-14/9*a^4/b^6*f/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))+7/9*a^4/b^6*f/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a
/b)^(2/3))+11/9*a^3/b^5*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-11/18*a^3
/b^5*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-8/9*a^2/b^4*
d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+4/9*a^2/b^4*d/(a/b)^(1/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))+5/9*a/b^3*c/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^7/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264341, size = 645, normalized size = 1.93

$$\frac{\sqrt{3}\left(220\sqrt{3}\left(5\,ab^3c - 8\,a^2b^2d + 11\,a^3be - 14\,a^4f + \left(5\,b^4c - 8\,ab^3d + 11\,a^2b^2e - 14\,a^3bf\right)x^3\right)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2 - bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^7/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/11880*sqrt(3)*(220*sqrt(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*
e - 14*a^4*f + (5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*
x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/
b^2)^(1/3)) - 440*sqrt(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e -
 14*a^4*f + (5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3
)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3)) - 1320*(5*a*b^3*
c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a*b^3*d +
11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*arctan(-1/3*(2*s
qrt(3)*a*x - sqrt(3)*b*(-a^2/b^2)^(2/3))/(b*(-a^2/b^2)^(2/3))) +
3*sqrt(3)*(120*b^4*f*x^14 + 15*(11*b^4*e - 14*a*b^3*f)*x^11 + 33*
(8*b^4*d - 11*a*b^3*e + 14*a^2*b^2*f)*x^8 + 132*(5*b^4*c - 8*a*b^
3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^5 + 220*(5*a*b^3*c - 8*a^2*b^2
*d + 11*a^3*b*e - 14*a^4*f)*x^2))/(b^6*x^3 + a*b^5)
```

Sympy [A] time = 54.3489, size = 530, normalized size = 1.58

$$-\frac{x^2\left(a^4f - a^3be + a^2b^2d - ab^3c\right)}{3ab^5 + 3b^6x^3}$$
$$+\operatorname{RootSum}\left(729t^3b^{17} + 2744a^{11}f^3 - 6468a^{10}bef^2 + 4704a^9b^2df^2 + 5082a^9b^2e^2f - 2940a^8b^3cf^2 - 7392a^8b^3def - 1331a^8b\right.$$
$$+\frac{fx^{11}}{11b^2} - \frac{x^8(2af - be)}{8b^3} + \frac{x^5\left(3a^2f - 2abe + b^2d\right)}{5b^4} - \frac{x^2\left(4a^3f - 3a^2be + 2ab^2d - b^3c\right)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x**2*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(3*a*b**5 + 3*
b**6*x**3) + RootSum(729*_t**3*b**17 + 2744*a**11*f**3 - 6468*a**
10*b*e*f**2 + 4704*a**9*b**2*d*f**2 + 5082*a**9*b**2*e**2*f - 294
0*a**8*b**3*c*f**2 - 7392*a**8*b**3*d*e*f - 1331*a**8*b**3*e**3 +
 4620*a**7*b**4*c*e*f + 2688*a**7*b**4*d**2*f + 2904*a**7*b**4*d*
e**2 - 3360*a**6*b**5*c*d*f - 1815*a**6*b**5*c*e**2 - 2112*a**6*b
**5*d**2*e + 1050*a**5*b**6*c**2*f + 2640*a**5*b**6*c*d*e + 512*a
**5*b**6*d**3 - 825*a**4*b**7*c**2*e - 960*a**4*b**7*c*d**2 + 600
*a**3*b**8*c**2*d - 125*a**2*b**9*c**3, Lambda(_t, _t*log(81*_t**
2*b**11/(196*a**7*f**2 - 308*a**6*b*e*f + 224*a**5*b**2*d*f + 121
*a**5*b**2*e**2 - 140*a**4*b**3*c*f - 176*a**4*b**3*d*e + 110*a**
3*b**4*c*e + 64*a**3*b**4*d**2 - 80*a**2*b**5*c*d + 25*a*b**6*c**
2) + x))) + f*x**11/(11*b**2) - x**8*(2*a*f - b*e)/(8*b**3) + x**
5*(3*a**2*f - 2*a*b*e + b**2*d)/(5*b**4) - x**2*(4*a**3*f - 3*a**
2*b*e + 2*a*b**2*d - b**3*c)/(2*b**5)
```

GIAC/XCAS [A] time = 0.21984, size = 597, normalized size = 1.78

$$\frac{\left(5\,ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-8\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+11\,a^3b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}+\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{2}{3}}b^3c-8\left(-ab^2\right)^{\frac{2}{3}}ab^2d-14\left(-ab^2\right)^{\frac{2}{3}}a^3f+11\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^7}+\frac{ab^3cx^2-a^2b^2dx^2-a^4fx^2+a^3bx^2e}{3\,(bx^3+a)b^5}-\frac{\left(5\left(-ab^2\right)^{\frac{2}{3}}b^3c-8\left(-ab^2\right)^{\frac{2}{3}}ab^2d-14\left(-ab^2\right)^{\frac{2}{3}}a^3f+11\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^7}+\frac{40\,b^{20}fx^{11}-110\,ab^{19}fx^8+55\,b^{20}x^8e+88\,b^{20}dx^5+264\,a^2b^{18}fx^5-176\,ab^{19}x^5e+220\,b^{20}cx^2-440\,ab^{19}dx^2-880\,a^3b^{17}f}{440\,b^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^7/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(5*a*b^3*c*(-a/b)^(1/3) - 8*a^2*b^2*d*(-a/b)^(1/3) - 14*a^4*f
*(-a/b)^(1/3) + 11*a^3*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x -
(-a/b)^(1/3)))/(a*b^5) + 1/9*sqrt(3)*(5*(-a*b^2)^(2/3)*b^3*c - 8*
(-a*b^2)^(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2
/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3)
)/b^7 + 1/3*(a*b^3*c*x^2 - a^2*b^2*d*x^2 - a^4*f*x^2 + a^3*b*x^2*
e)/((b*x^3 + a)*b^5) - 1/18*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a*b^2)^
(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2/3)*a^2*b
*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 + 1/440*(40*b^20*
f*x^11 - 110*a*b^19*f*x^8 + 55*b^20*x^8*e + 88*b^20*d*x^5 + 264*a
^2*b^18*f*x^5 - 176*a*b^19*x^5*e + 220*b^20*c*x^2 - 440*a*b^19*d*
x^2 - 880*a^3*b^17*f*x^2 + 660*a^2*b^18*x^2*e)/b^22
```

3.262 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=328

$$\frac{x^4(3a^2f-2abe+b^2d)}{4b^4}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{9b^{16/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{3\sqrt{3}b^{16/3}}+\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5(a+bx^3)}+\frac{x(-4a^3f+3a^2be-2ab^2d+b^3c)}{b^5}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{18b^{16/3}}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^{10}}{10b^2}$$

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a
*b*e + 3*a^2*f)*x^4)/(4*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x
^10)/(10*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*
(a + b*x^3)) + (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^
3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*b^(16/3)) - (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(16/3)) + (a^(1/3)*(4*b^3*c -
7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(18*b^(16/3))
```

Rubi [A] time = 0.803358, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{x^4(3a^2f-2abe+b^2d)}{4b^4}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{9b^{16/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{3\sqrt{3}b^{16/3}}+\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5(a+bx^3)}+\frac{x(-4a^3f+3a^2be-2ab^2d+b^3c)}{b^5}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{18b^{16/3}}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```


```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a
*b*e + 3*a^2*f)*x^4)/(4*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x
^10)/(10*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*
(a + b*x^3)) + (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^
3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*b^(16/3)) - (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(16/3)) + (a^(1/3)*(4*b^3*c -
7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(18*b^(16/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.516766, size = 315, normalized size = 0.96

$$315b^{4/3}x^4\left(3a^2f-2abe+b^2d\right)+\frac{420a\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a+bx^3}+1260\sqrt[3]{b}x\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)+140\sqrt[3]{a}\log\left(\sqrt[3]{\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x + 315*b
^(4/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^4 + 180*b^(7/3)*(b*e - 2*a*f
)*x^7 + 126*b^(10/3)*f*x^10 + (420*a*b^(1/3)*(b^3*c - a*b^2*d + a
^2*b*e - a^3*f)*x)/(a + b*x^3) - 140*Sqrt[3]*a^(1/3)*(-4*b^3*c +
7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] + 140*a^(1/3)*(-4*b^3*c + 7*a*b^2*d - 10*a^2*b*e +
13*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-4*b^3*c + 7*a*b
^2*d - 10*a^2*b*e + 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(1260*b^(16/3))
```

Maple [B] time = 0.016, size = 567, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] -4/9*a/b^3*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))+13/9*a^4/b^6*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))-10/9*a^3/b^5*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))+7/9*a^2/b^4*d/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*a^4/b^5*x/(b*x^3+a)*f+1/
3*a^3/b^4*x/(b*x^3+a)*e+1/7/b^2*x^7*e+1/4/b^2*x^4*d+1/b^2*c*x+1/1
0*f*x^10/b^2+1/3*a/b^2*x/(b*x^3+a)*c-10/9*a^3/b^5*e/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))+5/9*a^3/b^5*e/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(
a/b)^(2/3))+7/9*a^2/b^4*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-7/18*a^2/
b^4*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-4/9*a/b^3*c/(
a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9*a/b^3*c/(a/b)^(2/3)*ln(x^2-x*(a/
b)^(1/3)+(a/b)^(2/3))+13/9*a^4/b^6*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-13/18*a^4/b^6*f/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1
/3*a^2/b^3*x/(b*x^3+a)*d-2/7/b^3*x^7*a*f+3/4/b^4*x^4*a^2*f-1/2/b^
3*x^4*a*e-4/b^5*a^3*f*x+3/b^4*a^2*e*x-2/b^3*a*d*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^6/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240745, size = 590, normalized size = 1.8

$$\frac{\sqrt{3}\left(70\sqrt{3}\left(4\,ab^3c - 7\,a^2b^2d + 10\,a^3be - 13\,a^4f + \left(4\,b^4c - 7\,ab^3d + 10\,a^2b^2e - 13\,a^3bf\right)x^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^6/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/3780*sqrt(3)*(70*sqrt(3)*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e
- 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^
3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*sqrt(
3)*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c -
7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*log(x + (
a/b)^(1/3)) + 420*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*
f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^3)*(a/b)^
(1/3)*arctan(-1/3*(2*sqrt(3)*x - sqrt(3)*(a/b)^(1/3))/(a/b)^(1/3)
) + 3*sqrt(3)*(42*b^4*f*x^13 + 6*(10*b^4*e - 13*a*b^3*f)*x^10 + 1
5*(7*b^4*d - 10*a*b^3*e + 13*a^2*b^2*f)*x^7 + 105*(4*b^4*c - 7*a*
b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^4 + 140*(4*a*b^3*c - 7*a^2*b
^2*d + 10*a^3*b*e - 13*a^4*f)*x))/(b^6*x^3 + a*b^5)
```

Sympy [A] time = 18.5485, size = 440, normalized size = 1.34

$$-\frac{x\left(a^4f - a^3be + a^2b^2d - ab^3c\right)}{3ab^5 + 3b^6x^3}$$
$$+\mathrm{RootSum}\left(729t^3b^{16} - 2197a^{10}f^3 + 5070a^9bef^2 - 3549a^8b^2df^2 - 3900a^8b^2e^2f + 2028a^7b^3cf^2 + 5460a^7b^3def + 1000a^7b^3\right.$$
$$+\frac{fx^{10}}{10b^2} - \frac{x^7\left(2af - be\right)}{7b^3} + \frac{x^4\left(3a^2f - 2abe + b^2d\right)}{4b^4} - \frac{x\left(4a^3f - 3a^2be + 2ab^2d - b^3c\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(3*a*b**5 + 3*b**
6*x**3) + RootSum(729*_t**3*b**16 - 2197*a**10*f**3 + 5070*a**9*b
*e*f**2 - 3549*a**8*b**2*d*f**2 - 3900*a**8*b**2*e**2*f + 2028*a*
*7*b**3*c*f**2 + 5460*a**7*b**3*d*e*f + 1000*a**7*b**3*e**3 - 312
0*a**6*b**4*c*e*f - 1911*a**6*b**4*d**2*f - 2100*a**6*b**4*d*e**2
 + 2184*a**5*b**5*c*d*f + 1200*a**5*b**5*c*e**2 + 1470*a**5*b**5*
d**2*e - 624*a**4*b**6*c**2*f - 1680*a**4*b**6*c*d*e - 343*a**4*b
**6*d**3 + 480*a**3*b**7*c**2*e + 588*a**3*b**7*c*d**2 - 336*a**2
*b**8*c**2*d + 64*a*b**9*c**3, Lambda(_t, _t*log(9*_t*b**5/(13*a*
*3*f - 10*a**2*b*e + 7*a*b**2*d - 4*b**3*c) + x))) + f*x**10/(10*
b**2) - x**7*(2*a*f - b*e)/(7*b**3) + x**4*(3*a**2*f - 2*a*b*e +
b**2*d)/(4*b**4) - x*(4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c
)/b**5
```

GIAC/XCAS [A] time = 0.215678, size = 532, normalized size = 1.62

$$-\frac{\sqrt{3}\left(4\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-7\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-13\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+10\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^{6}}+\frac{\left(4\,ab^{3}c-7\,a^{2}b^{2}d-13\,a^{4}f+10\,a^{3}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{5}}-\frac{\left(4\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-7\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-13\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+10\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{6}}+\frac{ab^{3}cx-a^{2}b^{2}dx-a^{4}fx+a^{3}bxe}{3\,(bx^{3}+a)b^{5}}+\frac{14\,b^{18}fx^{10}-40\,ab^{17}fx^{7}+20\,b^{18}x^{7}e+35\,b^{18}dx^{4}+105\,a^{2}b^{16}fx^{4}-70\,ab^{17}x^{4}e+140\,b^{18}cx-280\,ab^{17}dx-560\,a^{3}b^{15}fx+\cdots}{140\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^6/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(4*(-a*b^2)^(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d -
 13*(-a*b^2)^(1/3)*a^3*f + 10*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 + 1/9*(4*a*b^3*c -
 7*a^2*b^2*d - 13*a^4*f + 10*a^3*b*e)*(-a/b)^(1/3)*ln(abs(x - (-a
/b)^(1/3)))/(a*b^5) - 1/18*(4*(-a*b^2)^(1/3)*b^3*c - 7*(-a*b^2)^(
1/3)*a*b^2*d - 13*(-a*b^2)^(1/3)*a^3*f + 10*(-a*b^2)^(1/3)*a^2*b*
e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^6 + 1/3*(a*b^3*c*x -
 a^2*b^2*d*x - a^4*f*x + a^3*b*x*e)/((b*x^3 + a)*b^5) + 1/140*(14
*b^18*f*x^10 - 40*a*b^17*f*x^7 + 20*b^18*x^7*e + 35*b^18*d*x^4 +
105*a^2*b^16*f*x^4 - 70*a*b^17*x^4*e + 140*b^18*c*x - 280*a*b^17*
d*x - 560*a^3*b^15*f*x + 420*a^2*b^16*x*e)/b^20
```

3.263 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=298

$$\frac{x^2\left(3a^2f-2abe+b^2d\right)}{2b^4}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-11a^3f+8a^2be-5ab^2d+2b^3c\right)}{9\sqrt[3]{a}b^{14/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-11a^3f+8a^2be-5ab^2d+2b^3c\right)}{3\sqrt{3}\sqrt[3]{a}b^{14/3}}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-11a^3f+8a^2be-5ab^2d+2b^3c\right)}{18\sqrt[3]{a}b^{14/3}}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^8}{8b^2}$$

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/(2*b^4) + ((b*e - 2*a*f)*x^5)/(
5*b^3) + (f*x^8)/(8*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x
^2)/(3*b^4*(a + b*x^3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt
[3]*a^(1/3)*b^(14/3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^
3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(14/3)) + ((2*b^3*c -
 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(18*a^(1/3)*b^(14/3))
```

Rubi [A] time = 0.968636, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2\left(3a^2f-2abe+b^2d\right)}{2b^4}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-11a^3f+8a^2be-5ab^2d+2b^3c\right)}{9\sqrt[3]{a}b^{14/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-11a^3f+8a^2be-5ab^2d+2b^3c\right)}{3\sqrt{3}\sqrt[3]{a}b^{14/3}}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-11a^3f+8a^2be-5ab^2d+2b^3c\right)}{18\sqrt[3]{a}b^{14/3}}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/(2*b^4) + ((b*e - 2*a*f)*x^5)/(
5*b^3) + (f*x^8)/(8*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x
^2)/(3*b^4*(a + b*x^3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt
[3]*a^(1/3)*b^(14/3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^
3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(14/3)) + ((2*b^3*c -
 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(18*a^(1/3)*b^(14/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.302505, size = 282, normalized size = 0.95

$$\frac{180b^{2/3}x^2\left(3a^2f-2abe+b^2d\right)+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(11a^3f-8a^2be+5ab^2d-2b^3c\right)}{\sqrt[3]{a}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(11a^3f-8a^2be+5ab^2d-2b^3c\right)}{\sqrt[3]{a}}-\frac{120b}{}}{360b^{14/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (180*b^(2/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2 + 72*b^(5/3)*(b*e -
2*a*f)*x^5 + 45*b^(8/3)*f*x^8 - (120*b^(2/3)*(b^3*c - a*b^2*d + a
^2*b*e - a^3*f)*x^2)/(a + b*x^3) + (40*Sqrt[3]*(-2*b^3*c + 5*a*b^
2*d - 8*a^2*b*e + 11*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/a^(1/3) + (40*(-2*b^3*c + 5*a*b^2*d - 8*a^2*b*e + 11*a^3*
f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(2*b^3*c - 5*a*b^2*d +
 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/a^(1/3))/(360*b^(14/3))
```

Maple [B] time = 0.015, size = 529, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/8*f*x^8/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+3/2/b^4*x^2*a^2*f-1/b
^3*x^2*a*e+1/2*x^2*d/b^2+1/3/b^4*x^2/(b*x^3+a)*a^3*f-1/3/b^3*x^2/
(b*x^3+a)*a^2*e+1/3/b^2*x^2/(b*x^3+a)*a*d-1/3/b*x^2/(b*x^3+a)*c+1
1/9/b^5*a^3*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-11/18/b^5*a^3*f/(a/b)
^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-11/9/b^5*a^3*f*3^(1/2)/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-8/9/b^4*a^2*e/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))+4/9/b^4*a^2*e/(a/b)^(1/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))+8/9/b^4*a^2*e*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+5/9/b^3*a*d/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))-5/18/b^3*a*d/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2
/3))-5/9/b^3*a*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))-2/9/b^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*c/(a/
b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9/b^2*c*3^(1/2)/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^4/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239034, size = 541, normalized size = 1.82

$$\frac{\sqrt{3}\Big(20\sqrt{3}\left(2\,ab^3c-5\,a^2b^2d+8\,a^3be-11\,a^4f+\left(2\,b^4c-5\,ab^3d+8\,a^2b^2e-11\,a^3bf\right)x^3\right)\log\left(\left(ab^2\right)^{\frac{1}{3}}bx^2+ab-\left(ab^2\right)^{\frac{2}{3}}x\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^4/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/1080*sqrt(3)*(20*sqrt(3)*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e -
 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)
*log((a*b^2)^(1/3)*b*x^2 + a*b - (a*b^2)^(2/3)*x) - 40*sqrt(3)*(2
*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^
3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*log(a*b + (a*b^2)^(2/3)*x) +
 120*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c -
 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*arctan(-1/3*(sqrt(3)*
a*b - 2*sqrt(3)*(a*b^2)^(2/3)*x)/(a*b)) + 3*sqrt(3)*(15*b^3*f*x^1
1 + 3*(8*b^3*e - 11*a*b^2*f)*x^8 + 12*(5*b^3*d - 8*a*b^2*e + 11*a
^2*b*f)*x^5 - 20*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*x^2
)*(a*b^2)^(1/3))/((b^5*x^3 + a*b^4)*(a*b^2)^(1/3))
```

Sympy [A] time = 51.1795, size = 484, normalized size = 1.62

$$\frac{x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{3ab^4+3b^5x^3}$$
$$+\mathrm{RootSum}\left(729t^3ab^{14}-1331a^9f^3+2904a^8bef^2-1815a^7b^2df^2-2112a^7b^2e^2f+726a^6b^3cf^2+2640a^6b^3def+512a^6b^3e^3\right.$$
$$+\frac{fx^8}{8b^2}-\frac{x^5\left(2af-be\right)}{5b^3}+\frac{x^2\left(3a^2f-2abe+b^2d\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x
**3) + RootSum(729*_t**3*a*b**14 - 1331*a**9*f**3 + 2904*a**8*b*e
*f**2 - 1815*a**7*b**2*d*f**2 - 2112*a**7*b**2*e**2*f + 726*a**6*
b**3*c*f**2 + 2640*a**6*b**3*d*e*f + 512*a**6*b**3*e**3 - 1056*a*
*5*b**4*c*e*f - 825*a**5*b**4*d**2*f - 960*a**5*b**4*d*e**2 + 660
*a**4*b**5*c*d*f + 384*a**4*b**5*c*e**2 + 600*a**4*b**5*d**2*e -
132*a**3*b**6*c**2*f - 480*a**3*b**6*c*d*e - 125*a**3*b**6*d**3 +
 96*a**2*b**7*c**2*e + 150*a**2*b**7*c*d**2 - 60*a*b**8*c**2*d +
8*b**9*c**3, Lambda(_t, _t*log(81*_t**2*a*b**9/(121*a**6*f**2 - 1
76*a**5*b*e*f + 110*a**4*b**2*d*f + 64*a**4*b**2*e**2 - 44*a**3*b
**3*c*f - 80*a**3*b**3*d*e + 32*a**2*b**4*c*e + 25*a**2*b**4*d**2
 - 20*a*b**5*c*d + 4*b**6*c**2) + x))) + f*x**8/(8*b**2) - x**5*(
2*a*f - b*e)/(5*b**3) + x**2*(3*a**2*f - 2*a*b*e + b**2*d)/(2*b**
4)
```

GIAC/XCAS [A] time = 0.219401, size = 537, normalized size = 1.8

$$-\frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-11\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4}$$
$$-\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e}{3\,(bx^3+a)b^4}$$
$$-\frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{2}{3}}b^3c-5\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d-11\,\left(-ab^2\right)^{\frac{2}{3}}a^3f+8\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^6}$$
$$+\frac{\left(2\,\left(-ab^2\right)^{\frac{2}{3}}b^3c-5\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d-11\,\left(-ab^2\right)^{\frac{2}{3}}a^3f+8\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^6}$$
$$+\frac{5\,b^{14}fx^8-16\,ab^{13}fx^5+8\,b^{14}x^5e+20\,b^{14}dx^2+60\,a^2b^{12}fx^2-40\,ab^{13}x^2e}{40\,b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^4/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b^3*c*(-a/b)^(1/3) - 5*a*b^2*d*(-a/b)^(1/3) - 11*a^3*f*(-
a/b)^(1/3) + 8*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/
b)^(1/3)))/(a*b^4) - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a
^2*b*x^2*e)/((b*x^3 + a)*b^4) - 1/9*sqrt(3)*(2*(-a*b^2)^(2/3)*b^3
*c - 5*(-a*b^2)^(2/3)*a*b^2*d - 11*(-a*b^2)^(2/3)*a^3*f + 8*(-a*b
^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(a*b^6) + 1/18*(2*(-a*b^2)^(2/3)*b^3*c - 5*(-a*b^2)^(2/3)
*a*b^2*d - 11*(-a*b^2)^(2/3)*a^3*f + 8*(-a*b^2)^(2/3)*a^2*b*e)*ln
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^6) + 1/40*(5*b^14*f*x^
8 - 16*a*b^13*f*x^5 + 8*b^14*x^5*e + 20*b^14*d*x^2 + 60*a^2*b^12*
f*x^2 - 40*a*b^13*x^2*e)/b^16
```

3.264 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=288

$$\frac{x\left(3a^2f-2abe+b^2d\right)}{b^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{18a^{2/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{9a^{2/3}b^{13/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{3\sqrt{3}a^{2/3}b^{13/3}}+\frac{x^4(be-2af)}{4b^3}+\frac{fx^7}{7b^2}$$

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^4)/(4*b^3)
 + (f*x^7)/(7*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b
^4*(a + b*x^3)) - ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3
)*b^(13/3)) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(13/3)) - ((b^3*c - 4*a*b^2*d + 7
*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(18*a^(2/3)*b^(13/3))
```

Rubi [A] time = 0.679117, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{x\left(3a^2f-2abe+b^2d\right)}{b^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{18a^{2/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{9a^{2/3}b^{13/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{3\sqrt{3}a^{2/3}b^{13/3}}+\frac{x^4(be-2af)}{4b^3}+\frac{fx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^4)/(4*b^3)
 + (f*x^7)/(7*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b
^4*(a + b*x^3)) - ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3
)*b^(13/3)) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(13/3)) - ((b^3*c - 4*a*b^2*d + 7
*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(18*a^(2/3)*b^(13/3))
```

Rubi in Sympy [A] time = 141.744, size = 284, normalized size = 0.99

$$\frac{fx^7}{7b^2} - \frac{x^4(2af-be)}{4b^3} + \frac{x(3a^2f-2abe+b^2d)}{b^4} + \frac{x(a^3f-a^2be+ab^2d-b^3c)}{3b^4(a+bx^3)} - \frac{(10a^3f-7a^2be+4ab^2d-b^3c)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{2}{3}}b^{\frac{13}{3}}} + \frac{(10a^3f-7a^2be+4ab^2d-b^3c)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{2}{3}}b^{\frac{13}{3}}} + \frac{\sqrt{3}(10a^3f-7a^2be+4ab^2d-b^3c)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{2}{3}}b^{\frac{13}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] f*x**7/(7*b**2) - x**4*(2*a*f - b*e)/(4*b**3) + x*(3*a**2*f - 2*a
*b*e + b**2*d)/b**4 + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(
3*b**4*(a + b*x**3)) - (10*a**3*f - 7*a**2*b*e + 4*a*b**2*d - b**
3*c)*log(a**(1/3) + b**(1/3)*x)/(9*a**(2/3)*b**(13/3)) + (10*a**3
*f - 7*a**2*b*e + 4*a*b**2*d - b**3*c)*log(a**(2/3) - a**(1/3)*b*
*(1/3)*x + b**(2/3)*x**2)/(18*a**(2/3)*b**(13/3)) + sqrt(3)*(10*a
**3*f - 7*a**2*b*e + 4*a*b**2*d - b**3*c)*atan(sqrt(3)*(a**(1/3)/
3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(2/3)*b**(13/3))
```

Mathematica [A] time = 0.299475, size = 277, normalized size = 0.96

$$\frac{252\sqrt[3]{b}x\left(3a^2f-2abe+b^2d\right)-\frac{84\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a+bx^3}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{a^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)(\ldots}{\ldots}}{252b^{13/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (252*b^(1/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x + 63*b^(4/3)*(b*e - 2*
a*f)*x^4 + 36*b^(7/3)*f*x^7 - (84*b^(1/3)*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x)/(a + b*x^3) + (28*Sqrt[3]*(-(b^3*c) + 4*a*b^2*d -
 7*a^2*b*e + 10*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/a^(2/3) + (28*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a
^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + 4*a*b^2*d - 7*a^2*
b*e + 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a
^(2/3))/(252*b^(13/3))
```

Maple [B] time = 0.015, size = 514, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/7*f*x^7/b^2-1/2/b^3*x^4*a*f+1/4/b^2*x^4*e+3/b^4*a^2*f*x-2/b^3*a
*e*x+x*d/b^2+1/3/b^4*x/(b*x^3+a)*a^3*f-1/3/b^3*x/(b*x^3+a)*a^2*e+
```

```
1/3/b^2*x/(b*x^3+a)*a*d-1/3/b*x/(b*x^3+a)*c-10/9/b^5*a^3*f/(a/b)^
(2/3)*ln(x+(a/b)^(1/3))+5/9/b^5*a^3*f/(a/b)^(2/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))-10/9/b^5*a^3*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))+7/9/b^4*a^2*e/(a/b)^(2/3)*ln(x+(a/b)
^(1/3))-7/18/b^4*a^2*e/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))+7/9/b^4*a^2*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))-4/9/b^3*a*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9/b^3*a*
d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-4/9/b^3*a*d/(a/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9/b^2*c/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2*c/(a/b)^(2/3)*ln(x^2-x*(a/
b)^(1/3)+(a/b)^(2/3))+1/9/b^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^3/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235551, size = 531, normalized size = 1.84

$$\frac{\sqrt{3}\left(14\,\sqrt{3}\left(ab^3c-4\,a^2b^2d+7\,a^3be-10\,a^4f+\left(b^4c-4\,ab^3d+7\,a^2b^2e-10\,a^3bf\right)x^3\right)\log\left(\left(-a^2b\right)^{\frac{2}{3}}x^2+\left(-a^2b\right)^{\frac{1}{3}}ax+a^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^3/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/756*sqrt(3)*(14*sqrt(3)*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10
*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*log(
(-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*a*x + a^2) - 28*sqrt(3)*(a*b^
3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7
*a^2*b^2*e - 10*a^3*b*f)*x^3)*log((-a^2*b)^(1/3)*x - a) + 84*(a*b
^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d +
7*a^2*b^2*e - 10*a^3*b*f)*x^3)*arctan(1/3*(2*sqrt(3)*(-a^2*b)^(1/
3)*x + sqrt(3)*a)/a) + 3*sqrt(3)*(12*b^3*f*x^10 + 3*(7*b^3*e - 10
*a*b^2*f)*x^7 + 21*(4*b^3*d - 7*a*b^2*e + 10*a^2*b*f)*x^4 - 28*(b
^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*x)*(-a^2*b)^(1/3))/((b^5
*x^3 + a*b^4)*(-a^2*b)^(1/3))
```

Sympy [A] time = 17.6266, size = 398, normalized size = 1.38

$$\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{3ab^4+3b^5x^3}$$
$$+\mathrm{RootSum}\left(729t^3a^2b^{13}+1000a^9f^3-2100a^8bef^2+1200a^7b^2df^2+1470a^7b^2e^2f-300a^6b^3cf^2-1680a^6b^3def-343a^6b^3e\right.$$
$$+\frac{fx^7}{7b^2}-\frac{x^4\left(2af-be\right)}{4b^3}+\frac{x\left(3a^2f-2abe+b^2d\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3
) + RootSum(729*_t**3*a**2*b**13 + 1000*a**9*f**3 - 2100*a**8*b*e
```

```
*f**2 + 1200*a**7*b**2*d*f**2 + 1470*a**7*b**2*e**2*f - 300*a**6*
b**3*c*f**2 - 1680*a**6*b**3*d*e*f - 343*a**6*b**3*e**3 + 420*a**
5*b**4*c*e*f + 480*a**5*b**4*d**2*f + 588*a**5*b**4*d*e**2 - 240*
a**4*b**5*c*d*f - 147*a**4*b**5*c*e**2 - 336*a**4*b**5*d**2*e + 3
0*a**3*b**6*c**2*f + 168*a**3*b**6*c*d*e + 64*a**3*b**6*d**3 - 21
*a**2*b**7*c**2*e - 48*a**2*b**7*c*d**2 + 12*a*b**8*c**2*d - b**9
*c**3, Lambda(_t, _t*log(-9*_t*a*b**4/(10*a**3*f - 7*a**2*b*e + 4
*a*b**2*d - b**3*c) + x))) + f*x**7/(7*b**2) - x**4*(2*a*f - b*e)
/(4*b**3) + x*(3*a**2*f - 2*a*b*e + b**2*d)/b**4
```

GIAC/XCAS [A] time = 0.219428, size = 471, normalized size = 1.64

$$-\frac{\left(b^{3}c-4\,ab^{2}d-10\,a^{3}f+7\,a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{4}}+\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-4\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-10\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+7\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{5}}-\frac{b^{3}cx-ab^{2}dx-a^{3}fx+a^{2}bxe}{3\,(bx^{3}+a)b^{4}}+\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-4\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-10\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+7\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{5}}+\frac{4\,b^{12}fx^{7}-14\,ab^{11}fx^{4}+7\,b^{12}x^{4}e+28\,b^{12}dx+84\,a^{2}b^{10}fx-56\,ab^{11}xe}{28\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^3/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*(-a/b)^(1/3)*ln(a
bs(x - (-a/b)^(1/3)))/(a*b^4) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b^3*c
 - 4*(-a*b^2)^(1/3)*a*b^2*d - 10*(-a*b^2)^(1/3)*a^3*f + 7*(-a*b^2
)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a*b^5) - 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/(
(b*x^3 + a)*b^4) + 1/18*((-a*b^2)^(1/3)*b^3*c - 4*(-a*b^2)^(1/3)*
a*b^2*d - 10*(-a*b^2)^(1/3)*a^3*f + 7*(-a*b^2)^(1/3)*a^2*b*e)*ln(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5) + 1/28*(4*b^12*f*x^7
 - 14*a*b^11*f*x^4 + 7*b^12*x^4*e + 28*b^12*d*x + 84*a^2*b^10*f*x
 - 56*a*b^11*x*e)/b^14
```

3.265 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=271

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{18a^{4/3}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{9a^{4/3}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{3\sqrt{3}a^{4/3}b^{11/3}}+\frac{x^2(be-2af)}{2b^3}+\frac{fx^5}{5b^2}$$

```
[Out] ((b*e - 2*a*f)*x^2)/(2*b^3) + (f*x^5)/(5*b^2) + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) - ((b^3*c + 2*a*b^
2*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(11/3)) - ((b^3*c + 2*a*b^2*d -
 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(11/
3)) + ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(11/3))
```

Rubi [A] time = 0.617647, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{18a^{4/3}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{9a^{4/3}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{3\sqrt{3}a^{4/3}b^{11/3}}+\frac{x^2(be-2af)}{2b^3}+\frac{fx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x^2)/(2*b^3) + (f*x^5)/(5*b^2) + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) - ((b^3*c + 2*a*b^
2*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(11/3)) - ((b^3*c + 2*a*b^2*d -
 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(11/
3)) + ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(11/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{fx^5}{5b^2}-\frac{(2af-be)\int x\,dx}{b^3}-\frac{x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{3ab^3\left(a+bx^3\right)}-\frac{\left(8a^3f-5a^2be+2ab^2d+b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{4}{3}}b^{\frac{11}{3}}}+\frac{\left(8a^3f-5a^2be+2ab^2d+b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{4}{3}}b^{\frac{11}{3}}}-\frac{\sqrt{3}\left(8a^3f-5a^2be+2ab^2d+b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{4}{3}}b^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] f*x**5/(5*b**2) - (2*a*f - b*e)*Integral(x, x)/b**3 - x**2*(a**3*
f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**3*(a + b*x**3)) - (8*a*
*3*f - 5*a**2*b*e + 2*a*b**2*d + b**3*c)*log(a**(1/3) + b**(1/3)*
x)/(9*a**(4/3)*b**(11/3)) + (8*a**3*f - 5*a**2*b*e + 2*a*b**2*d +
 b**3*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*
a**(4/3)*b**(11/3)) - sqrt(3)*(8*a**3*f - 5*a**2*b*e + 2*a*b**2*d
 + b**3*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(
9*a**(4/3)*b**(11/3))
```

Mathematica [A] time = 0.275454, size = 255, normalized size = 0.94

$$\frac{\frac{30b^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a(a+bx^3)}-\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{a^{4/3}}-\frac{10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{a^{4/3}}+\frac{5\log\left(a^{2/3}-\cdots\right.}{\cdots}}{90b^{11/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(b*e - 2*a*f)*x^2 + 18*b^(5/3)*f*x^5 + (30*b^(2/3)*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt
[3]*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(1 - (2*b^(1
/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) - (10*(b^3*c + 2*a*b^2*d - 5*a^
2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + (5*(b^3*c +
2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/a^(4/3))/(90*b^(11/3))
```

Maple [B] time = 0.014, size = 495, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/5*f*x^5/b^2-1/b^3*x^2*a*f+1/2/b^2*x^2*e-1/3/b^3*x^2*a^2/(b*x^3+
a)*f+1/3/b^2*x^2*a/(b*x^3+a)*e-1/3*x^2*d/(b*x^3+a)/b+1/3*x^2/a/(b
*x^3+a)*c-8/9/b^4*a^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+5/9/b^3*a/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-2/9/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))*d-1/9/b/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+4/9/b^4*a^2/(a/b)^(
1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f-5/18/b^3*a/(a/b)^(1/3)*l
n(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/9/b^2/(a/b)^(1/3)*ln(x^2-x*(
a/b)^(1/3)+(a/b)^(2/3))*d+1/18/b/a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/
3)+(a/b)^(2/3))*c+8/9/b^4*a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*f-5/9/b^3*a*3^(1/2)/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+2/9/b^2*3^(1/2)/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/9/b/a*3^(1/2)/(a/b)^(1/3
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23619, size = 513, normalized size = 1.89

$$-\frac{\sqrt{3}\left(5\sqrt{3}\left(ab^3c+2a^2b^2d-5a^3be+8a^4f+\left(b^4c+2ab^3d-5a^2b^2e+8a^3bf\right)x^3\right)\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2-ab+\left(-ab^2\right)^{\frac{2}{3}}x\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/270*sqrt(3)*(5*sqrt(3)*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b*e + 8*
a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*log((-
a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 10*sqrt(3)*(a*b^3*
c + 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^
2*b^2*e + 8*a^3*b*f)*x^3)*log(a*b + (-a*b^2)^(2/3)*x) + 30*(a*b^3
*c + 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a
^2*b^2*e + 8*a^3*b*f)*x^3)*arctan(-1/3*(sqrt(3)*a*b - 2*sqrt(3)*(
-a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(6*a*b^2*f*x^8 + 3*(5*a*b^2*e
 - 8*a^2*b*f)*x^5 + 5*(2*b^3*c - 2*a*b^2*d + 5*a^2*b*e - 8*a^3*f)
*x^2)*(-a*b^2)^(1/3))/((a*b^4*x^3 + a^2*b^3)*(-a*b^2)^(1/3))
```

Sympy [A] time = 28.3682, size = 461, normalized size = 1.7

$$-\frac{x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^2b^3+3ab^4x^3}$$
$$+\mathrm{RootSum}\left(729t^3a^4b^{11}+512a^9f^3-960a^8bef^2+384a^7b^2df^2+600a^7b^2e^2f+192a^6b^3cf^2-480a^6b^3def-125a^6b^3e^3-24\right.$$
$$+\frac{fx^5}{5b^2}-\frac{x^2\left(2af-be\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**2*b**3 + 3*a*
b**4*x**3) + RootSum(729*_t**3*a**4*b**11 + 512*a**9*f**3 - 960*a
**8*b*e*f**2 + 384*a**7*b**2*d*f**2 + 600*a**7*b**2*e**2*f + 192*
a**6*b**3*c*f**2 - 480*a**6*b**3*d*e*f - 125*a**6*b**3*e**3 - 240
*a**5*b**4*c*e*f + 96*a**5*b**4*d**2*f + 150*a**5*b**4*d*e**2 + 9
6*a**4*b**5*c*d*f + 75*a**4*b**5*c*e**2 - 60*a**4*b**5*d**2*e + 2
4*a**3*b**6*c**2*f - 60*a**3*b**6*c*d*e + 8*a**3*b**6*d**3 - 15*a
**2*b**7*c**2*e + 12*a**2*b**7*c*d**2 + 6*a*b**8*c**2*d + b**9*c*
*3, Lambda(_t, _t*log(81*_t**2*a**3*b**7/(64*a**6*f**2 - 80*a**5*
b*e*f + 32*a**4*b**2*d*f + 25*a**4*b**2*e**2 + 16*a**3*b**3*c*f -
 20*a**3*b**3*d*e - 10*a**2*b**4*c*e + 4*a**2*b**4*d**2 + 4*a*b**
5*c*d + b**6*c**2) + x))) + f*x**5/(5*b**2) - x**2*(2*a*f - b*e)/
(2*b**3)
```

GIAC/XCAS [A] time = 0.218952, size = 494, normalized size = 1.82

$$-\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3}+\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e}{3\,(bx^3+a)ab^3}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c+2\left(-ab^2\right)^{\frac{2}{3}}ab^2d+8\left(-ab^2\right)^{\frac{2}{3}}a^3f-5\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^5}+\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c+2\left(-ab^2\right)^{\frac{2}{3}}ab^2d+8\left(-ab^2\right)^{\frac{2}{3}}a^3f-5\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^5}+\frac{2\,b^8fx^5-10\,ab^7fx^2+5\,b^8x^2e}{10\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(b^3*c*(-a/b)^(1/3) + 2*a*b^2*d*(-a/b)^(1/3) + 8*a^3*f*(-a/b
)^(1/3) - 5*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^
(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^
2*b*x^2*e)/((b*x^3 + a)*a*b^3) - 1/9*sqrt(3)*((-a*b^2)^(2/3)*b^3*
c + 2*(-a*b^2)^(2/3)*a*b^2*d + 8*(-a*b^2)^(2/3)*a^3*f - 5*(-a*b^2
)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a^2*b^5) + 1/18*((-a*b^2)^(2/3)*b^3*c + 2*(-a*b^2)^(2/3)*a
*b^2*d + 8*(-a*b^2)^(2/3)*a^3*f - 5*(-a*b^2)^(2/3)*a^2*b*e)*ln(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^5) + 1/10*(2*b^8*f*x^5
- 10*a*b^7*f*x^2 + 5*b^8*x^2*e)/b^10
```

3.266 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=264

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{18a^{5/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{9a^{5/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{3\sqrt{3}a^{5/3}b^{10/3}}+\frac{x(be-2af)}{b^3}+\frac{fx^4}{4b^2}$$

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^4)/(4*b^2) + ((b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x)/(3*a*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d - 4*
a^2*b*e + 7*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((2*b^3*c + a*b^2*d - 4*a^2*b
*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((
2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))
```

Rubi [A] time = 0.597331, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{18a^{5/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{9a^{5/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{3\sqrt{3}a^{5/3}b^{10/3}}+\frac{x(be-2af)}{b^3}+\frac{fx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^4)/(4*b^2) + ((b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x)/(3*a*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d - 4*
a^2*b*e + 7*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((2*b^3*c + a*b^2*d - 4*a^2*b
*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((
2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))
```

Rubi in Sympy [A] time = 105.473, size = 258, normalized size = 0.98

$$\frac{fx^4}{4b^2}-\frac{x\left(2af-be\right)}{b^3}-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{3ab^3\left(a+bx^3\right)}+\frac{\left(7a^3f-4a^2be+ab^2d+2b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{10}{3}}}-\frac{\left(7a^3f-4a^2be+ab^2d+2b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{5}{3}}b^{\frac{10}{3}}}-\frac{\sqrt{3}\left(7a^3f-4a^2be+ab^2d+2b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] f*x**4/(4*b**2) - x*(2*a*f - b*e)/b**3 - x*(a**3*f - a**2*b*e + a
*b**2*d - b**3*c)/(3*a*b**3*(a + b*x**3)) + (7*a**3*f - 4*a**2*b*
e + a*b**2*d + 2*b**3*c)*log(a**(1/3) + b**(1/3)*x)/(9*a**(5/3)*b
**(10/3)) - (7*a**3*f - 4*a**2*b*e + a*b**2*d + 2*b**3*c)*log(a**
(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*a**(5/3)*b**(10/
3)) - sqrt(3)*(7*a**3*f - 4*a**2*b*e + a*b**2*d + 2*b**3*c)*atan(
sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*b**(1
0/3))
```

Mathematica [A] time = 0.281821, size = 251, normalized size = 0.95

$$\frac{\frac{12\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a\left(a+bx^3\right)}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{a^{5/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{a^{5/3}}-\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\right.}{}}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(b*e - 2*a*f)*x + 9*b^(4/3)*f*x^4 + (12*b^(1/3)*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(2
*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]])/a^(5/3) + (4*(2*b^3*c + a*b^2*d - 4*a^2*b*e +
 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(2*b^3*c + a*b^2
*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(5/3))/(36*b^(10/3))
```

Maple [B] time = 0.013, size = 482, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/4*f*x^4/b^2-2/b^3*a*f*x+1/b^2*e*x-1/3/b^3*x*a^2/(b*x^3+a)*f+1/3
/b^2*x*a/(b*x^3+a)*e-1/3/b*x/(b*x^3+a)*d+1/3*c*x/a/(b*x^3+a)+7/9/
b^4*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-4/9/b^3*a/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))*e+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+2/9*c/a/
b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-7/18/b^4*a^2/(a/b)^(2/3)*ln(x^2-x
*(a/b)^(1/3)+(a/b)^(2/3))*f+2/9/b^3*a/(a/b)^(2/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))*e-1/18/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a
/b)^(2/3))*d-1/9*c/a/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))+7/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*f-4/9/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))*e+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))*d+2/9*c/a/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235543, size = 494, normalized size = 1.87

$$-\frac{\sqrt{3}\left(2\sqrt{3}\left(2\,ab^3c + a^2b^2d - 4\,a^3be + 7\,a^4f + \left(2\,b^4c + ab^3d - 4\,a^2b^2e + 7\,a^3bf\right)x^3\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2 - \left(a^2b\right)^{\frac{1}{3}}ax + a^2\right) - 4\,\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/108*sqrt(3)*(2*sqrt(3)*(2*a*b^3*c + a^2*b^2*d - 4*a^3*b*e + 7*
a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^3)*log((a
^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 4*sqrt(3)*(2*a*b^3*c
 + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b
^2*e + 7*a^3*b*f)*x^3)*log((a^2*b)^(1/3)*x + a) - 12*(2*a*b^3*c +
 a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2
*e + 7*a^3*b*f)*x^3)*arctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)*x - sqrt
(3)*a)/a) - 3*sqrt(3)*(3*a*b^2*f*x^7 + 3*(4*a*b^2*e - 7*a^2*b*f)*
x^4 + 4*(b^3*c - a*b^2*d + 4*a^2*b*e - 7*a^3*f)*x)*(a^2*b)^(1/3))
/((a*b^4*x^3 + a^2*b^3)*(a^2*b)^(1/3))
```

Sympy [A] time = 11.7539, size = 376, normalized size = 1.42

$$-\frac{x\left(a^3f - a^2be + ab^2d - b^3c\right)}{3a^2b^3 + 3ab^4x^3}$$
$$+\operatorname{RootSum}\left(729t^3a^5b^{10} - 343a^9f^3 + 588a^8bef^2 - 147a^7b^2df^2 - 336a^7b^2e^2f - 294a^6b^3cf^2 + 168a^6b^3def + 64a^6b^3e^3 + 336\right.$$
$$+\frac{fx^4}{4b^2} - \frac{x\left(2af - be\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**2*b**3 + 3*a*b**
4*x**3) + RootSum(729*_t**3*a**5*b**10 - 343*a**9*f**3 + 588*a**8
*b*e*f**2 - 147*a**7*b**2*d*f**2 - 336*a**7*b**2*e**2*f - 294*a**
6*b**3*c*f**2 + 168*a**6*b**3*d*e*f + 64*a**6*b**3*e**3 + 336*a**
5*b**4*c*e*f - 21*a**5*b**4*d**2*f - 48*a**5*b**4*d*e**2 - 84*a**
4*b**5*c*d*f - 96*a**4*b**5*c*e**2 + 12*a**4*b**5*d**2*e - 84*a**
3*b**6*c**2*f + 48*a**3*b**6*c*d*e - a**3*b**6*d**3 + 48*a**2*b**
7*c**2*e - 6*a**2*b**7*c*d**2 - 12*a*b**8*c**2*d - 8*b**9*c**3, L
ambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*f - 4*a**2*b*e + a*b**2*d
 + 2*b**3*c) + x))) + f*x**4/(4*b**2) - x*(2*a*f - b*e)/b**3
```

GIAC/XCAS [A] time = 0.217001, size = 433, normalized size = 1.64

$$-\frac{\left(2\,b^{3}c+ab^{2}d+7\,a^{3}f-4\,a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}b^{3}}+\frac{\sqrt{3}\left(2\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c+\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d+7\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f-4\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b^{4}}+\frac{b^{3}cx-ab^{2}dx-a^{3}fx+a^{2}bxe}{3\,(bx^{3}+a)ab^{3}}+\frac{\left(2\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c+\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d+7\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f-4\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b^{4}}+\frac{b^{6}fx^{4}-8\,ab^{5}fx+4\,b^{6}xe}{4\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b^3*c + a*b^2*d + 7*a^3*f - 4*a^2*b*e)*(-a/b)^(1/3)*ln(ab
s(x - (-a/b)^(1/3)))/(a^2*b^3) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b^
3*c + (-a*b^2)^(1/3)*a*b^2*d + 7*(-a*b^2)^(1/3)*a^3*f - 4*(-a*b^2
)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a^2*b^4) + 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)
/((b*x^3 + a)*a*b^3) + 1/18*(2*(-a*b^2)^(1/3)*b^3*c + (-a*b^2)^(1
/3)*a*b^2*d + 7*(-a*b^2)^(1/3)*a^3*f - 4*(-a*b^2)^(1/3)*a^2*b*e)*
ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4) + 1/4*(b^6*f*x^
4 - 8*a*b^5*f*x + 4*b^6*x*e)/b^8
```

3.267 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=265

$$-\frac{c}{a^2x}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{18a^{7/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{9a^{7/3}b^{8/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{3\sqrt{3}a^{7/3}b^{8/3}}+\frac{fx^2}{2b^2}$$

```
[Out] -(c/(a^2*x)) + (f*x^2)/(2*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^2)/(3*a^2*b^2*(a + b*x^3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b
*e + 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(3*Sqrt[3]*a^(7/3)*b^(8/3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5
*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(8/3)) - ((4*b^3*c
 - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(18*a^(7/3)*b^(8/3))
```

Rubi [A] time = 0.594864, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{c}{a^2x}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{18a^{7/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{9a^{7/3}b^{8/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{3\sqrt{3}a^{7/3}b^{8/3}}+\frac{fx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```


```
[Out] -(c/(a^2*x)) + (f*x^2)/(2*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^2)/(3*a^2*b^2*(a + b*x^3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b
*e + 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(3*Sqrt[3]*a^(7/3)*b^(8/3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5
*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(8/3)) - ((4*b^3*c
 - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(18*a^(7/3)*b^(8/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{f\int x\,dx}{b^2}-\frac{x\left(\frac{a^3f}{x^2}-\frac{a^2be}{x^2}+\frac{ab^2d}{x^2}-\frac{b^3c}{x^2}\right)}{3ab^3\left(a+bx^3\right)}-\frac{a^2f-abe+b^2d}{ab^3x}+\frac{\left(3a^2f-2abe+b^2d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{4}{3}}b^{\frac{8}{3}}}-\frac{\left(3a^2f-2abe+b^2d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{4}{3}}b^{\frac{8}{3}}}+\frac{\sqrt{3}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{4}{3}}b^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] f*Integral(x, x)/b**2 - x*(a**3*f/x**2 - a**2*b*e/x**2 + a*b**2*d
/x**2 - b**3*c/x**2)/(3*a*b**3*(a + b*x**3)) - (a**2*f - a*b*e +
b**2*d)/(a*b**3*x) + (3*a**2*f - 2*a*b*e + b**2*d)*log(a**(1/3) +
 b**(1/3)*x)/(3*a**(4/3)*b**(8/3)) - (3*a**2*f - 2*a*b*e + b**2*d
)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(4/3)
*b**(8/3)) + sqrt(3)*(3*a**2*f - 2*a*b*e + b**2*d)*atan(sqrt(3)*(
a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(4/3)*b**(8/3))
```

Mathematica [A] time = 0.290292, size = 255, normalized size = 0.96

$$\frac{1}{18}\left(-\frac{18c}{a^2x}+\frac{6x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{a^{7/3}b^{8/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{a^{7/3}b^{8/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{a^{7/3}b^{8/3}}+\frac{9fx^2}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*c)/(a^2*x) + (9*f*x^2)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b
*e + a^3*f)*x^2)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(4*b^3*c - a*
b^2*d - 2*a^2*b*e + 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]])/(a^(7/3)*b^(8/3)) + (2*(4*b^3*c - a*b^2*d - 2*a^2*b*e +
5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(7/3)*b^(8/3)) - ((4*b^3*c
- a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(a^(7/3)*b^(8/3)))/18
```

Maple [B] time = 0.018, size = 474, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] 1/2*f*x^2/b^2-1/a^2*c/x+1/3*a/b^2*x^2/(b*x^3+a)*f-1/3/b*x^2/(b*x^
3+a)*e+1/3*d*x^2/a/(b*x^3+a)-1/3/a^2*b*x^2/(b*x^3+a)*c+5/9*a/b^3*
f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18*a/b^3*f/(a/b)^(1/3)*ln(x^2-x
*(a/b)^(1/3)+(a/b)^(2/3))-5/9*a/b^3*f*3^(1/2)/(a/b)^(1/3)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9*d/a/b/(a/b)^(1/3)*ln(x+(a/b)
```

```
^(1/3))+1/18*d/a/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+
1/9*d/a*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))+4/9/a^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*c/(a/b)^(1/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-4/9/a^2*c*3^(1/2)/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/9/b^2*e/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))+1/9/b^2*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^
(2/3))+2/9/b^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238255, size = 510, normalized size = 1.92

$$-\frac{\sqrt{3}\left(\sqrt{3}\left(\left(4\,b^4c-ab^3d-2\,a^2b^2e+5\,a^3bf\right)x^4+\left(4\,ab^3c-a^2b^2d-2\,a^3be+5\,a^4f\right)x\right)\log\left(\left(ab^2\right)^{\frac{1}{3}}bx^2+ab-\left(ab^2\right)^{\frac{2}{3}}x\right)-2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/54*sqrt(3)*(sqrt(3)*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*
b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*log((
a*b^2)^(1/3)*b*x^2 + a*b - (a*b^2)^(2/3)*x) - 2*sqrt(3)*((4*b^4*c
 - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*
d - 2*a^3*b*e + 5*a^4*f)*x)*log(a*b + (a*b^2)^(2/3)*x) + 6*((4*b^
4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b
^2*d - 2*a^3*b*e + 5*a^4*f)*x)*arctan(-1/3*(sqrt(3)*a*b - 2*sqrt(
3)*(a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(3*a^2*b*f*x^6 - 6*a*b^2*c
 - (8*b^3*c - 2*a*b^2*d + 2*a^2*b*e - 5*a^3*f)*x^3)*(a*b^2)^(1/3)
)/((a^2*b^3*x^4 + a^3*b^2*x)*(a*b^2)^(1/3))
```

Sympy [A] time = 49.411, size = 457, normalized size = 1.72

$$\frac{-3ab^2c+x^3\left(a^3f-a^2be+ab^2d-4b^3c\right)}{3a^3b^2x+3a^2b^3x^4}$$
$$+\operatorname{RootSum}\left(729t^3a^7b^8-125a^9f^3+150a^8bef^2+75a^7b^2df^2-60a^7b^2e^2f-300a^6b^3cf^2-60a^6b^3def+8a^6b^3e^3+240a^5b^4c\cdots\right.$$
$$+\frac{fx^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*a*b**2*c + x**3*(a**3*f - a**2*b*e + a*b**2*d - 4*b**3*c))/(3
*a**3*b**2*x + 3*a**2*b**3*x**4) + RootSum(729*_t**3*a**7*b**8 -
125*a**9*f**3 + 150*a**8*b*e*f**2 + 75*a**7*b**2*d*f**2 - 60*a**7
*b**2*e**2*f - 300*a**6*b**3*c*f**2 - 60*a**6*b**3*d*e*f + 8*a**6
*b**3*e**3 + 240*a**5*b**4*c*e*f - 15*a**5*b**4*d**2*f + 12*a**5*
b**4*d*e**2 + 120*a**4*b**5*c*d*f - 48*a**4*b**5*c*e**2 + 6*a**4*
```

```
b**5*d**2*e - 240*a**3*b**6*c**2*f - 48*a**3*b**6*c*d*e + a**3*b*
*6*d**3 + 96*a**2*b**7*c**2*e - 12*a**2*b**7*c*d**2 + 48*a*b**8*c
**2*d - 64*b**9*c**3, Lambda(_t, _t*log(81*_t**2*a**5*b**5/(25*a*
*6*f**2 - 20*a**5*b*e*f - 10*a**4*b**2*d*f + 4*a**4*b**2*e**2 + 4
0*a**3*b**3*c*f + 4*a**3*b**3*d*e - 16*a**2*b**4*c*e + a**2*b**4*
d**2 - 8*a*b**5*c*d + 16*b**6*c**2) + x))) + f*x**2/(2*b**2)
```

GIAC/XCAS [A] time = 0.219279, size = 477, normalized size = 1.8

$$\frac{fx^2}{2\,b^2}+\frac{\left(4\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^2}-\frac{4\,b^3cx^3-ab^2dx^3-a^3fx^3+a^2bx^3e+3\,ab^2c}{3\,(bx^4+ax)a^2b^2}+\frac{\sqrt{3}\left(4\,\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d+5\,\left(-ab^2\right)^{\frac{2}{3}}a^3f-2\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^4}-\frac{\left(4\,\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d+5\,\left(-ab^2\right)^{\frac{2}{3}}a^3f-2\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^2 + 1/9*(4*b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3)
+ 5*a^3*f*(-a/b)^(1/3) - 2*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(
abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/3*(4*b^3*c*x^3 - a*b^2*d*x^3
 - a^3*f*x^3 + a^2*b*x^3*e + 3*a*b^2*c)/((b*x^4 + a*x)*a^2*b^2) +
 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + 5
*(-a*b^2)^(2/3)*a^3*f - 2*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^4) - 1/18*(4*(-a*b^
2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + 5*(-a*b^2)^(2/3)*a^3*f
- 2*(-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a^3*b^4)
```

3.268 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=260

$$-\frac{c}{2a^2x^2}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{18a^{8/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{9a^{8/3}b^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{3\sqrt{3}a^{8/3}b^{7/3}}+\frac{fx}{b^2}$$

```
[Out] -c/(2*a^2*x^2) + (f*x)/b^2 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*x)/(3*a^2*b^2*(a + b*x^3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4
*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqr
t[3]*a^(8/3)*b^(7/3)) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f
)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^(7/3)) + ((5*b^3*c - 2*a
*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*a^(8/3)*b^(7/3))
```

Rubi [A] time = 0.573303, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{c}{2a^2x^2}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{18a^{8/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{9a^{8/3}b^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{3\sqrt{3}a^{8/3}b^{7/3}}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -c/(2*a^2*x^2) + (f*x)/b^2 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*x)/(3*a^2*b^2*(a + b*x^3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4
*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqr
t[3]*a^(8/3)*b^(7/3)) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f
)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^(7/3)) + ((5*b^3*c - 2*a
*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*a^(8/3)*b^(7/3))
```

Rubi in Sympy [A] time = 132.498, size = 250, normalized size = 0.96

$$\frac{fx}{b^2}-\frac{x\left(\frac{a^3f}{x^3}-\frac{a^2be}{x^3}+\frac{ab^2d}{x^3}-\frac{b^3c}{x^3}\right)}{3ab^3\left(a+bx^3\right)}-\frac{a^2f-abe+b^2d}{2ab^3x^2}-\frac{\left(3a^2f-2abe+b^2d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{5}{3}}b^{\frac{7}{3}}}+\frac{\left(3a^2f-2abe+b^2d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{5}{3}}b^{\frac{7}{3}}}+\frac{\sqrt{3}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{5}{3}}b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] f*x/b**2 - x*(a**3*f/x**3 - a**2*b*e/x**3 + a*b**2*d/x**3 - b**3*
c/x**3)/(3*a*b**3*(a + b*x**3)) - (a**2*f - a*b*e + b**2*d)/(2*a*
b**3*x**2) - (3*a**2*f - 2*a*b*e + b**2*d)*log(a**(1/3) + b**(1/3
)*x)/(3*a**(5/3)*b**(7/3)) + (3*a**2*f - 2*a*b*e + b**2*d)*log(a*
*(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(5/3)*b**(7/3
)) + sqrt(3)*(3*a**2*f - 2*a*b*e + b**2*d)*atan(sqrt(3)*(a**(1/3)
/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(5/3)*b**(7/3))
```

Mathematica [A]   time = 0.282511, size = 250, normalized size = 0.96

$$\frac{1}{18}\left(-\frac{9c}{a^2x^2}+\frac{6x\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{a^{8/3}b^{7/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{a^{8/3}b^{7/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{a^{8/3}b^{7/3}}+\frac{18fx}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*c)/(a^2*x^2) + (18*f*x)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b
*e + a^3*f)*x)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(5*b^3*c - 2*a*
b^2*d - a^2*b*e + 4*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/(a^(8/3)*b^(7/3)) - (2*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*
a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(8/3)*b^(7/3)) + ((5*b^3*c -
2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(a^(8/3)*b^(7/3)))/18
```

Maple [B]   time = 0.016, size = 463, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] f*x/b^2-1/2*c/a^2/x^2+1/3*a/b^2*x/(b*x^3+a)*f-1/3/b*x/(b*x^3+a)*e
+1/3/a*x/(b*x^3+a)*d-1/3/a^2*b*x/(b*x^3+a)*c-4/9*a/b^3*f/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))+2/9*a/b^3*f/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3
)+(a/b)^(2/3))-4/9*a/b^3*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))+2/9/a/b*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9
```

```
/a/b*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9/a/b*d/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-5/9/a^2
*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/18/a^2*c/(a/b)^(2/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))-5/9/a^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9/b^2*e/(a/b)^(2/3)*ln(x+(a/b)^(1
/3))-1/18/b^2*e/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/9
/b^2*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239142, size = 522, normalized size = 2.01

$$-\frac{\sqrt{3}\left(\sqrt{3}\left(\left(5\,b^4c-2\,ab^3d-a^2b^2e+4\,a^3bf\right)x^5+\left(5\,ab^3c-2\,a^2b^2d-a^3be+4\,a^4f\right)x^2\right)\log\left(\left(-a^2b\right)^{\frac{2}{3}}x^2+\left(-a^2b\right)^{\frac{1}{3}}ax+a^2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/54*sqrt(3)*(sqrt(3)*((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*
b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*log
((-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*a*x + a^2) - 2*sqrt(3)*((5*b
^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^
2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*log((-a^2*b)^(1/3)*x - a) + 6*(
(5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c -
2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*arctan(1/3*(2*sqrt(3)*(-a^2
*b)^(1/3)*x + sqrt(3)*a)/a) - 3*sqrt(3)*(6*a^2*b*f*x^6 - 3*a*b^2*
c - (5*b^3*c - 2*a*b^2*d + 2*a^2*b*e - 8*a^3*f)*x^3)*(-a^2*b)^(1/
3))/((a^2*b^3*x^5 + a^3*b^2*x^2)*(-a^2*b)^(1/3))
```

Sympy [A] time = 107.283, size = 381, normalized size = 1.47

$$\frac{-3ab^2c+x^3\left(2a^3f-2a^2be+2ab^2d-5b^3c\right)}{6a^3b^2x^2+6a^2b^3x^5}$$
$$+\operatorname{RootSum}\left(729t^3a^8b^7+64a^9f^3-48a^8bef^2-96a^7b^2df^2+12a^7b^2e^2f+240a^6b^3cf^2+48a^6b^3def-a^6b^3e^3-120a^5b^4cef\right.$$
$$+\frac{fx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*a*b**2*c + x**3*(2*a**3*f - 2*a**2*b*e + 2*a*b**2*d - 5*b**3*
c))/(6*a**3*b**2*x**2 + 6*a**2*b**3*x**5) + RootSum(729*_t**3*a**
8*b**7 + 64*a**9*f**3 - 48*a**8*b*e*f**2 - 96*a**7*b**2*d*f**2 +
12*a**7*b**2*e**2*f + 240*a**6*b**3*c*f**2 + 48*a**6*b**3*d*e*f -
 a**6*b**3*e**3 - 120*a**5*b**4*c*e*f + 48*a**5*b**4*d**2*f - 6*a
**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f + 15*a**4*b**5*c*e**2 - 12*
```

```
a**4*b**5*d**2*e + 300*a**3*b**6*c**2*f + 60*a**3*b**6*c*d*e - 8*
a**3*b**6*d**3 - 75*a**2*b**7*c**2*e + 60*a**2*b**7*c*d**2 - 150*
a*b**8*c**2*d + 125*b**9*c**3, Lambda(_t, _t*log(-9*_t*a**3*b**2/
(4*a**3*f - a**2*b*e - 2*a*b**2*d + 5*b**3*c) + x))) + f*x/b**2
```

GIAC/XCAS [A] time = 0.217019, size = 417, normalized size = 1.6

$$\frac{fx}{b^2} + \frac{\left(5\,b^3c - 2\,ab^2d + 4\,a^3f - a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^2} - \frac{c}{2\,a^2x^2}$$
$$- \frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^3c - 2\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d + 4\,\left(-ab^2\right)^{\frac{1}{3}}a^3f - \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^3}$$
$$- \frac{b^3cx - ab^2dx - a^3fx + a^2bxe}{3\,(bx^3 + a)a^2b^2}$$
$$- \frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^3c - 2\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d + 4\,\left(-ab^2\right)^{\frac{1}{3}}a^3f - \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] f*x/b^2 + 1/9*(5*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e)*(-a/b)^(1
/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/2*c/(a^2*x^2) - 1/9*s
qrt(3)*(5*(-a*b^2)^(1/3)*b^3*c - 2*(-a*b^2)^(1/3)*a*b^2*d + 4*(-a
*b^2)^(1/3)*a^3*f - (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) - 1/3*(b^3*c*x - a*b^2
*d*x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a^2*b^2) - 1/18*(5*(-a*b
^2)^(1/3)*b^3*c - 2*(-a*b^2)^(1/3)*a*b^2*d + 4*(-a*b^2)^(1/3)*a^3
*f - (-a*b^2)^(1/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/(a^3*b^3)
```

3.269 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx$

Optimal. Leaf size=269

$$\frac{2bc-ad}{a^3x}-\frac{c}{4a^2x^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{18a^{10/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{9a^{10/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{3\sqrt{3}a^{10/3}b^{5/3}}$$

```
[Out] -c/(4*a^2*x^4) + (2*b*c - a*d)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^2)/(3*a^3*b*(a + b*x^3)) - ((7*b^3*c - 4*a*b^2*d +
 a^2*b*e + 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(3*Sqrt[3]*a^(10/3)*b^(5/3)) - ((7*b^3*c - 4*a*b^2*d + a^2*
b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(10/3)*b^(5/3)) + (
(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3)*b^(5/3))
```

Rubi [A] time = 0.656291, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{2bc-ad}{a^3x}-\frac{c}{4a^2x^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{18a^{10/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{9a^{10/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{3\sqrt{3}a^{10/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] -c/(4*a^2*x^4) + (2*b*c - a*d)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^2)/(3*a^3*b*(a + b*x^3)) - ((7*b^3*c - 4*a*b^2*d +
 a^2*b*e + 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(3*Sqrt[3]*a^(10/3)*b^(5/3)) - ((7*b^3*c - 4*a*b^2*d + a^2*
b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(10/3)*b^(5/3)) + (
(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3)*b^(5/3))
```

Rubi in Sympy [A] time = 139.282, size = 270, normalized size = 1.

$$-\frac{x\left(\frac{a^3f}{x^5}-\frac{a^2be}{x^5}+\frac{ab^2d}{x^5}-\frac{b^3c}{x^5}\right)}{3ab^3\left(a+bx^3\right)}-\frac{a^2f-abe+b^2d}{4ab^3x^4}+\frac{2a^2f-2abe+b^2d}{a^2b^2x}-\frac{\left(3a^2f-2abe+b^2d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{7}{3}}b^{\frac{5}{3}}}+\frac{\left(3a^2f-2abe+b^2d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{7}{3}}b^{\frac{5}{3}}}-\frac{\sqrt{3}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{7}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*f/x**5 - a**2*b*e/x**5 + a*b**2*d/x**5 - b**3*c/x**5)/(3
*a*b**3*(a + b*x**3)) - (a**2*f - a*b*e + b**2*d)/(4*a*b**3*x**4)
 + (2*a**2*f - 2*a*b*e + b**2*d)/(a**2*b**2*x) - (3*a**2*f - 2*a*
b*e + b**2*d)*log(a**(1/3) + b**(1/3)*x)/(3*a**(7/3)*b**(5/3)) +
(3*a**2*f - 2*a*b*e + b**2*d)*log(a**(2/3) - a**(1/3)*b**(1/3)*x
+ b**(2/3)*x**2)/(6*a**(7/3)*b**(5/3)) - sqrt(3)*(3*a**2*f - 2*a*
b*e + b**2*d)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3)
)/(3*a**(7/3)*b**(5/3))
```

Mathematica [A] time = 0.301137, size = 255, normalized size = 0.95

$$\frac{-\frac{9a^{4/3}c}{x^4}-\frac{12\sqrt[3]{a}x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{b\left(a+bx^3\right)}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{b^{5/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{b^{5/3}}+2\log\cdots}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(4/3)*c)/x^4 - (36*a^(1/3)*(-2*b*c + a*d))/x - (12*a^(1/3)
*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4
*Sqrt[3]*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) - (4*(7*b^3*c - 4*a*b^2*d
+ a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) + (2*(7*b^
3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/b^(5/3))/(36*a^(10/3))
```

Maple [B] time = 0.022, size = 486, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x)
```

```
[Out] -1/4*c/x^4/a^2-d/a^2/x+2/a^3/x*b*c-1/3/b*x^2/(b*x^3+a)*f+1/3/a*x^
2/(b*x^3+a)*e-1/3/a^2*b*x^2/(b*x^3+a)*d+1/3/a^3*b^2*x^2/(b*x^3+a)
*c-2/9/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-1/9/a/b/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))*e+4/9/a^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-7/9/a^3
*b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/9/b^2/(a/b)^(1/3)*ln(x^2-x*(
a/b)^(1/3)+(a/b)^(2/3))*f+1/18/a/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/
```

```
3)+(a/b)^(2/3))*e-2/9/a^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^
(2/3))*d+7/18/a^3*b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
*c+2/9/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))*f+1/9/a/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*e-4/9/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))*d+7/9/a^3*b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23711, size = 532, normalized size = 1.98

$$-\frac{\sqrt{3}\left(2\sqrt{3}\left(\left(7\,b^4c-4\,ab^3d+a^2b^2e+2\,a^3bf\right)x^7+\left(7\,ab^3c-4\,a^2b^2d+a^3be+2\,a^4f\right)x^4\right)\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2-ab+\left(-ab^2\right)^{\frac{2}{3}}x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^5),x, algorithm="fricas")
```

```
[Out] -1/108*sqrt(3)*(2*sqrt(3)*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a
^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*
log((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 4*sqrt(3)*((
7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4
*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*log(a*b + (-a*b^2)^(2/3)*x)
+ 12*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^
3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*arctan(-1/3*(sqrt(3)*
a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(4*(7*b^3*c
- 4*a*b^2*d + a^2*b*e - a^3*f)*x^6 - 3*a^2*b*c + 3*(7*a*b^2*c - 4
*a^2*b*d)*x^3)*(-a*b^2)^(1/3))/((a^3*b^2*x^7 + a^4*b*x^4)*(-a*b^2
)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.220432, size = 483, normalized size = 1.8

$$-\frac{\left(7\,b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{4}b}+\frac{b^{3}cx^{2}-ab^{2}dx^{2}-a^{3}fx^{2}+a^{2}bx^{2}e}{3\,(bx^{3}+a)a^{3}b}-\frac{\sqrt{3}\left(7\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-4\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d+2\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{4}b^{3}}+\frac{\left(7\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-4\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d+2\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{4}b^{3}}+\frac{8\,bcx^{3}-4\,adx^{3}-ac}{4\,a^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^5),x, algorithm="giac")
```

```
[Out] -1/9*(7*b^3*c*(-a/b)^(1/3) - 4*a*b^2*d*(-a/b)^(1/3) + 2*a^3*f*(-a
/b)^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^
(1/3)))/(a^4*b) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*
b*x^2*e)/((b*x^3 + a)*a^3*b) - 1/9*sqrt(3)*(7*(-a*b^2)^(2/3)*b^3*
c - 4*(-a*b^2)^(2/3)*a*b^2*d + 2*(-a*b^2)^(2/3)*a^3*f + (-a*b^2)^
(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(a^4*b^3) + 1/18*(7*(-a*b^2)^(2/3)*b^3*c - 4*(-a*b^2)^(2/3)*a
*b^2*d + 2*(-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*ln(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^3) + 1/4*(8*b*c*x^3 - 4*a
*d*x^3 - a*c)/(a^3*x^4)
```

3.270 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx$

Optimal. Leaf size=270

$$\frac{2bc-ad}{2a^3x^2}-\frac{c}{5a^2x^5}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\,(a+bx^3)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{18a^{11/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{9a^{11/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{3\sqrt{3}a^{11/3}b^{4/3}}$$

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(2*a^3*x^2) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x)/(3*a^3*b*(a + b*x^3)) - ((8*b^3*c - 5*a*b^2*d
 + 2*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(3*Sqrt[3]*a^(11/3)*b^(4/3)) + ((8*b^3*c - 5*a*b^2*d + 2*
a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(11/3)*b^(4/3)) -
 ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3)*b^(4/3))
```

Rubi [A] time = 0.646128, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{2bc-ad}{2a^3x^2}-\frac{c}{5a^2x^5}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\,(a+bx^3)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{18a^{11/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{9a^{11/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{3\sqrt{3}a^{11/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(2*a^3*x^2) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x)/(3*a^3*b*(a + b*x^3)) - ((8*b^3*c - 5*a*b^2*d
 + 2*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(3*Sqrt[3]*a^(11/3)*b^(4/3)) + ((8*b^3*c - 5*a*b^2*d + 2*
a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(11/3)*b^(4/3)) -
 ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3)*b^(4/3))
```

Rubi in Sympy [A] time = 141.426, size = 274, normalized size = 1.01

$$-\frac{x\left(\frac{a^3f}{x^6}-\frac{a^2be}{x^6}+\frac{ab^2d}{x^6}-\frac{b^3c}{x^6}\right)}{3ab^3\left(a+bx^3\right)}-\frac{a^2f-abe+b^2d}{5ab^3x^5}+\frac{2a^2f-2abe+b^2d}{2a^2b^2x^2}+\frac{\left(3a^2f-2abe+b^2d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{8}{3}}b^{\frac{4}{3}}}-\frac{\left(3a^2f-2abe+b^2d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{8}{3}}b^{\frac{4}{3}}}-\frac{\sqrt{3}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{8}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*f/x**6 - a**2*b*e/x**6 + a*b**2*d/x**6 - b**3*c/x**6)/(3
*a*b**3*(a + b*x**3)) - (a**2*f - a*b*e + b**2*d)/(5*a*b**3*x**5)
 + (2*a**2*f - 2*a*b*e + b**2*d)/(2*a**2*b**2*x**2) + (3*a**2*f -
 2*a*b*e + b**2*d)*log(a**(1/3) + b**(1/3)*x)/(3*a**(8/3)*b**(4/3
)) - (3*a**2*f - 2*a*b*e + b**2*d)*log(a**(2/3) - a**(1/3)*b**(1/
3)*x + b**(2/3)*x**2)/(6*a**(8/3)*b**(4/3)) - sqrt(3)*(3*a**2*f -
 2*a*b*e + b**2*d)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**
(1/3))/(3*a**(8/3)*b**(4/3))
```

Mathematica [A] time = 0.293454, size = 253, normalized size = 0.94

$$\frac{-\frac{45a^{2/3}(ad-2bc)}{x^2}-\frac{18a^{5/3}c}{x^5}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{b^{4/3}}-\frac{10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{b^{4/3}}-\frac{30a^{2/3}x\left(a^3f-a^2b\ldots\right.}{b(a+b\ldots}}{90a^{11/3}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*c)/x^5 - (45*a^(2/3)*(-2*b*c + a*d))/x^2 - (30*a^(2
/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (
10*Sqrt[3]*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3) + (10*(8*b^3*c - 5*a*b^2
*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (5*(8
*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/b^(4/3))/(90*a^(11/3))
```

Maple [B] time = 0.019, size = 477, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x)
```

```
[Out] -1/5*c/a^2/x^5-1/2*d/a^2/x^2+1/a^3/x^2*b*c-1/3*x/(b*x^3+a)/b*f+1/
3/a*x/(b*x^3+a)*e-1/3/a^2*x/(b*x^3+a)*b*d+1/3/a^3*x/(b*x^3+a)*b^2
*c+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+2/9/a/b/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))*e-5/9/a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+8/9/a^3
*b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/18/b^2/(a/b)^(2/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))*f-1/9/a/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/
```

```
3)+(a/b)^(2/3))*e+5/18/a^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)
^(2/3))*d-4/9/a^3*b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
*c+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))*f+2/9/a/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*e-5/9/a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))*d+8/9/a^3*b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.246023, size = 518, normalized size = 1.92

$$-\frac{\sqrt{3}\left(5\sqrt{3}\left(\left(8\,b^4c-5\,ab^3d+2\,a^2b^2e+a^3bf\right)x^8+\left(8\,ab^3c-5\,a^2b^2d+2\,a^3be+a^4f\right)x^5\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^6),x, algorithm="fricas")
```

```
[Out] -1/270*sqrt(3)*(5*sqrt(3)*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a
^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*
log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 10*sqrt(3)*((8
*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*
a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*log((a^2*b)^(1/3)*x + a) - 30
*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c
- 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*arctan(1/3*(2*sqrt(3)*(a^
2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*(5*(8*b^3*c - 5*a*b^2*d
+ 2*a^2*b*e - 2*a^3*f)*x^6 - 6*a^2*b*c + 3*(8*a*b^2*c - 5*a^2*b*d
)*x^3)*(a^2*b)^(1/3))/((a^3*b^2*x^8 + a^4*b*x^5)*(a^2*b)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219709, size = 429, normalized size = 1.59

$$-\frac{\left(8\,b^{3}c-5\,ab^{2}d+a^{3}f+2\,a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{4}b}+\frac{\sqrt{3}\left(8\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-5\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d+\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+2\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{4}b^{2}}+\frac{b^{3}cx-ab^{2}dx-a^{3}fx+a^{2}bxe}{3\,(bx^{3}+a)a^{3}b}+\frac{\left(8\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-5\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d+\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+2\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{4}b^{2}}+\frac{10\,bcx^{3}-5\,adx^{3}-2\,ac}{10\,a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^6),x, algorithm="giac")
```

```
[Out] -1/9*(8*b^3*c - 5*a*b^2*d + a^3*f + 2*a^2*b*e)*(-a/b)^(1/3)*ln(ab
s(x - (-a/b)^(1/3)))/(a^4*b) + 1/9*sqrt(3)*(8*(-a*b^2)^(1/3)*b^3*
c - 5*(-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(1/3)*a^3*f + 2*(-a*b^2)^
(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(a^4*b^2) + 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/(
(b*x^3 + a)*a^3*b) + 1/18*(8*(-a*b^2)^(1/3)*b^3*c - 5*(-a*b^2)^(1
/3)*a*b^2*d + (-a*b^2)^(1/3)*a^3*f + 2*(-a*b^2)^(1/3)*a^2*b*e)*ln
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^2) + 1/10*(10*b*c*x^
3 - 5*a*d*x^3 - 2*a*c)/(a^3*x^5)
```

3.271 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$

Optimal. Leaf size=297

$$\frac{2bc-ad}{4a^3x^4}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{a^4x}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{18a^{13/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{9a^{13/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{3\sqrt{3}a^{13/3}b^{2/3}}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}$$

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(4*a^3*x^4) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^
4*(a + b*x^3)) + ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcT
an[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(13/3
)*b^(2/3)) + ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(1
/3) + b^(1/3)*x])/(9*a^(13/3)*b^(2/3)) - ((10*b^3*c - 7*a*b^2*d +
 4*a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(13/3)*b^(2/3))
```

Rubi [A] time = 0.784527, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{2bc-ad}{4a^3x^4}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{a^4x}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{18a^{13/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{9a^{13/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{3\sqrt{3}a^{13/3}b^{2/3}}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```



```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(4*a^3*x^4) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^
4*(a + b*x^3)) + ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcT
an[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(13/3
)*b^(2/3)) + ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(1
/3) + b^(1/3)*x])/(9*a^(13/3)*b^(2/3)) - ((10*b^3*c - 7*a*b^2*d +
 4*a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(13/3)*b^(2/3))
```

Rubi in Sympy [A] time = 147.304, size = 299, normalized size = 1.01

$$-\frac{x\left(\frac{a^3f}{x^8}-\frac{a^2be}{x^8}+\frac{ab^2d}{x^8}-\frac{b^3c}{x^8}\right)}{3ab^3\left(a+bx^3\right)}-\frac{a^2f-abe+b^2d}{7ab^3x^7}+\frac{2a^2f-2abe+b^2d}{4a^2b^2x^4}-\frac{3a^2f-2abe+b^2d}{a^3bx}+\frac{\left(3a^2f-2abe+b^2d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{10}{3}}b^{\frac{2}{3}}}-\frac{\left(3a^2f-2abe+b^2d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{10}{3}}b^{\frac{2}{3}}}+\frac{\sqrt{3}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{10}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*f/x**8 - a**2*b*e/x**8 + a*b**2*d/x**8 - b**3*c/x**8)/(3
*a*b**3*(a + b*x**3)) - (a**2*f - a*b*e + b**2*d)/(7*a*b**3*x**7)
 + (2*a**2*f - 2*a*b*e + b**2*d)/(4*a**2*b**2*x**4) - (3*a**2*f -
 2*a*b*e + b**2*d)/(a**3*b*x) + (3*a**2*f - 2*a*b*e + b**2*d)*log
(a**(1/3) + b**(1/3)*x)/(3*a**(10/3)*b**(2/3)) - (3*a**2*f - 2*a*
b*e + b**2*d)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)
/(6*a**(10/3)*b**(2/3)) + sqrt(3)*(3*a**2*f - 2*a*b*e + b**2*d)*a
tan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(10/3)*
b**(2/3))
```

Mathematica [A] time = 0.340983, size = 281, normalized size = 0.95

$$\frac{-\frac{63a^{4/3}(ad-2bc)}{x^4}-\frac{36a^{7/3}c}{x^7}-\frac{252\sqrt[3]{a}\left(a^2e-2abd+3b^2c\right)}{x}+\frac{84\sqrt[3]{a}x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{b^{2/3}}+-}{252a^{13/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] ((-36*a^(7/3)*c)/x^7 - (63*a^(4/3)*(-2*b*c + a*d))/x^4 - (252*a^(
1/3)*(3*b^2*c - 2*a*b*d + a^2*e))/x + (84*a^(1/3)*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(10*b^3*c
- 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/b^(2/3) + (28*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^
3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-10*b^3*c + 7*a*b^2
*d - 4*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/b^(2/3))/(252*a^(13/3))
```

Maple [B] time = 0.021, size = 529, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x)
```

```
[Out] -1/7*c/a^2/x^7-1/4/a^2/x^4*d+1/2/a^3/x^4*b*c-e/a^2/x+2/a^3/x*b*d-
3/a^4/x*b^2*c+1/3/a*x^2/(b*x^3+a)*f-1/3/a^2*x^2/(b*x^3+a)*b*e+1/3
/a^3*x^2/(b*x^3+a)*b^2*d-1/3/a^4*x^2/(b*x^3+a)*b^3*c+4/9/a^2*e/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^
```

```
(1/3)+(a/b)^(2/3))-4/9/a^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))-7/9/a^3*b*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
+7/18/a^3*b*d/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+7/9/a
^3*b*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)+10/9/a^4*b^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/9/a^4*b^2*c/(a/b
)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-10/9/a^4*b^2*c*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a*f/b/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))+1/18/a*f/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))+1/9/a*f*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248018, size = 570, normalized size = 1.92

$$\frac{\sqrt{3}\left(14\sqrt{3}\left(\left(10\,b^4c - 7\,ab^3d + 4\,a^2b^2e - a^3bf\right)x^{10} + \left(10\,ab^3c - 7\,a^2b^2d + 4\,a^3be - a^4f\right)x^7\right)\log\left(\left(-ab^2\right)^{\frac{1}{3}}bx^2 - ab + \left(-ab^2\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^8),x, algorithm="fricas")
```

```
[Out] 1/756*sqrt(3)*(14*sqrt(3)*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e -
a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^
7)*log((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 28*sqrt(3
)*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^
3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*log(a*b + (-a*b^2)^(2
/3)*x) + 84*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10
+ (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*arctan(-1/3
*(sqrt(3)*a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(2
8*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*x^9 + 21*(10*a*b^2*c
 - 7*a^2*b*d + 4*a^3*e)*x^6 + 12*a^3*c - 3*(10*a^2*b*c - 7*a^3*d)
*x^3)*(-a*b^2)^(1/3))/((a^4*b*x^10 + a^5*x^7)*(-a*b^2)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218856, size = 522, normalized size = 1.76

$$\frac{\left(10\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5}-\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e}{3\,(bx^3+a)a^4}+\frac{\sqrt{3}\left(10\left(-ab^2\right)^{\frac{2}{3}}b^3c-7\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\left(-ab^2\right)^{\frac{2}{3}}a^3f+4\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5b^2}-\frac{\left(10\left(-ab^2\right)^{\frac{2}{3}}b^3c-7\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\left(-ab^2\right)^{\frac{2}{3}}a^3f+4\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5b^2}-\frac{84\,b^2cx^6-56\,abdx^6+28\,a^2x^6e-14\,abcx^3+7\,a^2dx^3+4\,a^2c}{28\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^8),x, algorithm="giac")
```

```
[Out] 1/9*(10*b^3*c*(-a/b)^(1/3) - 7*a*b^2*d*(-a/b)^(1/3) - a^3*f*(-a/b
)^(1/3) + 4*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^
(1/3)))/a^5 - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^
2*e)/((b*x^3 + a)*a^4) + 1/9*sqrt(3)*(10*(-a*b^2)^(2/3)*b^3*c - 7
*(-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + 4*(-a*b^2)^(2/3)
*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(
a^5*b^2) - 1/18*(10*(-a*b^2)^(2/3)*b^3*c - 7*(-a*b^2)^(2/3)*a*b^2
*d - (-a*b^2)^(2/3)*a^3*f + 4*(-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^2) - 1/28*(84*b^2*c*x^6 - 56*
a*b*d*x^6 + 28*a^2*x^6*e - 14*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/
(a^4*x^7)
```

3.272 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^2}\,dx$

Optimal. Leaf size=297

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{8a^2x^8}-\frac{a^2e-2abd+3b^2c}{2a^4x^2}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{9a^{14/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{3\sqrt{3}a^{14/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{18a^{14/3}\sqrt[3]{b}}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}$$

```
[Out] -c/(8*a^2*x^8) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(2*a^4*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*
a^4*(a + b*x^3)) + ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
14/3)*b^(1/3)) - ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Lo
g[a^(1/3) + b^(1/3)*x])/(9*a^(14/3)*b^(1/3)) + ((11*b^3*c - 8*a*b
^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*a^(14/3)*b^(1/3))
```

Rubi [A] time = 0.758633, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{8a^2x^8}-\frac{a^2e-2abd+3b^2c}{2a^4x^2}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{9a^{14/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{3\sqrt{3}a^{14/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{18a^{14/3}\sqrt[3]{b}}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```


```
[Out] -c/(8*a^2*x^8) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(2*a^4*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*
a^4*(a + b*x^3)) + ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
14/3)*b^(1/3)) - ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Lo
g[a^(1/3) + b^(1/3)*x])/(9*a^(14/3)*b^(1/3)) + ((11*b^3*c - 8*a*b
^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*a^(14/3)*b^(1/3))
```

Rubi in Sympy [A] time = 153.905, size = 303, normalized size = 1.02

$$-\frac{x\left(\frac{a^3f}{x^9}-\frac{a^2be}{x^9}+\frac{ab^2d}{x^9}-\frac{b^3c}{x^9}\right)}{3ab^3\left(a+bx^3\right)}-\frac{a^2f-abe+b^2d}{8ab^3x^8}+\frac{2a^2f-2abe+b^2d}{5a^2b^2x^5}-\frac{3a^2f-2abe+b^2d}{2a^3bx^2}-\frac{\left(3a^2f-2abe+b^2d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{11}{3}}\sqrt[3]{b}}+\frac{\left(3a^2f-2abe+b^2d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{11}{3}}\sqrt[3]{b}}+\frac{\sqrt{3}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{11}{3}}\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*f/x**9 - a**2*b*e/x**9 + a*b**2*d/x**9 - b**3*c/x**9)/(3
*a*b**3*(a + b*x**3)) - (a**2*f - a*b*e + b**2*d)/(8*a*b**3*x**8)
 + (2*a**2*f - 2*a*b*e + b**2*d)/(5*a**2*b**2*x**5) - (3*a**2*f -
 2*a*b*e + b**2*d)/(2*a**3*b*x**2) - (3*a**2*f - 2*a*b*e + b**2*d
)*log(a**(1/3) + b**(1/3)*x)/(3*a**(11/3)*b**(1/3)) + (3*a**2*f -
 2*a*b*e + b**2*d)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*
x**2)/(6*a**(11/3)*b**(1/3)) + sqrt(3)*(3*a**2*f - 2*a*b*e + b**2
*d)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(1
1/3)*b**(1/3))
```

Mathematica [A] time = 0.335302, size = 280, normalized size = 0.94

$$\frac{-\frac{72a^{5/3}(ad-2bc)}{x^5}-\frac{45a^{8/3}c}{x^8}-\frac{180a^{2/3}\left(a^2e-2abd+3b^2c\right)}{x^2}+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^3f-5a^2be+8ab^2d-11b^3c\right)}{\sqrt[3]{b}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-2a^3f+5a^2be-\cdots\right.}{\sqrt[3]{b}}}{360a^{14/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] ((-45*a^(8/3)*c)/x^8 - (72*a^(5/3)*(-2*b*c + a*d))/x^5 - (180*a^(
2/3)*(3*b^2*c - 2*a*b*d + a^2*e))/x^2 + (120*a^(2/3)*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(11*b^3*c
 - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/b^(1/3) + (40*(-11*b^3*c + 8*a*b^2*d - 5*a^2*b*e
+ 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20*(11*b^3*c - 8*
a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/b^(1/3))/(360*a^(14/3))
```

Maple [B] time = 0.02, size = 520, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x)
```

```
[Out] -1/8*c/a^2/x^8-1/5/a^2/x^5*d+2/5/a^3/x^5*b*c-1/2/a^2/x^2*e+1/a^3/
x^2*b*d-3/2/a^4/x^2*b^2*c+1/3/a*x/(b*x^3+a)*f-1/3/a^2*x/(b*x^3+a)
*b*e+1/3/a^3*x/(b*x^3+a)*b^2*d-1/3/a^4*x/(b*x^3+a)*b^3*c-5/9/a^2*
e/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/18/a^2*e/(a/b)^(2/3)*ln(x^2-x*(
```

```
a/b)^(1/3)+(a/b)^(2/3))-5/9/a^2*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))+8/9/a^3*b*d/(a/b)^(2/3)*ln(x+(a/b)^(
1/3))-4/9/a^3*b*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+8
/9/a^3*b*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))-11/9/a^4*b^2*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+11/18/a^4*b^2*
c/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-11/9/a^4*b^2*c/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/a*f
/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9/a*f/b/(a/b)^(2/3)*ln(x^2-x*(
a/b)^(1/3)+(a/b)^(2/3))+2/9/a*f/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^9),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25194, size = 555, normalized size = 1.87

$$\frac{\sqrt{3}\left(20\sqrt{3}\left(\left(11\,b^4c-8\,ab^3d+5\,a^2b^2e-2\,a^3bf\right)x^{11}+\left(11\,ab^3c-8\,a^2b^2d+5\,a^3be-2\,a^4f\right)x^8\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^9),x, algorithm="fricas")
```

```
[Out] 1/1080*sqrt(3)*(20*sqrt(3)*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e -
 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*
f)*x^8)*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 40*sqr
t(3)*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11
*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*log((a^2*b)^(1
/3)*x + a) - 120*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f
)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*ar
ctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*(
20*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^9 + 12*(11*a*b^
2*c - 8*a^2*b*d + 5*a^3*e)*x^6 + 15*a^3*c - 3*(11*a^2*b*c - 8*a^3
*d)*x^3)*(a^2*b)^(1/3))/((a^4*b*x^11 + a^5*x^8)*(a^2*b)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217116, size = 468, normalized size = 1.58

$$\frac{\left(11\,b^3c-8\,ab^2d-2\,a^3f+5\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5}-\frac{\sqrt{3}\left(11\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-8\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d-2\,\left(-ab^2\right)^{\frac{1}{3}}a^3f+5\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5b}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{3\,(bx^3+a)a^4}-\frac{\left(11\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-8\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d-2\,\left(-ab^2\right)^{\frac{1}{3}}a^3f+5\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5b}-\frac{60\,b^2cx^6-40\,abdx^6+20\,a^2x^6e-16\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^9),x, algorithm="giac")
```

```
[Out] 1/9*(11*b^3*c - 8*a*b^2*d - 2*a^3*f + 5*a^2*b*e)*(-a/b)^(1/3)*ln(
abs(x - (-a/b)^(1/3)))/a^5 - 1/9*sqrt(3)*(11*(-a*b^2)^(1/3)*b^3*c
 - 8*(-a*b^2)^(1/3)*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)
^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^5*b) - 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((
b*x^3 + a)*a^4) - 1/18*(11*(-a*b^2)^(1/3)*b^3*c - 8*(-a*b^2)^(1/3
)*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)^(1/3)*a^2*b*e)*ln
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/40*(60*b^2*c*x^
6 - 40*a*b*d*x^6 + 20*a^2*x^6*e - 16*a*b*c*x^3 + 8*a^2*d*x^3 + 5*
a^2*c)/(a^4*x^8)
```

3.273 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2}\,dx$

Optimal. Leaf size=334

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{10a^2x^{10}}-\frac{a^2e-2abd+3b^2c}{4a^4x^4}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{9a^{16/3}}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{3\sqrt{3}a^{16/3}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{18a^{16/3}}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

```
[Out] -c/(10*a^2*x^10) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d
 + a^2*e)/(4*a^4*x^4) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)
/(a^5*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a
+ b*x^3)) - (b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a
^(16/3)) - (b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(16/3)) + (b^(1/3)*(13*b^3*c - 10
*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(18*a^(16/3))
```

Rubi [A] time = 0.948896, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{10a^2x^{10}}-\frac{a^2e-2abd+3b^2c}{4a^4x^4}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{9a^{16/3}}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{3\sqrt{3}a^{16/3}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{18a^{16/3}}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```


```
[Out] -c/(10*a^2*x^10) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d
 + a^2*e)/(4*a^4*x^4) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)
/(a^5*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a
+ b*x^3)) - (b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a
^(16/3)) - (b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(16/3)) + (b^(1/3)*(13*b^3*c - 10
*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(18*a^(16/3))
```

Rubi in Sympy [A] time = 162.534, size = 326, normalized size = 0.98

$$-\frac{x\left(\frac{a^3f}{x^{11}}-\frac{a^2be}{x^{11}}+\frac{ab^2d}{x^{11}}-\frac{b^3c}{x^{11}}\right)}{3ab^3\left(a+bx^3\right)}-\frac{a^2f-abe+b^2d}{10ab^3x^{10}}+\frac{2a^2f-2abe+b^2d}{7a^2b^2x^7}-\frac{3a^2f-2abe+b^2d}{4a^3bx^4}+\frac{3a^2f-2abe+b^2d}{a^4x}-\frac{\sqrt[3]{b}\left(3a^2f-2abe+b^2d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{13}{3}}}+\frac{\sqrt[3]{b}\left(3a^2f-2abe+b^2d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{13}{3}}}-\frac{\sqrt{3}\sqrt[3]{b}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{13}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*f/x**11 - a**2*b*e/x**11 + a*b**2*d/x**11 - b**3*c/x**11
)/(3*a*b**3*(a + b*x**3)) - (a**2*f - a*b*e + b**2*d)/(10*a*b**3*
x**10) + (2*a**2*f - 2*a*b*e + b**2*d)/(7*a**2*b**2*x**7) - (3*a*
*2*f - 2*a*b*e + b**2*d)/(4*a**3*b*x**4) + (3*a**2*f - 2*a*b*e +
b**2*d)/(a**4*x) - b**(1/3)*(3*a**2*f - 2*a*b*e + b**2*d)*log(a**
(1/3) + b**(1/3)*x)/(3*a**(13/3)) + b**(1/3)*(3*a**2*f - 2*a*b*e
+ b**2*d)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*
a**(13/3)) - sqrt(3)*b**(1/3)*(3*a**2*f - 2*a*b*e + b**2*d)*atan(
sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(13/3))
```

Mathematica [A] time = 0.343361, size = 319, normalized size = 0.96

$$-\frac{180a^{7/3}(ad-2bc)}{x^7}-\frac{126a^{10/3}c}{x^{10}}-\frac{315a^{4/3}\left(a^2e-2abd+3b^2c\right)}{x^4}-\frac{420\sqrt[3]{a}bx^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}-\frac{1260\sqrt[3]{a}\left(a^3f-2a^2be+3ab^2d-4b^3c\right)}{x}+140\sqrt[3]{b}\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

```
[Out] ((-126*a^(10/3)*c)/x^10 - (180*a^(7/3)*(-2*b*c + a*d))/x^7 - (315
*a^(4/3)*(3*b^2*c - 2*a*b*d + a^2*e))/x^4 - (1260*a^(1/3)*(-4*b^3
*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/x - (420*a^(1/3)*b*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1
/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-13*b^3*c + 10*a*b^2*
d - 7*a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/3)*(1
3*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(1260*a^(16/3))
```

Maple [A] time = 0.025, size = 575, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x)
```

```
[Out] -1/3*b/a^2*x^2/(b*x^3+a)*f+1/3*b^2/a^3*x^2/(b*x^3+a)*e+13/18*b^3/
a^5*c/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*b^4/a^5*x
^2/(b*x^3+a)*c-7/9*b/a^3*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/18*b/a
^3*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-4/9/a^2*f*3^(1
/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/7/a^2/x
^7*d-1/4/a^2/x^4*e-1/a^2/x*f-1/3*b^3/a^4*x^2/(b*x^3+a)*d+10/9*b^2
/a^4*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/9*b^2/a^4*d/(a/b)^(1/3)*ln
(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-13/9*b^3/a^5*c/(a/b)^(1/3)*ln(x+(
a/b)^(1/3))+2/7/a^3/x^7*b*c+1/2/a^3/x^4*b*d-3/4/a^4/x^4*b^2*c+2/a
^3/x*b*e-3/a^4/x*b^2*d+4/a^5/x*b^3*c+4/9/a^2*f/(a/b)^(1/3)*ln(x+(
a/b)^(1/3))-2/9/a^2*f/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3
))+7/9*b/a^3*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-10/9*b^2/a^4*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))+13/9*b^3/a^5*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/10*c/a^2/x^10
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^11),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247194, size = 632, normalized size = 1.89

$$\frac{\sqrt{3}\left(70\,\sqrt{3}\left(\left(13\,b^{4}c-10\,ab^{3}d+7\,a^{2}b^{2}e-4\,a^{3}bf\right)x^{13}+\left(13\,ab^{3}c-10\,a^{2}b^{2}d+7\,a^{3}be-4\,a^{4}f\right)x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^11),x, algorithm="fricas")
```

```
[Out] 1/3780*sqrt(3)*(70*sqrt(3)*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e
- 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^
4*f)*x^10)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3
)) - 140*sqrt(3)*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*
f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)
*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) - 420*((13*b^4*c - 10*a*b^3
*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d +
 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*arctan(-1/3*(2*sqrt(3)*b*
x - sqrt(3)*a*(b/a)^(2/3))/(a*(b/a)^(2/3))) + 3*sqrt(3)*(140*(13*
b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^12 + 105*(13*a*b^
3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^9 - 15*(13*a^2*b^2*c
- 10*a^3*b*d + 7*a^4*e)*x^6 - 42*a^4*c + 6*(13*a^3*b*c - 10*a^4*d
)*x^3))/(a^5*b*x^13 + a^6*x^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217663, size = 590, normalized size = 1.77

$$-\frac{\left(13\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-10\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^6}-\frac{\sqrt{3}\left(13\,\left(-ab^2\right)^{\frac{2}{3}}b^3c-10\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d-4\,\left(-ab^2\right)^{\frac{2}{3}}a^3f+7\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^6b}+\frac{b^4cx^2-ab^3dx^2-a^3bfx^2+a^2b^2x^2e}{3\,(bx^3+a)a^5}+\frac{\left(13\,\left(-ab^2\right)^{\frac{2}{3}}b^3c-10\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d-4\,\left(-ab^2\right)^{\frac{2}{3}}a^3f+7\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^6b}+\frac{560\,b^3cx^9-420\,ab^2dx^9-140\,a^3fx^9+280\,a^2bx^9e-105\,ab^2cx^6+70\,a^2bdx^6-35\,a^3x^6e+40\,a^2bcx^3-20\,a^3dx^3-14\,a^3c}{140\,a^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^11),x, algorithm="giac")
```

```
[Out] -1/9*(13*b^4*c*(-a/b)^(1/3) - 10*a*b^3*d*(-a/b)^(1/3) - 4*a^3*b*f
*(-a/b)^(1/3) + 7*a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x -
 (-a/b)^(1/3)))/a^6 - 1/9*sqrt(3)*(13*(-a*b^2)^(2/3)*b^3*c - 10*(
-a*b^2)^(2/3)*a*b^2*d - 4*(-a*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)
*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(
a^6*b) + 1/3*(b^4*c*x^2 - a*b^3*d*x^2 - a^3*b*f*x^2 + a^2*b^2*x^2
*e)/((b*x^3 + a)*a^5) + 1/18*(13*(-a*b^2)^(2/3)*b^3*c - 10*(-a*b^
2)^(2/3)*a*b^2*d - 4*(-a*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)*a^2*
b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) + 1/140*(560
*b^3*c*x^9 - 420*a*b^2*d*x^9 - 140*a^3*f*x^9 + 280*a^2*b*x^9*e -
105*a*b^2*c*x^6 + 70*a^2*b*d*x^6 - 35*a^3*x^6*e + 40*a^2*b*c*x^3
- 20*a^3*d*x^3 - 14*a^3*c)/(a^5*x^10)
```

3.274 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx$

Optimal. Leaf size=335

$$\frac{2bc-ad}{8a^3x^8}-\frac{c}{11a^2x^{11}}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{18a^{17/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{9a^{17/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{3\sqrt{3}a^{17/3}}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{2a^5x^2}$$

```
[Out] -c/(11*a^2*x^11) + (2*b*c - a*d)/(8*a^3*x^8) - (3*b^2*c - 2*a*b*d
 + a^2*e)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)
/(2*a^5*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(
a + b*x^3)) - (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3
*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]
*a^(17/3)) + (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*
f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(17/3)) - (b^(2/3)*(14*b^3*c -
11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(18*a^(17/3))
```

Rubi [A] time = 0.965721, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{2bc-ad}{8a^3x^8}-\frac{c}{11a^2x^{11}}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{18a^{17/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{9a^{17/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{3\sqrt{3}a^{17/3}}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{2a^5x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```



```
[Out] -c/(11*a^2*x^11) + (2*b*c - a*d)/(8*a^3*x^8) - (3*b^2*c - 2*a*b*d
 + a^2*e)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)
/(2*a^5*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(
a + b*x^3)) - (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3
*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]
*a^(17/3)) + (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*
f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(17/3)) - (b^(2/3)*(14*b^3*c -
11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(18*a^(17/3))
```

Rubi in Sympy [A] time = 172.126, size = 330, normalized size = 0.99

$$-\frac{x\left(\frac{a^3f}{x^{12}}-\frac{a^2be}{x^{12}}+\frac{ab^2d}{x^{12}}-\frac{b^3c}{x^{12}}\right)}{3ab^3\left(a+bx^3\right)}-\frac{a^2f-abe+b^2d}{11ab^3x^{11}}+\frac{2a^2f-2abe+b^2d}{8a^2b^2x^8}-\frac{3a^2f-2abe+b^2d}{5a^3bx^5}+\frac{3a^2f-2abe+b^2d}{2a^4x^2}+\frac{b^{\frac{2}{3}}\left(3a^2f-2abe+b^2d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{14}{3}}}-\frac{b^{\frac{2}{3}}\left(3a^2f-2abe+b^2d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{14}{3}}}-\frac{\sqrt{3}b^{\frac{2}{3}}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{14}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*f/x**12 - a**2*b*e/x**12 + a*b**2*d/x**12 - b**3*c/x**12
)/(3*a*b**3*(a + b*x**3)) - (a**2*f - a*b*e + b**2*d)/(11*a*b**3*
x**11) + (2*a**2*f - 2*a*b*e + b**2*d)/(8*a**2*b**2*x**8) - (3*a*
*2*f - 2*a*b*e + b**2*d)/(5*a**3*b*x**5) + (3*a**2*f - 2*a*b*e +
b**2*d)/(2*a**4*x**2) + b**(2/3)*(3*a**2*f - 2*a*b*e + b**2*d)*lo
g(a**(1/3) + b**(1/3)*x)/(3*a**(14/3)) - b**(2/3)*(3*a**2*f - 2*a
*b*e + b**2*d)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2
)/(6*a**(14/3)) - sqrt(3)*b**(2/3)*(3*a**2*f - 2*a*b*e + b**2*d)*
atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(14/3)
)
```

Mathematica [A] time = 0.345357, size = 317, normalized size = 0.95

$$-\frac{495a^{8/3}(ad-2bc)}{x^8}-\frac{360a^{11/3}c}{x^{11}}-\frac{792a^{5/3}\left(a^2e-2abd+3b^2c\right)}{x^5}+440b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)-440\sqrt{3}b$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

```
[Out] ((-360*a^(11/3)*c)/x^11 - (495*a^(8/3)*(-2*b*c + a*d))/x^8 - (792
*a^(5/3)*(3*b^2*c - 2*a*b*d + a^2*e))/x^5 - (1980*a^(2/3)*(-4*b^3
*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/x^2 - (1320*a^(2/3)*b*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(a + b*x^3) - 440*Sqrt[3]*b^(
2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(14*b^3*c - 11*a*b^2*
d + 8*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2/3)*(
-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(3960*a^(17/3))
```

Maple [A] time = 0.023, size = 566, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x)
```

```
[Out] 1/3*b^2/a^3*x/(b*x^3+a)*e-1/3*b^3/a^4*x/(b*x^3+a)*d-1/8/a^2/x^8*d
-1/5/a^2/x^5*e-1/2/a^2/x^2*f-11/9*b^2/a^4*d/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))+11/18*b^2/a^4*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(
2/3))+14/9*b^3/a^5*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-7/9*b^3/a^5*c/
(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1/3*b/a^2*x/(b*x^3+
a)*f+8/9*b/a^3*e/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-4/9*b/a^3*e/(a/b)^
(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*b^4/a^5*x/(b*x^3+a)*c
-5/9/a^2*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))+1/4/a^3/x^8*b*c+2/5/a^3/x^5*b*d-3/5/a^4/x^5*b^2*c+1/a^3/x^2
*b*e-3/2/a^4/x^2*b^2*d+2/a^5/x^2*b^3*c-5/9/a^2*f/(a/b)^(2/3)*ln(x
+(a/b)^(1/3))+5/18/a^2*f/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(
2/3))+8/9*b/a^3*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))-11/9*b^2/a^4*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))+14/9*b^3/a^5*c/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/11*c/a^2/x^11
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^12),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23277, size = 670, normalized size = 2.

$$\frac{\sqrt{3}\left(220\sqrt{3}\left(\left(14\,b^4c-11\,ab^3d+8\,a^2b^2e-5\,a^3bf\right)x^{14}+\left(14\,ab^3c-11\,a^2b^2d+8\,a^3be-5\,a^4f\right)x^{11}\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^12),x, algorithm="fricas")
```

```
[Out] 1/11880*sqrt(3)*(220*sqrt(3)*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*
e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*
a^4*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3
) + a^2*(-b^2/a^2)^(2/3)) - 440*sqrt(3)*((14*b^4*c - 11*a*b^3*d +
 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a
^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(
1/3)) + 1320*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x
^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b
^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x + sqrt(3)*a*(-b^2/a^2)^(1
/3))/(a*(-b^2/a^2)^(1/3))) + 3*sqrt(3)*(220*(14*b^4*c - 11*a*b^3*
d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 132*(14*a*b^3*c - 11*a^2*b^2*
d + 8*a^3*b*e - 5*a^4*f)*x^9 - 33*(14*a^2*b^2*c - 11*a^3*b*d + 8*
a^4*e)*x^6 - 120*a^4*c + 15*(14*a^3*b*c - 11*a^4*d)*x^3))/(a^5*b*
x^14 + a^6*x^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217836, size = 528, normalized size = 1.58

$$\frac{\sqrt{3}\left(14\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-11\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-5\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+8\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^{6}}-\frac{\left(14b^{4}c-11ab^{3}d-5a^{3}bf+8a^{2}b^{2}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^{6}}+\frac{\left(14\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-11\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-5\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+8\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^{6}}+\frac{b^{4}cx-ab^{3}dx-a^{3}bfx+a^{2}b^{2}xe}{3\left(bx^{3}+a\right)a^{5}}+\frac{880b^{3}cx^{9}-660ab^{2}dx^{9}-220a^{3}fx^{9}+440a^{2}bx^{9}e-264ab^{2}cx^{6}+176a^{2}bdx^{6}-88a^{3}x^{6}e+110a^{2}bcx^{3}-55a^{3}dx^{3}-40a^{3}}{440a^{5}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^12),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d
- 5*(-a*b^2)^(1/3)*a^3*f + 8*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*s
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^6 - 1/9*(14*b^4*c - 1
1*a*b^3*d - 5*a^3*b*f + 8*a^2*b^2*e)*(-a/b)^(1/3)*ln(abs(x - (-a/
b)^(1/3)))/a^6 + 1/18*(14*(-a*b^2)^(1/3)*b^3*c - 11*(-a*b^2)^(1/3
)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 8*(-a*b^2)^(1/3)*a^2*b*e)*ln
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 + 1/3*(b^4*c*x - a*b^3*
d*x - a^3*b*f*x + a^2*b^2*x*e)/((b*x^3 + a)*a^5) + 1/440*(880*b^3
*c*x^9 - 660*a*b^2*d*x^9 - 220*a^3*f*x^9 + 440*a^2*b*x^9*e - 264*
a*b^2*c*x^6 + 176*a^2*b*d*x^6 - 88*a^3*x^6*e + 110*a^2*b*c*x^3 -
55*a^3*d*x^3 - 40*a^3*c)/(a^5*x^11)
```

3.275 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2}\,dx$

Optimal. Leaf size=375

$$\frac{2bc-ad}{10a^3x^{10}}-\frac{c}{13a^2x^{13}}-\frac{a^2e-2abd+3b^2c}{7a^4x^7}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{18a^{19/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{9a^{19/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{3\sqrt{3}a^{19/3}}-\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6\left(a+bx^3\right)}-\frac{b\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{4a^5x^4}$$

```
[Out] -c/(13*a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b
*d + a^2*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*
f)/(4*a^5*x^4) - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/
(a^6*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^6*(a
 + b*x^3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3
*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]
*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) - (b^(4/3)*(16*b^3*c -
 13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(18*a^(19/3))
```

Rubi [A] time = 1.13424, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{2bc-ad}{10a^3x^{10}}-\frac{c}{13a^2x^{13}}-\frac{a^2e-2abd+3b^2c}{7a^4x^7}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{18a^{19/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{9a^{19/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{3\sqrt{3}a^{19/3}}-\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6\left(a+bx^3\right)}-\frac{b\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{4a^5x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] -c/(13*a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b
*d + a^2*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*
f)/(4*a^5*x^4) - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/
(a^6*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^6*(a
 + b*x^3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3
*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]
*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) - (b^(4/3)*(16*b^3*c -
 13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)
```

```
*x + b^(2/3)*x^2])/(18*a^(19/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.793809, size = 370, normalized size = 0.99

$$\frac{2bc-ad}{10a^3x^{10}}-\frac{c}{13a^2x^{13}}-\frac{a^2e-2abd+3b^2c}{7a^4x^7}+\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^3f-10a^2be+13ab^2d-16b^3c\right)}{18a^{19/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{9a^{19/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{3\sqrt{3}a^{19/3}}+\frac{b^2x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{b\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{4a^5x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] -c/(13*a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b
*d + a^2*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*
f)/(4*a^5*x^4) + (b*(-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))
/(a^6*x) + (b^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(3*a^
6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7
*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(3*Sqrt[3]*a
^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3*f
)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) + (b^(4/3)*(-16*b^3*c +
13*a*b^2*d - 10*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(18*a^(19/3))
```

Maple [A] time = 0.024, size = 631, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x)
```

```
[Out] 7/9*b/a^3*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))-10/9*b^2/a^4*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))+13/9*b^3/a^5*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))-16/9*b^4/a^6*c*3^(1/2)/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*b^4/a^5*x^2/(b*x^3+a)*d-
3/7/a^4/x^7*b^2*c-1/4/a^2/x^4*f-1/10/a^2/x^10*d-1/7/a^2/x^7*e+1/2
```

```
/a^3/x^4*b*e-3/4/a^4/x^4*b^2*d+1/a^5/x^4*b^3*c+2/a^3*b/x*f-3/a^4*
b^2/x*e+4/a^5*b^3/x*d-5/a^6*b^4/x*c+1/5/a^3/x^10*b*c+2/7/a^3/x^7*
b*d-1/3*b^5/a^6*x^2/(b*x^3+a)*c+13/18*b^3/a^5*d/(a/b)^(1/3)*ln(x^
2-x*(a/b)^(1/3)+(a/b)^(2/3))+16/9*b^4/a^6*c/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))-8/9*b^4/a^6*c/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))-7/9*b/a^3*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/18*b/a^3*f/(a/b)^
(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*b^2/a^3*x^2/(b*x^3+a)
*f-1/3*b^3/a^4*x^2/(b*x^3+a)*e+10/9*b^2/a^4*e/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))-5/9*b^2/a^4*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(
2/3))-13/9*b^3/a^5*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/13*c/a^2/x^1
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^14),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.229478, size = 721, normalized size = 1.92

$$\frac{\sqrt{3}\left(910\sqrt{3}\left(\left(16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f\right)x^{16}+\left(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf\right)x^{13}\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^14),x, algorithm="fricas")
```

```
[Out] 1/49140*sqrt(3)*(910*sqrt(3)*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3
*e - 7*a^3*b^2*f)*x^16 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*
e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) -
a*(-b/a)^(1/3)) - 1820*sqrt(3)*((16*b^5*c - 13*a*b^4*d + 10*a^2*b
^3*e - 7*a^3*b^2*f)*x^16 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^
2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) - 5
460*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 +
(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/
a)^(1/3)*arctan(-1/3*(2*sqrt(3)*b*x - sqrt(3)*a*(-b/a)^(2/3))/(a*
(-b/a)^(2/3))) - 3*sqrt(3)*(1820*(16*b^5*c - 13*a*b^4*d + 10*a^2*
b^3*e - 7*a^3*b^2*f)*x^15 + 1365*(16*a*b^4*c - 13*a^2*b^3*d + 10*
a^3*b^2*e - 7*a^4*b*f)*x^12 - 195*(16*a^2*b^3*c - 13*a^3*b^2*d +
10*a^4*b*e - 7*a^5*f)*x^9 + 78*(16*a^3*b^2*c - 13*a^4*b*d + 10*a^
5*e)*x^6 + 420*a^5*c - 42*(16*a^4*b*c - 13*a^5*d)*x^3))/(a^6*b*x^
16 + a^7*x^13)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218039, size = 651, normalized size = 1.74

$$\frac{\sqrt{3}\left(16\left(-ab^2\right)^{\frac{2}{3}}b^3c-13\left(-ab^2\right)^{\frac{2}{3}}ab^2d-7\left(-ab^2\right)^{\frac{2}{3}}a^3f+10\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^7}+\frac{\left(16b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-13ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10a^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^7}-\frac{\left(16\left(-ab^2\right)^{\frac{2}{3}}b^3c-13\left(-ab^2\right)^{\frac{2}{3}}ab^2d-7\left(-ab^2\right)^{\frac{2}{3}}a^3f+10\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^7}-\frac{b^5cx^2-ab^4dx^2-a^3b^2fx^2+a^2b^3x^2e}{3\left(bx^3+a\right)a^6}-\frac{9100b^4cx^{12}-7280ab^3dx^{12}-3640a^3bfx^{12}+5460a^2b^2x^{12}e-1820ab^3cx^9+1365a^2b^2dx^9+455a^4fx^9-910a^3bx^9e+7}{1820a^6x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^2*x^14),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d
- 7*(-a*b^2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^7 + 1/9*(16*b^5*c*(-
a/b)^(1/3) - 13*a*b^4*d*(-a/b)^(1/3) - 7*a^3*b^2*f*(-a/b)^(1/3) +
 10*a^2*b^3*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3))
)/a^7 - 1/18*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d
 - 7*(-a*b^2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 - 1/3*(b^5*c*x^2 - a*b^4*d*x^2
- a^3*b^2*f*x^2 + a^2*b^3*x^2*e)/((b*x^3 + a)*a^6) - 1/1820*(9100
*b^4*c*x^12 - 7280*a*b^3*d*x^12 - 3640*a^3*b*f*x^12 + 5460*a^2*b^
2*x^12*e - 1820*a*b^3*c*x^9 + 1365*a^2*b^2*d*x^9 + 455*a^4*f*x^9
- 910*a^3*b*x^9*e + 780*a^2*b^2*c*x^6 - 520*a^3*b*d*x^6 + 260*a^4
*x^6*e - 364*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^6*x^13)
```

3.276 $\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=266

$$\frac{x^9\left(6a^2f-3abe+b^2d\right)}{9b^5}+\frac{a^3\left(-7a^3f+6a^2be-5ab^2d+4b^3c\right)}{3b^8\left(a+bx^3\right)}+\frac{a^2\log\left(a+bx^3\right)\left(-21a^3f+15a^2be-10ab^2d+6b^3c\right)}{3b^8}-\frac{ax^3\left(-15a^3f+10a^2be-6ab^2d+3b^3c\right)}{3b^7}+\frac{x^6\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{6b^6}-\frac{a^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^8\left(a+bx^3\right)^2}+\frac{x^{12}(be-3af)}{12b^4}+\frac{fx^{15}}{15b^3}$$

```
[Out] -(a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x^3)/(3*b^7) +
((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6)/(6*b^6) + ((b^2*
d - 3*a*b*e + 6*a^2*f)*x^9)/(9*b^5) + ((b*e - 3*a*f)*x^12)/(12*b^
4) + (f*x^15)/(15*b^3) - (a^4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/(6*b^8*(a + b*x^3)^2) + (a^3*(4*b^3*c - 5*a*b^2*d + 6*a^2*b*e -
 7*a^3*f))/(3*b^8*(a + b*x^3)) + (a^2*(6*b^3*c - 10*a*b^2*d + 15*
a^2*b*e - 21*a^3*f)*Log[a + b*x^3])/(3*b^8)
```

Rubi [A] time = 0.866896, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x^9\left(6a^2f-3abe+b^2d\right)}{9b^5}+\frac{a^3\left(-7a^3f+6a^2be-5ab^2d+4b^3c\right)}{3b^8\left(a+bx^3\right)}+\frac{a^2\log\left(a+bx^3\right)\left(-21a^3f+15a^2be-10ab^2d+6b^3c\right)}{3b^8}-\frac{ax^3\left(-15a^3f+10a^2be-6ab^2d+3b^3c\right)}{3b^7}+\frac{x^6\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{6b^6}-\frac{a^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^8\left(a+bx^3\right)^2}+\frac{x^{12}(be-3af)}{12b^4}+\frac{fx^{15}}{15b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```



```
[Out] -(a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x^3)/(3*b^7) +
((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6)/(6*b^6) + ((b^2*
d - 3*a*b*e + 6*a^2*f)*x^9)/(9*b^5) + ((b*e - 3*a*f)*x^12)/(12*b^
4) + (f*x^15)/(15*b^3) - (a^4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/(6*b^8*(a + b*x^3)^2) + (a^3*(4*b^3*c - 5*a*b^2*d + 6*a^2*b*e -
 7*a^3*f))/(3*b^8*(a + b*x^3)) + (a^2*(6*b^3*c - 10*a*b^2*d + 15*
a^2*b*e - 21*a^3*f)*Log[a + b*x^3])/(3*b^8)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^8\left(a+bx^3\right)^2}-\frac{a^3\left(7a^3f-6a^2be+5ab^2d-4b^3c\right)}{3b^8\left(a+bx^3\right)}-\frac{a^2\left(21a^3f-15a^2be+10ab^2d-6b^3c\right)\log\left(a+bx^3\right)}{3b^8}+\frac{fx^{15}}{15b^3}-\frac{x^{12}\left(3af-be\right)}{12b^4}+\frac{x^9\left(6a^2f-3abe+b^2d\right)}{9b^5}-\frac{\left(10a^3f-6a^2be+3ab^2d-b^3c\right)\int^{x^3}x\,dx}{3b^6}+\frac{\left(15a^3f-10a^2be+6ab^2d-3b^3c\right)\int^{x^3}a\,dx}{3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**14*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] a**4*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*b**8*(a + b*x**3)
**2) - a**3*(7*a**3*f - 6*a**2*b*e + 5*a*b**2*d - 4*b**3*c)/(3*b*
```

```
*8*(a + b*x**3)) - a**2*(21*a**3*f - 15*a**2*b*e + 10*a*b**2*d -
6*b**3*c)*log(a + b*x**3)/(3*b**8) + f*x**15/(15*b**3) - x**12*(3
*a*f - b*e)/(12*b**4) + x**9*(6*a**2*f - 3*a*b*e + b**2*d)/(9*b**
5) - (10*a**3*f - 6*a**2*b*e + 3*a*b**2*d - b**3*c)*Integral(x, (
x, x**3))/(3*b**6) + (15*a**3*f - 10*a**2*b*e + 6*a*b**2*d - 3*b*
*3*c)*Integral(a, (x, x**3))/(3*b**7)
```

Mathematica [A] time = 0.271543, size = 246, normalized size = 0.92

$$\frac{20b^3x^9\left(6a^2f-3abe+b^2d\right)+30b^2x^6\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)+60abx^3\left(15a^3f-10a^2be+6ab^2d-3b^3c\right)-\frac{60a^3(}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (60*a*b*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x^3 + 30*b
^2*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6 + 20*b^3*(b^2*d
 - 3*a*b*e + 6*a^2*f)*x^9 + 15*b^4*(b*e - 3*a*f)*x^12 + 12*b^5*f*
x^15 + (30*a^4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3
)^2 - (60*a^3*(-4*b^3*c + 5*a*b^2*d - 6*a^2*b*e + 7*a^3*f))/(a +
b*x^3) + 60*a^2*(6*b^3*c - 10*a*b^2*d + 15*a^2*b*e - 21*a^3*f)*Lo
g[a + b*x^3])/(180*b^8)
```

Maple [A] time = 0.021, size = 361, normalized size = 1.4

$$\frac{a^5d}{6\,b^6\left(bx^3+a\right)^2}-\frac{a^4c}{6\,b^5\left(bx^3+a\right)^2}-\frac{7\,a^6f}{3\,b^8\left(bx^3+a\right)}+2\,\frac{a^5e}{b^7\left(bx^3+a\right)}-\frac{5\,a^4d}{3\,b^6\left(bx^3+a\right)}+\frac{4\,a^3c}{3\,b^5\left(bx^3+a\right)}-7\,\frac{a^5\ln\left(bx^3+a\right)f}{b^8}+5\,\frac{a^4\ln\left(bx^3+a\right)e}{b^7}-\frac{10\,a^3\ln\left(bx^3+a\right)d}{3\,b^6}+2\,\frac{a^2\ln\left(bx^3+a\right)c}{b^5}-\frac{x^{12}af}{4\,b^4}+\frac{2\,x^9a^2f}{3\,b^5}-\frac{x^9ae}{3\,b^4}-\frac{5\,a^3fx^6}{3\,b^6}+\frac{a^2ex^6}{b^5}-\frac{adx^6}{2\,b^4}+5\,\frac{a^4fx^3}{b^7}-\frac{10\,a^3ex^3}{3\,b^6}+2\,\frac{a^2dx^3}{b^5}-\frac{acx^3}{b^4}+\frac{x^{12}e}{12\,b^3}+\frac{x^9d}{9\,b^3}+\frac{x^6c}{6\,b^3}-\frac{a^6e}{6\,b^7\left(bx^3+a\right)^2}+\frac{a^7f}{6\,b^8\left(bx^3+a\right)^2}+\frac{fx^{15}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*a^5/b^6/(b*x^3+a)^2*d-1/6*a^4/b^5/(b*x^3+a)^2*c-7/3*a^6/b^8/(
b*x^3+a)*f+2*a^5/b^7/(b*x^3+a)*e-5/3*a^4/b^6/(b*x^3+a)*d+4/3*a^3/
b^5/(b*x^3+a)*c-7*a^5/b^8*ln(b*x^3+a)*f+5*a^4/b^7*ln(b*x^3+a)*e-1
0/3*a^3/b^6*ln(b*x^3+a)*d+2*a^2/b^5*ln(b*x^3+a)*c-1/4/b^4*x^12*a*
f+2/3/b^5*x^9*a^2*f-1/3/b^4*x^9*a*e-5/3/b^6*x^6*a^3*f+1/b^5*x^6*a
^2*e-1/2/b^4*x^6*a*d+5/b^7*a^4*f*x^3-10/3/b^6*a^3*e*x^3+2/b^5*a^2
*d*x^3-1/b^4*a*c*x^3+1/12/b^3*x^12*e+1/9/b^3*x^9*d+1/6/b^3*x^6*c-
1/6*a^6/b^7/(b*x^3+a)^2*e+1/6*a^7/b^8/(b*x^3+a)^2*f+1/15*f*x^15/b
^3
```

Maxima [A] time = 1.37891, size = 371, normalized size = 1.39

$$\frac{7\,a^4b^3c-9\,a^5b^2d+11\,a^6be-13\,a^7f+2\left(4\,a^3b^4c-5\,a^4b^3d+6\,a^5b^2e-7\,a^6bf\right)x^3}{6\left(b^{10}x^6+2\,ab^9x^3+a^2b^8\right)}+\frac{12\,b^4fx^{15}+15\left(b^4e-3\,ab^3f\right)x^{12}+20\left(b^4d-3\,ab^3e+6\,a^2b^2f\right)x^9+30\left(b^4c-3\,ab^3d+6\,a^2b^2e-10\,a^3bf\right)x^6-60\left(3\,ab^3c\right.}{180\,b^7}+\frac{\left(6\,a^2b^3c-10\,a^3b^2d+15\,a^4be-21\,a^5f\right)\log\left(bx^3+a\right)}{3\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^14/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(7*a^4*b^3*c - 9*a^5*b^2*d + 11*a^6*b*e - 13*a^7*f + 2*(4*a^3
*b^4*c - 5*a^4*b^3*d + 6*a^5*b^2*e - 7*a^6*b*f)*x^3)/(b^10*x^6 +
2*a*b^9*x^3 + a^2*b^8) + 1/180*(12*b^4*f*x^15 + 15*(b^4*e - 3*a*b
^3*f)*x^12 + 20*(b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^9 + 30*(b^4*c
 - 3*a*b^3*d + 6*a^2*b^2*e - 10*a^3*b*f)*x^6 - 60*(3*a*b^3*c - 6*
a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*x^3)/b^7 + 1/3*(6*a^2*b^3*c -
10*a^3*b^2*d + 15*a^4*b*e - 21*a^5*f)*log(b*x^3 + a)/b^8
```

Fricas [A] time = 0.205264, size = 535, normalized size = 2.01

$$\frac{12\,b^7fx^{21} + 3\left(5\,b^7e - 7\,ab^6f\right)x^{18} + 2\left(10\,b^7d - 15\,ab^6e + 21\,a^2b^5f\right)x^{15} + 5\left(6\,b^7c - 10\,ab^6d + 15\,a^2b^5e - 21\,a^3b^4f\right)x^{12} - 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^14/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^7*f*x^21 + 3*(5*b^7*e - 7*a*b^6*f)*x^18 + 2*(10*b^7*d
 - 15*a*b^6*e + 21*a^2*b^5*f)*x^15 + 5*(6*b^7*c - 10*a*b^6*d + 15
*a^2*b^5*e - 21*a^3*b^4*f)*x^12 - 20*(6*a*b^6*c - 10*a^2*b^5*d +
15*a^3*b^4*e - 21*a^4*b^3*f)*x^9 + 210*a^4*b^3*c - 270*a^5*b^2*d
+ 330*a^6*b*e - 390*a^7*f - 30*(11*a^2*b^5*c - 21*a^3*b^4*d + 34*
a^4*b^3*e - 50*a^5*b^2*f)*x^6 + 60*(a^3*b^4*c + a^4*b^3*d - 4*a^5
*b^2*e + 8*a^6*b*f)*x^3 + 60*(6*a^4*b^3*c - 10*a^5*b^2*d + 15*a^6
*b*e - 21*a^7*f + (6*a^2*b^5*c - 10*a^3*b^4*d + 15*a^4*b^3*e - 21
*a^5*b^2*f)*x^6 + 2*(6*a^3*b^4*c - 10*a^4*b^3*d + 15*a^5*b^2*e -
21*a^6*b*f)*x^3)*log(b*x^3 + a))/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^
8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218528, size = 471, normalized size = 1.77

$$\frac{\left(6\,a^2b^3c - 10\,a^3b^2d - 21\,a^5f + 15\,a^4be\right)\ln\left(\left|bx^3 + a\right|\right)}{3\,b^8} - \frac{18\,a^2b^5cx^6 - 30\,a^3b^4dx^6 - 63\,a^5b^2fx^6 + 45\,a^4b^3x^6e + 28\,a^3b^4cx^3 - 50\,a^4b^3dx^3 - 112\,a^6bfx^3 + 78\,a^5b^2x^3e + 11\,a^4b^3c - 2}{6\left(bx^3 + a\right)^2b^8} + \frac{12\,b^{12}fx^{15} - 45\,ab^{11}fx^{12} + 15\,b^{12}x^{12}e + 20\,b^{12}dx^9 + 120\,a^2b^{10}fx^9 - 60\,ab^{11}x^9e + 30\,b^{12}cx^6 - 90\,ab^{11}dx^6 - 300\,a^3b^9fx^6 +}{180\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^14/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*b^3*c - 10*a^3*b^2*d - 21*a^5*f + 15*a^4*b*e)*ln(abs(b
*x^3 + a))/b^8 - 1/6*(18*a^2*b^5*c*x^6 - 30*a^3*b^4*d*x^6 - 63*a^
5*b^2*f*x^6 + 45*a^4*b^3*x^6*e + 28*a^3*b^4*c*x^3 - 50*a^4*b^3*d*
x^3 - 112*a^6*b*f*x^3 + 78*a^5*b^2*x^3*e + 11*a^4*b^3*c - 21*a^5*
```

```
b^2*d - 50*a^7*f + 34*a^6*b*e)/((b*x^3 + a)^2*b^8) + 1/180*(12*b^
12*f*x^15 - 45*a*b^11*f*x^12 + 15*b^12*x^12*e + 20*b^12*d*x^9 + 1
20*a^2*b^10*f*x^9 - 60*a*b^11*x^9*e + 30*b^12*c*x^6 - 90*a*b^11*d
*x^6 - 300*a^3*b^9*f*x^6 + 180*a^2*b^10*x^6*e - 180*a*b^11*c*x^3
+ 360*a^2*b^10*d*x^3 + 900*a^4*b^8*f*x^3 - 600*a^3*b^9*x^3*e)/b^1
5
```

3.277 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=226

$$\frac{x^6\left(6a^2f-3abe+b^2d\right)}{6b^5}-\frac{a^2\left(-6a^3f+5a^2be-4ab^2d+3b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{a\log\left(a+bx^3\right)\left(-15a^3f+10a^2be-6ab^2d+3b^3c\right)}{3b^7}+\frac{x^3\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{3b^6}+\frac{x^9(be-3af)}{9b^4}+\frac{fx^{12}}{12b^3}$$

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*
d - 3*a*b*e + 6*a^2*f)*x^6)/(6*b^5) + ((b*e - 3*a*f)*x^9)/(9*b^4)
 + (f*x^12)/(12*b^3) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/
(6*b^7*(a + b*x^3)^2) - (a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6
*a^3*f))/(3*b^7*(a + b*x^3)) - (a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b
*e - 15*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rubi [A] time = 0.698882, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x^6\left(6a^2f-3abe+b^2d\right)}{6b^5}-\frac{a^2\left(-6a^3f+5a^2be-4ab^2d+3b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{a\log\left(a+bx^3\right)\left(-15a^3f+10a^2be-6ab^2d+3b^3c\right)}{3b^7}+\frac{x^3\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{3b^6}+\frac{x^9(be-3af)}{9b^4}+\frac{fx^{12}}{12b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*
d - 3*a*b*e + 6*a^2*f)*x^6)/(6*b^5) + ((b*e - 3*a*f)*x^9)/(9*b^4)
 + (f*x^12)/(12*b^3) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/
(6*b^7*(a + b*x^3)^2) - (a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6
*a^3*f))/(3*b^7*(a + b*x^3)) - (a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b
*e - 15*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^7\left(a+bx^3\right)^2}+\frac{a^2\left(6a^3f-5a^2be+4ab^2d-3b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a\left(15a^3f-10a^2be+6ab^2d-3b^3c\right)\log\left(a+bx^3\right)}{3b^7}-\left(\frac{10a^3f}{3}-2a^2be+ab^2d-\frac{b^3c}{3}\right)\int^{x^3}\frac{1}{b^6}\,dx+\frac{fx^{12}}{12b^3}-\frac{x^9(3af-be)}{9b^4}+\frac{\left(6a^2f-3abe+b^2d\right)\int^{x^3}x\,dx}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] -a**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*b**7*(a + b*x**3
)**2) + a**2*(6*a**3*f - 5*a**2*b*e + 4*a*b**2*d - 3*b**3*c)/(3*b
**7*(a + b*x**3)) + a*(15*a**3*f - 10*a**2*b*e + 6*a*b**2*d - 3*b
**3*c)*log(a + b*x**3)/(3*b**7) - (10*a**3*f/3 - 2*a**2*b*e + a*b
**2*d - b**3*c/3)*Integral(b**(-6), (x, x**3)) + f*x**12/(12*b**3
) - x**9*(3*a*f - b*e)/(9*b**4) + (6*a**2*f - 3*a*b*e + b**2*d)*I
```

```
ntegral(x, (x, x**3))/(3*b**5)
```

Mathematica [A] time = 0.230815, size = 208, normalized size = 0.92

$$\frac{6b^2x^6\left(6a^2f-3abe+b^2d\right)+12bx^3\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)+\frac{12a^2\left(6a^3f-5a^2be+4ab^2d-3b^3c\right)}{a+bx^3}+\frac{6a^3\left(a^3(-f)+a^2be-ab^2d+b^3\right.}{\left(a+bx^3\right)^2}}{36b^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (12*b*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3 + 6*b^2*(b^2
*d - 3*a*b*e + 6*a^2*f)*x^6 + 4*b^3*(b*e - 3*a*f)*x^9 + 3*b^4*f*x
^12 + (6*a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3)^2 +
 (12*a^2*(-3*b^3*c + 4*a*b^2*d - 5*a^2*b*e + 6*a^3*f))/(a + b*x^3
) + 12*a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*Log[a + b
*x^3])/(36*b^7)
```

Maple [A] time = 0.022, size = 313, normalized size = 1.4

$$\frac{fx^{12}}{12\,b^3}-\frac{x^9af}{3\,b^4}+\frac{x^9e}{9\,b^3}+\frac{a^2fx^6}{b^5}-\frac{aex^6}{2\,b^4}+\frac{dx^6}{6\,b^3}-\frac{10\,a^3fx^3}{3\,b^6}+2\,\frac{a^2ex^3}{b^5}-\frac{adx^3}{b^4}+\frac{cx^3}{3\,b^3}+5\,\frac{a^4\ln\left(bx^3+a\right)f}{b^7}-\frac{10\,a^3\ln\left(bx^3+a\right)e}{3\,b^6}+2\,\frac{a^2\ln\left(bx^3+a\right)d}{b^5}-\frac{a\ln\left(bx^3+a\right)c}{b^4}-\frac{a^6f}{6\,b^7\left(bx^3+a\right)^2}+\frac{a^5e}{6\,b^6\left(bx^3+a\right)^2}-\frac{a^4d}{6\,b^5\left(bx^3+a\right)^2}+\frac{a^3c}{6\,b^4\left(bx^3+a\right)^2}+2\,\frac{a^5f}{b^7\left(bx^3+a\right)}-\frac{5\,a^4e}{3\,b^6\left(bx^3+a\right)}+\frac{4\,a^3d}{3\,b^5\left(bx^3+a\right)}-\frac{a^2c}{b^4\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/12*f*x^12/b^3-1/3/b^4*x^9*a*f+1/9/b^3*x^9*e+1/b^5*x^6*a^2*f-1/2
/b^4*x^6*a*e+1/6/b^3*x^6*d-10/3/b^6*a^3*f*x^3+2/b^5*a^2*e*x^3-1/b
^4*a*d*x^3+1/3/b^3*c*x^3+5*a^4/b^7*ln(b*x^3+a)*f-10/3*a^3/b^6*ln(
b*x^3+a)*e+2*a^2/b^5*ln(b*x^3+a)*d-a/b^4*ln(b*x^3+a)*c-1/6*a^6/b^
7/(b*x^3+a)^2*f+1/6*a^5/b^6/(b*x^3+a)^2*e-1/6*a^4/b^5/(b*x^3+a)^2
*d+1/6*a^3/b^4/(b*x^3+a)^2*c+2*a^5/b^7/(b*x^3+a)*f-5/3*a^4/b^6/(b
*x^3+a)*e+4/3*a^3/b^5/(b*x^3+a)*d-a^2/b^4/(b*x^3+a)*c
```

Maxima [A] time = 1.44419, size = 315, normalized size = 1.39

$$-\frac{5\,a^3b^3c-7\,a^4b^2d+9\,a^5be-11\,a^6f+2\left(3\,a^2b^4c-4\,a^3b^3d+5\,a^4b^2e-6\,a^5bf\right)x^3}{6\left(b^9x^6+2\,ab^8x^3+a^2b^7\right)}+\frac{3\,b^3fx^{12}+4\left(b^3e-3\,ab^2f\right)x^9+6\left(b^3d-3\,ab^2e+6\,a^2bf\right)x^6+12\left(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f\right)x^3}{36\,b^6}-\frac{\left(3\,ab^3c-6\,a^2b^2d+10\,a^3be-15\,a^4f\right)\log\left(bx^3+a\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^11/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(5*a^3*b^3*c - 7*a^4*b^2*d + 9*a^5*b*e - 11*a^6*f + 2*(3*a^2
*b^4*c - 4*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)/(b^9*x^6 + 2
*a*b^8*x^3 + a^2*b^7) + 1/36*(3*b^3*f*x^12 + 4*(b^3*e - 3*a*b^2*f
)*x^9 + 6*(b^3*d - 3*a*b^2*e + 6*a^2*b*f)*x^6 + 12*(b^3*c - 3*a*b
^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/b^6 - 1/3*(3*a*b^3*c - 6*a^2*b^
```

```
2*d + 10*a^3*b*e - 15*a^4*f)*log(b*x^3 + a)/b^7
```

Fricas [A] time = 0.206411, size = 477, normalized size = 2.11

$$\frac{3\,b^{6}fx^{18} + 2\left(2\,b^{6}e - 3\,ab^{5}f\right)x^{15} + \left(6\,b^{6}d - 10\,ab^{5}e + 15\,a^{2}b^{4}f\right)x^{12} + 4\left(3\,b^{6}c - 6\,ab^{5}d + 10\,a^{2}b^{4}e - 15\,a^{3}b^{3}f\right)x^{9} - 30\,a^{3}b^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^11/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^6*f*x^18 + 2*(2*b^6*e - 3*a*b^5*f)*x^15 + (6*b^6*d - 10
*a*b^5*e + 15*a^2*b^4*f)*x^12 + 4*(3*b^6*c - 6*a*b^5*d + 10*a^2*b
^4*e - 15*a^3*b^3*f)*x^9 - 30*a^3*b^3*c + 42*a^4*b^2*d - 54*a^5*b
*e + 66*a^6*f + 6*(4*a*b^5*c - 11*a^2*b^4*d + 21*a^3*b^3*e - 34*a
^4*b^2*f)*x^6 - 12*(2*a^2*b^4*c - a^3*b^3*d - a^4*b^2*e + 4*a^5*b
*f)*x^3 - 12*(3*a^3*b^3*c - 6*a^4*b^2*d + 10*a^5*b*e - 15*a^6*f +
 (3*a*b^5*c - 6*a^2*b^4*d + 10*a^3*b^3*e - 15*a^4*b^2*f)*x^6 + 2*
(3*a^2*b^4*c - 6*a^3*b^3*d + 10*a^4*b^2*e - 15*a^5*b*f)*x^3)*log(
b*x^3 + a))/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.215971, size = 402, normalized size = 1.78

$$-\frac{\left(3\,ab^{3}c - 6\,a^{2}b^{2}d - 15\,a^{4}f + 10\,a^{3}be\right)\ln\left(\left|bx^{3} + a\right|\right)}{3\,b^{7}} + \frac{9\,ab^{5}cx^{6} - 18\,a^{2}b^{4}dx^{6} - 45\,a^{4}b^{2}fx^{6} + 30\,a^{3}b^{3}x^{6}e + 12\,a^{2}b^{4}cx^{3} - 28\,a^{3}b^{3}dx^{3} - 78\,a^{5}bfx^{3} + 50\,a^{4}b^{2}x^{3}e + 4\,a^{3}b^{3}c - 11\,a^{4}b}{6\left(bx^{3} + a\right)^{2}b^{7}} + \frac{3\,b^{9}fx^{12} - 12\,ab^{8}fx^{9} + 4\,b^{9}x^{9}e + 6\,b^{9}dx^{6} + 36\,a^{2}b^{7}fx^{6} - 18\,ab^{8}x^{6}e + 12\,b^{9}cx^{3} - 36\,ab^{8}dx^{3} - 120\,a^{3}b^{6}fx^{3} + 72\,a^{2}b^{7}x^{3}e}{36\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^11/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b^3*c - 6*a^2*b^2*d - 15*a^4*f + 10*a^3*b*e)*ln(abs(b*x
^3 + a))/b^7 + 1/6*(9*a*b^5*c*x^6 - 18*a^2*b^4*d*x^6 - 45*a^4*b^2
*f*x^6 + 30*a^3*b^3*x^6*e + 12*a^2*b^4*c*x^3 - 28*a^3*b^3*d*x^3 -
 78*a^5*b*f*x^3 + 50*a^4*b^2*x^3*e + 4*a^3*b^3*c - 11*a^4*b^2*d -
 34*a^6*f + 21*a^5*b*e)/((b*x^3 + a)^2*b^7) + 1/36*(3*b^9*f*x^12
- 12*a*b^8*f*x^9 + 4*b^9*x^9*e + 6*b^9*d*x^6 + 36*a^2*b^7*f*x^6 -
 18*a*b^8*x^6*e + 12*b^9*c*x^3 - 36*a*b^8*d*x^3 - 120*a^3*b^6*f*x
^3 + 72*a^2*b^7*x^3*e)/b^12
```

3.278 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=186

$$\frac{x^3(6a^2f-3abe+b^2d)}{3b^5}+\frac{a(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6(a+bx^3)}-\frac{a^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6(a+bx^3)^2}+\frac{\log(a+bx^3)(-10a^3f+6a^2be-3ab^2d+b^3c)}{3b^6}+\frac{x^6(be-3af)}{6b^4}+\frac{fx^9}{9b^3}$$

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^6)/(
6*b^4) + (f*x^9)/(9*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f))/(6*b^6*(a + b*x^3)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e -
 5*a^3*f))/(3*b^6*(a + b*x^3)) + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e
- 10*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rubi [A] time = 0.549245, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^3(6a^2f-3abe+b^2d)}{3b^5}+\frac{a(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6(a+bx^3)}-\frac{a^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6(a+bx^3)^2}+\frac{\log(a+bx^3)(-10a^3f+6a^2be-3ab^2d+b^3c)}{3b^6}+\frac{x^6(be-3af)}{6b^4}+\frac{fx^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^6)/(
6*b^4) + (f*x^9)/(9*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f))/(6*b^6*(a + b*x^3)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e -
 5*a^3*f))/(3*b^6*(a + b*x^3)) + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e
- 10*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2(a^3f-a^2be+ab^2d-b^3c)}{6b^6(a+bx^3)^2}-\frac{a(5a^3f-4a^2be+3ab^2d-2b^3c)}{3b^6(a+bx^3)}+\left(2a^2f-abe+\frac{b^2d}{3}\right)\int^{x^3}\frac{1}{b^5}\,dx+\frac{fx^9}{9b^3}-\frac{(3af-be)\int^{x^3}x\,dx}{3b^4}-\frac{(10a^3f-6a^2be+3ab^2d-b^3c)\log(a+bx^3)}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] a**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*b**6*(a + b*x**3)
**2) - a*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)/(3*b**6*
(a + b*x**3)) + (2*a**2*f - a*b*e + b**2*d/3)*Integral(b**(-5), (
x, x**3)) + f*x**9/(9*b**3) - (3*a*f - b*e)*Integral(x, (x, x**3)
)/(3*b**4) - (10*a**3*f - 6*a**2*b*e + 3*a*b**2*d - b**3*c)*log(a
 + b*x**3)/(3*b**6)
```

Mathematica [A] time = 0.189258, size = 170, normalized size = 0.91

$$\frac{6bx^3(6a^2f-3abe+b^2d)-\frac{6a(5a^3f-4a^2be+3ab^2d-2b^3c)}{a+bx^3}+\frac{3a^2(a^3f-a^2be+ab^2d-b^3c)}{(a+bx^3)^2}+6\log(a+bx^3)(-10a^3f+6a^2be-3ab^2d}{18b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (6*b*(b^2*d - 3*a*b*e + 6*a^2*f)*x^3 + 3*b^2*(b*e - 3*a*f)*x^6 +
2*b^3*f*x^9 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a +
 b*x^3)^2 - (6*a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f))/(a
 + b*x^3) + 6*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*Log[a +
b*x^3])/(18*b^6)
```

Maple [A] time = 0.02, size = 266, normalized size = 1.4

$$\frac{fx^9}{9\,b^3}-\frac{x^6af}{2\,b^4}+\frac{ex^6}{6\,b^3}+2\,\frac{a^2fx^3}{b^5}-\frac{aex^3}{b^4}+\frac{dx^3}{3\,b^3}-\frac{10\,\ln\left(bx^3+a\right)a^3f}{3\,b^6}+2\,\frac{\ln\left(bx^3+a\right)a^2e}{b^5}-\frac{\ln\left(bx^3+a\right)ad}{b^4}+\frac{\ln\left(bx^3+a\right)c}{3\,b^3}+\frac{a^5f}{6\,b^6\left(bx^3+a\right)^2}-\frac{a^4e}{6\,b^5\left(bx^3+a\right)^2}+\frac{a^3d}{6\,b^4\left(bx^3+a\right)^2}-\frac{a^2c}{6\,b^3\left(bx^3+a\right)^2}-\frac{5\,a^4f}{3\,b^6\left(bx^3+a\right)}+\frac{4\,a^3e}{3\,b^5\left(bx^3+a\right)}-\frac{a^2d}{b^4\left(bx^3+a\right)}+\frac{2\,ac}{3\,b^3\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/9*f*x^9/b^3-1/2/b^4*x^6*a*f+1/6/b^3*x^6*e+2/b^5*a^2*f*x^3-1/b^4
*a*e*x^3+1/3/b^3*d*x^3-10/3/b^6*ln(b*x^3+a)*a^3*f+2/b^5*ln(b*x^3+
a)*a^2*e-1/b^4*ln(b*x^3+a)*a*d+1/3/b^3*ln(b*x^3+a)*c+1/6/b^6*a^5/
(b*x^3+a)^2*f-1/6/b^5*a^4/(b*x^3+a)^2*e+1/6/b^4*a^3/(b*x^3+a)^2*d
-1/6/b^3*a^2/(b*x^3+a)^2*c-5/3/b^6*a^4/(b*x^3+a)*f+4/3/b^5*a^3/(b
*x^3+a)*e-1/b^4*a^2/(b*x^3+a)*d+2/3/b^3*a/(b*x^3+a)*c
```

Maxima [A] time = 1.44621, size = 258, normalized size = 1.39

$$\frac{3\,a^2b^3c-5\,a^3b^2d+7\,a^4be-9\,a^5f+2\left(2\,ab^4c-3\,a^2b^3d+4\,a^3b^2e-5\,a^4bf\right)x^3}{6\left(b^8x^6+2\,ab^7x^3+a^2b^6\right)}+\frac{2\,b^2fx^9+3\left(b^2e-3\,abf\right)x^6+6\left(b^2d-3\,abe+6\,a^2f\right)x^3}{18\,b^5}+\frac{\left(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f\right)\log\left(bx^3+a\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^8/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*b^3*c - 5*a^3*b^2*d + 7*a^4*b*e - 9*a^5*f + 2*(2*a*b^4
*c - 3*a^2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)/(b^8*x^6 + 2*a*b
^7*x^3 + a^2*b^6) + 1/18*(2*b^2*f*x^9 + 3*(b^2*e - 3*a*b*f)*x^6 +
 6*(b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/b^5 + 1/3*(b^3*c - 3*a*b^2*d
+ 6*a^2*b*e - 10*a^3*f)*log(b*x^3 + a)/b^6
```

Fricas [A] time = 0.206585, size = 398, normalized size = 2.14

$$\frac{2\,b^5fx^{15}+\left(3\,b^5e-5\,ab^4f\right)x^{12}+2\left(3\,b^5d-6\,ab^4e+10\,a^2b^3f\right)x^9+3\left(4\,ab^4d-11\,a^2b^3e+21\,a^3b^2f\right)x^6+9\,a^2b^3c-15\,a^3b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^8/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*f*x^15 + (3*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 6*
a*b^4*e + 10*a^2*b^3*f)*x^9 + 3*(4*a*b^4*d - 11*a^2*b^3*e + 21*a^
3*b^2*f)*x^6 + 9*a^2*b^3*c - 15*a^3*b^2*d + 21*a^4*b*e - 27*a^5*f
 + 6*(2*a*b^4*c - 2*a^2*b^3*d + a^3*b^2*e + a^4*b*f)*x^3 + 6*((b^
5*c - 3*a*b^4*d + 6*a^2*b^3*e - 10*a^3*b^2*f)*x^6 + a^2*b^3*c - 3
*a^3*b^2*d + 6*a^4*b*e - 10*a^5*f + 2*(a*b^4*c - 3*a^2*b^3*d + 6*
a^3*b^2*e - 10*a^4*b*f)*x^3)*log(b*x^3 + a))/(b^8*x^6 + 2*a*b^7*x
^3 + a^2*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218015, size = 319, normalized size = 1.72

$$\frac{\left(b^3c - 3\,ab^2d - 10\,a^3f + 6\,a^2be\right)\ln\left(\left|bx^3 + a\right|\right)}{3\,b^6} - \frac{3\,b^5cx^6 - 9\,ab^4dx^6 - 30\,a^3b^2fx^6 + 18\,a^2b^3x^6e + 2\,ab^4cx^3 - 12\,a^2b^3dx^3 - 50\,a^4bfx^3 + 28\,a^3b^2x^3e - 4\,a^3b^2d - 21\,a^5f + 11}{6\,(bx^3 + a)^2b^6} + \frac{2\,b^6fx^9 - 9\,ab^5fx^6 + 3\,b^6x^6e + 6\,b^6dx^3 + 36\,a^2b^4fx^3 - 18\,ab^5x^3e}{18\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^8/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - 3*a*b^2*d - 10*a^3*f + 6*a^2*b*e)*ln(abs(b*x^3 + a))
/b^6 - 1/6*(3*b^5*c*x^6 - 9*a*b^4*d*x^6 - 30*a^3*b^2*f*x^6 + 18*a
^2*b^3*x^6*e + 2*a*b^4*c*x^3 - 12*a^2*b^3*d*x^3 - 50*a^4*b*f*x^3
+ 28*a^3*b^2*x^3*e - 4*a^3*b^2*d - 21*a^5*f + 11*a^4*b*e)/((b*x^3
 + a)^2*b^6) + 1/18*(2*b^6*f*x^9 - 9*a*b^5*f*x^6 + 3*b^6*x^6*e +
6*b^6*d*x^3 + 36*a^2*b^4*f*x^3 - 18*a*b^5*x^3*e)/b^9
```

3.279 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=146

$$\frac{\log\left(a+bx^3\right)\left(6a^2f-3abe+b^2d\right)}{3b^5}-\frac{-4a^3f+3a^2be-2ab^2d+b^3c}{3b^5\left(a+bx^3\right)}+\frac{a\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}+\frac{x^3(be-3af)}{3b^4}+\frac{fx^6}{6b^3}$$

```
[Out] ((b*e - 3*a*f)*x^3)/(3*b^4) + (f*x^6)/(6*b^3) + (a*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(6*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d
 + 3*a^2*b*e - 4*a^3*f)/(3*b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e +
 6*a^2*f)*Log[a + b*x^3])/(3*b^5)
```

Rubi [A] time = 0.387975, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{\log\left(a+bx^3\right)\left(6a^2f-3abe+b^2d\right)}{3b^5}-\frac{-4a^3f+3a^2be-2ab^2d+b^3c}{3b^5\left(a+bx^3\right)}+\frac{a\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}+\frac{x^3(be-3af)}{3b^4}+\frac{fx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^3)/(3*b^4) + (f*x^6)/(6*b^3) + (a*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(6*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d
 + 3*a^2*b*e - 4*a^3*f)/(3*b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e +
 6*a^2*f)*Log[a + b*x^3])/(3*b^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\left(af-\frac{be}{3}\right)\int^{x^3}\frac{1}{b^4}\,dx+\frac{f\int^{x^3}x\,dx}{3b^3}+\frac{\left(6a^2f-3abe+b^2d\right)\log\left(a+bx^3\right)}{3b^5}+\frac{4a^3f-3a^2be+2ab^2d-b^3c}{3b^5\left(a+bx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] -a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*b**5*(a + b*x**3)**
2) - (a*f - b*e/3)*Integral(b**(-4), (x, x**3)) + f*Integral(x, (
x, x**3))/(3*b**3) + (6*a**2*f - 3*a*b*e + b**2*d)*log(a + b*x**3
)/(3*b**5) + (4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(3*b**
5*(a + b*x**3))
```

Mathematica [A] time = 0.135377, size = 145, normalized size = 0.99

$$\frac{7a^4f+a^3b\left(2fx^3-5e\right)+2\left(a+bx^3\right)^2\log\left(a+bx^3\right)\left(6a^2f-3abe+b^2d\right)+a^2b^2\left(3d-4ex^3-11fx^6\right)-ab^3\left(c-4x^3\left(d+\right.\right.}{6b^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (7*a^4*f + a^3*b*(-5*e + 2*f*x^3) + a^2*b^2*(3*d - 4*e*x^3 - 11*f
*x^6) + b^4*x^3*(-2*c + 2*e*x^6 + f*x^9) - a*b^3*(c - 4*x^3*(d +
e*x^3 - f*x^6)) + 2*(b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^3)^2*Log
[a + b*x^3])/(6*b^5*(a + b*x^3)^2)
```

Maple [A] time = 0.017, size = 213, normalized size = 1.5

$$\frac{fx^6}{6\,b^3}-\frac{ax^3f}{b^4}+\frac{x^3e}{3\,b^3}+2\,\frac{\ln\left(bx^3+a\right)a^2f}{b^5}-\frac{\ln\left(bx^3+a\right)ae}{b^4}+\frac{\ln\left(bx^3+a\right)d}{3\,b^3}-\frac{a^4f}{6\,b^5\left(bx^3+a\right)^2}+\frac{a^3e}{6\,b^4\left(bx^3+a\right)^2}-\frac{a^2d}{6\,b^3\left(bx^3+a\right)^2}+\frac{ac}{6\,b^2\left(bx^3+a\right)^2}+\frac{4\,a^3f}{3\,b^5\left(bx^3+a\right)}-\frac{a^2e}{b^4\left(bx^3+a\right)}+\frac{2\,ad}{3\,b^3\left(bx^3+a\right)}-\frac{c}{3\,b^2\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*f*x^6/b^3-1/b^4*x^3*a*f+1/3/b^3*x^3*e+2/b^5*ln(b*x^3+a)*a^2*f
-1/b^4*ln(b*x^3+a)*a*e+1/3/b^3*ln(b*x^3+a)*d-1/6/b^5*a^4/(b*x^3+a
)^2*f+1/6/b^4*a^3/(b*x^3+a)^2*e-1/6/b^3*a^2/(b*x^3+a)^2*d+1/6/b^2
*a/(b*x^3+a)^2*c+4/3/b^5/(b*x^3+a)*a^3*f-1/b^4/(b*x^3+a)*a^2*e+2/
3/b^3/(b*x^3+a)*a*d-1/3/b^2/(b*x^3+a)*c
```

Maxima [A] time = 1.38214, size = 198, normalized size = 1.36

$$-\frac{ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+2\left(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf\right)x^3}{6\left(b^7x^6+2\,ab^6x^3+a^2b^5\right)}+\frac{bfx^6+2\,(be-3\,af)x^3}{6\,b^4}+\frac{\left(b^2d-3\,abe+6\,a^2f\right)\log\left(bx^3+a\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^5/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + 2*(b^4*c - 2*
a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)/(b^7*x^6 + 2*a*b^6*x^3 +
a^2*b^5) + 1/6*(b*f*x^6 + 2*(b*e - 3*a*f)*x^3)/b^4 + 1/3*(b^2*d -
 3*a*b*e + 6*a^2*f)*log(b*x^3 + a)/b^5
```

Fricas [A] time = 0.20742, size = 304, normalized size = 2.08

$$\frac{b^4fx^{12}+2\left(b^4e-2\,ab^3f\right)x^9+\left(4\,ab^3e-11\,a^2b^2f\right)x^6-ab^3c+3\,a^2b^2d-5\,a^3be+7\,a^4f-2\left(b^4c-2\,ab^3d+2\,a^2b^2e-a^3bf\right.}{6\left(b^7x^6+2\,ab^6x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^5/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*f*x^12 + 2*(b^4*e - 2*a*b^3*f)*x^9 + (4*a*b^3*e - 11*a^2
*b^2*f)*x^6 - a*b^3*c + 3*a^2*b^2*d - 5*a^3*b*e + 7*a^4*f - 2*(b^
4*c - 2*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^3 + 2*((b^4*d - 3*a*b^
3*e + 6*a^2*b^2*f)*x^6 + a^2*b^2*d - 3*a^3*b*e + 6*a^4*f + 2*(a*b
^3*d - 3*a^2*b^2*e + 6*a^3*b*f)*x^3)*log(b*x^3 + a))/(b^7*x^6 + 2
*a*b^6*x^3 + a^2*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.21479, size = 197, normalized size = 1.35

$$\frac{(b^2d + 6a^2f - 3abe)\ln(|bx^3 + a|)}{3b^5} + \frac{b^3fx^6 - 6ab^2fx^3 + 2b^3x^3e}{6b^6} - \frac{ab^3c - 3a^2b^2d - 7a^4f + 5a^3be + 2(b^4c - 2ab^3d - 4a^3bf + 3a^2b^2e)x^3}{6(bx^3 + a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^5/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*d + 6*a^2*f - 3*a*b*e)*ln(abs(b*x^3 + a))/b^5 + 1/6*(b^3
*f*x^6 - 6*a*b^2*f*x^3 + 2*b^3*x^3*e)/b^6 - 1/6*(a*b^3*c - 3*a^2*
b^2*d - 7*a^4*f + 5*a^3*b*e + 2*(b^4*c - 2*a*b^3*d - 4*a^3*b*f +
3*a^2*b^2*e)*x^3)/((b*x^3 + a)^2*b^5)
```

3.280 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=109

$$-\frac{3a^2f-2abe+b^2d}{3b^4(a+bx^3)}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6b^4(a+bx^3)^2}+\frac{(be-3af)\log(a+bx^3)}{3b^4}+\frac{fx^3}{3b^3}$$

```
[Out] (f*x^3)/(3*b^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*b^4*(a +
 b*x^3)^2) - (b^2*d - 2*a*b*e + 3*a^2*f)/(3*b^4*(a + b*x^3)) + ((
b*e - 3*a*f)*Log[a + b*x^3])/(3*b^4)
```

Rubi [A] time = 0.303931, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{3a^2f-2abe+b^2d}{3b^4(a+bx^3)}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6b^4(a+bx^3)^2}+\frac{(be-3af)\log(a+bx^3)}{3b^4}+\frac{fx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (f*x^3)/(3*b^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*b^4*(a +
 b*x^3)^2) - (b^2*d - 2*a*b*e + 3*a^2*f)/(3*b^4*(a + b*x^3)) + ((
b*e - 3*a*f)*Log[a + b*x^3])/(3*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^3} f\,dx}{3b^3}-\frac{(3af-be)\log(a+bx^3)}{3b^4}-\frac{3a^2f-2abe+b^2d}{3b^4(a+bx^3)}+\frac{a^3f-a^2be+ab^2d-b^3c}{6b^4(a+bx^3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Integral(f, (x, x**3))/(3*b**3) - (3*a*f - b*e)*log(a + b*x**3)/(
3*b**4) - (3*a**2*f - 2*a*b*e + b**2*d)/(3*b**4*(a + b*x**3)) + (
a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*b**4*(a + b*x**3)**2)
```

Mathematica [A] time = 0.101789, size = 105, normalized size = 0.96

$$\frac{-5a^3f+a^2b(3e-4fx^3)+ab^2(-d+4ex^3+4fx^6)+2(a+bx^3)^2(be-3af)\log(a+bx^3)-b^3(c+2dx^3-2fx^9)}{6b^4(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (-5*a^3*f + a^2*b*(3*e - 4*f*x^3) + a*b^2*(-d + 4*e*x^3 + 4*f*x^6
) - b^3*(c + 2*d*x^3 - 2*f*x^9) + 2*(b*e - 3*a*f)*(a + b*x^3)^2*L
og[a + b*x^3])/(6*b^4*(a + b*x^3)^2)
```

Maple [A] time = 0.016, size = 156, normalized size = 1.4

$$\frac{fx^3}{3b^3}-\frac{\ln(bx^3+a)af}{b^4}+\frac{\ln(bx^3+a)e}{3b^3}+\frac{a^3f}{6b^4(bx^3+a)^2}-\frac{a^2e}{6b^3(bx^3+a)^2}+\frac{ad}{6b^2(bx^3+a)^2}-\frac{c}{6b(bx^3+a)^2}-\frac{a^2f}{b^4(bx^3+a)}+\frac{2ae}{3b^3(bx^3+a)}-\frac{d}{3b^2(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/3*f*x^3/b^3-1/b^4*ln(b*x^3+a)*a*f+1/3/b^3*ln(b*x^3+a)*e+1/6/b^4
/(b*x^3+a)^2*a^3*f-1/6/b^3/(b*x^3+a)^2*a^2*e+1/6/b^2/(b*x^3+a)^2*
a*d-1/6/b/(b*x^3+a)^2*c-1/b^4/(b*x^3+a)*a^2*f+2/3/b^3/(b*x^3+a)*a
*e-1/3/b^2/(b*x^3+a)*d
```

Maxima [A] time = 1.38247, size = 147, normalized size = 1.35

$$\frac{fx^3}{3b^3}-\frac{b^3c+ab^2d-3a^2be+5a^3f+2(b^3d-2ab^2e+3a^2bf)x^3}{6(b^6x^6+2ab^5x^3+a^2b^4)}+\frac{(be-3af)\log(bx^3+a)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^2/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b^3 - 1/6*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f + 2*(b
^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b
^4) + 1/3*(b*e - 3*a*f)*log(b*x^3 + a)/b^4
```

Fricas [A] time = 0.202961, size = 213, normalized size = 1.95

$$\frac{2b^3fx^9+4ab^2fx^6-b^3c-ab^2d+3a^2be-5a^3f-2(b^3d-2ab^2e+2a^2bf)x^3+2((b^3e-3ab^2f)x^6+a^2be-3a^3f+2}{6(b^6x^6+2ab^5x^3+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^2/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*f*x^9 + 4*a*b^2*f*x^6 - b^3*c - a*b^2*d + 3*a^2*b*e -
5*a^3*f - 2*(b^3*d - 2*a*b^2*e + 2*a^2*b*f)*x^3 + 2*((b^3*e - 3*a
*b^2*f)*x^6 + a^2*b*e - 3*a^3*f + 2*(a*b^2*e - 3*a^2*b*f)*x^3)*lo
g(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.215713, size = 135, normalized size = 1.24

$$\frac{fx^3}{3\,b^3} - \frac{(3\,af - be)\ln\left(\left|bx^3 + a\right|\right)}{3\,b^4} - \frac{b^3c + ab^2d + 5\,a^3f + 2\,\left(b^3d + 3\,a^2bf - 2\,ab^2e\right)x^3 - 3\,a^2be}{6\,(bx^3 + a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^2/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b^3 - 1/3*(3*a*f - b*e)*ln(abs(b*x^3 + a))/b^4 - 1/6*(b
^3*c + a*b^2*d + 5*a^3*f + 2*(b^3*d + 3*a^2*b*f - 2*a*b^2*e)*x^3
- 3*a^2*b*e)/((b*x^3 + a)^2*b^4)
```

3.281 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3}\,dx$

Optimal. Leaf size=114

$$-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)+\frac{c\log(x)}{a^3}+\frac{2a^3f-a^2be+b^3c}{3a^2b^3\left(a+bx^3\right)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6ab^3\left(a+bx^3\right)^2}$$

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a*b^3*(a + b*x^3)^2) + (b^
3*c - a^2*b*e + 2*a^3*f)/(3*a^2*b^3*(a + b*x^3)) + (c*Log[x])/a^3
 - ((c/a^3 - f/b^3)*Log[a + b*x^3])/3
```

Rubi [A] time = 0.297897, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)+\frac{c\log(x)}{a^3}+\frac{2a^3f-a^2be+b^3c}{3a^2b^3\left(a+bx^3\right)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6ab^3\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a*b^3*(a + b*x^3)^2) + (b^
3*c - a^2*b*e + 2*a^3*f)/(3*a^2*b^3*(a + b*x^3)) + (c*Log[x])/a^3
 - ((c/a^3 - f/b^3)*Log[a + b*x^3])/3
```

Rubi in Sympy [A] time = 54.6483, size = 105, normalized size = 0.92

$$-\left(-\frac{f}{3b^3}+\frac{c}{3a^3}\right)\log\left(a+bx^3\right)-\frac{a^3f-a^2be+ab^2d-b^3c}{6ab^3\left(a+bx^3\right)^2}+\frac{2a^3f-a^2be+b^3c}{3a^2b^3\left(a+bx^3\right)}+\frac{c\log\left(x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**3,x)
```

```
[Out] -(-f/(3*b**3) + c/(3*a**3))*log(a + b*x**3) - (a**3*f - a**2*b*e
+ a*b**2*d - b**3*c)/(6*a*b**3*(a + b*x**3)**2) + (2*a**3*f - a**
2*b*e + b**3*c)/(3*a**2*b**3*(a + b*x**3)) + c*log(x**3)/(3*a**3)
```

Mathematica [A] time = 0.198968, size = 104, normalized size = 0.91

$$\frac{\frac{2\left(a^3f-b^3c\right)\log\left(a+bx^3\right)+\frac{a\left(3a^4f-a^3b\left(e-4fx^3\right)-a^2b^2\left(d+2ex^3\right)+3ab^3c+2b^4cx^3\right)}{\left(a+bx^3\right)^2}}{b^3}+6c\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (6*c*Log[x] + ((a*(3*a*b^3*c + 3*a^4*f + 2*b^4*c*x^3 - a^2*b^2*(d
 + 2*e*x^3) - a^3*b*(e - 4*f*x^3)))/(a + b*x^3)^2 + 2*(-(b^3*c) +
 a^3*f)*Log[a + b*x^3])/b^3)/(6*a^3)
```

Maple [A] time = 0.02, size = 147, normalized size = 1.3

$$\frac{c\ln(x)}{a^3}+\frac{\ln\left(bx^3+a\right)f}{3\,b^3}-\frac{c\ln\left(bx^3+a\right)}{3\,a^3}-\frac{a^2f}{6\,b^3\left(bx^3+a\right)^2}+\frac{ae}{6\,b^2\left(bx^3+a\right)^2}-\frac{d}{6\,b\left(bx^3+a\right)^2}+\frac{c}{6\,a\left(bx^3+a\right)^2}+\frac{2\,af}{3\,b^3\left(bx^3+a\right)}-\frac{e}{3\,b^2\left(bx^3+a\right)}+\frac{c}{3\,a^2\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x)
```

```
[Out] c*ln(x)/a^3+1/3/b^3*ln(b*x^3+a)*f-1/3*c*ln(b*x^3+a)/a^3-1/6*a^2/b
^3/(b*x^3+a)^2*f+1/6*a/b^2/(b*x^3+a)^2*e-1/6/b/(b*x^3+a)^2*d+1/6/
a/(b*x^3+a)^2*c+2/3*a/b^3/(b*x^3+a)*f-1/3/b^2/(b*x^3+a)*e+1/3/a^2
/(b*x^3+a)*c
```

Maxima [A] time = 1.38861, size = 174, normalized size = 1.53

$$\frac{3\,ab^3c-a^2b^2d-a^3be+3\,a^4f+2\left(b^4c-a^2b^2e+2\,a^3bf\right)x^3}{6\left(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3\right)}+\frac{c\log\left(x^3\right)}{3\,a^3}-\frac{\left(b^3c-a^3f\right)\log\left(bx^3+a\right)}{3\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f + 2*(b^4*c - a^2*b
^2*e + 2*a^3*b*f)*x^3)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3) +
1/3*c*log(x^3)/a^3 - 1/3*(b^3*c - a^3*f)*log(b*x^3 + a)/(a^3*b^3)
```

Fricas [A] time = 0.223264, size = 252, normalized size = 2.21

$$\frac{3\,a^2b^3c-a^3b^2d-a^4be+3\,a^5f+2\left(ab^4c-a^3b^2e+2\,a^4bf\right)x^3-2\left(\left(b^5c-a^3b^2f\right)x^6+a^2b^3c-a^5f+2\left(ab^4c-a^4bf\right)x^3\right)}{6\left(a^3b^5x^6+2\,a^4b^4x^3+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*b^3*c - a^3*b^2*d - a^4*b*e + 3*a^5*f + 2*(a*b^4*c - a
^3*b^2*e + 2*a^4*b*f)*x^3 - 2*((b^5*c - a^3*b^2*f)*x^6 + a^2*b^3*
c - a^5*f + 2*(a*b^4*c - a^4*b*f)*x^3)*log(b*x^3 + a) + 6*(b^5*c*
x^6 + 2*a*b^4*c*x^3 + a^2*b^3*c)*log(x))/(a^3*b^5*x^6 + 2*a^4*b^4
*x^3 + a^5*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.215346, size = 173, normalized size = 1.52

$$\frac{c\ln\left(|x|\right)}{a^3} - \frac{\left(b^3c - a^3f\right)\ln\left(\left|bx^3 + a\right|\right)}{3\,a^3b^3} + \frac{3\,b^4cx^6 - 3\,a^3bfx^6 + 8\,ab^3cx^3 - 2\,a^4fx^3 - 2\,a^3bx^3e + 6\,a^2b^2c - a^3bd - a^4e}{6\,(bx^3 + a)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x),x, algorithm="giac")
```

```
[Out] c*ln(abs(x))/a^3 - 1/3*(b^3*c - a^3*f)*ln(abs(b*x^3 + a))/(a^3*b^
3) + 1/6*(3*b^4*c*x^6 - 3*a^3*b*f*x^6 + 8*a*b^3*c*x^3 - 2*a^4*f*x
^3 - 2*a^3*b*x^3*e + 6*a^2*b^2*c - a^3*b*d - a^4*e)/((b*x^3 + a)^
2*a^3*b^2)
```

3.282 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=134

$$\frac{(3bc-ad)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-ad)}{a^4}-\frac{a^3f-ab^2d+2b^3c}{3a^3b^2\left(a+bx^3\right)}-\frac{c}{3a^3x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^2b^2\left(a+bx^3\right)^2}$$

```
[Out] -c/(3*a^3*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^2*b^2*(
a + b*x^3)^2) - (2*b^3*c - a*b^2*d + a^3*f)/(3*a^3*b^2*(a + b*x^3
)) - ((3*b*c - a*d)*Log[x])/a^4 + ((3*b*c - a*d)*Log[a + b*x^3])/
(3*a^4)
```

Rubi [A] time = 0.34449, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{(3bc-ad)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-ad)}{a^4}-\frac{a^3f-ab^2d+2b^3c}{3a^3b^2\left(a+bx^3\right)}-\frac{c}{3a^3x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^2b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -c/(3*a^3*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^2*b^2*(
a + b*x^3)^2) - (2*b^3*c - a*b^2*d + a^3*f)/(3*a^3*b^2*(a + b*x^3
)) - ((3*b*c - a*d)*Log[x])/a^4 + ((3*b*c - a*d)*Log[a + b*x^3])/
(3*a^4)
```

Rubi in Sympy [A] time = 57.1606, size = 124, normalized size = 0.93

$$\frac{a^3f-a^2be+ab^2d-b^3c}{6a^2b^2\left(a+bx^3\right)^2}-\frac{c}{3a^3x^3}-\frac{a^3f-ab^2d+2b^3c}{3a^3b^2\left(a+bx^3\right)}+\frac{(ad-3bc)\log\left(x^3\right)}{3a^4}-\frac{(ad-3bc)\log\left(a+bx^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*a**2*b**2*(a + b*x**3)
**2) - c/(3*a**3*x**3) - (a**3*f - a*b**2*d + 2*b**3*c)/(3*a**3*b
**2*(a + b*x**3)) + (a*d - 3*b*c)*log(x**3)/(3*a**4) - (a*d - 3*b
*c)*log(a + b*x**3)/(3*a**4)
```

Mathematica [A] time = 0.172494, size = 121, normalized size = 0.9

$$\frac{-\frac{2a\left(a^3f-ab^2d+2b^3c\right)}{b^2\left(a+bx^3\right)}+\frac{a^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^2\left(a+bx^3\right)^2}+2(3bc-ad)\log\left(a+bx^3\right)+6\log(x)(ad-3bc)-\frac{2ac}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a*c)/x^3 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2
*(a + b*x^3)^2) - (2*a*(2*b^3*c - a*b^2*d + a^3*f))/(b^2*(a + b*x
^3)) + 6*(-3*b*c + a*d)*Log[x] + 2*(3*b*c - a*d)*Log[a + b*x^3])/
(6*a^4)
```

Maple [A] time = 0.025, size = 163, normalized size = 1.2

$$-\frac{c}{3\,a^3x^3}+\frac{d\ln(x)}{a^3}-3\,\frac{bc\ln(x)}{a^4}-\frac{d\ln\left(bx^3+a\right)}{3\,a^3}+\frac{bc\ln\left(bx^3+a\right)}{a^4}+\frac{af}{6\,b^2\,(bx^3+a)^2}-\frac{e}{6\,b\,(bx^3+a)^2}+\frac{d}{6\,a\,(bx^3+a)^2}-\frac{bc}{6\,a^2\,(bx^3+a)^2}-\frac{f}{3\,b^2\,(bx^3+a)}+\frac{d}{3\,a^2\,(bx^3+a)}-\frac{2\,bc}{3\,a^3\,(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x)
```

```
[Out] -1/3*c/a^3/x^3+d*ln(x)/a^3-3*b*c*ln(x)/a^4-1/3*d*ln(b*x^3+a)/a^3+
b*c*ln(b*x^3+a)/a^4+1/6*a/b^2/(b*x^3+a)^2*f-1/6/b/(b*x^3+a)^2*e+1
/6/a/(b*x^3+a)^2*d-1/6/a^2*b/(b*x^3+a)^2*c-1/3/b^2/(b*x^3+a)*f+1/
3/a^2/(b*x^3+a)*d-2/3/a^3*b/(b*x^3+a)*c
```

Maxima [A] time = 1.38988, size = 194, normalized size = 1.45

$$-\frac{2\left(3\,b^4c-ab^3d+a^3bf\right)x^6+2\,a^2b^2c+\left(9\,ab^3c-3\,a^2b^2d+a^3be+a^4f\right)x^3}{6\,(a^3b^4x^9+2\,a^4b^3x^6+a^5b^2x^3)}+\frac{(3\,bc-ad)\log\left(bx^3+a\right)}{3\,a^4}-\frac{(3\,bc-ad)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^4),x, algorithm="maxima")
```

```
[Out] -1/6*(2*(3*b^4*c - a*b^3*d + a^3*b*f)*x^6 + 2*a^2*b^2*c + (9*a*b^
3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)/(a^3*b^4*x^9 + 2*a^4*b^
3*x^6 + a^5*b^2*x^3) + 1/3*(3*b*c - a*d)*log(b*x^3 + a)/a^4 - 1/3
*(3*b*c - a*d)*log(x^3)/a^4
```

Fricas [A] time = 0.273458, size = 338, normalized size = 2.52

$$-\frac{2\left(3\,ab^4c-a^2b^3d+a^4bf\right)x^6+2\,a^3b^2c+\left(9\,a^2b^3c-3\,a^3b^2d+a^4be+a^5f\right)x^3-2\left(\left(3\,b^5c-ab^4d\right)x^9+2\left(3\,ab^4c-a^2b^3d\right)x\right.}{6\,(a^4b^4x^9+2\,a^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^4),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*a*b^4*c - a^2*b^3*d + a^4*b*f)*x^6 + 2*a^3*b^2*c + (9*
a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e + a^5*f)*x^3 - 2*((3*b^5*c - a*
b^4*d)*x^9 + 2*(3*a*b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a^3*b
^2*d)*x^3)*log(b*x^3 + a) + 6*((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*b
^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a^3*b^2*d)*x^3)*log(x))/(a
^4*b^4*x^9 + 2*a^5*b^3*x^6 + a^6*b^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.215929, size = 234, normalized size = 1.75

$$-\frac{(3\,bc-ad)\ln(|x|)}{a^4}+\frac{\left(3\,b^2c-abd\right)\ln\left(\left|bx^3+a\right|\right)}{3\,a^4b}+\frac{3\,bcx^3-adx^3-ac}{3\,a^4x^3}-\frac{9\,b^5cx^6-3\,ab^4dx^6+22\,ab^4cx^3-8\,a^2b^3dx^3+2\,a^4bfx^3+14\,a^2b^3c-6\,a^3b^2d+a^5f+a^4be}{6\,(bx^3+a)^2a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^4),x, algorithm="giac")
```

```
[Out] -(3*b*c - a*d)*ln(abs(x))/a^4 + 1/3*(3*b^2*c - a*b*d)*ln(abs(b*x^
3 + a))/(a^4*b) + 1/3*(3*b*c*x^3 - a*d*x^3 - a*c)/(a^4*x^3) - 1/6
*(9*b^5*c*x^6 - 3*a*b^4*d*x^6 + 22*a*b^4*c*x^3 - 8*a^2*b^3*d*x^3
+ 2*a^4*b*f*x^3 + 14*a^2*b^3*c - 6*a^3*b^2*d + a^5*f + a^4*b*e)/(
(b*x^3 + a)^2*a^4*b^2)
```

3.283 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx$

Optimal. Leaf size=163

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{6a^3x^6}-\frac{\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)}{3a^5}+\frac{\log(x)\left(a^2e-3abd+6b^2c\right)}{a^5}+\frac{a^2e-2abd+3b^2c}{3a^4\left(a+bx^3\right)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^3b\left(a+bx^3\right)^2}$$

```
[Out] -c/(6*a^3*x^6) + (3*b*c - a*d)/(3*a^4*x^3) + (b^3*c - a*b^2*d + a
^2*b*e - a^3*f)/(6*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^
2*e)/(3*a^4*(a + b*x^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*Log[x])/a
^5 - ((6*b^2*c - 3*a*b*d + a^2*e)*Log[a + b*x^3])/(3*a^5)
```

Rubi [A] time = 0.402378, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{6a^3x^6}-\frac{\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)}{3a^5}+\frac{\log(x)\left(a^2e-3abd+6b^2c\right)}{a^5}+\frac{a^2e-2abd+3b^2c}{3a^4\left(a+bx^3\right)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^3b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -c/(6*a^3*x^6) + (3*b*c - a*d)/(3*a^4*x^3) + (b^3*c - a*b^2*d + a
^2*b*e - a^3*f)/(6*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^
2*e)/(3*a^4*(a + b*x^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*Log[x])/a
^5 - ((6*b^2*c - 3*a*b*d + a^2*e)*Log[a + b*x^3])/(3*a^5)
```

Rubi in Sympy [A] time = 61.9752, size = 156, normalized size = 0.96

$$-\frac{c}{6a^3x^6}-\frac{a^3f-a^2be+ab^2d-b^3c}{6a^3b\left(a+bx^3\right)^2}+\frac{a^2e-2abd+3b^2c}{3a^4\left(a+bx^3\right)}-\frac{ad-3bc}{3a^4x^3}+\frac{\left(a^2e-3abd+6b^2c\right)\log\left(x^3\right)}{3a^5}-\frac{\left(a^2e-3abd+6b^2c\right)\log\left(a+bx^3\right)}{3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**3,x)
```

```
[Out] -c/(6*a**3*x**6) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*a**
3*b*(a + b*x**3)**2) + (a**2*e - 2*a*b*d + 3*b**2*c)/(3*a**4*(a +
 b*x**3)) - (a*d - 3*b*c)/(3*a**4*x**3) + (a**2*e - 3*a*b*d + 6*b
**2*c)*log(x**3)/(3*a**5) - (a**2*e - 3*a*b*d + 6*b**2*c)*log(a +
 b*x**3)/(3*a**5)
```

Mathematica [A] time = 0.212299, size = 149, normalized size = 0.91

$$\frac{\frac{2a\left(a^2e-2abd+3b^2c\right)}{a+bx^3}-2\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)+6\log(x)\left(a^2e-3abd+6b^2c\right)-\frac{a^2c}{x^6}+\frac{a^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b\left(a+bx^3\right)^2}-2}{6a^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] (-((a^2*c)/x^6) - (2*a*(-3*b*c + a*d))/x^3 + (a^2*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f))/(b*(a + b*x^3)^2) + (2*a*(3*b^2*c - 2*a*b*d
 + a^2*e))/(a + b*x^3) + 6*(6*b^2*c - 3*a*b*d + a^2*e)*Log[x] - 2
*(6*b^2*c - 3*a*b*d + a^2*e)*Log[a + b*x^3])/(6*a^5)
```

Maple [A] time = 0.025, size = 213, normalized size = 1.3

$$-\frac{c}{6\,a^3x^6}-\frac{d}{3\,a^3x^3}+\frac{bc}{a^4x^3}+\frac{e\ln(x)}{a^3}-3\,\frac{\ln(x)\,bd}{a^4}+6\,\frac{\ln(x)\,b^2c}{a^5}-\frac{e\ln\left(bx^3+a\right)}{3\,a^3}+\frac{\ln\left(bx^3+a\right)\,bd}{a^4}-2\,\frac{\ln\left(bx^3+a\right)\,b^2c}{a^5}-\frac{f}{6\,b\,(bx^3+a)^2}+\frac{e}{6\,a\,(bx^3+a)^2}-\frac{bd}{6\,a^2\,(bx^3+a)^2}+\frac{b^2c}{6\,a^3\,(bx^3+a)^2}+\frac{e}{3\,a^2\,(bx^3+a)}-\frac{2\,bd}{3\,a^3\,(bx^3+a)}+\frac{b^2c}{a^4\,(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x)
```

```
[Out] -1/6*c/a^3/x^6-1/3/a^3/x^3*d+1/a^4/x^3*b*c+e*ln(x)/a^3-3/a^4*ln(x
)*b*d+6/a^5*ln(x)*b^2*c-1/3*e*ln(b*x^3+a)/a^3+1/a^4*ln(b*x^3+a)*b
*d-2/a^5*ln(b*x^3+a)*b^2*c-1/6/b/(b*x^3+a)^2*f+1/6/a/(b*x^3+a)^2*
e-1/6/a^2*b/(b*x^3+a)^2*d+1/6/a^3*b^2/(b*x^3+a)^2*c+1/3/a^2/(b*x^
3+a)*e-2/3/a^3/(b*x^3+a)*b*d+1/a^4/(b*x^3+a)*b^2*c
```

Maxima [A] time = 1.38375, size = 246, normalized size = 1.51

$$\frac{2\left(6\,b^4c-3\,ab^3d+a^2b^2e\right)x^9+\left(18\,ab^3c-9\,a^2b^2d+3\,a^3be-a^4f\right)x^6-a^3bc+2\left(2\,a^2b^2c-a^3bd\right)x^3}{6\left(a^4b^3x^{12}+2\,a^5b^2x^9+a^6bx^6\right)}-\frac{\left(6\,b^2c-3\,abd+a^2e\right)\log\left(bx^3+a\right)}{3\,a^5}+\frac{\left(6\,b^2c-3\,abd+a^2e\right)\log\left(x^3\right)}{3\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^7),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(6*b^4*c - 3*a*b^3*d + a^2*b^2*e)*x^9 + (18*a*b^3*c - 9*a^
2*b^2*d + 3*a^3*b*e - a^4*f)*x^6 - a^3*b*c + 2*(2*a^2*b^2*c - a^3
*b*d)*x^3)/(a^4*b^3*x^12 + 2*a^5*b^2*x^9 + a^6*b*x^6) - 1/3*(6*b^
2*c - 3*a*b*d + a^2*e)*log(b*x^3 + a)/a^5 + 1/3*(6*b^2*c - 3*a*b*
d + a^2*e)*log(x^3)/a^5
```

Fricas [A] time = 0.279899, size = 427, normalized size = 2.62

$$\frac{2\left(6\,ab^4c-3\,a^2b^3d+a^3b^2e\right)x^9+\left(18\,a^2b^3c-9\,a^3b^2d+3\,a^4be-a^5f\right)x^6-a^4bc+2\left(2\,a^3b^2c-a^4bd\right)x^3-2\left(\left(6\,b^5c-3\,ab^4d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^7),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (18*a^2*b^3*c
- 9*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^6 - a^4*b*c + 2*(2*a^3*b^2*c
 - a^4*b*d)*x^3 - 2*((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(
6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b
^2*d + a^4*b*e)*x^6)*log(b*x^3 + a) + 6*((6*b^5*c - 3*a*b^4*d + a
^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (6
*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(x))/(a^5*b^3*x^12 +
2*a^6*b^2*x^9 + a^7*b*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.216499, size = 255, normalized size = 1.56

$$\frac{\left(6\,b^{2}c-3\,abd+a^{2}e\right)\ln\left(|x|\right)}{a^{5}}-\frac{\left(6\,b^{3}c-3\,ab^{2}d+a^{2}be\right)\ln\left(\left|bx^{3}+a\right|\right)}{3\,a^{5}b}+\frac{12\,b^{4}cx^{9}-6\,ab^{3}dx^{9}+2\,a^{2}b^{2}x^{9}e+18\,ab^{3}cx^{6}-9\,a^{2}b^{2}dx^{6}-a^{4}fx^{6}+3\,a^{3}bx^{6}e+4\,a^{2}b^{2}cx^{3}-2\,a^{3}bdx^{3}-a^{3}bc}{6\,\left(bx^{6}+ax^{3}\right)^{2}a^{4}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^7),x, algorithm="giac")
```

```
[Out] (6*b^2*c - 3*a*b*d + a^2*e)*ln(abs(x))/a^5 - 1/3*(6*b^3*c - 3*a*b
^2*d + a^2*b*e)*ln(abs(b*x^3 + a))/(a^5*b) + 1/6*(12*b^4*c*x^9 -
6*a*b^3*d*x^9 + 2*a^2*b^2*x^9*e + 18*a*b^3*c*x^6 - 9*a^2*b^2*d*x^
6 - a^4*f*x^6 + 3*a^3*b*x^6*e + 4*a^2*b^2*c*x^3 - 2*a^3*b*d*x^3 -
 a^3*b*c)/((b*x^6 + a*x^3)^2*a^4*b)
```

3.284 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx$

Optimal. Leaf size=218

$$\frac{3bc-ad}{6a^4x^6}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{3a^6}-\frac{\log(x)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{a^6}-\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5\left(a+bx^3\right)}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^4\left(a+bx^3\right)^2}$$

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(6*a^4*x^6) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(3*a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*
(a + b*x^3)^2) - (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5
*(a + b*x^3)) - ((10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[x
])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[a + b*x^
3])/(3*a^6)
```

Rubi [A] time = 0.54307, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{3bc-ad}{6a^4x^6}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{3a^6}-\frac{\log(x)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{a^6}-\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5\left(a+bx^3\right)}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^4\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(6*a^4*x^6) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(3*a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*
(a + b*x^3)^2) - (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5
*(a + b*x^3)) - ((10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[x
])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[a + b*x^
3])/(3*a^6)
```

Rubi in Sympy [A] time = 79.3933, size = 212, normalized size = 0.97

$$-\frac{c}{9a^3x^9}+\frac{a^3f-a^2be+ab^2d-b^3c}{6a^4\left(a+bx^3\right)^2}-\frac{ad-3bc}{6a^4x^6}+\frac{a^3f-2a^2be+3ab^2d-4b^3c}{3a^5\left(a+bx^3\right)}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\left(a^3f-3a^2be+6ab^2d-10b^3c\right)\log\left(x^3\right)}{3a^6}-\frac{\left(a^3f-3a^2be+6ab^2d-10b^3c\right)\log\left(a+bx^3\right)}{3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**3,x)
```

```
[Out] -c/(9*a**3*x**9) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*a**
4*(a + b*x**3)**2) - (a*d - 3*b*c)/(6*a**4*x**6) + (a**3*f - 2*a*
*2*b*e + 3*a*b**2*d - 4*b**3*c)/(3*a**5*(a + b*x**3)) - (a**2*e -
 3*a*b*d + 6*b**2*c)/(3*a**5*x**3) + (a**3*f - 3*a**2*b*e + 6*a*b
**2*d - 10*b**3*c)*log(x**3)/(3*a**6) - (a**3*f - 3*a**2*b*e + 6*
a*b**2*d - 10*b**3*c)*log(a + b*x**3)/(3*a**6)
```

Mathematica [A] time = 0.279585, size = 200, normalized size = 0.92

$$\frac{-\frac{2a^3c}{x^9}-\frac{6a(a^2e-3abd+6b^2c)}{x^3}-\frac{3a^2(ad-3bc)}{x^6}+\frac{3a^2(a^3f-a^2be+ab^2d-b^3c)}{(a+bx^3)^2}+\frac{6a(a^3f-2a^2be+3ab^2d-4b^3c)}{a+bx^3}+6\log(a+bx^3)(a^3(-f)+3a^2\dots}{18a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-3*b*c + a*d))/x^6 - (6*a*(6*b^2*c - 3*
a*b*d + a^2*e))/x^3 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f))/(a + b*x^3)^2 + (6*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*
f))/(a + b*x^3) + 18*(-10*b^3*c + 6*a*b^2*d - 3*a^2*b*e + a^3*f)*
Log[x] + 6*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[a + b*x
^3])/(18*a^6)
```

Maple [A] time = 0.03, size = 293, normalized size = 1.3

$$-\frac{c}{9\,a^3x^9}-\frac{d}{6\,a^3x^6}+\frac{bc}{2\,a^4x^6}-\frac{e}{3\,a^3x^3}+\frac{bd}{a^4x^3}-2\,\frac{b^2c}{a^5x^3}+\frac{\ln(x)\,f}{a^3}-3\,\frac{\ln(x)\,be}{a^4}$$
$$+6\,\frac{\ln(x)\,b^2d}{a^5}-10\,\frac{\ln(x)\,b^3c}{a^6}-\frac{\ln(bx^3+a)\,f}{3\,a^3}+\frac{b\ln(bx^3+a)\,e}{a^4}-2\,\frac{b^2\ln(bx^3+a)\,d}{a^5}$$
$$+\frac{10\,b^3\ln(bx^3+a)\,c}{3\,a^6}+\frac{f}{6\,a\,(bx^3+a)^2}-\frac{be}{6\,a^2\,(bx^3+a)^2}+\frac{b^2d}{6\,a^3\,(bx^3+a)^2}$$
$$-\frac{b^3c}{6\,a^4\,(bx^3+a)^2}+\frac{f}{3\,a^2\,(bx^3+a)}-\frac{2\,be}{3\,a^3\,(bx^3+a)}+\frac{b^2d}{a^4\,(bx^3+a)}-\frac{4\,b^3c}{3\,a^5\,(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x)
```

```
[Out] -1/9*c/a^3/x^9-1/6/a^3/x^6*d+1/2/a^4/x^6*b*c-1/3/a^3/x^3*e+1/a^4/
x^3*b*d-2/a^5/x^3*b^2*c+1/a^3*ln(x)*f-3/a^4*ln(x)*b*e+6/a^5*ln(x)
*b^2*d-10/a^6*ln(x)*b^3*c-1/3/a^3*ln(b*x^3+a)*f+b/a^4*ln(b*x^3+a)
*e-2*b^2/a^5*ln(b*x^3+a)*d+10/3*b^3/a^6*ln(b*x^3+a)*c+1/6/a/(b*x^
3+a)^2*f-1/6*b/a^2/(b*x^3+a)^2*e+1/6*b^2/a^3/(b*x^3+a)^2*d-1/6*b^
3/a^4/(b*x^3+a)^2*c+1/3/a^2/(b*x^3+a)*f-2/3*b/a^3/(b*x^3+a)*e+b^2
/a^4/(b*x^3+a)*d-4/3*b^3/a^5/(b*x^3+a)*c
```

Maxima [A] time = 1.43119, size = 313, normalized size = 1.44

$$-\frac{6\left(10\,b^4c-6\,ab^3d+3\,a^2b^2e-a^3bf\right)x^{12}+9\left(10\,ab^3c-6\,a^2b^2d+3\,a^3be-a^4f\right)x^9+2\left(10\,a^2b^2c-6\,a^3bd+3\,a^4e\right)x^6+2\,a^4\dots}{18\,(a^5b^2x^{15}+2\,a^6bx^{12}+a^7x^9)}$$
$$+\frac{\left(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,a^6}-\frac{\left(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f\right)\log\left(x^3\right)}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^10),x, algorithm="maxima")
```

```
[Out] -1/18*(6*(10*b^4*c - 6*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^12 + 9*
(10*a*b^3*c - 6*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^9 + 2*(10*a^2*b^
2*c - 6*a^3*b*d + 3*a^4*e)*x^6 + 2*a^4*c - (5*a^3*b*c - 3*a^4*d)*
x^3)/(a^5*b^2*x^15 + 2*a^6*b*x^12 + a^7*x^9) + 1/3*(10*b^3*c - 6*
a*b^2*d + 3*a^2*b*e - a^3*f)*log(b*x^3 + a)/a^6 - 1/3*(10*b^3*c -
 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(x^3)/a^6
```

Fricas [A] time = 0.31021, size = 535, normalized size = 2.45

$$-\frac{6\left(10\,ab^4c-6\,a^2b^3d+3\,a^3b^2e-a^4bf\right)x^{12}+9\left(10\,a^2b^3c-6\,a^3b^2d+3\,a^4be-a^5f\right)x^9+2\left(10\,a^3b^2c-6\,a^4bd+3\,a^5e\right)x^6+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^10),x, algorithm="fricas")
```

```
[Out] -1/18*(6*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12
+ 9*(10*a^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9 + 2*(10*
a^3*b^2*c - 6*a^4*b*d + 3*a^5*e)*x^6 + 2*a^5*c - (5*a^4*b*c - 3*a
^5*d)*x^3 - 6*((10*b^5*c - 6*a*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*x
^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 +
 (10*a^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9)*log(b*x^3
+ a) + 18*((10*b^5*c - 6*a*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*x^15
+ 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + (10
*a^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9)*log(x))/(a^6*b
^2*x^15 + 2*a^7*b*x^12 + a^8*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217954, size = 437, normalized size = 2.

$$-\frac{\left(10\,b^3c-6\,ab^2d-a^3f+3\,a^2be\right)\ln\left(|x|\right)}{a^6}+\frac{\left(10\,b^4c-6\,ab^3d-a^3bf+3\,a^2b^2e\right)\ln\left(\left|bx^3+a\right|\right)}{3\,a^6b}-\frac{30\,b^5cx^6-18\,ab^4dx^6-3\,a^3b^2fx^6+9\,a^2b^3x^6e+68\,ab^4cx^3-42\,a^2b^3dx^3-8\,a^4bfx^3+22\,a^3b^2x^3e+39\,a^2b^3c-25\,a^3b^2d-}{6\left(bx^3+a\right)^2a^6}+\frac{110\,b^3cx^9-66\,ab^2dx^9-11\,a^3fx^9+33\,a^2bx^9e-36\,ab^2cx^6+18\,a^2bdx^6-6\,a^3x^6e+9\,a^2bcx^3-3\,a^3dx^3-2\,a^3c}{18\,a^6x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^10),x, algorithm="giac")
```

```
[Out] -(10*b^3*c - 6*a*b^2*d - a^3*f + 3*a^2*b*e)*ln(abs(x))/a^6 + 1/3*
(10*b^4*c - 6*a*b^3*d - a^3*b*f + 3*a^2*b^2*e)*ln(abs(b*x^3 + a))
/(a^6*b) - 1/6*(30*b^5*c*x^6 - 18*a*b^4*d*x^6 - 3*a^3*b^2*f*x^6 +
 9*a^2*b^3*x^6*e + 68*a*b^4*c*x^3 - 42*a^2*b^3*d*x^3 - 8*a^4*b*f*
x^3 + 22*a^3*b^2*x^3*e + 39*a^2*b^3*c - 25*a^3*b^2*d - 6*a^5*f +
14*a^4*b*e)/((b*x^3 + a)^2*a^6) + 1/18*(110*b^3*c*x^9 - 66*a*b^2*
d*x^9 - 11*a^3*f*x^9 + 33*a^2*b*x^9*e - 36*a*b^2*c*x^6 + 18*a^2*b
*d*x^6 - 6*a^3*x^6*e + 9*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^
6*x^9)
```

3.285 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^3}\,dx$

Optimal. Leaf size=258

$$\frac{3bc-ad}{9a^4x^9}-\frac{c}{12a^3x^{12}}-\frac{a^2e-3abd+6b^2c}{6a^5x^6}-\frac{b\log\left(a+bx^3\right)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{3a^7}+\frac{b\log(x)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7}+\frac{b\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3}+\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

```
[Out] -c/(12*a^3*x^12) + (3*b*c - a*d)/(9*a^4*x^9) - (6*b^2*c - 3*a*b*d
 + a^2*e)/(6*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f
)/(3*a^6*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*a^5*(a
 + b*x^3)^2) + (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(3
*a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3
*f)*Log[x])/a^7 - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f
)*Log[a + b*x^3])/(3*a^7)
```

Rubi [A] time = 0.670518, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{3bc-ad}{9a^4x^9}-\frac{c}{12a^3x^{12}}-\frac{a^2e-3abd+6b^2c}{6a^5x^6}-\frac{b\log\left(a+bx^3\right)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{3a^7}+\frac{b\log(x)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7}+\frac{b\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3}+\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```


```
[Out] -c/(12*a^3*x^12) + (3*b*c - a*d)/(9*a^4*x^9) - (6*b^2*c - 3*a*b*d
 + a^2*e)/(6*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f
)/(3*a^6*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*a^5*(a
 + b*x^3)^2) + (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(3
*a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3
*f)*Log[x])/a^7 - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f
)*Log[a + b*x^3])/(3*a^7)
```

Rubi in Sympy [A] time = 91.9342, size = 262, normalized size = 1.02

$$-\frac{c}{12a^3x^{12}}-\frac{ad-3bc}{9a^4x^9}-\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{6a^5\left(a+bx^3\right)^2}-\frac{a^2e-3abd+6b^2c}{6a^5x^6}-\frac{b\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)}{3a^6\left(a+bx^3\right)}-\frac{a^3f-3a^2be+6ab^2d-10b^3c}{3a^6x^3}-\frac{b\left(3a^3f-6a^2be+10ab^2d-15b^3c\right)\log\left(x^3\right)}{3a^7}+\frac{b\left(3a^3f-6a^2be+10ab^2d-15b^3c\right)\log\left(a+bx^3\right)}{3a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**3,x)
```

```
[Out] -c/(12*a**3*x**12) - (a*d - 3*b*c)/(9*a**4*x**9) - b*(a**3*f - a*
*2*b*e + a*b**2*d - b**3*c)/(6*a**5*(a + b*x**3)**2) - (a**2*e -
3*a*b*d + 6*b**2*c)/(6*a**5*x**6) - b*(2*a**3*f - 3*a**2*b*e + 4*
a*b**2*d - 5*b**3*c)/(3*a**6*(a + b*x**3)) - (a**3*f - 3*a**2*b*e
```

```
 + 6*a*b**2*d - 10*b**3*c)/(3*a**6*x**3) - b*(3*a**3*f - 6*a**2*b
*e + 10*a*b**2*d - 15*b**3*c)*log(x**3)/(3*a**7) + b*(3*a**3*f -
6*a**2*b*e + 10*a*b**2*d - 15*b**3*c)*log(a + b*x**3)/(3*a**7)
```

Mathematica [A] time = 0.415773, size = 238, normalized size = 0.92

$$\frac{12b\log\left(a+bx^3\right)\left(3a^3f-6a^2be+10ab^2d-15b^3c\right)+36b\log(x)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)-\frac{a(a^5(3c+4dx^3+6ex^6+12f}{}}{36a^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

```
[Out] (-((a*(-180*b^5*c*x^15 + 30*a*b^4*x^12*(-9*c + 4*d*x^3) - 12*a^2*
b^3*x^9*(5*c - 15*d*x^3 + 6*e*x^6) - 2*a^4*b*x^3*(3*c + 5*d*x^3 +
 12*e*x^6 - 27*f*x^9) + a^5*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9)
+ a^3*b^2*x^6*(15*c + 40*d*x^3 - 108*e*x^6 + 36*f*x^9)))/(x^12*(a
 + b*x^3)^2)) + 36*b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f
)*Log[x] + 12*b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f)*Lo
g[a + b*x^3])/(36*a^7)
```

Maple [A] time = 0.03, size = 349, normalized size = 1.4

$$-\frac{c}{12\,a^3x^{12}}-\frac{d}{9\,a^3x^9}-\frac{e}{6\,a^3x^6}-\frac{f}{3\,a^3x^3}+\frac{bc}{3\,a^4x^9}+\frac{bd}{2\,a^4x^6}-\frac{b^2c}{a^5x^6}-10\,\frac{b^3\ln(x)\,d}{a^6}-3\,\frac{b\ln(x)\,f}{a^4}+6\,\frac{b^2\ln(x)\,e}{a^5}+\frac{be}{a^4x^3}-2\,\frac{b^2d}{a^5x^3}+\frac{10\,b^3c}{3\,a^6x^3}-\frac{fb}{6\,a^2\,(bx^3+a)^2}+\frac{eb^2}{6\,a^3\,(bx^3+a)^2}-\frac{b^3d}{6\,a^4\,(bx^3+a)^2}+\frac{b^4c}{6\,a^5\,(bx^3+a)^2}-\frac{2\,fb}{3\,a^3\,(bx^3+a)}+\frac{eb^2}{a^4\,(bx^3+a)}-\frac{4\,b^3d}{3\,a^5\,(bx^3+a)}+\frac{5\,b^4c}{3\,a^6\,(bx^3+a)}+\frac{b\ln\left(bx^3+a\right)f}{a^4}-2\,\frac{b^2\ln\left(bx^3+a\right)e}{a^5}+\frac{10\,b^3\ln\left(bx^3+a\right)d}{3\,a^6}-5\,\frac{b^4\ln\left(bx^3+a\right)c}{a^7}+15\,\frac{b^4\ln(x)\,c}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x)
```

```
[Out] -1/12*c/a^3/x^12-1/9/a^3/x^9*d-1/6/a^3/x^6*e-1/3/a^3/x^3*f+1/3/a^
4/x^9*b*c+1/2/a^4/x^6*b*d-1/a^5/x^6*b^2*c-10*b^3/a^6*ln(x)*d-3*b/
a^4*ln(x)*f+6*b^2/a^5*ln(x)*e+1/a^4/x^3*b*e-2/a^5/x^3*b^2*d+10/3/
a^6/x^3*b^3*c-1/6*b/a^2/(b*x^3+a)^2*f+1/6*b^2/a^3/(b*x^3+a)^2*e-1
/6*b^3/a^4/(b*x^3+a)^2*d+1/6*b^4/a^5/(b*x^3+a)^2*c-2/3*b/a^3/(b*x
^3+a)*f+b^2/a^4/(b*x^3+a)*e-4/3*b^3/a^5/(b*x^3+a)*d+5/3*b^4/a^6/(
b*x^3+a)*c+b/a^4*ln(b*x^3+a)*f-2*b^2/a^5*ln(b*x^3+a)*e+10/3*b^3/a
^6*ln(b*x^3+a)*d-5*b^4/a^7*ln(b*x^3+a)*c+15*b^4/a^7*ln(x)*c
```

Maxima [A] time = 1.39292, size = 378, normalized size = 1.47

$$\frac{12\left(15\,b^5c-10\,ab^4d+6\,a^2b^3e-3\,a^3b^2f\right)x^{15}+18\left(15\,ab^4c-10\,a^2b^3d+6\,a^3b^2e-3\,a^4bf\right)x^{12}+4\left(15\,a^2b^3c-10\,a^3b^2d+6\right.}{36\left(a^6b^2x^{18}+2\,a^7bx^{15}+a^8x^{12}\right)}-\frac{\left(15\,b^4c-10\,ab^3d+6\,a^2b^2e-3\,a^3bf\right)\log\left(bx^3+a\right)}{3\,a^7}+\frac{\left(15\,b^4c-10\,ab^3d+6\,a^2b^2e-3\,a^3bf\right)\log\left(x^3\right)}{3\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^13),x, algorithm="maxima")
```

```
[Out] 1/36*(12*(15*b^5*c - 10*a*b^4*d + 6*a^2*b^3*e - 3*a^3*b^2*f)*x^15
 + 18*(15*a*b^4*c - 10*a^2*b^3*d + 6*a^3*b^2*e - 3*a^4*b*f)*x^12
```

```
+ 4*(15*a^2*b^3*c - 10*a^3*b^2*d + 6*a^4*b*e - 3*a^5*f)*x^9 - (15
*a^3*b^2*c - 10*a^4*b*d + 6*a^5*e)*x^6 - 3*a^5*c + 2*(3*a^4*b*c -
 2*a^5*d)*x^3)/(a^6*b^2*x^18 + 2*a^7*b*x^15 + a^8*x^12) - 1/3*(15
*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log(b*x^3 + a)/a^7
 + 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log(x^3)
/a^7
```

Fricas [A] time = 0.299195, size = 605, normalized size = 2.34

$$\frac{12\left(15\,ab^5c - 10\,a^2b^4d + 6\,a^3b^3e - 3\,a^4b^2f\right)x^{15} + 18\left(15\,a^2b^4c - 10\,a^3b^3d + 6\,a^4b^2e - 3\,a^5bf\right)x^{12} + 4\left(15\,a^3b^3c - 10\,a^4b^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^13),x, algorithm="fricas")
```

```
[Out] 1/36*(12*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a^4*b^2*f)*
x^15 + 18*(15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)
*x^12 + 4*(15*a^3*b^3*c - 10*a^4*b^2*d + 6*a^5*b*e - 3*a^6*f)*x^9
 - 3*a^6*c - (15*a^4*b^2*c - 10*a^5*b*d + 6*a^6*e)*x^6 + 2*(3*a^5
*b*c - 2*a^6*d)*x^3 - 12*((15*b^6*c - 10*a*b^5*d + 6*a^2*b^4*e -
3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e -
3*a^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e -
3*a^5*b*f)*x^12)*log(b*x^3 + a) + 36*((15*b^6*c - 10*a*b^5*d + 6*
a^2*b^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d + 6*
a^3*b^3*e - 3*a^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*
a^4*b^2*e - 3*a^5*b*f)*x^12)*log(x))/(a^7*b^2*x^18 + 2*a^8*b*x^15
 + a^9*x^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217273, size = 513, normalized size = 1.99

$$\frac{\left(15\,b^4c - 10\,ab^3d - 3\,a^3bf + 6\,a^2b^2e\right)\ln\left(|x|\right)}{a^7} - \frac{\left(15\,b^5c - 10\,ab^4d - 3\,a^3b^2f + 6\,a^2b^3e\right)\ln\left(\left|bx^3 + a\right|\right)}{3\,a^7b} + \frac{45\,b^6cx^6 - 30\,ab^5dx^6 - 9\,a^3b^3fx^6 + 18\,a^2b^4x^6e + 100\,ab^5cx^3 - 68\,a^2b^4dx^3 - 22\,a^4b^2fx^3 + 42\,a^3b^3x^3e + 56\,a^2b^4c - 39\,a^3b}{6\,(bx^3 + a)^2a^7} - \frac{375\,b^4cx^{12} - 250\,ab^3dx^{12} - 75\,a^3bfx^{12} + 150\,a^2b^2x^{12}e - 120\,ab^3cx^9 + 72\,a^2b^2dx^9 + 12\,a^4fx^9 - 36\,a^3bx^9e + 36\,a^2b^2cx^6 -}{36\,a^7x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^13),x, algorithm="giac")
```

```
[Out] (15*b^4*c - 10*a*b^3*d - 3*a^3*b*f + 6*a^2*b^2*e)*ln(abs(x))/a^7
- 1/3*(15*b^5*c - 10*a*b^4*d - 3*a^3*b^2*f + 6*a^2*b^3*e)*ln(abs(
b*x^3 + a))/(a^7*b) + 1/6*(45*b^6*c*x^6 - 30*a*b^5*d*x^6 - 9*a^3*
b^3*f*x^6 + 18*a^2*b^4*x^6*e + 100*a*b^5*c*x^3 - 68*a^2*b^4*d*x^3
 - 22*a^4*b^2*f*x^3 + 42*a^3*b^3*x^3*e + 56*a^2*b^4*c - 39*a^3*b^
3*d - 14*a^5*b*f + 25*a^4*b^2*e)/((b*x^3 + a)^2*a^7) - 1/36*(375*
b^4*c*x^12 - 250*a*b^3*d*x^12 - 75*a^3*b*f*x^12 + 150*a^2*b^2*x^1
```

```
2*e - 120*a*b^3*c*x^9 + 72*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 36*a^3*
b*x^9*e + 36*a^2*b^2*c*x^6 - 18*a^3*b*d*x^6 + 6*a^4*x^6*e - 12*a^
3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4*c)/(a^7*x^12)
```

3.286 $\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=416

$$\frac{x^7\left(6a^2f-3abe+b^2d\right)}{7b^5}-\frac{a^2x\left(-37a^3f+31a^2be-25ab^2d+19b^3c\right)}{18b^7\left(a+bx^3\right)}+\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{ax\left(-15a^3f+10a^2be-6ab^2d+3b^3c\right)}{b^7}+\frac{x^4\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{4b^6}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-152a^3f+104a^2be-65ab^2d+35b^3c\right)}{54b^{22/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-152a^3f+104a^2be-65ab^2d+35b^3c\right)}{27b^{22/3}}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-152a^3f+104a^2be-65ab^2d+35b^3c\right)}{9\sqrt{3}b^{22/3}}+\frac{x^{10}(be-3af)}{10b^4}+\frac{fx^{13}}{13b^3}$$

```
[Out] -((a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x)/b^7) + ((b^
3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d -
3*a*b*e + 6*a^2*f)*x^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) +
 (f*x^13)/(13*b^3) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/
(6*b^7*(a + b*x^3)^2) - (a^2*(19*b^3*c - 25*a*b^2*d + 31*a^2*b*e
- 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) - (a^(4/3)*(35*b^3*c - 65*a*b
^2*d + 104*a^2*b*e - 152*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(22/3)) + (a^(4/3)*(35*b^3*c - 65*
a*b^2*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*
b^(22/3)) - (a^(4/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a
^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3
))
```

Rubi [A] time = 1.4612, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{x^7\left(6a^2f-3abe+b^2d\right)}{7b^5}-\frac{a^2x\left(-37a^3f+31a^2be-25ab^2d+19b^3c\right)}{18b^7\left(a+bx^3\right)}+\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{ax\left(-15a^3f+10a^2be-6ab^2d+3b^3c\right)}{b^7}+\frac{x^4\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{4b^6}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-152a^3f+104a^2be-65ab^2d+35b^3c\right)}{54b^{22/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-152a^3f+104a^2be-65ab^2d+35b^3c\right)}{27b^{22/3}}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-152a^3f+104a^2be-65ab^2d+35b^3c\right)}{9\sqrt{3}b^{22/3}}+\frac{x^{10}(be-3af)}{10b^4}+\frac{fx^{13}}{13b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] -((a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x)/b^7) + ((b^
3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d -
3*a*b*e + 6*a^2*f)*x^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) +
 (f*x^13)/(13*b^3) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/
(6*b^7*(a + b*x^3)^2) - (a^2*(19*b^3*c - 25*a*b^2*d + 31*a^2*b*e
- 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) - (a^(4/3)*(35*b^3*c - 65*a*b
^2*d + 104*a^2*b*e - 152*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
```

```
qrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(22/3)) + (a^(4/3)*(35*b^3*c - 65*
a*b^2*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*
b^(22/3)) - (a^(4/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a
^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3
))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**12*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.973115, size = 411, normalized size = 0.99

$$\frac{x^7\left(6a^2f-3abe+b^2d\right)}{7b^5}+\frac{a^2x\left(37a^3f-31a^2be+25ab^2d-19b^3c\right)}{18b^7\left(a+bx^3\right)}+\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}+\frac{ax\left(15a^3f-10a^2be+6ab^2d-3b^3c\right)}{b^7}+\frac{x^4\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{4b^6}+\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(152a^3f-104a^2be+65ab^2d-35b^3c\right)}{54b^{22/3}}-\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(152a^3f-104a^2be+65ab^2d-35b^3c\right)}{27b^{22/3}}+\frac{a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(152a^3f-104a^2be+65ab^2d-35b^3c\right)}{9\sqrt{3}b^{22/3}}+\frac{x^{10}(be-3af)}{10b^4}+\frac{fx^{13}}{13b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*
c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*
a*b*e + 6*a^2*f)*x^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (
f*x^13)/(13*b^3) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6
*b^7*(a + b*x^3)^2) + (a^2*(-19*b^3*c + 25*a*b^2*d - 31*a^2*b*e +
 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) + (a^(4/3)*(-35*b^3*c + 65*a*b
^2*d - 104*a^2*b*e + 152*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/(9*Sqrt[3]*b^(22/3)) - (a^(4/3)*(-35*b^3*c + 65*a*b^2
*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22
/3)) + (a^(4/3)*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))
```

Maple [A] time = 0.021, size = 706, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 35/27*a^2/b^5*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))-152/27*a^5/b^8*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))+104/27*a^4/b^7*e/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-65/27*a^3/b^6*d/(a/b)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/9*a^5/b^6/(b*x^3
+a)^2*e*x+11/9*a^4/b^5/(b*x^3+a)^2*d*x+6/7/b^5*x^7*a^2*f-3/7/b^4*
x^7*a*e-5/2/b^6*x^4*a^3*f+3/2/b^5*x^4*a^2*e-3/4/b^4*x^4*a*d+15/b^
7*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a^2*d*x-3/b^4*a*c*x+1/13*f*x^13/b^
3-3/10/b^4*x^10*a*f+37/18*a^5/b^6/(b*x^3+a)^2*x^4*f-31/18*a^4/b^5
/(b*x^3+a)^2*x^4*e+25/18*a^3/b^4/(b*x^3+a)^2*x^4*d-19/18*a^2/b^3/
(b*x^3+a)^2*x^4*c+104/27*a^4/b^7*e/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-
8/9*a^3/b^4/(b*x^3+a)^2*c*x-152/27*a^5/b^8*f/(a/b)^(2/3)*ln(x+(a/
b)^(1/3))-52/27*a^4/b^7*e/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^
(2/3))-65/27*a^3/b^6*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+65/54*a^3/b^
6*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+35/27*a^2/b^5*c
/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+76/27*a^5/b^8*f/(a/b)^(2/3)*ln(x^2
-x*(a/b)^(1/3)+(a/b)^(2/3))+1/7/b^3*x^7*d+1/4/b^3*x^4*c+1/10/b^3*
x^10*e-35/54*a^2/b^5*c/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))+17/9*a^6/b^7/(b*x^3+a)^2*f*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^12/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.245609, size = 919, normalized size = 2.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^12/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/147420*sqrt(3)*(910*sqrt(3)*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*
a^5*b*e - 152*a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e
- 152*a^4*b^2*f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b
^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3)) - 1820*sqrt(3)*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a
^5*b*e - 152*a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e -
 152*a^4*b^2*f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^
2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) + 5460
*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*a^6*f + (35*a*b
^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2*(35*
a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/
b)^(1/3)*arctan(1/3*(2*sqrt(3)*x + sqrt(3)*(-a/b)^(1/3))/(-a/b)^(
1/3)) + 3*sqrt(3)*(1260*b^6*f*x^19 + 126*(13*b^6*e - 19*a*b^5*f)*
x^16 + 36*(65*b^6*d - 104*a*b^5*e + 152*a^2*b^4*f)*x^13 + 117*(35
*b^6*c - 65*a*b^5*d + 104*a^2*b^4*e - 152*a^3*b^3*f)*x^10 - 1170*
(35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^7 -
 3185*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)
*x^4 - 1820*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*a^6*
f)*x))/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**12*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217243, size = 675, normalized size = 1.62

$$\frac{\sqrt{3}\left(35\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}c-65\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{2}d-152\left(-ab^{2}\right)^{\frac{1}{3}}a^{4}f+104\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^{8}}-\frac{\left(35a^{2}b^{3}c-65a^{3}b^{2}d-152a^{5}f+104a^{4}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^{7}}+\frac{\left(35\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}c-65\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{2}d-152\left(-ab^{2}\right)^{\frac{1}{3}}a^{4}f+104\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b^{8}}-\frac{19a^{2}b^{4}cx^{4}-25a^{3}b^{3}dx^{4}-37a^{5}bfx^{4}+31a^{4}b^{2}x^{4}e+16a^{3}b^{3}cx-22a^{4}b^{2}dx-34a^{6}fx+28a^{5}bxe}{18\left(bx^{3}+a\right)^{2}b^{7}}+\frac{140b^{36}fx^{13}-546ab^{35}fx^{10}+182b^{36}x^{10}e+260b^{36}dx^{7}+1560a^{2}b^{34}fx^{7}-780ab^{35}x^{7}e+455b^{36}cx^{4}-1365ab^{35}dx^{4}-455}{1820b^{39}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^12/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b
^2*d - 152*(-a*b^2)^(1/3)*a^4*f + 104*(-a*b^2)^(1/3)*a^3*b*e)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/27*(35
*a^2*b^3*c - 65*a^3*b^2*d - 152*a^5*f + 104*a^4*b*e)*(-a/b)^(1/3)
*ln(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/54*(35*(-a*b^2)^(1/3)*a*b^
3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d - 152*(-a*b^2)^(1/3)*a^4*f + 10
4*(-a*b^2)^(1/3)*a^3*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/b^8 - 1/18*(19*a^2*b^4*c*x^4 - 25*a^3*b^3*d*x^4 - 37*a^5*b*f*x^4
 + 31*a^4*b^2*x^4*e + 16*a^3*b^3*c*x - 22*a^4*b^2*d*x - 34*a^6*f*
x + 28*a^5*b*x*e)/((b*x^3 + a)^2*b^7) + 1/1820*(140*b^36*f*x^13 -
 546*a*b^35*f*x^10 + 182*b^36*x^10*e + 260*b^36*d*x^7 + 1560*a^2*
b^34*f*x^7 - 780*a*b^35*x^7*e + 455*b^36*c*x^4 - 1365*a*b^35*d*x^
4 - 4550*a^3*b^33*f*x^4 + 2730*a^2*b^34*x^4*e - 5460*a*b^35*c*x +
 10920*a^2*b^34*d*x + 27300*a^4*b^32*f*x - 18200*a^3*b^33*x*e)/b^
39
```

3.287 $\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=384

$$\frac{x^5(6a^2f-3abe+b^2d)}{5b^5}+\frac{x^2(-10a^3f+6a^2be-3ab^2d+b^3c)}{2b^6}+\frac{ax^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{9b^6(a+bx^3)}-\frac{a^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6(a+bx^3)^2}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{54b^{20/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{27b^{20/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{9\sqrt{3}b^{20/3}}+\frac{x^8(be-3af)}{8b^4}+\frac{fx^{11}}{11b^3}$$

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*
d - 3*a*b*e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4)
 + (f*x^11)/(11*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x
^2)/(6*b^6*(a + b*x^3)^2) + (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e
 - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) + (a^(2/3)*(20*b^3*c - 44*a
*b^2*d + 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(20/3)) + (a^(2/3)*(20*b^3*c - 44
*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*
b^(20/3)) - (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3)
)
```

Rubi [A] time = 2.05217, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^5(6a^2f-3abe+b^2d)}{5b^5}+\frac{x^2(-10a^3f+6a^2be-3ab^2d+b^3c)}{2b^6}+\frac{ax^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{9b^6(a+bx^3)}-\frac{a^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6(a+bx^3)^2}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{54b^{20/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{27b^{20/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{9\sqrt{3}b^{20/3}}+\frac{x^8(be-3af)}{8b^4}+\frac{fx^{11}}{11b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*
d - 3*a*b*e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4)
 + (f*x^11)/(11*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x
^2)/(6*b^6*(a + b*x^3)^2) + (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e
 - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) + (a^(2/3)*(20*b^3*c - 44*a
*b^2*d + 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(20/3)) + (a^(2/3)*(20*b^3*c - 44
*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*
b^(20/3)) - (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3)
)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.977661, size = 380, normalized size = 0.99

$$\frac{x^5\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{x^2\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{2b^6}+\frac{ax^2\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{9b^6\left(a+bx^3\right)}+\frac{a^2x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^6\left(a+bx^3\right)^2}+\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(119a^3f-77a^2be+44ab^2d-20b^3c\right)}{54b^{20/3}}-\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(119a^3f-77a^2be+44ab^2d-20b^3c\right)}{27b^{20/3}}-\frac{a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(119a^3f-77a^2be+44ab^2d-20b^3c\right)}{9\sqrt{3}b^{20/3}}+\frac{x^8(be-3af)}{8b^4}+\frac{fx^{11}}{11b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*
d - 3*a*b*e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4)
 + (f*x^11)/(11*b^3) + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x^2)/(6*b^6*(a + b*x^3)^2) + (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*
b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) - (a^(2/3)*(-20*b^3*c +
44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(20/3)) - (a^(2/3)*(-20*b^3*c + 44*
a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b
^(20/3)) + (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3)
)
```

Maple [B] time = 0.021, size = 668, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -3/8/b^4*x^8*a*f+6/5/b^5*x^5*a^2*f-3/5/b^4*x^5*a*e-5/b^6*x^2*a^3*
f+3/b^5*x^2*a^2*e-3/2/b^4*x^2*a*d+1/11*f*x^11/b^3+20/27*a/b^4*c/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))-119/27*a^4/b^7*f/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))+119/54*a^4/b^7*f/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b
)^(2/3))+1/8/b^3*x^8*e+1/5/b^3*x^5*d+1/2/b^3*x^2*c+7/9*a/b^2/(b*x
^3+a)^2*x^5*c-29/18*a^5/b^6/(b*x^3+a)^2*x^2*f+23/18*a^4/b^5/(b*x^
3+a)^2*x^2*e+77/27*a^3/b^6*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-77/54*
```

```
a^3/b^6*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-10/27*a/b
^4*c/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-16/9*a^4/b^5/(
b*x^3+a)^2*x^5*f+13/9*a^3/b^4/(b*x^3+a)^2*x^5*e-10/9*a^2/b^3/(b*x
^3+a)^2*x^5*d-44/27*a^2/b^5*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+22/27
*a^2/b^5*d/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+119/27*a
^4/b^7*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))-77/27*a^3/b^6*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))+44/27*a^2/b^5*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))-20/27*a/b^4*c*3^(1/2)/(a/b)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-17/18*a^3/b^4/(b*x^3+a)^2*x^2*
d+11/18*a^2/b^3/(b*x^3+a)^2*x^2*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^10/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240825, size = 887, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^10/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/35640*sqrt(3)*(220*sqrt(3)*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3
*e - 119*a^3*b^2*f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*
e - 119*a^5*f + 2*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 119
*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3)
- a*(-a^2/b^2)^(1/3)) - 440*sqrt(3)*((20*b^5*c - 44*a*b^4*d + 77*
a^2*b^3*e - 119*a^3*b^2*f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77
*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*
e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/
3)) - 1320*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f
)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*
(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a
^2/b^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*a*x - sqrt(3)*b*(-a^2/b^2)^(
2/3))/(b*(-a^2/b^2)^(2/3))) + 3*sqrt(3)*(360*b^5*f*x^17 + 45*(11*
b^5*e - 17*a*b^4*f)*x^14 + 18*(44*b^5*d - 77*a*b^4*e + 119*a^2*b^
3*f)*x^11 + 99*(20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^
2*f)*x^8 + 352*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^
4*b*f)*x^5 + 220*(20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*
a^5*f)*x^2))/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.223775, size = 663, normalized size = 1.73

$$\frac{\left(20\,ab^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 44\,a^{2}b^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 119\,a^{4}f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 77\,a^{3}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^{6}}$$
$$+\frac{\sqrt{3}\left(20\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c - 44\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d - 119\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f + 77\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^{8}}$$
$$-\frac{\left(20\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c - 44\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d - 119\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f + 77\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\ln\left(x^{2} + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^{8}}$$
$$+\frac{14\,ab^{4}cx^{5} - 20\,a^{2}b^{3}dx^{5} - 32\,a^{4}bfx^{5} + 26\,a^{3}b^{2}x^{5}e + 11\,a^{2}b^{3}cx^{2} - 17\,a^{3}b^{2}dx^{2} - 29\,a^{5}fx^{2} + 23\,a^{4}bx^{2}e}{18\left(bx^{3} + a\right)^{2}b^{6}}$$
$$+\frac{40\,b^{30}fx^{11} - 165\,ab^{29}fx^{8} + 55\,b^{30}x^{8}e + 88\,b^{30}dx^{5} + 528\,a^{2}b^{28}fx^{5} - 264\,ab^{29}x^{5}e + 220\,b^{30}cx^{2} - 660\,ab^{29}dx^{2} - 2200\,a^{3}b^{27}}{440\,b^{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^10/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(20*a*b^3*c*(-a/b)^(1/3) - 44*a^2*b^2*d*(-a/b)^(1/3) - 119*a
^4*f*(-a/b)^(1/3) + 77*a^3*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(
x - (-a/b)^(1/3)))/(a*b^6) + 1/27*sqrt(3)*(20*(-a*b^2)^(2/3)*b^3*
c - 44*(-a*b^2)^(2/3)*a*b^2*d - 119*(-a*b^2)^(2/3)*a^3*f + 77*(-a
*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/b^8 - 1/54*(20*(-a*b^2)^(2/3)*b^3*c - 44*(-a*b^2)^(2/3)
*a*b^2*d - 119*(-a*b^2)^(2/3)*a^3*f + 77*(-a*b^2)^(2/3)*a^2*b*e)*
ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^8 + 1/18*(14*a*b^4*c*x^
5 - 20*a^2*b^3*d*x^5 - 32*a^4*b*f*x^5 + 26*a^3*b^2*x^5*e + 11*a^2
*b^3*c*x^2 - 17*a^3*b^2*d*x^2 - 29*a^5*f*x^2 + 23*a^4*b*x^2*e)/((
b*x^3 + a)^2*b^6) + 1/440*(40*b^30*f*x^11 - 165*a*b^29*f*x^8 + 55
*b^30*x^8*e + 88*b^30*d*x^5 + 528*a^2*b^28*f*x^5 - 264*a*b^29*x^5
*e + 220*b^30*c*x^2 - 660*a*b^29*d*x^2 - 2200*a^3*b^27*f*x^2 + 13
20*a^2*b^28*x^2*e)/b^33
```

3.288 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=375

$$\frac{x^4\left(6a^2f-3abe+b^2d\right)}{4b^5}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-104a^3f+65a^2be-35ab^2d+14b^3c\right)}{27b^{19/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-104a^3f+65a^2be-35ab^2d+14b^3c\right)}{9\sqrt{3}b^{19/3}}+\frac{ax\left(-31a^3f+25a^2be-19ab^2d+13b^3c\right)}{18b^6\left(a+bx^3\right)}-\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}+\frac{x\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{b^6}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-104a^3f+65a^2be-35ab^2d+14b^3c\right)}{54b^{19/3}}+\frac{x^7(be-3af)}{7b^4}+\frac{fx^{10}}{10b^3}$$

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*
a*b*e + 6*a^2*f)*x^4)/(4*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*
x^10)/(10*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b
^6*(a + b*x^3)^2) + (a*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a
^3*f)*x)/(18*b^6*(a + b*x^3)) + (a^(1/3)*(14*b^3*c - 35*a*b^2*d +
 65*a^2*b*e - 104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*b^(19/3)) - (a^(1/3)*(14*b^3*c - 35*a*b^2*d
 + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(19/3)
) + (a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(19/3))
```

Rubi [A] time = 1.25337, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{x^4\left(6a^2f-3abe+b^2d\right)}{4b^5}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-104a^3f+65a^2be-35ab^2d+14b^3c\right)}{27b^{19/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-104a^3f+65a^2be-35ab^2d+14b^3c\right)}{9\sqrt{3}b^{19/3}}+\frac{ax\left(-31a^3f+25a^2be-19ab^2d+13b^3c\right)}{18b^6\left(a+bx^3\right)}-\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}+\frac{x\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{b^6}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-104a^3f+65a^2be-35ab^2d+14b^3c\right)}{54b^{19/3}}+\frac{x^7(be-3af)}{7b^4}+\frac{fx^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*
a*b*e + 6*a^2*f)*x^4)/(4*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*
x^10)/(10*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b
^6*(a + b*x^3)^2) + (a*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a
^3*f)*x)/(18*b^6*(a + b*x^3)) + (a^(1/3)*(14*b^3*c - 35*a*b^2*d +
 65*a^2*b*e - 104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*b^(19/3)) - (a^(1/3)*(14*b^3*c - 35*a*b^2*d
 + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(19/3)
) + (a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(19/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.960212, size = 362, normalized size = 0.97

$$945b^{4/3}x^4\left(6a^2f-3abe+b^2d\right)+\frac{210a\sqrt[3]{b}x\left(-31a^3f+25a^2be-19ab^2d+13b^3c\right)}{a+bx^3}+\frac{630a^2\sqrt[3]{b}x\left(a^3f-a^2be+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+3780\sqrt[3]{b}x\left(-10a^3f+\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (3780*b^(1/3)*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x + 945*
b^(4/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x^4 + 540*b^(7/3)*(b*e - 3*a*
f)*x^7 + 378*b^(10/3)*f*x^10 + (630*a^2*b^(1/3)*(-(b^3*c) + a*b^2
*d - a^2*b*e + a^3*f)*x)/(a + b*x^3)^2 + (210*a*b^(1/3)*(13*b^3*c
 - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x)/(a + b*x^3) - 140*Sqrt[
3]*a^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*ArcT
an[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-14*b^3*c
+ 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x] -
 70*a^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*b^(19/3))
```

Maple [A] time = 0.02, size = 651, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 104/27*a^4/b^7*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))-65/27*a^3/b^6*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))+35/27*a^2/b^5*d/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/27*a/b^4*c/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/10*f*x^10/b^3-3/7/b^
4*x^7*a*f+3/2/b^5*x^4*a^2*f-3/4/b^4*x^4*a*e-10/b^6*a^3*f*x+6/b^5*
a^2*e*x-3/b^4*a*d*x-31/18*a^4/b^5/(b*x^3+a)^2*x^4*f+25/18*a^3/b^4
/(b*x^3+a)^2*x^4*e-19/18*a^2/b^3/(b*x^3+a)^2*x^4*d+13/18*a/b^2/(b
*x^3+a)^2*x^4*c+65/54*a^3/b^6*e/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+
(a/b)^(2/3))+35/27*a^2/b^5*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-35/54*
a^2/b^5*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-14/27*a/b
^4*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27*a/b^4*c/(a/b)^(2/3)*ln(x^
2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/7/b^3*x^7*e+1/4/b^3*x^4*d+1/b^3*c*
x+104/27*a^4/b^7*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-52/27*a^4/b^7*f/
(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-65/27*a^3/b^6*e/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))-14/9*a^5/b^6/(b*x^3+a)^2*f*x+11/9*a^4/
b^5/(b*x^3+a)^2*e*x-8/9*a^3/b^4/(b*x^3+a)^2*d*x+5/9*a^2/b^3/(b*x^
3+a)^2*c*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^9/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241863, size = 832, normalized size = 2.22

$$\frac{\sqrt{3}\left(70\sqrt{3}\left(\left(14\,b^{5}c - 35\,ab^{4}d + 65\,a^{2}b^{3}e - 104\,a^{3}b^{2}f\right)x^{6} + 14\,a^{2}b^{3}c - 35\,a^{3}b^{2}d + 65\,a^{4}be - 104\,a^{5}f + 2\left(14\,ab^{4}c - 35\,a^{2}b^{3}d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^9/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/11340*sqrt(3)*(70*sqrt(3)*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*
e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b*e
 - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*
a^4*b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
- 140*sqrt(3)*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^
2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f +
 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b*f)*x^3)*
(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*((14*b^5*c - 35*a*b^4*d +
65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d +
 65*a^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b
^2*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*arctan(-1/3*(2*sqrt(3)*x - s
qrt(3)*(a/b)^(1/3))/(a/b)^(1/3)) + 3*sqrt(3)*(126*b^5*f*x^16 + 36
*(5*b^5*e - 8*a*b^4*f)*x^13 + 9*(35*b^5*d - 65*a*b^4*e + 104*a^2*
b^3*f)*x^10 + 90*(14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*
b^2*f)*x^7 + 245*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*
a^4*b*f)*x^4 + 140*(14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b*e - 10
4*a^5*f)*x))/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217133, size = 598, normalized size = 1.59

$$-\frac{\sqrt{3}\left(14\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-35\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-104\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+65\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^{7}}+\frac{\left(14\,ab^{3}c-35\,a^{2}b^{2}d-104\,a^{4}f+65\,a^{3}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^{6}}-\frac{\left(14\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-35\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-104\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+65\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^{7}}+\frac{13\,ab^{4}cx^{4}-19\,a^{2}b^{3}dx^{4}-31\,a^{4}bfx^{4}+25\,a^{3}b^{2}x^{4}e+10\,a^{2}b^{3}cx-16\,a^{3}b^{2}dx-28\,a^{5}fx+22\,a^{4}bxe}{18\left(bx^{3}+a\right)^{2}b^{6}}+\frac{14\,b^{27}fx^{10}-60\,ab^{26}fx^{7}+20\,b^{27}x^{7}e+35\,b^{27}dx^{4}+210\,a^{2}b^{25}fx^{4}-105\,ab^{26}x^{4}e+140\,b^{27}cx-420\,ab^{26}dx-1400\,a^{3}b^{24}fx}{140\,b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^9/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*
d - 104*(-a*b^2)^(1/3)*a^3*f + 65*(-a*b^2)^(1/3)*a^2*b*e)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/27*(14*a*b
^3*c - 35*a^2*b^2*d - 104*a^4*f + 65*a^3*b*e)*(-a/b)^(1/3)*ln(abs
(x - (-a/b)^(1/3)))/(a*b^6) - 1/54*(14*(-a*b^2)^(1/3)*b^3*c - 35*
(-a*b^2)^(1/3)*a*b^2*d - 104*(-a*b^2)^(1/3)*a^3*f + 65*(-a*b^2)^(
1/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 + 1/18*
(13*a*b^4*c*x^4 - 19*a^2*b^3*d*x^4 - 31*a^4*b*f*x^4 + 25*a^3*b^2*
x^4*e + 10*a^2*b^3*c*x - 16*a^3*b^2*d*x - 28*a^5*f*x + 22*a^4*b*x
*e)/((b*x^3 + a)^2*b^6) + 1/140*(14*b^27*f*x^10 - 60*a*b^26*f*x^7
 + 20*b^27*x^7*e + 35*b^27*d*x^4 + 210*a^2*b^25*f*x^4 - 105*a*b^2
6*x^4*e + 140*b^27*c*x - 420*a*b^26*d*x - 1400*a^3*b^24*f*x + 840
*a^2*b^25*x*e)/b^30
```

3.289 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=345

$$\frac{x^2(6a^2f-3abe+b^2d)}{2b^5}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{27\sqrt[3]{a}b^{17/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{9\sqrt{3}\sqrt[3]{a}b^{17/3}}-\frac{x^2(-13a^3f+10a^2be-7ab^2d+4b^3c)}{9b^5(a+bx^3)}+\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{54\sqrt[3]{a}b^{17/3}}+\frac{x^5(be-3af)}{5b^4}+\frac{fx^8}{8b^3}$$

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/(2*b^5) + ((b*e - 3*a*f)*x^5)/(
5*b^4) + (f*x^8)/(8*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*x^2)/(6*b^5*(a + b*x^3)^2) - ((4*b^3*c - 7*a*b^2*d + 10*a^2*b*e
- 13*a^3*f)*x^2)/(9*b^5*(a + b*x^3)) - ((5*b^3*c - 20*a*b^2*d + 4
4*a^2*b*e - 77*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(1/3)*b^(17/3)) - ((5*b^3*c - 20*a*b^2*d + 4
4*a^2*b*e - 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(17
/3)) + ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(17/3))
```

Rubi [A] time = 1.52835, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x^2(6a^2f-3abe+b^2d)}{2b^5}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{27\sqrt[3]{a}b^{17/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{9\sqrt{3}\sqrt[3]{a}b^{17/3}}-\frac{x^2(-13a^3f+10a^2be-7ab^2d+4b^3c)}{9b^5(a+bx^3)}+\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{54\sqrt[3]{a}b^{17/3}}+\frac{x^5(be-3af)}{5b^4}+\frac{fx^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/(2*b^5) + ((b*e - 3*a*f)*x^5)/(
5*b^4) + (f*x^8)/(8*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*x^2)/(6*b^5*(a + b*x^3)^2) - ((4*b^3*c - 7*a*b^2*d + 10*a^2*b*e
- 13*a^3*f)*x^2)/(9*b^5*(a + b*x^3)) - ((5*b^3*c - 20*a*b^2*d + 4
4*a^2*b*e - 77*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(1/3)*b^(17/3)) - ((5*b^3*c - 20*a*b^2*d + 4
4*a^2*b*e - 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(17
/3)) + ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(17/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A]   time = 0.415864, size = 329, normalized size = 0.95

$$540b^{2/3}x^2\left(6a^2f-3abe+b^2d\right)+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(77a^3f-44a^2be+20ab^2d-5b^3c\right)}{\sqrt[3]{a}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(77a^3f-44a^2be+20ab^2d-5b^3c\right)}{\sqrt[3]{a}}-$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (540*b^(2/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2 + 216*b^(5/3)*(b*e -
 3*a*f)*x^5 + 135*b^(8/3)*f*x^8 + (180*a*b^(2/3)*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^2)/(a + b*x^3)^2 - (120*b^(2/3)*(4*b^3*c -
7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(a + b*x^3) + (40*Sqrt[3]
*(-5*b^3*c + 20*a*b^2*d - 44*a^2*b*e + 77*a^3*f)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-5*b^3*c + 20*a*b^2*d
 - 44*a^2*b*e + 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20
*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(1080*b^(17/3))
```

Maple [B]   time = 0.019, size = 611, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/8*f*x^8/b^3+5/54/b^3*c/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(
2/3))+1/2/b^3*x^2*d+1/5/b^3*x^5*e-3/5/b^4*x^5*a*f+3/b^5*x^2*a^2*f
-3/2/b^4*x^2*a*e-4/9/b/(b*x^3+a)^2*x^5*c-5/27/b^3*c/(a/b)^(1/3)*l
n(x+(a/b)^(1/3))+44/27/b^5*a^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))-20/27/b^4*a*d*3^(1/2)/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-77/27/b^6*a^3*f*3^(1/2)/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-17/18/b^4/(b*x^3+
a)^2*x^2*a^3*e+11/18/b^3/(b*x^3+a)^2*x^2*a^2*d-5/18/b^2/(b*x^3+a)
^2*x^2*a*c+77/27/b^6*a^3*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-77/54/b^
6*a^3*f/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-44/27/b^5*a
^2*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+22/27/b^5*a^2*e/(a/b)^(1/3)*ln
(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+20/27/b^4*a*d/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))-10/27/b^4*a*d/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(
2/3))+5/27/b^3*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))+13/9/b^4/(b*x^3+a)^2*x^5*f*a^3-10/9/b^3/(b*x^3+a)^2*x
^5*a^2*e+7/9/b^2/(b*x^3+a)^2*x^5*a*d+23/18/b^5/(b*x^3+a)^2*x^2*a^
4*f
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^7/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238246, size = 780, normalized size = 2.26

$$\sqrt{3}\left(20\,\sqrt{3}\left(\left(5\,b^{5}c-20\,ab^{4}d+44\,a^{2}b^{3}e-77\,a^{3}b^{2}f\right)x^{6}+5\,a^{2}b^{3}c-20\,a^{3}b^{2}d+44\,a^{4}be-77\,a^{5}f+2\left(5\,ab^{4}c-20\,a^{2}b^{3}d+44\,a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^7/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/3240*sqrt(3)*(20*sqrt(3)*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e
- 77*a^3*b^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 7
7*a^5*f + 2*(5*a*b^4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f
)*x^3)*log((a*b^2)^(1/3)*b*x^2 + a*b - (a*b^2)^(2/3)*x) - 40*sqrt
(3)*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6 + 5
*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c
- 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*log(a*b + (a*b^2
)^(2/3)*x) + 120*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f +
2*(5*a*b^4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*arc
tan(-1/3*(sqrt(3)*a*b - 2*sqrt(3)*(a*b^2)^(2/3)*x)/(a*b)) + 3*sqr
t(3)*(45*b^4*f*x^14 + 18*(4*b^4*e - 7*a*b^3*f)*x^11 + 9*(20*b^4*d
 - 44*a*b^3*e + 77*a^2*b^2*f)*x^8 - 32*(5*b^4*c - 20*a*b^3*d + 44
*a^2*b^2*e - 77*a^3*b*f)*x^5 - 20*(5*a*b^3*c - 20*a^2*b^2*d + 44*
a^3*b*e - 77*a^4*f)*x^2)*(a*b^2)^(1/3))/((b^7*x^6 + 2*a*b^6*x^3 +
 a^2*b^5)*(a*b^2)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.221355, size = 601, normalized size = 1.74

$$-\frac{\left(5\,b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-77\,a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^{5}}-\frac{\sqrt{3}\left(5\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-20\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-77\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+44\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{7}}-\frac{8\,b^{4}cx^{5}-14\,ab^{3}dx^{5}-26\,a^{3}bfx^{5}+20\,a^{2}b^{2}x^{5}e+5\,ab^{3}cx^{2}-11\,a^{2}b^{2}dx^{2}-23\,a^{4}fx^{2}+17\,a^{3}bx^{2}e}{18\left(bx^{3}+a\right)^{2}b^{5}}+\frac{\left(5\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-20\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-77\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+44\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^{7}}+\frac{5\,b^{21}fx^{8}-24\,ab^{20}fx^{5}+8\,b^{21}x^{5}e+20\,b^{21}dx^{2}+120\,a^{2}b^{19}fx^{2}-60\,ab^{20}x^{2}e}{40\,b^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^7/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(5*b^3*c*(-a/b)^(1/3) - 20*a*b^2*d*(-a/b)^(1/3) - 77*a^3*f*
(-a/b)^(1/3) + 44*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (
-a/b)^(1/3)))/(a*b^5) - 1/27*sqrt(3)*(5*(-a*b^2)^(2/3)*b^3*c - 20
*(-a*b^2)^(2/3)*a*b^2*d - 77*(-a*b^2)^(2/3)*a^3*f + 44*(-a*b^2)^(
2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(a*b^7) - 1/18*(8*b^4*c*x^5 - 14*a*b^3*d*x^5 - 26*a^3*b*f*x^5
+ 20*a^2*b^2*x^5*e + 5*a*b^3*c*x^2 - 11*a^2*b^2*d*x^2 - 23*a^4*f*
x^2 + 17*a^3*b*x^2*e)/((b*x^3 + a)^2*b^5) + 1/54*(5*(-a*b^2)^(2/3
)*b^3*c - 20*(-a*b^2)^(2/3)*a*b^2*d - 77*(-a*b^2)^(2/3)*a^3*f + 4
4*(-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a*b^7) + 1/40*(5*b^21*f*x^8 - 24*a*b^20*f*x^5 + 8*b^21*x^5*e +
20*b^21*d*x^2 + 120*a^2*b^19*f*x^2 - 60*a*b^20*x^2*e)/b^24
```

3.290 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=336

$$\frac{x(6a^2f-3abe+b^2d)}{b^5}-\frac{x(-25a^3f+19a^2be-13ab^2d+7b^3c)}{18b^5(a+bx^3)}+\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{54a^{2/3}b^{16/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{27a^{2/3}b^{16/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{9\sqrt{3}a^{2/3}b^{16/3}}+\frac{x^4(be-3af)}{4b^4}+\frac{fx^7}{7b^3}$$

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x)/b^5 + ((b*e - 3*a*f)*x^4)/(4*b^4)
 + (f*x^7)/(7*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6
*b^5*(a + b*x^3)^2) - ((7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^
3*f)*x)/(18*b^5*(a + b*x^3)) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*
e - 65*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(2/3)*b^(16/3)) + ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*
e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(16/3)) - (
(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(16/3))
```

Rubi [A] time = 1.03791, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{x(6a^2f-3abe+b^2d)}{b^5}-\frac{x(-25a^3f+19a^2be-13ab^2d+7b^3c)}{18b^5(a+bx^3)}+\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{54a^{2/3}b^{16/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{27a^{2/3}b^{16/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{9\sqrt{3}a^{2/3}b^{16/3}}+\frac{x^4(be-3af)}{4b^4}+\frac{fx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x)/b^5 + ((b*e - 3*a*f)*x^4)/(4*b^4)
 + (f*x^7)/(7*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6
*b^5*(a + b*x^3)^2) - ((7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^
3*f)*x)/(18*b^5*(a + b*x^3)) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*
e - 65*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(2/3)*b^(16/3)) + ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*
e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(16/3)) - (
(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(16/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.755561, size = 323, normalized size = 0.96

$$\frac{756\sqrt[3]{b}x\left(6a^2f-3abe+b^2d\right)-\frac{42\sqrt[3]{b}x\left(-25a^3f+19a^2be-13ab^2d+7b^3c\right)}{a+bx^3}+\frac{126a\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-65a^3f+3\right.}{a^{2/3}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (756*b^(1/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x + 189*b^(4/3)*(b*e - 3
*a*f)*x^4 + 108*b^(7/3)*f*x^7 + (126*a*b^(1/3)*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*x)/(a + b*x^3)^2 - (42*b^(1/3)*(7*b^3*c - 13*a*
b^2*d + 19*a^2*b*e - 25*a^3*f)*x)/(a + b*x^3) + (28*Sqrt[3]*(-2*b
^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*f)*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(2*b^3*c - 14*a*b^2*d + 35*a
^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-2*b^
3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(756*b^(16/3))
```

Maple [B] time = 0.019, size = 596, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -65/27/b^6*a^3*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))+35/27/b^5*a^2*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))-14/27/b^4*a*d/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-19/18/b^3/(b*x^3+a)^2*x^4*a^2*e+1
/4/b^3*x^4*e+1/b^3*d*x-3/b^4*a*e*x-7/18/b/(b*x^3+a)^2*x^4*c-1/27/
b^3*c/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/27/b^3*c/(a
/b)^(2/3)*ln(x+(a/b)^(1/3))+1/7*f*x^7/b^3-3/4/b^4*x^4*a*f+6/b^5*a
^2*f*x+25/18/b^4/(b*x^3+a)^2*x^4*a^3*f-8/9/b^4/(b*x^3+a)^2*a^3*e*
x+5/9/b^3/(b*x^3+a)^2*a^2*d*x-2/9/b^2/(b*x^3+a)^2*a*c*x-65/27/b^6
*a^3*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+65/54/b^6*a^3*f/(a/b)^(2/3)*
ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+35/27/b^5*a^2*e/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))-35/54/b^5*a^2*e/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(
a/b)^(2/3))-14/27/b^4*a*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27/b^4*
a*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+13/18/b^2/(b*x^
3+a)^2*x^4*a*d+11/9/b^5/(b*x^3+a)^2*a^4*f*x+2/27/b^3*c/(a/b)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^6/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234425, size = 779, normalized size = 2.32

$$\sqrt{3}\Big(14\sqrt{3}\big(\left(2\,b^5c - 14\,ab^4d + 35\,a^2b^3e - 65\,a^3b^2f\right)x^6 + 2\,a^2b^3c - 14\,a^3b^2d + 35\,a^4be - 65\,a^5f + 2\,\big(2\,ab^4c - 14\,a^2b^3d + 35\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^6/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/2268*sqrt(3)*(14*sqrt(3)*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e
- 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 6
5*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f
)*x^3)*log((-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*a*x + a^2) - 28*sq
rt(3)*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 +
 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*
c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*log((-a^2*b)^(
1/3)*x - a) + 84*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b
^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f +
2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*arc
tan(1/3*(2*sqrt(3)*(-a^2*b)^(1/3)*x + sqrt(3)*a)/a) + 3*sqrt(3)*(
36*b^4*f*x^13 + 9*(7*b^4*e - 13*a*b^3*f)*x^10 + 18*(14*b^4*d - 35
*a*b^3*e + 65*a^2*b^2*f)*x^7 - 49*(2*b^4*c - 14*a*b^3*d + 35*a^2*
b^2*e - 65*a^3*b*f)*x^4 - 28*(2*a*b^3*c - 14*a^2*b^2*d + 35*a^3*b
*e - 65*a^4*f)*x)*(-a^2*b)^(1/3))/((b^7*x^6 + 2*a*b^6*x^3 + a^2*b
^5)*(-a^2*b)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218029, size = 539, normalized size = 1.6

$$-\frac{\left(2\,b^3c - 14\,ab^2d - 65\,a^3f + 35\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^5} + \frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}b^3c - 14\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d - 65\,\left(-ab^2\right)^{\frac{1}{3}}a^3f + 35\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^6} + \frac{\left(2\,\left(-ab^2\right)^{\frac{1}{3}}b^3c - 14\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d - 65\,\left(-ab^2\right)^{\frac{1}{3}}a^3f + 35\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^6} - \frac{7\,b^4cx^4 - 13\,ab^3dx^4 - 25\,a^3bfx^4 + 19\,a^2b^2x^4e + 4\,ab^3cx - 10\,a^2b^2dx - 22\,a^4fx + 16\,a^3bxe}{18\,\left(bx^3 + a\right)^2b^5} + \frac{4\,b^{18}fx^7 - 21\,ab^{17}fx^4 + 7\,b^{18}x^4e + 28\,b^{18}dx + 168\,a^2b^{16}fx - 84\,ab^{17}xe}{28\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^6/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35*a^2*b*e)*(-a/b)^(1/3)
*ln(abs(x - (-a/b)^(1/3)))/(a*b^5) + 1/27*sqrt(3)*(2*(-a*b^2)^(1/
3)*b^3*c - 14*(-a*b^2)^(1/3)*a*b^2*d - 65*(-a*b^2)^(1/3)*a^3*f +
35*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a*b^6) + 1/54*(2*(-a*b^2)^(1/3)*b^3*c - 14*(-a*b
^2)^(1/3)*a*b^2*d - 65*(-a*b^2)^(1/3)*a^3*f + 35*(-a*b^2)^(1/3)*a
^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^6) - 1/18*(7
*b^4*c*x^4 - 13*a*b^3*d*x^4 - 25*a^3*b*f*x^4 + 19*a^2*b^2*x^4*e +
 4*a*b^3*c*x - 10*a^2*b^2*d*x - 22*a^4*f*x + 16*a^3*b*x*e)/((b*x^
3 + a)^2*b^5) + 1/28*(4*b^18*f*x^7 - 21*a*b^17*f*x^4 + 7*b^18*x^4
*e + 28*b^18*d*x + 168*a^2*b^16*f*x - 84*a*b^17*x*e)/b^21
```

3.291 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

Optimal. Leaf size=316

$$\frac{x^2\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{9ab^4\left(a+bx^3\right)}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{54a^{4/3}b^{14/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{27a^{4/3}b^{14/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{9\sqrt{3}a^{4/3}b^{14/3}}+\frac{x^2(be-3af)}{2b^4}+\frac{fx^5}{5b^3}$$

```
[Out] ((b*e - 3*a*f)*x^2)/(2*b^4) + (f*x^5)/(5*b^3) - ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^2)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 4*a*b^
2*d + 7*a^2*b*e - 10*a^3*f)*x^2)/(9*a*b^4*(a + b*x^3)) - ((b^3*c
+ 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(14/3)) - ((b^3*c + 5
*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(4/3)*b^(14/3)) + ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(14/
3))
```

Rubi [A] time = 1.01798, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{9ab^4\left(a+bx^3\right)}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{54a^{4/3}b^{14/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{27a^{4/3}b^{14/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{9\sqrt{3}a^{4/3}b^{14/3}}+\frac{x^2(be-3af)}{2b^4}+\frac{fx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^2)/(2*b^4) + (f*x^5)/(5*b^3) - ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^2)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 4*a*b^
2*d + 7*a^2*b*e - 10*a^3*f)*x^2)/(9*a*b^4*(a + b*x^3)) - ((b^3*c
+ 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(14/3)) - ((b^3*c + 5
*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(4/3)*b^(14/3)) + ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(14/
3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.394382, size = 300, normalized size = 0.95

$$\frac{\frac{30b^{2/3}x^2\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{a\left(a+bx^3\right)}-\frac{45b^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2}-\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{a^{4/3}}-10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{270b^{14/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(b*e - 3*a*f)*x^2 + 54*b^(5/3)*f*x^5 - (45*b^(2/3)*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b*x^3)^2 + (30*b^(2/
3)*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*x^2)/(a*(a + b*x^3)
) - (10*Sqrt[3]*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) - (10*(b^3*c + 5*
a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3
) + (5*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(270*b^(14/3))
```

Maple [B] time = 0.017, size = 574, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/5*f*x^5/b^3-3/2/b^4*x^2*a*f+1/2/b^3*x^2*e-10/9/b^3/(b*x^3+a)^2*
a^2*x^5*f+7/9/b^2/(b*x^3+a)^2*a*x^5*e-4/9/b/(b*x^3+a)^2*x^5*d+1/9
/(b*x^3+a)^2/a*x^5*c-17/18/b^4/(b*x^3+a)^2*x^2*a^3*f+11/18/b^3/(b
*x^3+a)^2*x^2*a^2*e-5/18/b^2/(b*x^3+a)^2*x^2*a*d-1/18/b/(b*x^3+a)
^2*x^2*c-44/27/b^5*a^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+20/27/b^4*
a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-5/27/b^3/(a/b)^(1/3)*ln(x+(a/b)
^(1/3))*d-1/27/b^2/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+22/27/b^5*a^
2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f-10/27/b^4*a/(a/
b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+5/54/b^3/(a/b)^(1/3)
*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/54/b^2/a/(a/b)^(1/3)*ln(x^
2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+44/27/b^5*a^2*3^(1/2)/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-20/27/b^4*a*3^(1/2)/(a/
b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+5/27/b^3*3^(1/
2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/27/b^2
/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^4/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236581, size = 747, normalized size = 2.36

$$-\frac{\sqrt{3}\left(5\,\sqrt{3}\left(\left(b^5c+5\,ab^4d-20\,a^2b^3e+44\,a^3b^2f\right)x^6+a^2b^3c+5\,a^3b^2d-20\,a^4be+44\,a^5f+2\,\left(ab^4c+5\,a^2b^3d-20\,a^3b^2e+4\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^4/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] -1/810*sqrt(3)*(5*sqrt(3)*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44
*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f
 + 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*log
((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 10*sqrt(3)*((b^
5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c +
5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*a^2*b^3*d -
20*a^3*b^2*e + 44*a^4*b*f)*x^3)*log(a*b + (-a*b^2)^(2/3)*x) + 30*
((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*
c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*a^2*b^3*
d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*arctan(-1/3*(sqrt(3)*a*b - 2*
sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(18*a*b^3*f*x^11 + 9
*(5*a*b^3*e - 11*a^2*b^2*f)*x^8 + 2*(5*b^4*c - 20*a*b^3*d + 80*a^
2*b^2*e - 176*a^3*b*f)*x^5 - 5*(a*b^3*c + 5*a^2*b^2*d - 20*a^3*b*
e + 44*a^4*f)*x^2)*(-a*b^2)^(1/3))/((a*b^6*x^6 + 2*a^2*b^5*x^3 +
a^3*b^4)*(-a*b^2)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.221729, size = 558, normalized size = 1.77

$$-\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c+5\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d+44\,\left(-ab^2\right)^{\frac{2}{3}}a^3f-20\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^6}+\frac{2\,b^4cx^5-8\,ab^3dx^5-20\,a^3bfx^5+14\,a^2b^2x^5e-ab^3cx^2-5\,a^2b^2dx^2-17\,a^4fx^2+11\,a^3bx^2e}{18\,(bx^3+a)^2ab^4}+\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c+5\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d+44\,\left(-ab^2\right)^{\frac{2}{3}}a^3f-20\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^6}+\frac{2\,b^{12}fx^5-15\,ab^{11}fx^2+5\,b^{12}x^2e}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^4/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(b^3*c*(-a/b)^(1/3) + 5*a*b^2*d*(-a/b)^(1/3) + 44*a^3*f*(-a
/b)^(1/3) - 20*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/
```

```
b)^(1/3)))/(a^2*b^4) - 1/27*sqrt(3)*((-a*b^2)^(2/3)*b^3*c + 5*(-a
*b^2)^(2/3)*a*b^2*d + 44*(-a*b^2)^(2/3)*a^3*f - 20*(-a*b^2)^(2/3)
*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(
a^2*b^6) + 1/18*(2*b^4*c*x^5 - 8*a*b^3*d*x^5 - 20*a^3*b*f*x^5 + 1
4*a^2*b^2*x^5*e - a*b^3*c*x^2 - 5*a^2*b^2*d*x^2 - 17*a^4*f*x^2 +
11*a^3*b*x^2*e)/((b*x^3 + a)^2*a*b^4) + 1/54*((-a*b^2)^(2/3)*b^3*
c + 5*(-a*b^2)^(2/3)*a*b^2*d + 44*(-a*b^2)^(2/3)*a^3*f - 20*(-a*b
^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b
^6) + 1/10*(2*b^12*f*x^5 - 15*a*b^11*f*x^2 + 5*b^12*x^2*e)/b^15
```

3.292 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=307

$$\frac{x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{18ab^4\left(a+bx^3\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{54a^{5/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{27a^{5/3}b^{13/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{9\sqrt{3}a^{5/3}b^{13/3}}+\frac{x(be-3af)}{b^4}+\frac{fx^4}{4b^3}$$

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^4)/(4*b^3) - ((b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 7*a*b^2*d + 13
*a^2*b*e - 19*a^3*f)*x)/(18*a*b^4*(a + b*x^3)) - ((b^3*c + 2*a*b^
2*d - 14*a^2*b*e + 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(13/3)) + ((b^3*c + 2*a*b^2*d
 - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b
^(13/3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(13/3))
```

Rubi [A] time = 0.87972, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{18ab^4\left(a+bx^3\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{54a^{5/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{27a^{5/3}b^{13/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{9\sqrt{3}a^{5/3}b^{13/3}}+\frac{x(be-3af)}{b^4}+\frac{fx^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^4)/(4*b^3) - ((b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 7*a*b^2*d + 13
*a^2*b*e - 19*a^3*f)*x)/(18*a*b^4*(a + b*x^3)) - ((b^3*c + 2*a*b^
2*d - 14*a^2*b*e + 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(13/3)) + ((b^3*c + 2*a*b^2*d
 - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b
^(13/3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(13/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

[Out] Timed out

Mathematica [A] time = 0.373461, size = 294, normalized size = 0.96

$$\frac{\frac{6\sqrt[3]{b}x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{a(a+bx^3)}-\frac{18\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{(a+bx^3)^2}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{a^{5/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)(35}{a}}{108b^{13/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(b*e - 3*a*f)*x + 27*b^(4/3)*f*x^4 - (18*b^(1/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)^2 + (6*b^(1/3)*(b
^3*c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)*x)/(a*(a + b*x^3)) - (4
*Sqrt[3]*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (4*(b^3*c + 2*a*b^2*d
- 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(
b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(108*b^(13/3))
```

Maple [B] time = 0.017, size = 561, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/4*f*x^4/b^3-3/b^4*a*f*x+1/b^3*e*x-19/18/b^3/(b*x^3+a)^2*x^4*a^2
*f+13/18/b^2/(b*x^3+a)^2*x^4*a*e-7/18/b/(b*x^3+a)^2*x^4*d+1/18/(b
*x^3+a)^2/a*x^4*c-8/9/b^4/(b*x^3+a)^2*a^3*f*x+5/9/b^3/(b*x^3+a)^2
*a^2*e*x-2/9/b^2/(b*x^3+a)^2*a*d*x-1/9/b/(b*x^3+a)^2*c*x+35/27/b^
5*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-14/27/b^4*a/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))*e+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+1/27/b^
2/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-35/54/b^5*a^2/(a/b)^(2/3)*ln(
x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+7/27/b^4*a/(a/b)^(2/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))*e-1/27/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))*d-1/54/b^2/a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a
/b)^(2/3))*c+35/27/b^5*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*f-14/27/b^4*a/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+2/27/b^3/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/27/b^2/a/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^3/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.230494, size = 728, normalized size = 2.37

$$-\frac{\sqrt{3}\left(2\sqrt{3}\left(\left(b^5c+2\,ab^4d-14\,a^2b^3e+35\,a^3b^2f\right)x^6+a^2b^3c+2\,a^3b^2d-14\,a^4be+35\,a^5f+2\left(ab^4c+2\,a^2b^3d-14\,a^3b^2e+3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^3/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] -1/324*sqrt(3)*(2*sqrt(3)*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*e + 35
*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f
 + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*log
((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 4*sqrt(3)*((b^5*c
 + 2*a*b^4*d - 14*a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a
^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*
a^3*b^2*e + 35*a^4*b*f)*x^3)*log((a^2*b)^(1/3)*x + a) - 12*((b^5*
c + 2*a*b^4*d - 14*a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*
a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14
*a^3*b^2*e + 35*a^4*b*f)*x^3)*arctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)
*x - sqrt(3)*a)/a) - 3*sqrt(3)*(9*a*b^3*f*x^10 + 18*(2*a*b^3*e -
5*a^2*b^2*f)*x^7 + (2*b^4*c - 14*a*b^3*d + 98*a^2*b^2*e - 245*a^3
*b*f)*x^4 - 4*(a*b^3*c + 2*a^2*b^2*d - 14*a^3*b*e + 35*a^4*f)*x)*
(a^2*b)^(1/3))/((a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4)*(a^2*b)^(1/
3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219125, size = 495, normalized size = 1.61

$$-\frac{\left(b^3c+2\,ab^2d+35\,a^3f-14\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c+2\left(-ab^2\right)^{\frac{1}{3}}ab^2d+35\left(-ab^2\right)^{\frac{1}{3}}a^3f-14\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^5}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c+2\left(-ab^2\right)^{\frac{1}{3}}ab^2d+35\left(-ab^2\right)^{\frac{1}{3}}a^3f-14\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^5}+\frac{b^4cx^4-7\,ab^3dx^4-19\,a^3bfx^4+13\,a^2b^2x^4e-2\,ab^3cx-4\,a^2b^2dx-16\,a^4fx+10\,a^3bxe}{18\left(bx^3+a\right)^2ab^4}+\frac{b^9fx^4-12\,ab^8fx+4\,b^9xe}{4\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x^3/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(b^3*c + 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)*(-a/b)^(1/3)*ln
(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/27*sqrt(3)*((-a*b^2)^(1/3)*
b^3*c + 2*(-a*b^2)^(1/3)*a*b^2*d + 35*(-a*b^2)^(1/3)*a^3*f - 14*(
```

```
-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(a^2*b^5) + 1/54*((-a*b^2)^(1/3)*b^3*c + 2*(-a*b^2)^(
1/3)*a*b^2*d + 35*(-a*b^2)^(1/3)*a^3*f - 14*(-a*b^2)^(1/3)*a^2*b*
e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^5) + 1/18*(b^4*
c*x^4 - 7*a*b^3*d*x^4 - 19*a^3*b*f*x^4 + 13*a^2*b^2*x^4*e - 2*a*b
^3*c*x - 4*a^2*b^2*d*x - 16*a^4*f*x + 10*a^3*b*x*e)/((b*x^3 + a)^
2*a*b^4) + 1/4*(b^9*f*x^4 - 12*a*b^8*f*x + 4*b^9*x*e)/b^12
```

3.293 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=301

$$\frac{x^2(7a^3f-4a^2be+ab^2d+2b^3c)}{9a^2b^3(a+bx^3)}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3(a+bx^3)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{54a^{7/3}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{27a^{7/3}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{9\sqrt{3}a^{7/3}b^{11/3}}+\frac{fx^2}{2b^3}$$

```
[Out] (f*x^2)/(2*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*
b^3*(a + b*x^3)^2) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x
^2)/(9*a^2*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 2
0*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sq
rt[3]*a^(7/3)*b^(11/3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^
3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(11/3)) + ((2*b^3*c
+ a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(54*a^(7/3)*b^(11/3))
```

Rubi [A] time = 0.805486, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{x^2(7a^3f-4a^2be+ab^2d+2b^3c)}{9a^2b^3(a+bx^3)}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3(a+bx^3)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{54a^{7/3}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{27a^{7/3}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{9\sqrt{3}a^{7/3}b^{11/3}}+\frac{fx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (f*x^2)/(2*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*
b^3*(a + b*x^3)^2) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x
^2)/(9*a^2*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 2
0*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sq
rt[3]*a^(7/3)*b^(11/3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^
3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(11/3)) + ((2*b^3*c
+ a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(54*a^(7/3)*b^(11/3))
```

Rubi in Sympy [A] time = 128.405, size = 291, normalized size = 0.97

$$\frac{fx^2}{2b^3} - \frac{x^2\left(a^3f - a^2be + ab^2d - b^3c\right)}{6ab^3\left(a + bx^3\right)^2} + \frac{x^2\left(7a^3f - 4a^2be + ab^2d + 2b^3c\right)}{9a^2b^3\left(a + bx^3\right)} + \frac{\left(a\left(20a^2f - 5abe - b^2d\right) - 2b^3c\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{\frac{7}{3}}b^{\frac{11}{3}}} - \frac{\left(a\left(20a^2f - 5abe - b^2d\right) - 2b^3c\right)\log\left(a^{\frac{2}{3}} - \sqrt[3]{a}\sqrt[3]{b}x + b^{\frac{2}{3}}x^2\right)}{54a^{\frac{7}{3}}b^{\frac{11}{3}}} + \frac{\sqrt{3}\left(a\left(20a^2f - 5abe - b^2d\right) - 2b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - \frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{7}{3}}b^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] f*x**2/(2*b**3) - x**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6
*a*b**3*(a + b*x**3)**2) + x**2*(7*a**3*f - 4*a**2*b*e + a*b**2*d
 + 2*b**3*c)/(9*a**2*b**3*(a + b*x**3)) + (a*(20*a**2*f - 5*a*b*e
 - b**2*d) - 2*b**3*c)*log(a**(1/3) + b**(1/3)*x)/(27*a**(7/3)*b*
*(11/3)) - (a*(20*a**2*f - 5*a*b*e - b**2*d) - 2*b**3*c)*log(a**(
2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(54*a**(7/3)*b**(11/3
)) + sqrt(3)*(a*(20*a**2*f - 5*a*b*e - b**2*d) - 2*b**3*c)*atan(s
qrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a**(7/3)*b**(1
1/3))
```

Mathematica [A] time = 0.379933, size = 284, normalized size = 0.94

$$\frac{\frac{6b^{2/3}x^2\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{a^2\left(a+bx^3\right)} + \frac{9b^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a\left(a+bx^3\right)^2} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-20a^3f+5a^2be+ab^2d+2b^3c\right)}{a^{7/3}} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)(-20}{a^{7/}}}{54b^{11/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*f*x^2 + (9*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*x^2)/(a*(a + b*x^3)^2) + (6*b^(2/3)*(2*b^3*c + a*b^2*d - 4*a^2*
b*e + 7*a^3*f)*x^2)/(a^2*(a + b*x^3)) - (2*Sqrt[3]*(2*b^3*c + a*b
^2*d + 5*a^2*b*e - 20*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]])/a^(7/3) - (2*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*
Log[a^(1/3) + b^(1/3)*x])/a^(7/3) + ((2*b^3*c + a*b^2*d + 5*a^2*b
*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^
(7/3))/(54*b^(11/3))
```

Maple [B] time = 0.017, size = 550, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/2*f*x^2/b^3+7/9/b^2/(b*x^3+a)^2*a*x^5*f-4/9/b/(b*x^3+a)^2*x^5*e
+1/9/(b*x^3+a)^2/a*x^5*d+2/9*b/(b*x^3+a)^2/a^2*x^5*c+11/18/b^3/(b
```

```
*x^3+a)^2*a^2*x^2*f-5/18/b^2/(b*x^3+a)^2*a*x^2*e-1/18/b/(b*x^3+a)
^2*x^2*d+7/18/(b*x^3+a)^2/a*x^2*c+20/27/b^4*a/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))*f-5/27/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-1/27/b^2/a/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-2/27/b/a^2/(a/b)^(1/3)*ln(x+(a/b)
^(1/3))*c-10/27/b^4*a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3
))*f+5/54/b^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/5
4/b^2/a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/27/b/a^
2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-20/27/b^4*a*3^(
1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+5/27/b
^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+
1/27/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))*d+2/27/b/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.222284, size = 711, normalized size = 2.36

$$\frac{\sqrt{3}\Big(\sqrt{3}\big(\big(2\,b^5c + ab^4d + 5\,a^2b^3e - 20\,a^3b^2f\big)x^6 + 2\,a^2b^3c + a^3b^2d + 5\,a^4be - 20\,a^5f + 2\,\big(2\,ab^4c + a^2b^3d + 5\,a^3b^2e - 20\,a^4bf}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*(sqrt(3)*((2*b^5*c + a*b^4*d + 5*a^2*b^3*e - 20*a^3
*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*a^5*f + 2*
(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*log((a*b^
2)^(1/3)*b*x^2 + a*b - (a*b^2)^(2/3)*x) - 2*sqrt(3)*((2*b^5*c + a
*b^4*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*
d + 5*a^4*b*e - 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e
 - 20*a^4*b*f)*x^3)*log(a*b + (a*b^2)^(2/3)*x) + 6*((2*b^5*c + a*
b^4*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d
 + 5*a^4*b*e - 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e
- 20*a^4*b*f)*x^3)*arctan(-1/3*(sqrt(3)*a*b - 2*sqrt(3)*(a*b^2)^(
2/3)*x)/(a*b)) + 3*sqrt(3)*(9*a^2*b^2*f*x^8 + 2*(2*b^4*c + a*b^3*
d - 4*a^2*b^2*e + 16*a^3*b*f)*x^5 + (7*a*b^3*c - a^2*b^2*d - 5*a^
3*b*e + 20*a^4*f)*x^2)*(a*b^2)^(1/3))/((a^2*b^5*x^6 + 2*a^3*b^4*x
^3 + a^4*b^3)*(a*b^2)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.220603, size = 522, normalized size = 1.73

$$\frac{fx^2}{2\,b^3} - \frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 20\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3} - \frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{2}{3}}b^3c + \left(-ab^2\right)^{\frac{2}{3}}ab^2d - 20\,\left(-ab^2\right)^{\frac{2}{3}}a^3f + 5\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^5} + \frac{4\,b^4cx^5 + 2\,ab^3dx^5 + 14\,a^3bfx^5 - 8\,a^2b^2x^5e + 7\,ab^3cx^2 - a^2b^2dx^2 + 11\,a^4fx^2 - 5\,a^3bx^2e}{18\,(bx^3 + a)^2a^2b^3} + \frac{\left(2\,\left(-ab^2\right)^{\frac{2}{3}}b^3c + \left(-ab^2\right)^{\frac{2}{3}}ab^2d - 20\,\left(-ab^2\right)^{\frac{2}{3}}a^3f + 5\,\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)*x/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^3 - 1/27*(2*b^3*c*(-a/b)^(1/3) + a*b^2*d*(-a/b)^(1/3)
 - 20*a^3*f*(-a/b)^(1/3) + 5*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*l
n(abs(x - (-a/b)^(1/3)))/(a^3*b^3) - 1/27*sqrt(3)*(2*(-a*b^2)^(2/
3)*b^3*c + (-a*b^2)^(2/3)*a*b^2*d - 20*(-a*b^2)^(2/3)*a^3*f + 5*(
-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(a^3*b^5) + 1/18*(4*b^4*c*x^5 + 2*a*b^3*d*x^5 + 14*a^
3*b*f*x^5 - 8*a^2*b^2*x^5*e + 7*a*b^3*c*x^2 - a^2*b^2*d*x^2 + 11*
a^4*f*x^2 - 5*a^3*b*x^2*e)/((b*x^3 + a)^2*a^2*b^3) + 1/54*(2*(-a*
b^2)^(2/3)*b^3*c + (-a*b^2)^(2/3)*a*b^2*d - 20*(-a*b^2)^(2/3)*a^3
*f + 5*(-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^3*b^5)
```

3.294 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=292

$$\frac{x\left(13a^3f-7a^2be+ab^2d+5b^3c\right)}{18a^2b^3\left(a+bx^3\right)}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{54a^{8/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{27a^{8/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{9\sqrt{3}a^{8/3}b^{10/3}}+\frac{fx}{b^3}$$

```
[Out] (f*x)/b^3 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a +
 b*x^3)^2) + ((5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(18*a
^2*b^3*(a + b*x^3)) - ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^
(8/3)*b^(10/3)) + ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log
[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(10/3)) - ((5*b^3*c + a*b^2*
d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(54*a^(8/3)*b^(10/3))
```

Rubi [A] time = 0.684743, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x\left(13a^3f-7a^2be+ab^2d+5b^3c\right)}{18a^2b^3\left(a+bx^3\right)}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{54a^{8/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{27a^{8/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{9\sqrt{3}a^{8/3}b^{10/3}}+\frac{fx}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```


```
[Out] (f*x)/b^3 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a +
 b*x^3)^2) + ((5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(18*a
^2*b^3*(a + b*x^3)) - ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^
(8/3)*b^(10/3)) + ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log
[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(10/3)) - ((5*b^3*c + a*b^2*
d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(54*a^(8/3)*b^(10/3))
```

Rubi in Sympy [A] time = 116.6, size = 289, normalized size = 0.99

$$\frac{fx}{b^3}-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{6ab^3\left(a+bx^3\right)^2}+\frac{x\left(13a^3f-7a^2be+ab^2d+5b^3c\right)}{18a^2b^3\left(a+bx^3\right)}-\frac{\left(14a^3f-2a^2be-ab^2d-5b^3c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{8}{3}}b^{\frac{10}{3}}}+\frac{\left(14a^3f-2a^2be-ab^2d-5b^3c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{8}{3}}b^{\frac{10}{3}}}+\frac{\sqrt{3}\left(14a^3f-2a^2be-ab^2d-5b^3c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{8}{3}}b^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] f*x/b**3 - x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*a*b**3*(a
 + b*x**3)**2) + x*(13*a**3*f - 7*a**2*b*e + a*b**2*d + 5*b**3*c)
/(18*a**2*b**3*(a + b*x**3)) - (14*a**3*f - 2*a**2*b*e - a*b**2*d
 - 5*b**3*c)*log(a**(1/3) + b**(1/3)*x)/(27*a**(8/3)*b**(10/3)) +
 (14*a**3*f - 2*a**2*b*e - a*b**2*d - 5*b**3*c)*log(a**(2/3) - a*
*(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(54*a**(8/3)*b**(10/3)) + sqrt
(3)*(14*a**3*f - 2*a**2*b*e - a*b**2*d - 5*b**3*c)*atan(sqrt(3)*(
a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a**(8/3)*b**(10/3))
```

Mathematica [A] time = 0.355843, size = 279, normalized size = 0.96

$$\frac{\frac{3\sqrt[3]{b}x\left(13a^3f-7a^2be+ab^2d+5b^3c\right)}{a^2\left(a+bx^3\right)}+\frac{9\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a\left(a+bx^3\right)^2}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{a^{8/3}}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-14a^3\right.}{a^{8/3}}}{54b^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*f*x + (9*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x)/(a*(a + b*x^3)^2) + (3*b^(1/3)*(5*b^3*c + a*b^2*d - 7*a^2*b*e
+ 13*a^3*f)*x)/(a^2*(a + b*x^3)) - (2*Sqrt[3]*(5*b^3*c + a*b^2*d
+ 2*a^2*b*e - 14*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/a^(8/3) + (2*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a
^(1/3) + b^(1/3)*x])/a^(8/3) - ((5*b^3*c + a*b^2*d + 2*a^2*b*e -
14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3)
)/(54*b^(10/3))
```

Maple [B] time = 0.017, size = 539, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] f*x/b^3+13/18/b^2/(b*x^3+a)^2*x^4*a*f-7/18/b/(b*x^3+a)^2*x^4*e+1/
18/(b*x^3+a)^2/a*x^4*d+5/18*b/(b*x^3+a)^2/a^2*x^4*c+5/9/b^3/(b*x^
```

```
3+a)^2*a^2*f*x-2/9/b^2/(b*x^3+a)^2*a*e*x-1/9/b/(b*x^3+a)^2*d*x+4/
9*c/a*x/(b*x^3+a)^2-14/27/b^4*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+2
/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1/27/b^2/a/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))*d+5/27*c/a^2/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27
/b^4*a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f-1/27/b^3/(
a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e-1/54/b^2/a/(a/b)^(
2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-5/54*c/a^2/b/(a/b)^(2/3)
*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-14/27/b^4*a/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+2/27/b^3/(a/b)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/27/b^2/a/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+5/27*c/
a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219346, size = 710, normalized size = 2.43

$$\sqrt{3}\left(\sqrt{3}\left(\left(5 b^5 c + a b^4 d + 2 a^2 b^3 e - 14 a^3 b^2 f\right) x^6 + 5 a^2 b^3 c + a^3 b^2 d + 2 a^4 b e - 14 a^5 f + 2 \left(5 a b^4 c + a^2 b^3 d + 2 a^3 b^2 e - 14 a^4 b f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*(sqrt(3)*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3
*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*
(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*log((-a^2
*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*a*x + a^2) - 2*sqrt(3)*((5*b^5*c +
 a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^
2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2
*e - 14*a^4*b*f)*x^3)*log((-a^2*b)^(1/3)*x - a) + 6*((5*b^5*c + a
*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*
d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e
 - 14*a^4*b*f)*x^3)*arctan(1/3*(2*sqrt(3)*(-a^2*b)^(1/3)*x + sqrt
(3)*a)/a) + 3*sqrt(3)*(18*a^2*b^2*f*x^7 + (5*b^4*c + a*b^3*d - 7*
a^2*b^2*e + 49*a^3*b*f)*x^4 + 2*(4*a*b^3*c - a^2*b^2*d - 2*a^3*b*
e + 14*a^4*f)*x)*(-a^2*b)^(1/3))/((a^2*b^5*x^6 + 2*a^3*b^4*x^3 +
a^4*b^3)*(-a^2*b)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219098, size = 463, normalized size = 1.59

$$\frac{fx}{b^3} - \frac{\left(5\,b^3c + ab^2d - 14\,a^3f + 2\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3} + \frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^3c + \left(-ab^2\right)^{\frac{1}{3}}ab^2d - 14\,\left(-ab^2\right)^{\frac{1}{3}}a^3f + 2\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^4} + \frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^3c + \left(-ab^2\right)^{\frac{1}{3}}ab^2d - 14\,\left(-ab^2\right)^{\frac{1}{3}}a^3f + 2\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^4} + \frac{5\,b^4cx^4 + ab^3dx^4 + 13\,a^3bfx^4 - 7\,a^2b^2x^4e + 8\,ab^3cx - 2\,a^2b^2dx + 10\,a^4fx - 4\,a^3bxe}{18\,(bx^3+a)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 - 1/27*(5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*e)*(-a/b)^
(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/27*sqrt(3)*(5*(-a*b
^2)^(1/3)*b^3*c + (-a*b^2)^(1/3)*a*b^2*d - 14*(-a*b^2)^(1/3)*a^3*
f + 2*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/(a^3*b^4) + 1/54*(5*(-a*b^2)^(1/3)*b^3*c + (-a
*b^2)^(1/3)*a*b^2*d - 14*(-a*b^2)^(1/3)*a^3*f + 2*(-a*b^2)^(1/3)*
a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4) + 1/18
*(5*b^4*c*x^4 + a*b^3*d*x^4 + 13*a^3*b*f*x^4 - 7*a^2*b^2*x^4*e +
8*a*b^3*c*x - 2*a^2*b^2*d*x + 10*a^4*f*x - 4*a^3*b*x*e)/((b*x^3 +
 a)^2*a^2*b^3)
```

3.295 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=303

$$-\frac{c}{a^3x}-\frac{x^2\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{9a^3b^2\left(a+bx^3\right)}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^3f-a^2be-2ab^2d+14b^3c\right)}{54a^{10/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^3f-a^2be-2ab^2d+14b^3c\right)}{27a^{10/3}b^{8/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^3f-a^2be-2ab^2d+14b^3c\right)}{9\sqrt{3}a^{10/3}b^{8/3}}$$

```
[Out] -(c/(a^3*x)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^2*b
^2*(a + b*x^3)^2) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^
2)/(9*a^3*b^2*(a + b*x^3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5
*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqr
t[3]*a^(10/3)*b^(8/3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(8/3)) - ((14*b^3*c
- 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(10/3)*b^(8/3))
```

Rubi [A] time = 0.805318, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{c}{a^3x}-\frac{x^2\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{9a^3b^2\left(a+bx^3\right)}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^3f-a^2be-2ab^2d+14b^3c\right)}{54a^{10/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^3f-a^2be-2ab^2d+14b^3c\right)}{27a^{10/3}b^{8/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^3f-a^2be-2ab^2d+14b^3c\right)}{9\sqrt{3}a^{10/3}b^{8/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^2*b
^2*(a + b*x^3)^2) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^
2)/(9*a^3*b^2*(a + b*x^3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5
*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqr
t[3]*a^(10/3)*b^(8/3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(8/3)) - ((14*b^3*c
- 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(10/3)*b^(8/3))
```

Rubi in Sympy [A] time = 145.367, size = 286, normalized size = 0.94

$$-\frac{x\left(\frac{a^3f}{x^2}-\frac{a^2be}{x^2}+\frac{ab^2d}{x^2}-\frac{b^3c}{x^2}\right)}{6ab^3(a+bx^3)^2}-\frac{x^2\left(3a^2f-2abe+b^2d\right)}{3a^2b^2(a+bx^3)}-\frac{a^2f-abe+b^2d}{a^2b^3x}+\frac{\left(3a^2f-5abe+4b^2d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{7}{3}}b^{\frac{8}{3}}}-\frac{\left(3a^2f-5abe+4b^2d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{7}{3}}b^{\frac{8}{3}}}+\frac{\sqrt{3}\left(3a^2f-5abe+4b^2d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{7}{3}}b^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] -x*(a**3*f/x**2 - a**2*b*e/x**2 + a*b**2*d/x**2 - b**3*c/x**2)/(6
*a*b**3*(a + b*x**3)**2) - x**2*(3*a**2*f - 2*a*b*e + b**2*d)/(3*
a**2*b**2*(a + b*x**3)) - (a**2*f - a*b*e + b**2*d)/(a**2*b**3*x)
 + (3*a**2*f - 5*a*b*e + 4*b**2*d)*log(a**(1/3) + b**(1/3)*x)/(9*
a**(7/3)*b**(8/3)) - (3*a**2*f - 5*a*b*e + 4*b**2*d)*log(a**(2/3)
 - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*a**(7/3)*b**(8/3)) +
sqrt(3)*(3*a**2*f - 5*a*b*e + 4*b**2*d)*atan(sqrt(3)*(a**(1/3)/3
- 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(7/3)*b**(8/3))
```

Mathematica [A] time = 0.41739, size = 286, normalized size = 0.94

$$\frac{-\frac{6\sqrt[3]{a}x^2(4a^3f-a^2be-2ab^2d+5b^3c)}{b^2(a+bx^3)}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(5a^3f+a^2be+2ab^2d-14b^3c)}{b^{8/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)(-5a^3f-a^2be-2ab^2d+14b^3c)}{b^{8/3}}+\frac{9a^{4/3}x^2(\cdots}{\cdots}}{54a^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(1/3)*c)/x + (9*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a
^3*f)*x^2)/(b^2*(a + b*x^3)^2) - (6*a^(1/3)*(5*b^3*c - 2*a*b^2*d
- a^2*b*e + 4*a^3*f)*x^2)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(14*b^3*
c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(8/3) - (2*(-14*b^3*c + 2*a*b^2*d + a^2*b*e + 5*
a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(8/3) + ((-14*b^3*c + 2*a*b^2*
d + a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/b^(8/3))/(54*a^(10/3))
```

Maple [B] time = 0.02, size = 547, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x)
```

```
[Out] -c/a^3/x-4/9/(b*x^3+a)^2/b*x^5*f+1/9/a/(b*x^3+a)^2*x^5*e+2/9/a^2/
(b*x^3+a)^2*b*x^5*d-5/9/a^3/(b*x^3+a)^2*b^2*x^5*c-5/18*a/(b*x^3+a
)^2/b^2*x^2*f-1/18/(b*x^3+a)^2/b*x^2*e+7/18*d/a*x^2/(b*x^3+a)^2-1
3/18/a^2/(b*x^3+a)^2*x^2*b*c-5/27/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))*f-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-2/27*d/a^2/b/(a/b
```

```
)^(1/3)*ln(x+(a/b)^(1/3))+14/27/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
*c+5/54/b^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/54/
a/b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/27*d/a^2/
b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-7/27/a^3/(a/b)^(1
/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+5/27/b^3*3^(1/2)/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+1/27/a/b^2*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+2/27*d/a^2*
3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/
27/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.222033, size = 741, normalized size = 2.45

$$\frac{\sqrt{3}\left(\sqrt{3}\left(\left(14\,b^5c-2\,ab^4d-a^2b^3e-5\,a^3b^2f\right)x^7+2\left(14\,ab^4c-2\,a^2b^3d-a^3b^2e-5\,a^4bf\right)x^4+\left(14\,a^2b^3c-2\,a^3b^2d-a^4be-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^2),x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*(sqrt(3)*((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3
*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f
)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*a^5*f)*x)*log((
-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 2*sqrt(3)*((14*b^
5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c -
2*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*
b^2*d - a^4*b*e - 5*a^5*f)*x)*log(a*b + (-a*b^2)^(2/3)*x) + 6*((1
4*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*
c - 2*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*
a^3*b^2*d - a^4*b*e - 5*a^5*f)*x)*arctan(-1/3*(sqrt(3)*a*b - 2*sq
rt(3)*(-a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(2*(14*b^4*c - 2*a*b^3
*d - a^2*b^2*e + 4*a^3*b*f)*x^6 + 18*a^2*b^2*c + (49*a*b^3*c - 7*
a^2*b^2*d + a^3*b*e + 5*a^4*f)*x^3)*(-a*b^2)^(1/3))/((a^3*b^4*x^7
 + 2*a^4*b^3*x^4 + a^5*b^2*x)*(-a*b^2)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.222157, size = 525, normalized size = 1.73

$$-\frac{c}{a^{3}x}+\frac{\left(14\,b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{4}b^{2}}+\frac{\sqrt{3}\left(14\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-2\,\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-5\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f-\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{4}b^{4}}-\frac{10\,b^{4}cx^{5}-4\,ab^{3}dx^{5}+8\,a^{3}bfx^{5}-2\,a^{2}b^{2}x^{5}e+13\,ab^{3}cx^{2}-7\,a^{2}b^{2}dx^{2}+5\,a^{4}fx^{2}+a^{3}bx^{2}e}{18\,(bx^{3}+a)^{2}a^{3}b^{2}}-\frac{\left(14\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-2\,\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-5\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f-\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{4}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^2),x, algorithm="giac")
```

```
[Out] -c/(a^3*x) + 1/27*(14*b^3*c*(-a/b)^(1/3) - 2*a*b^2*d*(-a/b)^(1/3)
 - 5*a^3*f*(-a/b)^(1/3) - a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(a
bs(x - (-a/b)^(1/3)))/(a^4*b^2) + 1/27*sqrt(3)*(14*(-a*b^2)^(2/3)
*b^3*c - 2*(-a*b^2)^(2/3)*a*b^2*d - 5*(-a*b^2)^(2/3)*a^3*f - (-a*
b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b
)^(1/3))/(a^4*b^4) - 1/18*(10*b^4*c*x^5 - 4*a*b^3*d*x^5 + 8*a^3*b
*f*x^5 - 2*a^2*b^2*x^5*e + 13*a*b^3*c*x^2 - 7*a^2*b^2*d*x^2 + 5*a
^4*f*x^2 + a^3*b*x^2*e)/((b*x^3 + a)^2*a^3*b^2) - 1/54*(14*(-a*b^
2)^(2/3)*b^3*c - 2*(-a*b^2)^(2/3)*a*b^2*d - 5*(-a*b^2)^(2/3)*a^3*
f - (-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a^4*b^4)
```

3.296 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3}\,dx$

Optimal. Leaf size=301

$$-\frac{c}{2a^3x^2}-\frac{x\left(7a^3f-a^2be-5ab^2d+11b^3c\right)}{18a^3b^2\left(a+bx^3\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{54a^{11/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{27a^{11/3}b^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{9\sqrt{3}a^{11/3}b^{7/3}}$$

```
[Out] -c/(2*a^3*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^2*b
^2*(a + b*x^3)^2) - ((11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x
)/(18*a^3*b^2*(a + b*x^3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2
*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqr
t[3]*a^(11/3)*b^(7/3)) - ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(7/3)) + ((20*b^3*c
- 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(11/3)*b^(7/3))
```

Rubi [A] time = 0.786445, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{c}{2a^3x^2}-\frac{x\left(7a^3f-a^2be-5ab^2d+11b^3c\right)}{18a^3b^2\left(a+bx^3\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{54a^{11/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{27a^{11/3}b^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{9\sqrt{3}a^{11/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -c/(2*a^3*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^2*b
^2*(a + b*x^3)^2) - ((11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x
)/(18*a^3*b^2*(a + b*x^3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2
*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqr
t[3]*a^(11/3)*b^(7/3)) - ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(7/3)) + ((20*b^3*c
- 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(11/3)*b^(7/3))
```

Rubi in Sympy [A] time = 148.259, size = 287, normalized size = 0.95

$$-\frac{x\left(\frac{a^3 f}{x^3}-\frac{a^2 b e}{x^3}+\frac{a b^2 d}{x^3}-\frac{b^3 c}{x^3}\right)}{6 a b^3\left(a+b x^3\right)^2}-\frac{x\left(3 a^2 f-2 a b e+b^2 d\right)}{3 a^2 b^2\left(a+b x^3\right)}-\frac{a^2 f-a b e+b^2 d}{2 a^2 b^3 x^2}-\frac{\left(6 a^2 f-7 a b e+5 b^2 d\right) \log\left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{9 a^{\frac{8}{3}} b^{\frac{7}{3}}}+\frac{\left(6 a^2 f-7 a b e+5 b^2 d\right) \log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b} x+b^{\frac{2}{3}} x^2\right)}{18 a^{\frac{8}{3}} b^{\frac{7}{3}}}+\frac{\sqrt{3}\left(6 a^2 f-7 a b e+5 b^2 d\right) \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b} x}{3}\right)}{\sqrt[3]{a}}\right)}{9 a^{\frac{8}{3}} b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] -x*(a**3*f/x**3 - a**2*b*e/x**3 + a*b**2*d/x**3 - b**3*c/x**3)/(6
*a*b**3*(a + b*x**3)**2) - x*(3*a**2*f - 2*a*b*e + b**2*d)/(3*a**
2*b**2*(a + b*x**3)) - (a**2*f - a*b*e + b**2*d)/(2*a**2*b**3*x**
2) - (6*a**2*f - 7*a*b*e + 5*b**2*d)*log(a**(1/3) + b**(1/3)*x)/(
9*a**(8/3)*b**(7/3)) + (6*a**2*f - 7*a*b*e + 5*b**2*d)*log(a**(2/
3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*a**(8/3)*b**(7/3))
+ sqrt(3)*(6*a**2*f - 7*a*b*e + 5*b**2*d)*atan(sqrt(3)*(a**(1/3)/
3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(8/3)*b**(7/3))
```

Mathematica [A] time = 0.409312, size = 283, normalized size = 0.94

$$\frac{-\frac{27 a^{2/3} c}{x^2}+\frac{2 \log\left(\sqrt[3]{a}+\sqrt[3]{b} x\right)\left(2 a^3 f+a^2 b e+5 a b^2 d-20 b^3 c\right)}{b^{7/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b} x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-2 a^3 f-a^2 b e-5 a b^2 d+20 b^3 c\right)}{b^{7/3}}+\frac{9 a^{5/3} x\left(a^3 f-a^2 b e+a b^2 d-b^3 c\right)}{b^2\left(a+b x^3\right)^2}-3 \cdots}{54 a^{11/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(2/3)*c)/x^2 + (9*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*x)/(b^2*(a + b*x^3)^2) - (3*a^(2/3)*(11*b^3*c - 5*a*b^2*d
 - a^2*b*e + 7*a^3*f)*x)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(20*b^3*c
 - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/b^(7/3) + (2*(-20*b^3*c + 5*a*b^2*d + a^2*b*e + 2*a
^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(7/3) - ((-20*b^3*c + 5*a*b^2*d
 + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/b^(7/3))/(54*a^(11/3))
```

Maple [B] time = 0.02, size = 539, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -1/2*c/a^3/x^2-7/18/(b*x^3+a)^2/b*x^4*f+1/18/a/(b*x^3+a)^2*x^4*e+
5/18/a^2/(b*x^3+a)^2*x^4*b*d-11/18/a^3/(b*x^3+a)^2*x^4*b^2*c-2/9*
a/(b*x^3+a)^2/b^2*x*f-1/9/(b*x^3+a)^2/b*x*e+4/9/a/(b*x^3+a)^2*x*d
-7/9/a^2/(b*x^3+a)^2*x*b*c+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
*f+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+5/27/a^2/b/(a/b)^(2
```

```
/3)*ln(x+(a/b)^(1/3))*d-20/27/a^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c
-1/27/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f-1/54/a/
b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e-5/54/a^2/b/(a
/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+10/27/a^3/(a/b)^(2/
3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+2/27/b^3/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+1/27/a/b^2/(a/b)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+5/27/a^2/b/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-20/2
7/a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266457, size = 737, normalized size = 2.45

$$\sqrt{3}\Big(\sqrt{3}\left(\left(20\,b^5c - 5\,ab^4d - a^2b^3e - 2\,a^3b^2f\right)x^8 + 2\left(20\,ab^4c - 5\,a^2b^3d - a^3b^2e - 2\,a^4bf\right)x^5 + \left(20\,a^2b^3c - 5\,a^3b^2d - a^4be - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^3),x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*(sqrt(3)*((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3
*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f
)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*log
((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 2*sqrt(3)*((20*b^
5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c -
5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*
b^2*d - a^4*b*e - 2*a^5*f)*x^2)*log((a^2*b)^(1/3)*x + a) - 6*((20
*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c
 - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a
^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*arctan(1/3*(2*sqrt(3)*(a^2*b)^
(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*((20*b^4*c - 5*a*b^3*d - a^2*
b^2*e + 7*a^3*b*f)*x^6 + 9*a^2*b^2*c + 2*(16*a*b^3*c - 4*a^2*b^2*
d + a^3*b*e + 2*a^4*f)*x^3)*(a^2*b)^(1/3))/((a^3*b^4*x^8 + 2*a^4*
b^3*x^5 + a^5*b^2*x^2)*(a^2*b)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218071, size = 486, normalized size = 1.61

$$\frac{\left(20\,b^{3}c-5\,ab^{2}d-2\,a^{3}f-a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{4}b^{2}}-\frac{\sqrt{3}\left(20\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-5\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-2\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f-\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{4}b^{3}}-\frac{\left(20\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-5\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-2\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f-\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{4}b^{3}}-\frac{20\,b^{4}cx^{6}-5\,ab^{3}dx^{6}+7\,a^{3}bfx^{6}-a^{2}b^{2}x^{6}e+32\,ab^{3}cx^{3}-8\,a^{2}b^{2}dx^{3}+4\,a^{4}fx^{3}+2\,a^{3}bx^{3}e+9\,a^{2}b^{2}c}{18\left(bx^{4}+ax\right)^{2}a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^3),x, algorithm="giac")
```

```
[Out] 1/27*(20*b^3*c - 5*a*b^2*d - 2*a^3*f - a^2*b*e)*(-a/b)^(1/3)*ln(a
bs(x - (-a/b)^(1/3)))/(a^4*b^2) - 1/27*sqrt(3)*(20*(-a*b^2)^(1/3)
*b^3*c - 5*(-a*b^2)^(1/3)*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f - (-a*
b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b
)^(1/3))/(a^4*b^3) - 1/54*(20*(-a*b^2)^(1/3)*b^3*c - 5*(-a*b^2)^(
1/3)*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f - (-a*b^2)^(1/3)*a^2*b*e)*l
n(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^3) - 1/18*(20*b^4*c
*x^6 - 5*a*b^3*d*x^6 + 7*a^3*b*f*x^6 - a^2*b^2*x^6*e + 32*a*b^3*c
*x^3 - 8*a^2*b^2*d*x^3 + 4*a^4*f*x^3 + 2*a^3*b*x^3*e + 9*a^2*b^2*
c)/((b*x^4 + a*x)^2*a^3*b^2)
```

3.297 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx$

Optimal. Leaf size=317

$$\frac{3bc-ad}{a^4x}-\frac{c}{4a^3x^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{54a^{13/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{27a^{13/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{9\sqrt{3}a^{13/3}b^{5/3}}+\frac{x^2\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{9a^4b\left(a+bx^3\right)}$$

```
[Out] -c/(4*a^3*x^4) + (3*b*c - a*d)/(a^4*x) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((8*b^3*c - 5*a*b^2*d
 + 2*a^2*b*e + a^3*f)*x^2)/(9*a^4*b*(a + b*x^3)) - ((35*b^3*c - 1
4*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(13/3)*b^(5/3)) - ((35*b^3*c - 14*a
*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(13/3
)*b^(5/3)) + ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3)*b^(5/3))
```

Rubi [A] time = 0.899201, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{3bc-ad}{a^4x}-\frac{c}{4a^3x^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{54a^{13/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{27a^{13/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{9\sqrt{3}a^{13/3}b^{5/3}}+\frac{x^2\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{9a^4b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] -c/(4*a^3*x^4) + (3*b*c - a*d)/(a^4*x) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((8*b^3*c - 5*a*b^2*d
 + 2*a^2*b*e + a^3*f)*x^2)/(9*a^4*b*(a + b*x^3)) - ((35*b^3*c - 1
4*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(13/3)*b^(5/3)) - ((35*b^3*c - 14*a
*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(13/3
)*b^(5/3)) + ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3)*b^(5/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.43246, size = 303, normalized size = 0.96

$$\frac{-\frac{27a^{4/3}c}{x^4}+\frac{12\sqrt[3]{a}x^2\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{b\left(a+bx^3\right)}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{b^{5/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{b^{5/3}}}{108a^{13/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(4/3)*c)/x^4 - (108*a^(1/3)*(-3*b*c + a*d))/x - (18*a^(4/
3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)^2)
+ (12*a^(1/3)*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(b*(
a + b*x^3)) - (4*Sqrt[3]*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3
*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) - (4*(35
*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/b^(5/3) + (2*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(108*a^(13/3))
```

Maple [B] time = 0.023, size = 574, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x)
```

```
[Out] -1/4*c/a^3/x^4-d/a^3/x+3/a^4/x*b*c+1/9/a/(b*x^3+a)^2*x^5*f+2/9/a^
2/(b*x^3+a)^2*x^5*b*e-5/9/a^3/(b*x^3+a)^2*x^5*b^2*d+8/9/a^4/(b*x^
3+a)^2*x^5*b^3*c-1/18/(b*x^3+a)^2*x^2/b*f+7/18/a/(b*x^3+a)^2*x^2*
e-13/18/a^2/(b*x^3+a)^2*x^2*b*d+19/18/a^3/(b*x^3+a)^2*c*x^2*b^2-1
/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-2/27/a^2/b/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))*e+14/27/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-35/
27/a^4*b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/54/a/b^2/(a/b)^(1/3)*l
n(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/27/a^2/b/(a/b)^(1/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))*e-7/27/a^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))*d+35/54/a^4*b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)
+(a/b)^(2/3))*c+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*f+2/27/a^2/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-14/27/a^3*3^(1/2)/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+35/27/a^4*b*3^(1/2)/(a/b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240055, size = 765, normalized size = 2.41

$$-\frac{\sqrt{3}\left(2\sqrt{3}\left(\left(35\,b^5c-14\,ab^4d+2\,a^2b^3e+a^3b^2f\right)x^{10}+2\left(35\,ab^4c-14\,a^2b^3d+2\,a^3b^2e+a^4bf\right)x^7+\left(35\,a^2b^3c-14\,a^3b^2d+2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^5),x, algorithm="fricas")
```

```
[Out] -1/324*sqrt(3)*(2*sqrt(3)*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e +
 a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a
^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x
^4)*log((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x) - 4*sqrt(3
)*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35
*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^
3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*log(a*b + (-a*b^2)^(
2/3)*x) + 12*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x
^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 +
 (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*arctan(-1
/3*(sqrt(3)*a*b - 2*sqrt(3)*(-a*b^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*
(4*(35*b^4*c - 14*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^9 + (245*a*b
^3*c - 98*a^2*b^2*d + 14*a^3*b*e - 2*a^4*f)*x^6 - 9*a^3*b*c + 18*
(5*a^2*b^2*c - 2*a^3*b*d)*x^3)*(-a*b^2)^(1/3))/((a^4*b^3*x^10 + 2
*a^5*b^2*x^7 + a^6*b*x^4)*(-a*b^2)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.221169, size = 547, normalized size = 1.73

$$-\frac{\left(35\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5b}-\frac{\sqrt{3}\left(35\left(-ab^2\right)^{\frac{2}{3}}b^3c-14\left(-ab^2\right)^{\frac{2}{3}}ab^2d+\left(-ab^2\right)^{\frac{2}{3}}a^3f+2\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b^3}+\frac{16\,b^4cx^5-10\,ab^3dx^5+2\,a^3bfx^5+4\,a^2b^2x^5e+19\,ab^3cx^2-13\,a^2b^2dx^2-a^4fx^2+7\,a^3bx^2e}{18\left(bx^3+a\right)^2a^4b}+\frac{\left(35\left(-ab^2\right)^{\frac{2}{3}}b^3c-14\left(-ab^2\right)^{\frac{2}{3}}ab^2d+\left(-ab^2\right)^{\frac{2}{3}}a^3f+2\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b^3}+\frac{12\,bcx^3-4\,adx^3-ac}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^5),x, algorithm="giac")
```

```
[Out] -1/27*(35*b^3*c*(-a/b)^(1/3) - 14*a*b^2*d*(-a/b)^(1/3) + a^3*f*(-
a/b)^(1/3) + 2*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/
b)^(1/3)))/(a^5*b) - 1/27*sqrt(3)*(35*(-a*b^2)^(2/3)*b^3*c - 14*(
```

```
-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(2/3)*a^3*f + 2*(-a*b^2)^(2/3)*a
^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^
5*b^3) + 1/18*(16*b^4*c*x^5 - 10*a*b^3*d*x^5 + 2*a^3*b*f*x^5 + 4*
a^2*b^2*x^5*e + 19*a*b^3*c*x^2 - 13*a^2*b^2*d*x^2 - a^4*f*x^2 + 7
*a^3*b*x^2*e)/((b*x^3 + a)^2*a^4*b) + 1/54*(35*(-a*b^2)^(2/3)*b^3
*c - 14*(-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(2/3)*a^3*f + 2*(-a*b^2
)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^3
) + 1/4*(12*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^4*x^4)
```

3.298 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3}\,dx$

Optimal. Leaf size=316

$$\frac{3bc-ad}{2a^4x^2}-\frac{c}{5a^3x^5}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{54a^{14/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{27a^{14/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{9\sqrt{3}a^{14/3}b^{4/3}}+\frac{x\left(a^3f+5a^2be-11ab^2d+17b^3c\right)}{18a^4b\left(a+bx^3\right)}$$

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(2*a^4*x^2) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((17*b^3*c - 11*a*b
^2*d + 5*a^2*b*e + a^3*f)*x)/(18*a^4*b*(a + b*x^3)) - ((44*b^3*c
- 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/
(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)*b^(4/3)) + ((44*b^3*c - 2
0*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1
4/3)*b^(4/3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(14/3)*b^(4/3))
```

Rubi [A] time = 0.879611, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{3bc-ad}{2a^4x^2}-\frac{c}{5a^3x^5}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{54a^{14/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{27a^{14/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{9\sqrt{3}a^{14/3}b^{4/3}}+\frac{x\left(a^3f+5a^2be-11ab^2d+17b^3c\right)}{18a^4b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(2*a^4*x^2) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((17*b^3*c - 11*a*b
^2*d + 5*a^2*b*e + a^3*f)*x)/(18*a^4*b*(a + b*x^3)) - ((44*b^3*c
- 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/
(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)*b^(4/3)) + ((44*b^3*c - 2
0*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1
4/3)*b^(4/3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(14/3)*b^(4/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.426866, size = 299, normalized size = 0.95

$$\frac{-\frac{135a^{2/3}(ad-3bc)}{x^2}-\frac{54a^{5/3}c}{x^5}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{b^{4/3}}-\frac{10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{b^{4/3}}-\frac{45a^{5/3}x\left(a^3f\right.}{b}}{270a^{14/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*c)/x^5 - (135*a^(2/3)*(-3*b*c + a*d))/x^2 - (45*a^(
5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)^2)
+ (15*a^(2/3)*(17*b^3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x)/(b*(
a + b*x^3)) - (10*Sqrt[3]*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^
3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3) + (10*(
44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/b^(4/3) - (5*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(270*a^(14/3
))
```

Maple [B] time = 0.023, size = 566, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x)
```

```
[Out] -1/5*c/a^3/x^5-1/2*d/a^3/x^2+3/2/a^4/x^2*b*c+1/18/a/(b*x^3+a)^2*x
^4*f+5/18/a^2/(b*x^3+a)^2*x^4*b*e-11/18/a^3/(b*x^3+a)^2*x^4*b^2*d
+17/18/a^4/(b*x^3+a)^2*x^4*b^3*c-1/9/(b*x^3+a)^2*x/b*f+4/9/a/(b*x
^3+a)^2*x*e-7/9/a^2/(b*x^3+a)^2*x*b*d+10/9/a^3/(b*x^3+a)^2*x*b^2*
c+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+5/27/a^2/b/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))*e-20/27/a^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+
44/27/a^4*b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/54/a/b^2/(a/b)^(2/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f-5/54/a^2/b/(a/b)^(2/3)*ln(x
^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+10/27/a^3/(a/b)^(2/3)*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))*d-22/27/a^4*b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(
1/3)+(a/b)^(2/3))*c+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*f+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-20/27/a^3/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+44/27/a^4*b/(a/b)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.22673, size = 752, normalized size = 2.38

$$-\frac{\sqrt{3}\left(5\sqrt{3}\left(\left(44\,b^5c-20\,ab^4d+5\,a^2b^3e+a^3b^2f\right)x^{11}+2\left(44\,ab^4c-20\,a^2b^3d+5\,a^3b^2e+a^4bf\right)x^8+\left(44\,a^2b^3c-20\,a^3b^2d+5\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^6),x, algorithm="fricas")
```

```
[Out] -1/810*sqrt(3)*(5*sqrt(3)*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e +
 a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e + a
^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x
^5)*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 10*sqrt(3)
*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*
a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3
*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*log((a^2*b)^(1/3)*x +
 a) - 30*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11
+ 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8 + (44
*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*arctan(1/3*(2
*sqrt(3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*(5*(44*b^4*c
 - 20*a*b^3*d + 5*a^2*b^2*e + a^3*b*f)*x^9 + 2*(176*a*b^3*c - 80*
a^2*b^2*d + 20*a^3*b*e - 5*a^4*f)*x^6 - 18*a^3*b*c + 9*(11*a^2*b^
2*c - 5*a^3*b*d)*x^3)*(a^2*b)^(1/3))/((a^4*b^3*x^11 + 2*a^5*b^2*x
^8 + a^6*b*x^5)*(a^2*b)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218416, size = 491, normalized size = 1.55

$$-\frac{\left(44\,b^3c-20\,ab^2d+a^3f+5\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5b}+\frac{\sqrt{3}\left(44\left(-ab^2\right)^{\frac{1}{3}}b^3c-20\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^3f+5\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b^2}+\frac{\left(44\left(-ab^2\right)^{\frac{1}{3}}b^3c-20\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^3f+5\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b^2}+\frac{17\,b^4cx^4-11\,ab^3dx^4+a^3bfx^4+5\,a^2b^2x^4e+20\,ab^3cx-14\,a^2b^2dx-2\,a^4fx+8\,a^3bxe}{18\left(bx^3+a\right)^2a^4b}+\frac{15\,bcx^3-5\,adx^3-2\,ac}{10\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^6),x, algorithm="giac")
```

```
[Out] -1/27*(44*b^3*c - 20*a*b^2*d + a^3*f + 5*a^2*b*e)*(-a/b)^(1/3)*ln
(abs(x - (-a/b)^(1/3)))/(a^5*b) + 1/27*sqrt(3)*(44*(-a*b^2)^(1/3)
*b^3*c - 20*(-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(1/3)*a^3*f + 5*(-a
```

```
*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a^5*b^2) + 1/54*(44*(-a*b^2)^(1/3)*b^3*c - 20*(-a*b^2)
^(1/3)*a*b^2*d + (-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)^(1/3)*a^2*b*e)
*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^2) + 1/18*(17*b^4
*c*x^4 - 11*a*b^3*d*x^4 + a^3*b*f*x^4 + 5*a^2*b^2*x^4*e + 20*a*b^
3*c*x - 14*a^2*b^2*d*x - 2*a^4*f*x + 8*a^3*b*x*e)/((b*x^3 + a)^2*
a^4*b) + 1/10*(15*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^4*x^5)
```

3.299 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3}\,dx$

Optimal. Leaf size=343

$$\frac{3bc-ad}{4a^4x^4}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{a^5x}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{54a^{16/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{27a^{16/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{9\sqrt{3}a^{16/3}b^{2/3}}-\frac{x^2\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{9a^5\left(a+bx^3\right)}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(4*a^4*x^4) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^
4*(a + b*x^3)^2) - ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*
x^2)/(9*a^5*(a + b*x^3)) + ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e -
 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*S
qrt[3]*a^(16/3)*b^(2/3)) + ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e -
 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(16/3)*b^(2/3)) - ((65*
b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(54*a^(16/3)*b^(2/3))
```

Rubi [A] time = 1.14773, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{3bc-ad}{4a^4x^4}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{a^5x}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{54a^{16/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{27a^{16/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{9\sqrt{3}a^{16/3}b^{2/3}}-\frac{x^2\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{9a^5\left(a+bx^3\right)}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```



```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(4*a^4*x^4) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^
4*(a + b*x^3)^2) - ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*
x^2)/(9*a^5*(a + b*x^3)) + ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e -
 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*S
qrt[3]*a^(16/3)*b^(2/3)) + ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e -
 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(16/3)*b^(2/3)) - ((65*
b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(54*a^(16/3)*b^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.461329, size = 328, normalized size = 0.96

$$-\frac{189a^{4/3}(ad-3bc)}{x^4}-\frac{108a^{7/3}c}{x^7}-\frac{756\sqrt[3]{a}(a^2e-3abd+6b^2c)}{x}+\frac{84\sqrt[3]{a}x^2(2a^3f-5a^2be+8ab^2d-11b^3c)}{a+bx^3}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-2a^3f+14a^2be-35ab^2d+65}{b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

```
[Out] ((-108*a^(7/3)*c)/x^7 - (189*a^(4/3)*(-3*b*c + a*d))/x^4 - (756*a
^(1/3)*(6*b^2*c - 3*a*b*d + a^2*e))/x + (126*a^(4/3)*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f)*x^2)/(a + b*x^3)^2 + (84*a^(1/3)*(-11*
b^3*c + 8*a*b^2*d - 5*a^2*b*e + 2*a^3*f)*x^2)/(a + b*x^3) + (28*S
qrt[3]*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(65*b^3*c - 35*a*
b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) +
 (14*(-65*b^3*c + 35*a*b^2*d - 14*a^2*b*e + 2*a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(756*a^(16/3))
```

Maple [B] time = 0.024, size = 611, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x)
```

```
[Out] 7/18/a/(b*x^3+a)^2*x^2*f+14/27/a^3*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3)
)-7/27/a^3*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+3/4/a^
4/x^4*b*c+3/a^4/x*b*d-6/a^5/x*b^2*c-13/18/a^2/(b*x^3+a)^2*x^2*b*e
+19/18/a^3/(b*x^3+a)^2*x^2*b^2*d-25/18/a^4/(b*x^3+a)^2*x^2*b^3*c+
1/27/a^2*f/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-14/27/
a^3*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-35/27/a^4*b*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+35/54/a^4*b*d/(a/b)^
(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+65/27/a^5*b^2*c/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))-65/54/a^5*b^2*c/(a/b)^(1/3)*ln(x^2-x*(a/b)^(
1/3)+(a/b)^(2/3))-1/7*c/a^3/x^7+2/27/a^2*f*3^(1/2)/b/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+35/27/a^4*b*d*3^(1/2)/(a/
b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-65/27/a^5*b^2*c*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/4/a
^3/x^4*d-e/a^3/x-5/9/a^3/(b*x^3+a)^2*x^5*e*b^2+8/9/a^4/(b*x^3+a)^
2*x^5*d*b^3-2/27/a^2*f/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+2/9/a^2/(b
*x^3+a)^2*x^5*f*b-11/9/a^5/(b*x^3+a)^2*x^5*b^4*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.222887, size = 809, normalized size = 2.36

$$\sqrt{3}\Big(14\,\sqrt{3}\big(\left(65\,b^{5}c - 35\,ab^{4}d + 14\,a^{2}b^{3}e - 2\,a^{3}b^{2}f\right)x^{13} + 2\,\left(65\,ab^{4}c - 35\,a^{2}b^{3}d + 14\,a^{3}b^{2}e - 2\,a^{4}bf\right)x^{10} + \left(65\,a^{2}b^{3}c - 35\,a^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^8),x, algorithm="fricas")
```

```
[Out] 1/2268*sqrt(3)*(14*sqrt(3)*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e
 - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*
e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e -
 2*a^5*f)*x^7)*log((-a*b^2)^(1/3)*b*x^2 - a*b + (-a*b^2)^(2/3)*x)
 - 28*sqrt(3)*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*
f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f
)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7
)*log(a*b + (-a*b^2)^(2/3)*x) + 84*((65*b^5*c - 35*a*b^4*d + 14*a
^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*
a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a
^4*b*e - 2*a^5*f)*x^7)*arctan(-1/3*(sqrt(3)*a*b - 2*sqrt(3)*(-a*b
^2)^(2/3)*x)/(a*b)) - 3*sqrt(3)*(28*(65*b^4*c - 35*a*b^3*d + 14*a
^2*b^2*e - 2*a^3*b*f)*x^12 + 49*(65*a*b^3*c - 35*a^2*b^2*d + 14*a
^3*b*e - 2*a^4*f)*x^9 + 18*(65*a^2*b^2*c - 35*a^3*b*d + 14*a^4*e)
*x^6 + 36*a^4*c - 9*(13*a^3*b*c - 7*a^4*d)*x^3)*(-a*b^2)^(1/3))/(
(a^5*b^2*x^13 + 2*a^6*b*x^10 + a^7*x^7)*(-a*b^2)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.221184, size = 586, normalized size = 1.71

$$\frac{\left(65\,b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 35\,ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 14\,a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{6}} + \frac{\sqrt{3}\left(65\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c - 35\,\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d - 2\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f + 14\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{6}b^{2}} - \frac{22\,b^{4}cx^{5} - 16\,ab^{3}dx^{5} - 4\,a^{3}bfx^{5} + 10\,a^{2}b^{2}x^{5}e + 25\,ab^{3}cx^{2} - 19\,a^{2}b^{2}dx^{2} - 7\,a^{4}fx^{2} + 13\,a^{3}bx^{2}e}{18\,(bx^{3} + a)^{2}a^{5}} - \frac{\left(65\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c - 35\,\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d - 2\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f + 14\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\ln\left(x^{2} + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{6}b^{2}} - \frac{168\,b^{2}cx^{6} - 84\,abdx^{6} + 28\,a^{2}x^{6}e - 21\,abcx^{3} + 7\,a^{2}dx^{3} + 4\,a^{2}c}{28\,a^{5}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^8),x, algorithm="giac")
```

```
[Out] 1/27*(65*b^3*c*(-a/b)^(1/3) - 35*a*b^2*d*(-a/b)^(1/3) - 2*a^3*f*(
-a/b)^(1/3) + 14*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-
a/b)^(1/3)))/a^6 + 1/27*sqrt(3)*(65*(-a*b^2)^(2/3)*b^3*c - 35*(-a
*b^2)^(2/3)*a*b^2*d - 2*(-a*b^2)^(2/3)*a^3*f + 14*(-a*b^2)^(2/3)*
a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^6*b^2) - 1/18*(22*b^4*c*x^5 - 16*a*b^3*d*x^5 - 4*a^3*b*f*x^5 + 1
0*a^2*b^2*x^5*e + 25*a*b^3*c*x^2 - 19*a^2*b^2*d*x^2 - 7*a^4*f*x^2
 + 13*a^3*b*x^2*e)/((b*x^3 + a)^2*a^5) - 1/54*(65*(-a*b^2)^(2/3)*
b^3*c - 35*(-a*b^2)^(2/3)*a*b^2*d - 2*(-a*b^2)^(2/3)*a^3*f + 14*(
-a*b^2)^(2/3)*a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a
^6*b^2) - 1/28*(168*b^2*c*x^6 - 84*a*b*d*x^6 + 28*a^2*x^6*e - 21*
a*b*c*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^5*x^7)
```

3.300 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx$

Optimal. Leaf size=341

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{8a^3x^8}-\frac{a^2e-3abd+6b^2c}{2a^5x^2}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{27a^{17/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{9\sqrt{3}a^{17/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{54a^{17/3}\sqrt[3]{b}}-\frac{x\left(-5a^3f+11a^2be-17ab^2d+23b^3c\right)}{18a^5\left(a+bx^3\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

```
[Out] -c/(8*a^3*x^8) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(2*a^5*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*
a^4*(a + b*x^3)^2) - ((23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3
*f)*x)/(18*a^5*(a + b*x^3)) + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*
e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
9*Sqrt[3]*a^(17/3)*b^(1/3)) - ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*
e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(17/3)*b^(1/3)) + ((
77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(17/3)*b^(1/3))
```

Rubi [A] time = 1.12027, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{8a^3x^8}-\frac{a^2e-3abd+6b^2c}{2a^5x^2}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{27a^{17/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{9\sqrt{3}a^{17/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{54a^{17/3}\sqrt[3]{b}}-\frac{x\left(-5a^3f+11a^2be-17ab^2d+23b^3c\right)}{18a^5\left(a+bx^3\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] -c/(8*a^3*x^8) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(2*a^5*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*
a^4*(a + b*x^3)^2) - ((23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3
*f)*x)/(18*a^5*(a + b*x^3)) + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*
e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
9*Sqrt[3]*a^(17/3)*b^(1/3)) - ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*
e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(17/3)*b^(1/3)) + ((
77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(17/3)*b^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.481443, size = 324, normalized size = 0.95

$$-\frac{216a^{5/3}(ad-3bc)}{x^5}-\frac{135a^{8/3}c}{x^8}-\frac{540a^{2/3}\left(a^2e-3abd+6b^2c\right)}{x^2}+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^3f-20a^2be+44ab^2d-77b^3c\right)}{\sqrt[3]{b}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-5a^3f+20a\right.}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] ((-135*a^(8/3)*c)/x^8 - (216*a^(5/3)*(-3*b*c + a*d))/x^5 - (540*a
^(2/3)*(6*b^2*c - 3*a*b*d + a^2*e))/x^2 + (180*a^(5/3)*(-(b^3*c)
+ a*b^2*d - a^2*b*e + a^3*f)*x)/(a + b*x^3)^2 + (60*a^(2/3)*(-23*
b^3*c + 17*a*b^2*d - 11*a^2*b*e + 5*a^3*f)*x)/(a + b*x^3) + (40*S
qrt[3]*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-77*b^3*c + 44*a
*b^2*d - 20*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
+ (20*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(1080*a^(17/3))
```

Maple [B] time = 0.025, size = 603, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x)
```

```
[Out] 5/27/a^2*f/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))+44/27/a^4*b*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))-77/27/a^5*b^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))+4/9/a/(b*x^3+a)^2*f*x-20/27/a^3*e/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))+10/27/a^3*e/(a/b)^(2/3)*ln(x^2-x*(a/b)
^(1/3)+(a/b)^(2/3))+3/5/a^4/x^5*b*c+3/2/a^4/x^2*b*d-3/a^5/x^2*b^2
*c+10/9/a^3/(b*x^3+a)^2*b^2*d*x-13/9/a^4/(b*x^3+a)^2*b^3*c*x+5/27
/a^2*f/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-5/54/a^2*f/b/(a/b)^(2/3)*l
n(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-20/27/a^3*e/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+44/27/a^4*b*d/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))-1/8*c/a^3/x^8-1/5/a^3/x^5*d-1/2/a^3/x^2*e-11/1
8/a^3/(b*x^3+a)^2*x^4*b^2*e-22/27/a^4*b*d/(a/b)^(2/3)*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))-77/27/a^5*b^2*c/(a/b)^(2/3)*ln(x+(a/b)^(1/
3))+77/54/a^5*b^2*c/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
-23/18/a^5/(b*x^3+a)^2*x^4*b^4*c-7/9/a^2/(b*x^3+a)^2*b*e*x+17/18/
a^4/(b*x^3+a)^2*x^4*b^3*d+5/18/a^2/(b*x^3+a)^2*x^4*b*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^9),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.224288, size = 794, normalized size = 2.33

$$\frac{\sqrt{3}\left(20\,\sqrt{3}\left(\left(77\,b^{5}c-44\,ab^{4}d+20\,a^{2}b^{3}e-5\,a^{3}b^{2}f\right)x^{14}+2\,\left(77\,ab^{4}c-44\,a^{2}b^{3}d+20\,a^{3}b^{2}e-5\,a^{4}bf\right)x^{11}+\left(77\,a^{2}b^{3}c-44\,a^{3}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^9),x, algorithm="fricas")
```

```
[Out] 1/3240*sqrt(3)*(20*sqrt(3)*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e
 - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*
e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e -
 5*a^5*f)*x^8)*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) -
 40*sqrt(3)*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)
*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*
x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*
log((a^2*b)^(1/3)*x + a) - 120*((77*b^5*c - 44*a*b^4*d + 20*a^2*b
^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*
b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b
*e - 5*a^5*f)*x^8)*arctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)*x - sqrt(3
)*a)/a) - 3*sqrt(3)*(20*(77*b^4*c - 44*a*b^3*d + 20*a^2*b^2*e - 5
*a^3*b*f)*x^12 + 32*(77*a*b^3*c - 44*a^2*b^2*d + 20*a^3*b*e - 5*a
^4*f)*x^9 + 9*(77*a^2*b^2*c - 44*a^3*b*d + 20*a^4*e)*x^6 + 45*a^4
*c - 18*(7*a^3*b*c - 4*a^4*d)*x^3)*(a^2*b)^(1/3))/((a^5*b^2*x^14
+ 2*a^6*b*x^11 + a^7*x^8)*(a^2*b)^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219569, size = 532, normalized size = 1.56

$$\frac{\left(77\,b^{3}c-44\,ab^{2}d-5\,a^{3}f+20\,a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{6}}-\frac{\sqrt{3}\left(77\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-44\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-5\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+20\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{6}b}-\frac{\left(77\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-44\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-5\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+20\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{6}b}-\frac{23\,b^{4}cx^{4}-17\,ab^{3}dx^{4}-5\,a^{3}bfx^{4}+11\,a^{2}b^{2}x^{4}e+26\,ab^{3}cx-20\,a^{2}b^{2}dx-8\,a^{4}fx+14\,a^{3}bxe}{18\,\left(bx^{3}+a\right)^{2}a^{5}}-\frac{120\,b^{2}cx^{6}-60\,abdx^{6}+20\,a^{2}x^{6}e-24\,abcx^{3}+8\,a^{2}dx^{3}+5\,a^{2}c}{40\,a^{5}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^9),x, algorithm="giac")
```

```
[Out] 1/27*(77*b^3*c - 44*a*b^2*d - 5*a^3*f + 20*a^2*b*e)*(-a/b)^(1/3)*
ln(abs(x - (-a/b)^(1/3)))/a^6 - 1/27*sqrt(3)*(77*(-a*b^2)^(1/3)*b
^3*c - 44*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 20*(-
a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/(a^6*b) - 1/54*(77*(-a*b^2)^(1/3)*b^3*c - 44*(-a*b^2)^
(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 20*(-a*b^2)^(1/3)*a^2*b*
e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) - 1/18*(23*b^4
*c*x^4 - 17*a*b^3*d*x^4 - 5*a^3*b*f*x^4 + 11*a^2*b^2*x^4*e + 26*a
*b^3*c*x - 20*a^2*b^2*d*x - 8*a^4*f*x + 14*a^3*b*x*e)/((b*x^3 + a
)^2*a^5) - 1/40*(120*b^2*c*x^6 - 60*a*b*d*x^6 + 20*a^2*x^6*e - 24
*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^5*x^8)
```

3.301 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx$

Optimal. Leaf size=381

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{10a^3x^{10}}-\frac{a^2e-3abd+6b^2c}{4a^5x^4}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{27a^{19/3}}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{9\sqrt{3}a^{19/3}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{54a^{19/3}}+\frac{bx^2\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{9a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

```
[Out] -c/(10*a^3*x^10) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d
 + a^2*e)/(4*a^5*x^4) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f
)/(a^6*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a
 + b*x^3)^2) + (b*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x
^2)/(9*a^6*(a + b*x^3)) - (b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a
^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(19/3)) - (b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35
*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(19/3)) + (b
^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(19/3))
```

Rubi [A] time = 1.40591, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{10a^3x^{10}}-\frac{a^2e-3abd+6b^2c}{4a^5x^4}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{27a^{19/3}}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{9\sqrt{3}a^{19/3}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{54a^{19/3}}+\frac{bx^2\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{9a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] -c/(10*a^3*x^10) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d
 + a^2*e)/(4*a^5*x^4) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f
)/(a^6*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a
 + b*x^3)^2) + (b*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x
^2)/(9*a^6*(a + b*x^3)) - (b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a
^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(19/3)) - (b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35
*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(19/3)) + (b
^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2/
```

```
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(19/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.850349, size = 366, normalized size = 0.96

$$\frac{-\frac{540a^{7/3}(ad-3bc)}{x^7}-\frac{378a^{10/3}c}{x^{10}}-\frac{945a^{4/3}(a^2e-3abd+6b^2c)}{x^4}-\frac{420\sqrt[3]{a}bx^2(5a^3f-8a^2be+11ab^2d-14b^3c)}{a+bx^3}-\frac{3780\sqrt[3]{a}(a^3f-3a^2be+6ab^2d-10b^3c)}{x}+1}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] ((-378*a^(10/3)*c)/x^10 - (540*a^(7/3)*(-3*b*c + a*d))/x^7 - (945
*a^(4/3)*(6*b^2*c - 3*a*b*d + a^2*e))/x^4 - (3780*a^(1/3)*(-10*b^
3*c + 6*a*b^2*d - 3*a^2*b*e + a^3*f))/x - (630*a^(4/3)*b*(-(b^3*c
) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(a + b*x^3)^2 - (420*a^(1/3)*
b*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e + 5*a^3*f)*x^2)/(a + b*x^3)
 - 140*Sqrt[3]*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*
a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*
(-104*b^3*c + 65*a*b^2*d - 35*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b
^(1/3)*x] + 70*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*
a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*a^(1
9/3))
```

Maple [A] time = 0.025, size = 659, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x)
```

```
[Out] 8/9*b^3/a^4/(b*x^3+a)^2*x^5*e-11/9*b^4/a^5/(b*x^3+a)^2*x^5*d+14/9
*b^5/a^6/(b*x^3+a)^2*x^5*c-13/18*b/a^2/(b*x^3+a)^2*x^2*f-65/54*b^
2/a^5*d/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-104/27*b^3/
a^6*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+35/27*b/a^4*e*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-65/27*b^2/a^5*d*3^(1
/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+104/27*b^
3/a^6*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))+65/27*b^2/a^5*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/9*b^2/a^3/(b*x
^3+a)^2*x^5*f-1/4/a^3/x^4*e-1/a^3/x*f-1/7/a^3/x^7*d-1/10*c/a^3/x^
10+14/27/a^3*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-7/27/a^3*f/(a/b)^(1/
3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+3/7/a^4/x^7*b*c+3/4/a^4/x^4*
b*d-3/2/a^5/x^4*b^2*c+3/a^4/x*b*e-6/a^5/x*b^2*d+10/a^6/x*b^3*c+52
/27*b^3/a^6*c/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+19/18
*b^2/a^3/(b*x^3+a)^2*x^2*e-25/18*b^3/a^4/(b*x^3+a)^2*x^2*d+35/54*
b/a^4*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+31/18*b^4/a
^5/(b*x^3+a)^2*x^2*c-35/27*b/a^4*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-
```

```
14/27/a^3*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^11),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.230684, size = 873, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^11),x, algorithm="fricas")
```

```
[Out] 1/11340*sqrt(3)*(70*sqrt(3)*((104*b^5*c - 65*a*b^4*d + 35*a^2*b^3
*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*
b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4
*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) +
a*(b/a)^(1/3)) - 140*sqrt(3)*((104*b^5*c - 65*a*b^4*d + 35*a^2*b^
3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3
*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^2*d + 35*a^
4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) - 42
0*((104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 +
2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 +
 (104*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/
a)^(1/3)*arctan(-1/3*(2*sqrt(3)*b*x - sqrt(3)*a*(b/a)^(2/3))/(a*(
b/a)^(2/3))) + 3*sqrt(3)*(140*(104*b^5*c - 65*a*b^4*d + 35*a^2*b^
3*e - 14*a^3*b^2*f)*x^15 + 245*(104*a*b^4*c - 65*a^2*b^3*d + 35*a
^3*b^2*e - 14*a^4*b*f)*x^12 + 90*(104*a^2*b^3*c - 65*a^3*b^2*d +
35*a^4*b*e - 14*a^5*f)*x^9 - 9*(104*a^3*b^2*c - 65*a^4*b*d + 35*a
^5*e)*x^6 - 126*a^5*c + 36*(8*a^4*b*c - 5*a^5*d)*x^3))/(a^6*b^2*x
^16 + 2*a^7*b*x^13 + a^8*x^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219558, size = 656, normalized size = 1.72

$$-\frac{\left(104\,b^{4}c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 65\,ab^{3}d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14\,a^{3}bf\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 35\,a^{2}b^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{7}} - \frac{\sqrt{3}\left(104\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c - 65\,\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d - 14\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f + 35\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{7}b} + \frac{\left(104\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c - 65\,\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d - 14\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f + 35\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\ln\left(x^{2} + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{7}b} + \frac{28\,b^{5}cx^{5} - 22\,ab^{4}dx^{5} - 10\,a^{3}b^{2}fx^{5} + 16\,a^{2}b^{3}x^{5}e + 31\,ab^{4}cx^{2} - 25\,a^{2}b^{3}dx^{2} - 13\,a^{4}bfx^{2} + 19\,a^{3}b^{2}x^{2}e}{18\,\left(bx^{3} + a\right)^{2}a^{6}} + \frac{1400\,b^{3}cx^{9} - 840\,ab^{2}dx^{9} - 140\,a^{3}fx^{9} + 420\,a^{2}bx^{9}e - 210\,ab^{2}cx^{6} + 105\,a^{2}bdx^{6} - 35\,a^{3}x^{6}e + 60\,a^{2}bcx^{3} - 20\,a^{3}dx^{3} - 14\,a^{3}c}{140\,a^{6}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^11),x, algorithm="giac")
```

```
[Out] -1/27*(104*b^4*c*(-a/b)^(1/3) - 65*a*b^3*d*(-a/b)^(1/3) - 14*a^3*
b*f*(-a/b)^(1/3) + 35*a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs
(x - (-a/b)^(1/3)))/a^7 - 1/27*sqrt(3)*(104*(-a*b^2)^(2/3)*b^3*c
- 65*(-a*b^2)^(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 35*(-a*b^
2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/(a^7*b) + 1/54*(104*(-a*b^2)^(2/3)*b^3*c - 65*(-a*b^2)^(2/
3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 35*(-a*b^2)^(2/3)*a^2*b*e)
*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^7*b) + 1/18*(28*b^5*c
*x^5 - 22*a*b^4*d*x^5 - 10*a^3*b^2*f*x^5 + 16*a^2*b^3*x^5*e + 31*
a*b^4*c*x^2 - 25*a^2*b^3*d*x^2 - 13*a^4*b*f*x^2 + 19*a^3*b^2*x^2*
e)/((b*x^3 + a)^2*a^6) + 1/140*(1400*b^3*c*x^9 - 840*a*b^2*d*x^9
- 140*a^3*f*x^9 + 420*a^2*b*x^9*e - 210*a*b^2*c*x^6 + 105*a^2*b*d
*x^6 - 35*a^3*x^6*e + 60*a^2*b*c*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(
a^6*x^10)
```

3.302 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3}\,dx$

Optimal. Leaf size=380

$$\frac{3bc-ad}{8a^4x^8}-\frac{c}{11a^3x^{11}}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{54a^{20/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{27a^{20/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{9\sqrt{3}a^{20/3}}+\frac{bx\left(-11a^3f+17a^2be-23ab^2d+29b^3c\right)}{18a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{2a^6x^2}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

```
[Out] -c/(11*a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d
 + a^2*e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f
)/(2*a^6*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*
(a + b*x^3)^2) + (b*(29*b^3*c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*
f)*x)/(18*a^6*(a + b*x^3)) - (b^(2/3)*(119*b^3*c - 77*a*b^2*d + 4
4*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^2*d +
 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) -
 (b^(2/3)*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Rubi [A] time = 1.39485, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3bc-ad}{8a^4x^8}-\frac{c}{11a^3x^{11}}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{54a^{20/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{27a^{20/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{9\sqrt{3}a^{20/3}}+\frac{bx\left(-11a^3f+17a^2be-23ab^2d+29b^3c\right)}{18a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{2a^6x^2}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```


```
[Out] -c/(11*a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d
 + a^2*e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f
)/(2*a^6*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*
(a + b*x^3)^2) + (b*(29*b^3*c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*
f)*x)/(18*a^6*(a + b*x^3)) - (b^(2/3)*(119*b^3*c - 77*a*b^2*d + 4
4*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^2*d +
 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) -
 (b^(2/3)*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^
```

```
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.869465, size = 376, normalized size = 0.99

$$\frac{3bc-ad}{8a^4x^8}-\frac{c}{11a^3x^{11}}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(20a^3f-44a^2be+77ab^2d-119b^3c\right)}{54a^{20/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{27a^{20/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(20a^3f-44a^2be+77ab^2d-119b^3c\right)}{9\sqrt{3}a^{20/3}}+\frac{bx\left(-11a^3f+17a^2be-23ab^2d+29b^3c\right)}{18a^6\left(a+bx^3\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{2a^6x^2}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

```
[Out] -c/(11*a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d
 + a^2*e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f
)/(2*a^6*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*
(a + b*x^3)^2) + (b*(29*b^3*c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*
f)*x)/(18*a^6*(a + b*x^3)) + (b^(2/3)*(-119*b^3*c + 77*a*b^2*d -
44*a^2*b*e + 20*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^2*d + 44*a
^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) + (b^(
2/3)*(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Maple [A] time = 0.026, size = 651, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x)
```

```
[Out] 17/18*b^3/a^4/(b*x^3+a)^2*x^4*e+44/27*b/a^4*e/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-77/27*b^2/a^5*d/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+119/27*b^3/a^
```

```
6*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1
/5/a^3/x^5*e-1/2/a^3/x^2*f-1/8/a^3/x^8*d-1/11*c/a^3/x^11-20/27/a^
3*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+10/27/a^3*f/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))+3/8/a^4/x^8*b*c+3/5/a^4/x^5*b*d-6/5/a^
5/x^5*b^2*c+3/2/a^4/x^2*b*e-3/a^5/x^2*b^2*d+5/a^6/x^2*b^3*c-23/18
*b^4/a^5/(b*x^3+a)^2*x^4*d-22/27*b/a^4*e/(a/b)^(2/3)*ln(x^2-x*(a/
b)^(1/3)+(a/b)^(2/3))-77/27*b^2/a^5*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))+77/54*b^2/a^5*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-
11/18*b^2/a^3/(b*x^3+a)^2*x^4*f+29/18*b^5/a^6/(b*x^3+a)^2*x^4*c-7
/9*b/a^2/(b*x^3+a)^2*f*x+10/9*b^2/a^3/(b*x^3+a)^2*e*x-13/9*b^3/a^
4/(b*x^3+a)^2*d*x+16/9*b^4/a^5/(b*x^3+a)^2*c*x+119/27*b^3/a^6*c/(
a/b)^(2/3)*ln(x+(a/b)^(1/3))-119/54*b^3/a^6*c/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))-20/27/a^3*f/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+44/27*b/a^4*e/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^12),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221767, size = 911, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^12),x, algorithm="fricas")
```

```
[Out] 1/35640*sqrt(3)*(220*sqrt(3)*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^
3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3
*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^
4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^
2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 440*sqrt(3)*((119*b^5*c -
77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c -
 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c
- 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*lo
g(b*x - a*(-b^2/a^2)^(1/3)) + 1320*((119*b^5*c - 77*a*b^4*d + 44*
a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d +
44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d +
 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt
(3)*b*x + sqrt(3)*a*(-b^2/a^2)^(1/3))/(a*(-b^2/a^2)^(1/3))) + 3*s
qrt(3)*(220*(119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f
)*x^15 + 352*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*
b*f)*x^12 + 99*(119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^
5*f)*x^9 - 18*(119*a^3*b^2*c - 77*a^4*b*d + 44*a^5*e)*x^6 - 360*a
^5*c + 45*(17*a^4*b*c - 11*a^5*d)*x^3))/(a^6*b^2*x^17 + 2*a^7*b*x
^14 + a^8*x^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.21837, size = 594, normalized size = 1.56

$$\frac{\sqrt{3}\left(119\left(-ab^2\right)^{\frac{1}{3}}b^3c-77\left(-ab^2\right)^{\frac{1}{3}}ab^2d-20\left(-ab^2\right)^{\frac{1}{3}}a^3f+44\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^7}-\frac{\left(119b^4c-77ab^3d-20a^3bf+44a^2b^2e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^7}+\frac{\left(119\left(-ab^2\right)^{\frac{1}{3}}b^3c-77\left(-ab^2\right)^{\frac{1}{3}}ab^2d-20\left(-ab^2\right)^{\frac{1}{3}}a^3f+44\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^7}+\frac{29b^5cx^4-23ab^4dx^4-11a^3b^2fx^4+17a^2b^3x^4e+32ab^4cx-26a^2b^3dx-14a^4bfx+20a^3b^2xe}{18\left(bx^3+a\right)^2a^6}+\frac{2200b^3cx^9-1320ab^2dx^9-220a^3fx^9+660a^2bx^9e-528ab^2cx^6+264a^2bdx^6-88a^3x^6e+165a^2bcx^3-55a^3dx^3-40a^3c}{440a^6x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^12),x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(119*(-a*b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*
d - 20*(-a*b^2)^(1/3)*a^3*f + 44*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^7 - 1/27*(119*b^4
*c - 77*a*b^3*d - 20*a^3*b*f + 44*a^2*b^2*e)*(-a/b)^(1/3)*ln(abs(
x - (-a/b)^(1/3)))/a^7 + 1/54*(119*(-a*b^2)^(1/3)*b^3*c - 77*(-a*
b^2)^(1/3)*a*b^2*d - 20*(-a*b^2)^(1/3)*a^3*f + 44*(-a*b^2)^(1/3)*
a^2*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 + 1/18*(29*b
^5*c*x^4 - 23*a*b^4*d*x^4 - 11*a^3*b^2*f*x^4 + 17*a^2*b^3*x^4*e +
 32*a*b^4*c*x - 26*a^2*b^3*d*x - 14*a^4*b*f*x + 20*a^3*b^2*x*e)/(
(b*x^3 + a)^2*a^6) + 1/440*(2200*b^3*c*x^9 - 1320*a*b^2*d*x^9 - 2
20*a^3*f*x^9 + 660*a^2*b*x^9*e - 528*a*b^2*c*x^6 + 264*a^2*b*d*x^
6 - 88*a^3*x^6*e + 165*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^
6*x^11)
```

3.303 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3}\,dx$

Optimal. Leaf size=424

$$\frac{3bc-ad}{10a^4x^{10}}-\frac{c}{13a^3x^{13}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{54a^{22/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{27a^{22/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{9\sqrt{3}a^{22/3}}-\frac{b^2x^2\left(-8a^3f+11a^2be-14ab^2d+17b^3c\right)}{9a^7\left(a+bx^3\right)}-\frac{b\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{4a^6x^4}-\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

```
[Out] -c/(13*a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b
*d + a^2*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3
*f)/(4*a^6*x^4) - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f
))/(a^7*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^6
*(a + b*x^3)^2) - (b^2*(17*b^3*c - 14*a*b^2*d + 11*a^2*b*e - 8*a^
3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d
 + 65*a^2*b*e - 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^
2*d + 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/
3)) - (b^(4/3)*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Rubi [A] time = 1.70244, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3bc-ad}{10a^4x^{10}}-\frac{c}{13a^3x^{13}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{54a^{22/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{27a^{22/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{9\sqrt{3}a^{22/3}}-\frac{b^2x^2\left(-8a^3f+11a^2be-14ab^2d+17b^3c\right)}{9a^7\left(a+bx^3\right)}-\frac{b\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{4a^6x^4}-\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```


```
[Out] -c/(13*a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b
*d + a^2*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3
*f)/(4*a^6*x^4) - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f
))/(a^7*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^6
*(a + b*x^3)^2) - (b^2*(17*b^3*c - 14*a*b^2*d + 11*a^2*b*e - 8*a^
3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d
 + 65*a^2*b*e - 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^
```

```
2*d + 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/
3)) - (b^(4/3)*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.08831, size = 419, normalized size = 0.99

$$\frac{3bc-ad}{10a^4x^{10}}-\frac{c}{13a^3x^{13}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}+\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^3f-65a^2be+104ab^2d-152b^3c\right)}{54a^{22/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{27a^{22/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{9\sqrt{3}a^{22/3}}+\frac{b^2x^2\left(8a^3f-11a^2be+14ab^2d-17b^3c\right)}{9a^7\left(a+bx^3\right)}+\frac{b\left(3a^3f-6a^2be+10ab^2d-15b^3c\right)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{4a^6x^4}+\frac{b^2x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

```
[Out] -c/(13*a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b
*d + a^2*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3
*f)/(4*a^6*x^4) + (b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*
f))/(a^7*x) + (b^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6
*a^6*(a + b*x^3)^2) + (b^2*(-17*b^3*c + 14*a*b^2*d - 11*a^2*b*e +
 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152*b^3*c - 104*a*
b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2
*d + 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3
)) + (b^(4/3)*(-152*b^3*c + 104*a*b^2*d - 65*a^2*b*e + 35*a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Maple [A] time = 0.03, size = 716, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x)
```

```
[Out] 35/27*b/a^4*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))-65/27*b^2/a^5*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))+104/27*b^3/a^6*d*3^(1/2)/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-152/27*b^4/a^7*c*3^(1/2)/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-104/27*b^3/a^6*d/(a/b
)^(1/3)*ln(x+(a/b)^(1/3))+14/9*b^5/a^6/(b*x^3+a)^2*x^5*d+31/18*b^
4/a^5/(b*x^3+a)^2*x^2*d-37/18*b^5/a^6/(b*x^3+a)^2*x^2*c-35/27*b/a
^4*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+52/27*b^3/a^6*d/(a/b)^(1/3)*ln
(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+152/27*b^4/a^7*c/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))-76/27*b^4/a^7*c/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a
/b)^(2/3))+19/18*b^2/a^3/(b*x^3+a)^2*x^2*f-25/18*b^3/a^4/(b*x^3+a
)^2*x^2*e-17/9*b^6/a^7/(b*x^3+a)^2*x^5*c+3*b/a^4/x*f-6*b^2/a^5/x*
e+10*b^3/a^6/x*d-15*b^4/a^7/x*c+3/10/a^4/x^10*b*c+3/7/a^4/x^7*b*d
-6/7/a^5/x^7*b^2*c+3/4/a^4/x^4*b*e+35/54*b/a^4*f/(a/b)^(1/3)*ln(x
^2-x*(a/b)^(1/3)+(a/b)^(2/3))+65/27*b^2/a^5*e/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))-65/54*b^2/a^5*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)
^(2/3))-3/2/a^5/x^4*b^2*d+5/2/a^6/x^4*b^3*c-1/13*c/a^3/x^13-1/10/
a^3/x^10*d-1/7/a^3/x^7*e-1/4/a^3/x^4*f+8/9*b^3/a^4/(b*x^3+a)^2*x^
5*f-11/9*b^4/a^5/(b*x^3+a)^2*x^5*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^14),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243658, size = 963, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^14),x, algorithm="fricas")
```

```
[Out] 1/147420*sqrt(3)*(910*sqrt(3)*((152*b^6*c - 104*a*b^5*d + 65*a^2*
b^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*
a^3*b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d +
 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-
b/a)^(2/3) - a*(-b/a)^(1/3)) - 1820*sqrt(3)*((152*b^6*c - 104*a*b
^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a
^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c - 1
04*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*log(
b*x + a*(-b/a)^(2/3)) - 5460*((152*b^6*c - 104*a*b^5*d + 65*a^2*b
^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a
^3*b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d +
65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*arctan(-1/3*(2*sqrt
(3)*b*x - sqrt(3)*a*(-b/a)^(2/3))/(a*(-b/a)^(2/3))) - 3*sqrt(3)*(
1820*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^18
 + 3185*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*
f)*x^15 + 1170*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35
*a^5*b*f)*x^12 - 117*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e
- 35*a^6*f)*x^9 + 1260*a^6*c + 36*(152*a^4*b^2*c - 104*a^5*b*d +
65*a^6*e)*x^6 - 126*(19*a^5*b*c - 13*a^6*d)*x^3))/(a^7*b^2*x^19 +
 2*a^8*b*x^16 + a^9*x^13)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.221781, size = 717, normalized size = 1.69

$$\frac{\sqrt{3}\left(152\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-104\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-35\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+65\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^{8}}+\frac{\left(152b^{5}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-104ab^{4}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-35a^{3}b^{2}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+65a^{2}b^{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^{8}}-\frac{\left(152\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-104\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d-35\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f+65\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^{8}}-\frac{34b^{6}cx^{5}-28ab^{5}dx^{5}-16a^{3}b^{3}fx^{5}+22a^{2}b^{4}x^{5}e+37ab^{5}cx^{2}-31a^{2}b^{4}dx^{2}-19a^{4}b^{2}fx^{2}+25a^{3}b^{3}x^{2}e}{18\left(bx^{3}+a\right)^{2}a^{7}}-\frac{27300b^{4}cx^{12}-18200ab^{3}dx^{12}-5460a^{3}bfx^{12}+10920a^{2}b^{2}x^{12}e-4550ab^{3}cx^{9}+2730a^{2}b^{2}dx^{9}+455a^{4}fx^{9}-1365a^{3}bx^{9}}{1820a^{7}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^6 + d*x^3 + c)/((b*x^3 + a)^3*x^14),x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(152*(-a*b^2)^(2/3)*b^3*c - 104*(-a*b^2)^(2/3)*a*b^2
*d - 35*(-a*b^2)^(2/3)*a^3*f + 65*(-a*b^2)^(2/3)*a^2*b*e)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^8 + 1/27*(152*b^
5*c*(-a/b)^(1/3) - 104*a*b^4*d*(-a/b)^(1/3) - 35*a^3*b^2*f*(-a/b)
^(1/3) + 65*a^2*b^3*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b
)^(1/3)))/a^8 - 1/54*(152*(-a*b^2)^(2/3)*b^3*c - 104*(-a*b^2)^(2/
3)*a*b^2*d - 35*(-a*b^2)^(2/3)*a^3*f + 65*(-a*b^2)^(2/3)*a^2*b*e)
*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^8 - 1/18*(34*b^6*c*x^5
 - 28*a*b^5*d*x^5 - 16*a^3*b^3*f*x^5 + 22*a^2*b^4*x^5*e + 37*a*b^
5*c*x^2 - 31*a^2*b^4*d*x^2 - 19*a^4*b^2*f*x^2 + 25*a^3*b^3*x^2*e)
/((b*x^3 + a)^2*a^7) - 1/1820*(27300*b^4*c*x^12 - 18200*a*b^3*d*x
^12 - 5460*a^3*b*f*x^12 + 10920*a^2*b^2*x^12*e - 4550*a*b^3*c*x^9
 + 2730*a^2*b^2*d*x^9 + 455*a^4*f*x^9 - 1365*a^3*b*x^9*e + 1560*a
^2*b^2*c*x^6 - 780*a^3*b*d*x^6 + 260*a^4*x^6*e - 546*a^3*b*c*x^3
+ 182*a^4*d*x^3 + 140*a^4*c)/(a^7*x^13)
```

3.304 $\int \frac{(1-x)x^4}{1+x^3}\,dx$

Optimal. Leaf size=54

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x^2/2 - x^3/3 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x]
)/3 + Log[1 - x + x^2]/6
```

Rubi [A] time = 0.136182, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 - x^3/3 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x]
)/3 + Log[1 - x + x^2]/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x^3}{3}+\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1-x)*x**4/(x**3+1),x)
```

```
[Out] -x**3/3 + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(sqr
t(3)*(2*x/3 - 1/3))/3 + Integral(x, x)
```

Mathematica [A] time = 0.0256297, size = 59, normalized size = 1.09

$$\frac{1}{6}\left(-2x^3+2\log\left(x^3+1\right)+3x^2-\log\left(x^2-x+1\right)+2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] (3*x^2 - 2*x^3 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 +
 x] - Log[1 - x + x^2] + 2*Log[1 + x^3])/6
```

Maple [A] time = 0.009, size = 45, normalized size = 0.8

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((1-x)*x^4/(x^3+1),x)
```

```
[Out] -1/3*x^3+1/2*x^2+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3
^(1/2))+2/3*ln(1+x)
```

Maxima [A] time = 1.52153, size = 59, normalized size = 1.09

$$-\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x - 1)*x^4/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) +
1/6*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Fricas [A] time = 0.215939, size = 77, normalized size = 1.43

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\left(2x^3-3x^2\right)-\sqrt{3}\log\left(x^2-x+1\right)-4\sqrt{3}\log\left(x+1\right)+6\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x - 1)*x^4/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*(2*x^3 - 3*x^2) - sqrt(3)*log(x^2 - x + 1)
 - 4*sqrt(3)*log(x + 1) + 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.158589, size = 53, normalized size = 0.98

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((1-x)*x**4/(x**3+1),x)
```

```
[Out] -x**3/3 + x**2/2 + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)
*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.212363, size = 61, normalized size = 1.13

$$-\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\ln\left(x^2-x+1\right)+\frac{2}{3}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x - 1)*x^4/(x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) +
1/6*ln(x^2 - x + 1) + 2/3*ln(abs(x + 1))
```

3.305 $\int \frac{(1-x)x^3}{1+x^3}\,dx$

Optimal. Leaf size=30

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

```
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rubi [A] time = 0.0687791, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[((1 - x)*x^3)/(1 + x^3),x]

[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x-\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{3}-\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)*x**3/(x**3+1),x)

[Out] x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - Integral(x, x)
```

Mathematica [A] time = 0.00712826, size = 30, normalized size = 1.

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 - x)*x^3)/(1 + x^3),x]

[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Maple [A] time = 0.007, size = 25, normalized size = 0.8

$$x-\frac{x^2}{2}-\frac{2\ln\left(1+x\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)*x^3/(x^3+1),x)

[Out] x-1/2*x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Maxima [A] time = 1.52492, size = 32, normalized size = 1.07

$$-\frac{1}{2}x^2 + x + \frac{1}{3}\log\left(x^2 - x + 1\right) - \frac{2}{3}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)*x^3/(x^3 + 1),x, algorithm="maxima")

[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Fricas [A] time = 0.208356, size = 32, normalized size = 1.07

$$-\frac{1}{2}x^2 + x + \frac{1}{3}\log\left(x^2 - x + 1\right) - \frac{2}{3}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)*x^3/(x^3 + 1),x, algorithm="fricas")

[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Sympy [A] time = 0.094271, size = 24, normalized size = 0.8

$$-\frac{x^2}{2} + x - \frac{2\log\left(x + 1\right)}{3} + \frac{\log\left(x^2 - x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)*x**3/(x**3+1),x)

[Out] -x**2/2 + x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3
```

GIAC/XCAS [A] time = 0.210913, size = 34, normalized size = 1.13

$$-\frac{1}{2}x^2 + x + \frac{1}{3}\ln\left(x^2 - x + 1\right) - \frac{2}{3}\ln\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)*x^3/(x^3 + 1),x, algorithm="giac")

[Out] -1/2*x^2 + x + 1/3*ln(x^2 - x + 1) - 2/3*ln(abs(x + 1))
```

3.306 $\int \frac{(1-x)x^2}{1+x^3}\,dx$

Optimal. Leaf size=44

$$\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -x - ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[1
 - x + x^2]/6
```

Rubi [A] time = 0.112368, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x - ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[1
 - x + x^2]/6
```

Rubi in Sympy [A] time = 17.7968, size = 41, normalized size = 0.93

$$-x+\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)*x**2/(x**3+1),x)
```

```
[Out] -x + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(sqrt(3)*
(2*x/3 - 1/3))/3
```

Mathematica [A] time = 0.015404, size = 53, normalized size = 1.2

$$\frac{1}{3}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x + ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x]/3 - Log[1 -
x + x^2]/6 + Log[1 + x^3]/3
```

Maple [A] time = 0.008, size = 38, normalized size = 0.9

$$-x+\frac{\ln\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)*x^2/(x^3+1),x)
```

```
[Out] -x+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+2/3*ln
(1+x)
```

Maxima [A] time = 1.53376, size = 50, normalized size = 1.14

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)*x^2/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x +
 1) + 2/3*log(x + 1)
```

Fricas [A] time = 0.21263, size = 65, normalized size = 1.48

$$-\frac{1}{18}\sqrt{3}\left(6\sqrt{3}x-\sqrt{3}\log\left(x^2-x+1\right)-4\sqrt{3}\log(x+1)-6\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)*x^2/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(6*sqrt(3)*x - sqrt(3)*log(x^2 - x + 1) - 4*sqrt(3)
*log(x + 1) - 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.172118, size = 44, normalized size = 1.

$$-x+\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)*x**2/(x**3+1),x)
```

```
[Out] -x + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3
)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.209723, size = 51, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x+\frac{1}{6}\ln\left(x^2-x+1\right)+\frac{2}{3}\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)*x^2/(x^3 + 1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*ln(x^2 - x +
1) + 2/3*ln(abs(x + 1))
```

3.307 $\int \frac{(1-x)x}{1+x^3} dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - (2*Log[1 + x])/3 - Log[1 -
 x + x^2]/6
```

Rubi [A] time = 0.0826407, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 - x)*x)/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - (2*Log[1 + x])/3 - Log[1 -
 x + x^2]/6
```

Rubi in Sympy [A] time = 12.45, size = 39, normalized size = 0.95

$$-\frac{2\log(x+1)}{3}-\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)*x/(x**3+1),x)
```

```
[Out] -2*log(x + 1)/3 - log(x**2 - x + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x
/3 - 1/3))/3
```

Mathematica [A] time = 0.0132038, size = 50, normalized size = 1.22

$$-\frac{1}{3}\log\left(x^3+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 - x)*x)/(1 + x^3),x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x
^2]/6 - Log[1 + x^3]/3
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$-\frac{\ln\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((1-x)*x/(x^3+1),x)
```

```
[Out] -1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-2/3*ln(1
+x)
```

Maxima [A] time = 1.51995, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x - 1)*x/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1)
- 2/3*log(x + 1)
```

Fricas [A] time = 0.214494, size = 55, normalized size = 1.34

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^2-x+1\right)+4\sqrt{3}\log\left(x+1\right)-6\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x - 1)*x/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(x^2 - x + 1) + 4*sqrt(3)*log(x + 1) -
6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.166668, size = 42, normalized size = 1.02

$$-\frac{2\log\left(x+1\right)}{3}-\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((1-x)*x/(x**3+1),x)
```

```
[Out] -2*log(x + 1)/3 - log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/
3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.21205, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\ln\left(x^2-x+1\right)-\frac{2}{3}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x - 1)*x/(x^3 + 1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*ln(x^2 - x + 1) -
 2/3*ln(abs(x + 1))
```

3.308 $\int \frac{1-x}{x(1+x^3)}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - L
og[1 - x + x^2]/6
```

Rubi [A] time = 0.100178, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - L
og[1 - x + x^2]/6
```

Rubi in Sympy [A] time = 14.5417, size = 42, normalized size = 1.

$$\log(x)-\frac{2\log(x+1)}{3}-\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)/x/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1)/3 - log(x**2 - x + 1)/6 - sqrt(3)*atan(sqrt
(3)*(2*x/3 - 1/3))/3
```

Mathematica [A] time = 0.0140908, size = 53, normalized size = 1.26

$$-\frac{1}{3}\log\left(x^3+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x]/3 + L
og[1 - x + x^2]/6 - Log[1 + x^3]/3
```

Maple [A] time = 0.01, size = 37, normalized size = 0.9

$$\ln(x)-\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((1-x)/x/(x^3+1),x)
```

```
[Out] ln(x)-1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-2/3
*ln(1+x)
```

Maxima [A] time = 1.51761, size = 49, normalized size = 1.17

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x - 1)/((x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1)
 - 2/3*log(x + 1) + log(x)
```

Fricas [A] time = 0.215446, size = 65, normalized size = 1.55

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^2-x+1\right)+4\sqrt{3}\log\left(x+1\right)-6\sqrt{3}\log\left(x\right)+6\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x - 1)/((x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(x^2 - x + 1) + 4*sqrt(3)*log(x + 1) -
6*sqrt(3)*log(x) + 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.270013, size = 46, normalized size = 1.1

$$\log\left(x\right)-\frac{2\log\left(x+1\right)}{3}-\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((1-x)/x/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1)/3 - log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sq
rt(3)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.212307, size = 51, normalized size = 1.21

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\ln\left(x^2-x+1\right)-\frac{2}{3}\ln\left(|x+1|\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x - 1)/((x^3 + 1)*x),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*ln(x^2 - x + 1)
- 2/3*ln(abs(x + 1)) + ln(abs(x))
```

3.309 $\int \frac{1-x}{x^2(1+x^3)}\,dx$

Optimal. Leaf size=49

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + (2*Log[1 +
 x])/3 + Log[1 - x + x^2]/6
```

Rubi [A] time = 0.102185, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x)/(x^2*(1 + x^3)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + (2*Log[1 +
 x])/3 + Log[1 - x + x^2]/6
```

Rubi in Sympy [A] time = 14.7888, size = 46, normalized size = 0.94

$$-\log(x)+\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)/x**2/(x**3+1),x)
```

```
[Out] -log(x) + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(sqr
t(3)*(2*x/3 - 1/3))/3 - 1/x
```

Mathematica [A] time = 0.0284103, size = 60, normalized size = 1.22

$$\frac{1}{3}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x)/(x^2*(1 + x^3)),x]
```

```
[Out] -x^(-1) - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + Log[1 + x
]/3 - Log[1 - x + x^2]/6 + Log[1 + x^3]/3
```

Maple [A] time = 0.011, size = 44, normalized size = 0.9

$$-x^{-1}-\ln(x)+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)/x^2/(x^3+1),x)
```

```
[Out] -1/x-ln(x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2)
)+2/3*ln(1+x)
```

Maxima [A] time = 1.5259, size = 58, normalized size = 1.18

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/((x^3 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 -
x + 1) + 2/3*log(x + 1) - log(x)
```

Fricas [A] time = 0.217175, size = 81, normalized size = 1.65

$$\frac{\sqrt{3}\left(\sqrt{3}x\log\left(x^2-x+1\right)+4\sqrt{3}x\log\left(x+1\right)-6\sqrt{3}x\log\left(x\right)-6x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6\sqrt{3}\right)}{18x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/((x^3 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*x*log(x^2 - x + 1) + 4*sqrt(3)*x*log(x + 1)
 - 6*sqrt(3)*x*log(x) - 6*x*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*sqr
t(3))/x
```

Sympy [A] time = 0.279235, size = 49, normalized size = 1.

$$-\log\left(x\right)+\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)/x**2/(x**3+1),x)
```

```
[Out] -log(x) + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*s
qrt(3)*x/3 - sqrt(3)/3)/3 - 1/x
```

GIAC/XCAS [A] time = 0.210805, size = 61, normalized size = 1.24

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}+\frac{1}{6}\ln\left(x^2-x+1\right)+\frac{2}{3}\ln\left(|x+1|\right)-\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/((x^3 + 1)*x^2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*ln(x^2 - x
 + 1) + 2/3*ln(abs(x + 1)) - ln(abs(x))
```

3.310 $\int \frac{1-x}{x^3(1+x^3)}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

```
[Out] -1/(2*x^2) + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rubi [A] time = 0.0623125, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x)/(x^3*(1 + x^3)),x]

[Out] -1/(2*x^2) + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rubi in Sympy [A] time = 12.3339, size = 27, normalized size = 0.84

$$-\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{3}+\frac{1}{x}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)/x**3/(x**3+1),x)

[Out] -2*log(x + 1)/3 + log(x**2 - x + 1)/3 + 1/x - 1/(2*x**2)
```

Mathematica [A] time = 0.00754776, size = 32, normalized size = 1.

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x)/(x^3*(1 + x^3)),x]

[Out] -1/(2*x^2) + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Maple [A] time = 0.011, size = 27, normalized size = 0.8

$$-\frac{1}{2\,x^2}+x^{-1}-\frac{2\,\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)/x^3/(x^3+1),x)

[Out] -1/2/x^2+1/x-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Maxima [A] time = 1.52555, size = 38, normalized size = 1.19

$$\frac{2x-1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/((x^3 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Fricas [A] time = 0.22255, size = 45, normalized size = 1.41

$$\frac{2x^2\log\left(x^2-x+1\right)-4x^2\log\left(x+1\right)+6x-3}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/((x^3 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2*log(x^2 - x + 1) - 4*x^2*log(x + 1) + 6*x - 3)/x^2
```

Sympy [A] time = 0.131823, size = 27, normalized size = 0.84

$$-\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{3}+\frac{2x-1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)/x**3/(x**3+1),x)
```

```
[Out] -2*log(x + 1)/3 + log(x**2 - x + 1)/3 + (2*x - 1)/(2*x**2)
```

GIAC/XCAS [A] time = 0.21133, size = 39, normalized size = 1.22

$$\frac{2x-1}{2x^2}+\frac{1}{3}\ln\left(x^2-x+1\right)-\frac{2}{3}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/((x^3 + 1)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*x - 1)/x^2 + 1/3*ln(x^2 - x + 1) - 2/3*ln(abs(x + 1))
```

3.311 $\int \frac{x(1+2x)}{1+x^3} \, dx$

Optimal. Leaf size=41

$$\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x]/3 + (5*Log[1 -
x + x^2])/6
```

Rubi [A] time = 0.0814405, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(1 + 2*x))/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x]/3 + (5*Log[1 -
x + x^2])/6
```

Rubi in Sympy [A] time = 11.5199, size = 39, normalized size = 0.95

$$\frac{\log(x+1)}{3}+\frac{5\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+2*x)/(x**3+1),x)
```

```
[Out] log(x + 1)/3 + 5*log(x**2 - x + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x/
3 - 1/3))/3
```

Mathematica [A] time = 0.0139612, size = 47, normalized size = 1.15

$$\frac{1}{6}\left(4\log\left(x^3+1\right)+\log\left(x^2-x+1\right)-2\log(x+1)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(1 + 2*x))/(1 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + Log[1 - x
+ x^2] + 4*Log[1 + x^3])/6
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$\frac{5\ln\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(1+2*x)/(x^3+1),x)
```

```
[Out] 5/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/3*ln(1+
x)
```

Maxima [A] time = 1.58207, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x + 1)*x/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1)
+ 1/3*log(x + 1)
```

Fricas [A] time = 0.23471, size = 57, normalized size = 1.39

$$\frac{1}{18}\sqrt{3}\left(5\sqrt{3}\log\left(x^2-x+1\right)+2\sqrt{3}\log\left(x+1\right)+6\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x + 1)*x/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(5*sqrt(3)*log(x^2 - x + 1) + 2*sqrt(3)*log(x + 1) +
 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.166632, size = 42, normalized size = 1.02

$$\frac{\log\left(x+1\right)}{3}+\frac{5\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(1+2*x)/(x**3+1),x)
```

```
[Out] log(x + 1)/3 + 5*log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3
 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.211482, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5}{6}\ln\left(x^2-x+1\right)+\frac{1}{3}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x + 1)*x/(x^3 + 1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*ln(x^2 - x + 1) +
 1/3*ln(abs(x + 1))
```

3.312 $\int \frac{x(1+2x)}{1-x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x] - Log[1 + x + x
^2]/2
```

Rubi [A] time = 0.0832375, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(1 + 2*x))/(1 - x^3),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x] - Log[1 + x + x
^2]/2
```

Rubi in Sympy [A] time = 11.1168, size = 37, normalized size = 0.95

$$-\log(-x+1)-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+2*x)/(-x**3+1),x)
```

```
[Out] -log(-x + 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(sqrt(3)*(2*x/3
+ 1/3))/3
```

Mathematica [A] time = 0.0206975, size = 53, normalized size = 1.36

$$-\frac{2}{3}\log\left(1-x^3\right)+\frac{1}{6}\log\left(x^2+x+1\right)-\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(1 + 2*x))/(1 - x^3),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x]/3 + Log[1 + x +
 x^2]/6 - (2*Log[1 - x^3])/3
```

Maple [A] time = 0.008, size = 33, normalized size = 0.9

$$-\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+2*x)/(-x^3+1),x)
```

```
[Out] -1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-ln(-1+x)
```

Maxima [A] time = 1.52304, size = 43, normalized size = 1.1

$$-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{2}\,\log\left(x^2+x+1\right)-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x + 1)*x/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(x - 1)
```

Fricas [A] time = 0.238685, size = 53, normalized size = 1.36

$$-\frac{1}{6}\,\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)+2\,\sqrt{3}\log\left(x-1\right)+2\,\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x + 1)*x/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) + 2*sqrt(3)*log(x - 1) + 2
*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.155739, size = 41, normalized size = 1.05

$$-\log\left(x-1\right)-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+2*x)/(-x**3+1),x)
```

```
[Out] -log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.211462, size = 45, normalized size = 1.15

$$-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{2}\,\ln\left(x^2+x+1\right)-\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x + 1)*x/(x^3 - 1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*ln(x^2 + x + 1)
- ln(abs(x - 1))
```

3.313 $\int (c + dx + ex^2) (a + bx^3)\, dx$

Optimal. Leaf size=50

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}bcx^4 + \frac{1}{5}bdx^5 + \frac{1}{6}bex^6$$

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (
b*e*x^6)/6
```

Rubi [A] time = 0.0585857, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}bcx^4 + \frac{1}{5}bdx^5 + \frac{1}{6}bex^6$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (
b*e*x^6)/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ad \int x\, dx + \frac{aex^3}{3} + \frac{bcx^4}{4} + \frac{bdx^5}{5} + \frac{bex^6}{6} + c \int a\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*d*Integral(x, x) + a*e*x**3/3 + b*c*x**4/4 + b*d*x**5/5 + b*e*x
**6/6 + c*Integral(a, x)
```

Mathematica [A] time = 0.00581697, size = 50, normalized size = 1.

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}bcx^4 + \frac{1}{5}bdx^5 + \frac{1}{6}bex^6$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (
b*e*x^6)/6
```

Maple [A] time = 0.002, size = 41, normalized size = 0.8

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{bcx^4}{4} + \frac{bdx^5}{5} + \frac{bex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)*(b*x^3+a),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6
```

Maxima [A] time = 1.37736, size = 54, normalized size = 1.08

$$\frac{1}{6}\,bex^6 + \frac{1}{5}\,bdx^5 + \frac{1}{4}\,bcx^4 + \frac{1}{3}\,aex^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x
^2 + a*c*x
```

Fricas [A] time = 0.203941, size = 1, normalized size = 0.02

$$\frac{1}{6}x^6eb + \frac{1}{5}x^5db + \frac{1}{4}x^4cb + \frac{1}{3}x^3ea + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*b + 1/5*x^5*d*b + 1/4*x^4*c*b + 1/3*x^3*e*a + 1/2*x^2*d
*a + x*c*a
```

Sympy [A] time = 0.047076, size = 46, normalized size = 0.92

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{bcx^4}{4} + \frac{bdx^5}{5} + \frac{bex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*c*x**4/4 + b*d*x**5/5 + b*e*x
**6/6
```

GIAC/XCAS [A] time = 0.206528, size = 57, normalized size = 1.14

$$\frac{1}{6}\,bx^6e + \frac{1}{5}\,bdx^5 + \frac{1}{4}\,bcx^4 + \frac{1}{3}\,ax^3e + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6*e + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*x^3*e + 1/2*a*d*x
^2 + a*c*x
```

3.314 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)
/6 + (b*e*x^7)/7
```

Rubi [A] time = 0.0767854, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

Antiderivative was successfully verified.

```
[In]  Int[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)
/6 + (b*e*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ac\int x\,dx+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*Integral(x, x) + a*d*x**3/3 + a*e*x**4/4 + b*c*x**5/5 + b*d*x
**6/6 + b*e*x**7/7
```

Mathematica [A] time = 0.00397227, size = 55, normalized size = 1.

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)
/6 + (b*e*x^7)/7
```

Maple [A] time = 0.001, size = 44, normalized size = 0.8

$$\frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)*(b*x^3+a),x)
```

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b
*e*x^7
```

Maxima [A] time = 1.39773, size = 58, normalized size = 1.05

$$\frac{1}{7}\,bex^7 + \frac{1}{6}\,bdx^6 + \frac{1}{5}\,bcx^5 + \frac{1}{4}\,aex^4 + \frac{1}{3}\,adx^3 + \frac{1}{2}\,acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(e*x^2 + d*x + c)*x,x, algorithm="maxima")
```

```
[Out] 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x
^3 + 1/2*a*c*x^2
```

Fricas [A] time = 0.200415, size = 1, normalized size = 0.02

$$\frac{1}{7}x^7eb + \frac{1}{6}x^6db + \frac{1}{5}x^5cb + \frac{1}{4}x^4ea + \frac{1}{3}x^3da + \frac{1}{2}x^2ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(e*x^2 + d*x + c)*x,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e*b + 1/6*x^6*d*b + 1/5*x^5*c*b + 1/4*x^4*e*a + 1/3*x^3*d
*a + 1/2*x^2*c*a
```

Sympy [A] time = 0.048977, size = 49, normalized size = 0.89

$$\frac{acx^2}{2} + \frac{adx^3}{3} + \frac{aex^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*c*x**5/5 + b*d*x**6/6 +
b*e*x**7/7
```

GIAC/XCAS [A] time = 0.208031, size = 61, normalized size = 1.11

$$\frac{1}{7}\,bx^7e + \frac{1}{6}\,bdx^6 + \frac{1}{5}\,bcx^5 + \frac{1}{4}\,ax^4e + \frac{1}{3}\,adx^3 + \frac{1}{2}\,acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(e*x^2 + d*x + c)*x,x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*x^4*e + 1/3*a*d*x
^3 + 1/2*a*c*x^2
```

3.315 $\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)
/7 + (b*e*x^8)/8
```

Rubi [A] time = 0.101293, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)
/7 + (b*e*x^8)/8
```

Rubi in Sympy [A] time = 13.1799, size = 49, normalized size = 0.89

$$\frac{acx^3}{3}+\frac{adx^4}{4}+\frac{aex^5}{5}+\frac{bcx^6}{6}+\frac{bdx^7}{7}+\frac{bex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*c*x**6/6 + b*d*x**7/7 +
b*e*x**8/8
```

Mathematica [A] time = 0.00417322, size = 55, normalized size = 1.

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)
/7 + (b*e*x^8)/8
```

Maple [A] time = 0.002, size = 44, normalized size = 0.8

$$\frac{acx^3}{3}+\frac{adx^4}{4}+\frac{aex^5}{5}+\frac{bcx^6}{6}+\frac{bdx^7}{7}+\frac{bex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)*(b*x^3+a),x)
```

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b
*e*x^8
```

Maxima [A] time = 1.42165, size = 58, normalized size = 1.05

$$\frac{1}{8}\,bex^{8} + \frac{1}{7}\,bdx^{7} + \frac{1}{6}\,bcx^{6} + \frac{1}{5}\,aex^{5} + \frac{1}{4}\,adx^{4} + \frac{1}{3}\,acx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(e*x^2 + d*x + c)*x^2,x, algorithm="maxima")
```

```
[Out] 1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x
^4 + 1/3*a*c*x^3
```

Fricas [A] time = 0.19167, size = 1, normalized size = 0.02

$$\frac{1}{8}x^{8}eb + \frac{1}{7}x^{7}db + \frac{1}{6}x^{6}cb + \frac{1}{5}x^{5}ea + \frac{1}{4}x^{4}da + \frac{1}{3}x^{3}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(e*x^2 + d*x + c)*x^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*b + 1/7*x^7*d*b + 1/6*x^6*c*b + 1/5*x^5*e*a + 1/4*x^4*d
*a + 1/3*x^3*c*a
```

Sympy [A] time = 0.049732, size = 49, normalized size = 0.89

$$\frac{acx^{3}}{3} + \frac{adx^{4}}{4} + \frac{aex^{5}}{5} + \frac{bcx^{6}}{6} + \frac{bdx^{7}}{7} + \frac{bex^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*c*x**6/6 + b*d*x**7/7 +
b*e*x**8/8
```

GIAC/XCAS [A] time = 0.210504, size = 61, normalized size = 1.11

$$\frac{1}{8}\,bx^{8}e + \frac{1}{7}\,bdx^{7} + \frac{1}{6}\,bcx^{6} + \frac{1}{5}\,ax^{5}e + \frac{1}{4}\,adx^{4} + \frac{1}{3}\,acx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(e*x^2 + d*x + c)*x^2,x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*e + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*x^5*e + 1/4*a*d*x
^4 + 1/3*a*c*x^3
```

3.316 $\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx$

Optimal. Leaf size=77

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{e\left(a+bx^3\right)^3}{9b}+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*
c*x^7)/7 + (b^2*d*x^8)/8 + (e*(a + b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.114055, antiderivative size = 92, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{3}abex^6+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{1}{9}b^2ex^9$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a*b*c*x^4)/2 + (2*a*b*
d*x^5)/5 + (a*b*e*x^6)/3 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (b^2*e
*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2d\int x\,dx+a^2\int c\,dx+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}+\frac{e\left(a+bx^3\right)^3}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*d*Integral(x, x) + a**2*Integral(c, x) + a*b*c*x**4/2 + 2*a*
b*d*x**5/5 + b**2*c*x**7/7 + b**2*d*x**8/8 + e*(a + b*x**3)**3/(9
*b)
```

Mathematica [A] time = 0.00542147, size = 92, normalized size = 1.19

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{3}abex^6+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{1}{9}b^2ex^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a*b*c*x^4)/2 + (2*a*b*
d*x^5)/5 + (a*b*e*x^6)/3 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (b^2*e
*x^9)/9
```

Maple [A] time = 0.002, size = 77, normalized size = 1.

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{abcx^4}{2}+\frac{2\,abdx^5}{5}+\frac{abex^6}{3}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}+\frac{b^2ex^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)*(b*x^3+a)^2,x)
```

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1
/3*a*b*e*x^6+1/7*b^2*c*x^7+1/8*b^2*d*x^8+1/9*b^2*e*x^9
```

Maxima [A] time = 1.41973, size = 103, normalized size = 1.34

$$\frac{1}{9}\,b^2ex^9 + \frac{1}{8}\,b^2dx^8 + \frac{1}{7}\,b^2cx^7 + \frac{1}{3}\,abex^6 + \frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4 + \frac{1}{3}\,a^2ex^3 + \frac{1}{2}\,a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2
/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^
2*c*x
```

Fricas [A] time = 0.191922, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9eb^2 + \frac{1}{8}x^8db^2 + \frac{1}{7}x^7cb^2 + \frac{1}{3}x^6eba + \frac{2}{5}x^5dba + \frac{1}{2}x^4cba + \frac{1}{3}x^3ea^2 + \frac{1}{2}x^2da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e*b^2 + 1/8*x^8*d*b^2 + 1/7*x^7*c*b^2 + 1/3*x^6*e*b*a + 2
/5*x^5*d*b*a + 1/2*x^4*c*b*a + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*
c*a^2
```

Sympy [A] time = 0.065512, size = 88, normalized size = 1.14

$$a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{abcx^4}{2} + \frac{2abdx^5}{5} + \frac{abex^6}{3} + \frac{b^2cx^7}{7} + \frac{b^2dx^8}{8} + \frac{b^2ex^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a*b*c*x**4/2 + 2*a*b*d
*x**5/5 + a*b*e*x**6/3 + b**2*c*x**7/7 + b**2*d*x**8/8 + b**2*e*x
**9/9
```

GIAC/XCAS [A] time = 0.207988, size = 107, normalized size = 1.39

$$\frac{1}{9}\,b^2x^9e + \frac{1}{8}\,b^2dx^8 + \frac{1}{7}\,b^2cx^7 + \frac{1}{3}\,abx^6e + \frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4 + \frac{1}{3}\,a^2x^3e + \frac{1}{2}\,a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9*e + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*x^6*e + 2
/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^
2*c*x
```

3.317 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx$

Optimal. Leaf size=97

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{9}b^2dx^9+\frac{1}{10}b^2ex^{10}$$

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^8)/8 + (b^2*d*x^9)/9
+ (b^2*e*x^10)/10
```

Rubi [A] time = 0.145058, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{9}b^2dx^9+\frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^8)/8 + (b^2*d*x^9)/9
+ (b^2*e*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2c\int x\,dx+\frac{a^2ex^4}{4}+\frac{2abcx^5}{5}+\frac{2abex^7}{7}+\frac{b^2cx^8}{8}+\frac{b^2ex^{10}}{10}+\frac{d\left(a+bx^3\right)^3}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*Integral(x, x) + a**2*e*x**4/4 + 2*a*b*c*x**5/5 + 2*a*b*e*
x**7/7 + b**2*c*x**8/8 + b**2*e*x**10/10 + d*(a + b*x**3)**3/(9*b
)
```

Mathematica [A] time = 0.00573634, size = 97, normalized size = 1.

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{9}b^2dx^9+\frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^8)/8 + (b^2*d*x^9)/9
+ (b^2*e*x^10)/10
```

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{a^2cx^2}{2}+\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}+\frac{2\,abcx^5}{5}+\frac{abdx^6}{3}+\frac{2\,abex^7}{7}+\frac{b^2cx^8}{8}+\frac{b^2dx^9}{9}+\frac{b^2ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x)
```

```
[Out] 1/2*a^2*c*x^2+1/3*a^2*d*x^3+1/4*a^2*e*x^4+2/5*a*b*c*x^5+1/3*a*b*d
*x^6+2/7*a*b*e*x^7+1/8*b^2*c*x^8+1/9*b^2*d*x^9+1/10*b^2*e*x^10
```

Maxima [A] time = 1.37669, size = 107, normalized size = 1.1

$$\frac{1}{10}\,b^{2}ex^{10}+\frac{1}{9}\,b^{2}dx^{9}+\frac{1}{8}\,b^{2}cx^{8}+\frac{2}{7}\,abex^{7}+\frac{1}{3}\,abdx^{6}+\frac{2}{5}\,abcx^{5}+\frac{1}{4}\,a^{2}ex^{4}+\frac{1}{3}\,a^{2}dx^{3}+\frac{1}{2}\,a^{2}cx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(e*x^2 + d*x + c)*x,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 +
 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 +
1/2*a^2*c*x^2
```

Fricas [A] time = 0.188728, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10}eb^{2}+\frac{1}{9}x^{9}db^{2}+\frac{1}{8}x^{8}cb^{2}+\frac{2}{7}x^{7}eba+\frac{1}{3}x^{6}dba+\frac{2}{5}x^{5}cba+\frac{1}{4}x^{4}ea^{2}+\frac{1}{3}x^{3}da^{2}+\frac{1}{2}x^{2}ca^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(e*x^2 + d*x + c)*x,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*b^2 + 1/9*x^9*d*b^2 + 1/8*x^8*c*b^2 + 2/7*x^7*e*b*a +
 1/3*x^6*d*b*a + 2/5*x^5*c*b*a + 1/4*x^4*e*a^2 + 1/3*x^3*d*a^2 +
1/2*x^2*c*a^2
```

Sympy [A] time = 0.069293, size = 94, normalized size = 0.97

$$\frac{a^{2}cx^{2}}{2}+\frac{a^{2}dx^{3}}{3}+\frac{a^{2}ex^{4}}{4}+\frac{2abcx^{5}}{5}+\frac{abdx^{6}}{3}+\frac{2abex^{7}}{7}+\frac{b^{2}cx^{8}}{8}+\frac{b^{2}dx^{9}}{9}+\frac{b^{2}ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + 2*a*b*c*x**5/5 +
a*b*d*x**6/3 + 2*a*b*e*x**7/7 + b**2*c*x**8/8 + b**2*d*x**9/9 + b
**2*e*x**10/10
```

GIAC/XCAS [A] time = 0.208943, size = 111, normalized size = 1.14

$$\frac{1}{10}\,b^{2}x^{10}e+\frac{1}{9}\,b^{2}dx^{9}+\frac{1}{8}\,b^{2}cx^{8}+\frac{2}{7}\,abx^{7}e+\frac{1}{3}\,abdx^{6}+\frac{2}{5}\,abcx^{5}+\frac{1}{4}\,a^{2}x^{4}e+\frac{1}{3}\,a^{2}dx^{3}+\frac{1}{2}\,a^{2}cx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(e*x^2 + d*x + c)*x,x, algorithm="giac")
```

```
[Out] 1/10*b^2*x^10*e + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*x^7*e +
 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 +
1/2*a^2*c*x^2
```

3.318 $\int x^2 \left(c + dx + ex^2\right) \left(a + bx^3\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{1}{3}a^2cx^3 + \frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{1}{3}abcx^6 + \frac{2}{7}abdx^7 + \frac{1}{4}abex^8 + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

```
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a*b*c*x^6)/3 + (
2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10
+ (b^2*e*x^11)/11
```

Rubi [A] time = 0.161161, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{1}{3}a^2cx^3 + \frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{1}{3}abcx^6 + \frac{2}{7}abdx^7 + \frac{1}{4}abex^8 + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a*b*c*x^6)/3 + (
2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10
+ (b^2*e*x^11)/11
```

Rubi in Sympy [A] time = 35.1027, size = 75, normalized size = 0.77

$$\frac{a^2dx^4}{4} + \frac{a^2ex^5}{5} + \frac{2abdx^7}{7} + \frac{abex^8}{4} + \frac{b^2dx^{10}}{10} + \frac{b^2ex^{11}}{11} + \frac{c\left(a + bx^3\right)^3}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*d*x**4/4 + a**2*e*x**5/5 + 2*a*b*d*x**7/7 + a*b*e*x**8/4 + b
**2*d*x**10/10 + b**2*e*x**11/11 + c*(a + b*x**3)**3/(9*b)
```

Mathematica [A] time = 0.00599648, size = 97, normalized size = 1.

$$\frac{1}{3}a^2cx^3 + \frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{1}{3}abcx^6 + \frac{2}{7}abdx^7 + \frac{1}{4}abex^8 + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a*b*c*x^6)/3 + (
2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10
+ (b^2*e*x^11)/11
```

Maple [A] time = 0., size = 80, normalized size = 0.8

$$\frac{a^2cx^3}{3} + \frac{a^2dx^4}{4} + \frac{a^2ex^5}{5} + \frac{abcx^6}{3} + \frac{2\,abdx^7}{7} + \frac{abex^8}{4} + \frac{b^2cx^9}{9} + \frac{b^2dx^{10}}{10} + \frac{b^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x)
```

```
[Out] 1/3*a^2*c*x^3+1/4*a^2*d*x^4+1/5*a^2*e*x^5+1/3*a*b*c*x^6+2/7*a*b*d
*x^7+1/4*a*b*e*x^8+1/9*b^2*c*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11
```

Maxima [A] time = 1.37591, size = 107, normalized size = 1.1

$$\frac{1}{11}\,b^2ex^{11} + \frac{1}{10}\,b^2dx^{10} + \frac{1}{9}\,b^2cx^9 + \frac{1}{4}\,abex^8 + \frac{2}{7}\,abdx^7 + \frac{1}{3}\,abcx^6 + \frac{1}{5}\,a^2ex^5 + \frac{1}{4}\,a^2dx^4 + \frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(e*x^2 + d*x + c)*x^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8
 + 2/7*a*b*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4
+ 1/3*a^2*c*x^3
```

Fricas [A] time = 0.183414, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}eb^2 + \frac{1}{10}x^{10}db^2 + \frac{1}{9}x^9cb^2 + \frac{1}{4}x^8eba + \frac{2}{7}x^7dba + \frac{1}{3}x^6cba + \frac{1}{5}x^5ea^2 + \frac{1}{4}x^4da^2 + \frac{1}{3}x^3ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(e*x^2 + d*x + c)*x^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b^2 + 1/4*x^8*e*b*a
 + 2/7*x^7*d*b*a + 1/3*x^6*c*b*a + 1/5*x^5*e*a^2 + 1/4*x^4*d*a^2
+ 1/3*x^3*c*a^2
```

Sympy [A] time = 0.065967, size = 92, normalized size = 0.95

$$\frac{a^2cx^3}{3} + \frac{a^2dx^4}{4} + \frac{a^2ex^5}{5} + \frac{abcx^6}{3} + \frac{2abdx^7}{7} + \frac{abex^8}{4} + \frac{b^2cx^9}{9} + \frac{b^2dx^{10}}{10} + \frac{b^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + a*b*c*x**6/3 + 2*
a*b*d*x**7/7 + a*b*e*x**8/4 + b**2*c*x**9/9 + b**2*d*x**10/10 + b
**2*e*x**11/11
```

GIAC/XCAS [A] time = 0.209191, size = 111, normalized size = 1.14

$$\frac{1}{11}\,b^2x^{11}e + \frac{1}{10}\,b^2dx^{10} + \frac{1}{9}\,b^2cx^9 + \frac{1}{4}\,abx^8e + \frac{2}{7}\,abdx^7 + \frac{1}{3}\,abcx^6 + \frac{1}{5}\,a^2x^5e + \frac{1}{4}\,a^2dx^4 + \frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(e*x^2 + d*x + c)*x^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*x^8*e
 + 2/7*a*b*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4
+ 1/3*a^2*c*x^3
```

3.319 $\int (c + dx + ex^2) (a + bx^3)^3 \, dx$

Optimal. Leaf size=134

$$a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{1}{2}a^2bex^6 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{3}ab^2ex^9 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11} + \frac{1}{12}b^3ex^{12}$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^4)/4 + (3*
a^2*b*d*x^5)/5 + (a^2*b*e*x^6)/2 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d
*x^8)/8 + (a*b^2*e*x^9)/3 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (
b^3*e*x^12)/12
```

Rubi [A] time = 0.183741, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{1}{2}a^2bex^6 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{3}ab^2ex^9 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11} + \frac{1}{12}b^3ex^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^4)/4 + (3*
a^2*b*d*x^5)/5 + (a^2*b*e*x^6)/2 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d
*x^8)/8 + (a*b^2*e*x^9)/3 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (
b^3*e*x^12)/12
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3d\int x\,dx + a^3\int c\,dx + \frac{3a^2bcx^4}{4} + \frac{3a^2bdx^5}{5} + \frac{3ab^2cx^7}{7} + \frac{3ab^2dx^8}{8} + \frac{b^3cx^{10}}{10} + \frac{b^3dx^{11}}{11} + \frac{e(a + bx^3)^4}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*d*Integral(x, x) + a**3*Integral(c, x) + 3*a**2*b*c*x**4/4 +
 3*a**2*b*d*x**5/5 + 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3
*c*x**10/10 + b**3*d*x**11/11 + e*(a + b*x**3)**4/(12*b)
```

Mathematica [A] time = 0.00637086, size = 134, normalized size = 1.

$$a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{1}{2}a^2bex^6 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{3}ab^2ex^9 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11} + \frac{1}{12}b^3ex^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^4)/4 + (3*
a^2*b*d*x^5)/5 + (a^2*b*e*x^6)/2 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d
*x^8)/8 + (a*b^2*e*x^9)/3 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (
```

```
b^3*e*x^12)/12
```

Maple [A] time = 0.002, size = 113, normalized size = 0.8

$$a^{3}cx+\frac{a^{3}dx^{2}}{2}+\frac{a^{3}ex^{3}}{3}+\frac{3\,a^{2}bcx^{4}}{4}+\frac{3\,a^{2}bdx^{5}}{5}+\frac{a^{2}bex^{6}}{2}+\frac{3\,ab^{2}cx^{7}}{7}+\frac{3\,ab^{2}dx^{8}}{8}+\frac{ab^{2}ex^{9}}{3}+\frac{b^{3}cx^{10}}{10}+\frac{b^{3}dx^{11}}{11}+\frac{b^{3}ex^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)*(b*x^3+a)^3,x)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x
^5+1/2*a^2*b*e*x^6+3/7*a*b^2*c*x^7+3/8*a*b^2*d*x^8+1/3*a*b^2*e*x^
9+1/10*b^3*c*x^10+1/11*b^3*d*x^11+1/12*b^3*e*x^12
```

Maxima [A] time = 1.40354, size = 151, normalized size = 1.13

$$\frac{1}{12}\,b^{3}ex^{12}+\frac{1}{11}\,b^{3}dx^{11}+\frac{1}{10}\,b^{3}cx^{10}+\frac{1}{3}\,ab^{2}ex^{9}+\frac{3}{8}\,ab^{2}dx^{8}+\frac{3}{7}\,ab^{2}cx^{7}+\frac{1}{2}\,a^{2}bex^{6}+\frac{3}{5}\,a^{2}bdx^{5}+\frac{3}{4}\,a^{2}bcx^{4}+\frac{1}{3}\,a^{3}ex^{3}+\frac{1}{2}\,a^{3}dx^{2}+a^{3}cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3*(e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e
*x^9 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*
a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a
^3*c*x
```

Fricas [A] time = 0.185684, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}eb^{3}+\frac{1}{11}x^{11}db^{3}+\frac{1}{10}x^{10}cb^{3}+\frac{1}{3}x^{9}eb^{2}a+\frac{3}{8}x^{8}db^{2}a+\frac{3}{7}x^{7}cb^{2}a+\frac{1}{2}x^{6}eba^{2}+\frac{3}{5}x^{5}dba^{2}+\frac{3}{4}x^{4}cba^{2}+\frac{1}{3}x^{3}ea^{3}+\frac{1}{2}x^{2}da^{3}+xca^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3*(e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e*b^3 + 1/11*x^11*d*b^3 + 1/10*x^10*c*b^3 + 1/3*x^9*e*b
^2*a + 3/8*x^8*d*b^2*a + 3/7*x^7*c*b^2*a + 1/2*x^6*e*b*a^2 + 3/5*
x^5*d*b*a^2 + 3/4*x^4*c*b*a^2 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x
*c*a^3
```

Sympy [A] time = 0.075547, size = 134, normalized size = 1.

$$a^{3}cx+\frac{a^{3}dx^{2}}{2}+\frac{a^{3}ex^{3}}{3}+\frac{3a^{2}bcx^{4}}{4}+\frac{3a^{2}bdx^{5}}{5}+\frac{a^{2}bex^{6}}{2}+\frac{3ab^{2}cx^{7}}{7}+\frac{3ab^{2}dx^{8}}{8}+\frac{ab^{2}ex^{9}}{3}+\frac{b^{3}cx^{10}}{10}+\frac{b^{3}dx^{11}}{11}+\frac{b^{3}ex^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + 3*a**2*b*c*x**4/4 + 3*
a**2*b*d*x**5/5 + a**2*b*e*x**6/2 + 3*a*b**2*c*x**7/7 + 3*a*b**2*
d*x**8/8 + a*b**2*e*x**9/3 + b**3*c*x**10/10 + b**3*d*x**11/11 +
b**3*e*x**12/12
```

GIAC/XCAS [A] time = 0.209451, size = 157, normalized size = 1.17

$$\frac{1}{12}\,b^3x^{12}e + \frac{1}{11}\,b^3dx^{11} + \frac{1}{10}\,b^3cx^{10} + \frac{1}{3}\,ab^2x^9e + \frac{3}{8}\,ab^2dx^8 + \frac{3}{7}\,ab^2cx^7 + \frac{1}{2}\,a^2bx^6e + \frac{3}{5}\,a^2bdx^5 + \frac{3}{4}\,a^2bcx^4 + \frac{1}{3}\,a^3x^3e + \frac{1}{2}\,a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3*(e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12*e + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*x
^9*e + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*x^6*e + 3/5*
a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a
^3*c*x
```

3.320 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{1}{3}ab^2dx^9+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{12}b^3dx^{12}+\frac{1}{13}b^3ex^{13}$$

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (a*b
^2*d*x^9)/3 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*d*x^12
)/12 + (b^3*e*x^13)/13
```

Rubi [A] time = 0.213196, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{1}{3}ab^2dx^9+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{12}b^3dx^{12}+\frac{1}{13}b^3ex^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (a*b
^2*d*x^9)/3 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*d*x^12
)/12 + (b^3*e*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3c\int x\,dx+\frac{a^3ex^4}{4}+\frac{3a^2bcx^5}{5}+\frac{3a^2bex^7}{7}+\frac{3ab^2cx^8}{8}+\frac{3ab^2ex^{10}}{10}+\frac{b^3cx^{11}}{11}+\frac{b^3ex^{13}}{13}+\frac{d\left(a+bx^3\right)^4}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*Integral(x, x) + a**3*e*x**4/4 + 3*a**2*b*c*x**5/5 + 3*a**
2*b*e*x**7/7 + 3*a*b**2*c*x**8/8 + 3*a*b**2*e*x**10/10 + b**3*c*x
**11/11 + b**3*e*x**13/13 + d*(a + b*x**3)**4/(12*b)
```

Mathematica [A] time = 0.00643486, size = 139, normalized size = 1.

$$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{1}{3}ab^2dx^9+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{12}b^3dx^{12}+\frac{1}{13}b^3ex^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (a*b
^2*d*x^9)/3 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*d*x^12
```

```
)/12 + (b^3*e*x^13)/13
```

Maple [A] time = 0.001, size = 116, normalized size = 0.8

$$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{3\,a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3\,a^2bex^7}{7}+\frac{3\,ab^2cx^8}{8}+\frac{ab^2dx^9}{3}+\frac{3\,ab^2ex^{10}}{10}+\frac{b^3cx^{11}}{11}+\frac{b^3dx^{12}}{12}+\frac{b^3ex^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x)
```

```
[Out] 1/2*a^3*c*x^2+1/3*a^3*d*x^3+1/4*a^3*e*x^4+3/5*a^2*b*c*x^5+1/2*a^2
*b*d*x^6+3/7*a^2*b*e*x^7+3/8*a*b^2*c*x^8+1/3*a*b^2*d*x^9+3/10*a*b
^2*e*x^10+1/11*b^3*c*x^11+1/12*b^3*d*x^12+1/13*b^3*e*x^13
```

Maxima [A] time = 1.41346, size = 155, normalized size = 1.12

$$\frac{1}{13}\,b^3ex^{13}+\frac{1}{12}\,b^3dx^{12}+\frac{1}{11}\,b^3cx^{11}+\frac{3}{10}\,ab^2ex^{10}+\frac{1}{3}\,ab^2dx^9+\frac{3}{8}\,ab^2cx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3ex^4+\frac{1}{3}\,a^3dx^3+\frac{1}{2}\,a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3*(e*x^2 + d*x + c)*x,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*
e*x^10 + 1/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/
2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 +
 1/2*a^3*c*x^2
```

Fricas [A] time = 0.186303, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}eb^3+\frac{1}{12}x^{12}db^3+\frac{1}{11}x^{11}cb^3+\frac{3}{10}x^{10}eb^2a+\frac{1}{3}x^9db^2a+\frac{3}{8}x^8cb^2a+\frac{3}{7}x^7eba^2+\frac{1}{2}x^6dba^2+\frac{3}{5}x^5cba^2+\frac{1}{4}x^4ea^3+\frac{1}{3}x^3da^3+\frac{1}{2}x^2ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3*(e*x^2 + d*x + c)*x,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e*b^3 + 1/12*x^12*d*b^3 + 1/11*x^11*c*b^3 + 3/10*x^10*e
*b^2*a + 1/3*x^9*d*b^2*a + 3/8*x^8*c*b^2*a + 3/7*x^7*e*b*a^2 + 1/
2*x^6*d*b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*x^4*e*a^3 + 1/3*x^3*d*a^3 +
 1/2*x^2*c*a^3
```

Sympy [A] time = 0.077943, size = 138, normalized size = 0.99

$$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3ab^2cx^8}{8}+\frac{ab^2dx^9}{3}+\frac{3ab^2ex^{10}}{10}+\frac{b^3cx^{11}}{11}+\frac{b^3dx^{12}}{12}+\frac{b^3ex^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + 3*a**2*b*c*x**5/5
 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a*b**2*c*x**8/8 + a*b*
*2*d*x**9/3 + 3*a*b**2*e*x**10/10 + b**3*c*x**11/11 + b**3*d*x**1
2/12 + b**3*e*x**13/13
```

GIAC/XCAS [A] time = 0.209888, size = 161, normalized size = 1.16

$$\frac{1}{13}\,b^3x^{13}e + \frac{1}{12}\,b^3dx^{12} + \frac{1}{11}\,b^3cx^{11} + \frac{3}{10}\,ab^2x^{10}e + \frac{1}{3}\,ab^2dx^9 + \frac{3}{8}\,ab^2cx^8 + \frac{3}{7}\,a^2bx^7e + \frac{1}{2}\,a^2bdx^6 + \frac{3}{5}\,a^2bcx^5 + \frac{1}{4}\,a^3x^4e + \frac{1}{3}\,a^3dx^3 + \frac{1}{2}\,a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3*(e*x^2 + d*x + c)*x,x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*
x^10*e + 1/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*x^7*e + 1/
2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 +
 1/2*a^3*c*x^2
```

3.321 $\int x^2 (c + dx + ex^2) (a + bx^3)^3 \, dx$

Optimal. Leaf size=139

$$\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{2}a^2bcx^6+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{12}b^3cx^{12}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*b*c*x^6)/2 +
 (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b
^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^12)/12 + (b^3*d*x^
13)/13 + (b^3*e*x^14)/14
```

Rubi [A] time = 0.251333, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{2}a^2bcx^6+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{12}b^3cx^{12}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*b*c*x^6)/2 +
 (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b
^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^12)/12 + (b^3*d*x^
13)/13 + (b^3*e*x^14)/14
```

Rubi in Sympy [A] time = 34.6413, size = 107, normalized size = 0.77

$$\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{3a^2bdx^7}{7}+\frac{3a^2bex^8}{8}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{b^3dx^{13}}{13}+\frac{b^3ex^{14}}{14}+\frac{c\left(a+bx^3\right)^4}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*d*x**4/4 + a**3*e*x**5/5 + 3*a**2*b*d*x**7/7 + 3*a**2*b*e*x*
*8/8 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + b**3*d*x**13/1
3 + b**3*e*x**14/14 + c*(a + b*x**3)**4/(12*b)
```

Mathematica [A] time = 0.00765015, size = 139, normalized size = 1.

$$\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{2}a^2bcx^6+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{12}b^3cx^{12}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*b*c*x^6)/2 +
 (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b
^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^12)/12 + (b^3*d*x^
```

```
13)/13 + (b^3*e*x^14)/14
```

Maple [A] time = 0.001, size = 116, normalized size = 0.8

$$\frac{a^3cx^3}{3}+\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{a^2bcx^6}{2}+\frac{3\,a^2bdx^7}{7}+\frac{3\,a^2bex^8}{8}+\frac{ab^2cx^9}{3}+\frac{3\,ab^2dx^{10}}{10}+\frac{3\,ab^2ex^{11}}{11}+\frac{b^3cx^{12}}{12}+\frac{b^3dx^{13}}{13}+\frac{b^3ex^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x)
```

```
[Out] 1/3*a^3*c*x^3+1/4*a^3*d*x^4+1/5*a^3*e*x^5+1/2*a^2*b*c*x^6+3/7*a^2
*b*d*x^7+3/8*a^2*b*e*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a
*b^2*e*x^11+1/12*b^3*c*x^12+1/13*b^3*d*x^13+1/14*b^3*e*x^14
```

Maxima [A] time = 1.37136, size = 155, normalized size = 1.12

$$\frac{1}{14}\,b^3ex^{14}+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bex^8+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3ex^5+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3*(e*x^2 + d*x + c)*x^2,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*
e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 +
3/7*a^2*b*d*x^7 + 1/2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4
 + 1/3*a^3*c*x^3
```

Fricas [A] time = 0.185472, size = 1, normalized size = 0.01

$$\frac{1}{14}x^{14}eb^3+\frac{1}{13}x^{13}db^3+\frac{1}{12}x^{12}cb^3+\frac{3}{11}x^{11}eb^2a+\frac{3}{10}x^{10}db^2a+\frac{1}{3}x^9cb^2a+\frac{3}{8}x^8eba^2+\frac{3}{7}x^7dba^2+\frac{1}{2}x^6cba^2+\frac{1}{5}x^5ea^3+\frac{1}{4}x^4da^3+\frac{1}{3}x^3ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3*(e*x^2 + d*x + c)*x^2,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*e*b^3 + 1/13*x^13*d*b^3 + 1/12*x^12*c*b^3 + 3/11*x^11*e
*b^2*a + 3/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3/8*x^8*e*b*a^2 +
3/7*x^7*d*b*a^2 + 1/2*x^6*c*b*a^2 + 1/5*x^5*e*a^3 + 1/4*x^4*d*a^3
 + 1/3*x^3*c*a^3
```

Sympy [A] time = 0.07861, size = 138, normalized size = 0.99

$$\frac{a^3cx^3}{3}+\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{a^2bcx^6}{2}+\frac{3a^2bdx^7}{7}+\frac{3a^2bex^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{b^3cx^{12}}{12}+\frac{b^3dx^{13}}{13}+\frac{b^3ex^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + a**2*b*c*x**6/2 +
 3*a**2*b*d*x**7/7 + 3*a**2*b*e*x**8/8 + a*b**2*c*x**9/3 + 3*a*b*
*2*d*x**10/10 + 3*a*b**2*e*x**11/11 + b**3*c*x**12/12 + b**3*d*x*
*13/13 + b**3*e*x**14/14
```

GIAC/XCAS [A] time = 0.208852, size = 161, normalized size = 1.16

$$\frac{1}{14}\,b^3x^{14}e+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2x^{11}e+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bx^8e+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3x^5e+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3*(e*x^2 + d*x + c)*x^2,x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*
x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*x^8*e +
3/7*a^2*b*d*x^7 + 1/2*a^2*b*c*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4
 + 1/3*a^3*c*x^3
```

3.322 $\int (c + dx + ex^2) (a + bx^3)^4 \, dx$

Optimal. Leaf size=173

$$a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + \frac{2}{3}a^3bex^6 + \frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + \frac{2}{3}a^2b^2ex^9 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{1}{3}ab^3ex^{12} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14} + \frac{1}{15}b^4ex^{15}$$

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + a^3*b*c*x^4 + (4*a^3*b*
d*x^5)/5 + (2*a^3*b*e*x^6)/3 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d
*x^8)/4 + (2*a^2*b^2*e*x^9)/3 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x
^11)/11 + (a*b^3*e*x^12)/3 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14 +
(b^4*e*x^15)/15
```

Rubi [A] time = 0.259694, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + \frac{2}{3}a^3bex^6 + \frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + \frac{2}{3}a^2b^2ex^9 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{1}{3}ab^3ex^{12} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14} + \frac{1}{15}b^4ex^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + a^3*b*c*x^4 + (4*a^3*b*
d*x^5)/5 + (2*a^3*b*e*x^6)/3 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d
*x^8)/4 + (2*a^2*b^2*e*x^9)/3 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x
^11)/11 + (a*b^3*e*x^12)/3 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14 +
(b^4*e*x^15)/15
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4d\int x\,dx + a^4\int c\,dx + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} + \frac{b^4cx^{13}}{13} + \frac{b^4dx^{14}}{14} + \frac{e(a + bx^3)^5}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*d*Integral(x, x) + a**4*Integral(c, x) + a**3*b*c*x**4 + 4*a
**3*b*d*x**5/5 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*
a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11 + b**4*c*x**13/13 + b**4*d
*x**14/14 + e*(a + b*x**3)**5/(15*b)
```

Mathematica [A] time = 0.00801525, size = 173, normalized size = 1.

$$a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + \frac{2}{3}a^3bex^6 + \frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + \frac{2}{3}a^2b^2ex^9 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{1}{3}ab^3ex^{12} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14} + \frac{1}{15}b^4ex^{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + a^3*b*c*x^4 + (4*a^3*b*
d*x^5)/5 + (2*a^3*b*e*x^6)/3 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d
*x^8)/4 + (2*a^2*b^2*e*x^9)/3 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x
^11)/11 + (a*b^3*e*x^12)/3 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14 +
(b^4*e*x^15)/15
```

Maple [A] time = 0.002, size = 148, normalized size = 0.9

$$a^4cx+\frac{a^4dx^2}{2}+\frac{a^4ex^3}{3}+a^3bcx^4+\frac{4\,a^3bdx^5}{5}+\frac{2\,a^3bex^6}{3}+\frac{6\,a^2b^2cx^7}{7}+\frac{3\,a^2b^2dx^8}{4}+\frac{2\,a^2b^2ex^9}{3}+\frac{2\,ab^3cx^{10}}{5}+\frac{4\,ab^3dx^{11}}{11}+\frac{ab^3ex^{12}}{3}+\frac{b^4cx^{13}}{13}+\frac{b^4dx^{14}}{14}+\frac{b^4ex^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)*(b*x^3+a)^4,x)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+a^3*b*c*x^4+4/5*a^3*b*d*x^5+2
/3*a^3*b*e*x^6+6/7*a^2*b^2*c*x^7+3/4*a^2*b^2*d*x^8+2/3*a^2*b^2*e*
x^9+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/3*a*b^3*e*x^12+1/13*b^4*
c*x^13+1/14*b^4*d*x^14+1/15*b^4*e*x^15
```

Maxima [A] time = 1.41452, size = 198, normalized size = 1.14

$$\frac{1}{15}\,b^4ex^{15}+\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{1}{3}\,ab^3ex^{12}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{2}{3}\,a^2b^2ex^9+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{2}{3}\,a^3bex^6+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^4*(e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e
*x^12 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9
+ 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a
^3*b*d*x^5 + a^3*b*c*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*
x
```

Fricas [A] time = 0.185243, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}eb^4+\frac{1}{14}x^{14}db^4+\frac{1}{13}x^{13}cb^4+\frac{1}{3}x^{12}eb^3a+\frac{4}{11}x^{11}db^3a+\frac{2}{5}x^{10}cb^3a+\frac{2}{3}x^9eb^2a^2+\frac{3}{4}x^8db^2a^2+\frac{6}{7}x^7cb^2a^2+\frac{2}{3}x^6eba^3+\frac{4}{5}x^5dba^3+x^4cba^3+\frac{1}{3}x^3ea^4+\frac{1}{2}x^2da^4+xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^4*(e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e*b^4 + 1/14*x^14*d*b^4 + 1/13*x^13*c*b^4 + 1/3*x^12*e*
b^3*a + 4/11*x^11*d*b^3*a + 2/5*x^10*c*b^3*a + 2/3*x^9*e*b^2*a^2
+ 3/4*x^8*d*b^2*a^2 + 6/7*x^7*c*b^2*a^2 + 2/3*x^6*e*b*a^3 + 4/5*x
^5*d*b*a^3 + x^4*c*b*a^3 + 1/3*x^3*e*a^4 + 1/2*x^2*d*a^4 + x*c*a^
4
```

Sympy [A] time = 0.088664, size = 178, normalized size = 1.03

$$a^4cx + \frac{a^4dx^2}{2} + \frac{a^4ex^3}{3} + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{2a^3bex^6}{3} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2a^2b^2ex^9}{3} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} + \frac{ab^3ex^{12}}{3} + \frac{b^4cx^{13}}{13} + \frac{b^4dx^{14}}{14} + \frac{b^4ex^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**3*b*c*x**4 + 4*a**3
*b*d*x**5/5 + 2*a**3*b*e*x**6/3 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b
**2*d*x**8/4 + 2*a**2*b**2*e*x**9/3 + 2*a*b**3*c*x**10/5 + 4*a*b*
*3*d*x**11/11 + a*b**3*e*x**12/3 + b**4*c*x**13/13 + b**4*d*x**14
/14 + b**4*e*x**15/15
```

GIAC/XCAS [A] time = 0.21047, size = 205, normalized size = 1.18

$$\frac{1}{15}b^4x^{15}e + \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{1}{3}ab^3x^{12}e + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{2}{3}a^2b^2x^9e + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{2}{3}a^3bx^6e + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{3}a^4x^3e + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^4*(e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/15*b^4*x^15*e + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*x
^12*e + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*x^9*e
+ 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*x^6*e + 4/5*a
^3*b*d*x^5 + a^3*b*c*x^4 + 1/3*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*
x
```

3.323 $\int x (c + dx + ex^2) (a + bx^3)^4 \, dx$

Optimal. Leaf size=181

$$\frac{1}{2}a^4cx^2 + \frac{1}{3}a^4dx^3 + \frac{1}{4}a^4ex^4 + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{3}{4}a^2b^2cx^8 + \frac{2}{3}a^2b^2dx^9 + \frac{3}{5}a^2b^2ex^{10} + \frac{4}{11}ab^3cx^{11} + \frac{1}{3}ab^3dx^{12} + \frac{4}{13}ab^3ex^{13} + \frac{1}{14}b^4cx^{14} + \frac{1}{15}b^4dx^{15} + \frac{1}{16}b^4ex^{16}$$

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5
 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (3*a^2*b^2*c*x^8)/4 +
(2*a^2*b^2*d*x^9)/3 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11
+ (a*b^3*d*x^12)/3 + (4*a*b^3*e*x^13)/13 + (b^4*c*x^14)/14 + (b^4
*d*x^15)/15 + (b^4*e*x^16)/16
```

Rubi [A] time = 0.306498, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{1}{2}a^4cx^2 + \frac{1}{3}a^4dx^3 + \frac{1}{4}a^4ex^4 + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{3}{4}a^2b^2cx^8 + \frac{2}{3}a^2b^2dx^9 + \frac{3}{5}a^2b^2ex^{10} + \frac{4}{11}ab^3cx^{11} + \frac{1}{3}ab^3dx^{12} + \frac{4}{13}ab^3ex^{13} + \frac{1}{14}b^4cx^{14} + \frac{1}{15}b^4dx^{15} + \frac{1}{16}b^4ex^{16}$$

Antiderivative was successfully verified.

```
[In]   Int[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5
 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (3*a^2*b^2*c*x^8)/4 +
(2*a^2*b^2*d*x^9)/3 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11
+ (a*b^3*d*x^12)/3 + (4*a*b^3*e*x^13)/13 + (b^4*c*x^14)/14 + (b^4
*d*x^15)/15 + (b^4*e*x^16)/16
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4c\int x\,dx + \frac{a^4ex^4}{4} + \frac{4a^3bcx^5}{5} + \frac{4a^3bex^7}{7} + \frac{3a^2b^2cx^8}{4} + \frac{3a^2b^2ex^{10}}{5} + \frac{4ab^3cx^{11}}{11} + \frac{4ab^3ex^{13}}{13} + \frac{b^4cx^{14}}{14} + \frac{b^4ex^{16}}{16} + \frac{d\left(a + bx^3\right)^5}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*Integral(x, x) + a**4*e*x**4/4 + 4*a**3*b*c*x**5/5 + 4*a**
3*b*e*x**7/7 + 3*a**2*b**2*c*x**8/4 + 3*a**2*b**2*e*x**10/5 + 4*a
*b**3*c*x**11/11 + 4*a*b**3*e*x**13/13 + b**4*c*x**14/14 + b**4*e
*x**16/16 + d*(a + b*x**3)**5/(15*b)
```

Mathematica [A] time = 0.00765655, size = 181, normalized size = 1.

$$\frac{1}{2}a^4cx^2 + \frac{1}{3}a^4dx^3 + \frac{1}{4}a^4ex^4 + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{3}{4}a^2b^2cx^8 + \frac{2}{3}a^2b^2dx^9 + \frac{3}{5}a^2b^2ex^{10} + \frac{4}{11}ab^3cx^{11} + \frac{1}{3}ab^3dx^{12} + \frac{4}{13}ab^3ex^{13} + \frac{1}{14}b^4cx^{14} + \frac{1}{15}b^4dx^{15} + \frac{1}{16}b^4ex^{16}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5
 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (3*a^2*b^2*c*x^8)/4 +
(2*a^2*b^2*d*x^9)/3 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11
+ (a*b^3*d*x^12)/3 + (4*a*b^3*e*x^13)/13 + (b^4*c*x^14)/14 + (b^4
*d*x^15)/15 + (b^4*e*x^16)/16
```

Maple [A] time = 0.001, size = 152, normalized size = 0.8

$$\frac{a^4cx^2}{2}+\frac{a^4dx^3}{3}+\frac{a^4ex^4}{4}+\frac{4\,a^3bcx^5}{5}+\frac{2\,a^3bdx^6}{3}+\frac{4\,a^3bex^7}{7}+\frac{3\,a^2b^2cx^8}{4}+\frac{2\,a^2b^2dx^9}{3}+\frac{3\,a^2b^2ex^{10}}{5}+\frac{4\,ab^3cx^{11}}{11}+\frac{ab^3dx^{12}}{3}+\frac{4\,ab^3ex^{13}}{13}+\frac{b^4cx^{14}}{14}+\frac{b^4dx^{15}}{15}+\frac{b^4ex^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x)
```

```
[Out] 1/2*a^4*c*x^2+1/3*a^4*d*x^3+1/4*a^4*e*x^4+4/5*a^3*b*c*x^5+2/3*a^3
*b*d*x^6+4/7*a^3*b*e*x^7+3/4*a^2*b^2*c*x^8+2/3*a^2*b^2*d*x^9+3/5*
a^2*b^2*e*x^10+4/11*a*b^3*c*x^11+1/3*a*b^3*d*x^12+4/13*a*b^3*e*x^
13+1/14*b^4*c*x^14+1/15*b^4*d*x^15+1/16*b^4*e*x^16
```

Maxima [A] time = 1.42459, size = 204, normalized size = 1.13

$$\frac{1}{16}\,b^4ex^{16}+\frac{1}{15}\,b^4dx^{15}+\frac{1}{14}\,b^4cx^{14}+\frac{4}{13}\,ab^3ex^{13}+\frac{1}{3}\,ab^3dx^{12}+\frac{4}{11}\,ab^3cx^{11}+\frac{3}{5}\,a^2b^2ex^{10}+\frac{2}{3}\,a^2b^2dx^9+\frac{3}{4}\,a^2b^2cx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4ex^4+\frac{1}{3}\,a^4dx^3+\frac{1}{2}\,a^4cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^4*(e*x^2 + d*x + c)*x,x, algorithm="maxima")
```

```
[Out] 1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*
e*x^13 + 1/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^1
0 + 2/3*a^2*b^2*d*x^9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3
*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 +
1/2*a^4*c*x^2
```

Fricas [A] time = 0.181537, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}eb^4+\frac{1}{15}x^{15}db^4+\frac{1}{14}x^{14}cb^4+\frac{4}{13}x^{13}eb^3a+\frac{1}{3}x^{12}db^3a+\frac{4}{11}x^{11}cb^3a+\frac{3}{5}x^{10}eb^2a^2+\frac{2}{3}x^9db^2a^2+\frac{3}{4}x^8cb^2a^2+\frac{4}{7}x^7eba^3+\frac{2}{3}x^6dba^3+\frac{4}{5}x^5cba^3+\frac{1}{4}x^4ea^4+\frac{1}{3}x^3da^4+\frac{1}{2}x^2ca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^4*(e*x^2 + d*x + c)*x,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e*b^4 + 1/15*x^15*d*b^4 + 1/14*x^14*c*b^4 + 4/13*x^13*e
*b^3*a + 1/3*x^12*d*b^3*a + 4/11*x^11*c*b^3*a + 3/5*x^10*e*b^2*a^
2 + 2/3*x^9*d*b^2*a^2 + 3/4*x^8*c*b^2*a^2 + 4/7*x^7*e*b*a^3 + 2/3
*x^6*d*b*a^3 + 4/5*x^5*c*b*a^3 + 1/4*x^4*e*a^4 + 1/3*x^3*d*a^4 +
1/2*x^2*c*a^4
```

Sympy [A] time = 0.089343, size = 185, normalized size = 1.02

$$\frac{a^4cx^2}{2}+\frac{a^4dx^3}{3}+\frac{a^4ex^4}{4}+\frac{4a^3bcx^5}{5}+\frac{2a^3bdx^6}{3}+\frac{4a^3bex^7}{7}+\frac{3a^2b^2cx^8}{4}+\frac{2a^2b^2dx^9}{3}+\frac{3a^2b^2ex^{10}}{5}+\frac{4ab^3cx^{11}}{11}+\frac{ab^3dx^{12}}{3}+\frac{4ab^3ex^{13}}{13}+\frac{b^4cx^{14}}{14}+\frac{b^4dx^{15}}{15}+\frac{b^4ex^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**2/2 + a**4*d*x**3/3 + a**4*e*x**4/4 + 4*a**3*b*c*x**5/5
 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + 3*a**2*b**2*c*x**8/4 +
 2*a**2*b**2*d*x**9/3 + 3*a**2*b**2*e*x**10/5 + 4*a*b**3*c*x**11/
11 + a*b**3*d*x**12/3 + 4*a*b**3*e*x**13/13 + b**4*c*x**14/14 + b
**4*d*x**15/15 + b**4*e*x**16/16
```

GIAC/XCAS [A] time = 0.210272, size = 211, normalized size = 1.17

$$\frac{1}{16}b^4x^{16}e+\frac{1}{15}b^4dx^{15}+\frac{1}{14}b^4cx^{14}+\frac{4}{13}ab^3x^{13}e+\frac{1}{3}ab^3dx^{12}+\frac{4}{11}ab^3cx^{11}+\frac{3}{5}a^2b^2x^{10}e+\frac{2}{3}a^2b^2dx^9+\frac{3}{4}a^2b^2cx^8+\frac{4}{7}a^3bx^7e+\frac{2}{3}a^3bdx^6+\frac{4}{5}a^3bcx^5+\frac{1}{4}a^4x^4e+\frac{1}{3}a^4dx^3+\frac{1}{2}a^4cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^4*(e*x^2 + d*x + c)*x,x, algorithm="giac")
```

```
[Out] 1/16*b^4*x^16*e + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*
x^13*e + 1/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*x^10*
e + 2/3*a^2*b^2*d*x^9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*x^7*e + 2/3
*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5 + 1/4*a^4*x^4*e + 1/3*a^4*d*x^3 +
1/2*a^4*c*x^2
```

3.324 $\int x^2 \left(c + dx + ex^2\right) \left(a + bx^3\right)^4 \, dx$

Optimal. Leaf size=181

$$\frac{1}{3}a^4cx^3 + \frac{1}{4}a^4dx^4 + \frac{1}{5}a^4ex^5 + \frac{2}{3}a^3bcx^6 + \frac{4}{7}a^3bdx^7 + \frac{1}{2}a^3bex^8 + \frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{1}{3}ab^3cx^{12} + \frac{4}{13}ab^3dx^{13} + \frac{2}{7}ab^3ex^{14} + \frac{1}{15}b^4cx^{15} + \frac{1}{16}b^4dx^{16} + \frac{1}{17}b^4ex^{17}$$

```
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (2*a^3*b*c*x^6)/3
 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (2*a^2*b^2*c*x^9)/3 + (3
*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a*b^3*c*x^12)/3 + (
4*a*b^3*d*x^13)/13 + (2*a*b^3*e*x^14)/7 + (b^4*c*x^15)/15 + (b^4*
d*x^16)/16 + (b^4*e*x^17)/17
```

Rubi [A] time = 0.333306, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{1}{3}a^4cx^3 + \frac{1}{4}a^4dx^4 + \frac{1}{5}a^4ex^5 + \frac{2}{3}a^3bcx^6 + \frac{4}{7}a^3bdx^7 + \frac{1}{2}a^3bex^8 + \frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{1}{3}ab^3cx^{12} + \frac{4}{13}ab^3dx^{13} + \frac{2}{7}ab^3ex^{14} + \frac{1}{15}b^4cx^{15} + \frac{1}{16}b^4dx^{16} + \frac{1}{17}b^4ex^{17}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (2*a^3*b*c*x^6)/3
 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (2*a^2*b^2*c*x^9)/3 + (3
*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a*b^3*c*x^12)/3 + (
4*a*b^3*d*x^13)/13 + (2*a*b^3*e*x^14)/7 + (b^4*c*x^15)/15 + (b^4*
d*x^16)/16 + (b^4*e*x^17)/17
```

Rubi in Sympy [A] time = 42.2621, size = 136, normalized size = 0.75

$$\frac{a^4dx^4}{4} + \frac{a^4ex^5}{5} + \frac{4a^3bdx^7}{7} + \frac{a^3bex^8}{2} + \frac{3a^2b^2dx^{10}}{5} + \frac{6a^2b^2ex^{11}}{11} + \frac{4ab^3dx^{13}}{13} + \frac{2ab^3ex^{14}}{7} + \frac{b^4dx^{16}}{16} + \frac{b^4ex^{17}}{17} + \frac{c\left(a + bx^3\right)^5}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*d*x**4/4 + a**4*e*x**5/5 + 4*a**3*b*d*x**7/7 + a**3*b*e*x**8
/2 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + 4*a*b**3*d*
x**13/13 + 2*a*b**3*e*x**14/7 + b**4*d*x**16/16 + b**4*e*x**17/17
 + c*(a + b*x**3)**5/(15*b)
```

Mathematica [A] time = 0.00786134, size = 181, normalized size = 1.

$$\frac{1}{3}a^4cx^3 + \frac{1}{4}a^4dx^4 + \frac{1}{5}a^4ex^5 + \frac{2}{3}a^3bcx^6 + \frac{4}{7}a^3bdx^7 + \frac{1}{2}a^3bex^8 + \frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{1}{3}ab^3cx^{12} + \frac{4}{13}ab^3dx^{13} + \frac{2}{7}ab^3ex^{14} + \frac{1}{15}b^4cx^{15} + \frac{1}{16}b^4dx^{16} + \frac{1}{17}b^4ex^{17}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (2*a^3*b*c*x^6)/3
 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (2*a^2*b^2*c*x^9)/3 + (3
*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a*b^3*c*x^12)/3 + (
4*a*b^3*d*x^13)/13 + (2*a*b^3*e*x^14)/7 + (b^4*c*x^15)/15 + (b^4*
d*x^16)/16 + (b^4*e*x^17)/17
```

Maple [A] time = 0.002, size = 152, normalized size = 0.8

$$\frac{a^4cx^3}{3}+\frac{a^4dx^4}{4}+\frac{a^4ex^5}{5}+\frac{2\,a^3bcx^6}{3}+\frac{4\,a^3bdx^7}{7}+\frac{a^3bex^8}{2}+\frac{2\,a^2b^2cx^9}{3}+\frac{3\,a^2b^2dx^{10}}{5}+\frac{6\,a^2b^2ex^{11}}{11}+\frac{ab^3cx^{12}}{3}+\frac{4\,ab^3dx^{13}}{13}+\frac{2\,ab^3ex^{14}}{7}+\frac{b^4cx^{15}}{15}+\frac{b^4dx^{16}}{16}+\frac{b^4ex^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x)
```

```
[Out] 1/3*a^4*c*x^3+1/4*a^4*d*x^4+1/5*a^4*e*x^5+2/3*a^3*b*c*x^6+4/7*a^3
*b*d*x^7+1/2*a^3*b*e*x^8+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10+6/1
1*a^2*b^2*e*x^11+1/3*a*b^3*c*x^12+4/13*a*b^3*d*x^13+2/7*a*b^3*e*x
^14+1/15*b^4*c*x^15+1/16*b^4*d*x^16+1/17*b^4*e*x^17
```

Maxima [A] time = 1.37662, size = 204, normalized size = 1.13

$$\frac{1}{17}\,b^4ex^{17}+\frac{1}{16}\,b^4dx^{16}+\frac{1}{15}\,b^4cx^{15}+\frac{2}{7}\,ab^3ex^{14}+\frac{4}{13}\,ab^3dx^{13}+\frac{1}{3}\,ab^3cx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bex^8+\frac{4}{7}\,a^3bdx^7+\frac{2}{3}\,a^3bcx^6+\frac{1}{5}\,a^4ex^5+\frac{1}{4}\,a^4dx^4+\frac{1}{3}\,a^4cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^4*(e*x^2 + d*x + c)*x^2,x, algorithm="maxima")
```

```
[Out] 1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e
*x^14 + 4/13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^1
1 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/
7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 +
 1/3*a^4*c*x^3
```

Fricas [A] time = 0.18471, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}eb^4+\frac{1}{16}x^{16}db^4+\frac{1}{15}x^{15}cb^4+\frac{2}{7}x^{14}eb^3a+\frac{4}{13}x^{13}db^3a+\frac{1}{3}x^{12}cb^3a+\frac{6}{11}x^{11}eb^2a^2+\frac{3}{5}x^{10}db^2a^2+\frac{2}{3}x^9cb^2a^2+\frac{1}{2}x^8eba^3+\frac{4}{7}x^7dba^3+\frac{2}{3}x^6cba^3+\frac{1}{5}x^5ea^4+\frac{1}{4}x^4da^4+\frac{1}{3}x^3ca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^4*(e*x^2 + d*x + c)*x^2,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*e*b^4 + 1/16*x^16*d*b^4 + 1/15*x^15*c*b^4 + 2/7*x^14*e*
b^3*a + 4/13*x^13*d*b^3*a + 1/3*x^12*c*b^3*a + 6/11*x^11*e*b^2*a^
2 + 3/5*x^10*d*b^2*a^2 + 2/3*x^9*c*b^2*a^2 + 1/2*x^8*e*b*a^3 + 4/
7*x^7*d*b*a^3 + 2/3*x^6*c*b*a^3 + 1/5*x^5*e*a^4 + 1/4*x^4*d*a^4 +
 1/3*x^3*c*a^4
```

Sympy [A] time = 0.088768, size = 184, normalized size = 1.02

$$\frac{a^4cx^3}{3}+\frac{a^4dx^4}{4}+\frac{a^4ex^5}{5}+\frac{2a^3bcx^6}{3}+\frac{4a^3bdx^7}{7}+\frac{a^3bex^8}{2}+\frac{2a^2b^2cx^9}{3}+\frac{3a^2b^2dx^{10}}{5}+\frac{6a^2b^2ex^{11}}{11}+\frac{ab^3cx^{12}}{3}+\frac{4ab^3dx^{13}}{13}+\frac{2ab^3ex^{14}}{7}+\frac{b^4cx^{15}}{15}+\frac{b^4dx^{16}}{16}+\frac{b^4ex^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**3/3 + a**4*d*x**4/4 + a**4*e*x**5/5 + 2*a**3*b*c*x**6/3
 + 4*a**3*b*d*x**7/7 + a**3*b*e*x**8/2 + 2*a**2*b**2*c*x**9/3 + 3
*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + a*b**3*c*x**12/3
+ 4*a*b**3*d*x**13/13 + 2*a*b**3*e*x**14/7 + b**4*c*x**15/15 + b*
*4*d*x**16/16 + b**4*e*x**17/17
```

GIAC/XCAS [A] time = 0.209389, size = 211, normalized size = 1.17

$$\frac{1}{17}\,b^4x^{17}e+\frac{1}{16}\,b^4dx^{16}+\frac{1}{15}\,b^4cx^{15}+\frac{2}{7}\,ab^3x^{14}e+\frac{4}{13}\,ab^3dx^{13}+\frac{1}{3}\,ab^3cx^{12}+\frac{6}{11}\,a^2b^2x^{11}e+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bx^8e+\frac{4}{7}\,a^3bdx^7+\frac{2}{3}\,a^3bcx^6+\frac{1}{5}\,a^4x^5e+\frac{1}{4}\,a^4dx^4+\frac{1}{3}\,a^4cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^4*(e*x^2 + d*x + c)*x^2,x, algorithm="giac")
```

```
[Out] 1/17*b^4*x^17*e + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*x
^14*e + 4/13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*x^11*
e + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*x^8*e + 4/
7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^6 + 1/5*a^4*x^5*e + 1/4*a^4*d*x^4 +
 1/3*a^4*c*x^3
```

3.325 $\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}+\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}$$

```
[Out] (c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a
^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^
(1/3)*x])/(3*b^(5/3)) + (a^(1/3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)) - (a*e*Log[
a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.529306, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}+\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a
^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^
(1/3)*x])/(3*b^(5/3)) + (a^(1/3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)) - (a*e*Log[
a + b*x^3])/(3*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[3]{a}\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{5}{3}}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{5}{3}}}+\frac{\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{5}{3}}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}+\frac{d\int x\,dx}{b}+\frac{ex^3}{3b}+\frac{\int c\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] a**(1/3)*(a**(1/3)*d - b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x)/(3*
b**(5/3)) - a**(1/3)*(a**(1/3)*d - b**(1/3)*c)*log(a**(2/3) - a**
(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*b**(5/3)) + sqrt(3)*a**(1/3)
*(a**(1/3)*d + b**(1/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*
x/3)/a**(1/3))/(3*b**(5/3)) - a*e*log(a + b*x**3)/(3*b**2) + d*In
```

```
tegral(x, x)/b + e*x**3/(3*b) + Integral(c, x)/b
```

Mathematica [A] time = 0.226753, size = 191, normalized size = 0.93

$$\frac{\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(a^{2/3}d-\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b*c*x + 3*b*d*x^2 + 2*b*e*x^3 + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(
1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] +
 2*b^(1/3)*(-(a^(1/3)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/
3)*x] + b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*e*Log[a + b*x^3])/(6*b^2)
```

Maple [A] time = 0.005, size = 231, normalized size = 1.1

$$\frac{ex^3}{3\,b}+\frac{dx^2}{2\,b}+\frac{cx}{b}-\frac{ac}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ac}{6\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{a\sqrt{3}c}{3\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ad}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{ad}{6\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{a\sqrt{3}d}{3\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{ae\ln\left(bx^3+a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*e*x^3/b+1/2*d*x^2/b+c*x/b-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1
/3))*c+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-
1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*c+1/3/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*a*d-1/6/b^2/(a/b)^(1
/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a*d-1/3/b^2*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*d-1/3*a*e*ln(b*x^3
+a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.88853, size = 178, normalized size = 0.87

$$\operatorname{RootSum}\left(27t^3b^6 + 27t^2ab^4e + t\left(9a^2b^2e^2 + 9ab^3cd\right) + a^3e^3 + 3a^2bcde - a^2bd^3 + ab^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2b^4d + 6tab^2de - \right.\right.\right.$$
$$+ \frac{cx}{b} + \frac{dx^2}{2b} + \frac{ex^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**6 + 27*_t**2*a*b**4*e + _t*(9*a**2*b**2*e**2
+ 9*a*b**3*c*d) + a**3*e**3 + 3*a**2*b*c*d*e - a**2*b*d**3 + a*b*
*2*c**3, Lambda(_t, _t*log(x + (9*_t**2*b**4*d + 6*_t*a*b**2*d*e
- 3*_t*b**3*c**2 + a**2*d*e**2 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*
d**3 + b**2*c**3)))) + c*x/b + d*x**2/(2*b) + e*x**3/(3*b)
```

GIAC/XCAS [A] time = 0.215968, size = 302, normalized size = 1.47

$$-\frac{ae\ln\left(\left|bx^3+a\right|\right)}{3\,b^2} + \frac{2\,b^2x^3e + 3\,b^2dx^2 + 6\,b^2cx}{6\,b^3} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2c - \left(-ab^2\right)^{\frac{2}{3}}abd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2c + \left(-ab^2\right)^{\frac{2}{3}}abd\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^4} + \frac{\left(ab^6d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ab^6c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*a*e*ln(abs(b*x^3 + a))/b^2 + 1/6*(2*b^2*x^3*e + 3*b^2*d*x^2
+ 6*b^2*c*x)/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^2*c - (-a*b^2)
^(2/3)*a*b*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(a*b^4) - 1/6*((-a*b^2)^(1/3)*a*b^2*c + (-a*b^2)^(2/3)*a*b*d)*
ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) + 1/3*(a*b^6*d*(-
a/b)^(1/3) + a*b^6*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b
^7)
```

3.326 $\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}+\frac{c\log\left(a+bx^3\right)}{3b}+\frac{dx}{b}+\frac{ex^2}{2b}$$

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) -
(a^(1/3)*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
5/3)) + (a^(1/3)*(d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)) + (c*Log[a + b*x^3])/(3*b)
```

Rubi [A] time = 0.49715, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}+\frac{c\log\left(a+bx^3\right)}{3b}+\frac{dx}{b}+\frac{ex^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) -
(a^(1/3)*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
5/3)) + (a^(1/3)*(d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)) + (c*Log[a + b*x^3])/(3*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[3]{a}\left(\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{5}{3}}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{5}{3}}}+\frac{\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{5}{3}}}+\frac{c\log\left(a+bx^3\right)}{3b}+\frac{e\int x\,dx}{b}+\frac{\int d\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] a**(1/3)*(a**(1/3)*e - b**(1/3)*d)*log(a**(1/3) + b**(1/3)*x)/(3*
b**(5/3)) - a**(1/3)*(a**(1/3)*e - b**(1/3)*d)*log(a**(2/3) - a**
(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*b**(5/3)) + sqrt(3)*a**(1/3)
*(a**(1/3)*e + b**(1/3)*d)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*
x/3)/a**(1/3))/(3*b**(5/3)) + c*log(a + b*x**3)/(3*b) + e*Integra
l(x, x)/b + Integral(d, x)/b
```

Mathematica [A] time = 0.165772, size = 184, normalized size = 0.95

$$\frac{-\left(a^{2/3}e-\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\left(a^{2/3}e-\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2b^{2/3}c\log\left(a+bx^3\right)+2\sqrt{3}\sqrt[3]{a}\left(\cdots\right.}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*d*x + 3*b^(2/3)*e*x^2 + 2*Sqrt[3]*a^(1/3)*(b^(1/3)*d +
 a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-(a^
(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(1/
3)*b^(1/3)*d) + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2] + 2*b^(2/3)*c*Log[a + b*x^3])/(6*b^(5/3))
```

Maple [A] time = 0.006, size = 221, normalized size = 1.2

$$\frac{ex^2}{2b}+\frac{dx}{b}-\frac{ad}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ad}{6b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{a\sqrt{3}d}{3b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ae}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{ae}{6b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{a\sqrt{3}e}{3b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{c\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/2*e*x^2/b+d*x/b-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*d+1/6/b
^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a*d-1/3/b^2/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*d+1/3*a
/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-1/6*a/b^2/(a/b)^(1/3)*ln(x^2
-x*(a/b)^(1/3)+(a/b)^(2/3))*e-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3*c*ln(b*x^3+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.82039, size = 150, normalized size = 0.78

$$\operatorname{RootSum}\left(27t^3b^5 - 27t^2b^4c + t\left(9ab^2de + 9b^3c^2\right) - a^2e^3 - 3abcde + abd^3 - b^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2b^3e - 6tb^2ce - 3tb^2d^2}{ae^3 +}\right.\right.\right.$$
$$+ \frac{dx}{b} + \frac{ex^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c + _t*(9*a*b**2*d*e + 9*b*
*3*c**2) - a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda
(_t, _t*log(x + (9*_t**2*b**3*e - 6*_t*b**2*c*e - 3*_t*b**2*d**2
+ 2*a*d*e**2 + b*c**2*e + b*c*d**2)/(a*e**3 + b*d**3)))) + d*x/b
+ e*x**2/(2*b)
```

GIAC/XCAS [A] time = 0.215265, size = 285, normalized size = 1.48

$$\frac{c\ln\left(\left|bx^3 + a\right|\right)}{3b} + \frac{bx^2e + 2bdx}{2b^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}abe\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^4}$$
$$- \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2d + \left(-ab^2\right)^{\frac{2}{3}}abe\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^4}$$
$$+ \frac{\left(ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + ab^4d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*c*ln(abs(b*x^3 + a))/b + 1/2*(b*x^2*e + 2*b*d*x)/b^2 - 1/3*sq
rt(3)*((-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(2/3)*a*b*e)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/6*((-a*b^2
)^(1/3)*a*b^2*d + (-a*b^2)^(2/3)*a*b*e)*ln(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/(a*b^4) + 1/3*(a*b^4*(-a/b)^(1/3)*e + a*b^4*d)*(-a
/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

3.327 $\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=183

$$\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}+\frac{d\log\left(a+bx^3\right)}{3b}+\frac{ex}{b}$$

```
[Out] (e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^
(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(4/3)) + ((b^(2/3
)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x^3])/(3*b)
```

Rubi [A] time = 0.469453, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}+\frac{d\log\left(a+bx^3\right)}{3b}+\frac{ex}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^
(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(4/3)) + ((b^(2/3
)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x^3])/(3*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d\log\left(a+bx^3\right)}{3b}+\frac{\int e\,dx}{b}+\frac{\sqrt{3}\left(a^{\frac{2}{3}}e-b^{\frac{2}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}b^{\frac{4}{3}}}-\frac{\left(a^{\frac{2}{3}}e+b^{\frac{2}{3}}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{\frac{4}{3}}}+\frac{\left(a^{\frac{2}{3}}e+b^{\frac{2}{3}}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6\sqrt[3]{a}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] d*log(a + b*x**3)/(3*b) + Integral(e, x)/b + sqrt(3)*(a**(2/3)*e
- b**(2/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3)
)/(3*a**(1/3)*b**(4/3)) - (a**(2/3)*e + b**(2/3)*c)*log(a**(1/3)
+ b**(1/3)*x)/(3*a**(1/3)*b**(4/3)) + (a**(2/3)*e + b**(2/3)*c)*l
og(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(1/3)*b*
*(4/3))
```

Mathematica [A] time = 0.126427, size = 200, normalized size = 1.09

$$-\frac{\left(a^{4/3}\left(-\sqrt[3]{b}\right)e-a^{2/3}bc\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6ab^{5/3}}+\frac{\left(a^{4/3}\left(-\sqrt[3]{b}\right)e-a^{2/3}bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3ab^{5/3}}+\frac{\left(a^{2/3}bc-a^{4/3}\sqrt[3]{b}e\right)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}ab^{5/3}}+\frac{d\log\left(a+bx^3\right)}{3b}+\frac{ex}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b + ((a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*ArcTan[(-a^(1/3) + 2
*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b^(5/3)) + ((-(a^(2/3)
*b*c) - a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a*b^(5/3)
) - ((-(a^(2/3)*b*c) - a^(4/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(5/3)) + (d*Log[a + b*x^3])/(3*b)
```

Maple [A] time = 0.004, size = 209, normalized size = 1.1

$$\frac{ex}{b}-\frac{ae}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ae}{6\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{a\sqrt{3}e}{3\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{c}{6\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{c\sqrt{3}}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d\ln\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] e*x/b-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1/6*a/b^2/(a/b)^(
2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e-1/3*a/b^2/(a/b)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/3/b/(a/b)^(1/3)
*ln(x+(a/b)^(1/3))*c+1/6/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)
^(2/3))*c+1/3/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*c+1/3*d*ln(b*x^3+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.95915, size = 160, normalized size = 0.87

$$\operatorname{RootSum}\left(27t^3ab^4 - 27t^2ab^3d + t\left(-9ab^2ce + 9ab^2d^2\right) + a^2e^3 + 3abcde - abd^3 + b^2c^3, \left(t \mapsto t\log\left(x + \frac{-9t^2ab^3c - 3ta^2be^2}{}\right.\right.\right.$$
$$+ \frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**4 - 27*_t**2*a*b**3*d + _t*(-9*a*b**2*c*e +
 9*a*b**2*d**2) + a**2*e**3 + 3*a*b*c*d*e - a*b*d**3 + b**2*c**3,
 Lambda(_t, _t*log(x + (-9*_t**2*a*b**3*c - 3*_t*a**2*b*e**2 + 6*
_t*a*b**2*c*d + a**2*d*e**2 + 2*a*b*c**2*e - a*b*c*d**2)/(a**2*e*
*3 - b**2*c**3)))) + e*x/b
```

GIAC/XCAS [A] time = 0.216157, size = 257, normalized size = 1.4

$$\frac{xe}{b} + \frac{d\ln\left(\left|bx^3 + a\right|\right)}{3\,b} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae + \left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}$$
$$- \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae - \left(-ab^2\right)^{\frac{2}{3}}c\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2}$$
$$- \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] x*e/b + 1/3*d*ln(abs(b*x^3 + a))/b - 1/3*sqrt(3)*((-a*b^2)^(1/3)*
a*e + (-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a*b^2) - 1/6*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c
)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2) - 1/3*(b^3*c*(-
a/b)^(1/3) - a*b^2*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b
^3)
```

3.328 $\int \frac{c+dx+ex^2}{a+bx^3}\,dx$

Optimal. Leaf size=177

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*
d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b)
```

Rubi [A] time = 0.290702, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*
d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b)
```

Rubi in Sympy [A] time = 43.4738, size = 163, normalized size = 0.92

$$\frac{e\log\left(a+bx^3\right)}{3b}-\frac{\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}+\frac{\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] e*log(a + b*x**3)/(3*b) - (a**(1/3)*d - b**(1/3)*c)*log(a**(1/3)
+ b**(1/3)*x)/(3*a**(2/3)*b**(2/3)) + (a**(1/3)*d - b**(1/3)*c)*l
og(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*b*
*(2/3)) - sqrt(3)*(a**(1/3)*d + b**(1/3)*c)*atan(sqrt(3)*(a**(1/3
)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*b**(2/3))
```

Mathematica [A] time = 0.18897, size = 176, normalized size = 0.99

$$\frac{-\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\mathrm{ta}}{6ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(a^(1/3)*b^(1/3)*c - a
^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] - b^(1/3)*(a^(1/3)*b^(1/3)*c -
 a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*
e*Log[a + b*x^3])/(6*a*b)
```

Maple [A] time = 0.003, size = 200, normalized size = 1.1

$$\frac{c}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{6\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c\sqrt{3}}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d}{6\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d\sqrt{3}}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{e\ln\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*c/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*c/b/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*c/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*d/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))+1/6*d/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*d*3^
(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*e
*ln(b*x^3+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.25022, size = 160, normalized size = 0.9

$$\text{RootSum}\left(27t^3a^2b^3 - 27t^2a^2b^2e + t\left(9a^2be^2 + 9ab^2cd\right) - a^2e^3 - 3abcde + abd^3 - b^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2b^2d - 6ta^2bd\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3 - 27*_t**2*a**2*b**2*e + _t*(9*a**2*b*
e**2 + 9*a*b**2*c*d) - a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*
c**3, Lambda(_t, _t*log(x + (9*_t**2*a**2*b**2*d - 6*_t*a**2*b*d*
e + 3*_t*a*b**2*c**2 + a**2*d*e**2 - a*b*c**2*e + 2*a*b*c*d**2)/(
a*b*d**3 + b**2*c**3))))
```

GIAC/XCAS [A] time = 0.216631, size = 246, normalized size = 1.39

$$\frac{e\ln\left(\left|bx^3 + a\right|\right)}{3\,b} - \frac{\left(bd\left(-\frac{a}{b}\right)^{\frac{1}{3}} + bc\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*e*ln(abs(b*x^3 + a))/b - 1/3*(b*d*(-a/b)^(1/3) + b*c)*(-a/b)^
(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*((-a*b^2)^(1/
3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^
(2/3)*a*b^2*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.329 $\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx$

Optimal. Leaf size=184

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{c\log(x)}{a}$$

```
[Out] -(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + (c*Log[x])/a + ((b^(1/3
)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) -
((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*Log[a + b*x^3])/(3*a)
```

Rubi [A] time = 0.432794, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{c\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] -(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + (c*Log[x])/a + ((b^(1/3
)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) -
((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*Log[a + b*x^3])/(3*a)
```

Rubi in Sympy [A] time = 72.5788, size = 170, normalized size = 0.92

$$\frac{c\log(x)}{a}-\frac{c\log\left(a+bx^3\right)}{3a}-\frac{\left(\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}+\frac{\left(\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] c*log(x)/a - c*log(a + b*x**3)/(3*a) - (a**(1/3)*e - b**(1/3)*d)*
log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(2/3)) + (a**(1/3)*e -
b**(1/3)*d)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
6*a**(2/3)*b**(2/3)) - sqrt(3)*(a**(1/3)*e + b**(1/3)*d)*atan(sqr
t(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*b**(2/3)
)
```

Mathematica [A] time = 0.187235, size = 176, normalized size = 0.96

$$\frac{\left(a^{2/3}e-\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\left(\sqrt[3]{a}\sqrt[3]{b}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2b^{2/3}c\log\left(a+bx^3\right)-2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{\cdots}\right.}{6ab^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 6*b^(2/3)*c*Log[x] + 2*(a^(1/3)*b^(1/3)*
d - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] + (-(a^(1/3)*b^(1/3)*d) +
 a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^
(2/3)*c*Log[a + b*x^3])/(6*a*b^(2/3))
```

Maple [A] time = 0.008, size = 207, normalized size = 1.1

$$\frac{c\ln(x)}{a}+\frac{d}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{d\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{e}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\sqrt{3}e}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c\ln\left(bx^3+a\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x/(b*x^3+a),x)
```

```
[Out] c*ln(x)/a+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-1/6/b/(a/b)^(2/3)
*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/3/b/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3/b/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))*e+1/6/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*
e+1/3/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))*e-1/3*c*ln(b*x^3+a)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)*x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.215654, size = 267, normalized size = 1.45

$$-\frac{c\ln\left(\left|bx^3+a\right|\right)}{3\,a}+\frac{c\ln\left(\left|x\right|\right)}{a}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd-\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+a^2bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3d+\left(-ab^2\right)^{\frac{2}{3}}ab^2e\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)*x),x, algorithm="giac")
```

```
[Out] -1/3*c*ln(abs(b*x^3 + a))/a + c*ln(abs(x))/a + 1/3*sqrt(3)*((-a*b
^2)^(1/3)*b*d - (-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(a^2*b*(-a/b)^(1/3)*e + a^2*
b*d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/6*((-a*b^
2)^(1/3)*a*b^3*d + (-a*b^2)^(2/3)*a*b^2*e)*ln(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/(a^2*b^4)
```

3.330 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=192

$$-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a}-\frac{c}{ax}+\frac{d\log(x)}{a}$$

```
[Out] -(c/(a*x)) + ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(1/3)) + (d*Log[x])/a
+ ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b
^(1/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*a^(4/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a)
```

Rubi [A] time = 0.460485, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a}-\frac{c}{ax}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(1/3)) + (d*Log[x])/a
+ ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b
^(1/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*a^(4/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a)
```

Rubi in Sympy [A] time = 77.108, size = 175, normalized size = 0.91

$$-\frac{c}{ax}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3\right)}{3a}-\frac{\sqrt{3}\left(a^{\frac{2}{3}}e-b^{\frac{2}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{4}{3}}\sqrt[3]{b}}+\frac{\left(a^{\frac{2}{3}}e+b^{\frac{2}{3}}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{4}{3}}\sqrt[3]{b}}-\frac{\left(a^{\frac{2}{3}}e+b^{\frac{2}{3}}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{4}{3}}\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] -c/(a*x) + d*log(x)/a - d*log(a + b*x**3)/(3*a) - sqrt(3)*(a**(2/
3)*e - b**(2/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**
(1/3))/(3*a**(4/3)*b**(1/3)) + (a**(2/3)*e + b**(2/3)*c)*log(a**(
1/3) + b**(1/3)*x)/(3*a**(4/3)*b**(1/3)) - (a**(2/3)*e + b**(2/3)
*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(4/
3)*b**(1/3))
```

Mathematica [A] time = 0.501373, size = 184, normalized size = 0.96

$$-\frac{\frac{\left(a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-\frac{2\left(a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}a^{2/3}\left(a^{2/3}e-b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+2ad\log\left(a+b\right.}{6a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -((6*a*c)/x + (2*Sqrt[3]*a^(2/3)*(-(b^(2/3)*c) + a^(2/3)*e)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 6*a*d*Log[x] -
(2*(a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1
/3) + ((a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*a*d*Log[a + b*x^3])/(6*a^2)
```

Maple [A] time = 0.01, size = 216, normalized size = 1.1

$$\frac{d\ln(x)}{a}-\frac{c}{ax}+\frac{e}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{\sqrt{3}e}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c}{3a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{c}{6a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c\sqrt{3}}{3a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{d\ln\left(bx^3+a\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^2/(b*x^3+a),x)
```

```
[Out] d*ln(x)/a-c/a/x+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e-1/6/b/(a/b)
^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/3/b/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3/a/(a/b)^(1/3)*l
n(x+(a/b)^(1/3))*c-1/6/a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(
2/3))*c-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))*c-1/3*d*ln(b*x^3+a)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.216593, size = 279, normalized size = 1.45

$$-\frac{d\ln\left(\left|bx^3+a\right|\right)}{3\,a}+\frac{d\ln\left(\left|x\right|\right)}{a}-\frac{c}{ax}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae-\left(-ab^2\right)^{\frac{2}{3}}c\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b}+\frac{\left(ab^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2e+\left(-ab^2\right)^{\frac{2}{3}}b^2c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)*x^2),x, algorithm="giac")
```

```
[Out] -1/3*d*ln(abs(b*x^3 + a))/a + d*ln(abs(x))/a - c/(a*x) + 1/6*((-a
*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)*ln(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/(a^2*b) + 1/3*(a*b^2*c*(-a/b)^(1/3) - a^2*b*e)*(-a/b)^
(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/3*sqrt(3)*((-a*b^2)^(
1/3)*a*b^2*e + (-a*b^2)^(2/3)*b^2*c)*arctan(1/3*sqrt(3)*(2*x + (-
a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3)
```

3.331 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=203

$$\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{e\log\left(a+bx^3\right)}{3a}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

```
[Out] -c/(2*a*x^2) - d/(a*x) + (b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (
e*Log[x])/a - (b^(1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(5/3)) + (b^(2/3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)) - (e*Log[a +
b*x^3])/(3*a)
```

Rubi [A] time = 0.426781, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{e\log\left(a+bx^3\right)}{3a}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -c/(2*a*x^2) - d/(a*x) + (b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (
e*Log[x])/a - (b^(1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(5/3)) + (b^(2/3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)) - (e*Log[a +
b*x^3])/(3*a)
```

Rubi in Sympy [A] time = 67.2907, size = 184, normalized size = 0.91

$$-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{e\log\left(a+bx^3\right)}{3a}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{5}{3}}}-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{5}{3}}}+\frac{\sqrt{3}\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] -c/(2*a*x**2) - d/(a*x) + e*log(x)/a - e*log(a + b*x**3)/(3*a) +
b**(1/3)*(a**(1/3)*d - b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x)/(3*
a**(5/3)) - b**(1/3)*(a**(1/3)*d - b**(1/3)*c)*log(a**(2/3) - a**
(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(5/3)) + sqrt(3)*b**(1/3)
*(a**(1/3)*d + b**(1/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*
```

```
x/3)/a**(1/3))/(3*a**(5/3))
```

Mathematica [A] time = 0.373687, size = 192, normalized size = 0.95

$$\frac{\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(a^{2/3}d-\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan}{6a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a*c)/x^2 - (6*a*d)/x + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c
+ a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 6*a*e*
Log[x] + 2*b^(1/3)*(-(a^(1/3)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3)
 + b^(1/3)*x] + b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*e*Log[a + b*x^3])/(6*
a^2)
```

Maple [A] time = 0.01, size = 225, normalized size = 1.1

$$-\frac{d}{ax}+\frac{e\ln(x)}{a}-\frac{c}{2\,ax^2}-\frac{c}{3\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c}{6\,a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{c\sqrt{3}}{3\,a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{d}{3\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{d}{6\,a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{d\sqrt{3}}{3\,a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{e\ln\left(bx^3+a\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^3/(b*x^3+a),x)
```

```
[Out] -d/a/x+e*ln(x)/a-1/2*c/a/x^2-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*
c+1/6/a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-1/3/a/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/3/a/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-1/6/a/(a/b)^(1/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))*d-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*d-1/3*e*ln(b*x^3+a)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)*x^3),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.217112, size = 285, normalized size = 1.4

$$-\frac{e\ln\left(\left|bx^3+a\right|\right)}{3\,a}+\frac{e\ln\left(\left|x\right|\right)}{a}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}d\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b}+\frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}-\frac{2\,dx+c}{2\,ax^2}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)*x^3),x, algorithm="giac")
```

```
[Out] -1/3*e*ln(abs(b*x^3 + a))/a + e*ln(abs(x))/a - 1/6*((-a*b^2)^(1/3
)*b*c + (-a*b^2)^(2/3)*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^2*b) + 1/3*(a*b^2*d*(-a/b)^(1/3) + a*b^2*c)*(-a/b)^(1/3)*ln(a
bs(x - (-a/b)^(1/3)))/(a^3*b) - 1/2*(2*d*x + c)/(a*x^2) - 1/3*sqr
t(3)*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(2/3)*a*b^2*d)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3)
```

3.332 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=190

$$-\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{5/3}}-\frac{\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}}-\frac{c+dx+ex^2}{3b\,(a+bx^3)}$$

```
[Out] -(c + d*x + e*x^2)/(3*b*(a + b*x^3)) - ((b^(1/3)*d + 2*a^(1/3)*e)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^
(2/3)*b^(5/3)) + ((b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)
*x])/(9*a^(2/3)*b^(5/3)) - ((d - (2*a^(1/3)*e)/b^(1/3))*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*b^(4/3))
```

Rubi [A] time = 0.364204, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304

$$-\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{5/3}}-\frac{\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}}-\frac{c+dx+ex^2}{3b\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] -(c + d*x + e*x^2)/(3*b*(a + b*x^3)) - ((b^(1/3)*d + 2*a^(1/3)*e)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^
(2/3)*b^(5/3)) + ((b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)
*x])/(9*a^(2/3)*b^(5/3)) - ((d - (2*a^(1/3)*e)/b^(1/3))*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*b^(4/3))
```

Rubi in Sympy [A] time = 49.2203, size = 175, normalized size = 0.92

$$-\frac{c+dx+ex^2}{3b\,(a+bx^3)}+\frac{\left(\sqrt[3]{a}e-\frac{\sqrt[3]{b}d}{2}\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{9a^{\frac{2}{3}}b^{\frac{5}{3}}}-\frac{\left(2\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{2}{3}}b^{\frac{5}{3}}}-\frac{\sqrt{3}\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{2}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] -(c + d*x + e*x**2)/(3*b*(a + b*x**3)) + (a**(1/3)*e - b**(1/3)*d
/2)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(9*a**(2/
3)*b**(5/3)) - (2*a**(1/3)*e - b**(1/3)*d)*log(a**(1/3) + b**(1/3
)*x)/(9*a**(2/3)*b**(5/3)) - sqrt(3)*(2*a**(1/3)*e + b**(1/3)*d)*
atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(2/3)*
b**(5/3))
```

Mathematica [A] time = 0.275483, size = 174, normalized size = 0.92

$$\frac{\frac{\left(2\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{2\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}-\frac{2\sqrt{3}\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}-\frac{6b^{2/3}(c+x(d+ex))}{a+bx^3}}{18b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*(b^(1/3)
*d + 2*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^
(2/3) + (2*(b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^
(2/3) + ((-(b^(1/3)*d) + 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(5/3))
```

Maple [A] time = 0.012, size = 219, normalized size = 1.2

$$\frac{1}{bx^3+a}\left(-\frac{ex^2}{3\,b}-\frac{dx}{3\,b}-\frac{c}{3\,b}\right)+\frac{d}{9\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{18\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{d\sqrt{3}}{9\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{2\,e}{9\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{e}{9\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{2\,e\sqrt{3}}{9\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (-1/3*e*x^2/b-1/3*d*x/b-1/3*c/b)/(b*x^3+a)+1/9/b^2/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))*d-1/18/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)
^(2/3))*d+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*d-2/9/b^2*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*e/
(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9/b^2*e*3^(1/2)/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 4.31458, size = 109, normalized size = 0.57

$$\operatorname{RootSum}\left(729t^3a^2b^5 + 54tab^2de + 8ae^3 - bd^3, \left(t \mapsto t\log\left(x + \frac{162t^2a^2b^3e + 9tab^2d^2 + 8ade^2}{8ae^3 + bd^3}\right)\right)\right) - \frac{c + dx + ex^2}{3ab + 3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**2*b**5 + 54*_t*a*b**2*d*e + 8*a*e**3 - b*d**
3, Lambda(_t, _t*log(x + (162*_t**2*a**2*b**3*e + 9*_t*a*b**2*d**
2 + 8*a*d*e**2)/(8*a*e**3 + b*d**3)))) - (c + d*x + e*x**2)/(3*a*
b + 3*b**2*x**3)
```

GIAC/XCAS [A] time = 0.216969, size = 258, normalized size = 1.36

$$-\frac{\left(2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab} - \frac{x^2e + dx + c}{3\,(bx^3 + a)b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd - 2\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2d + 2\left(-ab^2\right)^{\frac{2}{3}}abe\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(-a/b)^(1/3)*e + d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3))
)/(a*b) - 1/3*(x^2*e + d*x + c)/((b*x^3 + a)*b) + 1/9*sqrt(3)*((-
a*b^2)^(1/3)*b*d - 2*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/18*((-a*b^2)^(1/3)*a*b^2*
d + 2*(-a*b^2)^(2/3)*a*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a^2*b^4)
```

3.333 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=200

$$\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(3*a*b*(a + b*x^3)) - ((b^(2/3)*c +
a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*
Sqrt[3]*a^(4/3)*b^(4/3)) - ((b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) +
 b^(1/3)*x])/(9*a^(4/3)*b^(4/3)) + ((b^(2/3)*c - a^(2/3)*e)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(4/3))
```

Rubi [A] time = 0.36624, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(3*a*b*(a + b*x^3)) - ((b^(2/3)*c +
a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*
Sqrt[3]*a^(4/3)*b^(4/3)) - ((b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) +
 b^(1/3)*x])/(9*a^(4/3)*b^(4/3)) + ((b^(2/3)*c - a^(2/3)*e)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(4/3))
```

Rubi in Sympy [A] time = 53.9996, size = 178, normalized size = 0.89

$$-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}+\frac{\left(a^{\frac{2}{3}}e-b^{\frac{2}{3}}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{4}{3}}b^{\frac{4}{3}}}-\frac{\left(a^{\frac{2}{3}}e-b^{\frac{2}{3}}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{4}{3}}b^{\frac{4}{3}}}-\frac{\sqrt{3}\left(a^{\frac{2}{3}}e+b^{\frac{2}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{4}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] -x*(a*e - b*c*x - b*d*x**2)/(3*a*b*(a + b*x**3)) + (a**(2/3)*e -
b**(2/3)*c)*log(a**(1/3) + b**(1/3)*x)/(9*a**(4/3)*b**(4/3)) - (a
**(2/3)*e - b**(2/3)*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(
2/3)*x**2)/(18*a**(4/3)*b**(4/3)) - sqrt(3)*(a**(2/3)*e + b**(2/3
)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(
4/3)*b**(4/3))
```

Mathematica [A] time = 0.45468, size = 186, normalized size = 0.93

$$\frac{-\left(a^{4/3}\sqrt[3]{b}e-a^{2/3}bc\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\left(a^{4/3}\sqrt[3]{b}e-a^{2/3}bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\left(a^{2/3}bc+a^{4/3}\sqrt[3]{b}e\right)\tan}{18a^2b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a*b^(2/3)*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) - 2*Sqrt[3
]*(a^(2/3)*b*c + a^(4/3)*b^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] + 2*(-(a^(2/3)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(1/
3) + b^(1/3)*x] - (-(a^(2/3)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^2*b^(5/3))
```

Maple [A] time = 0.01, size = 228, normalized size = 1.1

$$\frac{1}{bx^3+a}\left(\frac{cx^2}{3a}-\frac{ex}{3b}-\frac{d}{3b}\right)+\frac{e}{9b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{18b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{e\sqrt{3}}{9b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{c}{18ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{c\sqrt{3}}{9ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (1/3/a*c*x^2-1/3*e*x/b-1/3/b*d)/(b*x^3+a)+1/9/b^2*e/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))-1/18/b^2*e/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b
)^(2/3))+1/9/b^2*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))-1/9/b/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/18/b/a/(a
/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+1/9/b/a*3^(1/2)/(a/
b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.5051, size = 124, normalized size = 0.62

$$\operatorname{RootSum}\left(729t^3a^4b^4 + 27ta^2b^2ce - a^2e^3 + b^2c^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^3b^3c + 9ta^3be^2 + 2abc^2e}{a^2e^3 + b^2c^3}\right)\right)\right) + \frac{-ad - aex + bcx^2}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**4*b**4 + 27*_t*a**2*b**2*c*e - a**2*e**3 + b
**2*c**3, Lambda(_t, _t*log(x + (81*_t**2*a**3*b**3*c + 9*_t*a**3
*b*e**2 + 2*a*b*c**2*e)/(a**2*e**3 + b**2*c**3)))) + (-a*d - a*e*
x + b*c*x**2)/(3*a**2*b + 3*a*b**2*x**3)
```

GIAC/XCAS [A] time = 0.215455, size = 273, normalized size = 1.36

$$-\frac{\left(bc\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae - \left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{bcx^2 - axe - ad}{3\left(bx^3 + a\right)ab} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2e + \left(-ab^2\right)^{\frac{2}{3}}b^2c\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(b*c*(-a/b)^(1/3) + a*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3
)))/(a^2*b) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2)
+ 1/3*(b*c*x^2 - a*x*e - a*d)/((b*x^3 + a)*a*b) + 1/18*((-a*b^2)^
(1/3)*a*b^2*e + (-a*b^2)^(2/3)*b^2*c)*ln(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/(a^2*b^4)
```

3.334 $\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=199

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

```
[Out] -(a*e - b*x*(c + d*x))/(3*a*b*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1
/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt
[3]*a^(5/3)*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b
^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3))
```

Rubi [A] time = 0.314682, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] -(a*e - b*x*(c + d*x))/(3*a*b*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1
/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt
[3]*a^(5/3)*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b
^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3))
```

Rubi in Sympy [A] time = 46.5138, size = 178, normalized size = 0.89

$$-\frac{ae-bx\left(c+dx\right)}{3ab\left(a+bx^3\right)}+\frac{\left(\frac{\sqrt[3]{a}d}{2}-\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] -(a*e - b*x*(c + d*x))/(3*a*b*(a + b*x**3)) + (a**(1/3)*d/2 - b**
(1/3)*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(9*a
**(5/3)*b**(2/3)) - (a**(1/3)*d - 2*b**(1/3)*c)*log(a**(1/3) + b*
*(1/3)*x)/(9*a**(5/3)*b**(2/3)) - sqrt(3)*(a**(1/3)*d + 2*b**(1/3
)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(
5/3)*b**(2/3))
```

Mathematica [A] time = 0.382753, size = 189, normalized size = 0.95

$$\frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\left(4\sqrt[3]{a}b^{2/3}c-2a^{2/3}\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{6a(bx(c+dx)-ae)}{a+bx^3}-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{}}{18a^2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] ((6*a*(-(a*e) + b*x*(c + d*x)))/(a + b*x^3) - 2*Sqrt[3]*a^(1/3)*b
^(1/3)*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]] + (4*a^(1/3)*b^(2/3)*c - 2*a^(2/3)*b^(1/3)*d)*Log[a^(
1/3) + b^(1/3)*x] + a^(1/3)*b^(1/3)*(-2*b^(1/3)*c + a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^2*b)
```

Maple [A] time = 0.006, size = 253, normalized size = 1.3

$$\frac{cx}{3\,a\,(bx^3+a)}+\frac{2\,c}{9\,ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{9\,ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{2\,c\sqrt{3}}{9\,ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{dx^2}{3\,a\,(bx^3+a)}-\frac{d}{9\,ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{d}{18\,ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d\sqrt{3}}{9\,ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{e}{3\,b\,(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*c*x/a/(b*x^3+a)+2/9*c/a/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9*c
/a/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9*c/a/b/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d*x^2
/a/(b*x^3+a)-1/9*d/a/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18*d/a/b/(
a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/9*d/a*3^(1/2)/b/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/b/(b*x^3+a)
*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.61076, size = 116, normalized size = 0.58

$$\operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{-ae + bcx + bdx^2}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**
3, Lambda(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 +
 4*a*c*d**2)/(a*d**3 + 8*b*c**3)))) + (-a*e + b*c*x + b*d*x**2)/(
3*a**2*b + 3*a*b**2*x**3)
```

GIAC/XCAS [A] time = 0.216552, size = 266, normalized size = 1.34

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{bdx^2 + bcx - ae}{3\,(bx^3 + a)ab} + \frac{\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(d*(-a/b)^(1/3) + 2*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3))
)/a^2 + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/
3*(b*d*x^2 + b*c*x - a*e)/((b*x^3 + a)*a*b) + 1/18*(2*(-a*b^2)^(1
/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*ln(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/(a^3*b^4)
```

3.335 $\int \frac{c+dx+ex^2}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=222

$$-\frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^2}+\frac{c\log(x)}{a^2}$$

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) - ((2*b^(1/3)*d +
 a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3
*Sqrt[3]*a^(5/3)*b^(2/3)) + (c*Log[x])/a^2 + ((2*b^(1/3)*d - a^(1
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3
)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(18*a^(5/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 0.616815, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$-\frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^2}+\frac{c\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) - ((2*b^(1/3)*d +
 a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3
*Sqrt[3]*a^(5/3)*b^(2/3)) + (c*Log[x])/a^2 + ((2*b^(1/3)*d - a^(1
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3
)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(18*a^(5/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^2)
```

Rubi in Sympy [A] time = 58.7766, size = 175, normalized size = 0.79

$$\frac{x\left(\frac{c}{x}+d+ex\right)}{3a\left(a+bx^3\right)}-\frac{\left(\sqrt[3]{a}e-2\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}+\frac{\left(\sqrt[3]{a}e-2\sqrt[3]{b}d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] x*(c/x + d + e*x)/(3*a*(a + b*x**3)) - (a**(1/3)*e - 2*b**(1/3)*d
)*log(a**(1/3) + b**(1/3)*x)/(9*a**(5/3)*b**(2/3)) + (a**(1/3)*e
- 2*b**(1/3)*d)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**
2)/(18*a**(5/3)*b**(2/3)) - sqrt(3)*(a**(1/3)*e + 2*b**(1/3)*d)*a
tan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*b
**(2/3))
```

Mathematica [A] time = 0.329429, size = 199, normalized size = 0.9

$$\frac{\frac{\left(a^{2/3}e-2\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6a(c+x(d+ex))}{a+bx^3}}{18a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((6*a*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1
/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b
^(2/3) + 18*c*Log[x] + (2*(2*a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a
^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3)*d + a^(2/3)*e
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 6*c*L
og[a + b*x^3])/(18*a^2)
```

Maple [A] time = 0.017, size = 274, normalized size = 1.2

$$\frac{c\ln(x)}{a^2}+\frac{ex^2}{3a(bx^3+a)}+\frac{dx}{3a(bx^3+a)}+\frac{c}{3a(bx^3+a)}+\frac{2d}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{d}{9ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2d\sqrt{3}}{9ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{e}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{e}{18ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{\sqrt{3}e}{9ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c\ln(bx^3+a)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x/(b*x^3+a)^2,x)
```

```
[Out] c*ln(x)/a^2+1/3/a*x^2/(b*x^3+a)*e+1/3/a*x/(b*x^3+a)*d+1/3/a/(b*x^
3+a)*c+2/9/a/b*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9/a/b*d/(a/b)^(2
/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9/a/b*d/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a/b/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))*e+1/18/a/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)
^(2/3))*e+1/9/a/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*e-1/3*c*ln(b*x^3+a)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218173, size = 311, normalized size = 1.4

$$-\frac{c\ln\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{c\ln\left(|x|\right)}{a^2}+\frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}bd-\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+\frac{ax^2e+adx+ac}{3\,(bx^3+a)a^2}-\frac{\left(a^3b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+2\,a^3bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}+\frac{\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ab^3d+\left(-ab^2\right)^{\frac{2}{3}}ab^2e\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x),x, algorithm="giac")
```

```
[Out] -1/3*c*ln(abs(b*x^3 + a))/a^2 + c*ln(abs(x))/a^2 + 1/9*sqrt(3)*(2
*(-a*b^2)^(1/3)*b*d - (-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/3*(a*x^2*e + a*d*x + a
*c)/((b*x^3 + a)*a^2) - 1/9*(a^3*b*(-a/b)^(1/3)*e + 2*a^3*b*d)*(-
a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^5*b) + 1/18*(2*(-a*b^2)^(
1/3)*a*b^3*d + (-a*b^2)^(2/3)*a*b^2*e)*ln(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a^3*b^4)
```

3.336 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=231

$$-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(a^{2/3}e+2b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(2b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}}+\frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^2}-\frac{c}{a^2x}+\frac{d\log(x)}{a^2}$$

```
[Out] -(c/(a^2*x)) + (x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) +
(2*(2*b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(1/3)) + (d*Log[x])/a^2 + (2*
(2*b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^
(1/3)) - ((2*b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(9*a^(7/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a^
2)
```

Rubi [A] time = 0.680472, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(a^{2/3}e+2b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(2b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}}+\frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^2}-\frac{c}{a^2x}+\frac{d\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```



```
[Out] -(c/(a^2*x)) + (x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) +
(2*(2*b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(1/3)) + (d*Log[x])/a^2 + (2*
(2*b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^
(1/3)) - ((2*b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(9*a^(7/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a^
2)
```

Rubi in Sympy [A] time = 49.6303, size = 139, normalized size = 0.6

$$\frac{x\left(\frac{c}{x^2}+\frac{d}{x}+e\right)}{3a\left(a+bx^3\right)}+\frac{2e\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}\sqrt[3]{b}}-\frac{e\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{9a^{\frac{5}{3}}\sqrt[3]{b}}-\frac{2\sqrt{3}e\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] x*(c/x**2 + d/x + e)/(3*a*(a + b*x**3)) + 2*e*log(a**(1/3) + b**(
1/3)*x)/(9*a**(5/3)*b**(1/3)) - e*log(a**(2/3) - a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(9*a**(5/3)*b**(1/3)) - 2*sqrt(3)*e*atan(sqr
t(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*b**(1/3)
)
```

Mathematica [A] time = 0.522439, size = 213, normalized size = 0.92

$$-\frac{\frac{\left(2a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-\frac{2\left(2a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}a^{2/3}\left(a^{2/3}e-2b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{3a(a(d+ex)-}{a+bx^3}}{9a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((9*a*c)/x - (3*a*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) + (2*S
qrt[3]*a^(2/3)*(-2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 9*a*d*Log[x] - (2*(2*a^(2/3)*b^(2
/3)*c + a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((2*a^(2/3
)*b^(2/3)*c + a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/b^(1/3) + 3*a*d*Log[a + b*x^3])/(9*a^3)
```

Maple [A] time = 0.021, size = 275, normalized size = 1.2

$$\frac{d\ln(x)}{a^2}-\frac{c}{a^2x}-\frac{bx^2c}{3a^2(bx^3+a)}+\frac{ex}{3a(bx^3+a)}+\frac{d}{3a(bx^3+a)}+\frac{2e}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{9ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\sqrt{3}e}{9ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{4c}{9a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{2c}{9a^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{4c\sqrt{3}}{9a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{d\ln\left(bx^3+a\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] d*ln(x)/a^2-1/a^2*c/x-1/3/a^2*b*x^2/(b*x^3+a)*c+1/3/a*x/(b*x^3+a)
*e+1/3/a/(b*x^3+a)*d+2/9/a/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e-1/9/
a/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+2/9/a/b/(a/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+4/9/a^2*
c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*c/(a/b)^(1/3)*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))-4/9/a^2*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*d*ln(b*x^3+a)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.216959, size = 328, normalized size = 1.42

$$-\frac{d\ln\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{d\ln\left(\left|x\right|\right)}{a^2}-\frac{4\,bcx^3-ax^2e-adx+3\,ac}{3\,(bx^4+ax)a^2}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae-2\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b}+\frac{2\,\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2e+2\,\left(-ab^2\right)^{\frac{2}{3}}b^2c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^3}+\frac{2\left(2\,a^2b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] -1/3*d*ln(abs(b*x^3 + a))/a^2 + d*ln(abs(x))/a^2 - 1/3*(4*b*c*x^3
 - a*x^2*e - a*d*x + 3*a*c)/((b*x^4 + a*x)*a^2) + 1/9*((-a*b^2)^(
1/3)*a*e - 2*(-a*b^2)^(2/3)*c)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^3*b) + 2/9*sqrt(3)*((-a*b^2)^(1/3)*a*b^2*e + 2*(-a*b^2)^
(2/3)*b^2*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3)
)/(a^3*b^3) + 2/9*(2*a^2*b^2*c*(-a/b)^(1/3) - a^3*b*e)*(-a/b)^(1/
3)*ln(abs(x - (-a/b)^(1/3)))/(a^5*b)
```

3.337 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}-\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^2}-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\log(x)}{a^2}$$

```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c + b*d*x + b*e*x^2))/(3*a^2*(
a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (e*Log
[x])/a^2 - (b^(1/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(
1/3)*x])/(9*a^(8/3)) + (b^(1/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)) - (e*Log[
a + b*x^3])/(3*a^2)
```

Rubi [A] time = 0.686778, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}-\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^2}-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c + b*d*x + b*e*x^2))/(3*a^2*(
a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (e*Log
[x])/a^2 - (b^(1/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(
1/3)*x])/(9*a^(8/3)) + (b^(1/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)) - (e*Log[
a + b*x^3])/(3*a^2)
```

Rubi in Sympy [A] time = 13.5804, size = 24, normalized size = 0.1

$$\frac{x\left(\frac{c}{x^3}+\frac{d}{x^2}+\frac{e}{x}\right)}{3a\left(a+bx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] x*(c/x**3 + d/x**2 + e/x)/(3*a*(a + b*x**3))
```

Mathematica [A] time = 0.328512, size = 221, normalized size = 0.91

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}c-4a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(4a^{2/3}d-5\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{6a(ae-bx(c+dx))}{a+bx^3}+2\sqrt{3}\sqrt[3]{}}{18a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a*c)/x^2 - (18*a*d)/x + (6*a*(a*e - b*x*(c + d*x)))/(a + b*x
^3) + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 18*a*e*Log[x] + 2*b^(1/3
)*(-5*a^(1/3)*b^(1/3)*c + 4*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] +
 b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 4*a^(2/3)*d)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2] - 6*a*e*Log[a + b*x^3])/(18*a^3)
```

Maple [A] time = 0.019, size = 276, normalized size = 1.1

$$-\frac{d}{a^2x}+\frac{e\ln(x)}{a^2}-\frac{c}{2\,a^2x^2}-\frac{bx^2d}{3\,a^2\,(bx^3+a)}-\frac{bcx}{3\,a^2\,(bx^3+a)}+\frac{e}{3\,a\,(bx^3+a)}$$
$$-\frac{5\,c}{9\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5\,c}{18\,a^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{5\,c\sqrt{3}}{9\,a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{4\,d}{9\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{2\,d}{9\,a^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{4\,d\sqrt{3}}{9\,a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{e\ln\left(bx^3+a\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] -d/a^2/x+e*ln(x)/a^2-1/2*c/a^2/x^2-1/3/a^2*b*x^2/(b*x^3+a)*d-1/3/
a^2*b*x/(b*x^3+a)*c+1/3/a/(b*x^3+a)*e-5/9/a^2*c/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))+5/18/a^2*c/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2
/3))-5/9/a^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))+4/9/a^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-2/9/a^2/(a/b)^(
1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-4/9/a^2*3^(1/2)/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3*e*ln(b*x^3+a)/a
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.216707, size = 344, normalized size = 1.42

$$-\frac{e\ln\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{e\ln\left(\left|x\right|\right)}{a^2}-\frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}bc+4\,\left(-ab^2\right)^{\frac{2}{3}}d\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b}+\frac{\left(4\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}-\frac{8\,bdx^4+5\,bcx^3-2\,ax^2e+6\,adx+3\,ac}{6\,(bx^3+a)a^2x^2}-\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ab^3c-4\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] -1/3*e*ln(abs(b*x^3 + a))/a^2 + e*ln(abs(x))/a^2 - 1/18*(5*(-a*b^
2)^(1/3)*b*c + 4*(-a*b^2)^(2/3)*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/(a^3*b) + 1/9*(4*a^2*b^2*d*(-a/b)^(1/3) + 5*a^2*b^2*c)*
(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/6*(8*b*d*x^4 +
 5*b*c*x^3 - 2*a*x^2*e + 6*a*d*x + 3*a*c)/((b*x^3 + a)*a^2*x^2) -
 1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*a*b^3*c - 4*(-a*b^2)^(2/3)*a*b^2*d
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^3)
```

3.338 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx$

Optimal. Leaf size=262

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}+\frac{2bc\log\left(a+bx^3\right)}{3a^3}-\frac{2bc\log(x)}{a^3}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}$$

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d + b*e*x - (b
^2*c*x^2)/a))/(3*a^2*(a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*d + 4*a^(
1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqr
t[3]*a^(8/3)) - (2*b*c*Log[x])/a^3 - (b^(1/3)*(5*b^(1/3)*d - 4*a^
(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)) + (b^(1/3)*(5*b^(1
/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(18*a^(8/3)) + (2*b*c*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.792762, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}+\frac{2bc\log\left(a+bx^3\right)}{3a^3}-\frac{2bc\log(x)}{a^3}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d + b*e*x - (b
^2*c*x^2)/a))/(3*a^2*(a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*d + 4*a^(
1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqr
t[3]*a^(8/3)) - (2*b*c*Log[x])/a^3 - (b^(1/3)*(5*b^(1/3)*d - 4*a^
(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)) + (b^(1/3)*(5*b^(1
/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(18*a^(8/3)) + (2*b*c*Log[a + b*x^3])/(3*a^3)
```

Rubi in Sympy [A] time = 13.7963, size = 26, normalized size = 0.1

$$\frac{x\left(\frac{c}{x^4}+\frac{d}{x^3}+\frac{e}{x^2}\right)}{3a\left(a+bx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] x*(c/x**4 + d/x**3 + e/x**2)/(3*a*(a + b*x**3))
```

Mathematica [A] time = 0.3521, size = 225, normalized size = 0.86

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}d-4a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(4a^{2/3}e-5\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\frac{6ab(c+x(d+ex))}{a+bx^3}+12bc\log}{18a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x - (6*a*b*(c + x*(d + e*x
)))/(a + b*x^3) + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*d + 4*a^(1
/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 36*b*c*Log[x
] + 2*b^(1/3)*(-5*a^(1/3)*b^(1/3)*d + 4*a^(2/3)*e)*Log[a^(1/3) +
b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*d - 4*a^(2/3)*e)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 12*b*c*Log[a + b*x^3])/(
18*a^3)
```

Maple [A] time = 0.02, size = 289, normalized size = 1.1

$$-\frac{d}{2\,a^2x^2}-\frac{e}{a^2x}-\frac{c}{3\,a^2x^3}-2\,\frac{bc\ln(x)}{a^3}-\frac{bex^2}{3\,a^2\,(bx^3+a)}-\frac{bxd}{3\,a^2\,(bx^3+a)}-\frac{bc}{3\,a^2\,(bx^3+a)}-\frac{5\,d}{9\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5\,d}{18\,a^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5\,d\sqrt{3}}{9\,a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{4\,e}{9\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{2\,e}{9\,a^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{4\,e\sqrt{3}}{9\,a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{2\,bc\ln\left(bx^3+a\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/2*d/a^2/x^2-e/a^2/x-1/3*c/a^2/x^3-2*b*c*ln(x)/a^3-1/3/a^2*x^2/
(b*x^3+a)*b*e-1/3/a^2*x/(b*x^3+a)*b*d-1/3/a^2*b/(b*x^3+a)*c-5/9/a
^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+5/18/a^2/(a/b)^(2/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))*d-5/9/a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+4/9/a^2*e/(a/b)^(1/3)*ln(x+(a/b)^
(1/3))-2/9/a^2*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-4/
9/a^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))+2/3*b*c*ln(b*x^3+a)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.21582, size = 373, normalized size = 1.42

$$\frac{2\,bc\ln\left(\left|bx^3+a\right|\right)}{3\,a^3}-\frac{2\,bc\ln\left(\left|x\right|\right)}{a^3}-\frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}bd+4\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b}-\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ab^3d-4\,\left(-ab^2\right)^{\frac{2}{3}}ab^2e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b^3}+\frac{\left(4\,a^4b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5\,a^4b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7b}-\frac{8\,abx^5e+5\,abdx^4+4\,abcx^3+6\,a^2x^2e+3\,a^2dx+2\,a^2c}{6\,(bx^3+a)a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] 2/3*b*c*ln(abs(b*x^3 + a))/a^3 - 2*b*c*ln(abs(x))/a^3 - 1/18*(5*(
-a*b^2)^(1/3)*b*d + 4*(-a*b^2)^(2/3)*e)*ln(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/(a^3*b) - 1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*a*b^3*d -
4*(-a*b^2)^(2/3)*a*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^4*b^3) + 1/9*(4*a^4*b^2*(-a/b)^(1/3)*e + 5*a^4*
b^2*d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^7*b) - 1/6*(8*a*
b*x^5*e + 5*a*b*d*x^4 + 4*a*b*c*x^3 + 6*a^2*x^2*e + 3*a^2*d*x + 2
*a^2*c)/((b*x^3 + a)*a^3*x^3)
```

3.339 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=215

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{5/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}}-\frac{c+dx+ex^2}{6b\,(a+bx^3)^2}+\frac{x(d+2ex)}{18ab\,(a+bx^3)}$$

```
[Out] -(c + d*x + e*x^2)/(6*b*(a + b*x^3)^2) + (x*(d + 2*e*x))/(18*a*b*
(a + b*x^3)) - ((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)
*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(5/3)) -
((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(54*a^(5/3)*b^(4/3))
```

Rubi [A] time = 0.441702, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{5/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}}-\frac{c+dx+ex^2}{6b\,(a+bx^3)^2}+\frac{x(d+2ex)}{18ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] -(c + d*x + e*x^2)/(6*b*(a + b*x^3)^2) + (x*(d + 2*e*x))/(18*a*b*
(a + b*x^3)) - ((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)
*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(5/3)) -
((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(54*a^(5/3)*b^(4/3))
```

Rubi in Sympy [A] time = 61.8711, size = 192, normalized size = 0.89

$$-\frac{c+dx+ex^2}{6b\,(a+bx^3)^2}+\frac{x\,(d+2ex)}{18ab\,(a+bx^3)}-\frac{\left(\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{5}{3}}b^{\frac{5}{3}}}+\frac{\left(\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{5}{3}}b^{\frac{5}{3}}}-\frac{\sqrt{3}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{5}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] -(c + d*x + e*x**2)/(6*b*(a + b*x**3)**2) + x*(d + 2*e*x)/(18*a*b
*(a + b*x**3)) - (a**(1/3)*e - b**(1/3)*d)*log(a**(1/3) + b**(1/3
)*x)/(27*a**(5/3)*b**(5/3)) + (a**(1/3)*e - b**(1/3)*d)*log(a**(2
/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(54*a**(5/3)*b**(5/3))
 - sqrt(3)*(a**(1/3)*e + b**(1/3)*d)*atan(sqrt(3)*(a**(1/3)/3 - 2
```

```
*b**(1/3)*x/3)/a**(1/3))/(27*a**(5/3)*b**(5/3))
```

Mathematica [A] time = 0.4436, size = 198, normalized size = 0.92

$$\frac{\frac{\left(\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{2\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{2\sqrt{3}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}-\frac{9b^{2/3}(c+x(d+ex))}{(a+bx^3)^2}+\frac{3b^{2/3}x(d\cdots}{a(a+b\cdots}}{54b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] ((3*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)) - (9*b^(2/3)*(c + x*(d
 + e*x)))/(a + b*x^3)^2 - (2*Sqrt[3]*(b^(1/3)*d + a^(1/3)*e)*ArcT
an[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(1/3)*d
- a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + ((-(b^(1/3)*d) +
 a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/
3))/(54*b^(5/3))
```

Maple [A] time = 0.013, size = 255, normalized size = 1.2

$$\frac{1}{(bx^3+a)^2}\left(\frac{ex^5}{9a}+\frac{dx^4}{18a}-\frac{ex^2}{18b}-\frac{dx}{9b}-\frac{c}{6b}\right)+\frac{d}{27ab^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{54ab^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{d\sqrt{3}}{27ab^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{27ab^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{e}{54ab^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\sqrt{3}e}{27ab^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (1/9/a*e*x^5+1/18*d/a*x^4-1/18*e*x^2/b-1/9*d*x/b-1/6*c/b)/(b*x^3+
a)^2+1/27/b^2/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-1/54/b^2/a/(a/b)^
(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/27/b^2/a/(a/b)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/27/a/b^2/(a/b
)^(1/3)*ln(x+(a/b)^(1/3))*e+1/54/a/b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)
^(1/3)+(a/b)^(2/3))*e+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 10.6732, size = 148, normalized size = 0.69

$$\operatorname{RootSum}\left(19683t^3a^5b^5 + 81ta^2b^2de + ae^3 - bd^3, \left(t \mapsto t\log\left(x + \frac{729t^2a^4b^3e + 27ta^2b^2d^2 + 2ade^2}{ae^3 + bd^3}\right)\right)\right) + \frac{-3ac - 2adx - aex^2 + bdx^4 + 2bex^5}{18a^3b + 36a^2b^2x^3 + 18ab^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**5*b**5 + 81*_t*a**2*b**2*d*e + a*e**3 - b*
d**3, Lambda(_t, _t*log(x + (729*_t**2*a**4*b**3*e + 27*_t*a**2*b
**2*d**2 + 2*a*d*e**2)/(a*e**3 + b*d**3)))) + (-3*a*c - 2*a*d*x -
 a*e*x**2 + b*d*x**4 + 2*b*e*x**5)/(18*a**3*b + 36*a**2*b**2*x**3
 + 18*a*b**3*x**6)
```

GIAC/XCAS [A] time = 0.217986, size = 288, normalized size = 1.34

$$-\frac{\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd - \left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3} + \frac{2\,bx^5e + bdx^4 - ax^2e - 2\,adx - 3\,ac}{18\left(bx^3 + a\right)^2ab} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2d + \left(-ab^2\right)^{\frac{2}{3}}abe\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/27*((-a/b)^(1/3)*e + d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))
/(a^2*b) + 1/27*sqrt(3)*((-a*b^2)^(1/3)*b*d - (-a*b^2)^(2/3)*e)*a
rctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) +
1/18*(2*b*x^5*e + b*d*x^4 - a*x^2*e - 2*a*d*x - 3*a*c)/((b*x^3 +
a)^2*a*b) + 1/54*((-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(2/3)*a*b*e)*
ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)
```

3.340 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=239

$$\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}}-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}}-\frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)}-\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(6*a*b*(a + b*x^3)^2) - (3*a*d - x*(
a*e + 4*b*c*x))/(18*a^2*b*(a + b*x^3)) - ((2*b^(2/3)*c + a^(2/3)*
e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*
a^(7/3)*b^(4/3)) - ((2*b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/
3)*x])/(27*a^(7/3)*b^(4/3)) + ((2*b^(2/3)*c - a^(2/3)*e)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(4/3))
```

Rubi [A] time = 0.460489, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}}-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}}-\frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)}-\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(6*a*b*(a + b*x^3)^2) - (3*a*d - x*(
a*e + 4*b*c*x))/(18*a^2*b*(a + b*x^3)) - ((2*b^(2/3)*c + a^(2/3)*
e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*
a^(7/3)*b^(4/3)) - ((2*b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/
3)*x])/(27*a^(7/3)*b^(4/3)) + ((2*b^(2/3)*c - a^(2/3)*e)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(4/3))
```

Rubi in Sympy [A] time = 73.7936, size = 216, normalized size = 0.9

$$-\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}-\frac{3ad-x\left(ae+4bcx\right)}{18a^2b\left(a+bx^3\right)}+\frac{\left(a^{\frac{2}{3}}e-2b^{\frac{2}{3}}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{7}{3}}b^{\frac{4}{3}}}-\frac{\left(a^{\frac{2}{3}}e-2b^{\frac{2}{3}}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{7}{3}}b^{\frac{4}{3}}}-\frac{\sqrt{3}\left(a^{\frac{2}{3}}e+2b^{\frac{2}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{7}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] -x*(a*e - b*c*x - b*d*x**2)/(6*a*b*(a + b*x**3)**2) - (3*a*d - x*
(a*e + 4*b*c*x))/(18*a**2*b*(a + b*x**3)) + (a**(2/3)*e - 2*b**(2
/3)*c)*log(a**(1/3) + b**(1/3)*x)/(27*a**(7/3)*b**(4/3)) - (a**(2
/3)*e - 2*b**(2/3)*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/
3)*x**2)/(54*a**(7/3)*b**(4/3)) - sqrt(3)*(a**(2/3)*e + 2*b**(2/3
)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a**
```

```
(7/3)*b**(4/3))
```

Mathematica [A] time = 0.556766, size = 214, normalized size = 0.9

$$\frac{\left(2a^{2/3}bc-a^{4/3}\sqrt[3]{b}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt{3}a^{2/3}\sqrt[3]{b}\left(a^{2/3}e+2b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(a^{4/3}\sqrt[3]{b}e-2a^{2/3}bc\right)}{54a^3b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] ((3*a*b^(2/3)*(4*b^2*c*x^5 - a^2*(3*d + 2*e*x) + a*b*x^2*(7*c + e
*x^2)))/(a + b*x^3)^2 - 2*Sqrt[3]*a^(2/3)*b^(1/3)*(2*b^(2/3)*c +
a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*a^
(2/3)*b*c + a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] + (2*a^(2
/3)*b*c - a^(4/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(54*a^3*b^(5/3))
```

Maple [A] time = 0.014, size = 256, normalized size = 1.1

$$\begin{aligned}&\frac{1}{(bx^3+a)^2}\left(\frac{2\,bcx^5}{9\,a^2}+\frac{ex^4}{18\,a}+\frac{7\,cx^2}{18\,a}-\frac{ex}{9\,b}-\frac{d}{6\,b}\right)\\&+\frac{e}{27\,ab^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{54\,ab^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\&+\frac{\sqrt{3}e}{27\,ab^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{2\,c}{27\,a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\&+\frac{c}{27\,a^2b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{2\,c\sqrt{3}}{27\,a^2b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (2/9/a^2*c*b*x^5+1/18/a*e*x^4+7/18/a*c*x^2-1/9*e*x/b-1/6/b*d)/(b*
x^3+a)^2+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e-1/54/a/b^2/(a
/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/27/a/b^2/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-2/27/b/a^2/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/27/b/a^2/(a/b)^(1/3)*ln(x^2-x*(
a/b)^(1/3)+(a/b)^(2/3))*c+2/27/b/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 7.1414, size = 170, normalized size = 0.71

$$\mathrm{RootSum}\left(19683t^3a^7b^4 + 162ta^3b^2ce - a^2e^3 + 8b^2c^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^5b^3c + 27ta^4be^2 + 8abc^2e}{a^2e^3 + 8b^2c^3}\right)\right)\right) + \frac{-3a^2d - 2a^2ex + 7abcx^2 + abex^4 + 4b^2cx^5}{18a^4b + 36a^3b^2x^3 + 18a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**7*b**4 + 162*_t*a**3*b**2*c*e - a**2*e**3
+ 8*b**2*c**3, Lambda(_t, _t*log(x + (1458*_t**2*a**5*b**3*c + 27
*_t*a**4*b*e**2 + 8*a*b*c**2*e)/(a**2*e**3 + 8*b**2*c**3)))) + (-
3*a**2*d - 2*a**2*e*x + 7*a*b*c*x**2 + a*b*e*x**4 + 4*b**2*c*x**5
)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*x**6)
```

GIAC/XCAS [A] time = 0.217957, size = 306, normalized size = 1.28

$$-\frac{\left(2\,bc\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae - 2\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2} + \frac{4\,b^2cx^5 + abx^4e + 7\,abcx^2 - 2\,a^2xe - 3\,a^2d}{18\,(bx^3 + a)^2a^2b} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2e + 2\,\left(-ab^2\right)^{\frac{2}{3}}b^2c\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*b*c*(-a/b)^(1/3) + a*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(
1/3)))/(a^3*b) + 1/27*sqrt(3)*((-a*b^2)^(1/3)*a*e - 2*(-a*b^2)^(2
/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3
*b^2) + 1/18*(4*b^2*c*x^5 + a*b*x^4*e + 7*a*b*c*x^2 - 2*a^2*x*e -
 3*a^2*d)/((b*x^3 + a)^2*a^2*b) + 1/54*((-a*b^2)^(1/3)*a*b^2*e +
2*(-a*b^2)^(2/3)*b^2*c)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(
a^3*b^4)
```

3.341 $\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

```
[Out] (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(6
*a*b*(a + b*x^3)^2) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3))
+ ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8
/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rubi [A] time = 0.415306, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(6
*a*b*(a + b*x^3)^2) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3))
+ ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8
/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rubi in Sympy [A] time = 59.8174, size = 207, normalized size = 0.92

$$-\frac{ae-bx\left(c+dx\right)}{6ab\left(a+bx^3\right)^2}+\frac{x\left(5c+4dx\right)}{18a^2\left(a+bx^3\right)}-\frac{\left(2\sqrt[3]{a}d-5\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{8}{3}}b^{\frac{2}{3}}}+\frac{\left(2\sqrt[3]{a}d-5\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{8}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{8}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] -(a*e - b*x*(c + d*x))/(6*a*b*(a + b*x**3)**2) + x*(5*c + 4*d*x)/
(18*a**2*(a + b*x**3)) - (2*a**(1/3)*d - 5*b**(1/3)*c)*log(a**(1/
3) + b**(1/3)*x)/(27*a**(8/3)*b**(2/3)) + (2*a**(1/3)*d - 5*b**(1
/3)*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(54*a*
*(8/3)*b**(2/3)) - sqrt(3)*(2*a**(1/3)*d + 5*b**(1/3)*c)*atan(sqr
t(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a**(8/3)*b**(2/3
```

```
))
```

Mathematica [A] time = 0.562098, size = 213, normalized size = 0.95

$$\frac{\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{a}d-5\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{3a\left(-3a^2e+abx(8c+7dx)+b^2x^4\right.}{(a+bx^3)^2}}{54a^3b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] ((3*a*(-3*a^2*e + b^2*x^4*(5*c + 4*d*x) + a*b*x*(8*c + 7*d*x)))/(
a + b*x^3)^2 - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)
*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*a^
(1/3)*b^(1/3)*c - 2*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)
*b^(1/3)*(-5*b^(1/3)*c + 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^3*b)
```

Maple [A] time = 0.006, size = 308, normalized size = 1.4

$$\frac{cx}{6\,a\left(bx^3+a\right)^2}+\frac{5\,cx}{18\,a^2\left(bx^3+a\right)}+\frac{5\,c}{27\,a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5\,c}{54\,a^2b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5\,c\sqrt{3}}{27\,a^2b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{dx^2}{6\,a\left(bx^3+a\right)^2}+\frac{2\,dx^2}{9\,a^2\left(bx^3+a\right)}-\frac{2\,d}{27\,a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d}{27\,a^2b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{2\,d\sqrt{3}}{27\,a^2b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{ex^3}{6\,a\left(bx^3+a\right)^2}-\frac{e}{6\,ab\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*c/a*x/(b*x^3+a)^2+5/18*c/a^2*x/(b*x^3+a)+5/27*c/a^2/b/(a/b)^(
2/3)*ln(x+(a/b)^(1/3))-5/54*c/a^2/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))+5/27*c/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))+1/6*d/a*x^2/(b*x^3+a)^2+2/9*d/a^2*x^2/(b
*x^3+a)-2/27*d/a^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27*d/a^2/b/(
a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/27*d/a^2*3^(1/2)/b
/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/6*e/a*x^3/
(b*x^3+a)^2-1/6*e/a/b/(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 4.37777, size = 163, normalized size = 0.72

$$\mathrm{RootSum}\left(19683t^3a^8b^2 + 810ta^3bcd + 8ad^3 - 125bc^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^6bd + 675ta^3bc^2 + 40acd^2}{8ad^3 + 125bc^3}\right)\right)\right) + \frac{-3a^2e + 8abcx + 7abdx^2 + 5b^2cx^4 + 4b^2dx^5}{18a^4b + 36a^3b^2x^3 + 18a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 12
5*b*c**3, Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**
3*b*c**2 + 40*a*c*d**2)/(8*a*d**3 + 125*b*c**3)))) + (-3*a**2*e +
 8*a*b*c*x + 7*a*b*d*x**2 + 5*b**2*c*x**4 + 4*b**2*d*x**5)/(18*a*
*4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*x**6)
```

GIAC/XCAS [A] time = 0.220394, size = 301, normalized size = 1.34

$$-\frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}bc - 2\,\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2} + \frac{4\,b^2dx^5 + 5\,b^2cx^4 + 7\,abdx^2 + 8\,abcx - 3\,a^2e}{18\,(bx^3+a)^2a^2b} + \frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ab^3c + 2\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/
3)))/a^3 + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(2/3)*
d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2
) + 1/18*(4*b^2*d*x^5 + 5*b^2*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3
*a^2*e)/((b*x^3 + a)^2*a^2*b) + 1/54*(5*(-a*b^2)^(1/3)*a*b^3*c +
2*(-a*b^2)^(2/3)*a*b^2*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^4*b^4)
```

3.342 $\int \frac{c+dx+ex^2}{x(a+bx^3)^3} \, dx$

Optimal. Leaf size=257

$$-\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+\frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{c\log(x)}{a^3}+\frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2}$$

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*d + 4
*a*e*x - 9*b*c*x^2))/(18*a^3*(a + b*x^3)) - ((5*b^(1/3)*d + 2*a^(
1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqr
t[3]*a^(8/3)*b^(2/3)) + (c*Log[x])/a^3 + ((5*b^(1/3)*d - 2*a^(1/3
)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)
*d - 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(54*a^(8/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.816306, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$-\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+\frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{c\log(x)}{a^3}+\frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*d + 4
*a*e*x - 9*b*c*x^2))/(18*a^3*(a + b*x^3)) - ((5*b^(1/3)*d + 2*a^(
1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqr
t[3]*a^(8/3)*b^(2/3)) + (c*Log[x])/a^3 + ((5*b^(1/3)*d - 2*a^(1/3
)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)
*d - 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(54*a^(8/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rubi in Sympy [A] time = 73.9252, size = 204, normalized size = 0.79

$$\frac{x\left(\frac{c}{x}+d+ex\right)}{6a\left(a+bx^3\right)^2}+\frac{x\left(5d+4ex\right)}{18a^2\left(a+bx^3\right)}-\frac{\left(2\sqrt[3]{a}e-5\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{8}{3}}b^{\frac{2}{3}}}+\frac{\left(2\sqrt[3]{a}e-5\sqrt[3]{b}d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{8}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{8}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] x*(c/x + d + e*x)/(6*a*(a + b*x**3)**2) + x*(5*d + 4*e*x)/(18*a**
2*(a + b*x**3)) - (2*a**(1/3)*e - 5*b**(1/3)*d)*log(a**(1/3) + b*
*(1/3)*x)/(27*a**(8/3)*b**(2/3)) + (2*a**(1/3)*e - 5*b**(1/3)*d)*
log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(54*a**(8/3)*
b**(2/3)) - sqrt(3)*(2*a**(1/3)*e + 5*b**(1/3)*d)*atan(sqrt(3)*(a
**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a**(8/3)*b**(2/3))
```

Mathematica [A] time = 0.357454, size = 229, normalized size = 0.89

$$\frac{\frac{\left(2a^{2/3}e-5\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}d-2a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{9a^2(c+x(d+ex))}{(a+bx^3)^2}-\frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}}{54a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((9*a^2*(c + x*(d + e*x)))/(a + b*x^3)^2 + (3*a*(6*c + x*(5*d + 4
*e*x)))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(5*b^(1/3)*d + 2*a^(1/3)
*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*c*L
og[x] + (2*(5*a^(1/3)*b^(1/3)*d - 2*a^(2/3)*e)*Log[a^(1/3) + b^(1
/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*d + 2*a^(2/3)*e)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 18*c*Log[a + b*x
^3])/(54*a^3)
```

Maple [A] time = 0.019, size = 331, normalized size = 1.3

$$\frac{c\ln(x)}{a^3}+\frac{2\,bx^5e}{9\,a^2\,(bx^3+a)^2}+\frac{5\,bdx^4}{18\,a^2\,(bx^3+a)^2}+\frac{cx^3b}{3\,a^2\,(bx^3+a)^2}+\frac{7\,ex^2}{18\,a\,(bx^3+a)^2}+\frac{4\,dx}{9\,a\,(bx^3+a)^2}$$
$$+\frac{c}{2\,a\,(bx^3+a)^2}+\frac{5\,d}{27\,a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5\,d}{54\,a^2b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{5\,d\sqrt{3}}{27\,a^2b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{2\,e}{27\,a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{e}{27\,a^2b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{2\,\sqrt{3}e}{27\,a^2b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c\ln\left(bx^3+a\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x/(b*x^3+a)^3,x)
```

```
[Out] c*ln(x)/a^3+2/9/a^2/(b*x^3+a)^2*x^5*b*e+5/18/a^2/(b*x^3+a)^2*x^4*
b*d+1/3/a^2/(b*x^3+a)^2*x^3*c*b+7/18/a/(b*x^3+a)^2*x^2*e+4/9/a/(b
*x^3+a)^2*x*d+1/2/a/(b*x^3+a)^2*c+5/27/a^2/b/(a/b)^(2/3)*ln(x+(a/
b)^(1/3))*d-5/54/a^2/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))*d+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))*d-2/27/a^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e+1/27/a^2
/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+2/27/a^2/b*3^(
1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/3*c*
ln(b*x^3+a)/a^3
```

Maple [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218052, size = 359, normalized size = 1.4

$$-\frac{c\ln\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{c\ln\left(\left|x\right|\right)}{a^3}+\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}bd-2\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2}+\frac{4\,abx^5e+5\,abdx^4+6\,abcx^3+7\,a^2x^2e+8\,a^2dx+9\,a^2c}{18\,(bx^3+a)^2a^3}-\frac{\left(2\,a^4b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5\,a^4bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}+\frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ab^3d+2\,\left(-ab^2\right)^{\frac{2}{3}}ab^2e\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x),x, algorithm="giac")
```

```
[Out] -1/3*c*ln(abs(b*x^3 + a))/a^3 + c*ln(abs(x))/a^3 + 1/27*sqrt(3)*(
5*(-a*b^2)^(1/3)*b*d - 2*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/18*(4*a*b*x^5*e + 5
*a*b*d*x^4 + 6*a*b*c*x^3 + 7*a^2*x^2*e + 8*a^2*d*x + 9*a^2*c)/((b
*x^3 + a)^2*a^3) - 1/27*(2*a^4*b*(-a/b)^(1/3)*e + 5*a^4*b*d)*(-a/
b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^7*b) + 1/54*(5*(-a*b^2)^(1/
3)*a*b^3*d + 2*(-a*b^2)^(2/3)*a*b^2*e)*ln(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a^4*b^4)
```

3.343 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=267

$$-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}}+\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}}+\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}}+\frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^3}-\frac{c}{a^3x}+\frac{d\log(x)}{a^3}+\frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2}$$

```
[Out] -(c/(a^3*x)) + (x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2)
+ (x*(5*a*e - 10*b*c*x - 9*b*d*x^2))/(18*a^3*(a + b*x^3)) + ((14*
b^(2/3)*c - 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(1/3)) + (d*Log[x])/a^3 + ((14*b
^(2/3)*c + 5*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^
(1/3)) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(1/3)) - (d*Log[a + b*x^3])/
(3*a^3)
```

Rubi [A] time = 0.908881, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}}+\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}}+\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}}+\frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^3}-\frac{c}{a^3x}+\frac{d\log(x)}{a^3}+\frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) + (x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2)
+ (x*(5*a*e - 10*b*c*x - 9*b*d*x^2))/(18*a^3*(a + b*x^3)) + ((14*
b^(2/3)*c - 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(1/3)) + (d*Log[x])/a^3 + ((14*b
^(2/3)*c + 5*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^
(1/3)) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(1/3)) - (d*Log[a + b*x^3])/
(3*a^3)
```

Rubi in Sympy [A] time = 57.4305, size = 160, normalized size = 0.6

$$\frac{x\left(\frac{c}{x^2}+\frac{d}{x}+e\right)}{6a\left(a+bx^3\right)^2}+\frac{5ex}{18a^2\left(a+bx^3\right)}+\frac{5e\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{8}{3}}\sqrt[3]{b}}-\frac{5e\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{8}{3}}\sqrt[3]{b}}-\frac{5\sqrt{3}e\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{8}{3}}\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] x*(c/x**2 + d/x + e)/(6*a*(a + b*x**3)**2) + 5*e*x/(18*a**2*(a +
b*x**3)) + 5*e*log(a**(1/3) + b**(1/3)*x)/(27*a**(8/3)*b**(1/3))
- 5*e*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(54*a**
(8/3)*b**(1/3)) - 5*sqrt(3)*e*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/
3)*x/3)/a**(1/3))/(27*a**(8/3)*b**(1/3))
```

Mathematica [A] time = 0.437054, size = 248, normalized size = 0.93

$$\frac{-\frac{\left(14a^{2/3}b^{2/3}c+5a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+\frac{2\left(14a^{2/3}b^{2/3}c+5a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\sqrt{3}a^{2/3}\left(5a^{2/3}e-14b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{9a^2(a(\cdots}{(\cdots}}{54a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a*c)/x + (3*a*(6*a*d + 5*a*e*x - 10*b*c*x^2))/(a + b*x^3) +
 (9*a^2*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3)^2 - (2*Sqrt[3]*a^
(2/3)*(-14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/b^(1/3) + 54*a*d*Log[x] + (2*(14*a^(2/3)*b^(2/3)*
c + 5*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - ((14*a^(2/3)
*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/b^(1/3) - 18*a*d*Log[a + b*x^3])/(54*a^4)
```

Maple [A] time = 0.023, size = 334, normalized size = 1.3

$$\frac{d\ln(x)}{a^3}-\frac{c}{a^3x}-\frac{5\,b^2x^5c}{9\,a^3\,(bx^3+a)^2}+\frac{5\,bx^4e}{18\,a^2\,(bx^3+a)^2}+\frac{bx^3d}{3\,a^2\,(bx^3+a)^2}$$
$$-\frac{13\,bx^2c}{18\,a^2\,(bx^3+a)^2}+\frac{4\,ex}{9\,a\,(bx^3+a)^2}+\frac{d}{2\,a\,(bx^3+a)^2}+\frac{5\,e}{27\,a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{5\,e}{54\,a^2b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5\,\sqrt{3}e}{27\,a^2b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{14\,c}{27\,a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{7\,c}{27\,a^3}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{14\,c\sqrt{3}}{27\,a^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{d\ln\left(bx^3+a\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x)
```

```
[Out] d*ln(x)/a^3-c/a^3/x-5/9/a^3/(b*x^3+a)^2*b^2*x^5*c+5/18/a^2/(b*x^3
+a)^2*x^4*b*e+1/3/a^2/(b*x^3+a)^2*x^3*b*d-13/18/a^2/(b*x^3+a)^2*x
^2*b*c+4/9/a/(b*x^3+a)^2*x*e+1/2/a/(b*x^3+a)^2*d+5/27/a^2/b/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))*e-5/54/a^2/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))*e+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))*e+14/27/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))*c-7/27/a^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-14
/27/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))*c-1/3*d*ln(b*x^3+a)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.220449, size = 377, normalized size = 1.41

$$-\frac{d\ln\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{d\ln\left(\left|x\right|\right)}{a^3}+\frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ae-14\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b}+\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ab^2e+14\,\left(-ab^2\right)^{\frac{2}{3}}b^2c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^3}-\frac{28\,b^2cx^6-5\,abx^5e-6\,abdx^4+49\,abcx^3-8\,a^2x^2e-9\,a^2dx+18\,a^2c}{18\left(bx^3+a\right)^2a^3x}+\frac{\left(14\,a^3b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^4be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x^2),x, algorithm="giac")
```

```
[Out] -1/3*d*ln(abs(b*x^3 + a))/a^3 + d*ln(abs(x))/a^3 + 1/54*(5*(-a*b^
2)^(1/3)*a*e - 14*(-a*b^2)^(2/3)*c)*ln(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/(a^4*b) + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*a*b^2*e + 14*
(-a*b^2)^(2/3)*b^2*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/(a^4*b^3) - 1/18*(28*b^2*c*x^6 - 5*a*b*x^5*e - 6*a*b*d
*x^4 + 49*a*b*c*x^3 - 8*a^2*x^2*e - 9*a^2*d*x + 18*a^2*c)/((b*x^3
 + a)^2*a^3*x) + 1/27*(14*a^3*b^2*c*(-a/b)^(1/3) - 5*a^4*b*e)*(-a
/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

3.344 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx$

Optimal. Leaf size=276

$$\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}}+\frac{2\sqrt[3]{b}\left(7\sqrt[3]{a}d+10\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}-\frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^3}-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\log(x)}{a^3}-\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2}$$

```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c + b*d*x + b*e*x^2))/(6*a^2*(
a + b*x^3)^2) - (x*(11*b*c + 10*b*d*x + 9*b*e*x^2))/(18*a^3*(a +
b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)) + (e*Log
[x])/a^3 - (2*b^(1/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) +
b^(1/3)*x])/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(11/3)) -
(e*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.976423, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}}+\frac{2\sqrt[3]{b}\left(7\sqrt[3]{a}d+10\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}-\frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^3}-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\log(x)}{a^3}-\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c + b*d*x + b*e*x^2))/(6*a^2*(
a + b*x^3)^2) - (x*(11*b*c + 10*b*d*x + 9*b*e*x^2))/(18*a^3*(a +
b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)) + (e*Log
[x])/a^3 - (2*b^(1/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) +
b^(1/3)*x])/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(11/3)) -
(e*Log[a + b*x^3])/(3*a^3)
```

Rubi in Sympy [A] time = 14.184, size = 26, normalized size = 0.09

$$\frac{x\left(\frac{c}{x^3}+\frac{d}{x^2}+\frac{e}{x}\right)}{6a\left(a+bx^3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] x*(c/x**3 + d/x**2 + e/x)/(6*a*(a + b*x**3)**2)
```

Mathematica [A] time = 0.413425, size = 253, normalized size = 0.92

$$\frac{2\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+4\sqrt[3]{b}\left(7a^{2/3}d-10\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{9a^2(ae-bx(c+dx))}{(a+bx^3)^2}+3a}{54a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a*c)/x^2 - (54*a*d)/x + (9*a^2*(a*e - b*x*(c + d*x)))/(a +
b*x^3)^2 + (3*a*(6*a*e - b*x*(11*c + 10*d*x)))/(a + b*x^3) + 4*Sq
rt[3]*a^(1/3)*b^(1/3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 54*a*e*Log[x] + 4*b^(1/3)*(-10*a^
(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/
3)*(10*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2] - 18*a*e*Log[a + b*x^3])/(54*a^4)
```

Maple [A] time = 0.025, size = 337, normalized size = 1.2

$$-\frac{d}{a^3x}+\frac{e\ln(x)}{a^3}-\frac{c}{2\,a^3x^2}-\frac{5\,x^5b^2d}{9\,a^3\,(bx^3+a)^2}-\frac{11\,x^4b^2c}{18\,a^3\,(bx^3+a)^2}+\frac{bex^3}{3\,a^2\,(bx^3+a)^2}-\frac{13\,bx^2d}{18\,a^2\,(bx^3+a)^2}-\frac{7\,bcx}{9\,a^2\,(bx^3+a)^2}+\frac{e}{2\,a\,(bx^3+a)^2}-\frac{20\,c}{27\,a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{10\,c}{27\,a^3}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{20\,c\sqrt{3}}{27\,a^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{14\,d}{27\,a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{7\,d}{27\,a^3}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{14\,d\sqrt{3}}{27\,a^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{e\ln\left(bx^3+a\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -d/a^3/x+e*ln(x)/a^3-1/2*c/a^3/x^2-5/9/a^3/(b*x^3+a)^2*x^5*b^2*d-
11/18/a^3/(b*x^3+a)^2*x^4*b^2*c+1/3*b/a^2/(b*x^3+a)^2*e*x^3-13/18
/a^2/(b*x^3+a)^2*x^2*b*d-7/9/a^2/(b*x^3+a)^2*x*b*c+1/2/a/(b*x^3+a
)^2*e-20/27/a^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c+10/27/a^3/(a/b)^(
2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-20/27/a^3/(a/b)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+14/27/a^3/(a/b)^(
1/3)*ln(x+(a/b)^(1/3))*d-7/27/a^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3
)+(a/b)^(2/3))*d-14/27/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*d-1/3*e*ln(b*x^3+a)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x^3),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218231, size = 390, normalized size = 1.41

$$-\frac{e\ln\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{e\ln\left(\left|x\right|\right)}{a^3}-\frac{\left(10\,\left(-ab^2\right)^{\frac{1}{3}}bc+7\,\left(-ab^2\right)^{\frac{2}{3}}d\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b}-\frac{28\,b^2dx^7+20\,b^2cx^6-6\,abx^5e+49\,abdx^4+32\,abcx^3-9\,a^2x^2e+18\,a^2dx+9\,a^2c}{18\,\left(bx^4+ax\right)^2a^3}-\frac{2\,\sqrt{3}\left(10\,\left(-ab^2\right)^{\frac{1}{3}}ab^3c-7\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b^3}+\frac{2\left(7\,a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10\,a^3b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x^3),x, algorithm="giac")
```

```
[Out] -1/3*e*ln(abs(b*x^3 + a))/a^3 + e*ln(abs(x))/a^3 - 1/27*(10*(-a*b
^2)^(1/3)*b*c + 7*(-a*b^2)^(2/3)*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/(a^4*b) - 1/18*(28*b^2*d*x^7 + 20*b^2*c*x^6 - 6*a*b*x^
5*e + 49*a*b*d*x^4 + 32*a*b*c*x^3 - 9*a^2*x^2*e + 18*a^2*d*x + 9*
a^2*c)/((b*x^4 + a*x)^2*a^3) - 2/27*sqrt(3)*(10*(-a*b^2)^(1/3)*a*
b^3*c - 7*(-a*b^2)^(2/3)*a*b^2*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^5*b^3) + 2/27*(7*a^3*b^2*d*(-a/b)^(1/3)
 + 10*a^3*b^2*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

3.345 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=298

$$\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}}+\frac{2\sqrt[3]{b}\left(7\sqrt[3]{a}e+10\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}+\frac{bc\log\left(a+bx^3\right)}{a^4}-\frac{3bc\log(x)}{a^4}-\frac{x\left(-\frac{15b^2cx^2}{a}+11bd+10bex\right)}{18a^3\left(a+bx^3\right)}-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2\left(a+bx^3\right)^2}$$

```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d + b*e*x - (b
^2*c*x^2)/a))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d + 10*b*e*x - (15
*b^2*c*x^2)/a))/(18*a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*d +
 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(11/3)) - (3*b*c*Log[x])/a^4 - (2*b^(1/3)*(10*b^(1/3
)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)) + (b^(
1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(27*a^(11/3)) + (b*c*Log[a + b*x^3])/a^4
```

Rubi [A] time = 1.11201, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}}+\frac{2\sqrt[3]{b}\left(7\sqrt[3]{a}e+10\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}+\frac{bc\log\left(a+bx^3\right)}{a^4}-\frac{3bc\log(x)}{a^4}-\frac{x\left(-\frac{15b^2cx^2}{a}+11bd+10bex\right)}{18a^3\left(a+bx^3\right)}-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```



```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d + b*e*x - (b
^2*c*x^2)/a))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d + 10*b*e*x - (15
*b^2*c*x^2)/a))/(18*a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*d +
 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(11/3)) - (3*b*c*Log[x])/a^4 - (2*b^(1/3)*(10*b^(1/3
)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)) + (b^(
1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(27*a^(11/3)) + (b*c*Log[a + b*x^3])/a^4
```

Rubi in Sympy [A] time = 14.1222, size = 27, normalized size = 0.09

$$\frac{x\left(\frac{c}{x^4}+\frac{d}{x^3}+\frac{e}{x^2}\right)}{6a\left(a+bx^3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] x*(c/x**4 + d/x**3 + e/x**2)/(6*a*(a + b*x**3)**2)
```

Mathematica [A] time = 0.698342, size = 255, normalized size = 0.86

$$-\frac{-2\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+4\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{9a^2b(c+x(d+ex))}{(a+bx^3)^2}+\cdots}{5\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -((18*a*c)/x^3 + (27*a*d)/x^2 + (54*a*e)/x + (9*a^2*b*(c + x*(d +
 e*x)))/(a + b*x^3)^2 + (3*a*b*(12*c + x*(11*d + 10*e*x)))/(a + b
*x^3) - 4*Sqrt[3]*a^(1/3)*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 162*b*c*Log[x] + 4*b^
(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*
x] - 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 54*b*c*Log[a + b*x^3])/(54*a^
4)
```

Maple [A] time = 0.025, size = 351, normalized size = 1.2

$$-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{c}{3a^3x^3}-3\frac{bc\ln(x)}{a^4}-\frac{5x^5eb^2}{9a^3(bx^3+a)^2}-\frac{11x^4b^2d}{18a^3(bx^3+a)^2}-\frac{2b^2x^3c}{3a^3(bx^3+a)^2}-\frac{13bex^2}{18a^2(bx^3+a)^2}-\frac{7bxd}{9a^2(bx^3+a)^2}-\frac{5bc}{6a^2(bx^3+a)^2}-\frac{20d}{27a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{10d}{27a^3}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{20d\sqrt{3}}{27a^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{14e}{27a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{7e}{27a^3}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{14e\sqrt{3}}{27a^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{bc\ln(bx^3+a)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x)
```

```
[Out] -1/2*d/a^3/x^2-e/a^3/x-1/3*c/a^3/x^3-3*b*c*ln(x)/a^4-5/9/a^3/(b*x
^3+a)^2*x^5*e*b^2-11/18/a^3/(b*x^3+a)^2*x^4*b^2*d-2/3/a^3*b^2/(b*
x^3+a)^2*x^3*c-13/18/a^2/(b*x^3+a)^2*x^2*b*e-7/9/a^2/(b*x^3+a)^2*
x*b*d-5/6/a^2*b/(b*x^3+a)^2*c-20/27/a^3/(a/b)^(2/3)*ln(x+(a/b)^(1
/3))*d+10/27/a^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-
20/27/a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*d+14/27/a^3*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-7/27/a^3*e/(a/b)
^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-14/27/a^3*e*3^(1/2)/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+b*c*ln(b*x^3+a)/a
^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x^4),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.222242, size = 421, normalized size = 1.41

$$\frac{bc\ln\left(\left|bx^3+a\right|\right)}{a^4}-\frac{3\,bc\ln\left(|x|\right)}{a^4}-\frac{\left(10\,\left(-ab^2\right)^{\frac{1}{3}}bd+7\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b}$$
$$-\frac{2\sqrt{3}\left(10\,\left(-ab^2\right)^{\frac{1}{3}}ab^3d-7\,\left(-ab^2\right)^{\frac{2}{3}}ab^2e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b^3}$$
$$+\frac{2\left(7\,a^5b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+10\,a^5b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^9b}$$
$$-\frac{28\,ab^2x^8e+20\,ab^2dx^7+18\,ab^2cx^6+49\,a^2bx^5e+32\,a^2bdx^4+27\,a^2bcx^3+18\,a^3x^2e+9\,a^3dx+6\,a^3c}{18\left(bx^3+a\right)^2a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^3*x^4),x, algorithm="giac")
```

```
[Out] b*c*ln(abs(b*x^3 + a))/a^4 - 3*b*c*ln(abs(x))/a^4 - 1/27*(10*(-a*
b^2)^(1/3)*b*d + 7*(-a*b^2)^(2/3)*e)*ln(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^4*b) - 2/27*sqrt(3)*(10*(-a*b^2)^(1/3)*a*b^3*d - 7
*(-a*b^2)^(2/3)*a*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/(a^5*b^3) + 2/27*(7*a^5*b^2*(-a/b)^(1/3)*e + 10*a^5
*b^2*d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^9*b) - 1/18*(28
*a*b^2*x^8*e + 20*a*b^2*d*x^7 + 18*a*b^2*c*x^6 + 49*a^2*b*x^5*e +
 32*a^2*b*d*x^4 + 27*a^2*b*c*x^3 + 18*a^3*x^2*e + 9*a^3*d*x + 6*a
^3*c)/((b*x^3 + a)^2*a^4*x^3)
```

3.346 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

Optimal. Leaf size=248

$$-\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}+\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}}-\frac{\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}}+\frac{x(5d+8ex)}{162a^2b\,(a+bx^3)}-\frac{c+dx+ex^2}{9b\,(a+bx^3)^3}+\frac{x(d+2ex)}{54ab\,(a+bx^3)^2}$$

```
[Out] -(c + d*x + e*x^2)/(9*b*(a + b*x^3)^3) + (x*(d + 2*e*x))/(54*a*b*
(a + b*x^3)^2) + (x*(5*d + 8*e*x))/(162*a^2*b*(a + b*x^3)) - ((5*
b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(81*Sqrt[3]*a^(8/3)*b^(5/3)) + ((5*b^(1/3)*d - 4*a^(1/
3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(8/3)*b^(5/3)) - ((5*b^(1/
3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(486*a^(8/3)*b^(5/3))
```

Rubi [A] time = 0.534477, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$-\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}+\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}}-\frac{\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}}+\frac{x(5d+8ex)}{162a^2b\,(a+bx^3)}-\frac{c+dx+ex^2}{9b\,(a+bx^3)^3}+\frac{x(d+2ex)}{54ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] -(c + d*x + e*x^2)/(9*b*(a + b*x^3)^3) + (x*(d + 2*e*x))/(54*a*b*
(a + b*x^3)^2) + (x*(5*d + 8*e*x))/(162*a^2*b*(a + b*x^3)) - ((5*
b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(81*Sqrt[3]*a^(8/3)*b^(5/3)) + ((5*b^(1/3)*d - 4*a^(1/
3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(8/3)*b^(5/3)) - ((5*b^(1/
3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(486*a^(8/3)*b^(5/3))
```

Rubi in Sympy [A] time = 75.8735, size = 228, normalized size = 0.92

$$-\frac{c+dx+ex^2}{9b\,(a+bx^3)^3}+\frac{x\,(d+2ex)}{54ab\,(a+bx^3)^2}+\frac{x\,(5d+8ex)}{162a^2b\,(a+bx^3)}-\frac{\left(4\sqrt[3]{a}e-5\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{\frac{8}{3}}b^{\frac{5}{3}}}+\frac{\left(4\sqrt[3]{a}e-5\sqrt[3]{b}d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{486a^{\frac{8}{3}}b^{\frac{5}{3}}}-\frac{\sqrt{3}\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{243a^{\frac{8}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] -(c + d*x + e*x**2)/(9*b*(a + b*x**3)**3) + x*(d + 2*e*x)/(54*a*b
*(a + b*x**3)**2) + x*(5*d + 8*e*x)/(162*a**2*b*(a + b*x**3)) - (
4*a**(1/3)*e - 5*b**(1/3)*d)*log(a**(1/3) + b**(1/3)*x)/(243*a**(
8/3)*b**(5/3)) + (4*a**(1/3)*e - 5*b**(1/3)*d)*log(a**(2/3) - a**
```

```
(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(486*a**(8/3)*b**(5/3)) - sqrt(
3)*(4*a**(1/3)*e + 5*b**(1/3)*d)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**
(1/3)*x/3)/a**(1/3))/(243*a**(8/3)*b**(5/3))
```

Mathematica [A] time = 0.478337, size = 230, normalized size = 0.93

$$\frac{\frac{\left(4\sqrt[3]{a}e-5\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}}+\frac{2\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}-\frac{2\sqrt{3}\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}+\frac{3b^{2/3}x(5d+8ex)}{a^2(a+bx^3)}-54b^2}{486b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((9*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)^2) + (3*b^(2/3)*x*(5*d
+ 8*e*x))/(a^2*(a + b*x^3)) - (54*b^(2/3)*(c + x*(d + e*x)))/(a +
 b*x^3)^3 - (2*Sqrt[3]*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) + (2*(5*b^(1/3)*d - 4*a^(1
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) + ((-5*b^(1/3)*d + 4*a^(
1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3))/
(486*b^(5/3))
```

Maple [A] time = 0.017, size = 275, normalized size = 1.1

$$\frac{1}{\left(bx^3+a\right)^3}\left(\frac{4\,bex^8}{81\,a^2}+\frac{5\,bdx^7}{162\,a^2}+\frac{11\,ex^5}{81\,a}+\frac{13\,dx^4}{162\,a}-\frac{2\,ex^2}{81\,b}-\frac{5\,dx}{81\,b}-\frac{c}{9\,b}\right)$$
$$+\frac{5\,d}{243\,a^2b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5\,d}{486\,a^2b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{5\,d\sqrt{3}}{243\,a^2b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{4\,e}{243\,a^2b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{2\,e}{243\,a^2b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{4\,e\sqrt{3}}{243\,a^2b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] (4/81/a^2*b*e*x^8+5/162/a^2*d*b*x^7+11/81/a*e*x^5+13/162*d/a*x^4-
2/81*e*x^2/b-5/81*d*x/b-1/9*c/b)/(b*x^3+a)^3+5/243*d/a^2/b^2/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))-5/486*d/a^2/b^2/(a/b)^(2/3)*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))+5/243*d/a^2/b^2/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-4/243*e/a^2/b^2/(a/b)^(1/3)*ln(
x+(a/b)^(1/3))+2/243*e/a^2/b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(
a/b)^(2/3))+4/243*e*3^(1/2)/a^2/b^2/(a/b)^(1/3)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 27.3954, size = 201, normalized size = 0.81

$$\operatorname{RootSum}\left(14348907t^3a^8b^5 + 14580ta^3b^2de + 64ae^3 - 125bd^3, \left(t \mapsto t\log\left(x + \frac{236196t^2a^6b^3e + 6075ta^3b^2d^2 + 160ade^2}{64ae^3 + 125bd^3}\right)\right)\right) + \frac{-18a^2c - 10a^2dx - 4a^2ex^2 + 13abdx^4 + 22abex^5 + 5b^2dx^7 + 8b^2ex^8}{162a^5b + 486a^4b^2x^3 + 486a^3b^3x^6 + 162a^2b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**8*b**5 + 14580*_t*a**3*b**2*d*e + 64*a*
e**3 - 125*b*d**3, Lambda(_t, _t*log(x + (236196*_t**2*a**6*b**3*
e + 6075*_t*a**3*b**2*d**2 + 160*a*d*e**2)/(64*a*e**3 + 125*b*d**
3)))) + (-18*a**2*c - 10*a**2*d*x - 4*a**2*e*x**2 + 13*a*b*d*x**4
 + 22*a*b*e*x**5 + 5*b**2*d*x**7 + 8*b**2*e*x**8)/(162*a**5*b + 4
86*a**4*b**2*x**3 + 486*a**3*b**3*x**6 + 162*a**2*b**4*x**9)
```

GIAC/XCAS [A] time = 0.218112, size = 333, normalized size = 1.34

$$-\frac{\left(4\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + 5d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^3b} + \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bd - 4\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b^3} + \frac{8\,b^2x^8e + 5\,b^2dx^7 + 22\,abx^5e + 13\,abdx^4 - 4\,a^2x^2e - 10\,a^2dx - 18\,a^2c}{162\,(bx^3 + a)^3a^2b} + \frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}ab^2d + 4\left(-ab^2\right)^{\frac{2}{3}}abe\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,a^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^4,x, algorithm="giac")
```

```
[Out] -1/243*(4*(-a/b)^(1/3)*e + 5*d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1
/3)))/(a^3*b) + 1/243*sqrt(3)*(5*(-a*b^2)^(1/3)*b*d - 4*(-a*b^2)^
(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^3*b^3) + 1/162*(8*b^2*x^8*e + 5*b^2*d*x^7 + 22*a*b*x^5*e + 13*a*
b*d*x^4 - 4*a^2*x^2*e - 10*a^2*d*x - 18*a^2*c)/((b*x^3 + a)^3*a^2
*b) + 1/486*(5*(-a*b^2)^(1/3)*a*b^2*d + 4*(-a*b^2)^(2/3)*a*b*e)*l
n(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^4)
```

3.347 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

Optimal. Leaf size=270

$$\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}}-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}}+\frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)}-\frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2}-\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(9*a*b*(a + b*x^3)^3) + (x*(5*a*e +
28*b*c*x))/(162*a^3*b*(a + b*x^3)) - (6*a*d - x*(a*e + 7*b*c*x))/
(54*a^2*b*(a + b*x^3)^2) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(10/3)*b
^(4/3)) - ((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])
/(243*a^(10/3)*b^(4/3)) + ((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(486*a^(10/3)*b^(4/3))
```

Rubi [A] time = 0.548044, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}}-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}}+\frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)}-\frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2}-\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(9*a*b*(a + b*x^3)^3) + (x*(5*a*e +
28*b*c*x))/(162*a^3*b*(a + b*x^3)) - (6*a*d - x*(a*e + 7*b*c*x))/
(54*a^2*b*(a + b*x^3)^2) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(10/3)*b
^(4/3)) - ((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])
/(243*a^(10/3)*b^(4/3)) + ((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(486*a^(10/3)*b^(4/3))
```

Rubi in Sympy [A] time = 91.2819, size = 250, normalized size = 0.93

$$-\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}-\frac{6ad-x\left(ae+7bcx\right)}{54a^2b\left(a+bx^3\right)^2}+\frac{x\left(5ae+28bcx\right)}{162a^3b\left(a+bx^3\right)}+\frac{\left(5a^{\frac{2}{3}}e-14b^{\frac{2}{3}}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{\frac{10}{3}}b^{\frac{4}{3}}}-\frac{\left(5a^{\frac{2}{3}}e-14b^{\frac{2}{3}}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{486a^{\frac{10}{3}}b^{\frac{4}{3}}}-\frac{\sqrt{3}\left(5a^{\frac{2}{3}}e+14b^{\frac{2}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{243a^{\frac{10}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] -x*(a*e - b*c*x - b*d*x**2)/(9*a*b*(a + b*x**3)**3) - (6*a*d - x*
(a*e + 7*b*c*x))/(54*a**2*b*(a + b*x**3)**2) + x*(5*a*e + 28*b*c*
x)/(162*a**3*b*(a + b*x**3)) + (5*a**(2/3)*e - 14*b**(2/3)*c)*log
(a**(1/3) + b**(1/3)*x)/(243*a**(10/3)*b**(4/3)) - (5*a**(2/3)*e
- 14*b**(2/3)*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x*
*2)/(486*a**(10/3)*b**(4/3)) - sqrt(3)*(5*a**(2/3)*e + 14*b**(2/3
)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(243*a*
*(10/3)*b**(4/3))
```

Mathematica [A] time = 0.713206, size = 241, normalized size = 0.89

$$\frac{a^{2/3}\sqrt[3]{b}\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt{3}a^{2/3}\sqrt[3]{b}\left(5a^{2/3}e+14b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(5a^{4/3}\sqrt[3]{b}e-\right.}{486a^4b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((3*a*b^(2/3)*(28*b^3*c*x^8 - 2*a^3*(9*d + 5*e*x) + a*b^2*x^5*(77
*c + 5*e*x^2) + a^2*b*x^2*(67*c + 13*e*x^2)))/(a + b*x^3)^3 - 2*S
qrt[3]*a^(2/3)*b^(1/3)*(14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-14*a^(2/3)*b*c + 5*a^(4/3)*b
^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] + a^(2/3)*b^(1/3)*(14*b^(2/3)*
c - 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(486*a^4*b^(5/3))
```

Maple [A] time = 0.016, size = 278, normalized size = 1.

$$\frac{1}{(bx^3+a)^3}\left(\frac{14\,b^2cx^8}{81\,a^3}+\frac{5\,bex^7}{162\,a^2}+\frac{77\,bcx^5}{162\,a^2}+\frac{13\,ex^4}{162\,a}+\frac{67\,cx^2}{162\,a}-\frac{5\,ex}{81\,b}-\frac{d}{9\,b}\right)$$
$$+\frac{5\,e}{243\,a^2b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5\,e}{486\,a^2b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{5\,e\sqrt{3}}{243\,a^2b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{14\,c}{243\,a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{7\,c}{243\,a^3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{14\,c\sqrt{3}}{243\,a^3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] (14/81*c/a^3*b^2*x^8+5/162/a^2*b*e*x^7+77/162/a^2*c*b*x^5+13/162/
a*e*x^4+67/162/a*c*x^2-5/81*e*x/b-1/9/b*d)/(b*x^3+a)^3+5/243/a^2*
e/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-5/486/a^2*e/b^2/(a/b)^(2/3)*l
n(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+5/243/a^2*e/b^2/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/243/b*c/a^3/(a/b)^
(1/3)*ln(x+(a/b)^(1/3))+7/243/b*c/a^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))+14/243/b*c*3^(1/2)/a^3/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 15.0171, size = 214, normalized size = 0.79

$$\operatorname{RootSum}\left(14348907t^3a^{10}b^4 + 51030ta^4b^2ce - 125a^2e^3 + 2744b^2c^3, \left(t \mapsto t\log\left(x + \frac{826686t^2a^7b^3c + 6075ta^5be^2 + 1960abc}{125a^2e^3 + 2744b^2c^3}\right.\right.\right.$$
$$+ \frac{-18a^3d - 10a^3ex + 67a^2bcx^2 + 13a^2bex^4 + 77ab^2cx^5 + 5ab^2ex^7 + 28b^3cx^8}{162a^6b + 486a^5b^2x^3 + 486a^4b^3x^6 + 162a^3b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**10*b**4 + 51030*_t*a**4*b**2*c*e - 125*
a**2*e**3 + 2744*b**2*c**3, Lambda(_t, _t*log(x + (826686*_t**2*a
**7*b**3*c + 6075*_t*a**5*b*e**2 + 1960*a*b*c**2*e)/(125*a**2*e**
3 + 2744*b**2*c**3)))) + (-18*a**3*d - 10*a**3*e*x + 67*a**2*b*c*
x**2 + 13*a**2*b*e*x**4 + 77*a*b**2*c*x**5 + 5*a*b**2*e*x**7 + 28
*b**3*c*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 486*a**4*b**3*x*
*6 + 162*a**3*b**4*x**9)
```

GIAC/XCAS [A] time = 0.217852, size = 346, normalized size = 1.28

$$-\frac{\left(14\,bc\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4b} + \frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ae - 14\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^4b^2} + \frac{28\,b^3cx^8 + 5\,ab^2x^7e + 77\,ab^2cx^5 + 13\,a^2bx^4e + 67\,a^2bcx^2 - 10\,a^3xe - 18\,a^3d}{162\,(bx^3+a)^3a^3b} + \frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ab^2e + 14\,\left(-ab^2\right)^{\frac{2}{3}}b^2c\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,a^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^4,x, algorithm="giac")
```

```
[Out] -1/243*(14*b*c*(-a/b)^(1/3) + 5*a*e)*(-a/b)^(1/3)*ln(abs(x - (-a/
b)^(1/3)))/(a^4*b) + 1/243*sqrt(3)*(5*(-a*b^2)^(1/3)*a*e - 14*(-a
*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
```

```
3))/(a^4*b^2) + 1/162*(28*b^3*c*x^8 + 5*a*b^2*x^7*e + 77*a*b^2*c*
x^5 + 13*a^2*b*x^4*e + 67*a^2*b*c*x^2 - 10*a^3*x*e - 18*a^3*d)/((
b*x^3 + a)^3*a^3*b) + 1/486*(5*(-a*b^2)^(1/3)*a*b^2*e + 14*(-a*b^
2)^(2/3)*b^2*c)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^4)
```

3.348 $\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$

Optimal. Leaf size=250

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

```
[Out] (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(
81*a^3*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(9*a*b*(a + b*x^3)^3)
 - (2*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/
3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3)*b^(2/
3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rubi [A] time = 0.499225, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(
81*a^3*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(9*a*b*(a + b*x^3)^3)
 - (2*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/
3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3)*b^(2/
3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rubi in Sympy [A] time = 71.0047, size = 235, normalized size = 0.94

$$-\frac{ae-bx\left(c+dx\right)}{9ab\left(a+bx^3\right)^3}+\frac{x\left(8c+7dx\right)}{54a^2\left(a+bx^3\right)^2}+\frac{x\left(40c+28dx\right)}{162a^3\left(a+bx^3\right)}-\frac{2\left(7\sqrt[3]{a}d-20\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{\frac{11}{3}}b^{\frac{2}{3}}}+\frac{\left(7\sqrt[3]{a}d-20\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{243a^{\frac{11}{3}}b^{\frac{2}{3}}}-\frac{2\sqrt{3}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{243a^{\frac{11}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] -(a*e - b*x*(c + d*x))/(9*a*b*(a + b*x**3)**3) + x*(8*c + 7*d*x)/
(54*a**2*(a + b*x**3)**2) + x*(40*c + 28*d*x)/(162*a**3*(a + b*x*
*3)) - 2*(7*a**(1/3)*d - 20*b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x
)/(243*a**(11/3)*b**(2/3)) + (7*a**(1/3)*d - 20*b**(1/3)*c)*log(a
```

```
**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(243*a**(11/3)*b**
(2/3)) - 2*sqrt(3)*(7*a**(1/3)*d + 20*b**(1/3)*c)*atan(sqrt(3)*(a
**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(243*a**(11/3)*b**(2/3))
```

Mathematica [A] time = 0.445695, size = 239, normalized size = 0.96

$$\frac{\frac{2\left(7a^{2/3}d-20\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{54a^3(ae-bx(c+dx))}{b(a+bx^3)^3}+\frac{9a^2x(8c+7dx)}{(a+bx^3)^2}-\frac{4\sqrt{3}\sqrt[3]{a}\left(7\sqrt[3]{\cdots}\right.}{\cdots}}{486a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] ((9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2 + (12*a*x*(10*c + 7*d*x))/
(a + b*x^3) - (54*a^3*(a*e - b*x*(c + d*x)))/(b*(a + b*x^3)^3) -
(4*Sqrt[3]*a^(1/3)*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(20*a^(1/3)*b^(1/3)*c -
7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-20*a^(1/3)*
b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/b^(2/3))/(486*a^4)
```

Maple [A] time = 0.007, size = 360, normalized size = 1.4

$$\frac{cx}{9\,a\left(bx^3+a\right)^3}+\frac{4\,cx}{27\,a^2\left(bx^3+a\right)^2}+\frac{20\,cx}{81\,a^3\left(bx^3+a\right)}+\frac{40\,c}{243\,a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{20\,c}{243\,a^3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{40\,c\sqrt{3}}{243\,a^3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{dx^2}{9\,a\left(bx^3+a\right)^3}+\frac{7\,dx^2}{54\,a^2\left(bx^3+a\right)^2}+\frac{14\,dx^2}{81\,a^3\left(bx^3+a\right)}$$
$$-\frac{14\,d}{243\,a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{7\,d}{243\,a^3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{14\,d\sqrt{3}}{243\,a^3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{ex^3}{9\,a\left(bx^3+a\right)^3}+\frac{ex^3}{9\,a^2\left(bx^3+a\right)^2}-\frac{e}{9\,a^2b\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] 1/9*c/a*x/(b*x^3+a)^3+4/27*c/a^2*x/(b*x^3+a)^2+20/81*c/a^3*x/(b*x
^3+a)+40/243*c/a^3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-20/243*c/a^3/b
/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+40/243*c/a^3/b/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9*d/a*
x^2/(b*x^3+a)^3+7/54*d/a^2*x^2/(b*x^3+a)^2+14/81*d/a^3*x^2/(b*x^3
+a)-14/243*d/a^3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/243*d/a^3/b/(a
/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+14/243*d/a^3*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9*e/a*x^3
/(b*x^3+a)^3+1/9*e/a^2*x^3/(b*x^3+a)^2-1/9*e/a^2/b/(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 8.3597, size = 202, normalized size = 0.81

$$\mathrm{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t\log\left(x + \frac{413343t^2a^8bd + 194400ta^4bc^2 + 7840a}{1372ad^3 + 32000bc^3}\right.\right.\right.$$
$$+ \frac{-18a^3e + 82a^2bcx + 67a^2bdx^2 + 104ab^2cx^4 + 77ab^2dx^5 + 40b^3cx^7 + 28b^3dx^8}{162a^6b + 486a^5b^2x^3 + 486a^4b^3x^6 + 162a^3b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a
*d**3 - 64000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*
d + 194400*_t*a**4*b*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b
*c**3)))) + (-18*a**3*e + 82*a**2*b*c*x + 67*a**2*b*d*x**2 + 104*
a*b**2*c*x**4 + 77*a*b**2*d*x**5 + 40*b**3*c*x**7 + 28*b**3*d*x**
8)/(162*a**6*b + 486*a**5*b**2*x**3 + 486*a**4*b**3*x**6 + 162*a*
*3*b**4*x**9)
```

GIAC/XCAS [A] time = 0.218322, size = 333, normalized size = 1.33

$$-\frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4} + \frac{2\sqrt{3}\left(20\left(-ab^2\right)^{\frac{1}{3}}bc - 7\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^4b^2} + \frac{28\,b^3dx^8 + 40\,b^3cx^7 + 77\,ab^2dx^5 + 104\,ab^2cx^4 + 67\,a^2bdx^2 + 82\,a^2bcx - 18\,a^3e}{162\left(bx^3 + a\right)^3a^3b} + \frac{\left(20\left(-ab^2\right)^{\frac{1}{3}}ab^3c + 7\left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^4,x, algorithm="giac")
```

```
[Out] -2/243*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(
1/3)))/a^4 + 2/243*sqrt(3)*(20*(-a*b^2)^(1/3)*b*c - 7*(-a*b^2)^(2
/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4
```

```
*b^2) + 1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104
*a*b^2*c*x^4 + 67*a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/((b*x^3
+ a)^3*a^3*b) + 1/243*(20*(-a*b^2)^(1/3)*a*b^3*c + 7*(-a*b^2)^(2/
3)*a*b^2*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^4)
```

3.349 $\int \frac{c+dx+ex^2}{x(a+bx^3)^4}\,dx$

Optimal. Leaf size=291

$$-\frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}e+20\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}+\frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^4}+\frac{c\log(x)}{a^4}+\frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2}+\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*d + 7
*a*e*x - 15*b*c*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*d + 28*a*
e*x - 99*b*c*x^2))/(162*a^4*(a + b*x^3)) - (2*(20*b^(1/3)*d + 7*a
^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*
Sqrt[3]*a^(11/3)*b^(2/3)) + (c*Log[x])/a^4 + (2*(20*b^(1/3)*d - 7
*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3)*b^(2/3)) - ((
20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(243*a^(11/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^4)
```

Rubi [A] time = 1.0053, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$-\frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}e+20\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}+\frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^4}+\frac{c\log(x)}{a^4}+\frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2}+\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*d + 7
*a*e*x - 15*b*c*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*d + 28*a*
e*x - 99*b*c*x^2))/(162*a^4*(a + b*x^3)) - (2*(20*b^(1/3)*d + 7*a
^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*
Sqrt[3]*a^(11/3)*b^(2/3)) + (c*Log[x])/a^4 + (2*(20*b^(1/3)*d - 7
*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3)*b^(2/3)) - ((
20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(243*a^(11/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^4)
```

Rubi in Sympy [A] time = 86.48, size = 231, normalized size = 0.79

$$\frac{x\left(\frac{c}{x}+d+ex\right)}{9a\left(a+bx^3\right)^3}+\frac{x\left(8d+7ex\right)}{54a^2\left(a+bx^3\right)^2}+\frac{x\left(40d+28ex\right)}{162a^3\left(a+bx^3\right)}-\frac{2\left(7\sqrt[3]{a}e-20\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{\frac{11}{3}}b^{\frac{2}{3}}}+\frac{\left(7\sqrt[3]{a}e-20\sqrt[3]{b}d\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{243a^{\frac{11}{3}}b^{\frac{2}{3}}}-\frac{2\sqrt{3}\left(7\sqrt[3]{a}e+20\sqrt[3]{b}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{243a^{\frac{11}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x/(b*x**3+a)**4,x)
```

```
[Out] x*(c/x + d + e*x)/(9*a*(a + b*x**3)**3) + x*(8*d + 7*e*x)/(54*a**
2*(a + b*x**3)**2) + x*(40*d + 28*e*x)/(162*a**3*(a + b*x**3)) -
2*(7*a**(1/3)*e - 20*b**(1/3)*d)*log(a**(1/3) + b**(1/3)*x)/(243*
a**(11/3)*b**(2/3)) + (7*a**(1/3)*e - 20*b**(1/3)*d)*log(a**(2/3)
 - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(243*a**(11/3)*b**(2/3))
- 2*sqrt(3)*(7*a**(1/3)*e + 20*b**(1/3)*d)*atan(sqrt(3)*(a**(1/3)
/3 - 2*b**(1/3)*x/3)/a**(1/3))/(243*a**(11/3)*b**(2/3))
```

Mathematica [A] time = 0.450075, size = 259, normalized size = 0.89

$$\frac{\frac{2\left(7a^{2/3}e-20\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{54a^3(c+x(d+ex))}{(a+bx^3)^3}+\frac{9a^2(9c+x(8d+7ex))}{(a+bx^3)^2}-\frac{4\sqrt{3}\sqrt[3]{a}\left(7\cdots\right.}{\cdots}}{486a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] ((54*a^3*(c + x*(d + e*x)))/(a + b*x^3)^3 + (9*a^2*(9*c + x*(8*d
+ 7*e*x)))/(a + b*x^3)^2 + (6*a*(27*c + 2*x*(10*d + 7*e*x)))/(a +
 b*x^3) - (4*Sqrt[3]*a^(1/3)*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 486*c*Log[x] + (4
*(20*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b
^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*d + 7*a^(2/3)*e)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 162*c*Log[a + b*x^3])/
(486*a^4)
```

Maple [A] time = 0.023, size = 394, normalized size = 1.4

$$\begin{aligned}
&\frac{c\ln(x)}{a^4}+\frac{14\,x^8b^2e}{81\,a^3\,(bx^3+a)^3}+\frac{20\,x^7b^2d}{81\,a^3\,(bx^3+a)^3}+\frac{b^2cx^6}{3\,a^3\,(bx^3+a)^3}\\
&+\frac{77\,bx^5e}{162\,a^2\,(bx^3+a)^3}+\frac{52\,bdx^4}{81\,a^2\,(bx^3+a)^3}+\frac{5\,bx^3c}{6\,a^2\,(bx^3+a)^3}+\frac{67\,ex^2}{162\,a\,(bx^3+a)^3}\\
&+\frac{41\,dx}{81\,a\,(bx^3+a)^3}+\frac{11\,c}{18\,a\,(bx^3+a)^3}+\frac{40\,d}{243\,a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&-\frac{20\,d}{243\,a^3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{40\,d\sqrt{3}}{243\,a^3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&-\frac{14\,e}{243\,a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{7\,e}{243\,a^3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\
&+\frac{14\,e\sqrt{3}}{243\,a^3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c\ln\left(bx^3+a\right)}{3\,a^4}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x/(b*x^3+a)^4,x)
```

```
[Out] c*ln(x)/a^4+14/81/a^3/(b*x^3+a)^3*x^8*b^2*e+20/81/a^3/(b*x^3+a)^3
*x^7*b^2*d+1/3/a^3/(b*x^3+a)^3*x^6*b^2*c+77/162/a^2/(b*x^3+a)^3*x
^5*b*e+52/81/a^2/(b*x^3+a)^3*x^4*b*d+5/6/a^2/(b*x^3+a)^3*x^3*b*c+
67/162/a/(b*x^3+a)^3*x^2*e+41/81/a/(b*x^3+a)^3*x*d+11/18/a/(b*x^3
```

```
+a)^3*c+40/243/a^3*d/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-20/243/a^3*d
/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+40/243/a^3*d/b/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/243
/a^3*e/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/243/a^3*e/b/(a/b)^(1/3)*
ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+14/243/a^3*e*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*c*ln(b*x^3+a)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219856, size = 409, normalized size = 1.41

$$-\frac{c\ln\left(\left|bx^3+a\right|\right)}{3\,a^4}+\frac{c\ln\left(\left|x\right|\right)}{a^4}+\frac{2\,\sqrt{3}\left(20\,\left(-ab^2\right)^{\frac{1}{3}}bd-7\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^4b^2}+\frac{28\,ab^2x^8e+40\,ab^2dx^7+54\,ab^2cx^6+77\,a^2bx^5e+104\,a^2bdx^4+135\,a^2bcx^3+67\,a^3x^2e+82\,a^3dx+99\,a^3c}{162\,(bx^3+a)^3a^4}+\frac{\left(20\,\left(-ab^2\right)^{\frac{1}{3}}ab^3d+7\,\left(-ab^2\right)^{\frac{2}{3}}ab^2e\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b^4}-\frac{2\left(7\,a^5b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+20\,a^5bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x),x, algorithm="giac")
```

```
[Out] -1/3*c*ln(abs(b*x^3 + a))/a^4 + c*ln(abs(x))/a^4 + 2/243*sqrt(3)*
(20*(-a*b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2) + 1/162*(28*a*b^2*x^8
*e + 40*a*b^2*d*x^7 + 54*a*b^2*c*x^6 + 77*a^2*b*x^5*e + 104*a^2*b
*d*x^4 + 135*a^2*b*c*x^3 + 67*a^3*x^2*e + 82*a^3*d*x + 99*a^3*c)/
((b*x^3 + a)^3*a^4) + 1/243*(20*(-a*b^2)^(1/3)*a*b^3*d + 7*(-a*b^
2)^(2/3)*a*b^2*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^
4) - 2/243*(7*a^5*b*(-a/b)^(1/3)*e + 20*a^5*b*d)*(-a/b)^(1/3)*ln(
abs(x - (-a/b)^(1/3)))/(a^9*b)
```

3.350 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4}\,dx$

Optimal. Leaf size=301

$$-\frac{10\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(7b^{2/3}c-2a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{13/3}\sqrt[3]{b}}+\frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^4}-\frac{c}{a^4x}+\frac{d\log(x)}{a^4}+\frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2}+\frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

```
[Out] -(c/(a^4*x)) + (x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3)
+ (x*(8*a*e - 16*b*c*x - 15*b*d*x^2))/(54*a^3*(a + b*x^3)^2) + (x
*(40*a*e - 118*b*c*x - 99*b*d*x^2))/(162*a^4*(a + b*x^3)) + (20*(
7*b^(2/3)*c - 2*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(81*Sqrt[3]*a^(13/3)*b^(1/3)) + (d*Log[x])/a^4 + (20
*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(13
/3)*b^(1/3)) - (10*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(13/3)*b^(1/3)) - (d*Log[a +
 b*x^3])/(3*a^4)
```

Rubi [A] time = 1.14169, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$-\frac{10\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(7b^{2/3}c-2a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{13/3}\sqrt[3]{b}}+\frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^4}-\frac{c}{a^4x}+\frac{d\log(x)}{a^4}+\frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2}+\frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```


```
[Out] -(c/(a^4*x)) + (x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3)
+ (x*(8*a*e - 16*b*c*x - 15*b*d*x^2))/(54*a^3*(a + b*x^3)^2) + (x
*(40*a*e - 118*b*c*x - 99*b*d*x^2))/(162*a^4*(a + b*x^3)) + (20*(
7*b^(2/3)*c - 2*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(81*Sqrt[3]*a^(13/3)*b^(1/3)) + (d*Log[x])/a^4 + (20
*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(13
/3)*b^(1/3)) - (10*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(13/3)*b^(1/3)) - (d*Log[a +
 b*x^3])/(3*a^4)
```

Rubi in Sympy [A] time = 63.2244, size = 178, normalized size = 0.59

$$\frac{x\left(\frac{c}{x^2}+\frac{d}{x}+e\right)}{9a\left(a+bx^3\right)^3}+\frac{4ex}{27a^2\left(a+bx^3\right)^2}+\frac{20ex}{81a^3\left(a+bx^3\right)}+\frac{40e\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{\frac{11}{3}}\sqrt[3]{b}}-\frac{20e\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{243a^{\frac{11}{3}}\sqrt[3]{b}}-\frac{40\sqrt{3}e\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{243a^{\frac{11}{3}}\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**4,x)
```

```
[Out] x*(c/x**2 + d/x + e)/(9*a*(a + b*x**3)**3) + 4*e*x/(27*a**2*(a +
b*x**3)**2) + 20*e*x/(81*a**3*(a + b*x**3)) + 40*e*log(a**(1/3) +
 b**(1/3)*x)/(243*a**(11/3)*b**(1/3)) - 20*e*log(a**(2/3) - a**(1
/3)*b**(1/3)*x + b**(2/3)*x**2)/(243*a**(11/3)*b**(1/3)) - 40*sqr
t(3)*e*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(243*
a**(11/3)*b**(1/3))
```

Mathematica [A] time = 0.553533, size = 279, normalized size = 0.93

$$\frac{-\frac{20\left(7a^{2/3}b^{2/3}c+2a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+\frac{40\left(7a^{2/3}b^{2/3}c+2a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{40\sqrt{3}a^{2/3}\left(2a^{2/3}e-7b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+54a^3(\ldots}{486a^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] ((-486*a*c)/x + (9*a^2*(9*a*d + 8*a*e*x - 16*b*c*x^2))/(a + b*x^3
)^2 + (6*a*(27*a*d + 20*a*e*x - 59*b*c*x^2))/(a + b*x^3) + (54*a^
3*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3)^3 - (40*Sqrt[3]*a^(2/3)
*(-7*b^(2/3)*c + 2*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(1/3) + 486*a*d*Log[x] + (40*(7*a^(2/3)*b^(2/3)*c + 2
*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (20*(7*a^(2/3)*b^
(2/3)*c + 2*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/b^(1/3) - 162*a*d*Log[a + b*x^3])/(486*a^5)
```

Maple [A] time = 0.027, size = 397, normalized size = 1.3

$$\begin{aligned}
&-\frac{c}{a^4x}+\frac{d\ln(x)}{a^4}-\frac{59\,x^8b^3c}{81\,a^4\left(bx^3+a\right)^3}+\frac{20\,x^7b^2e}{81\,a^3\left(bx^3+a\right)^3}+\frac{b^2dx^6}{3\,a^3\left(bx^3+a\right)^3}\\
&-\frac{142\,x^5b^2c}{81\,a^3\left(bx^3+a\right)^3}+\frac{52\,bx^4e}{81\,a^2\left(bx^3+a\right)^3}+\frac{5\,bx^3d}{6\,a^2\left(bx^3+a\right)^3}-\frac{92\,bx^2c}{81\,a^2\left(bx^3+a\right)^3}\\
&+\frac{41\,ex}{81\,a\left(bx^3+a\right)^3}+\frac{11\,d}{18\,a\left(bx^3+a\right)^3}+\frac{40\,e}{243\,a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&-\frac{20\,e}{243\,a^3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{40\,e\sqrt{3}}{243\,a^3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&+\frac{140\,c}{243\,a^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{70\,c}{243\,a^4}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\
&-\frac{140\,c\sqrt{3}}{243\,a^4}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{d\ln\left(bx^3+a\right)}{3\,a^4}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x)
```

```
[Out] -c/a^4/x+d*ln(x)/a^4-59/81/a^4/(b*x^3+a)^3*x^8*b^3*c+20/81/a^3/(b
*x^3+a)^3*x^7*b^2*e+1/3/a^3/(b*x^3+a)^3*x^6*b^2*d-142/81/a^3/(b*x
```

```
^3+a)^3*x^5*b^2*c+52/81/a^2/(b*x^3+a)^3*x^4*b*e+5/6/a^2/(b*x^3+a)
^3*x^3*b*d-92/81/a^2/(b*x^3+a)^3*x^2*b*c+41/81/a/(b*x^3+a)^3*x*e+
11/18/a/(b*x^3+a)^3*d+40/243/a^3*e/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-20/243/a^3*e/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+40
/243/a^3*e/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))+140/243/a^4*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-70/243/a^4*c/
(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-140/243/a^4*c*3^(1/
2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*d*ln(b
*x^3+a)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219696, size = 427, normalized size = 1.42

$$-\frac{d\ln\left(\left|bx^3+a\right|\right)}{3\,a^4}+\frac{d\ln\left(|x|\right)}{a^4}+\frac{10\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ae-7\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}$$
$$+\frac{20\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ab^2e+7\,\left(-ab^2\right)^{\frac{2}{3}}b^2c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b^3}$$
$$-\frac{280\,b^3cx^9-40\,ab^2x^8e-54\,ab^2dx^7+770\,ab^2cx^6-104\,a^2bx^5e-135\,a^2bdx^4+670\,a^2bcx^3-82\,a^3x^2e-99\,a^3dx+162\,a^3c}{162\,(bx^3+a)^3a^4x}$$
$$+\frac{20\left(7\,a^4b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^5be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x^2),x, algorithm="giac")
```

```
[Out] -1/3*d*ln(abs(b*x^3 + a))/a^4 + d*ln(abs(x))/a^4 + 10/243*(2*(-a*
b^2)^(1/3)*a*e - 7*(-a*b^2)^(2/3)*c)*ln(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^5*b) + 20/243*sqrt(3)*(2*(-a*b^2)^(1/3)*a*b^2*e +
7*(-a*b^2)^(2/3)*b^2*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a^5*b^3) - 1/162*(280*b^3*c*x^9 - 40*a*b^2*x^8*e -
54*a*b^2*d*x^7 + 770*a*b^2*c*x^6 - 104*a^2*b*x^5*e - 135*a^2*b*d*
x^4 + 670*a^2*b*c*x^3 - 82*a^3*x^2*e - 99*a^3*d*x + 162*a^3*c)/((
b*x^3 + a)^3*a^4*x) + 20/243*(7*a^4*b^2*c*(-a/b)^(1/3) - 2*a^5*b*
e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^9*b)
```

3.351 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx$

Optimal. Leaf size=310

$$\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}}+\frac{20\sqrt[3]{b}\left(7\sqrt[3]{a}d+11\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}}-\frac{x\left(139bc+118bdx+99bex^2\right)}{162a^4\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^4}-\frac{c}{2a^4x^2}-\frac{d}{a^4x}+\frac{e\log(x)}{a^4}-\frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3}$$

```
[Out] -c/(2*a^4*x^2) - d/(a^4*x) - (x*(b*c + b*d*x + b*e*x^2))/(9*a^2*(
a + b*x^3)^3) - (x*(17*b*c + 16*b*d*x + 15*b*e*x^2))/(54*a^3*(a +
 b*x^3)^2) - (x*(139*b*c + 118*b*d*x + 99*b*e*x^2))/(162*a^4*(a +
 b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(14/3)) + (e*
Log[x])/a^4 - (20*b^(1/3)*(11*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3
) + b^(1/3)*x])/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*c - 7*a^
(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(
14/3)) - (e*Log[a + b*x^3])/(3*a^4)
```

Rubi [A] time = 1.23059, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$

$$\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}}+\frac{20\sqrt[3]{b}\left(7\sqrt[3]{a}d+11\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}}-\frac{x\left(139bc+118bdx+99bex^2\right)}{162a^4\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^4}-\frac{c}{2a^4x^2}-\frac{d}{a^4x}+\frac{e\log(x)}{a^4}-\frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] -c/(2*a^4*x^2) - d/(a^4*x) - (x*(b*c + b*d*x + b*e*x^2))/(9*a^2*(
a + b*x^3)^3) - (x*(17*b*c + 16*b*d*x + 15*b*e*x^2))/(54*a^3*(a +
 b*x^3)^2) - (x*(139*b*c + 118*b*d*x + 99*b*e*x^2))/(162*a^4*(a +
 b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(14/3)) + (e*
Log[x])/a^4 - (20*b^(1/3)*(11*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3
) + b^(1/3)*x])/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*c - 7*a^
(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(
14/3)) - (e*Log[a + b*x^3])/(3*a^4)
```

Rubi in Sympy [A] time = 13.9475, size = 26, normalized size = 0.08

$$\frac{x\left(\frac{c}{x^3}+\frac{d}{x^2}+\frac{e}{x}\right)}{9a\left(a+bx^3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**4,x)
```

```
[Out] x*(c/x**3 + d/x**2 + e/x)/(9*a*(a + b*x**3)**3)
```

Mathematica [A] time = 0.608032, size = 284, normalized size = 0.92

$$20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+40\sqrt[3]{b}\left(7a^{2/3}d-11\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{54a^3(ae-bx(c+dx))}{(a+bx^3)^3}\cdot$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] ((-243*a*c)/x^2 - (486*a*d)/x + (54*a^3*(a*e - b*x*(c + d*x)))/(a
 + b*x^3)^3 + (9*a^2*(9*a*e - b*x*(17*c + 16*d*x)))/(a + b*x^3)^2
 + (3*a*(54*a*e - b*x*(139*c + 118*d*x)))/(a + b*x^3) + 40*Sqrt[3
]*a^(1/3)*b^(1/3)*(11*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(
1/3)*x)/a^(1/3))/Sqrt[3]] + 486*a*e*Log[x] + 40*b^(1/3)*(-11*a^(1
/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + 20*b^(1/3
)*(11*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2] - 162*a*e*Log[a + b*x^3])/(486*a^5)
```

Maple [A] time = 0.028, size = 400, normalized size = 1.3

$$-\frac{c}{2\,a^4x^2}-\frac{d}{a^4x}+\frac{e\ln(x)}{a^4}-\frac{59\,b^3dx^8}{81\,a^4\,(bx^3+a)^3}-\frac{139\,b^3cx^7}{162\,a^4\,(bx^3+a)^3}+\frac{b^2ex^6}{3\,a^3\,(bx^3+a)^3}$$
$$-\frac{142\,b^2dx^5}{81\,a^3\,(bx^3+a)^3}-\frac{329\,b^2cx^4}{162\,a^3\,(bx^3+a)^3}+\frac{5\,bex^3}{6\,a^2\,(bx^3+a)^3}-\frac{92\,bx^2d}{81\,a^2\,(bx^3+a)^3}$$
$$-\frac{104\,bcx}{81\,a^2\,(bx^3+a)^3}+\frac{11\,e}{18\,a\,(bx^3+a)^3}-\frac{220\,c}{243\,a^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{110\,c}{243\,a^4}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{220\,c\sqrt{3}}{243\,a^4}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{140\,d}{243\,a^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{70\,d}{243\,a^4}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{140\,d\sqrt{3}}{243\,a^4}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{e\ln\left(bx^3+a\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x)
```

```
[Out] -1/2*c/a^4/x^2-d/a^4/x+e*ln(x)/a^4-59/81*b^3/a^4/(b*x^3+a)^3*d*x^
8-139/162*b^3/a^4/(b*x^3+a)^3*c*x^7+1/3*b^2/a^3/(b*x^3+a)^3*e*x^6
-142/81*b^2/a^3/(b*x^3+a)^3*d*x^5-329/162*b^2/a^3/(b*x^3+a)^3*c*x
^4+5/6*b/a^2/(b*x^3+a)^3*e*x^3-92/81*b/a^2/(b*x^3+a)^3*x^2*d-104/
81*b/a^2/(b*x^3+a)^3*c*x+11/18/a/(b*x^3+a)^3*e-220/243/a^4*c/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))+110/243/a^4*c/(a/b)^(2/3)*ln(x^2-x*(a/b
)^(1/3)+(a/b)^(2/3))-220/243/a^4*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))+140/243/a^4*d/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))-70/243/a^4*d/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3
))-140/243/a^4*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))-1/3*e*ln(b*x^3+a)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x^3),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.21822, size = 441, normalized size = 1.42

$$-\frac{e\ln\left(\left|bx^3+a\right|\right)}{3\,a^4}+\frac{e\ln\left(\left|x\right|\right)}{a^4}-\frac{10\left(11\,\left(-ab^2\right)^{\frac{1}{3}}bc+7\,\left(-ab^2\right)^{\frac{2}{3}}d\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}$$
$$-\frac{20\,\sqrt{3}\left(11\,\left(-ab^2\right)^{\frac{1}{3}}ab^3c-7\,\left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^6b^3}$$
$$+\frac{20\left(7\,a^4b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+11\,a^4b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}$$
$$-\frac{280\,b^3dx^{10}+220\,b^3cx^9-54\,ab^2x^8e+770\,ab^2dx^7+572\,ab^2cx^6-135\,a^2bx^5e+670\,a^2bdx^4+451\,a^2bcx^3-99\,a^3x^2e+162}{162\left(bx^3+a\right)^3a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x^3),x, algorithm="giac")
```

```
[Out] -1/3*e*ln(abs(b*x^3 + a))/a^4 + e*ln(abs(x))/a^4 - 10/243*(11*(-a
*b^2)^(1/3)*b*c + 7*(-a*b^2)^(2/3)*d)*ln(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/(a^5*b) - 20/243*sqrt(3)*(11*(-a*b^2)^(1/3)*a*b^3*c
- 7*(-a*b^2)^(2/3)*a*b^2*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(a^6*b^3) + 20/243*(7*a^4*b^2*d*(-a/b)^(1/3) + 1
1*a^4*b^2*c)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^9*b) - 1/1
62*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*x^8*e + 770*a*b^2*d
*x^7 + 572*a*b^2*c*x^6 - 135*a^2*b*x^5*e + 670*a^2*b*d*x^4 + 451*
a^2*b*c*x^3 - 99*a^3*x^2*e + 162*a^3*d*x + 81*a^3*c)/((b*x^3 + a)
^3*a^4*x^2)
```

3.352 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx$

Optimal. Leaf size=340

$$\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}}+\frac{20\sqrt[3]{b}\left(7\sqrt[3]{a}e+11\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}}+\frac{4bc\log\left(a+bx^3\right)}{3a^5}-\frac{4bc\log(x)}{a^5}-\frac{x\left(-\frac{234b^2cx^2}{a}+139bd+118bex\right)}{162a^4\left(a+bx^3\right)}-\frac{c}{3a^4x^3}-\frac{d}{2a^4x^2}-\frac{e}{a^4x}-\frac{x\left(-\frac{24b^2cx^2}{a}+17bd+16bex\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2\left(a+bx^3\right)^3}$$

```
[Out] -c/(3*a^4*x^3) - d/(2*a^4*x^2) - e/(a^4*x) - (x*(b*d + b*e*x - (b
^2*c*x^2)/a))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*d + 16*b*e*x - (24
*b^2*c*x^2)/a))/(54*a^3*(a + b*x^3)^2) - (x*(139*b*d + 118*b*e*x
- (234*b^2*c*x^2)/a))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(
1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(81*Sqrt[3]*a^(14/3)) - (4*b*c*Log[x])/a^5 - (20*b^(1/3)*
(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(14
/3)) + (10*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)) + (4*b*c*Log[a + b*
x^3])/(3*a^5)
```

Rubi [A] time = 1.39981, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}}+\frac{20\sqrt[3]{b}\left(7\sqrt[3]{a}e+11\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}}+\frac{4bc\log\left(a+bx^3\right)}{3a^5}-\frac{4bc\log(x)}{a^5}-\frac{x\left(-\frac{234b^2cx^2}{a}+139bd+118bex\right)}{162a^4\left(a+bx^3\right)}-\frac{c}{3a^4x^3}-\frac{d}{2a^4x^2}-\frac{e}{a^4x}-\frac{x\left(-\frac{24b^2cx^2}{a}+17bd+16bex\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```


```
[Out] -c/(3*a^4*x^3) - d/(2*a^4*x^2) - e/(a^4*x) - (x*(b*d + b*e*x - (b
^2*c*x^2)/a))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*d + 16*b*e*x - (24
*b^2*c*x^2)/a))/(54*a^3*(a + b*x^3)^2) - (x*(139*b*d + 118*b*e*x
- (234*b^2*c*x^2)/a))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(
1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(81*Sqrt[3]*a^(14/3)) - (4*b*c*Log[x])/a^5 - (20*b^(1/3)*
(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(14
/3)) + (10*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)) + (4*b*c*Log[a + b*
x^3])/(3*a^5)
```

Rubi in Sympy [A] time = 14.1897, size = 27, normalized size = 0.08

$$\frac{x\left(\frac{c}{x^4}+\frac{d}{x^3}+\frac{e}{x^2}\right)}{9a\left(a+bx^3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**4,x)
```

```
[Out] x*(c/x**4 + d/x**3 + e/x**2)/(9*a*(a + b*x**3)**3)
```

Mathematica [A]   time = 0.990289, size = 284, normalized size = 0.84

$$-\frac{-20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+40\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{54a^3b(c+x(d+ex)}{(a+bx^3)^3}}{}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

```
[Out] -((162*a*c)/x^3 + (243*a*d)/x^2 + (486*a*e)/x + (54*a^3*b*(c + x*
(d + e*x)))/(a + b*x^3)^3 + (9*a^2*b*(18*c + x*(17*d + 16*e*x)))/
(a + b*x^3)^2 + (3*a*b*(162*c + x*(139*d + 118*e*x)))/(a + b*x^3)
 - 40*Sqrt[3]*a^(1/3)*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 1944*b*c*Log[x] + 40*b^(1
/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]
 - 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 648*b*c*Log[a + b*x^3])/(486*a
^5)
```

Maple [A]   time = 0.028, size = 415, normalized size = 1.2

$$
\begin{aligned}
&-\frac{c}{3\,a^4x^3}-\frac{d}{2\,a^4x^2}-\frac{e}{a^4x}-4\,\frac{bc\ln(x)}{a^5}-\frac{59\,b^3x^8e}{81\,a^4\,(bx^3+a)^3}-\frac{139\,b^3x^7d}{162\,a^4\,(bx^3+a)^3}\\
&-\frac{b^3x^6c}{a^4\,(bx^3+a)^3}-\frac{142\,b^2x^5e}{81\,a^3\,(bx^3+a)^3}-\frac{329\,b^2x^4d}{162\,a^3\,(bx^3+a)^3}-\frac{7\,b^2x^3c}{3\,a^3\,(bx^3+a)^3}\\
&-\frac{92\,bex^2}{81\,a^2\,(bx^3+a)^3}-\frac{104\,bxd}{81\,a^2\,(bx^3+a)^3}-\frac{13\,bc}{9\,a^2\,(bx^3+a)^3}-\frac{220\,d}{243\,a^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&+\frac{110\,d}{243\,a^4}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{220\,d\sqrt{3}}{243\,a^4}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&+\frac{140\,e}{243\,a^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{70\,e}{243\,a^4}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}\\
&-\frac{140\,e\sqrt{3}}{243\,a^4}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{4\,bc\ln\left(bx^3+a\right)}{3\,a^5}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x)
```

```
[Out] -1/3*c/a^4/x^3-1/2*d/a^4/x^2-e/a^4/x-4*b*c*ln(x)/a^5-59/81/a^4*b^
3/(b*x^3+a)^3*x^8*e-139/162/a^4*b^3/(b*x^3+a)^3*x^7*d-1/a^4*b^3/(
b*x^3+a)^3*x^6*c-142/81/a^3*b^2/(b*x^3+a)^3*x^5*e-329/162/a^3*b^2
/(b*x^3+a)^3*x^4*d-7/3/a^3*b^2/(b*x^3+a)^3*x^3*c-92/81/a^2*b/(b*x
^3+a)^3*x^2*e-104/81/a^2*b/(b*x^3+a)^3*x*d-13/9/a^2*b/(b*x^3+a)^3
*c-220/243/a^4*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+110/243/a^4*d/(a/b
)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-220/243/a^4*d/(a/b)^(2/
```

```
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+140/243/a^4*e/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))-70/243/a^4*e/(a/b)^(1/3)*ln(x^2-x*(
a/b)^(1/3)+(a/b)^(2/3))-140/243/a^4*e*3^(1/2)/(a/b)^(1/3)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+4/3*b*c*ln(b*x^3+a)/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x^4),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218495, size = 459, normalized size = 1.35

$$\frac{4\,bc\ln\left(\left|bx^3+a\right|\right)}{3\,a^5}-\frac{4\,bc\ln\left(\left|x\right|\right)}{a^5}-\frac{10\left(11\left(-ab^2\right)^{\frac{1}{3}}bd+7\left(-ab^2\right)^{\frac{2}{3}}e\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}-\frac{280\,b^3x^{11}e+220\,b^3dx^{10}+216\,b^3cx^9+770\,ab^2x^8e+572\,ab^2dx^7+540\,ab^2cx^6+670\,a^2bx^5e+451\,a^2bdx^4+396\,a^2bcx^3+1}{162\left(bx^4+ax\right)^3a^4}-\frac{20\sqrt{3}\left(11\left(-ab^2\right)^{\frac{1}{3}}ab^3d-7\left(-ab^2\right)^{\frac{2}{3}}ab^2e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^6b^3}+\frac{20\left(7\,a^6b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+11\,a^6b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{11}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^4*x^4),x, algorithm="giac")
```

```
[Out] 4/3*b*c*ln(abs(b*x^3 + a))/a^5 - 4*b*c*ln(abs(x))/a^5 - 10/243*(1
1*(-a*b^2)^(1/3)*b*d + 7*(-a*b^2)^(2/3)*e)*ln(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/(a^5*b) - 1/162*(280*b^3*x^11*e + 220*b^3*d*x^1
0 + 216*b^3*c*x^9 + 770*a*b^2*x^8*e + 572*a*b^2*d*x^7 + 540*a*b^2
*c*x^6 + 670*a^2*b*x^5*e + 451*a^2*b*d*x^4 + 396*a^2*b*c*x^3 + 16
2*a^3*x^2*e + 81*a^3*d*x + 54*a^3*c)/((b*x^4 + a*x)^3*a^4) - 20/2
43*sqrt(3)*(11*(-a*b^2)^(1/3)*a*b^3*d - 7*(-a*b^2)^(2/3)*a*b^2*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^6*b^3)
+ 20/243*(7*a^6*b^2*(-a/b)^(1/3)*e + 11*a^6*b^2*d)*(-a/b)^(1/3)*l
n(abs(x - (-a/b)^(1/3)))/(a^11*b)
```

3.353 $\int \frac{2ax-x^2}{a^3+x^3}\,dx$

Optimal. Leaf size=29

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rubi [A] time = 0.100566, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*a*x - x^2)/(a^3 + x^3),x]

[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rubi in Sympy [A] time = 11.0708, size = 31, normalized size = 1.07

$$-\log(a+x)-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}-\frac{2x}{3}\right)}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*a*x-x**2)/(a**3+x**3),x)

[Out] -log(a + x) - 2*sqrt(3)*atan(sqrt(3)*(a/3 - 2*x/3)/a)/3
```

Mathematica [A] time = 0.0240714, size = 57, normalized size = 1.97

$$\frac{1}{3}\left(-\log\left(a^3+x^3\right)+\log\left(a^2-ax+x^2\right)-2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*a*x - x^2)/(a^3 + x^3),x]

[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^
2 - a*x + x^2] - Log[a^3 + x^3])/3
```

Maple [A] time = 0.01, size = 29, normalized size = 1.

$$-\ln(a+x)+\frac{2\sqrt{3}}{3}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*a*x-x^2)/(a^3+x^3),x)
```

```
[Out] -ln(a+x)+2/3*3^(1/2)*arctan(1/3*(2*x-a)/a*3^(1/2))
```

Maxima [A] time = 1.5316, size = 35, normalized size = 1.21

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log\left(a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a*x - x^2)/(a^3 + x^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Fricas [A] time = 0.255039, size = 41, normalized size = 1.41

$$-\frac{1}{3}\sqrt{3}\left(\sqrt{3}\log\left(a+x\right)-2\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a*x - x^2)/(a^3 + x^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*log(a + x) - 2*arctan(-1/3*sqrt(3)*(a - 2*x
)/a))
```

Sympy [A] time = 0.588979, size = 54, normalized size = 1.86

$$-\log\left(a+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a*x-x**2)/(a**3+x**3),x)
```

```
[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)
*I*log(-a/2 + sqrt(3)*I*a/2 + x)/3
```

GIAC/XCAS [A] time = 0.212063, size = 36, normalized size = 1.24

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\ln\left(|a+x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a*x - x^2)/(a^3 + x^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - ln(abs(a + x))
```

3.354 $\int \frac{(2a-x)x}{a^3+x^3}\,dx$

Optimal. Leaf size=29

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rubi [A] time = 0.0692421, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2*a - x)*x)/(a^3 + x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rubi in Sympy [A] time = 9.62086, size = 31, normalized size = 1.07

$$-\log(a+x)-\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}-\frac{2x}{3}\right)}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*a-x)*x/(a**3+x**3),x)
```

```
[Out] -log(a + x) - 2*sqrt(3)*atan(sqrt(3)*(a/3 - 2*x/3)/a)/3
```

Mathematica [A] time = 0.010361, size = 57, normalized size = 1.97

$$\frac{1}{3}\left(-\log\left(a^3+x^3\right)+\log\left(a^2-ax+x^2\right)-2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2*a - x)*x)/(a^3 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^
2 - a*x + x^2] - Log[a^3 + x^3])/3
```

Maple [A] time = 0.005, size = 29, normalized size = 1.

$$-\ln(a+x)+\frac{2\sqrt{3}}{3}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*a-x)*x/(a^3+x^3),x)
```

```
[Out] -ln(a+x)+2/3*3^(1/2)*arctan(1/3*(2*x-a)/a*3^(1/2))
```

Maxima [A] time = 1.51933, size = 35, normalized size = 1.21

$$\frac{2}{3}\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)-\log\left(a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a - x)*x/(a^3 + x^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Fricas [A] time = 0.248579, size = 41, normalized size = 1.41

$$-\frac{1}{3}\,\sqrt{3}\left(\sqrt{3}\log\left(a+x\right)-2\,\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a - x)*x/(a^3 + x^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*log(a + x) - 2*arctan(-1/3*sqrt(3)*(a - 2*x
)/a))
```

Sympy [A] time = 0.592532, size = 54, normalized size = 1.86

$$-\log\left(a+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a-x)*x/(a**3+x**3),x)
```

```
[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)
*I*log(-a/2 + sqrt(3)*I*a/2 + x)/3
```

GIAC/XCAS [A] time = 0.21233, size = 36, normalized size = 1.24

$$\frac{2}{3}\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)-\ln\left(|a+x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a - x)*x/(a^3 + x^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - ln(abs(a + x))
```

3.355 $\int \frac{2ax+x^2}{a^3-x^3}\,dx$

Optimal. Leaf size=31

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rubi [A] time = 0.0977033, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a*x + x^2)/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rubi in Sympy [A] time = 11.6381, size = 31, normalized size = 1.

$$-\log(a-x)-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}+\frac{2x}{3}\right)}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*a*x+x**2)/(a**3-x**3),x)
```

```
[Out] -log(a - x) - 2*sqrt(3)*atan(sqrt(3)*(a/3 + 2*x/3)/a)/3
```

Mathematica [A] time = 0.0263551, size = 58, normalized size = 1.87

$$\frac{1}{3}\left(-\log\left(x^3-a^3\right)+\log\left(a^2+ax+x^2\right)-2\log(x-a)-2\sqrt{3}\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a*x + x^2)/(a^3 - x^3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a
^2 + a*x + x^2] - Log[-a^3 + x^3])/3
```

Maple [A] time = 0.012, size = 29, normalized size = 0.9

$$-\frac{2\sqrt{3}}{3}\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)-\ln(x-a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*a*x+x^2)/(a^3-x^3),x)
```

```
[Out] -2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)-ln(x-a)
```

Maxima [A] time = 1.53961, size = 38, normalized size = 1.23

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log\left(-a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a*x + x^2)/(a^3 - x^3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Fricas [A] time = 0.258938, size = 43, normalized size = 1.39

$$-\frac{1}{3}\sqrt{3}\left(\sqrt{3}\log\left(-a+x\right)+2\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a*x + x^2)/(a^3 - x^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*log(-a + x) + 2*arctan(1/3*sqrt(3)*(a + 2*x
)/a))
```

Sympy [A] time = 0.599156, size = 54, normalized size = 1.74

$$-\log\left(-a+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a*x+x**2)/(a**3-x**3),x)
```

```
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)
*I*log(a/2 + sqrt(3)*I*a/2 + x)/3
```

GIAC/XCAS [A] time = 0.21131, size = 39, normalized size = 1.26

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\ln\left(|-a+x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a*x + x^2)/(a^3 - x^3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - ln(abs(-a + x))
```

3.356 $\int \frac{x(2a+x)}{a^3-x^3}\,dx$

Optimal. Leaf size=31

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rubi [A] time = 0.0730572, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(2*a + x))/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rubi in Sympy [A] time = 10.7522, size = 31, normalized size = 1.

$$-\log\left(a-x\right)-\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{a}{3}+\frac{2x}{3}\right)}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*a+x)/(a**3-x**3),x)
```

```
[Out] -log(a - x) - 2*sqrt(3)*atan(sqrt(3)*(a/3 + 2*x/3)/a)/3
```

Mathematica [A] time = 0.0107569, size = 58, normalized size = 1.87

$$\frac{1}{3}\left(-\log\left(x^3-a^3\right)+\log\left(a^2+ax+x^2\right)-2\log(x-a)-2\sqrt{3}\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(2*a + x))/(a^3 - x^3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a
^2 + a*x + x^2] - Log[-a^3 + x^3])/3
```

Maple [A] time = 0.007, size = 29, normalized size = 0.9

$$-\frac{2\sqrt{3}}{3}\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)-\ln(x-a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*a+x)/(a^3-x^3),x)
```

```
[Out] -2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)-ln(x-a)
```

Maxima [A] time = 1.56903, size = 38, normalized size = 1.23

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log\left(-a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a + x)*x/(a^3 - x^3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Fricas [A] time = 0.245718, size = 43, normalized size = 1.39

$$-\frac{1}{3}\sqrt{3}\left(\sqrt{3}\log\left(-a+x\right)+2\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a + x)*x/(a^3 - x^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*log(-a + x) + 2*arctan(1/3*sqrt(3)*(a + 2*x
)/a))
```

Sympy [A] time = 0.590054, size = 54, normalized size = 1.74

$$-\log\left(-a+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*a+x)/(a**3-x**3),x)
```

```
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)
*I*log(a/2 + sqrt(3)*I*a/2 + x)/3
```

GIAC/XCAS [A] time = 0.213617, size = 39, normalized size = 1.26

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\ln\left(\left|-a+x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*a + x)*x/(a^3 - x^3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - ln(abs(-a + x))
```

3.357 $\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

Optimal. Leaf size=50

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Lo
g[(a/b)^(1/3) + x])/b
```

Rubi [A] time = 0.160007, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Lo
g[(a/b)^(1/3) + x])/b
```

Rubi in Sympy [A] time = 13.702, size = 46, normalized size = 0.92

$$\frac{C\log\left(x+\sqrt[3]{\frac{a}{b}}\right)}{b}+\frac{2\sqrt{3}C\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2x}{3\sqrt[3]{\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(-2*(a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*log(x + (a/b)**(1/3))/b + 2*sqrt(3)*C*atan(sqrt(3)*(-2*x/(3*(a/
b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.0916607, size = 146, normalized size = 2.92

$$\frac{C\left(-\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a+bx^3\right)+2\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] - (
a/b)^(1/3)*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
 + a^(1/3)*Log[a + b*x^3]))/(3*a^(1/3)*b)
```

Maple [A] time = 0.008, size = 87, normalized size = 1.7

$$\frac{2C}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)-\frac{C}{3b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)+\frac{C\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x)
```

```
[Out] 2/3*C*ln(x+(a/b)^(1/3))/b-1/3*C/b*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3
))-2/3*C/b*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/
b*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x - 2*C*(a/b)^(1/3))*x/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294038, size = 77, normalized size = 1.54

$$\frac{\sqrt{3}\left(\sqrt{3}C\log\left(bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+a\right)-2C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x - 2*C*(a/b)^(1/3))*x/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*C*log(b*x*(a/b)^(2/3) + a) - 2*C*arctan(1/3*
(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a))/b
```

Sympy [A] time = 0.847972, size = 100, normalized size = 2.

$$\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-2*(a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/
3)) - sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2
*b*(a/b)**(2/3)) + sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

GIAC/XCAS [A] time = 0.240343, size = 220, normalized size = 4.4

$$-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-2\left(ab^{2}\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}+\frac{\sqrt{3}\left(\sqrt{3}ab^{2}i-ab^{2}\right)C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{3}}+\frac{\left(\sqrt{3}ab^{2}i+3\,ab^{2}\right)C\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x - 2*C*(a/b)^(1/3))*x/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*(C*b*(-a/b)^(2/3) - 2*(a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(
1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(sqrt(3)*a*b^2
*i - a*b^2)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(a*b^3) + 1/6*(sqrt(3)*a*b^2*i + 3*a*b^2)*C*ln(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/(a*b^3)
```

3.358 $\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (
C*Log[(-(a/b))^(1/3) + x])/b
```

Rubi [A] time = 0.171559, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (
C*Log[(-(a/b))^(1/3) + x])/b
```

Rubi in Sympy [A] time = 14.0248, size = 51, normalized size = 0.96

$$-\frac{C\log\left(x+\sqrt[3]{-\frac{a}{b}}\right)}{b}-\frac{2\sqrt{3}C\,\mathrm{atan}\left(\sqrt{3}\left(-\frac{2x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-2*(-a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*log(x + (-a/b)**(1/3))/b - 2*sqrt(3)*C*atan(sqrt(3)*(-2*x/(3*(
-a/b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.133579, size = 149, normalized size = 2.81

$$-\frac{C\left(\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a-bx^3\right)-2\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right.\right.}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -(C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3
)*x] + (-(a/b))^(1/3)*b^(1/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2] + a^(1/3)*Log[a - b*x^3]))/(3*a^(1/3)*b)
```

Maple [B] time = 0.007, size = 135, normalized size = 2.6

$$\frac{2C}{3b}\sqrt[3]{-\frac{a}{b}}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{C}{3b}\sqrt[3]{-\frac{a}{b}}\ln\left(x^2+x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{2C\sqrt{3}}{3b}\sqrt[3]{-\frac{a}{b}}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{C\ln\left(bx^3-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x)
```

```
[Out] 2/3*C*(-a/b)^(1/3)/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))-1/3*C*(-a/b)^(
1/3)/b/(a/b)^(1/3)*ln(x^2+x*(a/b)^(1/3)+(a/b)^(2/3))+2/3*C*(-a/b)
^(1/3)*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/
2))-1/3*C/b*ln(b*x^3-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*x - 2*C*(-a/b)^(1/3))*x/(b*x^3 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29077, size = 81, normalized size = 1.53

$$-\frac{\sqrt{3}\left(\sqrt{3}C\log\left(bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-a\right)+2C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*x - 2*C*(-a/b)^(1/3))*x/(b*x^3 - a),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*C*log(b*x*(-a/b)^(2/3) - a) + 2*C*arctan(1/
3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a))/b
```

Sympy [A] time = 0.881824, size = 110, normalized size = 2.08

$$-\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-2*(-a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(-a/b)**
(2/3)) - sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a
/(2*b*(-a/b)**(2/3)) + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

GIAC/XCAS [A] time = 0.241539, size = 209, normalized size = 3.94

$$-\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\left(-ab^{2}\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}+\frac{\sqrt{3}\left(\sqrt{3}ab^{2}i+ab^{2}\right)C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{3}}+\frac{\left(\sqrt{3}ab^{2}i-3\,ab^{2}\right)C\ln\left(x^{2}+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(C*x - 2*C*(-a/b)^(1/3))*x/(b*x^3 - a),x, algorithm="giac")
```

```
[Out] -1/3*(C*b*(a/b)^(2/3) - 2*(-a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/
3)*ln(abs(x - (a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(sqrt(3)*a*b^2*i
+ a*b^2)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a
*b^3) + 1/6*(sqrt(3)*a*b^2*i - 3*a*b^2)*C*ln(x^2 + x*(a/b)^(1/3)
+ (a/b)^(2/3))/(a*b^3)
```

3.359 $\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

Optimal. Leaf size=54

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C
*Log[(-(a/b))^(1/3) - x])/b
```

Rubi [A] time = 0.14094, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C
*Log[(-(a/b))^(1/3) - x])/b
```

Rubi in Sympy [A] time = 14.1506, size = 49, normalized size = 0.91

$$\frac{C\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{b}+\frac{2\sqrt{3}C\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*(-a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*log(x - (-a/b)**(1/3))/b + 2*sqrt(3)*C*atan(sqrt(3)*(2*x/(3*(-a
/b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.0882398, size = 148, normalized size = 2.74

$$\frac{C\left(\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a+bx^3\right)-2\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)
*x] + (-(a/b))^(1/3)*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2] + a^(1/3)*Log[a + b*x^3]))/(3*a^(1/3)*b)
```

Maple [B] time = 0.005, size = 132, normalized size = 2.4

$$-\frac{2C}{3b}\sqrt[3]{-\frac{a}{b}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{C}{3b}\sqrt[3]{-\frac{a}{b}}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{2C\sqrt{3}}{3b}\sqrt[3]{-\frac{a}{b}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{C\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x)
```

```
[Out] -2/3*C*(-a/b)^(1/3)/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/3*C*(-a/b)^
(1/3)/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/3*C*(-a/b
)^(1/3)*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))+1/3*C/b*ln(b*x^3+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x + 2*C*(-a/b)^(1/3))*x/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278774, size = 80, normalized size = 1.48

$$\frac{\sqrt{3}\left(\sqrt{3}C\log\left(bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+a\right)-2C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x + 2*C*(-a/b)^(1/3))*x/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*C*log(b*x*(-a/b)^(2/3) + a) - 2*C*arctan(1/3
*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a))/b
```

Sympy [A] time = 0.840935, size = 109, normalized size = 2.02

$$\frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*(-a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(-a/b)**(
2/3)) - sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a
/(2*b*(-a/b)**(2/3)) + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

GIAC/XCAS [A] time = 0.216093, size = 131, normalized size = 2.43

$$\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x + 2*C*(-a/b)^(1/3))*x/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/b - 1/3*(C*b*(-a/b)^(2/3) + 2*(-a*b^2)^(1/3)*C*(-a/b)^(1/3))*(
-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b)
```

3.360 $\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*L
og[(a/b)^(1/3) - x])/b
```

Rubi [A] time = 0.143715, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*L
og[(a/b)^(1/3) - x])/b
```

Rubi in Sympy [A] time = 15.1221, size = 48, normalized size = 0.91

$$-\frac{C\log\left(x-\sqrt[3]{\frac{a}{b}}\right)}{b}-\frac{2\sqrt{3}C\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3\sqrt[3]{\frac{a}{b}}}+\frac{1}{3}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*(a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*log(x - (a/b)**(1/3))/b - 2*sqrt(3)*C*atan(sqrt(3)*(2*x/(3*(a/
b)**(1/3)) + 1/3))/(3*b)
```

Mathematica [B] time = 0.0989256, size = 147, normalized size = 2.77

$$-\frac{C\left(-\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a-bx^3\right)+2\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -(C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] -
(a/b)^(1/3)*b^(1/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
] + a^(1/3)*Log[a - b*x^3]))/(3*a^(1/3)*b)
```

Maple [A] time = 0.008, size = 90, normalized size = 1.7

$$-\frac{2C}{3b}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)+\frac{C}{3b}\ln\left(x^{2}+x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)-\frac{C\ln\left(bx^{3}-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x)
```

```
[Out] -2/3*C/b*ln(x-(a/b)^(1/3))+1/3*C/b*ln(x^2+x*(a/b)^(1/3)+(a/b)^(2/
3))-2/3*C*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2))/b*3^(1/2)-1/3*C
/b*ln(b*x^3-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*x + 2*C*(a/b)^(1/3))*x/(b*x^3 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275316, size = 78, normalized size = 1.47

$$-\frac{\sqrt{3}\left(\sqrt{3}C\log\left(bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-a\right)+2C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*x + 2*C*(a/b)^(1/3))*x/(b*x^3 - a),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*C*log(b*x*(a/b)^(2/3) - a) + 2*C*arctan(1/3
*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a))/b
```

Sympy [A] time = 0.861531, size = 102, normalized size = 1.92

$$-\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*(a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(a/b)**(2
/3)) - sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2
*b*(a/b)**(2/3)) + sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

GIAC/XCAS [A] time = 0.217312, size = 122, normalized size = 2.3

$$-\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(C*x + 2*C*(a/b)^(1/3))*x/(b*x^3 - a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3)
)/b - 1/3*(C*b*(a/b)^(2/3) + 2*(a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)
^(1/3)*ln(abs(x - (a/b)^(1/3)))/(a*b)
```

3.361 $\int x^4 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$

Optimal. Leaf size=97

$$\frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + (
(b*d + a*g)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g
*x^12)/12 + (b*h*x^13)/13
```

Rubi [A] time = 0.231481, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028

$$\frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + (
(b*d + a*g)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g
*x^12)/12 + (b*h*x^13)/13
```

Rubi in Sympy [A] time = 28.0393, size = 90, normalized size = 0.93

$$\frac{acx^5}{5}+\frac{adx^6}{6}+\frac{aex^7}{7}+\frac{bfx^{11}}{11}+\frac{bgx^{12}}{12}+\frac{bhx^{13}}{13}+x^{10}\left(\frac{ah}{10}+\frac{be}{10}\right)+x^9\left(\frac{ag}{9}+\frac{bd}{9}\right)+x^8\left(\frac{af}{8}+\frac{bc}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**4*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**5/5 + a*d*x**6/6 + a*e*x**7/7 + b*f*x**11/11 + b*g*x**12/1
2 + b*h*x**13/13 + x**10*(a*h/10 + b*e/10) + x**9*(a*g/9 + b*d/9)
 + x**8*(a*f/8 + b*c/8)
```

Mathematica [A] time = 0.0513963, size = 97, normalized size = 1.

$$\frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + (
(b*d + a*g)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g
*x^12)/12 + (b*h*x^13)/13
```

Maple [A] time = 0.002, size = 80, normalized size = 0.8

$$\frac{acx^5}{5}+\frac{adx^6}{6}+\frac{aex^7}{7}+\frac{(af+bc)\,x^8}{8}+\frac{(ag+bd)\,x^9}{9}+\frac{(ah+be)\,x^{10}}{10}+\frac{bfx^{11}}{11}+\frac{bgx^{12}}{12}+\frac{bhx^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*
d)*x^9+1/10*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x
^13
```

Maxima [A] time = 1.37896, size = 107, normalized size = 1.1

$$\frac{1}{13}\,bhx^{13}+\frac{1}{12}\,bgx^{12}+\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,(be+ah)x^{10}+\frac{1}{9}\,(bd+ag)x^{9}+\frac{1}{7}\,aex^{7}+\frac{1}{8}\,(bc+af)x^{8}+\frac{1}{6}\,adx^{6}+\frac{1}{5}\,acx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x^4,x, algorithm="
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(b*e + a*h)*
x^10 + 1/9*(b*d + a*g)*x^9 + 1/7*a*e*x^7 + 1/8*(b*c + a*f)*x^8 +
1/6*a*d*x^6 + 1/5*a*c*x^5
```

Fricas [A] time = 0.229551, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}hb+\frac{1}{12}x^{12}gb+\frac{1}{11}x^{11}fb+\frac{1}{10}x^{10}eb+\frac{1}{10}x^{10}ha+\frac{1}{9}x^{9}db+\frac{1}{9}x^{9}ga+\frac{1}{8}x^{8}cb+\frac{1}{8}x^{8}fa+\frac{1}{7}x^{7}ea+\frac{1}{6}x^{6}da+\frac{1}{5}x^{5}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x^4,x, algorithm="
```

```
[Out] 1/13*x^13*h*b + 1/12*x^12*g*b + 1/11*x^11*f*b + 1/10*x^10*e*b + 1
/10*x^10*h*a + 1/9*x^9*d*b + 1/9*x^9*g*a + 1/8*x^8*c*b + 1/8*x^8*
f*a + 1/7*x^7*e*a + 1/6*x^6*d*a + 1/5*x^5*c*a
```

Sympy [A] time = 0.071243, size = 90, normalized size = 0.93

$$\frac{acx^{5}}{5}+\frac{adx^{6}}{6}+\frac{aex^{7}}{7}+\frac{bfx^{11}}{11}+\frac{bgx^{12}}{12}+\frac{bhx^{13}}{13}+x^{10}\left(\frac{ah}{10}+\frac{be}{10}\right)+x^{9}\left(\frac{ag}{9}+\frac{bd}{9}\right)+x^{8}\left(\frac{af}{8}+\frac{bc}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**5/5 + a*d*x**6/6 + a*e*x**7/7 + b*f*x**11/11 + b*g*x**12/1
2 + b*h*x**13/13 + x**10*(a*h/10 + b*e/10) + x**9*(a*g/9 + b*d/9)
 + x**8*(a*f/8 + b*c/8)
```

GIAC/XCAS [A] time = 0.209735, size = 117, normalized size = 1.21

$$\frac{1}{13}\,bhx^{13}+\frac{1}{12}\,bgx^{12}+\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,ahx^{10}+\frac{1}{10}\,bx^{10}e+\frac{1}{9}\,bdx^{9}+\frac{1}{9}\,agx^{9}+\frac{1}{8}\,bcx^{8}+\frac{1}{8}\,afx^{8}+\frac{1}{7}\,ax^{7}e+\frac{1}{6}\,adx^{6}+\frac{1}{5}\,acx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x^4,x, algorithm="
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*a*h*x^10 + 1
/10*b*x^10*e + 1/9*b*d*x^9 + 1/9*a*g*x^9 + 1/8*b*c*x^8 + 1/8*a*f*
x^8 + 1/7*a*x^7*e + 1/6*a*d*x^6 + 1/5*a*c*x^5
```

3.362 $\int x^3 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$

Optimal. Leaf size=97

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + (
(b*d + a*g)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x
^11)/11 + (b*h*x^12)/12
```

Rubi [A] time = 0.208474, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + (
(b*d + a*g)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x
^11)/11 + (b*h*x^12)/12
```

Rubi in Sympy [A] time = 31.5982, size = 90, normalized size = 0.93

$$\frac{acx^4}{4}+\frac{adx^5}{5}+\frac{aex^6}{6}+\frac{bfx^{10}}{10}+\frac{bgx^{11}}{11}+\frac{bhx^{12}}{12}+x^9\left(\frac{ah}{9}+\frac{be}{9}\right)+x^8\left(\frac{ag}{8}+\frac{bd}{8}\right)+x^7\left(\frac{af}{7}+\frac{bc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + b*f*x**10/10 + b*g*x**11/1
1 + b*h*x**12/12 + x**9*(a*h/9 + b*e/9) + x**8*(a*g/8 + b*d/8) +
x**7*(a*f/7 + b*c/7)
```

Mathematica [A] time = 0.0513253, size = 97, normalized size = 1.

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + (
(b*d + a*g)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x
^11)/11 + (b*h*x^12)/12
```

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{acx^4}{4}+\frac{adx^5}{5}+\frac{aex^6}{6}+\frac{(af+bc)\,x^7}{7}+\frac{(ag+bd)\,x^8}{8}+\frac{(ah+be)\,x^9}{9}+\frac{bfx^{10}}{10}+\frac{bgx^{11}}{11}+\frac{bhx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*
d)*x^8+1/9*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^1
2
```

Maxima [A] time = 1.38121, size = 107, normalized size = 1.1

$$\frac{1}{12}\,bhx^{12}+\frac{1}{11}\,bgx^{11}+\frac{1}{10}\,bfx^{10}+\frac{1}{9}\,(be+ah)x^{9}+\frac{1}{8}\,(bd+ag)x^{8}+\frac{1}{6}\,aex^{6}+\frac{1}{7}\,(bc+af)x^{7}+\frac{1}{5}\,adx^{5}+\frac{1}{4}\,acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x^3,x, algorithm="
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(b*e + a*h)*x
^9 + 1/8*(b*d + a*g)*x^8 + 1/6*a*e*x^6 + 1/7*(b*c + a*f)*x^7 + 1/
5*a*d*x^5 + 1/4*a*c*x^4
```

Fricas [A] time = 0.213988, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}hb+\frac{1}{11}x^{11}gb+\frac{1}{10}x^{10}fb+\frac{1}{9}x^{9}eb+\frac{1}{9}x^{9}ha+\frac{1}{8}x^{8}db+\frac{1}{8}x^{8}ga+\frac{1}{7}x^{7}cb+\frac{1}{7}x^{7}fa+\frac{1}{6}x^{6}ea+\frac{1}{5}x^{5}da+\frac{1}{4}x^{4}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x^3,x, algorithm="
```

```
[Out] 1/12*x^12*h*b + 1/11*x^11*g*b + 1/10*x^10*f*b + 1/9*x^9*e*b + 1/9
*x^9*h*a + 1/8*x^8*d*b + 1/8*x^8*g*a + 1/7*x^7*c*b + 1/7*x^7*f*a
+ 1/6*x^6*e*a + 1/5*x^5*d*a + 1/4*x^4*c*a
```

Sympy [A] time = 0.068251, size = 90, normalized size = 0.93

$$\frac{acx^{4}}{4}+\frac{adx^{5}}{5}+\frac{aex^{6}}{6}+\frac{bfx^{10}}{10}+\frac{bgx^{11}}{11}+\frac{bhx^{12}}{12}+x^{9}\left(\frac{ah}{9}+\frac{be}{9}\right)+x^{8}\left(\frac{ag}{8}+\frac{bd}{8}\right)+x^{7}\left(\frac{af}{7}+\frac{bc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + b*f*x**10/10 + b*g*x**11/1
1 + b*h*x**12/12 + x**9*(a*h/9 + b*e/9) + x**8*(a*g/8 + b*d/8) +
x**7*(a*f/7 + b*c/7)
```

GIAC/XCAS [A] time = 0.2088, size = 117, normalized size = 1.21

$$\frac{1}{12}\,bhx^{12}+\frac{1}{11}\,bgx^{11}+\frac{1}{10}\,bfx^{10}+\frac{1}{9}\,ahx^{9}+\frac{1}{9}\,bx^{9}e+\frac{1}{8}\,bdx^{8}+\frac{1}{8}\,agx^{8}+\frac{1}{7}\,bcx^{7}+\frac{1}{7}\,afx^{7}+\frac{1}{6}\,ax^{6}e+\frac{1}{5}\,adx^{5}+\frac{1}{4}\,acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x^3,x, algorithm="
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*a*h*x^9 + 1/9
*b*x^9*e + 1/8*b*d*x^8 + 1/8*a*g*x^8 + 1/7*b*c*x^7 + 1/7*a*f*x^7
+ 1/6*a*x^6*e + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

3.363 $\int x^2 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$

Optimal. Leaf size=97

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + (
(b*d + a*g)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^1
0)/10 + (b*h*x^11)/11
```

Rubi [A] time = 0.209988, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + (
(b*d + a*g)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^1
0)/10 + (b*h*x^11)/11
```

Rubi in Sympy [A] time = 28.0042, size = 90, normalized size = 0.93

$$\frac{acx^3}{3}+\frac{adx^4}{4}+\frac{aex^5}{5}+\frac{bfx^9}{9}+\frac{bgx^{10}}{10}+\frac{bhx^{11}}{11}+x^8\left(\frac{ah}{8}+\frac{be}{8}\right)+x^7\left(\frac{ag}{7}+\frac{bd}{7}\right)+x^6\left(\frac{af}{6}+\frac{bc}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*f*x**9/9 + b*g*x**10/10
+ b*h*x**11/11 + x**8*(a*h/8 + b*e/8) + x**7*(a*g/7 + b*d/7) + x*
*6*(a*f/6 + b*c/6)
```

Mathematica [A] time = 0.0545123, size = 97, normalized size = 1.

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + (
(b*d + a*g)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^1
0)/10 + (b*h*x^11)/11
```

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{acx^3}{3}+\frac{adx^4}{4}+\frac{aex^5}{5}+\frac{(af+bc)\,x^6}{6}+\frac{(ag+bd)\,x^7}{7}+\frac{(ah+be)\,x^8}{8}+\frac{bfx^9}{9}+\frac{bgx^{10}}{10}+\frac{bhx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*
d)*x^7+1/8*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11
```

Maxima [A] time = 1.38367, size = 107, normalized size = 1.1

$$\frac{1}{11}bhx^{11} + \frac{1}{10}bgx^{10} + \frac{1}{9}bfx^9 + \frac{1}{8}(be+ah)x^8 + \frac{1}{7}(bd+ag)x^7 + \frac{1}{5}aex^5 + \frac{1}{6}(bc+af)x^6 + \frac{1}{4}adx^4 + \frac{1}{3}acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x^2,x, algorithm="
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(b*e + a*h)*x^8
 + 1/7*(b*d + a*g)*x^7 + 1/5*a*e*x^5 + 1/6*(b*c + a*f)*x^6 + 1/4*
a*d*x^4 + 1/3*a*c*x^3
```

Fricas [A] time = 0.229262, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}hb + \frac{1}{10}x^{10}gb + \frac{1}{9}x^9fb + \frac{1}{8}x^8eb + \frac{1}{8}x^8ha + \frac{1}{7}x^7db + \frac{1}{7}x^7ga + \frac{1}{6}x^6cb + \frac{1}{6}x^6fa + \frac{1}{5}x^5ea + \frac{1}{4}x^4da + \frac{1}{3}x^3ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x^2,x, algorithm="
```

```
[Out] 1/11*x^11*h*b + 1/10*x^10*g*b + 1/9*x^9*f*b + 1/8*x^8*e*b + 1/8*x
^8*h*a + 1/7*x^7*d*b + 1/7*x^7*g*a + 1/6*x^6*c*b + 1/6*x^6*f*a +
1/5*x^5*e*a + 1/4*x^4*d*a + 1/3*x^3*c*a
```

Sympy [A] time = 0.0684, size = 90, normalized size = 0.93

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{bfx^9}{9} + \frac{bgx^{10}}{10} + \frac{bhx^{11}}{11} + x^8\left(\frac{ah}{8} + \frac{be}{8}\right) + x^7\left(\frac{ag}{7} + \frac{bd}{7}\right) + x^6\left(\frac{af}{6} + \frac{bc}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*f*x**9/9 + b*g*x**10/10
+ b*h*x**11/11 + x**8*(a*h/8 + b*e/8) + x**7*(a*g/7 + b*d/7) + x*
*6*(a*f/6 + b*c/6)
```

GIAC/XCAS [A] time = 0.208694, size = 117, normalized size = 1.21

$$\frac{1}{11}bhx^{11} + \frac{1}{10}bgx^{10} + \frac{1}{9}bfx^9 + \frac{1}{8}ahx^8 + \frac{1}{8}bx^8e + \frac{1}{7}bdx^7 + \frac{1}{7}agx^7 + \frac{1}{6}bcx^6 + \frac{1}{6}afx^6 + \frac{1}{5}ax^5e + \frac{1}{4}adx^4 + \frac{1}{3}acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x^2,x, algorithm="
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*a*h*x^8 + 1/8*b
*x^8*e + 1/7*b*d*x^7 + 1/7*a*g*x^7 + 1/6*b*c*x^6 + 1/6*a*f*x^6 +
1/5*a*x^5*e + 1/4*a*d*x^4 + 1/3*a*c*x^3
```

3.364 $\int x\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=97

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + (
(b*d + a*g)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9
)/9 + (b*h*x^10)/10
```

Rubi [A] time = 0.191875, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + (
(b*d + a*g)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9
)/9 + (b*h*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ac\int x\,dx+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bfx^8}{8}+\frac{bgx^9}{9}+\frac{bhx^{10}}{10}+x^7\left(\frac{ah}{7}+\frac{be}{7}\right)+x^6\left(\frac{ag}{6}+\frac{bd}{6}\right)+x^5\left(\frac{af}{5}+\frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*Integral(x, x) + a*d*x**3/3 + a*e*x**4/4 + b*f*x**8/8 + b*g*x
**9/9 + b*h*x**10/10 + x**7*(a*h/7 + b*e/7) + x**6*(a*g/6 + b*d/6
) + x**5*(a*f/5 + b*c/5)
```

Mathematica [A] time = 0.037806, size = 97, normalized size = 1.

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + (
(b*d + a*g)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9
)/9 + (b*h*x^10)/10
```

Maple [A] time = 0.002, size = 80, normalized size = 0.8

$$\frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{(af+bc)\,x^5}{5}+\frac{(ag+bd)\,x^6}{6}+\frac{(ah+be)\,x^7}{7}+\frac{bfx^8}{8}+\frac{bgx^9}{9}+\frac{bhx^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*
d)*x^6+1/7*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10
```

Maxima [A] time = 1.42201, size = 107, normalized size = 1.1

$$\frac{1}{10}\,bhx^{10} + \frac{1}{9}\,bgx^{9} + \frac{1}{8}\,bfx^{8} + \frac{1}{7}\,(be + ah)x^{7} + \frac{1}{6}\,(bd + ag)x^{6} + \frac{1}{4}\,aex^{4} + \frac{1}{5}\,(bc + af)x^{5} + \frac{1}{3}\,adx^{3} + \frac{1}{2}\,acx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x,x, algorithm="ma
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(b*e + a*h)*x^7 +
 1/6*(b*d + a*g)*x^6 + 1/4*a*e*x^4 + 1/5*(b*c + a*f)*x^5 + 1/3*a*
d*x^3 + 1/2*a*c*x^2
```

Fricas [A] time = 0.207261, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10}hb + \frac{1}{9}x^{9}gb + \frac{1}{8}x^{8}fb + \frac{1}{7}x^{7}eb + \frac{1}{7}x^{7}ha + \frac{1}{6}x^{6}db + \frac{1}{6}x^{6}ga + \frac{1}{5}x^{5}cb + \frac{1}{5}x^{5}fa + \frac{1}{4}x^{4}ea + \frac{1}{3}x^{3}da + \frac{1}{2}x^{2}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x,x, algorithm="fr
```

```
[Out] 1/10*x^10*h*b + 1/9*x^9*g*b + 1/8*x^8*f*b + 1/7*x^7*e*b + 1/7*x^7
*h*a + 1/6*x^6*d*b + 1/6*x^6*g*a + 1/5*x^5*c*b + 1/5*x^5*f*a + 1/
4*x^4*e*a + 1/3*x^3*d*a + 1/2*x^2*c*a
```

Sympy [A] time = 0.067214, size = 90, normalized size = 0.93

$$\frac{acx^{2}}{2} + \frac{adx^{3}}{3} + \frac{aex^{4}}{4} + \frac{bfx^{8}}{8} + \frac{bgx^{9}}{9} + \frac{bhx^{10}}{10} + x^{7}\left(\frac{ah}{7} + \frac{be}{7}\right) + x^{6}\left(\frac{ag}{6} + \frac{bd}{6}\right) + x^{5}\left(\frac{af}{5} + \frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*f*x**8/8 + b*g*x**9/9 +
b*h*x**10/10 + x**7*(a*h/7 + b*e/7) + x**6*(a*g/6 + b*d/6) + x**5
*(a*f/5 + b*c/5)
```

GIAC/XCAS [A] time = 0.207944, size = 117, normalized size = 1.21

$$\frac{1}{10}\,bhx^{10} + \frac{1}{9}\,bgx^{9} + \frac{1}{8}\,bfx^{8} + \frac{1}{7}\,ahx^{7} + \frac{1}{7}\,bx^{7}e + \frac{1}{6}\,bdx^{6} + \frac{1}{6}\,agx^{6} + \frac{1}{5}\,bcx^{5} + \frac{1}{5}\,afx^{5} + \frac{1}{4}\,ax^{4}e + \frac{1}{3}\,adx^{3} + \frac{1}{2}\,acx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)*x,x, algorithm="gi
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*a*h*x^7 + 1/7*b*x
^7*e + 1/6*b*d*x^6 + 1/6*a*g*x^6 + 1/5*b*c*x^5 + 1/5*a*f*x^5 + 1/
4*a*x^4*e + 1/3*a*d*x^3 + 1/2*a*c*x^2
```

3.365 $\int (a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=92

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d +
 a*g)*x^5)/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 +
(b*h*x^9)/9
```

Rubi [A] time = 0.169736, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d +
 a*g)*x^5)/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 +
(b*h*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ad\int x\,dx+\frac{aex^3}{3}+\frac{bfx^7}{7}+\frac{bgx^8}{8}+\frac{bhx^9}{9}+c\int a\,dx+x^6\left(\frac{ah}{6}+\frac{be}{6}\right)+x^5\left(\frac{ag}{5}+\frac{bd}{5}\right)+x^4\left(\frac{af}{4}+\frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*d*Integral(x, x) + a*e*x**3/3 + b*f*x**7/7 + b*g*x**8/8 + b*h*x
**9/9 + c*Integral(a, x) + x**6*(a*h/6 + b*e/6) + x**5*(a*g/5 + b
*d/5) + x**4*(a*f/4 + b*c/4)
```

Mathematica [A] time = 0.0392504, size = 92, normalized size = 1.

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d +
 a*g)*x^5)/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 +
(b*h*x^9)/9
```

Maple [A] time = 0.001, size = 77, normalized size = 0.8

$$acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{(af+bc)\,x^4}{4}+\frac{(ag+bd)\,x^5}{5}+\frac{(ah+be)\,x^6}{6}+\frac{bfx^7}{7}+\frac{bgx^8}{8}+\frac{bhx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5
+1/6*(a*h+b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9
```

Maxima [A] time = 1.37556, size = 103, normalized size = 1.12

$$\frac{1}{9}bhx^9+\frac{1}{8}bgx^8+\frac{1}{7}bfx^7+\frac{1}{6}(be+ah)x^6+\frac{1}{5}(bd+ag)x^5+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{2}adx^2+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a),x, algorithm="maxi
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(b*e + a*h)*x^6 + 1
/5*(b*d + a*g)*x^5 + 1/3*a*e*x^3 + 1/4*(b*c + a*f)*x^4 + 1/2*a*d*
x^2 + a*c*x
```

Fricas [A] time = 0.207462, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9hb+\frac{1}{8}x^8gb+\frac{1}{7}x^7fb+\frac{1}{6}x^6eb+\frac{1}{6}x^6ha+\frac{1}{5}x^5db+\frac{1}{5}x^5ga+\frac{1}{4}x^4cb+\frac{1}{4}x^4fa+\frac{1}{3}x^3ea+\frac{1}{2}x^2da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a),x, algorithm="fric
```

```
[Out] 1/9*x^9*h*b + 1/8*x^8*g*b + 1/7*x^7*f*b + 1/6*x^6*e*b + 1/6*x^6*h
*a + 1/5*x^5*d*b + 1/5*x^5*g*a + 1/4*x^4*c*b + 1/4*x^4*f*a + 1/3*
x^3*e*a + 1/2*x^2*d*a + x*c*a
```

Sympy [A] time = 0.064743, size = 87, normalized size = 0.95

$$acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{bfx^7}{7}+\frac{bgx^8}{8}+\frac{bhx^9}{9}+x^6\left(\frac{ah}{6}+\frac{be}{6}\right)+x^5\left(\frac{ag}{5}+\frac{bd}{5}\right)+x^4\left(\frac{af}{4}+\frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*f*x**7/7 + b*g*x**8/8 + b*h*x
**9/9 + x**6*(a*h/6 + b*e/6) + x**5*(a*g/5 + b*d/5) + x**4*(a*f/4
 + b*c/4)
```

GIAC/XCAS [A] time = 0.20728, size = 113, normalized size = 1.23

$$\frac{1}{9}bhx^9+\frac{1}{8}bgx^8+\frac{1}{7}bfx^7+\frac{1}{6}ahx^6+\frac{1}{6}bx^6e+\frac{1}{5}bdx^5+\frac{1}{5}agx^5+\frac{1}{4}bcx^4+\frac{1}{4}afx^4+\frac{1}{3}ax^3e+\frac{1}{2}adx^2+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a),x, algorithm="giac
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*a*h*x^6 + 1/6*b*x^6
*e + 1/5*b*d*x^5 + 1/5*a*g*x^5 + 1/4*b*c*x^4 + 1/4*a*f*x^4 + 1/3*
a*x^3*e + 1/2*a*d*x^2 + a*c*x
```

3.366 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal. Leaf size=88

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 +
 ((b*e + a*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 +
a*c*Log[x]
```

Rubi [A] time = 0.122348, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 +
 ((b*e + a*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 +
a*c*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ac\log(x)+ae\int x\,dx+\frac{bfx^6}{6}+\frac{bgx^7}{7}+\frac{bhx^8}{8}+d\int a\,dx+x^5\left(\frac{ah}{5}+\frac{be}{5}\right)+x^4\left(\frac{ag}{4}+\frac{bd}{4}\right)+x^3\left(\frac{af}{3}+\frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a*c*log(x) + a*e*Integral(x, x) + b*f*x**6/6 + b*g*x**7/7 + b*h*x
**8/8 + d*Integral(a, x) + x**5*(a*h/5 + b*e/5) + x**4*(a*g/4 + b
*d/4) + x**3*(a*f/3 + b*c/3)
```

Mathematica [A] time = 0.0575435, size = 88, normalized size = 1.

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 +
 ((b*e + a*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 +
a*c*Log[x]
```

Maple [A] time = 0.004, size = 81, normalized size = 0.9

$$\frac{bhx^8}{8}+\frac{bgx^7}{7}+\frac{bfx^6}{6}+\frac{x^5ah}{5}+\frac{x^5be}{5}+\frac{x^4ag}{4}+\frac{bdx^4}{4}+\frac{x^3af}{3}+\frac{x^3bc}{3}+\frac{aex^2}{2}+adx+ac\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] 1/8*b*h*x^8+1/7*b*g*x^7+1/6*b*f*x^6+1/5*x^5*a*h+1/5*x^5*b*e+1/4*x
^4*a*g+1/4*b*d*x^4+1/3*x^3*a*f+1/3*x^3*b*c+1/2*a*e*x^2+a*d*x+a*c*
ln(x)
```

Maxima [A] time = 1.3777, size = 100, normalized size = 1.14

$$\frac{1}{8}bhx^8+\frac{1}{7}bgx^7+\frac{1}{6}bfx^6+\frac{1}{5}(be+ah)x^5+\frac{1}{4}(bd+ag)x^4+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+adx+ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x,x, algorithm="ma
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1
/4*(b*d + a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x +
a*c*log(x)
```

Fricas [A] time = 0.244673, size = 100, normalized size = 1.14

$$\frac{1}{8}bhx^8+\frac{1}{7}bgx^7+\frac{1}{6}bfx^6+\frac{1}{5}(be+ah)x^5+\frac{1}{4}(bd+ag)x^4+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+adx+ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x,x, algorithm="fr
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1
/4*(b*d + a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x +
a*c*log(x)
```

Sympy [A] time = 0.69921, size = 85, normalized size = 0.97

$$ac\log(x)+adx+\frac{aex^2}{2}+\frac{bfx^6}{6}+\frac{bgx^7}{7}+\frac{bhx^8}{8}+x^5\left(\frac{ah}{5}+\frac{be}{5}\right)+x^4\left(\frac{ag}{4}+\frac{bd}{4}\right)+x^3\left(\frac{af}{3}+\frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a*c*log(x) + a*d*x + a*e*x**2/2 + b*f*x**6/6 + b*g*x**7/7 + b*h*x
**8/8 + x**5*(a*h/5 + b*e/5) + x**4*(a*g/4 + b*d/4) + x**3*(a*f/3
 + b*c/3)
```

GIAC/XCAS [A] time = 0.210309, size = 112, normalized size = 1.27

$$\frac{1}{8}bhx^8+\frac{1}{7}bgx^7+\frac{1}{6}bfx^6+\frac{1}{5}ahx^5+\frac{1}{5}bx^5e+\frac{1}{4}bdx^4+\frac{1}{4}agx^4+\frac{1}{3}bcx^3+\frac{1}{3}afx^3+\frac{1}{2}ax^2e+adx+ac\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x,x, algorithm="gi
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*a*h*x^5 + 1/5*b*x^5
*e + 1/4*b*d*x^4 + 1/4*a*g*x^4 + 1/3*b*c*x^3 + 1/3*a*f*x^3 + 1/2*
a*x^2*e + a*d*x + a*c*ln(abs(x))
```

3.367 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx$

Optimal. Leaf size=86

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 +
((b*e + a*h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a
*d*Log[x]
```

Rubi [A] time = 0.148962, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 +
((b*e + a*h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a
*d*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ac}{x}+ad\log(x)+\frac{bfx^5}{5}+\frac{bgx^6}{6}+\frac{bhx^7}{7}+e\int a\,dx+x^4\left(\frac{ah}{4}+\frac{be}{4}\right)+x^3\left(\frac{ag}{3}+\frac{bd}{3}\right)+(af+bc)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a*c/x + a*d*log(x) + b*f*x**5/5 + b*g*x**6/6 + b*h*x**7/7 + e*In
tegral(a, x) + x**4*(a*h/4 + b*e/4) + x**3*(a*g/3 + b*d/3) + (a*f
 + b*c)*Integral(x, x)
```

Mathematica [A] time = 0.0774826, size = 86, normalized size = 1.

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 +
((b*e + a*h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a
*d*Log[x]
```

Maple [A] time = 0.009, size = 81, normalized size = 0.9

$$\frac{bhx^7}{7}+\frac{bgx^6}{6}+\frac{bfx^5}{5}+\frac{x^4ah}{4}+\frac{x^4be}{4}+\frac{x^3ag}{3}+\frac{x^3bd}{3}+\frac{x^2af}{2}+\frac{x^2bc}{2}+aex+ad\ln(x)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 1/7*b*h*x^7+1/6*b*g*x^6+1/5*b*f*x^5+1/4*x^4*a*h+1/4*x^4*b*e+1/3*x
^3*a*g+1/3*x^3*b*d+1/2*x^2*a*f+1/2*x^2*b*c+a*e*x+a*d*ln(x)-a*c/x
```

Maxima [A] time = 1.39431, size = 100, normalized size = 1.16

$$\frac{1}{7}bhx^7+\frac{1}{6}bgx^6+\frac{1}{5}bfx^5+\frac{1}{4}(be+ah)x^4+\frac{1}{3}(bd+ag)x^3+aex+\frac{1}{2}(bc+af)x^2+ad\log(x)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^2,x, algorithm="
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*(b*e + a*h)*x^4 + 1
/3*(b*d + a*g)*x^3 + a*e*x + 1/2*(b*c + a*f)*x^2 + a*d*log(x) - a
*c/x
```

Fricas [A] time = 0.239848, size = 109, normalized size = 1.27

$$\frac{60\,bhx^8+70\,bgx^7+84\,bfx^6+105\,(be+ah)x^5+140\,(bd+ag)x^4+420\,aex^2+210\,(bc+af)x^3+420\,adx\log(x)-420\,ac}{420\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^2,x, algorithm="
```

```
[Out] 1/420*(60*b*h*x^8 + 70*b*g*x^7 + 84*b*f*x^6 + 105*(b*e + a*h)*x^5
 + 140*(b*d + a*g)*x^4 + 420*a*e*x^2 + 210*(b*c + a*f)*x^3 + 420*
a*d*x*log(x) - 420*a*c)/x
```

Sympy [A] time = 0.734324, size = 82, normalized size = 0.95

$$-\frac{ac}{x}+ad\log(x)+aex+\frac{bfx^5}{5}+\frac{bgx^6}{6}+\frac{bhx^7}{7}+x^4\left(\frac{ah}{4}+\frac{be}{4}\right)+x^3\left(\frac{ag}{3}+\frac{bd}{3}\right)+x^2\left(\frac{af}{2}+\frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a*c/x + a*d*log(x) + a*e*x + b*f*x**5/5 + b*g*x**6/6 + b*h*x**7/
7 + x**4*(a*h/4 + b*e/4) + x**3*(a*g/3 + b*d/3) + x**2*(a*f/2 + b
*c/2)
```

GIAC/XCAS [A] time = 0.2087, size = 112, normalized size = 1.3

$$\frac{1}{7}bhx^7+\frac{1}{6}bgx^6+\frac{1}{5}bfx^5+\frac{1}{4}ahx^4+\frac{1}{4}bx^4e+\frac{1}{3}bdx^3+\frac{1}{3}agx^3+\frac{1}{2}bcx^2+\frac{1}{2}afx^2+axe+ad\ln(|x|)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^2,x, algorithm="
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*a*h*x^4 + 1/4*b*x^4
*e + 1/3*b*d*x^3 + 1/3*a*g*x^3 + 1/2*b*c*x^2 + 1/2*a*f*x^2 + a*x*
e + a*d*ln(abs(x)) - a*c/x
```

3.368 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=86

$$x(af+bc)+\frac{1}{2}x^2(ag+bd)+\frac{1}{3}x^3(ah+be)-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6$$

```
[Out] -(a*c)/(2*x^2) - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 +
((b*e + a*h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a
*e*Log[x]
```

Rubi [A] time = 0.15496, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028

$$x(af+bc)+\frac{1}{2}x^2(ag+bd)+\frac{1}{3}x^3(ah+be)-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -(a*c)/(2*x^2) - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 +
((b*e + a*h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a
*e*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{bfx^4}{4}+\frac{bgx^5}{5}+\frac{bhx^6}{6}+x^3\left(\frac{ah}{3}+\frac{be}{3}\right)+(ag+bd)\int x\,dx+\frac{(af+bc)\int a\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] -a*c/(2*x**2) - a*d/x + a*e*log(x) + b*f*x**4/4 + b*g*x**5/5 + b*
h*x**6/6 + x**3*(a*h/3 + b*e/3) + (a*g + b*d)*Integral(x, x) + (a
*f + b*c)*Integral(a, x)/a
```

Mathematica [A] time = 0.140529, size = 78, normalized size = 0.91

$$\frac{a(-3c-6dx+6fx^3+3gx^4+2hx^5)}{6x^2}+ae\log(x)+bcx+\frac{1}{60}bx^2\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] b*c*x + (a*(-3*c - 6*d*x + 6*f*x^3 + 3*g*x^4 + 2*h*x^5))/(6*x^2)
+ (b*x^2*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3)))/60 + a
*e*Log[x]
```

Maple [A] time = 0.009, size = 78, normalized size = 0.9

$$\frac{bhx^6}{6}+\frac{bgx^5}{5}+\frac{bfx^4}{4}+\frac{x^3ah}{3}+\frac{x^3be}{3}+\frac{x^2ag}{2}+\frac{x^2bd}{2}+afx+bcx+ae\ln(x)-\frac{ac}{2x^2}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] 1/6*b*h*x^6+1/5*b*g*x^5+1/4*b*f*x^4+1/3*x^3*a*h+1/3*x^3*b*e+1/2*x
^2*a*g+1/2*x^2*b*d+a*f*x+b*c*x+a*e*ln(x)-1/2*a*c/x^2-a*d/x
```

Maxima [A] time = 4.44427, size = 100, normalized size = 1.16

$$\frac{1}{6}\,bhx^{6}+\frac{1}{5}\,bgx^{5}+\frac{1}{4}\,bfx^{4}+\frac{1}{3}\,(be+ah)x^{3}+\frac{1}{2}\,(bd+ag)x^{2}+ae\log\left(x\right)+(bc+af)x-\frac{2\,adx+ac}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^3,x, algorithm="
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*(b*e + a*h)*x^3 + 1
/2*(b*d + a*g)*x^2 + a*e*log(x) + (b*c + a*f)*x - 1/2*(2*a*d*x +
a*c)/x^2
```

Fricas [A] time = 0.233984, size = 109, normalized size = 1.27

$$\frac{10\,bhx^{8}+12\,bgx^{7}+15\,bfx^{6}+20\,(be+ah)x^{5}+30\,(bd+ag)x^{4}+60\,aex^{2}\log\left(x\right)+60\,(bc+af)x^{3}-60\,adx-30\,ac}{60\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^3,x, algorithm="
```

```
[Out] 1/60*(10*b*h*x^8 + 12*b*g*x^7 + 15*b*f*x^6 + 20*(b*e + a*h)*x^5 +
 30*(b*d + a*g)*x^4 + 60*a*e*x^2*log(x) + 60*(b*c + a*f)*x^3 - 60
*a*d*x - 30*a*c)/x^2
```

Sympy [A] time = 0.859547, size = 82, normalized size = 0.95

$$ae\log\left(x\right)+\frac{bfx^{4}}{4}+\frac{bgx^{5}}{5}+\frac{bhx^{6}}{6}+x^{3}\left(\frac{ah}{3}+\frac{be}{3}\right)+x^{2}\left(\frac{ag}{2}+\frac{bd}{2}\right)+x\left(af+bc\right)-\frac{ac+2adx}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a*e*log(x) + b*f*x**4/4 + b*g*x**5/5 + b*h*x**6/6 + x**3*(a*h/3 +
 b*e/3) + x**2*(a*g/2 + b*d/2) + x*(a*f + b*c) - (a*c + 2*a*d*x)/
(2*x**2)
```

GIAC/XCAS [A] time = 0.210662, size = 108, normalized size = 1.26

$$\frac{1}{6}\,bhx^{6}+\frac{1}{5}\,bgx^{5}+\frac{1}{4}\,bfx^{4}+\frac{1}{3}\,ahx^{3}+\frac{1}{3}\,bx^{3}e+\frac{1}{2}\,bdx^{2}+\frac{1}{2}\,agx^{2}+bcx+afx+ae\ln\left(|x|\right)-\frac{2\,adx+ac}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^3,x, algorithm="
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*a*h*x^3 + 1/3*b*x^3
*e + 1/2*b*d*x^2 + 1/2*a*g*x^2 + b*c*x + a*f*x + a*e*ln(abs(x)) -
 1/2*(2*a*d*x + a*c)/x^2
```

3.369 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

Optimal. Leaf size=86

$$\log(x)(af+bc)+x(ag+bd)+\frac{1}{2}x^2(ah+be)-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5$$

```
[Out] -(a*c)/(3*x^3) - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e
+ a*h)*x^2)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c +
a*f)*Log[x]
```

Rubi [A] time = 0.155345, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$

$$\log(x)(af+bc)+x(ag+bd)+\frac{1}{2}x^2(ah+be)-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a*c)/(3*x^3) - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e
+ a*h)*x^2)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c +
a*f)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{bfx^3}{3}+\frac{bgx^4}{4}+\frac{bhx^5}{5}+(af+bc)\log(x)+(ah+be)\int x\,dx+\frac{(ag+bd)\int a\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] -a*c/(3*x**3) - a*d/(2*x**2) - a*e/x + b*f*x**3/3 + b*g*x**4/4 +
b*h*x**5/5 + (a*f + b*c)*log(x) + (a*h + b*e)*Integral(x, x) + (a
*g + b*d)*Integral(a, x)/a
```

Mathematica [A] time = 0.148415, size = 76, normalized size = 0.88

$$\log(x)(af+bc)-\frac{a\left(2c+3x\left(d+2ex+x^3(-(2g+hx))\right)\right)}{6x^3}+\frac{1}{60}bx\left(60d+x\left(30e+x\left(20f+15gx+12hx^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/(6*x^3) + (b*x*(60
*d + x*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/60 + (b*c + a*f)*L
og[x]
```

Maple [A] time = 0.01, size = 76, normalized size = 0.9

$$\frac{bhx^5}{5}+\frac{bgx^4}{4}+\frac{bfx^3}{3}+\frac{x^2ah}{2}+\frac{bex^2}{2}+xag+xbd+\ln(x)af+\ln(x)bc-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] 1/5*b*h*x^5+1/4*b*g*x^4+1/3*b*f*x^3+1/2*x^2*a*h+1/2*b*e*x^2+x*a*g
+x*b*d+ln(x)*a*f+ln(x)*b*c-1/3*a*c/x^3-1/2*a*d/x^2-a*e/x
```

Maxima [A] time = 1.3717, size = 101, normalized size = 1.17

$$\frac{1}{5}\,bhx^5+\frac{1}{4}\,bgx^4+\frac{1}{3}\,bfx^3+\frac{1}{2}\,(be+ah)x^2+(bd+ag)x+(bc+af)\log\left(x\right)-\frac{6\,aex^2+3\,adx+2\,ac}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^4,x, algorithm="
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*(b*e + a*h)*x^2 + (
b*d + a*g)*x + (b*c + a*f)*log(x) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*
a*c)/x^3
```

Fricas [A] time = 0.240668, size = 109, normalized size = 1.27

$$\frac{12\,bhx^8+15\,bgx^7+20\,bfx^6+30\,(be+ah)x^5+60\,(bd+ag)x^4+60\,(bc+af)x^3\log\left(x\right)-60\,aex^2-30\,adx-20\,ac}{60\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^4,x, algorithm="
```

```
[Out] 1/60*(12*b*h*x^8 + 15*b*g*x^7 + 20*b*f*x^6 + 30*(b*e + a*h)*x^5 +
 60*(b*d + a*g)*x^4 + 60*(b*c + a*f)*x^3*log(x) - 60*a*e*x^2 - 30
*a*d*x - 20*a*c)/x^3
```

Sympy [A] time = 1.51655, size = 82, normalized size = 0.95

$$\frac{bfx^3}{3}+\frac{bgx^4}{4}+\frac{bhx^5}{5}+x^2\left(\frac{ah}{2}+\frac{be}{2}\right)+x\left(ag+bd\right)+\left(af+bc\right)\log\left(x\right)-\frac{2ac+3adx+6aex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] b*f*x**3/3 + b*g*x**4/4 + b*h*x**5/5 + x**2*(a*h/2 + b*e/2) + x*(
a*g + b*d) + (a*f + b*c)*log(x) - (2*a*c + 3*a*d*x + 6*a*e*x**2)/
(6*x**3)
```

GIAC/XCAS [A] time = 0.21093, size = 107, normalized size = 1.24

$$\frac{1}{5}\,bhx^5+\frac{1}{4}\,bgx^4+\frac{1}{3}\,bfx^3+\frac{1}{2}\,ahx^2+\frac{1}{2}\,bx^2e+bdx+agx+(bc+af)\ln\left(|x|\right)-\frac{6\,ax^2e+3\,adx+2\,ac}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^4,x, algorithm="
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*a*h*x^2 + 1/2*b*x^2
*e + b*d*x + a*g*x + (b*c + a*f)*ln(abs(x)) - 1/6*(6*a*x^2*e + 3*
a*d*x + 2*a*c)/x^3
```

3.370 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$

Optimal. Leaf size=86

$$-\frac{af+bc}{x}+\log(x)(ag+bd)+x(ah+be)-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4$$

```
[Out] -(a*c)/(4*x^4) - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x +
(b*e + a*h)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d +
a*g)*Log[x]
```

Rubi [A] time = 0.160798, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028

$$-\frac{af+bc}{x}+\log(x)(ag+bd)+x(ah+be)-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -(a*c)/(4*x^4) - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x +
(b*e + a*h)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d +
a*g)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+bf\int x\,dx+\frac{bgx^3}{3}+\frac{bhx^4}{4}+(ag+bd)\log(x)-\frac{af+bc}{x}+\frac{(ah+be)\int a\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] -a*c/(4*x**4) - a*d/(3*x**3) - a*e/(2*x**2) + b*f*Integral(x, x)
+ b*g*x**3/3 + b*h*x**4/4 + (a*g + b*d)*log(x) - (a*f + b*c)/x +
(a*h + b*e)*Integral(a, x)/a
```

Mathematica [A] time = 0.118065, size = 77, normalized size = 0.9

$$\log(x)(ag+bd)-\frac{a\left(3c+4dx+6x^2\left(e+2fx-2hx^3\right)\right)}{12x^4}+b\left(-\frac{c}{x}+ex+\frac{1}{12}x^2\left(6f+4gx+3hx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] b*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) - (a*(3*c + 4
*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) + (b*d + a*g)*Log[x
]
```

Maple [A] time = 0.01, size = 76, normalized size = 0.9

$$\frac{bhx^4}{4}+\frac{bgx^3}{3}+\frac{bfx^2}{2}+xah+bex+\ln(x)ag+\ln(x)bd-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}-\frac{af}{x}-\frac{bc}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] 1/4*b*h*x^4+1/3*b*g*x^3+1/2*b*f*x^2+x*a*h+b*e*x+ln(x)*a*g+ln(x)*b
*d-1/4*a*c/x^4-1/3*a*d/x^3-1/2*a*e/x^2-1/x*a*f-1/x*b*c
```

Maxima [A] time = 1.38969, size = 101, normalized size = 1.17

$$\frac{1}{4}\,bhx^4 + \frac{1}{3}\,bgx^3 + \frac{1}{2}\,bfx^2 + (be + ah)x + (bd + ag)\log(x) - \frac{6\,aex^2 + 12\,(bc + af)x^3 + 4\,adx + 3\,ac}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^5,x, algorithm="
```

```
[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + (b*e + a*h)*x + (b*d +
a*g)*log(x) - 1/12*(6*a*e*x^2 + 12*(b*c + a*f)*x^3 + 4*a*d*x + 3*
a*c)/x^4
```

Fricas [A] time = 0.240342, size = 109, normalized size = 1.27

$$\frac{3\,bhx^8 + 4\,bgx^7 + 6\,bfx^6 + 12\,(be + ah)x^5 + 12\,(bd + ag)x^4\log(x) - 6\,aex^2 - 12\,(bc + af)x^3 - 4\,adx - 3\,ac}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^5,x, algorithm="
```

```
[Out] 1/12*(3*b*h*x^8 + 4*b*g*x^7 + 6*b*f*x^6 + 12*(b*e + a*h)*x^5 + 12
*(b*d + a*g)*x^4*log(x) - 6*a*e*x^2 - 12*(b*c + a*f)*x^3 - 4*a*d*
x - 3*a*c)/x^4
```

Sympy [A] time = 5.30844, size = 82, normalized size = 0.95

$$\frac{bfx^2}{2} + \frac{bgx^3}{3} + \frac{bhx^4}{4} + x\,(ah + be) + (ag + bd)\log(x) - \frac{3ac + 4adx + 6aex^2 + x^3\,(12af + 12bc)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] b*f*x**2/2 + b*g*x**3/3 + b*h*x**4/4 + x*(a*h + b*e) + (a*g + b*d
)*log(x) - (3*a*c + 4*a*d*x + 6*a*e*x**2 + x**3*(12*a*f + 12*b*c)
)/(12*x**4)
```

GIAC/XCAS [A] time = 0.212562, size = 104, normalized size = 1.21

$$\frac{1}{4}\,bhx^4 + \frac{1}{3}\,bgx^3 + \frac{1}{2}\,bfx^2 + ahx + bxe + (bd + ag)\ln(|x|) - \frac{12\,(bc + af)x^3 + 6\,ax^2e + 4\,adx + 3\,ac}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)/x^5,x, algorithm="
```

```
[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + a*h*x + b*x*e + (b*d +
a*g)*ln(abs(x)) - 1/12*(12*(b*c + a*f)*x^3 + 6*a*x^2*e + 4*a*d*x
+ 3*a*c)/x^4
```

3.371 $\int x^4 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=163

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2ah+be)+\frac{1}{10}ax^{10}(ah+2be)+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}$$

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*
x^8)/8 + (a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b
*(b*c + 2*a*f)*x^11)/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2
*a*h)*x^13)/13 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)
/16
```

Rubi [A] time = 0.42176, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2ah+be)+\frac{1}{10}ax^{10}(ah+2be)+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*
x^8)/8 + (a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b
*(b*c + 2*a*f)*x^11)/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2
*a*h)*x^13)/13 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)
/16
```

Rubi in Sympy [A] time = 44.9638, size = 151, normalized size = 0.93

$$\frac{a^2cx^5}{5}+\frac{a^2dx^6}{6}+\frac{a^2ex^7}{7}+\frac{ax^{10}\left(ah+2be\right)}{10}+\frac{ax^9\left(ag+2bd\right)}{9}+\frac{ax^8\left(af+2bc\right)}{8}+\frac{b^2fx^{14}}{14}+\frac{b^2gx^{15}}{15}+\frac{b^2hx^{16}}{16}+\frac{bx^{13}\left(2ah+be\right)}{13}+\frac{bx^{12}\left(2ag+bd\right)}{12}+\frac{bx^{11}\left(2af+bc\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**5/5 + a**2*d*x**6/6 + a**2*e*x**7/7 + a*x**10*(a*h + 2*
b*e)/10 + a*x**9*(a*g + 2*b*d)/9 + a*x**8*(a*f + 2*b*c)/8 + b**2*
f*x**14/14 + b**2*g*x**15/15 + b**2*h*x**16/16 + b*x**13*(2*a*h +
 b*e)/13 + b*x**12*(2*a*g + b*d)/12 + b*x**11*(2*a*f + b*c)/11
```

Mathematica [A] time = 0.0817406, size = 163, normalized size = 1.

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2ah+be)+\frac{1}{10}ax^{10}(ah+2be)+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*
x^8)/8 + (a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b
*(b*c + 2*a*f)*x^11)/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2
*a*h)*x^13)/13 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)
/16
```

Maple [A] time = 0.002, size = 152, normalized size = 0.9

$$\frac{b^2hx^{16}}{16}+\frac{b^2gx^{15}}{15}+\frac{b^2fx^{14}}{14}+\frac{\left(2\,abh+b^2e\right)x^{13}}{13}+\frac{\left(2\,abg+b^2d\right)x^{12}}{12}+\frac{\left(2\,abf+b^2c\right)x^{11}}{11}+\frac{\left(a^2h+2\,bea\right)x^{10}}{10}+\frac{\left(a^2g+2\,bda\right)x^9}{9}+\frac{\left(a^2f+2\,abc\right)x^8}{8}+\frac{a^2ex^7}{7}+\frac{a^2dx^6}{6}+\frac{a^2cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/16*b^2*h*x^16+1/15*b^2*g*x^15+1/14*b^2*f*x^14+1/13*(2*a*b*h+b^2
*e)*x^13+1/12*(2*a*b*g+b^2*d)*x^12+1/11*(2*a*b*f+b^2*c)*x^11+1/10
*(a^2*h+2*a*b*e)*x^10+1/9*(a^2*g+2*a*b*d)*x^9+1/8*(a^2*f+2*a*b*c)
*x^8+1/7*a^2*e*x^7+1/6*a^2*d*x^6+1/5*a^2*c*x^5
```

Maxima [A] time = 1.37527, size = 204, normalized size = 1.25

$$\frac{1}{16}\,b^2hx^{16}+\frac{1}{15}\,b^2gx^{15}+\frac{1}{14}\,b^2fx^{14}+\frac{1}{13}\left(b^2e+2\,abh\right)x^{13}+\frac{1}{12}\left(b^2d+2\,abg\right)x^{12}+\frac{1}{11}\left(b^2c+2\,abf\right)x^{11}+\frac{1}{10}\left(2\,abe+a^2h\right)x^{10}+\frac{1}{7}\,a^2ex^7+\frac{1}{9}\left(2\,abd+a^2g\right)x^9+\frac{1}{6}\,a^2dx^6+\frac{1}{8}\left(2\,abc+a^2f\right)x^8+\frac{1}{5}\,a^2cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x^4,x, algorithm
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(b^2*e
 + 2*a*b*h)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*
a*b*f)*x^11 + 1/10*(2*a*b*e + a^2*h)*x^10 + 1/7*a^2*e*x^7 + 1/9*(
2*a*b*d + a^2*g)*x^9 + 1/6*a^2*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8
+ 1/5*a^2*c*x^5
```

Fricas [A] time = 0.223275, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}hb^2+\frac{1}{15}x^{15}gb^2+\frac{1}{14}x^{14}fb^2+\frac{1}{13}x^{13}eb^2+\frac{2}{13}x^{13}hba+\frac{1}{12}x^{12}db^2+\frac{1}{6}x^{12}gba+\frac{1}{11}x^{11}cb^2+\frac{2}{11}x^{11}fba+\frac{1}{5}x^{10}eba+\frac{1}{10}x^{10}ha^2+\frac{2}{9}x^9dba+\frac{1}{9}x^9ga^2+\frac{1}{4}x^8cba+\frac{1}{8}x^8fa^2+\frac{1}{7}x^7ea^2+\frac{1}{6}x^6da^2+\frac{1}{5}x^5ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x^4,x, algorithm
```

```
[Out] 1/16*x^16*h*b^2 + 1/15*x^15*g*b^2 + 1/14*x^14*f*b^2 + 1/13*x^13*e
*b^2 + 2/13*x^13*h*b*a + 1/12*x^12*d*b^2 + 1/6*x^12*g*b*a + 1/11*
x^11*c*b^2 + 2/11*x^11*f*b*a + 1/5*x^10*e*b*a + 1/10*x^10*h*a^2 +
 2/9*x^9*d*b*a + 1/9*x^9*g*a^2 + 1/4*x^8*c*b*a + 1/8*x^8*f*a^2 +
1/7*x^7*e*a^2 + 1/6*x^6*d*a^2 + 1/5*x^5*c*a^2
```

Sympy [A] time = 0.099664, size = 167, normalized size = 1.02

$$\frac{a^2cx^5}{5} + \frac{a^2dx^6}{6} + \frac{a^2ex^7}{7} + \frac{b^2fx^{14}}{14} + \frac{b^2gx^{15}}{15} + \frac{b^2hx^{16}}{16} + x^{13}\left(\frac{2abh}{13} + \frac{b^2e}{13}\right) + x^{12}\left(\frac{abg}{6} + \frac{b^2d}{12}\right) + x^{11}\left(\frac{2abf}{11} + \frac{b^2c}{11}\right) + x^{10}\left(\frac{a^2h}{10} + \frac{abe}{5}\right) + x^9\left(\frac{a^2g}{9} + \frac{2abd}{9}\right) + x^8\left(\frac{a^2f}{8} + \frac{abc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**5/5 + a**2*d*x**6/6 + a**2*e*x**7/7 + b**2*f*x**14/14 +
 b**2*g*x**15/15 + b**2*h*x**16/16 + x**13*(2*a*b*h/13 + b**2*e/1
3) + x**12*(a*b*g/6 + b**2*d/12) + x**11*(2*a*b*f/11 + b**2*c/11)
 + x**10*(a**2*h/10 + a*b*e/5) + x**9*(a**2*g/9 + 2*a*b*d/9) + x*
*8*(a**2*f/8 + a*b*c/4)
```

GIAC/XCAS [A] time = 0.209728, size = 216, normalized size = 1.33

$$\frac{1}{16}\,b^2hx^{16} + \frac{1}{15}\,b^2gx^{15} + \frac{1}{14}\,b^2fx^{14} + \frac{2}{13}\,abhx^{13} + \frac{1}{13}\,b^2x^{13}e + \frac{1}{12}\,b^2dx^{12} + \frac{1}{6}\,abgx^{12} + \frac{1}{11}\,b^2cx^{11} + \frac{2}{11}\,abfx^{11} + \frac{1}{10}\,a^2hx^{10} + \frac{1}{5}\,abx^{10}e + \frac{2}{9}\,abdx^9 + \frac{1}{9}\,a^2gx^9 + \frac{1}{4}\,abcx^8 + \frac{1}{8}\,a^2fx^8 + \frac{1}{7}\,a^2x^7e + \frac{1}{6}\,a^2dx^6 + \frac{1}{5}\,a^2cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x^4,x, algorithm
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 2/13*a*b*h*
x^13 + 1/13*b^2*x^13*e + 1/12*b^2*d*x^12 + 1/6*a*b*g*x^12 + 1/11*
b^2*c*x^11 + 2/11*a*b*f*x^11 + 1/10*a^2*h*x^10 + 1/5*a*b*x^10*e +
 2/9*a*b*d*x^9 + 1/9*a^2*g*x^9 + 1/4*a*b*c*x^8 + 1/8*a^2*f*x^8 +
1/7*a^2*x^7*e + 1/6*a^2*d*x^6 + 1/5*a^2*c*x^5
```

3.372 $\int x^3 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\, dx$

Optimal. Leaf size=163

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah+be)+\frac{1}{9}ax^9(ah+2be)+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}$$

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*
x^7)/7 + (a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(
b*c + 2*a*f)*x^10)/10 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a
*h)*x^12)/12 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/1
5
```

Rubi [A] time = 0.388546, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah+be)+\frac{1}{9}ax^9(ah+2be)+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*
x^7)/7 + (a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(
b*c + 2*a*f)*x^10)/10 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a
*h)*x^12)/12 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/1
5
```

Rubi in Sympy [A] time = 48.5906, size = 151, normalized size = 0.93

$$\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{ax^9\left(ah+2be\right)}{9}+\frac{ax^8\left(ag+2bd\right)}{8}+\frac{ax^7\left(af+2bc\right)}{7}+\frac{b^2fx^{13}}{13}+\frac{b^2gx^{14}}{14}+\frac{b^2hx^{15}}{15}+\frac{bx^{12}\left(2ah+be\right)}{12}+\frac{bx^{11}\left(2ag+bd\right)}{11}+\frac{bx^{10}\left(2af+bc\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + a*x**9*(a*h + 2*b
*e)/9 + a*x**8*(a*g + 2*b*d)/8 + a*x**7*(a*f + 2*b*c)/7 + b**2*f*
x**13/13 + b**2*g*x**14/14 + b**2*h*x**15/15 + b*x**12*(2*a*h + b
*e)/12 + b*x**11*(2*a*g + b*d)/11 + b*x**10*(2*a*f + b*c)/10
```

Mathematica [A] time = 0.0743391, size = 163, normalized size = 1.

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah+be)+\frac{1}{9}ax^9(ah+2be)+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*
x^7)/7 + (a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(
b*c + 2*a*f)*x^10)/10 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a
*h)*x^12)/12 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/1
5
```

Maple [A] time = 0.001, size = 152, normalized size = 0.9

$$\frac{b^2hx^{15}}{15}+\frac{b^2gx^{14}}{14}+\frac{b^2fx^{13}}{13}+\frac{\left(2\,abh+b^2e\right)x^{12}}{12}+\frac{\left(2\,abg+b^2d\right)x^{11}}{11}+\frac{\left(2\,abf+b^2c\right)x^{10}}{10}+\frac{\left(a^2h+2\,bea\right)x^9}{9}+\frac{\left(a^2g+2\,bda\right)x^8}{8}+\frac{\left(a^2f+2\,abc\right)x^7}{7}+\frac{a^2ex^6}{6}+\frac{a^2dx^5}{5}+\frac{a^2cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/15*b^2*h*x^15+1/14*b^2*g*x^14+1/13*b^2*f*x^13+1/12*(2*a*b*h+b^2
*e)*x^12+1/11*(2*a*b*g+b^2*d)*x^11+1/10*(2*a*b*f+b^2*c)*x^10+1/9*
(a^2*h+2*a*b*e)*x^9+1/8*(a^2*g+2*a*b*d)*x^8+1/7*(a^2*f+2*a*b*c)*x
^7+1/6*a^2*e*x^6+1/5*a^2*d*x^5+1/4*a^2*c*x^4
```

Maxima [A] time = 1.41455, size = 204, normalized size = 1.25

$$\frac{1}{15}\,b^2hx^{15}+\frac{1}{14}\,b^2gx^{14}+\frac{1}{13}\,b^2fx^{13}+\frac{1}{12}\,\left(b^2e+2\,abh\right)x^{12}+\frac{1}{11}\,\left(b^2d+2\,abg\right)x^{11}+\frac{1}{10}\,\left(b^2c+2\,abf\right)x^{10}+\frac{1}{9}\,\left(2\,abe+a^2h\right)x^9+\frac{1}{6}\,a^2ex^6+\frac{1}{8}\,\left(2\,abd+a^2g\right)x^8+\frac{1}{5}\,a^2dx^5+\frac{1}{7}\,\left(2\,abc+a^2f\right)x^7+\frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x^3,x, algorithm
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(b^2*e
 + 2*a*b*h)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*
a*b*f)*x^10 + 1/9*(2*a*b*e + a^2*h)*x^9 + 1/6*a^2*e*x^6 + 1/8*(2*
a*b*d + a^2*g)*x^8 + 1/5*a^2*d*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 +
1/4*a^2*c*x^4
```

Fricas [A] time = 0.213034, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}hb^2+\frac{1}{14}x^{14}gb^2+\frac{1}{13}x^{13}fb^2+\frac{1}{12}x^{12}eb^2+\frac{1}{6}x^{12}hba+\frac{1}{11}x^{11}db^2+\frac{2}{11}x^{11}gba+\frac{1}{10}x^{10}cb^2+\frac{1}{5}x^{10}fba+\frac{2}{9}x^9eba+\frac{1}{9}x^9ha^2+\frac{1}{4}x^8dba+\frac{1}{8}x^8ga^2+\frac{2}{7}x^7cba+\frac{1}{7}x^7fa^2+\frac{1}{6}x^6ea^2+\frac{1}{5}x^5da^2+\frac{1}{4}x^4ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x^3,x, algorithm
```

```
[Out] 1/15*x^15*h*b^2 + 1/14*x^14*g*b^2 + 1/13*x^13*f*b^2 + 1/12*x^12*e
*b^2 + 1/6*x^12*h*b*a + 1/11*x^11*d*b^2 + 2/11*x^11*g*b*a + 1/10*
x^10*c*b^2 + 1/5*x^10*f*b*a + 2/9*x^9*e*b*a + 1/9*x^9*h*a^2 + 1/4
*x^8*d*b*a + 1/8*x^8*g*a^2 + 2/7*x^7*c*b*a + 1/7*x^7*f*a^2 + 1/6*
x^6*e*a^2 + 1/5*x^5*d*a^2 + 1/4*x^4*c*a^2
```

Sympy [A] time = 0.094831, size = 167, normalized size = 1.02

$$\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{b^2fx^{13}}{13}+\frac{b^2gx^{14}}{14}+\frac{b^2hx^{15}}{15}+x^{12}\left(\frac{abh}{6}+\frac{b^2e}{12}\right)+x^{11}\left(\frac{2abg}{11}+\frac{b^2d}{11}\right)+x^{10}\left(\frac{abf}{5}+\frac{b^2c}{10}\right)+x^9\left(\frac{a^2h}{9}+\frac{2abe}{9}\right)+x^8\left(\frac{a^2g}{8}+\frac{abd}{4}\right)+x^7\left(\frac{a^2f}{7}+\frac{2abc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + b**2*f*x**13/13 +
 b**2*g*x**14/14 + b**2*h*x**15/15 + x**12*(a*b*h/6 + b**2*e/12)
+ x**11*(2*a*b*g/11 + b**2*d/11) + x**10*(a*b*f/5 + b**2*c/10) +
x**9*(a**2*h/9 + 2*a*b*e/9) + x**8*(a**2*g/8 + a*b*d/4) + x**7*(a
**2*f/7 + 2*a*b*c/7)
```

GIAC/XCAS [A] time = 0.208828, size = 216, normalized size = 1.33

$$\frac{1}{15}b^2hx^{15}+\frac{1}{14}b^2gx^{14}+\frac{1}{13}b^2fx^{13}+\frac{1}{6}abhx^{12}+\frac{1}{12}b^2x^{12}e+\frac{1}{11}b^2dx^{11}+\frac{2}{11}abgx^{11}+\frac{1}{10}b^2cx^{10}+\frac{1}{5}abfx^{10}+\frac{1}{9}a^2hx^9+\frac{2}{9}abx^9e+\frac{1}{4}abdx^8+\frac{1}{8}a^2gx^8+\frac{2}{7}abcx^7+\frac{1}{7}a^2fx^7+\frac{1}{6}a^2x^6e+\frac{1}{5}a^2dx^5+\frac{1}{4}a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x^3,x, algorithm
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/6*a*b*h*x
^12 + 1/12*b^2*x^12*e + 1/11*b^2*d*x^11 + 2/11*a*b*g*x^11 + 1/10*
b^2*c*x^10 + 1/5*a*b*f*x^10 + 1/9*a^2*h*x^9 + 2/9*a*b*x^9*e + 1/4
*a*b*d*x^8 + 1/8*a^2*g*x^8 + 2/7*a*b*c*x^7 + 1/7*a^2*f*x^7 + 1/6*
a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

3.373 $\int x^2 (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\, dx$

Optimal. Leaf size=158

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{c(a+bx^3)^3}{9b}+\frac{1}{10}bx^{10}(2ag+bd)+\frac{1}{7}ax^7(ag+2bd)+\frac{1}{11}bx^{11}(2ah+be)+\frac{1}{8}ax^8(ah+2be)+\frac{2}{9}abfx^9+\frac{1}{12}b^2fx^{12}+\frac{1}{13}b^2gx^{13}+\frac{1}{14}b^2hx^{14}$$

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*
x^7)/7 + (a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*
a*g)*x^10)/10 + (b*(b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^
2*g*x^13)/13 + (b^2*h*x^14)/14 + (c*(a + b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.475947, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{c(a+bx^3)^3}{9b}+\frac{1}{10}bx^{10}(2ag+bd)+\frac{1}{7}ax^7(ag+2bd)+\frac{1}{11}bx^{11}(2ah+be)+\frac{1}{8}ax^8(ah+2be)+\frac{2}{9}abfx^9+\frac{1}{12}b^2fx^{12}+\frac{1}{13}b^2gx^{13}+\frac{1}{14}b^2hx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*
x^7)/7 + (a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*
a*g)*x^10)/10 + (b*(b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^
2*g*x^13)/13 + (b^2*h*x^14)/14 + (c*(a + b*x^3)^3)/(9*b)
```

Rubi in Sympy [A] time = 64.0786, size = 146, normalized size = 0.92

$$\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\frac{a^2fx^6}{6}+\frac{2abfx^9}{9}+\frac{ax^8(ah+2be)}{8}+\frac{ax^7(ag+2bd)}{7}+\frac{b^2fx^{12}}{12}+\frac{b^2gx^{13}}{13}+\frac{b^2hx^{14}}{14}+\frac{bx^{11}(2ah+be)}{11}+\frac{bx^{10}(2ag+bd)}{10}+\frac{c(a+bx^3)^3}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*d*x**4/4 + a**2*e*x**5/5 + a**2*f*x**6/6 + 2*a*b*f*x**9/9 +
a*x**8*(a*h + 2*b*e)/8 + a*x**7*(a*g + 2*b*d)/7 + b**2*f*x**12/12
 + b**2*g*x**13/13 + b**2*h*x**14/14 + b*x**11*(2*a*h + b*e)/11 +
 b*x**10*(2*a*g + b*d)/10 + c*(a + b*x**3)**3/(9*b)
```

Mathematica [A] time = 0.191079, size = 150, normalized size = 0.95

$$a^2\left(\frac{cx^3}{3}+\frac{dx^4}{4}+\frac{ex^5}{5}+\frac{fx^6}{6}+\frac{gx^7}{7}+\frac{hx^8}{8}\right)+ab\left(\frac{cx^6}{3}+\frac{2dx^7}{7}+\frac{ex^8}{4}+\frac{2fx^9}{9}+\frac{gx^{10}}{5}+\frac{2hx^{11}}{11}\right)+\frac{b^2x^9\left(20020c+3x\left(6006d+5460ex+55x^2\left(91f+84gx+78hx^2\right)\right)\right)}{180180}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*((c*x^3)/3 + (d*x^4)/4 + (e*x^5)/5 + (f*x^6)/6 + (g*x^7)/7 +
(h*x^8)/8) + a*b*((c*x^6)/3 + (2*d*x^7)/7 + (e*x^8)/4 + (2*f*x^9)
/9 + (g*x^10)/5 + (2*h*x^11)/11) + (b^2*x^9*(20020*c + 3*x*(6006*
d + 5460*e*x + 55*x^2*(91*f + 84*g*x + 78*h*x^2))))/180180
```

Maple [A] time = 0.002, size = 152, normalized size = 1.

$$\frac{b^2hx^{14}}{14}+\frac{b^2gx^{13}}{13}+\frac{b^2fx^{12}}{12}+\frac{\left(2\,abh+b^2e\right)x^{11}}{11}+\frac{\left(2\,abg+b^2d\right)x^{10}}{10}+\frac{\left(2\,abf+b^2c\right)x^9}{9}+\frac{\left(a^2h+2\,bea\right)x^8}{8}+\frac{\left(a^2g+2\,bda\right)x^7}{7}+\frac{\left(a^2f+2\,abc\right)x^6}{6}+\frac{a^2ex^5}{5}+\frac{a^2dx^4}{4}+\frac{a^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/14*b^2*h*x^14+1/13*b^2*g*x^13+1/12*b^2*f*x^12+1/11*(2*a*b*h+b^2
*e)*x^11+1/10*(2*a*b*g+b^2*d)*x^10+1/9*(2*a*b*f+b^2*c)*x^9+1/8*(a
^2*h+2*a*b*e)*x^8+1/7*(a^2*g+2*a*b*d)*x^7+1/6*(a^2*f+2*a*b*c)*x^6
+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1/3*a^2*c*x^3
```

Maxima [A] time = 1.43496, size = 204, normalized size = 1.29

$$\frac{1}{14}\,b^2hx^{14}+\frac{1}{13}\,b^2gx^{13}+\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\left(b^2e+2\,abh\right)x^{11}+\frac{1}{10}\left(b^2d+2\,abg\right)x^{10}+\frac{1}{9}\left(b^2c+2\,abf\right)x^9+\frac{1}{8}\left(2\,abe+a^2h\right)x^8+\frac{1}{5}\,a^2ex^5+\frac{1}{7}\left(2\,abd+a^2g\right)x^7+\frac{1}{4}\,a^2dx^4+\frac{1}{6}\left(2\,abc+a^2f\right)x^6+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x^2,x, algorithm
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(b^2*e
 + 2*a*b*h)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a
*b*f)*x^9 + 1/8*(2*a*b*e + a^2*h)*x^8 + 1/5*a^2*e*x^5 + 1/7*(2*a*
b*d + a^2*g)*x^7 + 1/4*a^2*d*x^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/
3*a^2*c*x^3
```

Fricas [A] time = 0.213634, size = 1, normalized size = 0.01

$$\frac{1}{14}x^{14}hb^2+\frac{1}{13}x^{13}gb^2+\frac{1}{12}x^{12}fb^2+\frac{1}{11}x^{11}eb^2+\frac{2}{11}x^{11}hba+\frac{1}{10}x^{10}db^2+\frac{1}{5}x^{10}gba+\frac{1}{9}x^9cb^2+\frac{2}{9}x^9fba+\frac{1}{4}x^8eba+\frac{1}{8}x^8ha^2+\frac{2}{7}x^7dba+\frac{1}{7}x^7ga^2+\frac{1}{3}x^6cba+\frac{1}{6}x^6fa^2+\frac{1}{5}x^5ea^2+\frac{1}{4}x^4da^2+\frac{1}{3}x^3ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x^2,x, algorithm
```

```
[Out] 1/14*x^14*h*b^2 + 1/13*x^13*g*b^2 + 1/12*x^12*f*b^2 + 1/11*x^11*e
*b^2 + 2/11*x^11*h*b*a + 1/10*x^10*d*b^2 + 1/5*x^10*g*b*a + 1/9*x
^9*c*b^2 + 2/9*x^9*f*b*a + 1/4*x^8*e*b*a + 1/8*x^8*h*a^2 + 2/7*x^
7*d*b*a + 1/7*x^7*g*a^2 + 1/3*x^6*c*b*a + 1/6*x^6*f*a^2 + 1/5*x^5
*e*a^2 + 1/4*x^4*d*a^2 + 1/3*x^3*c*a^2
```

Sympy [A] time = 0.093673, size = 167, normalized size = 1.06

$$\frac{a^2cx^3}{3} + \frac{a^2dx^4}{4} + \frac{a^2ex^5}{5} + \frac{b^2fx^{12}}{12} + \frac{b^2gx^{13}}{13} + \frac{b^2hx^{14}}{14} + x^{11}\left(\frac{2abh}{11} + \frac{b^2e}{11}\right) + x^{10}\left(\frac{abg}{5} + \frac{b^2d}{10}\right) + x^9\left(\frac{2abf}{9} + \frac{b^2c}{9}\right) + x^8\left(\frac{a^2h}{8} + \frac{abe}{4}\right) + x^7\left(\frac{a^2g}{7} + \frac{2abd}{7}\right) + x^6\left(\frac{a^2f}{6} + \frac{abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + b**2*f*x**12/12 +
 b**2*g*x**13/13 + b**2*h*x**14/14 + x**11*(2*a*b*h/11 + b**2*e/1
1) + x**10*(a*b*g/5 + b**2*d/10) + x**9*(2*a*b*f/9 + b**2*c/9) +
x**8*(a**2*h/8 + a*b*e/4) + x**7*(a**2*g/7 + 2*a*b*d/7) + x**6*(a
**2*f/6 + a*b*c/3)
```

GIAC/XCAS [A] time = 0.208823, size = 216, normalized size = 1.37

$$\frac{1}{14}\,b^2hx^{14} + \frac{1}{13}\,b^2gx^{13} + \frac{1}{12}\,b^2fx^{12} + \frac{2}{11}\,abhx^{11} + \frac{1}{11}\,b^2x^{11}e + \frac{1}{10}\,b^2dx^{10} + \frac{1}{5}\,abgx^{10} + \frac{1}{9}\,b^2cx^9 + \frac{2}{9}\,abfx^9 + \frac{1}{8}\,a^2hx^8 + \frac{1}{4}\,abx^8e + \frac{2}{7}\,abdx^7 + \frac{1}{7}\,a^2gx^7 + \frac{1}{3}\,abcx^6 + \frac{1}{6}\,a^2fx^6 + \frac{1}{5}\,a^2x^5e + \frac{1}{4}\,a^2dx^4 + \frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x^2,x, algorithm
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 2/11*a*b*h*
x^11 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/5*a*b*g*x^10 + 1/9*b
^2*c*x^9 + 2/9*a*b*f*x^9 + 1/8*a^2*h*x^8 + 1/4*a*b*x^8*e + 2/7*a*
b*d*x^7 + 1/7*a^2*g*x^7 + 1/3*a*b*c*x^6 + 1/6*a^2*f*x^6 + 1/5*a^2
*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

3.374 $\int x\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=158

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{6}a^2gx^6+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+\frac{2}{9}abgx^9+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}$$

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*
x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a
*b*g*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*
g*x^12)/12 + (b^2*h*x^13)/13 + (d*(a + b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.400245, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{6}a^2gx^6+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+\frac{2}{9}abgx^9+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*
x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a
*b*g*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*
g*x^12)/12 + (b^2*h*x^13)/13 + (d*(a + b*x^3)^3)/(9*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2c\int x\,dx+\frac{a^2ex^4}{4}+\frac{a^2gx^6}{6}+\frac{2abgx^9}{9}+\frac{ax^7\left(ah+2be\right)}{7}+\frac{ax^5\left(af+2bc\right)}{5}+\frac{b^2fx^{11}}{11}+\frac{b^2gx^{12}}{12}+\frac{b^2hx^{13}}{13}+\frac{bx^{10}\left(2ah+be\right)}{10}+\frac{bx^8\left(2af+bc\right)}{8}+\frac{d\left(a+bx^3\right)^3}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*Integral(x, x) + a**2*e*x**4/4 + a**2*g*x**6/6 + 2*a*b*g*x
**9/9 + a*x**7*(a*h + 2*b*e)/7 + a*x**5*(a*f + 2*b*c)/5 + b**2*f*
x**11/11 + b**2*g*x**12/12 + b**2*h*x**13/13 + b*x**10*(2*a*h + b
*e)/10 + b*x**8*(2*a*f + b*c)/8 + d*(a + b*x**3)**3/(9*b)
```

Mathematica [A] time = 0.0671372, size = 163, normalized size = 1.03

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{1}{9}bx^9(2ag+bd)+\frac{1}{6}ax^6(ag+2bd)+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*
x^5)/5 + (a*(2*b*d + a*g)*x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(
b*c + 2*a*f)*x^8)/8 + (b*(b*d + 2*a*g)*x^9)/9 + (b*(b*e + 2*a*h)*
x^10)/10 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

Maple [A] time = 0.001, size = 152, normalized size = 1.

$$\frac{b^2hx^{13}}{13}+\frac{b^2gx^{12}}{12}+\frac{b^2fx^{11}}{11}+\frac{\left(2\,abh+b^2e\right)x^{10}}{10}+\frac{\left(2\,abg+b^2d\right)x^9}{9}+\frac{\left(2\,abf+b^2c\right)x^8}{8}+\frac{\left(a^2h+2\,bea\right)x^7}{7}+\frac{\left(a^2g+2\,bda\right)x^6}{6}+\frac{\left(a^2f+2\,abc\right)x^5}{5}+\frac{a^2ex^4}{4}+\frac{a^2dx^3}{3}+\frac{a^2cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/13*b^2*h*x^13+1/12*b^2*g*x^12+1/11*b^2*f*x^11+1/10*(2*a*b*h+b^2
*e)*x^10+1/9*(2*a*b*g+b^2*d)*x^9+1/8*(2*a*b*f+b^2*c)*x^8+1/7*(a^2
*h+2*a*b*e)*x^7+1/6*(a^2*g+2*a*b*d)*x^6+1/5*(a^2*f+2*a*b*c)*x^5+1
/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2*a^2*c*x^2
```

Maxima [A] time = 1.41953, size = 204, normalized size = 1.29

$$\frac{1}{13}\,b^2hx^{13}+\frac{1}{12}\,b^2gx^{12}+\frac{1}{11}\,b^2fx^{11}+\frac{1}{10}\left(b^2e+2\,abh\right)x^{10}+\frac{1}{9}\left(b^2d+2\,abg\right)x^9+\frac{1}{8}\left(b^2c+2\,abf\right)x^8+\frac{1}{7}\left(2\,abe+a^2h\right)x^7+\frac{1}{4}\,a^2ex^4+\frac{1}{6}\left(2\,abd+a^2g\right)x^6+\frac{1}{3}\,a^2dx^3+\frac{1}{5}\left(2\,abc+a^2f\right)x^5+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x,x, algorithm="
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(b^2*e
 + 2*a*b*h)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b
*f)*x^8 + 1/7*(2*a*b*e + a^2*h)*x^7 + 1/4*a^2*e*x^4 + 1/6*(2*a*b*
d + a^2*g)*x^6 + 1/3*a^2*d*x^3 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*
a^2*c*x^2
```

Fricas [A] time = 0.216128, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}hb^2+\frac{1}{12}x^{12}gb^2+\frac{1}{11}x^{11}fb^2+\frac{1}{10}x^{10}eb^2+\frac{1}{5}x^{10}hba+\frac{1}{9}x^9db^2+\frac{2}{9}x^9gba+\frac{1}{8}x^8cb^2+\frac{1}{4}x^8fba+\frac{2}{7}x^7eba+\frac{1}{7}x^7ha^2+\frac{1}{3}x^6dba+\frac{1}{6}x^6ga^2+\frac{2}{5}x^5cba+\frac{1}{5}x^5fa^2+\frac{1}{4}x^4ea^2+\frac{1}{3}x^3da^2+\frac{1}{2}x^2ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x,x, algorithm="
```

```
[Out] 1/13*x^13*h*b^2 + 1/12*x^12*g*b^2 + 1/11*x^11*f*b^2 + 1/10*x^10*e
*b^2 + 1/5*x^10*h*b*a + 1/9*x^9*d*b^2 + 2/9*x^9*g*b*a + 1/8*x^8*c
*b^2 + 1/4*x^8*f*b*a + 2/7*x^7*e*b*a + 1/7*x^7*h*a^2 + 1/3*x^6*d*
b*a + 1/6*x^6*g*a^2 + 2/5*x^5*c*b*a + 1/5*x^5*f*a^2 + 1/4*x^4*e*a
^2 + 1/3*x^3*d*a^2 + 1/2*x^2*c*a^2
```

Sympy [A] time = 0.093544, size = 167, normalized size = 1.06

$$\frac{a^2cx^2}{2}+\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}+\frac{b^2fx^{11}}{11}+\frac{b^2gx^{12}}{12}+\frac{b^2hx^{13}}{13}+x^{10}\left(\frac{abh}{5}+\frac{b^2e}{10}\right)+x^9\left(\frac{2abg}{9}+\frac{b^2d}{9}\right)+x^8\left(\frac{abf}{4}+\frac{b^2c}{8}\right)+x^7\left(\frac{a^2h}{7}+\frac{2abe}{7}\right)+x^6\left(\frac{a^2g}{6}+\frac{abd}{3}\right)+x^5\left(\frac{a^2f}{5}+\frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + b**2*f*x**11/11 +
 b**2*g*x**12/12 + b**2*h*x**13/13 + x**10*(a*b*h/5 + b**2*e/10)
+ x**9*(2*a*b*g/9 + b**2*d/9) + x**8*(a*b*f/4 + b**2*c/8) + x**7*
(a**2*h/7 + 2*a*b*e/7) + x**6*(a**2*g/6 + a*b*d/3) + x**5*(a**2*f
/5 + 2*a*b*c/5)
```

GIAC/XCAS [A] time = 0.209178, size = 216, normalized size = 1.37

$$\frac{1}{13}\,b^2hx^{13}+\frac{1}{12}\,b^2gx^{12}+\frac{1}{11}\,b^2fx^{11}+\frac{1}{5}\,abhx^{10}+\frac{1}{10}\,b^2x^{10}e+\frac{1}{9}\,b^2dx^9+\frac{2}{9}\,abgx^9+\frac{1}{8}\,b^2cx^8+\frac{1}{4}\,abfx^8+\frac{1}{7}\,a^2hx^7+\frac{2}{7}\,abx^7e+\frac{1}{3}\,abdx^6+\frac{1}{6}\,a^2gx^6+\frac{2}{5}\,abcx^5+\frac{1}{5}\,a^2fx^5+\frac{1}{4}\,a^2x^4e+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2*x,x, algorithm="
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/5*a*b*h*x
^10 + 1/10*b^2*x^10*e + 1/9*b^2*d*x^9 + 2/9*a*b*g*x^9 + 1/8*b^2*c
*x^8 + 1/4*a*b*f*x^8 + 1/7*a^2*h*x^7 + 2/7*a*b*x^7*e + 1/3*a*b*d*
x^6 + 1/6*a^2*g*x^6 + 2/5*a*b*c*x^5 + 1/5*a^2*f*x^5 + 1/4*a^2*x^4
*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

3.375 $\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\, dx$

Optimal. Leaf size=153

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{6}a^2hx^6+\frac{1}{7}bx^7(2af+bc)+\frac{1}{4}ax^4(af+2bc)+\frac{1}{8}bx^8(2ag+bd)+\frac{1}{5}ax^5(ag+2bd)+\frac{e(a+bx^3)^3}{9b}+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}+\frac{1}{11}b^2gx^{11}+\frac{1}{12}b^2hx^{12}$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a
*g)*x^5)/5 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d +
2*a*g)*x^8)/8 + (2*a*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/
11 + (b^2*h*x^12)/12 + (e*(a + b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.349921, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{6}a^2hx^6+\frac{1}{7}bx^7(2af+bc)+\frac{1}{4}ax^4(af+2bc)+\frac{1}{8}bx^8(2ag+bd)+\frac{1}{5}ax^5(ag+2bd)+\frac{e(a+bx^3)^3}{9b}+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}+\frac{1}{11}b^2gx^{11}+\frac{1}{12}b^2hx^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a
*g)*x^5)/5 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d +
2*a*g)*x^8)/8 + (2*a*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/
11 + (b^2*h*x^12)/12 + (e*(a + b*x^3)^3)/(9*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2d\int x\,dx+\frac{a^2hx^6}{6}+a^2\int c\,dx+\frac{2abhx^9}{9}+\frac{ax^5(ag+2bd)}{5}+\frac{ax^4(af+2bc)}{4}+\frac{b^2fx^{10}}{10}+\frac{b^2gx^{11}}{11}+\frac{b^2hx^{12}}{12}+\frac{bx^8(2ag+bd)}{8}+\frac{bx^7(2af+bc)}{7}+\frac{e(a+bx^3)^3}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*d*Integral(x, x) + a**2*h*x**6/6 + a**2*Integral(c, x) + 2*a
*b*h*x**9/9 + a*x**5*(a*g + 2*b*d)/5 + a*x**4*(a*f + 2*b*c)/4 + b
**2*f*x**10/10 + b**2*g*x**11/11 + b**2*h*x**12/12 + b*x**8*(2*a*
g + b*d)/8 + b*x**7*(2*a*f + b*c)/7 + e*(a + b*x**3)**3/(9*b)
```

Mathematica [A] time = 0.172723, size = 125, normalized size = 0.82

$$\frac{462a^2x\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+22abx^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))+b}{27720}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (b^2*x^7*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x +
55*h*x^2))) + 462*a^2*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g
```

```
*x^2 + 10*h*x^3))) + 22*a*b*x^4*(630*c + x*(504*d + 5*x*(84*e + x
*(72*f + 7*x*(9*g + 8*h*x))))))/27720
```

Maple [A] time = 0., size = 149, normalized size = 1.

$$\frac{b^2hx^{12}}{12}+\frac{b^2gx^{11}}{11}+\frac{b^2fx^{10}}{10}+\frac{\left(2\,abh+b^2e\right)x^9}{9}+\frac{\left(2\,abg+b^2d\right)x^8}{8}+\frac{\left(2\,abf+b^2c\right)x^7}{7}+\frac{\left(a^2h+2\,bea\right)x^6}{6}+\frac{\left(a^2g+2\,bda\right)x^5}{5}+\frac{\left(a^2f+2\,abc\right)x^4}{4}+\frac{a^2ex^3}{3}+\frac{a^2dx^2}{2}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/12*b^2*h*x^12+1/11*b^2*g*x^11+1/10*b^2*f*x^10+1/9*(2*a*b*h+b^2*
e)*x^9+1/8*(2*a*b*g+b^2*d)*x^8+1/7*(2*a*b*f+b^2*c)*x^7+1/6*(a^2*h
+2*a*b*e)*x^6+1/5*(a^2*g+2*a*b*d)*x^5+1/4*(a^2*f+2*a*b*c)*x^4+1/3
*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x
```

Maxima [A] time = 1.37888, size = 200, normalized size = 1.31

$$\frac{1}{12}\,b^2hx^{12}+\frac{1}{11}\,b^2gx^{11}+\frac{1}{10}\,b^2fx^{10}+\frac{1}{9}\,\left(b^2e+2\,abh\right)x^9+\frac{1}{8}\,\left(b^2d+2\,abg\right)x^8+\frac{1}{7}\,\left(b^2c+2\,abf\right)x^7+\frac{1}{6}\,\left(2\,abe+a^2h\right)x^6+\frac{1}{3}\,a^2ex^3+\frac{1}{5}\,\left(2\,abd+a^2g\right)x^5+\frac{1}{2}\,a^2dx^2+\frac{1}{4}\,\left(2\,abc+a^2f\right)x^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2,x, algorithm="ma
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(b^2*e
+ 2*a*b*h)*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f
)*x^7 + 1/6*(2*a*b*e + a^2*h)*x^6 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*d
+ a^2*g)*x^5 + 1/2*a^2*d*x^2 + 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*
x
```

Fricas [A] time = 0.216842, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}hb^2+\frac{1}{11}x^{11}gb^2+\frac{1}{10}x^{10}fb^2+\frac{1}{9}x^9eb^2+\frac{2}{9}x^9hba+\frac{1}{8}x^8db^2+\frac{1}{4}x^8gba+\frac{1}{7}x^7cb^2+\frac{2}{7}x^7fba+\frac{1}{3}x^6eba+\frac{1}{6}x^6ha^2+\frac{2}{5}x^5dba+\frac{1}{5}x^5ga^2+\frac{1}{2}x^4cba+\frac{1}{4}x^4fa^2+\frac{1}{3}x^3ea^2+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2,x, algorithm="fr
```

```
[Out] 1/12*x^12*h*b^2 + 1/11*x^11*g*b^2 + 1/10*x^10*f*b^2 + 1/9*x^9*e*b
^2 + 2/9*x^9*h*b*a + 1/8*x^8*d*b^2 + 1/4*x^8*g*b*a + 1/7*x^7*c*b^
2 + 2/7*x^7*f*b*a + 1/3*x^6*e*b*a + 1/6*x^6*h*a^2 + 2/5*x^5*d*b*a
 + 1/5*x^5*g*a^2 + 1/2*x^4*c*b*a + 1/4*x^4*f*a^2 + 1/3*x^3*e*a^2
+ 1/2*x^2*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.091981, size = 163, normalized size = 1.07

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{b^2fx^{10}}{10}+\frac{b^2gx^{11}}{11}+\frac{b^2hx^{12}}{12}+x^9\left(\frac{2abh}{9}+\frac{b^2e}{9}\right)+x^8\left(\frac{abg}{4}+\frac{b^2d}{8}\right)+x^7\left(\frac{2abf}{7}+\frac{b^2c}{7}\right)+x^6\left(\frac{a^2h}{6}+\frac{abe}{3}\right)+x^5\left(\frac{a^2g}{5}+\frac{2abd}{5}\right)+x^4\left(\frac{a^2f}{4}+\frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + b**2*f*x**10/10 + b**2
*g*x**11/11 + b**2*h*x**12/12 + x**9*(2*a*b*h/9 + b**2*e/9) + x**
8*(a*b*g/4 + b**2*d/8) + x**7*(2*a*b*f/7 + b**2*c/7) + x**6*(a**2
*h/6 + a*b*e/3) + x**5*(a**2*g/5 + 2*a*b*d/5) + x**4*(a**2*f/4 +
a*b*c/2)
```

GIAC/XCAS [A] time = 0.2079, size = 212, normalized size = 1.39

$$\frac{1}{12}\,b^2hx^{12}+\frac{1}{11}\,b^2gx^{11}+\frac{1}{10}\,b^2fx^{10}+\frac{2}{9}\,abhx^9+\frac{1}{9}\,b^2x^9e+\frac{1}{8}\,b^2dx^8+\frac{1}{4}\,abgx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{7}\,abfx^7+\frac{1}{6}\,a^2hx^6+\frac{1}{3}\,abx^6e+\frac{2}{5}\,abdx^5+\frac{1}{5}\,a^2gx^5+\frac{1}{2}\,abcx^4+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2x^3e+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2,x, algorithm="gi
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 2/9*a*b*h*x
^9 + 1/9*b^2*x^9*e + 1/8*b^2*d*x^8 + 1/4*a*b*g*x^8 + 1/7*b^2*c*x^
7 + 2/7*a*b*f*x^7 + 1/6*a^2*h*x^6 + 1/3*a*b*x^6*e + 2/5*a*b*d*x^5
 + 1/5*a^2*g*x^5 + 1/2*a*b*c*x^4 + 1/4*a^2*f*x^4 + 1/3*a^2*x^3*e
+ 1/2*a^2*d*x^2 + a^2*c*x
```

3.376 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx$

Optimal. Leaf size=149

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{f(a+bx^3)^3}{9b}+\frac{1}{6}b^2cx^6+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)
/4 + (a*(2*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x
^7)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/
11 + (f*(a + b*x^3)^3)/(9*b) + a^2*c*Log[x]
```

Rubi [A] time = 0.242581, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{f(a+bx^3)^3}{9b}+\frac{1}{6}b^2cx^6+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)
/4 + (a*(2*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x
^7)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/
11 + (f*(a + b*x^3)^3)/(9*b) + a^2*c*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2c\log(x)+a^2e\int x\,dx+a^2\int d\,dx+\frac{ax^5(ah+2be)}{5}+\frac{ax^4(ag+2bd)}{4}+\frac{ax^3(af+2bc)}{3}+\frac{b^2fx^9}{9}+\frac{b^2gx^{10}}{10}+\frac{b^2hx^{11}}{11}+\frac{bx^8(2ah+be)}{8}+\frac{bx^7(2ag+bd)}{7}+\frac{bx^6(2af+bc)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**2*c*log(x) + a**2*e*Integral(x, x) + a**2*Integral(d, x) + a*x
**5*(a*h + 2*b*e)/5 + a*x**4*(a*g + 2*b*d)/4 + a*x**3*(a*f + 2*b*
c)/3 + b**2*f*x**9/9 + b**2*g*x**10/10 + b**2*h*x**11/11 + b*x**8
*(2*a*h + b*e)/8 + b*x**7*(2*a*g + b*d)/7 + b*x**6*(2*a*f + b*c)/
6
```

Mathematica [A] time = 0.113301, size = 154, normalized size = 1.03

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{6}bx^6(2af+bc)+\frac{1}{3}ax^3(af+2bc)+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{1}{9}b^2fx^9+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*c + a*f)*x^3)/3 + (a*(2*b*d + a
*g)*x^4)/4 + (a*(2*b*e + a*h)*x^5)/5 + (b*(b*c + 2*a*f)*x^6)/6 +
(b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*f*x^9)/9
 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + a^2*c*Log[x]
```

Maple [A] time = 0.004, size = 153, normalized size = 1.

$$\frac{b^2hx^{11}}{11}+\frac{b^2gx^{10}}{10}+\frac{x^9fb^2}{9}+\frac{x^8abh}{4}+\frac{x^8b^2e}{8}+\frac{2\,x^7abg}{7}+\frac{b^2dx^7}{7}+\frac{x^6abf}{3}+\frac{b^2cx^6}{6}+\frac{x^5a^2h}{5}+\frac{2\,x^5abe}{5}+\frac{x^4a^2g}{4}+\frac{x^4abd}{2}+\frac{a^2fx^3}{3}+\frac{2\,abcx^3}{3}+\frac{a^2ex^2}{2}+a^2dx+a^2c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] 1/11*b^2*h*x^11+1/10*b^2*g*x^10+1/9*x^9*f*b^2+1/4*x^8*a*b*h+1/8*x
^8*b^2*e+2/7*x^7*a*b*g+1/7*b^2*d*x^7+1/3*x^6*a*b*f+1/6*b^2*c*x^6+
1/5*x^5*a^2*h+2/5*x^5*a*b*e+1/4*x^4*a^2*g+1/2*x^4*a*b*d+1/3*a^2*f
*x^3+2/3*a*b*c*x^3+1/2*a^2*e*x^2+a^2*d*x+a^2*c*ln(x)
```

Maxima [A] time = 1.38052, size = 197, normalized size = 1.32

$$\frac{1}{11}\,b^2hx^{11}+\frac{1}{10}\,b^2gx^{10}+\frac{1}{9}\,b^2fx^9+\frac{1}{8}\left(b^2e+2\,abh\right)x^8+\frac{1}{7}\left(b^2d+2\,abg\right)x^7+\frac{1}{6}\left(b^2c+2\,abf\right)x^6+\frac{1}{5}\left(2\,abe+a^2h\right)x^5+\frac{1}{2}\,a^2ex^2+\frac{1}{4}\left(2\,abd+a^2g\right)x^4+a^2dx+\frac{1}{3}\left(2\,abc+a^2f\right)x^3+a^2c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x,x, algorithm="
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e +
2*a*b*h)*x^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*
x^6 + 1/5*(2*a*b*e + a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d +
a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

Fricas [A] time = 0.244357, size = 197, normalized size = 1.32

$$\frac{1}{11}\,b^2hx^{11}+\frac{1}{10}\,b^2gx^{10}+\frac{1}{9}\,b^2fx^9+\frac{1}{8}\left(b^2e+2\,abh\right)x^8+\frac{1}{7}\left(b^2d+2\,abg\right)x^7+\frac{1}{6}\left(b^2c+2\,abf\right)x^6+\frac{1}{5}\left(2\,abe+a^2h\right)x^5+\frac{1}{2}\,a^2ex^2+\frac{1}{4}\left(2\,abd+a^2g\right)x^4+a^2dx+\frac{1}{3}\left(2\,abc+a^2f\right)x^3+a^2c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x,x, algorithm="
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e +
2*a*b*h)*x^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*
x^6 + 1/5*(2*a*b*e + a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d +
a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

Sympy [A] time = 0.940603, size = 162, normalized size = 1.09

$$a^2c\log(x)+a^2dx+\frac{a^2ex^2}{2}+\frac{b^2fx^9}{9}+\frac{b^2gx^{10}}{10}+\frac{b^2hx^{11}}{11}+x^8\left(\frac{abh}{4}+\frac{b^2e}{8}\right)+x^7\left(\frac{2abg}{7}+\frac{b^2d}{7}\right)+x^6\left(\frac{abf}{3}+\frac{b^2c}{6}\right)+x^5\left(\frac{a^2h}{5}+\frac{2abe}{5}\right)+x^4\left(\frac{a^2g}{4}+\frac{abd}{2}\right)+x^3\left(\frac{a^2f}{3}+\frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + b**2*f*x**9/9 + b**2*g
*x**10/10 + b**2*h*x**11/11 + x**8*(a*b*h/4 + b**2*e/8) + x**7*(2
*a*b*g/7 + b**2*d/7) + x**6*(a*b*f/3 + b**2*c/6) + x**5*(a**2*h/5
 + 2*a*b*e/5) + x**4*(a**2*g/4 + a*b*d/2) + x**3*(a**2*f/3 + 2*a*
b*c/3)
```

GIAC/XCAS [A] time = 0.209234, size = 211, normalized size = 1.42

$$\frac{1}{11}\,b^2hx^{11} + \frac{1}{10}\,b^2gx^{10} + \frac{1}{9}\,b^2fx^9 + \frac{1}{4}\,abhx^8 + \frac{1}{8}\,b^2x^8e + \frac{1}{7}\,b^2dx^7 + \frac{2}{7}\,abgx^7 + \frac{1}{6}\,b^2cx^6 + \frac{1}{3}\,abfx^6 + \frac{1}{5}\,a^2hx^5 + \frac{2}{5}\,abx^5e + \frac{1}{2}\,abdx^4 + \frac{1}{4}\,a^2gx^4 + \frac{2}{3}\,abcx^3 + \frac{1}{3}\,a^2fx^3 + \frac{1}{2}\,a^2x^2e + a^2dx + a^2c\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x,x, algorithm="
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/4*a*b*h*x^8
 + 1/8*b^2*x^8*e + 1/7*b^2*d*x^7 + 2/7*a*b*g*x^7 + 1/6*b^2*c*x^6
+ 1/3*a*b*f*x^6 + 1/5*a^2*h*x^5 + 2/5*a*b*x^5*e + 1/2*a*b*d*x^4 +
 1/4*a^2*g*x^4 + 2/3*a*b*c*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e +
a^2*d*x + a^2*c*ln(abs(x))
```

3.377 $\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} \, dx$

Optimal. Leaf size=147

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{2}{3}abdx^3+\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{g\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b^2dx^6+\frac{1}{8}b^2fx^8+\frac{1}{10}b^2hx^{10}$$

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (2*a*b*d*x^3)/
3 + (a*(2*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b^2*d*x^
6)/6 + (b*(b*e + 2*a*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*h*x^10)/10
+ (g*(a + b*x^3)^3)/(9*b) + a^2*d*Log[x]
```

Rubi [A] time = 0.331635, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{2}{3}abdx^3+\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{g\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b^2dx^6+\frac{1}{8}b^2fx^8+\frac{1}{10}b^2hx^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (2*a*b*d*x^3)/
3 + (a*(2*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b^2*d*x^
6)/6 + (b*(b*e + 2*a*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*h*x^10)/10
+ (g*(a + b*x^3)^3)/(9*b) + a^2*d*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2\int e\,dx+\frac{ax^4(ah+2be)}{4}+\frac{ax^3(ag+2bd)}{3}+a(af+2bc)\int x\,dx+\frac{b^2fx^8}{8}+\frac{b^2gx^9}{9}+\frac{b^2hx^{10}}{10}+\frac{bx^7(2ah+be)}{7}+\frac{bx^6(2ag+bd)}{6}+\frac{bx^5(2af+bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*Integral(e, x) + a*x**4*(a*h + 2
*b*e)/4 + a*x**3*(a*g + 2*b*d)/3 + a*(a*f + 2*b*c)*Integral(x, x)
 + b**2*f*x**8/8 + b**2*g*x**9/9 + b**2*h*x**10/10 + b*x**7*(2*a*
h + b*e)/7 + b*x**6*(2*a*g + b*d)/6 + b*x**5*(2*a*f + b*c)/5
```

Mathematica [A] time = 0.15867, size = 152, normalized size = 1.03

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{1}{6}bx^6(2ag+bd)+\frac{1}{3}ax^3(ag+2bd)+\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{1}{8}b^2fx^8+\frac{1}{9}b^2gx^9+\frac{1}{10}b^2hx^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (a*(2*b*d + a*
g)*x^3)/3 + (a*(2*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (
b*(b*d + 2*a*g)*x^6)/6 + (b*(b*e + 2*a*h)*x^7)/7 + (b^2*f*x^8)/8
+ (b^2*g*x^9)/9 + (b^2*h*x^10)/10 + a^2*d*Log[x]
```

Maple [A] time = 0.009, size = 152, normalized size = 1.

$$\frac{b^2hx^{10}}{10}+\frac{b^2gx^9}{9}+\frac{b^2fx^8}{8}+\frac{2x^7abh}{7}+\frac{x^7b^2e}{7}+\frac{x^6abg}{3}+\frac{b^2dx^6}{6}+\frac{2x^5abf}{5}+\frac{x^5b^2c}{5}+\frac{x^4a^2h}{4}+\frac{x^4abe}{2}+\frac{x^3a^2g}{3}+\frac{2abdx^3}{3}+\frac{x^2a^2f}{2}+x^2abc+a^2ex+a^2d\ln(x)-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 1/10*b^2*h*x^10+1/9*b^2*g*x^9+1/8*b^2*f*x^8+2/7*x^7*a*b*h+1/7*x^7
*b^2*e+1/3*x^6*a*b*g+1/6*b^2*d*x^6+2/5*x^5*a*b*f+1/5*x^5*b^2*c+1/
4*x^4*a^2*h+1/2*x^4*a*b*e+1/3*x^3*a^2*g+2/3*a*b*d*x^3+1/2*x^2*a^2
*f+x^2*a*b*c+a^2*e*x+a^2*d*ln(x)-a^2*c/x
```

Maxima [A] time = 1.37963, size = 197, normalized size = 1.34

$$\frac{1}{10}b^2hx^{10}+\frac{1}{9}b^2gx^9+\frac{1}{8}b^2fx^8+\frac{1}{7}\left(b^2e+2abh\right)x^7+\frac{1}{6}\left(b^2d+2abg\right)x^6+\frac{1}{5}\left(b^2c+2abf\right)x^5+\frac{1}{4}\left(2abe+a^2h\right)x^4+a^2ex+\frac{1}{3}\left(2abd+a^2g\right)x^3+a^2d\log(x)+\frac{1}{2}\left(2abc+a^2f\right)x^2-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^2,x, algorithm
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 1/7*(b^2*e + 2*
a*b*h)*x^7 + 1/6*(b^2*d + 2*a*b*g)*x^6 + 1/5*(b^2*c + 2*a*b*f)*x^
5 + 1/4*(2*a*b*e + a^2*h)*x^4 + a^2*e*x + 1/3*(2*a*b*d + a^2*g)*x
^3 + a^2*d*log(x) + 1/2*(2*a*b*c + a^2*f)*x^2 - a^2*c/x
```

Fricas [A] time = 0.245877, size = 207, normalized size = 1.41

$$\frac{252\,b^2hx^{11}+280\,b^2gx^{10}+315\,b^2fx^9+360\left(b^2e+2\,abh\right)x^8+420\left(b^2d+2\,abg\right)x^7+504\left(b^2c+2\,abf\right)x^6+630\left(2\,abe+a^2h\right.}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^2,x, algorithm
```

```
[Out] 1/2520*(252*b^2*h*x^11 + 280*b^2*g*x^10 + 315*b^2*f*x^9 + 360*(b^
2*e + 2*a*b*h)*x^8 + 420*(b^2*d + 2*a*b*g)*x^7 + 504*(b^2*c + 2*a
*b*f)*x^6 + 630*(2*a*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2 + 840*(2*a
*b*d + a^2*g)*x^4 + 2520*a^2*d*x*log(x) + 1260*(2*a*b*c + a^2*f)*
x^3 - 2520*a^2*c)/x
```

Sympy [A] time = 0.983054, size = 156, normalized size = 1.06

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{b^2fx^8}{8}+\frac{b^2gx^9}{9}+\frac{b^2hx^{10}}{10}+x^7\left(\frac{2abh}{7}+\frac{b^2e}{7}\right)+x^6\left(\frac{abg}{3}+\frac{b^2d}{6}\right)+x^5\left(\frac{2abf}{5}+\frac{b^2c}{5}\right)+x^4\left(\frac{a^2h}{4}+\frac{abe}{2}\right)+x^3\left(\frac{a^2g}{3}+\frac{2abd}{3}\right)+x^2\left(\frac{a^2f}{2}+abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + b**2*f*x**8/8 + b**2*g*x**
9/9 + b**2*h*x**10/10 + x**7*(2*a*b*h/7 + b**2*e/7) + x**6*(a*b*g
/3 + b**2*d/6) + x**5*(2*a*b*f/5 + b**2*c/5) + x**4*(a**2*h/4 + a
*b*e/2) + x**3*(a**2*g/3 + 2*a*b*d/3) + x**2*(a**2*f/2 + a*b*c)
```

GIAC/XCAS [A] time = 0.208964, size = 209, normalized size = 1.42

$$\frac{1}{10}\,b^2hx^{10} + \frac{1}{9}\,b^2gx^9 + \frac{1}{8}\,b^2fx^8 + \frac{2}{7}\,abhx^7 + \frac{1}{7}\,b^2x^7e + \frac{1}{6}\,b^2dx^6 + \frac{1}{3}\,abgx^6 + \frac{1}{5}\,b^2cx^5 + \frac{2}{5}\,abfx^5 + \frac{1}{4}\,a^2hx^4 + \frac{1}{2}\,abx^4e + \frac{2}{3}\,abdx^3 + \frac{1}{3}\,a^2gx^3 + abcx^2 + \frac{1}{2}\,a^2fx^2 + a^2xe + a^2d\ln\left(|x|\right) - \frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^2,x, algorithm
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 2/7*a*b*h*x^7 +
 1/7*b^2*x^7*e + 1/6*b^2*d*x^6 + 1/3*a*b*g*x^6 + 1/5*b^2*c*x^5 +
2/5*a*b*f*x^5 + 1/4*a^2*h*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1
/3*a^2*g*x^3 + a*b*c*x^2 + 1/2*a^2*f*x^2 + a^2*x*e + a^2*d*ln(abs
(x)) - a^2*c/x
```

3.378 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{1}{4}bx^4(2af+bc)+ax(af+2bc)+\frac{1}{5}bx^5(2ag+bd)+\frac{1}{2}ax^2(ag+2bd)+\frac{2}{3}abex^3+\frac{h\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b^2ex^6+\frac{1}{7}b^2fx^7+\frac{1}{8}b^2gx^8$$

```
[Out] -(a^2*c)/(2*x^2) - (a^2*d)/x + a*(2*b*c + a*f)*x + (a*(2*b*d + a*
g)*x^2)/2 + (2*a*b*e*x^3)/3 + (b*(b*c + 2*a*f)*x^4)/4 + (b*(b*d +
 2*a*g)*x^5)/5 + (b^2*e*x^6)/6 + (b^2*f*x^7)/7 + (b^2*g*x^8)/8 +
(h*(a + b*x^3)^3)/(9*b) + a^2*e*Log[x]
```

Rubi [A] time = 0.338188, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{1}{4}bx^4(2af+bc)+ax(af+2bc)+\frac{1}{5}bx^5(2ag+bd)+\frac{1}{2}ax^2(ag+2bd)+\frac{2}{3}abex^3+\frac{h\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b^2ex^6+\frac{1}{7}b^2fx^7+\frac{1}{8}b^2gx^8$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -(a^2*c)/(2*x^2) - (a^2*d)/x + a*(2*b*c + a*f)*x + (a*(2*b*d + a*
g)*x^2)/2 + (2*a*b*e*x^3)/3 + (b*(b*c + 2*a*f)*x^4)/4 + (b*(b*d +
 2*a*g)*x^5)/5 + (b^2*e*x^6)/6 + (b^2*f*x^7)/7 + (b^2*g*x^8)/8 +
(h*(a + b*x^3)^3)/(9*b) + a^2*e*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{ax^3(ah+2be)}{3}+a(ag+2bd)\int x\,dx+\frac{a(af+2bc)\int f\,dx}{f}+\frac{b^2fx^7}{7}+\frac{b^2gx^8}{8}+\frac{b^2hx^9}{9}+\frac{bx^6(2ah+be)}{6}+\frac{bx^5(2ag+bd)}{5}+\frac{bx^4(2af+bc)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] -a**2*c/(2*x**2) - a**2*d/x + a**2*e*log(x) + a*x**3*(a*h + 2*b*e
)/3 + a*(a*g + 2*b*d)*Integral(x, x) + a*(a*f + 2*b*c)*Integral(f
, x)/f + b**2*f*x**7/7 + b**2*g*x**8/8 + b**2*h*x**9/9 + b*x**6*(
2*a*h + b*e)/6 + b*x**5*(2*a*g + b*d)/5 + b*x**4*(2*a*f + b*c)/4
```

Mathematica [A] time = 0.185175, size = 127, normalized size = 0.86

$$\frac{a^2\left(-3c-6dx+x^3\left(6f+3gx+2hx^2\right)\right)}{6x^2}+a^2e\log(x)+\frac{1}{30}abx\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+\frac{b^2x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^2*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (a*b
*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/3
0 + (b^2*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g +
 8*h*x))))))/2520 + a^2*e*Log[x]
```

Maple [A] time = 0.01, size = 150, normalized size = 1.

$$\frac{b^2hx^9}{9}+\frac{b^2gx^8}{8}+\frac{b^2fx^7}{7}+\frac{x^6abh}{3}+\frac{b^2ex^6}{6}+\frac{2\,x^5abg}{5}+\frac{b^2dx^5}{5}+\frac{x^4abf}{2}+\frac{x^4b^2c}{4}+\frac{x^3a^2h}{3}+\frac{2\,abex^3}{3}+\frac{x^2a^2g}{2}+x^2abd+a^2fx+2\,xabc+a^2e\ln(x)-\frac{a^2c}{2\,x^2}-\frac{a^2d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] 1/9*b^2*h*x^9+1/8*b^2*g*x^8+1/7*b^2*f*x^7+1/3*x^6*a*b*h+1/6*b^2*e
*x^6+2/5*x^5*a*b*g+1/5*b^2*d*x^5+1/2*x^4*a*b*f+1/4*x^4*b^2*c+1/3*
x^3*a^2*h+2/3*a*b*e*x^3+1/2*x^2*a^2*g+x^2*a*b*d+a^2*f*x+2*x*a*b*c
+a^2*e*ln(x)-1/2*a^2*c/x^2-a^2*d/x
```

Maxima [A] time = 1.38427, size = 197, normalized size = 1.34

$$\frac{1}{9}\,b^2hx^9+\frac{1}{8}\,b^2gx^8+\frac{1}{7}\,b^2fx^7+\frac{1}{6}\left(b^2e+2\,abh\right)x^6+\frac{1}{5}\left(b^2d+2\,abg\right)x^5+\frac{1}{4}\left(b^2c+2\,abf\right)x^4+\frac{1}{3}\left(2\,abe+a^2h\right)x^3+a^2e\log(x)+\frac{1}{2}\left(2\,abd+a^2g\right)x^2+\left(2\,abc+a^2f\right)x-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^3,x, algorithm
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/6*(b^2*e + 2*a*
b*h)*x^6 + 1/5*(b^2*d + 2*a*b*g)*x^5 + 1/4*(b^2*c + 2*a*b*f)*x^4
+ 1/3*(2*a*b*e + a^2*h)*x^3 + a^2*e*log(x) + 1/2*(2*a*b*d + a^2*g
)*x^2 + (2*a*b*c + a^2*f)*x - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

Fricas [A] time = 0.25107, size = 207, normalized size = 1.41

$$\frac{280\,b^2hx^{11}+315\,b^2gx^{10}+360\,b^2fx^9+420\left(b^2e+2\,abh\right)x^8+504\left(b^2d+2\,abg\right)x^7+630\left(b^2c+2\,abf\right)x^6+840\left(2\,abe+a^2h\right.}{2520\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^3,x, algorithm
```

```
[Out] 1/2520*(280*b^2*h*x^11 + 315*b^2*g*x^10 + 360*b^2*f*x^9 + 420*(b^
2*e + 2*a*b*h)*x^8 + 504*(b^2*d + 2*a*b*g)*x^7 + 630*(b^2*c + 2*a
*b*f)*x^6 + 840*(2*a*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2*log(x) + 1
260*(2*a*b*d + a^2*g)*x^4 - 2520*a^2*d*x + 2520*(2*a*b*c + a^2*f)
*x^3 - 1260*a^2*c)/x^2
```

Sympy [A] time = 1.11201, size = 156, normalized size = 1.06

$$a^2e\log(x)+\frac{b^2fx^7}{7}+\frac{b^2gx^8}{8}+\frac{b^2hx^9}{9}+x^6\left(\frac{abh}{3}+\frac{b^2e}{6}\right)+x^5\left(\frac{2abg}{5}+\frac{b^2d}{5}\right)+x^4\left(\frac{abf}{2}+\frac{b^2c}{4}\right)+x^3\left(\frac{a^2h}{3}+\frac{2abe}{3}\right)+x^2\left(\frac{a^2g}{2}+abd\right)+x\left(a^2f+2abc\right)-\frac{a^2c+2a^2dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**2*e*log(x) + b**2*f*x**7/7 + b**2*g*x**8/8 + b**2*h*x**9/9 + x
**6*(a*b*h/3 + b**2*e/6) + x**5*(2*a*b*g/5 + b**2*d/5) + x**4*(a*
b*f/2 + b**2*c/4) + x**3*(a**2*h/3 + 2*a*b*e/3) + x**2*(a**2*g/2
+ a*b*d) + x*(a**2*f + 2*a*b*c) - (a**2*c + 2*a**2*d*x)/(2*x**2)
```

GIAC/XCAS [A] time = 0.211995, size = 207, normalized size = 1.41

$$\frac{1}{9}b^2hx^9+\frac{1}{8}b^2gx^8+\frac{1}{7}b^2fx^7+\frac{1}{3}abhx^6+\frac{1}{6}b^2x^6e+\frac{1}{5}b^2dx^5+\frac{2}{5}abgx^5+\frac{1}{4}b^2cx^4+\frac{1}{2}abfx^4+\frac{1}{3}a^2hx^3+\frac{2}{3}abx^3e+abdx^2+\frac{1}{2}a^2gx^2+2\,abcx+a^2fx+a^2e\ln(|x|)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^3,x, algorithm
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/3*a*b*h*x^6 + 1
/6*b^2*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*b*g*x^5 + 1/4*b^2*c*x^4 + 1/
2*a*b*f*x^4 + 1/3*a^2*h*x^3 + 2/3*a*b*x^3*e + a*b*d*x^2 + 1/2*a^2
*g*x^2 + 2*a*b*c*x + a^2*f*x + a^2*e*ln(abs(x)) - 1/2*(2*a^2*d*x
+ a^2*c)/x^2
```

3.379 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

Optimal. Leaf size=152

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+\frac{1}{3}bx^3(2af+bc)+a\log(x)(af+2bc)+\frac{1}{4}bx^4(2ag+bd)$$
$$+ax(ag+2bd)+\frac{1}{5}bx^5(2ah+be)+\frac{1}{2}ax^2(ah+2be)+\frac{1}{6}b^2fx^6+\frac{1}{7}b^2gx^7+\frac{1}{8}b^2hx^8$$

```
[Out] -(a^2*c)/(3*x^3) - (a^2*d)/(2*x^2) - (a^2*e)/x + a*(2*b*d + a*g)*
x + (a*(2*b*e + a*h)*x^2)/2 + (b*(b*c + 2*a*f)*x^3)/3 + (b*(b*d +
 2*a*g)*x^4)/4 + (b*(b*e + 2*a*h)*x^5)/5 + (b^2*f*x^6)/6 + (b^2*g
*x^7)/7 + (b^2*h*x^8)/8 + a*(2*b*c + a*f)*Log[x]
```

Rubi [A] time = 0.307914, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+\frac{1}{3}bx^3(2af+bc)+a\log(x)(af+2bc)+\frac{1}{4}bx^4(2ag+bd)$$
$$+ax(ag+2bd)+\frac{1}{5}bx^5(2ah+be)+\frac{1}{2}ax^2(ah+2be)+\frac{1}{6}b^2fx^6+\frac{1}{7}b^2gx^7+\frac{1}{8}b^2hx^8$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^2*c)/(3*x^3) - (a^2*d)/(2*x^2) - (a^2*e)/x + a*(2*b*d + a*g)*
x + (a*(2*b*e + a*h)*x^2)/2 + (b*(b*c + 2*a*f)*x^3)/3 + (b*(b*d +
 2*a*g)*x^4)/4 + (b*(b*e + 2*a*h)*x^5)/5 + (b^2*f*x^6)/6 + (b^2*g
*x^7)/7 + (b^2*h*x^8)/8 + a*(2*b*c + a*f)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+a(af+2bc)\log(x)+a(ah+2be)\int x\,dx+\frac{a(ag+2bd)\int g\,dx}{g}$$
$$+\frac{b^2fx^6}{6}+\frac{b^2gx^7}{7}+\frac{b^2hx^8}{8}+\frac{bx^5(2ah+be)}{5}+\frac{bx^4(2ag+bd)}{4}+\frac{bx^3(2af+bc)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] -a**2*c/(3*x**3) - a**2*d/(2*x**2) - a**2*e/x + a*(a*f + 2*b*c)*l
og(x) + a*(a*h + 2*b*e)*Integral(x, x) + a*(a*g + 2*b*d)*Integral
(g, x)/g + b**2*f*x**6/6 + b**2*g*x**7/7 + b**2*h*x**8/8 + b*x**5
*(2*a*h + b*e)/5 + b*x**4*(2*a*g + b*d)/4 + b*x**3*(2*a*f + b*c)/
3
```

Mathematica [A] time = 0.183313, size = 123, normalized size = 0.81

$$-\frac{a^2\left(2c+3x\left(d+2ex+x^3(-(2g+hx))\right)\right)}{6x^3}+a\log(x)(af+2bc)$$
$$+\frac{1}{30}abx\left(60d+x\left(30e+x\left(20f+15gx+12hx^2\right)\right)\right)$$
$$+\frac{1}{840}b^2x^3\left(280c+x\left(210d+x\left(168e+140fx+120gx^2+105hx^3\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^2*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/(6*x^3) + (a*b*x
*(60*d + x*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/30 + (b^2*x^3*
(280*c + x*(210*d + x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3)))
)/840 + a*(2*b*c + a*f)*Log[x]
```

Maple [A] time = 0.011, size = 149, normalized size = 1.

$$\frac{b^2hx^8}{8}+\frac{b^2gx^7}{7}+\frac{b^2fx^6}{6}+\frac{2\,x^5abh}{5}+\frac{x^5b^2e}{5}+\frac{x^4abg}{2}+\frac{b^2dx^4}{4}+\frac{2\,x^3abf}{3}+\frac{x^3b^2c}{3}+\frac{x^2a^2h}{2}+x^2abe+xa^2g+2\,xabd+\ln\left(x\right)a^2f+2\,\ln\left(x\right)abc-\frac{a^2c}{3\,x^3}-\frac{a^2d}{2\,x^2}-\frac{ea^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] 1/8*b^2*h*x^8+1/7*b^2*g*x^7+1/6*b^2*f*x^6+2/5*x^5*a*b*h+1/5*x^5*b
^2*e+1/2*x^4*a*b*g+1/4*b^2*d*x^4+2/3*x^3*a*b*f+1/3*x^3*b^2*c+1/2*
x^2*a^2*h+x^2*a*b*e+x*a^2*g+2*x*a*b*d+ln(x)*a^2*f+2*ln(x)*a*b*c-1
/3*a^2*c/x^3-1/2*a^2*d/x^2-a^2*e/x
```

Maxima [A] time = 7.45838, size = 198, normalized size = 1.3

$$\frac{1}{8}\,b^2hx^8+\frac{1}{7}\,b^2gx^7+\frac{1}{6}\,b^2fx^6+\frac{1}{5}\,\left(b^2e+2\,abh\right)x^5+\frac{1}{4}\,\left(b^2d+2\,abg\right)x^4+\frac{1}{3}\,\left(b^2c+2\,abf\right)x^3+\frac{1}{2}\,\left(2\,abe+a^2h\right)x^2+\left(2\,abd+a^2g\right)x+\left(2\,abc+a^2f\right)\,\log\left(x\right)-\frac{6\,a^2ex^2+3\,a^2dx+2\,a^2c}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^4,x, algorithm
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 1/5*(b^2*e + 2*a*
b*h)*x^5 + 1/4*(b^2*d + 2*a*b*g)*x^4 + 1/3*(b^2*c + 2*a*b*f)*x^3
+ 1/2*(2*a*b*e + a^2*h)*x^2 + (2*a*b*d + a^2*g)*x + (2*a*b*c + a^
2*f)*log(x) - 1/6*(6*a^2*e*x^2 + 3*a^2*d*x + 2*a^2*c)/x^3
```

Fricas [A] time = 0.249359, size = 207, normalized size = 1.36

$$\frac{105\,b^2hx^{11}+120\,b^2gx^{10}+140\,b^2fx^9+168\,\left(b^2e+2\,abh\right)x^8+210\,\left(b^2d+2\,abg\right)x^7+280\,\left(b^2c+2\,abf\right)x^6+420\,(2\,abe+a^2h}{840\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^4,x, algorithm
```

```
[Out] 1/840*(105*b^2*h*x^11 + 120*b^2*g*x^10 + 140*b^2*f*x^9 + 168*(b^2
*e + 2*a*b*h)*x^8 + 210*(b^2*d + 2*a*b*g)*x^7 + 280*(b^2*c + 2*a*
b*f)*x^6 + 420*(2*a*b*e + a^2*h)*x^5 - 840*a^2*e*x^2 + 840*(2*a*b
*d + a^2*g)*x^4 + 840*(2*a*b*c + a^2*f)*x^3*log(x) - 420*a^2*d*x
- 280*a^2*c)/x^3
```

Sympy [A] time = 1.86847, size = 156, normalized size = 1.03

$$a\left(af+2bc\right)\log\left(x\right)+\frac{b^2fx^6}{6}+\frac{b^2gx^7}{7}+\frac{b^2hx^8}{8}+x^5\left(\frac{2abh}{5}+\frac{b^2e}{5}\right)+x^4\left(\frac{abg}{2}+\frac{b^2d}{4}\right)+x^3\left(\frac{2abf}{3}+\frac{b^2c}{3}\right)+x^2\left(\frac{a^2h}{2}+abe\right)+x\left(a^2g+2abd\right)-\frac{2a^2c+3a^2dx+6a^2ex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a*(a*f + 2*b*c)*log(x) + b**2*f*x**6/6 + b**2*g*x**7/7 + b**2*h*x
**8/8 + x**5*(2*a*b*h/5 + b**2*e/5) + x**4*(a*b*g/2 + b**2*d/4) +
 x**3*(2*a*b*f/3 + b**2*c/3) + x**2*(a**2*h/2 + a*b*e) + x*(a**2*
g + 2*a*b*d) - (2*a**2*c + 3*a**2*d*x + 6*a**2*e*x**2)/(6*x**3)
```

GIAC/XCAS [A] time = 0.219669, size = 207, normalized size = 1.36

$$\frac{1}{8}\,b^2hx^8+\frac{1}{7}\,b^2gx^7+\frac{1}{6}\,b^2fx^6+\frac{2}{5}\,abhx^5+\frac{1}{5}\,b^2x^5e+\frac{1}{4}\,b^2dx^4+\frac{1}{2}\,abgx^4+\frac{1}{3}\,b^2cx^3+\frac{2}{3}\,abfx^3+\frac{1}{2}\,a^2hx^2+abx^2e+2\,abdx+a^2gx+\left(2\,abc+a^2f\right)\ln\left(|x|\right)-\frac{6\,a^2x^2e+3\,a^2dx+2\,a^2c}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^4,x, algorithm
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 2/5*a*b*h*x^5 + 1
/5*b^2*x^5*e + 1/4*b^2*d*x^4 + 1/2*a*b*g*x^4 + 1/3*b^2*c*x^3 + 2/
3*a*b*f*x^3 + 1/2*a^2*h*x^2 + a*b*x^2*e + 2*a*b*d*x + a^2*g*x + (
2*a*b*c + a^2*f)*ln(abs(x)) - 1/6*(6*a^2*x^2*e + 3*a^2*d*x + 2*a^
2*c)/x^3
```

3.380 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

Optimal. Leaf size=152

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+\frac{1}{2}bx^2(2af+bc)-\frac{a(af+2bc)}{x}+\frac{1}{3}bx^3(2ag+bd)+a\log(x)(ag+2bd)+\frac{1}{4}bx^4(2ah+be)+ax(ah+2be)+\frac{1}{5}b^2fx^5+\frac{1}{6}b^2gx^6+\frac{1}{7}b^2hx^7$$

```
[Out] -(a^2*c)/(4*x^4) - (a^2*d)/(3*x^3) - (a^2*e)/(2*x^2) - (a*(2*b*c
+ a*f))/x + a*(2*b*e + a*h)*x + (b*(b*c + 2*a*f)*x^2)/2 + (b*(b*d
 + 2*a*g)*x^3)/3 + (b*(b*e + 2*a*h)*x^4)/4 + (b^2*f*x^5)/5 + (b^2
*g*x^6)/6 + (b^2*h*x^7)/7 + a*(2*b*d + a*g)*Log[x]
```

Rubi [A] time = 0.304658, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+\frac{1}{2}bx^2(2af+bc)-\frac{a(af+2bc)}{x}+\frac{1}{3}bx^3(2ag+bd)+a\log(x)(ag+2bd)+\frac{1}{4}bx^4(2ah+be)+ax(ah+2be)+\frac{1}{5}b^2fx^5+\frac{1}{6}b^2gx^6+\frac{1}{7}b^2hx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -(a^2*c)/(4*x^4) - (a^2*d)/(3*x^3) - (a^2*e)/(2*x^2) - (a*(2*b*c
+ a*f))/x + a*(2*b*e + a*h)*x + (b*(b*c + 2*a*f)*x^2)/2 + (b*(b*d
 + 2*a*g)*x^3)/3 + (b*(b*e + 2*a*h)*x^4)/4 + (b^2*f*x^5)/5 + (b^2
*g*x^6)/6 + (b^2*h*x^7)/7 + a*(2*b*d + a*g)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+a(ag+2bd)\log(x)-\frac{a(af+2bc)}{x}+\frac{a(ah+2be)\int h\,dx}{h}+\frac{b^2fx^5}{5}+\frac{b^2gx^6}{6}+\frac{b^2hx^7}{7}+\frac{bx^4(2ah+be)}{4}+\frac{bx^3(2ag+bd)}{3}+b(2af+bc)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] -a**2*c/(4*x**4) - a**2*d/(3*x**3) - a**2*e/(2*x**2) + a*(a*g + 2
*b*d)*log(x) - a*(a*f + 2*b*c)/x + a*(a*h + 2*b*e)*Integral(h, x)
/h + b**2*f*x**5/5 + b**2*g*x**6/6 + b**2*h*x**7/7 + b*x**4*(2*a*
h + b*e)/4 + b*x**3*(2*a*g + b*d)/3 + b*(2*a*f + b*c)*Integral(x,
 x)
```

Mathematica [A] time = 0.176475, size = 125, normalized size = 0.82

$$-\frac{a^2\left(3c+4dx+6x^2\left(e+2fx-2hx^3\right)\right)}{12x^4}-\frac{2abc}{x}+a\log(x)(ag+2bd)+\frac{1}{6}abx(12e+x(6f+x(4g+3hx)))+\frac{1}{420}b^2x^2\left(210c+x\left(140d+x\left(105e+84fx+70gx^2+60hx^3\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] (-2*a*b*c)/x - (a^2*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/
(12*x^4) + (a*b*x*(12*e + x*(6*f + x*(4*g + 3*h*x))))/6 + (b^2*x^
2*(210*c + x*(140*d + x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))
/420 + a*(2*b*d + a*g)*Log[x]
```

Maple [A] time = 0.011, size = 149, normalized size = 1.

$$\frac{b^2hx^7}{7}+\frac{b^2gx^6}{6}+\frac{b^2fx^5}{5}+\frac{x^4abh}{2}+\frac{x^4b^2e}{4}+\frac{2\,x^3abg}{3}+\frac{b^2dx^3}{3}+x^2abf+\frac{x^2b^2c}{2}+xa^2h+2\,abex+\ln(x)\,a^2g+2\,\ln(x)\,abd-\frac{a^2c}{4\,x^4}-\frac{a^2d}{3\,x^3}-\frac{ea^2}{2\,x^2}-\frac{a^2f}{x}-2\,\frac{abc}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] 1/7*b^2*h*x^7+1/6*b^2*g*x^6+1/5*b^2*f*x^5+1/2*x^4*a*b*h+1/4*x^4*b
^2*e+2/3*x^3*a*b*g+1/3*b^2*d*x^3+x^2*a*b*f+1/2*x^2*b^2*c+x*a^2*h+
2*a*b*e*x+ln(x)*a^2*g+2*ln(x)*a*b*d-1/4*a^2*c/x^4-1/3*a^2*d/x^3-1
/2*a^2*e/x^2-a^2/x*f-2*a/x*b*c
```

Maxima [A] time = 5.93386, size = 198, normalized size = 1.3

$$\frac{1}{7}\,b^2hx^7+\frac{1}{6}\,b^2gx^6+\frac{1}{5}\,b^2fx^5+\frac{1}{4}\,(b^2e+2\,abh)\,x^4+\frac{1}{3}\,(b^2d+2\,abg)\,x^3+\frac{1}{2}\,(b^2c+2\,abf)\,x^2+(2\,abe+a^2h)\,x+(2\,abd+a^2g)\,\log(x)-\frac{6\,a^2ex^2+4\,a^2dx+12\,(2\,abc+a^2f)\,x^3+3\,a^2c}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^5,x, algorithm
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/4*(b^2*e + 2*a*
b*h)*x^4 + 1/3*(b^2*d + 2*a*b*g)*x^3 + 1/2*(b^2*c + 2*a*b*f)*x^2
+ (2*a*b*e + a^2*h)*x + (2*a*b*d + a^2*g)*log(x) - 1/12*(6*a^2*e*
x^2 + 4*a^2*d*x + 12*(2*a*b*c + a^2*f)*x^3 + 3*a^2*c)/x^4
```

Fricas [A] time = 0.247182, size = 207, normalized size = 1.36

$$\frac{60\,b^2hx^{11}+70\,b^2gx^{10}+84\,b^2fx^9+105\,(b^2e+2\,abh)\,x^8+140\,(b^2d+2\,abg)\,x^7+210\,(b^2c+2\,abf)\,x^6+420\,(2\,abe+a^2h)\,x^5}{420\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^5,x, algorithm
```

```
[Out] 1/420*(60*b^2*h*x^11 + 70*b^2*g*x^10 + 84*b^2*f*x^9 + 105*(b^2*e
+ 2*a*b*h)*x^8 + 140*(b^2*d + 2*a*b*g)*x^7 + 210*(b^2*c + 2*a*b*f
)*x^6 + 420*(2*a*b*e + a^2*h)*x^5 + 420*(2*a*b*d + a^2*g)*x^4*log
(x) - 210*a^2*e*x^2 - 140*a^2*d*x - 420*(2*a*b*c + a^2*f)*x^3 - 1
05*a^2*c)/x^4
```

Sympy [A] time = 6.23856, size = 155, normalized size = 1.02

$$a\,(ag+2bd)\log(x)+\frac{b^2fx^5}{5}+\frac{b^2gx^6}{6}+\frac{b^2hx^7}{7}+x^4\left(\frac{abh}{2}+\frac{b^2e}{4}\right)+x^3\left(\frac{2abg}{3}+\frac{b^2d}{3}\right)+x^2\left(abf+\frac{b^2c}{2}\right)+x\,(a^2h+2abe)-\frac{3a^2c+4a^2dx+6a^2ex^2+x^3\,(12a^2f+24abc)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a*(a*g + 2*b*d)*log(x) + b**2*f*x**5/5 + b**2*g*x**6/6 + b**2*h*x
**7/7 + x**4*(a*b*h/2 + b**2*e/4) + x**3*(2*a*b*g/3 + b**2*d/3) +
 x**2*(a*b*f + b**2*c/2) + x*(a**2*h + 2*a*b*e) - (3*a**2*c + 4*a
**2*d*x + 6*a**2*e*x**2 + x**3*(12*a**2*f + 24*a*b*c))/(12*x**4)
```

GIAC/XCAS [A] time = 0.219114, size = 205, normalized size = 1.35

$$\frac{1}{7}b^2hx^7 + \frac{1}{6}b^2gx^6 + \frac{1}{5}b^2fx^5 + \frac{1}{2}abhx^4 + \frac{1}{4}b^2x^4e + \frac{1}{3}b^2dx^3 + \frac{2}{3}abgx^3 + \frac{1}{2}b^2cx^2 + abfx^2 + a^2hx + 2\,abxe + \left(2\,abd + a^2g\right)\ln\left(|x|\right) - \frac{6\,a^2x^2e + 4\,a^2dx + 12\,\left(2\,abc + a^2f\right)x^3 + 3\,a^2c}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^2/x^5,x, algorithm
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/2*a*b*h*x^4 + 1
/4*b^2*x^4*e + 1/3*b^2*d*x^3 + 2/3*a*b*g*x^3 + 1/2*b^2*c*x^2 + a*
b*f*x^2 + a^2*h*x + 2*a*b*x*e + (2*a*b*d + a^2*g)*ln(abs(x)) - 1/
12*(6*a^2*x^2*e + 4*a^2*d*x + 12*(2*a*b*c + a^2*f)*x^3 + 3*a^2*c)
/x^4
```

3.381 $\int x^4 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$

Optimal. Leaf size=223

$$\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x^{15}(3ag+bd)+\frac{1}{16}b^2x^{16}(3ah+be)+\frac{3}{11}abx^{11}(af+bc)+\frac{1}{4}abx^{12}(ag+bd)+\frac{3}{13}abx^{13}(ah+be)+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f
)*x^8)/8 + (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/1
0 + (3*a*b*(b*c + a*f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a
*b*(b*e + a*h)*x^13)/13 + (b^2*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d
 + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*x^16)/16 + (b^3*f*x^17)/1
7 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Rubi [A] time = 0.591195, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026

$$\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x^{15}(3ag+bd)+\frac{1}{16}b^2x^{16}(3ah+be)+\frac{3}{11}abx^{11}(af+bc)+\frac{1}{4}abx^{12}(ag+bd)+\frac{3}{13}abx^{13}(ah+be)+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f
)*x^8)/8 + (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/1
0 + (3*a*b*(b*c + a*f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a
*b*(b*e + a*h)*x^13)/13 + (b^2*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d
 + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*x^16)/16 + (b^3*f*x^17)/1
7 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Rubi in Sympy [A] time = 58.572, size = 211, normalized size = 0.95

$$\frac{a^3cx^5}{5}+\frac{a^3dx^6}{6}+\frac{a^3ex^7}{7}+\frac{a^2x^{10}\left(ah+3be\right)}{10}+\frac{a^2x^9\left(ag+3bd\right)}{9}+\frac{a^2x^8\left(af+3bc\right)}{8}+\frac{3abx^{13}\left(ah+be\right)}{13}+\frac{abx^{12}\left(ag+bd\right)}{4}+\frac{3abx^{11}\left(af+bc\right)}{11}+\frac{b^3fx^{17}}{17}+\frac{b^3gx^{18}}{18}+\frac{b^3hx^{19}}{19}+\frac{b^2x^{16}\left(3ah+be\right)}{16}+\frac{b^2x^{15}\left(3ag+bd\right)}{15}+\frac{b^2x^{14}\left(3af+bc\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**5/5 + a**3*d*x**6/6 + a**3*e*x**7/7 + a**2*x**10*(a*h +
 3*b*e)/10 + a**2*x**9*(a*g + 3*b*d)/9 + a**2*x**8*(a*f + 3*b*c)/
8 + 3*a*b*x**13*(a*h + b*e)/13 + a*b*x**12*(a*g + b*d)/4 + 3*a*b*
x**11*(a*f + b*c)/11 + b**3*f*x**17/17 + b**3*g*x**18/18 + b**3*h
*x**19/19 + b**2*x**16*(3*a*h + b*e)/16 + b**2*x**15*(3*a*g + b*d
)/15 + b**2*x**14*(3*a*f + b*c)/14
```

Mathematica [A] time = 0.111655, size = 223, normalized size = 1.

$$\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x^{15}(3ag+bd)+\frac{1}{16}b^2x^{16}(3ah+be)+\frac{3}{11}abx^{11}(af+bc)+\frac{1}{4}abx^{12}(ag+bd)+\frac{3}{13}abx^{13}(ah+be)+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f
)*x^8)/8 + (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/1
0 + (3*a*b*(b*c + a*f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a
*b*(b*e + a*h)*x^13)/13 + (b^2*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d
 + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*x^16)/16 + (b^3*f*x^17)/1
7 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Maple [A] time = 0.003, size = 224, normalized size = 1.

$$\frac{b^3hx^{19}}{19}+\frac{b^3gx^{18}}{18}+\frac{b^3fx^{17}}{17}+\frac{(3\,ab^2h+b^3e)\,x^{16}}{16}+\frac{(3\,ab^2g+b^3d)\,x^{15}}{15}+\frac{(3\,ab^2f+b^3c)\,x^{14}}{14}+\frac{(3\,a^2bh+3\,aeb^2)\,x^{13}}{13}+\frac{(3\,a^2bg+3\,ab^2d)\,x^{12}}{12}+\frac{(3\,a^2bf+3\,acb^2)\,x^{11}}{11}+\frac{(a^3h+3\,a^2be)\,x^{10}}{10}+\frac{(a^3g+3\,a^2bd)\,x^9}{9}+\frac{(a^3f+3\,a^2bc)\,x^8}{8}+\frac{a^3ex^7}{7}+\frac{a^3dx^6}{6}+\frac{a^3cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/19*b^3*h*x^19+1/18*b^3*g*x^18+1/17*b^3*f*x^17+1/16*(3*a*b^2*h+b
^3*e)*x^16+1/15*(3*a*b^2*g+b^3*d)*x^15+1/14*(3*a*b^2*f+b^3*c)*x^1
4+1/13*(3*a^2*b*h+3*a*b^2*e)*x^13+1/12*(3*a^2*b*g+3*a*b^2*d)*x^12
+1/11*(3*a^2*b*f+3*a*b^2*c)*x^11+1/10*(a^3*h+3*a^2*b*e)*x^10+1/9*
(a^3*g+3*a^2*b*d)*x^9+1/8*(a^3*f+3*a^2*b*c)*x^8+1/7*a^3*e*x^7+1/6
*a^3*d*x^6+1/5*a^3*c*x^5
```

Maxima [A] time = 1.43662, size = 293, normalized size = 1.31

$$\frac{1}{19}\,b^3hx^{19}+\frac{1}{18}\,b^3gx^{18}+\frac{1}{17}\,b^3fx^{17}+\frac{1}{16}\,(b^3e+3\,ab^2h)\,x^{16}+\frac{1}{15}\,(b^3d+3\,ab^2g)\,x^{15}+\frac{1}{14}\,(b^3c+3\,ab^2f)\,x^{14}+\frac{3}{13}\,(ab^2e+a^2bh)\,x^{13}+\frac{1}{4}\,(ab^2d+a^2bg)\,x^{12}+\frac{3}{11}\,(ab^2c+a^2bf)\,x^{11}+\frac{1}{7}\,a^3ex^7+\frac{1}{10}\,(3\,a^2be+a^3h)\,x^{10}+\frac{1}{6}\,a^3dx^6+\frac{1}{9}\,(3\,a^2bd+a^3g)\,x^9+\frac{1}{5}\,a^3cx^5+\frac{1}{8}\,(3\,a^2bc+a^3f)\,x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x^4,x, algorithm
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*(b^3*e
 + 3*a*b^2*h)*x^16 + 1/15*(b^3*d + 3*a*b^2*g)*x^15 + 1/14*(b^3*c
+ 3*a*b^2*f)*x^14 + 3/13*(a*b^2*e + a^2*b*h)*x^13 + 1/4*(a*b^2*d
+ a^2*b*g)*x^12 + 3/11*(a*b^2*c + a^2*b*f)*x^11 + 1/7*a^3*e*x^7 +
 1/10*(3*a^2*b*e + a^3*h)*x^10 + 1/6*a^3*d*x^6 + 1/9*(3*a^2*b*d +
 a^3*g)*x^9 + 1/5*a^3*c*x^5 + 1/8*(3*a^2*b*c + a^3*f)*x^8
```

Fricas [A] time = 0.223825, size = 1, normalized size = 0.

$$\frac{1}{19}x^{19}hb^3 + \frac{1}{18}x^{18}gb^3 + \frac{1}{17}x^{17}fb^3 + \frac{1}{16}x^{16}eb^3 + \frac{3}{16}x^{16}hb^2a + \frac{1}{15}x^{15}db^3 + \frac{1}{5}x^{15}gb^2a + \frac{1}{14}x^{14}cb^3 + \frac{3}{14}x^{14}fb^2a + \frac{3}{13}x^{13}eb^2a + \frac{3}{13}x^{13}hba^2 + \frac{1}{4}x^{12}db^2a + \frac{1}{4}x^{12}gba^2 + \frac{3}{11}x^{11}cb^2a + \frac{3}{11}x^{11}fba^2 + \frac{3}{10}x^{10}eba^2 + \frac{1}{10}x^{10}ha^3 + \frac{1}{3}x^9dba^2 + \frac{1}{9}x^9ga^3 + \frac{3}{8}x^8cba^2 + \frac{1}{8}x^8fa^3 + \frac{1}{7}x^7ea^3 + \frac{1}{6}x^6da^3 + \frac{1}{5}x^5ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x^4,x, algorithm
```

```
[Out] 1/19*x^19*h*b^3 + 1/18*x^18*g*b^3 + 1/17*x^17*f*b^3 + 1/16*x^16*e
*b^3 + 3/16*x^16*h*b^2*a + 1/15*x^15*d*b^3 + 1/5*x^15*g*b^2*a + 1
/14*x^14*c*b^3 + 3/14*x^14*f*b^2*a + 3/13*x^13*e*b^2*a + 3/13*x^1
3*h*b*a^2 + 1/4*x^12*d*b^2*a + 1/4*x^12*g*b*a^2 + 3/11*x^11*c*b^2
*a + 3/11*x^11*f*b*a^2 + 3/10*x^10*e*b*a^2 + 1/10*x^10*h*a^3 + 1/
3*x^9*d*b*a^2 + 1/9*x^9*g*a^3 + 3/8*x^8*c*b*a^2 + 1/8*x^8*f*a^3 +
 1/7*x^7*e*a^3 + 1/6*x^6*d*a^3 + 1/5*x^5*c*a^3
```

Sympy [A] time = 0.121624, size = 246, normalized size = 1.1

$$\frac{a^3cx^5}{5} + \frac{a^3dx^6}{6} + \frac{a^3ex^7}{7} + \frac{b^3fx^{17}}{17} + \frac{b^3gx^{18}}{18} + \frac{b^3hx^{19}}{19} + x^{16}\left(\frac{3ab^2h}{16} + \frac{b^3e}{16}\right) + x^{15}\left(\frac{ab^2g}{5} + \frac{b^3d}{15}\right) + x^{14}\left(\frac{3ab^2f}{14} + \frac{b^3c}{14}\right) + x^{13}\left(\frac{3a^2bh}{13} + \frac{3ab^2e}{13}\right) + x^{12}\left(\frac{a^2bg}{4} + \frac{ab^2d}{4}\right) + x^{11}\left(\frac{3a^2bf}{11} + \frac{3ab^2c}{11}\right) + x^{10}\left(\frac{a^3h}{10} + \frac{3a^2be}{10}\right) + x^9\left(\frac{a^3g}{9} + \frac{a^2bd}{3}\right) + x^8\left(\frac{a^3f}{8} + \frac{3a^2bc}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**5/5 + a**3*d*x**6/6 + a**3*e*x**7/7 + b**3*f*x**17/17 +
 b**3*g*x**18/18 + b**3*h*x**19/19 + x**16*(3*a*b**2*h/16 + b**3*
e/16) + x**15*(a*b**2*g/5 + b**3*d/15) + x**14*(3*a*b**2*f/14 + b
**3*c/14) + x**13*(3*a**2*b*h/13 + 3*a*b**2*e/13) + x**12*(a**2*b
*g/4 + a*b**2*d/4) + x**11*(3*a**2*b*f/11 + 3*a*b**2*c/11) + x**1
0*(a**3*h/10 + 3*a**2*b*e/10) + x**9*(a**3*g/9 + a**2*b*d/3) + x*
*8*(a**3*f/8 + 3*a**2*b*c/8)
```

GIAC/XCAS [A] time = 0.214309, size = 315, normalized size = 1.41

$$\frac{1}{19}b^3hx^{19} + \frac{1}{18}b^3gx^{18} + \frac{1}{17}b^3fx^{17} + \frac{3}{16}ab^2hx^{16} + \frac{1}{16}b^3x^{16}e + \frac{1}{15}b^3dx^{15} + \frac{1}{5}ab^2gx^{15} + \frac{1}{14}b^3cx^{14} + \frac{3}{14}ab^2fx^{14} + \frac{3}{13}a^2bhx^{13} + \frac{3}{13}ab^2x^{13}e + \frac{1}{4}ab^2dx^{12} + \frac{1}{4}a^2bgx^{12} + \frac{3}{11}ab^2cx^{11} + \frac{3}{11}a^2bfx^{11} + \frac{1}{10}a^3hx^{10} + \frac{3}{10}a^2bx^{10}e + \frac{1}{3}a^2bdx^9 + \frac{1}{9}a^3gx^9 + \frac{3}{8}a^2bcx^8 + \frac{1}{8}a^3fx^8 + \frac{1}{7}a^3x^7e + \frac{1}{6}a^3dx^6 + \frac{1}{5}a^3cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x^4,x, algorithm
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 3/16*a*b^2*
h*x^16 + 1/16*b^3*x^16*e + 1/15*b^3*d*x^15 + 1/5*a*b^2*g*x^15 + 1
/14*b^3*c*x^14 + 3/14*a*b^2*f*x^14 + 3/13*a^2*b*h*x^13 + 3/13*a*b
^2*x^13*e + 1/4*a*b^2*d*x^12 + 1/4*a^2*b*g*x^12 + 3/11*a*b^2*c*x^
11 + 3/11*a^2*b*f*x^11 + 1/10*a^3*h*x^10 + 3/10*a^2*b*x^10*e + 1/
3*a^2*b*d*x^9 + 1/9*a^3*g*x^9 + 3/8*a^2*b*c*x^8 + 1/8*a^3*f*x^8 +
 1/7*a^3*x^7*e + 1/6*a^3*d*x^6 + 1/5*a^3*c*x^5
```

3.382 $\int x^3 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=223

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3ag+bd)+\frac{1}{15}b^2x^{15}(3ah+be)+\frac{3}{10}abx^{10}(af+bc)+\frac{3}{11}abx^{11}(ag+bd)+\frac{1}{4}abx^{12}(ah+be)+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}$$

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f
)*x^7)/7 + (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9
+ (3*a*b*(b*c + a*f)*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*
b*(b*e + a*h)*x^12)/4 + (b^2*(b*c + 3*a*f)*x^13)/13 + (b^2*(b*d +
 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^15)/15 + (b^3*f*x^16)/16
+ (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Rubi [A] time = 0.584532, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3ag+bd)+\frac{1}{15}b^2x^{15}(3ah+be)+\frac{3}{10}abx^{10}(af+bc)+\frac{3}{11}abx^{11}(ag+bd)+\frac{1}{4}abx^{12}(ah+be)+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f
)*x^7)/7 + (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9
+ (3*a*b*(b*c + a*f)*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*
b*(b*e + a*h)*x^12)/4 + (b^2*(b*c + 3*a*f)*x^13)/13 + (b^2*(b*d +
 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^15)/15 + (b^3*f*x^16)/16
+ (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Rubi in Sympy [A] time = 61.6473, size = 211, normalized size = 0.95

$$\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{a^2x^9\left(ah+3be\right)}{9}+\frac{a^2x^8\left(ag+3bd\right)}{8}+\frac{a^2x^7\left(af+3bc\right)}{7}+\frac{abx^{12}\left(ah+be\right)}{4}+\frac{3abx^{11}\left(ag+bd\right)}{11}+\frac{3abx^{10}\left(af+bc\right)}{10}+\frac{b^3fx^{16}}{16}+\frac{b^3gx^{17}}{17}+\frac{b^3hx^{18}}{18}+\frac{b^2x^{15}\left(3ah+be\right)}{15}+\frac{b^2x^{14}\left(3ag+bd\right)}{14}+\frac{b^2x^{13}\left(3af+bc\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + a**2*x**9*(a*h +
3*b*e)/9 + a**2*x**8*(a*g + 3*b*d)/8 + a**2*x**7*(a*f + 3*b*c)/7
+ a*b*x**12*(a*h + b*e)/4 + 3*a*b*x**11*(a*g + b*d)/11 + 3*a*b*x*
*10*(a*f + b*c)/10 + b**3*f*x**16/16 + b**3*g*x**17/17 + b**3*h*x
**18/18 + b**2*x**15*(3*a*h + b*e)/15 + b**2*x**14*(3*a*g + b*d)/
14 + b**2*x**13*(3*a*f + b*c)/13
```

Mathematica [A] time = 0.105024, size = 223, normalized size = 1.

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3ag+bd)+\frac{1}{15}b^2x^{15}(3ah+be)+\frac{3}{10}abx^{10}(af+bc)+\frac{3}{11}abx^{11}(ag+bd)+\frac{1}{4}abx^{12}(ah+be)+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f
)*x^7)/7 + (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9
+ (3*a*b*(b*c + a*f)*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*
b*(b*e + a*h)*x^12)/4 + (b^2*(b*c + 3*a*f)*x^13)/13 + (b^2*(b*d +
 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^15)/15 + (b^3*f*x^16)/16
+ (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Maple [A] time = 0., size = 224, normalized size = 1.

$$\frac{b^3hx^{18}}{18}+\frac{b^3gx^{17}}{17}+\frac{b^3fx^{16}}{16}+\frac{\left(3\,ab^2h+b^3e\right)x^{15}}{15}+\frac{\left(3\,ab^2g+b^3d\right)x^{14}}{14}+\frac{\left(3\,ab^2f+b^3c\right)x^{13}}{13}+\frac{\left(3\,a^2bh+3\,aeb^2\right)x^{12}}{12}+\frac{\left(3\,a^2bg+3\,ab^2d\right)x^{11}}{11}+\frac{\left(3\,a^2bf+3\,acb^2\right)x^{10}}{10}+\frac{\left(a^3h+3\,a^2be\right)x^9}{9}+\frac{\left(a^3g+3\,a^2bd\right)x^8}{8}+\frac{\left(a^3f+3\,a^2bc\right)x^7}{7}+\frac{a^3ex^6}{6}+\frac{a^3dx^5}{5}+\frac{a^3cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/18*b^3*h*x^18+1/17*b^3*g*x^17+1/16*b^3*f*x^16+1/15*(3*a*b^2*h+b
^3*e)*x^15+1/14*(3*a*b^2*g+b^3*d)*x^14+1/13*(3*a*b^2*f+b^3*c)*x^1
3+1/12*(3*a^2*b*h+3*a*b^2*e)*x^12+1/11*(3*a^2*b*g+3*a*b^2*d)*x^11
+1/10*(3*a^2*b*f+3*a*b^2*c)*x^10+1/9*(a^3*h+3*a^2*b*e)*x^9+1/8*(a
^3*g+3*a^2*b*d)*x^8+1/7*(a^3*f+3*a^2*b*c)*x^7+1/6*a^3*e*x^6+1/5*a
^3*d*x^5+1/4*a^3*c*x^4
```

Maxima [A] time = 1.41636, size = 293, normalized size = 1.31

$$\frac{1}{18}\,b^3hx^{18}+\frac{1}{17}\,b^3gx^{17}+\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,\left(b^3e+3\,ab^2h\right)x^{15}+\frac{1}{14}\,\left(b^3d+3\,ab^2g\right)x^{14}+\frac{1}{13}\,\left(b^3c+3\,ab^2f\right)x^{13}+\frac{1}{4}\,\left(ab^2e+a^2bh\right)x^{12}+\frac{3}{11}\,\left(ab^2d+a^2bg\right)x^{11}+\frac{3}{10}\,\left(ab^2c+a^2bf\right)x^{10}+\frac{1}{6}\,a^3ex^6+\frac{1}{9}\,\left(3\,a^2be+a^3h\right)x^9+\frac{1}{5}\,a^3dx^5+\frac{1}{8}\,\left(3\,a^2bd+a^3g\right)x^8+\frac{1}{4}\,a^3cx^4+\frac{1}{7}\,\left(3\,a^2bc+a^3f\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x^3,x, algorithm
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*(b^3*e
 + 3*a*b^2*h)*x^15 + 1/14*(b^3*d + 3*a*b^2*g)*x^14 + 1/13*(b^3*c
+ 3*a*b^2*f)*x^13 + 1/4*(a*b^2*e + a^2*b*h)*x^12 + 3/11*(a*b^2*d
+ a^2*b*g)*x^11 + 3/10*(a*b^2*c + a^2*b*f)*x^10 + 1/6*a^3*e*x^6 +
 1/9*(3*a^2*b*e + a^3*h)*x^9 + 1/5*a^3*d*x^5 + 1/8*(3*a^2*b*d + a
^3*g)*x^8 + 1/4*a^3*c*x^4 + 1/7*(3*a^2*b*c + a^3*f)*x^7
```

Fricas [A] time = 0.222007, size = 1, normalized size = 0.

$$\frac{1}{18}x^{18}hb^3 + \frac{1}{17}x^{17}gb^3 + \frac{1}{16}x^{16}fb^3 + \frac{1}{15}x^{15}eb^3 + \frac{1}{5}x^{15}hb^2a + \frac{1}{14}x^{14}db^3 + \frac{3}{14}x^{14}gb^2a + \frac{1}{13}x^{13}cb^3 + \frac{3}{13}x^{13}fb^2a + \frac{1}{4}x^{12}eb^2a + \frac{1}{4}x^{12}hba^2 + \frac{3}{11}x^{11}db^2a + \frac{3}{11}x^{11}gba^2 + \frac{3}{10}x^{10}cb^2a + \frac{3}{10}x^{10}fba^2 + \frac{1}{3}x^9eba^2 + \frac{1}{9}x^9ha^3 + \frac{3}{8}x^8dba^2 + \frac{1}{8}x^8ga^3 + \frac{3}{7}x^7cba^2 + \frac{1}{7}x^7fa^3 + \frac{1}{6}x^6ea^3 + \frac{1}{5}x^5da^3 + \frac{1}{4}x^4ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x^3,x, algorithm
```

```
[Out] 1/18*x^18*h*b^3 + 1/17*x^17*g*b^3 + 1/16*x^16*f*b^3 + 1/15*x^15*e
*b^3 + 1/5*x^15*h*b^2*a + 1/14*x^14*d*b^3 + 3/14*x^14*g*b^2*a + 1
/13*x^13*c*b^3 + 3/13*x^13*f*b^2*a + 1/4*x^12*e*b^2*a + 1/4*x^12*
h*b*a^2 + 3/11*x^11*d*b^2*a + 3/11*x^11*g*b*a^2 + 3/10*x^10*c*b^2
*a + 3/10*x^10*f*b*a^2 + 1/3*x^9*e*b*a^2 + 1/9*x^9*h*a^3 + 3/8*x^
8*d*b*a^2 + 1/8*x^8*g*a^3 + 3/7*x^7*c*b*a^2 + 1/7*x^7*f*a^3 + 1/6
*x^6*e*a^3 + 1/5*x^5*d*a^3 + 1/4*x^4*c*a^3
```

Sympy [A] time = 0.117227, size = 246, normalized size = 1.1

$$\frac{a^3cx^4}{4} + \frac{a^3dx^5}{5} + \frac{a^3ex^6}{6} + \frac{b^3fx^{16}}{16} + \frac{b^3gx^{17}}{17} + \frac{b^3hx^{18}}{18} + x^{15}\left(\frac{ab^2h}{5} + \frac{b^3e}{15}\right) + x^{14}\left(\frac{3ab^2g}{14} + \frac{b^3d}{14}\right) + x^{13}\left(\frac{3ab^2f}{13} + \frac{b^3c}{13}\right) + x^{12}\left(\frac{a^2bh}{4} + \frac{ab^2e}{4}\right) + x^{11}\left(\frac{3a^2bg}{11} + \frac{3ab^2d}{11}\right) + x^{10}\left(\frac{3a^2bf}{10} + \frac{3ab^2c}{10}\right) + x^9\left(\frac{a^3h}{9} + \frac{a^2be}{3}\right) + x^8\left(\frac{a^3g}{8} + \frac{3a^2bd}{8}\right) + x^7\left(\frac{a^3f}{7} + \frac{3a^2bc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + b**3*f*x**16/16 +
 b**3*g*x**17/17 + b**3*h*x**18/18 + x**15*(a*b**2*h/5 + b**3*e/1
5) + x**14*(3*a*b**2*g/14 + b**3*d/14) + x**13*(3*a*b**2*f/13 + b
**3*c/13) + x**12*(a**2*b*h/4 + a*b**2*e/4) + x**11*(3*a**2*b*g/1
1 + 3*a*b**2*d/11) + x**10*(3*a**2*b*f/10 + 3*a*b**2*c/10) + x**9
*(a**3*h/9 + a**2*b*e/3) + x**8*(a**3*g/8 + 3*a**2*b*d/8) + x**7*
(a**3*f/7 + 3*a**2*b*c/7)
```

GIAC/XCAS [A] time = 0.215923, size = 315, normalized size = 1.41

$$\frac{1}{18}b^3hx^{18} + \frac{1}{17}b^3gx^{17} + \frac{1}{16}b^3fx^{16} + \frac{1}{5}ab^2hx^{15} + \frac{1}{15}b^3x^{15}e + \frac{1}{14}b^3dx^{14} + \frac{3}{14}ab^2gx^{14} + \frac{1}{13}b^3cx^{13} + \frac{3}{13}ab^2fx^{13} + \frac{1}{4}a^2bhx^{12} + \frac{1}{4}ab^2x^{12}e + \frac{3}{11}ab^2dx^{11} + \frac{3}{11}a^2bgx^{11} + \frac{3}{10}ab^2cx^{10} + \frac{3}{10}a^2bfx^{10} + \frac{1}{9}a^3hx^9 + \frac{1}{3}a^2bx^9e + \frac{3}{8}a^2bdx^8 + \frac{1}{8}a^3gx^8 + \frac{3}{7}a^2bcx^7 + \frac{1}{7}a^3fx^7 + \frac{1}{6}a^3x^6e + \frac{1}{5}a^3dx^5 + \frac{1}{4}a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x^3,x, algorithm
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/5*a*b^2*h
*x^15 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 3/14*a*b^2*g*x^14 + 1
/13*b^3*c*x^13 + 3/13*a*b^2*f*x^13 + 1/4*a^2*b*h*x^12 + 1/4*a*b^2
*x^12*e + 3/11*a*b^2*d*x^11 + 3/11*a^2*b*g*x^11 + 3/10*a*b^2*c*x^
10 + 3/10*a^2*b*f*x^10 + 1/9*a^3*h*x^9 + 1/3*a^2*b*x^9*e + 3/8*a^
2*b*d*x^8 + 1/8*a^3*g*x^8 + 3/7*a^2*b*c*x^7 + 1/7*a^3*f*x^7 + 1/6
*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

3.383 $\int x^2 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$

Optimal. Leaf size=212

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^2bfx^9+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+be)+\frac{1}{4}ab^2fx^{12}+\frac{c\left(a+bx^3\right)^4}{12b}+\frac{3}{10}abx^{10}(ag+bd)+\frac{3}{11}abx^{11}(ah+be)+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}$$

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^3*f*x^6)/6 + (a^2*(3*b*d + a*g
)*x^7)/7 + (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(
b*d + a*g)*x^10)/10 + (3*a*b*(b*e + a*h)*x^11)/11 + (a*b^2*f*x^12
)/4 + (b^2*(b*d + 3*a*g)*x^13)/13 + (b^2*(b*e + 3*a*h)*x^14)/14 +
 (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x^17)/17 + (c*(a + b*
x^3)^4)/(12*b)
```

Rubi [A] time = 0.612957, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^2bfx^9+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+be)+\frac{1}{4}ab^2fx^{12}+\frac{c\left(a+bx^3\right)^4}{12b}+\frac{3}{10}abx^{10}(ag+bd)+\frac{3}{11}abx^{11}(ah+be)+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^3*f*x^6)/6 + (a^2*(3*b*d + a*g
)*x^7)/7 + (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(
b*d + a*g)*x^10)/10 + (3*a*b*(b*e + a*h)*x^11)/11 + (a*b^2*f*x^12
)/4 + (b^2*(b*d + 3*a*g)*x^13)/13 + (b^2*(b*e + 3*a*h)*x^14)/14 +
 (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x^17)/17 + (c*(a + b*
x^3)^4)/(12*b)
```

Rubi in Sympy [A] time = 79.9396, size = 199, normalized size = 0.94

$$\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{a^3fx^6}{6}+\frac{a^2bfx^9}{3}+\frac{a^2x^8\left(ah+3be\right)}{8}+\frac{a^2x^7\left(ag+3bd\right)}{7}+\frac{ab^2fx^{12}}{4}+\frac{3abx^{11}\left(ah+be\right)}{11}+\frac{3abx^{10}\left(ag+bd\right)}{10}+\frac{b^3fx^{15}}{15}+\frac{b^3gx^{16}}{16}+\frac{b^3hx^{17}}{17}+\frac{b^2x^{14}\left(3ah+be\right)}{14}+\frac{b^2x^{13}\left(3ag+bd\right)}{13}+\frac{c\left(a+bx^3\right)^4}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*d*x**4/4 + a**3*e*x**5/5 + a**3*f*x**6/6 + a**2*b*f*x**9/3 +
 a**2*x**8*(a*h + 3*b*e)/8 + a**2*x**7*(a*g + 3*b*d)/7 + a*b**2*f
*x**12/4 + 3*a*b*x**11*(a*h + b*e)/11 + 3*a*b*x**10*(a*g + b*d)/1
0 + b**3*f*x**15/15 + b**3*g*x**16/16 + b**3*h*x**17/17 + b**2*x*
*14*(3*a*h + b*e)/14 + b**2*x**13*(3*a*g + b*d)/13 + c*(a + b*x**
3)**4/(12*b)
```

Mathematica [A] time = 0.121052, size = 223, normalized size = 1.05

$$\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^2x^6(af+3bc)+\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{12}b^2x^{12}(3af+bc)+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+be)+\frac{1}{3}abx^9(af+bc)+\frac{3}{10}abx^{10}(ag+bd)+\frac{3}{11}abx^{11}(ah+be)+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*(3*b*c + a*f
)*x^6)/6 + (a^2*(3*b*d + a*g)*x^7)/7 + (a^2*(3*b*e + a*h)*x^8)/8
+ (a*b*(b*c + a*f)*x^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10 + (3*a*b*
(b*e + a*h)*x^11)/11 + (b^2*(b*c + 3*a*f)*x^12)/12 + (b^2*(b*d +
3*a*g)*x^13)/13 + (b^2*(b*e + 3*a*h)*x^14)/14 + (b^3*f*x^15)/15 +
 (b^3*g*x^16)/16 + (b^3*h*x^17)/17
```

Maple [A] time = 0.001, size = 224, normalized size = 1.1

$$\frac{b^3hx^{17}}{17}+\frac{b^3gx^{16}}{16}+\frac{b^3fx^{15}}{15}+\frac{(3\,ab^2h+b^3e)\,x^{14}}{14}+\frac{(3\,ab^2g+b^3d)\,x^{13}}{13}+\frac{(3\,ab^2f+b^3c)\,x^{12}}{12}+\frac{(3\,a^2bh+3\,aeb^2)\,x^{11}}{11}+\frac{(3\,a^2bg+3\,ab^2d)\,x^{10}}{10}+\frac{(3\,a^2bf+3\,acb^2)\,x^9}{9}+\frac{(a^3h+3\,a^2be)\,x^8}{8}+\frac{(a^3g+3\,a^2bd)\,x^7}{7}+\frac{(a^3f+3\,a^2bc)\,x^6}{6}+\frac{a^3ex^5}{5}+\frac{a^3dx^4}{4}+\frac{a^3cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/17*b^3*h*x^17+1/16*b^3*g*x^16+1/15*b^3*f*x^15+1/14*(3*a*b^2*h+b
^3*e)*x^14+1/13*(3*a*b^2*g+b^3*d)*x^13+1/12*(3*a*b^2*f+b^3*c)*x^1
2+1/11*(3*a^2*b*h+3*a*b^2*e)*x^11+1/10*(3*a^2*b*g+3*a*b^2*d)*x^10
+1/9*(3*a^2*b*f+3*a*b^2*c)*x^9+1/8*(a^3*h+3*a^2*b*e)*x^8+1/7*(a^3
*g+3*a^2*b*d)*x^7+1/6*(a^3*f+3*a^2*b*c)*x^6+1/5*a^3*e*x^5+1/4*a^3
*d*x^4+1/3*a^3*c*x^3
```

Maxima [A] time = 1.42783, size = 293, normalized size = 1.38

$$\frac{1}{17}\,b^3hx^{17}+\frac{1}{16}\,b^3gx^{16}+\frac{1}{15}\,b^3fx^{15}+\frac{1}{14}\,\left(b^3e+3\,ab^2h\right)x^{14}+\frac{1}{13}\,\left(b^3d+3\,ab^2g\right)x^{13}+\frac{1}{12}\,\left(b^3c+3\,ab^2f\right)x^{12}+\frac{3}{11}\,\left(ab^2e+a^2bh\right)x^{11}+\frac{3}{10}\,\left(ab^2d+a^2bg\right)x^{10}+\frac{1}{3}\,\left(ab^2c+a^2bf\right)x^9+\frac{1}{5}\,a^3ex^5+\frac{1}{8}\,\left(3\,a^2be+a^3h\right)x^8+\frac{1}{4}\,a^3dx^4+\frac{1}{7}\,\left(3\,a^2bd+a^3g\right)x^7+\frac{1}{3}\,a^3cx^3+\frac{1}{6}\,\left(3\,a^2bc+a^3f\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x^2,x, algorithm
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*(b^3*e
 + 3*a*b^2*h)*x^14 + 1/13*(b^3*d + 3*a*b^2*g)*x^13 + 1/12*(b^3*c
+ 3*a*b^2*f)*x^12 + 3/11*(a*b^2*e + a^2*b*h)*x^11 + 3/10*(a*b^2*d
 + a^2*b*g)*x^10 + 1/3*(a*b^2*c + a^2*b*f)*x^9 + 1/5*a^3*e*x^5 +
1/8*(3*a^2*b*e + a^3*h)*x^8 + 1/4*a^3*d*x^4 + 1/7*(3*a^2*b*d + a^
3*g)*x^7 + 1/3*a^3*c*x^3 + 1/6*(3*a^2*b*c + a^3*f)*x^6
```

Fricas [A] time = 0.217897, size = 1, normalized size = 0.

$$\frac{1}{17}x^{17}hb^3 + \frac{1}{16}x^{16}gb^3 + \frac{1}{15}x^{15}fb^3 + \frac{1}{14}x^{14}eb^3 + \frac{3}{14}x^{14}hb^2a + \frac{1}{13}x^{13}db^3 + \frac{3}{13}x^{13}gb^2a + \frac{1}{12}x^{12}cb^3 + \frac{1}{4}x^{12}fb^2a + \frac{3}{11}x^{11}eb^2a + \frac{3}{11}x^{11}hba^2 + \frac{3}{10}x^{10}db^2a + \frac{3}{10}x^{10}gba^2 + \frac{1}{3}x^9cb^2a + \frac{1}{3}x^9fba^2 + \frac{3}{8}x^8eba^2 + \frac{1}{8}x^8ha^3 + \frac{3}{7}x^7dba^2 + \frac{1}{7}x^7ga^3 + \frac{1}{2}x^6cba^2 + \frac{1}{6}x^6fa^3 + \frac{1}{5}x^5ea^3 + \frac{1}{4}x^4da^3 + \frac{1}{3}x^3ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x^2,x, algorithm
```

```
[Out] 1/17*x^17*h*b^3 + 1/16*x^16*g*b^3 + 1/15*x^15*f*b^3 + 1/14*x^14*e
*b^3 + 3/14*x^14*h*b^2*a + 1/13*x^13*d*b^3 + 3/13*x^13*g*b^2*a +
1/12*x^12*c*b^3 + 1/4*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/11*x^1
1*h*b*a^2 + 3/10*x^10*d*b^2*a + 3/10*x^10*g*b*a^2 + 1/3*x^9*c*b^2
*a + 1/3*x^9*f*b*a^2 + 3/8*x^8*e*b*a^2 + 1/8*x^8*h*a^3 + 3/7*x^7*
d*b*a^2 + 1/7*x^7*g*a^3 + 1/2*x^6*c*b*a^2 + 1/6*x^6*f*a^3 + 1/5*x
^5*e*a^3 + 1/4*x^4*d*a^3 + 1/3*x^3*c*a^3
```

Sympy [A] time = 0.114923, size = 246, normalized size = 1.16

$$\frac{a^3cx^3}{3} + \frac{a^3dx^4}{4} + \frac{a^3ex^5}{5} + \frac{b^3fx^{15}}{15} + \frac{b^3gx^{16}}{16} + \frac{b^3hx^{17}}{17} + x^{14}\left(\frac{3ab^2h}{14} + \frac{b^3e}{14}\right) + x^{13}\left(\frac{3ab^2g}{13} + \frac{b^3d}{13}\right) + x^{12}\left(\frac{ab^2f}{4} + \frac{b^3c}{12}\right) + x^{11}\left(\frac{3a^2bh}{11} + \frac{3ab^2e}{11}\right) + x^{10}\left(\frac{3a^2bg}{10} + \frac{3ab^2d}{10}\right) + x^9\left(\frac{a^2bf}{3} + \frac{ab^2c}{3}\right) + x^8\left(\frac{a^3h}{8} + \frac{3a^2be}{8}\right) + x^7\left(\frac{a^3g}{7} + \frac{3a^2bd}{7}\right) + x^6\left(\frac{a^3f}{6} + \frac{a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + b**3*f*x**15/15 +
 b**3*g*x**16/16 + b**3*h*x**17/17 + x**14*(3*a*b**2*h/14 + b**3*
e/14) + x**13*(3*a*b**2*g/13 + b**3*d/13) + x**12*(a*b**2*f/4 + b
**3*c/12) + x**11*(3*a**2*b*h/11 + 3*a*b**2*e/11) + x**10*(3*a**2
*b*g/10 + 3*a*b**2*d/10) + x**9*(a**2*b*f/3 + a*b**2*c/3) + x**8*
(a**3*h/8 + 3*a**2*b*e/8) + x**7*(a**3*g/7 + 3*a**2*b*d/7) + x**6
*(a**3*f/6 + a**2*b*c/2)
```

GIAC/XCAS [A] time = 0.21377, size = 315, normalized size = 1.49

$$\frac{1}{17}b^3hx^{17} + \frac{1}{16}b^3gx^{16} + \frac{1}{15}b^3fx^{15} + \frac{3}{14}ab^2hx^{14} + \frac{1}{14}b^3x^{14}e + \frac{1}{13}b^3dx^{13} + \frac{3}{13}ab^2gx^{13} + \frac{1}{12}b^3cx^{12} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}a^2bhx^{11} + \frac{3}{11}ab^2x^{11}e + \frac{3}{10}ab^2dx^{10} + \frac{3}{10}a^2bgx^{10} + \frac{1}{3}ab^2cx^9 + \frac{1}{3}a^2bfx^9 + \frac{1}{8}a^3hx^8 + \frac{3}{8}a^2bx^8e + \frac{3}{7}a^2bdx^7 + \frac{1}{7}a^3gx^7 + \frac{1}{2}a^2bcx^6 + \frac{1}{6}a^3fx^6 + \frac{1}{5}a^3x^5e + \frac{1}{4}a^3dx^4 + \frac{1}{3}a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x^2,x, algorithm
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 3/14*a*b^2*
h*x^14 + 1/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 3/13*a*b^2*g*x^13 +
1/12*b^3*c*x^12 + 1/4*a*b^2*f*x^12 + 3/11*a^2*b*h*x^11 + 3/11*a*b
^2*x^11*e + 3/10*a*b^2*d*x^10 + 3/10*a^2*b*g*x^10 + 1/3*a*b^2*c*x
^9 + 1/3*a^2*b*f*x^9 + 1/8*a^3*h*x^8 + 3/8*a^2*b*x^8*e + 3/7*a^2*
b*d*x^7 + 1/7*a^3*g*x^7 + 1/2*a^2*b*c*x^6 + 1/6*a^3*f*x^6 + 1/5*a
^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

3.384 $\int x\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=212

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{6}a^3gx^6+\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{3}a^2bgx^9+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{13}b^2x^{13}(3ah+be)+\frac{1}{4}ab^2gx^{12}+\frac{3}{8}abx^8(af+bc)+\frac{d\left(a+bx^3\right)^4}{12b}+\frac{3}{10}abx^{10}(ah+be)+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}$$

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 + (a^3*
g*x^6)/6 + (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)*x^8)/8
+ (a^2*b*g*x^9)/3 + (3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c + 3*a
*f)*x^11)/11 + (a*b^2*g*x^12)/4 + (b^2*(b*e + 3*a*h)*x^13)/13 + (
b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x^16)/16 + (d*(a + b*x^
3)^4)/(12*b)
```

Rubi [A] time = 0.570306, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{6}a^3gx^6+\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{3}a^2bgx^9+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{13}b^2x^{13}(3ah+be)+\frac{1}{4}ab^2gx^{12}+\frac{3}{8}abx^8(af+bc)+\frac{d\left(a+bx^3\right)^4}{12b}+\frac{3}{10}abx^{10}(ah+be)+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 + (a^3*
g*x^6)/6 + (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)*x^8)/8
+ (a^2*b*g*x^9)/3 + (3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c + 3*a
*f)*x^11)/11 + (a*b^2*g*x^12)/4 + (b^2*(b*e + 3*a*h)*x^13)/13 + (
b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x^16)/16 + (d*(a + b*x^
3)^4)/(12*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3c\int x\,dx+\frac{a^3ex^4}{4}+\frac{a^3gx^6}{6}+\frac{a^2bgx^9}{3}+\frac{a^2x^7\left(ah+3be\right)}{7}+\frac{a^2x^5\left(af+3bc\right)}{5}+\frac{ab^2gx^{12}}{4}+\frac{3abx^{10}\left(ah+be\right)}{10}+\frac{3abx^8\left(af+bc\right)}{8}+\frac{b^3fx^{14}}{14}+\frac{b^3gx^{15}}{15}+\frac{b^3hx^{16}}{16}+\frac{b^2x^{13}\left(3ah+be\right)}{13}+\frac{b^2x^{11}\left(3af+bc\right)}{11}+\frac{d\left(a+bx^3\right)^4}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*Integral(x, x) + a**3*e*x**4/4 + a**3*g*x**6/6 + a**2*b*g*
x**9/3 + a**2*x**7*(a*h + 3*b*e)/7 + a**2*x**5*(a*f + 3*b*c)/5 +
a*b**2*g*x**12/4 + 3*a*b*x**10*(a*h + b*e)/10 + 3*a*b*x**8*(a*f +
 b*c)/8 + b**3*f*x**14/14 + b**3*g*x**15/15 + b**3*h*x**16/16 + b
**2*x**13*(3*a*h + b*e)/13 + b**2*x**11*(3*a*f + b*c)/11 + d*(a +
 b*x**3)**4/(12*b)
```

Mathematica [A] time = 0.0910579, size = 223, normalized size = 1.05

$$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{6}a^2x^6(ag+3bd)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{12}b^2x^{12}(3ag+bd)+\frac{1}{13}b^2x^{13}(3ah+be)+\frac{3}{8}abx^8(af+bc)+\frac{1}{3}abx^9(ag+bd)+\frac{3}{10}abx^{10}(ah+be)+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f
)*x^5)/5 + (a^2*(3*b*d + a*g)*x^6)/6 + (a^2*(3*b*e + a*h)*x^7)/7
+ (3*a*b*(b*c + a*f)*x^8)/8 + (a*b*(b*d + a*g)*x^9)/3 + (3*a*b*(b
*e + a*h)*x^10)/10 + (b^2*(b*c + 3*a*f)*x^11)/11 + (b^2*(b*d + 3*
a*g)*x^12)/12 + (b^2*(b*e + 3*a*h)*x^13)/13 + (b^3*f*x^14)/14 + (
b^3*g*x^15)/15 + (b^3*h*x^16)/16
```

Maple [A] time = 0.002, size = 224, normalized size = 1.1

$$\frac{b^3hx^{16}}{16}+\frac{b^3gx^{15}}{15}+\frac{b^3fx^{14}}{14}+\frac{\left(3\,ab^2h+b^3e\right)x^{13}}{13}+\frac{\left(3\,ab^2g+b^3d\right)x^{12}}{12}+\frac{\left(3\,ab^2f+b^3c\right)x^{11}}{11}+\frac{\left(3\,a^2bh+3\,aeb^2\right)x^{10}}{10}+\frac{\left(3\,a^2bg+3\,ab^2d\right)x^{9}}{9}+\frac{\left(3\,a^2bf+3\,acb^2\right)x^{8}}{8}+\frac{\left(a^3h+3\,a^2be\right)x^{7}}{7}+\frac{\left(a^3g+3\,a^2bd\right)x^{6}}{6}+\frac{\left(a^3f+3\,a^2bc\right)x^{5}}{5}+\frac{a^3ex^4}{4}+\frac{a^3dx^3}{3}+\frac{a^3cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/16*b^3*h*x^16+1/15*b^3*g*x^15+1/14*b^3*f*x^14+1/13*(3*a*b^2*h+b
^3*e)*x^13+1/12*(3*a*b^2*g+b^3*d)*x^12+1/11*(3*a*b^2*f+b^3*c)*x^1
1+1/10*(3*a^2*b*h+3*a*b^2*e)*x^10+1/9*(3*a^2*b*g+3*a*b^2*d)*x^9+1
/8*(3*a^2*b*f+3*a*b^2*c)*x^8+1/7*(a^3*h+3*a^2*b*e)*x^7+1/6*(a^3*g
+3*a^2*b*d)*x^6+1/5*(a^3*f+3*a^2*b*c)*x^5+1/4*a^3*e*x^4+1/3*a^3*d
*x^3+1/2*a^3*c*x^2
```

Maxima [A] time = 1.37465, size = 293, normalized size = 1.38

$$\frac{1}{16}\,b^3hx^{16}+\frac{1}{15}\,b^3gx^{15}+\frac{1}{14}\,b^3fx^{14}+\frac{1}{13}\left(b^3e+3\,ab^2h\right)x^{13}+\frac{1}{12}\left(b^3d+3\,ab^2g\right)x^{12}+\frac{1}{11}\left(b^3c+3\,ab^2f\right)x^{11}+\frac{3}{10}\left(ab^2e+a^2bh\right)x^{10}+\frac{1}{3}\left(ab^2d+a^2bg\right)x^{9}+\frac{3}{8}\left(ab^2c+a^2bf\right)x^{8}+\frac{1}{4}\,a^3ex^4+\frac{1}{7}\left(3\,a^2be+a^3h\right)x^7+\frac{1}{3}\,a^3dx^3+\frac{1}{6}\left(3\,a^2bd+a^3g\right)x^6+\frac{1}{2}\,a^3cx^2+\frac{1}{5}\left(3\,a^2bc+a^3f\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x,x, algorithm="
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*(b^3*e
 + 3*a*b^2*h)*x^13 + 1/12*(b^3*d + 3*a*b^2*g)*x^12 + 1/11*(b^3*c
+ 3*a*b^2*f)*x^11 + 3/10*(a*b^2*e + a^2*b*h)*x^10 + 1/3*(a*b^2*d
+ a^2*b*g)*x^9 + 3/8*(a*b^2*c + a^2*b*f)*x^8 + 1/4*a^3*e*x^4 + 1/
7*(3*a^2*b*e + a^3*h)*x^7 + 1/3*a^3*d*x^3 + 1/6*(3*a^2*b*d + a^3*
g)*x^6 + 1/2*a^3*c*x^2 + 1/5*(3*a^2*b*c + a^3*f)*x^5
```

Fricas [A] time = 0.221621, size = 1, normalized size = 0.

$$\frac{1}{16}x^{16}hb^3+\frac{1}{15}x^{15}gb^3+\frac{1}{14}x^{14}fb^3+\frac{1}{13}x^{13}eb^3+\frac{3}{13}x^{13}hb^2a+\frac{1}{12}x^{12}db^3+\frac{1}{4}x^{12}gb^2a+\frac{1}{11}x^{11}cb^3+\frac{3}{11}x^{11}fb^2a+\frac{3}{10}x^{10}eb^2a+\frac{3}{10}x^{10}hba^2+\frac{1}{3}x^9db^2a+\frac{1}{3}x^9gba^2+\frac{3}{8}x^8cb^2a+\frac{3}{8}x^8fba^2+\frac{3}{7}x^7eba^2+\frac{1}{7}x^7ha^3+\frac{1}{2}x^6dba^2+\frac{1}{6}x^6ga^3+\frac{3}{5}x^5cba^2+\frac{1}{5}x^5fa^3+\frac{1}{4}x^4ea^3+\frac{1}{3}x^3da^3+\frac{1}{2}x^2ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x,x, algorithm="
```

```
[Out] 1/16*x^16*h*b^3 + 1/15*x^15*g*b^3 + 1/14*x^14*f*b^3 + 1/13*x^13*e
*b^3 + 3/13*x^13*h*b^2*a + 1/12*x^12*d*b^3 + 1/4*x^12*g*b^2*a + 1
/11*x^11*c*b^3 + 3/11*x^11*f*b^2*a + 3/10*x^10*e*b^2*a + 3/10*x^1
0*h*b*a^2 + 1/3*x^9*d*b^2*a + 1/3*x^9*g*b*a^2 + 3/8*x^8*c*b^2*a +
 3/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/7*x^7*h*a^3 + 1/2*x^6*d*b*
a^2 + 1/6*x^6*g*a^3 + 3/5*x^5*c*b*a^2 + 1/5*x^5*f*a^3 + 1/4*x^4*e
*a^3 + 1/3*x^3*d*a^3 + 1/2*x^2*c*a^3
```

Sympy [A] time = 0.113549, size = 246, normalized size = 1.16

$$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{b^3fx^{14}}{14}+\frac{b^3gx^{15}}{15}+\frac{b^3hx^{16}}{16}+x^{13}\left(\frac{3ab^2h}{13}+\frac{b^3e}{13}\right)+x^{12}\left(\frac{ab^2g}{4}+\frac{b^3d}{12}\right)+x^{11}\left(\frac{3ab^2f}{11}+\frac{b^3c}{11}\right)+x^{10}\left(\frac{3a^2bh}{10}+\frac{3ab^2e}{10}\right)+x^9\left(\frac{a^2bg}{3}+\frac{ab^2d}{3}\right)+x^8\left(\frac{3a^2bf}{8}+\frac{3ab^2c}{8}\right)+x^7\left(\frac{a^3h}{7}+\frac{3a^2be}{7}\right)+x^6\left(\frac{a^3g}{6}+\frac{a^2bd}{2}\right)+x^5\left(\frac{a^3f}{5}+\frac{3a^2bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + b**3*f*x**14/14 +
 b**3*g*x**15/15 + b**3*h*x**16/16 + x**13*(3*a*b**2*h/13 + b**3*
e/13) + x**12*(a*b**2*g/4 + b**3*d/12) + x**11*(3*a*b**2*f/11 + b
**3*c/11) + x**10*(3*a**2*b*h/10 + 3*a*b**2*e/10) + x**9*(a**2*b*
g/3 + a*b**2*d/3) + x**8*(3*a**2*b*f/8 + 3*a*b**2*c/8) + x**7*(a*
*3*h/7 + 3*a**2*b*e/7) + x**6*(a**3*g/6 + a**2*b*d/2) + x**5*(a**
3*f/5 + 3*a**2*b*c/5)
```

GIAC/XCAS [A] time = 0.216325, size = 315, normalized size = 1.49

$$\frac{1}{16}b^3hx^{16}+\frac{1}{15}b^3gx^{15}+\frac{1}{14}b^3fx^{14}+\frac{3}{13}ab^2hx^{13}+\frac{1}{13}b^3x^{13}e+\frac{1}{12}b^3dx^{12}+\frac{1}{4}ab^2gx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{11}ab^2fx^{11}+\frac{3}{10}a^2bhx^{10}+\frac{3}{10}ab^2x^{10}e+\frac{1}{3}ab^2dx^9+\frac{1}{3}a^2bgx^9+\frac{3}{8}ab^2cx^8+\frac{3}{8}a^2bfx^8+\frac{1}{7}a^3hx^7+\frac{3}{7}a^2bx^7e+\frac{1}{2}a^2bdx^6+\frac{1}{6}a^3gx^6+\frac{3}{5}a^2bcx^5+\frac{1}{5}a^3fx^5+\frac{1}{4}a^3x^4e+\frac{1}{3}a^3dx^3+\frac{1}{2}a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3*x,x, algorithm="
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 3/13*a*b^2*
h*x^13 + 1/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/4*a*b^2*g*x^12 + 1
/11*b^3*c*x^11 + 3/11*a*b^2*f*x^11 + 3/10*a^2*b*h*x^10 + 3/10*a*b
^2*x^10*e + 1/3*a*b^2*d*x^9 + 1/3*a^2*b*g*x^9 + 3/8*a*b^2*c*x^8 +
 3/8*a^2*b*f*x^8 + 1/7*a^3*h*x^7 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*
x^6 + 1/6*a^3*g*x^6 + 3/5*a^2*b*c*x^5 + 1/5*a^3*f*x^5 + 1/4*a^3*x
^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

3.385 $\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=207

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{6}a^3hx^6+\frac{1}{4}a^2x^4(af+3bc)+\frac{1}{5}a^2x^5(ag+3bd)+\frac{1}{3}a^2bhx^9+\frac{1}{10}b^2x^{10}(3af+bc)+\frac{1}{11}b^2x^{11}(3ag+bd)+\frac{1}{4}ab^2hx^{12}+\frac{3}{7}abx^7(af+bc)+\frac{3}{8}abx^8(ag+bd)+\frac{e\left(a+bx^3\right)^4}{12b}+\frac{1}{13}b^3fx^{13}+\frac{1}{14}b^3gx^{14}+\frac{1}{15}b^3hx^{15}$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d
 + a*g)*x^5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a
*b*(b*d + a*g)*x^8)/8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^10
)/10 + (b^2*(b*d + 3*a*g)*x^11)/11 + (a*b^2*h*x^12)/4 + (b^3*f*x^
13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/15 + (e*(a + b*x^3)^4)/(1
2*b)
```

Rubi [A] time = 0.477985, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{6}a^3hx^6+\frac{1}{4}a^2x^4(af+3bc)+\frac{1}{5}a^2x^5(ag+3bd)+\frac{1}{3}a^2bhx^9+\frac{1}{10}b^2x^{10}(3af+bc)+\frac{1}{11}b^2x^{11}(3ag+bd)+\frac{1}{4}ab^2hx^{12}+\frac{3}{7}abx^7(af+bc)+\frac{3}{8}abx^8(ag+bd)+\frac{e\left(a+bx^3\right)^4}{12b}+\frac{1}{13}b^3fx^{13}+\frac{1}{14}b^3gx^{14}+\frac{1}{15}b^3hx^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d
 + a*g)*x^5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a
*b*(b*d + a*g)*x^8)/8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^10
)/10 + (b^2*(b*d + 3*a*g)*x^11)/11 + (a*b^2*h*x^12)/4 + (b^3*f*x^
13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/15 + (e*(a + b*x^3)^4)/(1
2*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3d\int x\,dx+\frac{a^3hx^6}{6}+a^3\int c\,dx+\frac{a^2bhx^9}{3}+\frac{a^2x^5\left(ag+3bd\right)}{5}+\frac{a^2x^4\left(af+3bc\right)}{4}+\frac{ab^2hx^{12}}{4}+\frac{3abx^8\left(ag+bd\right)}{8}+\frac{3abx^7\left(af+bc\right)}{7}+\frac{b^3fx^{13}}{13}+\frac{b^3gx^{14}}{14}+\frac{b^3hx^{15}}{15}+\frac{b^2x^{11}\left(3ag+bd\right)}{11}+\frac{b^2x^{10}\left(3af+bc\right)}{10}+\frac{e\left(a+bx^3\right)^4}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*d*Integral(x, x) + a**3*h*x**6/6 + a**3*Integral(c, x) + a**
2*b*h*x**9/3 + a**2*x**5*(a*g + 3*b*d)/5 + a**2*x**4*(a*f + 3*b*c
)/4 + a*b**2*h*x**12/4 + 3*a*b*x**8*(a*g + b*d)/8 + 3*a*b*x**7*(a
*f + b*c)/7 + b**3*f*x**13/13 + b**3*g*x**14/14 + b**3*h*x**15/15
 + b**2*x**11*(3*a*g + b*d)/11 + b**2*x**10*(3*a*f + b*c)/10 + e*
(a + b*x**3)**4/(12*b)
```

Mathematica [A] time = 0.220847, size = 170, normalized size = 0.82

$$\frac{x\left(2002a^3\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+143a^2bx^3(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (x*(13*a*b^2*x^6*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 6
0*g*x + 55*h*x^2))) + 2002*a^3*(60*c + x*(30*d + x*(20*e + 15*f*x
 + 12*g*x^2 + 10*h*x^3))) + 2*b^3*x^9*(6006*c + x*(5460*d + 11*x*
(455*e + 420*f*x + 390*g*x^2 + 364*h*x^3))) + 143*a^2*b*x^3*(630*
c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x)))))))/1201
20
```

Maple [A] time = 0.002, size = 221, normalized size = 1.1

$$\frac{b^3hx^{15}}{15}+\frac{b^3gx^{14}}{14}+\frac{b^3fx^{13}}{13}+\frac{\left(3\,ab^2h+b^3e\right)x^{12}}{12}+\frac{\left(3\,ab^2g+b^3d\right)x^{11}}{11}+\frac{\left(3\,ab^2f+b^3c\right)x^{10}}{10}+\frac{\left(3\,a^2bh+3\,aeb^2\right)x^9}{9}+\frac{\left(3\,a^2bg+3\,ab^2d\right)x^8}{8}+\frac{\left(3\,a^2bf+3\,acb^2\right)x^7}{7}+\frac{\left(a^3h+3\,a^2be\right)x^6}{6}+\frac{\left(a^3g+3\,a^2bd\right)x^5}{5}+\frac{\left(a^3f+3\,a^2bc\right)x^4}{4}+\frac{a^3ex^3}{3}+\frac{a^3dx^2}{2}+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/15*b^3*h*x^15+1/14*b^3*g*x^14+1/13*b^3*f*x^13+1/12*(3*a*b^2*h+b
^3*e)*x^12+1/11*(3*a*b^2*g+b^3*d)*x^11+1/10*(3*a*b^2*f+b^3*c)*x^1
0+1/9*(3*a^2*b*h+3*a*b^2*e)*x^9+1/8*(3*a^2*b*g+3*a*b^2*d)*x^8+1/7
*(3*a^2*b*f+3*a*b^2*c)*x^7+1/6*(a^3*h+3*a^2*b*e)*x^6+1/5*(a^3*g+3
*a^2*b*d)*x^5+1/4*(a^3*f+3*a^2*b*c)*x^4+1/3*a^3*e*x^3+1/2*a^3*d*x
^2+a^3*c*x
```

Maxima [A] time = 1.37433, size = 289, normalized size = 1.4

$$\frac{1}{15}\,b^3hx^{15}+\frac{1}{14}\,b^3gx^{14}+\frac{1}{13}\,b^3fx^{13}+\frac{1}{12}\left(b^3e+3\,ab^2h\right)x^{12}+\frac{1}{11}\left(b^3d+3\,ab^2g\right)x^{11}+\frac{1}{10}\left(b^3c+3\,ab^2f\right)x^{10}+\frac{1}{3}\left(ab^2e+a^2bh\right)x^9+\frac{3}{8}\left(ab^2d+a^2bg\right)x^8+\frac{3}{7}\left(ab^2c+a^2bf\right)x^7+\frac{1}{3}\,a^3ex^3+\frac{1}{6}\left(3\,a^2be+a^3h\right)x^6+\frac{1}{2}\,a^3dx^2+\frac{1}{5}\left(3\,a^2bd+a^3g\right)x^5+a^3cx+\frac{1}{4}\left(3\,a^2bc+a^3f\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3,x, algorithm="ma
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(b^3*e
 + 3*a*b^2*h)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c
+ 3*a*b^2*f)*x^10 + 1/3*(a*b^2*e + a^2*b*h)*x^9 + 3/8*(a*b^2*d +
a^2*b*g)*x^8 + 3/7*(a*b^2*c + a^2*b*f)*x^7 + 1/3*a^3*e*x^3 + 1/6*
(3*a^2*b*e + a^3*h)*x^6 + 1/2*a^3*d*x^2 + 1/5*(3*a^2*b*d + a^3*g)
*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

Fricas [A] time = 0.2204, size = 1, normalized size = 0.

$$\frac{1}{15}x^{15}hb^3+\frac{1}{14}x^{14}gb^3+\frac{1}{13}x^{13}fb^3+\frac{1}{12}x^{12}eb^3+\frac{1}{4}x^{12}hb^2a+\frac{1}{11}x^{11}db^3+\frac{3}{11}x^{11}gb^2a+\frac{1}{10}x^{10}cb^3+\frac{3}{10}x^{10}fb^2a+\frac{1}{3}x^9eb^2a+\frac{1}{3}x^9hba^2+\frac{3}{8}x^8db^2a+\frac{3}{8}x^8gba^2+\frac{3}{7}x^7cb^2a+\frac{3}{7}x^7fba^2+\frac{1}{2}x^6eba^2+\frac{1}{6}x^6ha^3+\frac{3}{5}x^5dba^2+\frac{1}{5}x^5ga^3+\frac{3}{4}x^4cba^2+\frac{1}{4}x^4fa^3+\frac{1}{3}x^3ea^3+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3,x, algorithm="fr
```

```
[Out] 1/15*x^15*h*b^3 + 1/14*x^14*g*b^3 + 1/13*x^13*f*b^3 + 1/12*x^12*e
*b^3 + 1/4*x^12*h*b^2*a + 1/11*x^11*d*b^3 + 3/11*x^11*g*b^2*a + 1
/10*x^10*c*b^3 + 3/10*x^10*f*b^2*a + 1/3*x^9*e*b^2*a + 1/3*x^9*h*
b*a^2 + 3/8*x^8*d*b^2*a + 3/8*x^8*g*b*a^2 + 3/7*x^7*c*b^2*a + 3/7
*x^7*f*b*a^2 + 1/2*x^6*e*b*a^2 + 1/6*x^6*h*a^3 + 3/5*x^5*d*b*a^2
+ 1/5*x^5*g*a^3 + 3/4*x^4*c*b*a^2 + 1/4*x^4*f*a^3 + 1/3*x^3*e*a^3
 + 1/2*x^2*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.110738, size = 243, normalized size = 1.17

$$a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{b^3fx^{13}}{13}+\frac{b^3gx^{14}}{14}+\frac{b^3hx^{15}}{15}+x^{12}\left(\frac{ab^2h}{4}+\frac{b^3e}{12}\right)+x^{11}\left(\frac{3ab^2g}{11}+\frac{b^3d}{11}\right)+x^{10}\left(\frac{3ab^2f}{10}+\frac{b^3c}{10}\right)+x^9\left(\frac{a^2bh}{3}+\frac{ab^2e}{3}\right)+x^8\left(\frac{3a^2bg}{8}+\frac{3ab^2d}{8}\right)+x^7\left(\frac{3a^2bf}{7}+\frac{3ab^2c}{7}\right)+x^6\left(\frac{a^3h}{6}+\frac{a^2be}{2}\right)+x^5\left(\frac{a^3g}{5}+\frac{3a^2bd}{5}\right)+x^4\left(\frac{a^3f}{4}+\frac{3a^2bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + b**3*f*x**13/13 + b**3
*g*x**14/14 + b**3*h*x**15/15 + x**12*(a*b**2*h/4 + b**3*e/12) +
x**11*(3*a*b**2*g/11 + b**3*d/11) + x**10*(3*a*b**2*f/10 + b**3*c
/10) + x**9*(a**2*b*h/3 + a*b**2*e/3) + x**8*(3*a**2*b*g/8 + 3*a*
b**2*d/8) + x**7*(3*a**2*b*f/7 + 3*a*b**2*c/7) + x**6*(a**3*h/6 +
 a**2*b*e/2) + x**5*(a**3*g/5 + 3*a**2*b*d/5) + x**4*(a**3*f/4 +
3*a**2*b*c/4)
```

GIAC/XCAS [A] time = 0.21609, size = 311, normalized size = 1.5

$$\frac{1}{15}b^3hx^{15}+\frac{1}{14}b^3gx^{14}+\frac{1}{13}b^3fx^{13}+\frac{1}{4}ab^2hx^{12}+\frac{1}{12}b^3x^{12}e+\frac{1}{11}b^3dx^{11}+\frac{3}{11}ab^2gx^{11}+\frac{1}{10}b^3cx^{10}+\frac{3}{10}ab^2fx^{10}+\frac{1}{3}a^2bhx^9+\frac{1}{3}ab^2x^9e+\frac{3}{8}ab^2dx^8+\frac{3}{8}a^2bgx^8+\frac{3}{7}ab^2cx^7+\frac{3}{7}a^2bfx^7+\frac{1}{6}a^3hx^6+\frac{1}{2}a^2bx^6e+\frac{3}{5}a^2bdx^5+\frac{1}{5}a^3gx^5+\frac{3}{4}a^2bcx^4+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3x^3e+\frac{1}{2}a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3,x, algorithm="gi
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/4*a*b^2*h
*x^12 + 1/12*b^3*x^12*e + 1/11*b^3*d*x^11 + 3/11*a*b^2*g*x^11 + 1
/10*b^3*c*x^10 + 3/10*a*b^2*f*x^10 + 1/3*a^2*b*h*x^9 + 1/3*a*b^2*
x^9*e + 3/8*a*b^2*d*x^8 + 3/8*a^2*b*g*x^8 + 3/7*a*b^2*c*x^7 + 3/7
*a^2*b*f*x^7 + 1/6*a^3*h*x^6 + 1/2*a^2*b*x^6*e + 3/5*a^2*b*d*x^5
+ 1/5*a^3*g*x^5 + 3/4*a^2*b*c*x^4 + 1/4*a^3*f*x^4 + 1/3*a^3*x^3*e
 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.386 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal. Leaf size=200

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+\frac{1}{2}ab^2cx^6+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3ah+be)+\frac{3}{7}abx^7(ag+bd)+\frac{3}{8}abx^8(ah+be)+\frac{f\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3cx^9+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}$$

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4
 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*
g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d
 + 3*a*g)*x^10)/10 + (b^2*(b*e + 3*a*h)*x^11)/11 + (b^3*g*x^13)/1
3 + (b^3*h*x^14)/14 + (f*(a + b*x^3)^4)/(12*b) + a^3*c*Log[x]
```

Rubi [A] time = 0.315798, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+\frac{1}{2}ab^2cx^6+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3ah+be)+\frac{3}{7}abx^7(ag+bd)+\frac{3}{8}abx^8(ah+be)+\frac{f\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3cx^9+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4
 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*
g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d
 + 3*a*g)*x^10)/10 + (b^2*(b*e + 3*a*h)*x^11)/11 + (b^3*g*x^13)/1
3 + (b^3*h*x^14)/14 + (f*(a + b*x^3)^4)/(12*b) + a^3*c*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3c\log(x)+a^3e\int x\,dx+a^3\int d\,dx+\frac{a^2x^5(ah+3be)}{5}+\frac{a^2x^4(ag+3bd)}{4}+\frac{a^2x^3(af+3bc)}{3}+\frac{3abx^8(ah+be)}{8}+\frac{3abx^7(ag+bd)}{7}+\frac{abx^6(af+bc)}{2}+\frac{b^3fx^{12}}{12}+\frac{b^3gx^{13}}{13}+\frac{b^3hx^{14}}{14}+\frac{b^2x^{11}(3ah+be)}{11}+\frac{b^2x^{10}(3ag+bd)}{10}+\frac{b^2x^9(3af+bc)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**3*c*log(x) + a**3*e*Integral(x, x) + a**3*Integral(d, x) + a**
2*x**5*(a*h + 3*b*e)/5 + a**2*x**4*(a*g + 3*b*d)/4 + a**2*x**3*(a
*f + 3*b*c)/3 + 3*a*b*x**8*(a*h + b*e)/8 + 3*a*b*x**7*(a*g + b*d)
/7 + a*b*x**6*(a*f + b*c)/2 + b**3*f*x**12/12 + b**3*g*x**13/13 +
 b**3*h*x**14/14 + b**2*x**11*(3*a*h + b*e)/11 + b**2*x**10*(3*a*
g + b*d)/10 + b**2*x**9*(3*a*f + b*c)/9
```

Mathematica [A] time = 0.195592, size = 214, normalized size = 1.07

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2x^3(af+3bc)+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+\frac{1}{9}b^2x^9(3af+bc)+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3ah+be)+\frac{1}{2}abx^6(af+bc)+\frac{3}{7}abx^7(ag+bd)+\frac{3}{8}abx^8(ah+be)+\frac{1}{12}b^3fx^{12}+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*c + a*f)*x^3)/3 + (a^2*(3*b*d
 + a*g)*x^4)/4 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b*(b*c + a*f)*x^6
)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^
2*(b*c + 3*a*f)*x^9)/9 + (b^2*(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e
+ 3*a*h)*x^11)/11 + (b^3*f*x^12)/12 + (b^3*g*x^13)/13 + (b^3*h*x^
14)/14 + a^3*c*Log[x]
```

Maple [A] time = 0.007, size = 224, normalized size = 1.1

$$\frac{b^3hx^{14}}{14}+\frac{b^3gx^{13}}{13}+\frac{fx^{12}b^3}{12}+\frac{3x^{11}ab^2h}{11}+\frac{x^{11}b^3e}{11}+\frac{3x^{10}ab^2g}{10}+\frac{x^{10}b^3d}{10}+\frac{x^9ab^2f}{3}+\frac{b^3cx^9}{9}+\frac{3x^8a^2bh}{8}+\frac{3x^8ab^2e}{8}+\frac{3x^7a^2bg}{7}+\frac{3x^7ab^2d}{7}+\frac{x^6a^2bf}{2}+\frac{ab^2cx^6}{2}+\frac{x^5a^3h}{5}+\frac{3x^5a^2be}{5}+\frac{x^4a^3g}{4}+\frac{3x^4a^2bd}{4}+\frac{a^3fx^3}{3}+a^2bcx^3+\frac{a^3ex^2}{2}+a^3dx+a^3c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] 1/14*b^3*h*x^14+1/13*b^3*g*x^13+1/12*f*x^12*b^3+3/11*x^11*a*b^2*h
+1/11*x^11*b^3*e+3/10*x^10*a*b^2*g+1/10*x^10*b^3*d+1/3*x^9*a*b^2*
f+1/9*b^3*c*x^9+3/8*x^8*a^2*b*h+3/8*x^8*a*b^2*e+3/7*x^7*a^2*b*g+3
/7*x^7*a*b^2*d+1/2*x^6*a^2*b*f+1/2*a*b^2*c*x^6+1/5*x^5*a^3*h+3/5*
x^5*a^2*b*e+1/4*x^4*a^3*g+3/4*x^4*a^2*b*d+1/3*a^3*f*x^3+a^2*b*c*x
^3+1/2*a^3*e*x^2+a^3*d*x+a^3*c*ln(x)
```

Maxima [A] time = 1.3949, size = 286, normalized size = 1.43

$$\frac{1}{14}b^3hx^{14}+\frac{1}{13}b^3gx^{13}+\frac{1}{12}b^3fx^{12}+\frac{1}{11}\left(b^3e+3ab^2h\right)x^{11}+\frac{1}{10}\left(b^3d+3ab^2g\right)x^{10}+\frac{1}{9}\left(b^3c+3ab^2f\right)x^9+\frac{3}{8}\left(ab^2e+a^2bh\right)x^8+\frac{3}{7}\left(ab^2d+a^2bg\right)x^7+\frac{1}{2}\left(ab^2c+a^2bf\right)x^6+\frac{1}{2}a^3ex^2+\frac{1}{5}\left(3a^2be+a^3h\right)x^5+a^3dx+\frac{1}{4}\left(3a^2bd+a^3g\right)x^4+a^3c\log(x)+\frac{1}{3}\left(3a^2bc+a^3f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x,x, algorithm="
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e
 + 3*a*b^2*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c +
 3*a*b^2*f)*x^9 + 3/8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^
2*b*g)*x^7 + 1/2*(a*b^2*c + a^2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3
*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3*a^2*b*d + a^3*g)*x^4 + a
^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

Fricas [A] time = 0.243257, size = 286, normalized size = 1.43

$$\frac{1}{14}b^3hx^{14}+\frac{1}{13}b^3gx^{13}+\frac{1}{12}b^3fx^{12}+\frac{1}{11}\left(b^3e+3ab^2h\right)x^{11}+\frac{1}{10}\left(b^3d+3ab^2g\right)x^{10}+\frac{1}{9}\left(b^3c+3ab^2f\right)x^9+\frac{3}{8}\left(ab^2e+a^2bh\right)x^8+\frac{3}{7}\left(ab^2d+a^2bg\right)x^7+\frac{1}{2}\left(ab^2c+a^2bf\right)x^6+\frac{1}{2}a^3ex^2+\frac{1}{5}\left(3a^2be+a^3h\right)x^5+a^3dx+\frac{1}{4}\left(3a^2bd+a^3g\right)x^4+a^3c\log(x)+\frac{1}{3}\left(3a^2bc+a^3f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x,x, algorithm="
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e
 + 3*a*b^2*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c +
 3*a*b^2*f)*x^9 + 3/8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^
2*b*g)*x^7 + 1/2*(a*b^2*c + a^2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3
*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3*a^2*b*d + a^3*g)*x^4 + a
^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

Sympy [A] time = 1.25182, size = 240, normalized size = 1.2

$$a^3c\log(x) + a^3dx + \frac{a^3ex^2}{2} + \frac{b^3fx^{12}}{12} + \frac{b^3gx^{13}}{13} + \frac{b^3hx^{14}}{14} + x^{11}\left(\frac{3ab^2h}{11} + \frac{b^3e}{11}\right) + x^{10}\left(\frac{3ab^2g}{10} + \frac{b^3d}{10}\right) + x^9\left(\frac{ab^2f}{3} + \frac{b^3c}{9}\right) + x^8\left(\frac{3a^2bh}{8} + \frac{3ab^2e}{8}\right) + x^7\left(\frac{3a^2bg}{7} + \frac{3ab^2d}{7}\right) + x^6\left(\frac{a^2bf}{2} + \frac{ab^2c}{2}\right) + x^5\left(\frac{a^3h}{5} + \frac{3a^2be}{5}\right) + x^4\left(\frac{a^3g}{4} + \frac{3a^2bd}{4}\right) + x^3\left(\frac{a^3f}{3} + a^2bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + b**3*f*x**12/12 + b**3
*g*x**13/13 + b**3*h*x**14/14 + x**11*(3*a*b**2*h/11 + b**3*e/11)
 + x**10*(3*a*b**2*g/10 + b**3*d/10) + x**9*(a*b**2*f/3 + b**3*c/
9) + x**8*(3*a**2*b*h/8 + 3*a*b**2*e/8) + x**7*(3*a**2*b*g/7 + 3*
a*b**2*d/7) + x**6*(a**2*b*f/2 + a*b**2*c/2) + x**5*(a**3*h/5 + 3
*a**2*b*e/5) + x**4*(a**3*g/4 + 3*a**2*b*d/4) + x**3*(a**3*f/3 +
a**2*b*c)
```

GIAC/XCAS [A] time = 0.220448, size = 308, normalized size = 1.54

$$\frac{1}{14}b^3hx^{14} + \frac{1}{13}b^3gx^{13} + \frac{1}{12}b^3fx^{12} + \frac{3}{11}ab^2hx^{11} + \frac{1}{11}b^3x^{11}e + \frac{1}{10}b^3dx^{10} + \frac{3}{10}ab^2gx^{10} + \frac{1}{9}b^3cx^9 + \frac{1}{3}ab^2fx^9 + \frac{3}{8}a^2bhx^8 + \frac{3}{8}ab^2x^8e + \frac{3}{7}ab^2dx^7 + \frac{3}{7}a^2bgx^7 + \frac{1}{2}ab^2cx^6 + \frac{1}{2}a^2bfx^6 + \frac{1}{5}a^3hx^5 + \frac{3}{5}a^2bx^5e + \frac{3}{4}a^2bdx^4 + \frac{1}{4}a^3gx^4 + a^2bcx^3 + \frac{1}{3}a^3fx^3 + \frac{1}{2}a^3x^2e + a^3dx + a^3c\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x,x, algorithm="
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 3/11*a*b^2*
h*x^11 + 1/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 3/10*a*b^2*g*x^10 +
1/9*b^3*c*x^9 + 1/3*a*b^2*f*x^9 + 3/8*a^2*b*h*x^8 + 3/8*a*b^2*x^8
*e + 3/7*a*b^2*d*x^7 + 3/7*a^2*b*g*x^7 + 1/2*a*b^2*c*x^6 + 1/2*a^
2*b*f*x^6 + 1/5*a^3*h*x^5 + 3/5*a^2*b*x^5*e + 3/4*a^2*b*d*x^4 + 1
/4*a^3*g*x^4 + a^2*b*c*x^3 + 1/3*a^3*f*x^3 + 1/2*a^3*x^2*e + a^3*
d*x + a^3*c*ln(abs(x))
```

3.387 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx$

Optimal. Leaf size=198

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3ah+be)+\frac{3}{5}abx^5(af+bc)+\frac{3}{7}abx^7(ah+be)+\frac{g\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3dx^9+\frac{1}{11}b^3fx^{11}+\frac{1}{13}b^3hx^{13}$$

```
[Out] -((a^3*c)/x) + a^3*e*x + (a^2*(3*b*c + a*f)*x^2)/2 + a^2*b*d*x^3
+ (a^2*(3*b*e + a*h)*x^4)/4 + (3*a*b*(b*c + a*f)*x^5)/5 + (a*b^2*
d*x^6)/2 + (3*a*b*(b*e + a*h)*x^7)/7 + (b^2*(b*c + 3*a*f)*x^8)/8
+ (b^3*d*x^9)/9 + (b^2*(b*e + 3*a*h)*x^10)/10 + (b^3*f*x^11)/11 +
 (b^3*h*x^13)/13 + (g*(a + b*x^3)^4)/(12*b) + a^3*d*Log[x]
```

Rubi [A] time = 0.463581, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3ah+be)+\frac{3}{5}abx^5(af+bc)+\frac{3}{7}abx^7(ah+be)+\frac{g\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3dx^9+\frac{1}{11}b^3fx^{11}+\frac{1}{13}b^3hx^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (a^2*(3*b*c + a*f)*x^2)/2 + a^2*b*d*x^3
+ (a^2*(3*b*e + a*h)*x^4)/4 + (3*a*b*(b*c + a*f)*x^5)/5 + (a*b^2*
d*x^6)/2 + (3*a*b*(b*e + a*h)*x^7)/7 + (b^2*(b*c + 3*a*f)*x^8)/8
+ (b^3*d*x^9)/9 + (b^2*(b*e + 3*a*h)*x^10)/10 + (b^3*f*x^11)/11 +
 (b^3*h*x^13)/13 + (g*(a + b*x^3)^4)/(12*b) + a^3*d*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3\int e\,dx+\frac{a^2x^4(ah+3be)}{4}+\frac{a^2x^3(ag+3bd)}{3}+a^2(af+3bc)\int x\,dx+\frac{3abx^7(ah+be)}{7}+\frac{abx^6(ag+bd)}{2}+\frac{3abx^5(af+bc)}{5}+\frac{b^3fx^{11}}{11}+\frac{b^3gx^{12}}{12}+\frac{b^3hx^{13}}{13}+\frac{b^2x^{10}(3ah+be)}{10}+\frac{b^2x^9(3ag+bd)}{9}+\frac{b^2x^8(3af+bc)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**3*c/x + a**3*d*log(x) + a**3*Integral(e, x) + a**2*x**4*(a*h
+ 3*b*e)/4 + a**2*x**3*(a*g + 3*b*d)/3 + a**2*(a*f + 3*b*c)*Integ
ral(x, x) + 3*a*b*x**7*(a*h + b*e)/7 + a*b*x**6*(a*g + b*d)/2 + 3
*a*b*x**5*(a*f + b*c)/5 + b**3*f*x**11/11 + b**3*g*x**12/12 + b**
3*h*x**13/13 + b**2*x**10*(3*a*h + b*e)/10 + b**2*x**9*(3*a*g + b
*d)/9 + b**2*x**8*(3*a*f + b*c)/8
```

Mathematica [A] time = 0.347019, size = 172, normalized size = 0.87

$$a^3\left(-\frac{c}{x}+ex+\frac{1}{12}x^2\left(6f+4gx+3hx^2\right)\right)+a^3d\log(x)+\frac{1}{140}a^2bx^2\left(210c+x\left(140d+x\left(105e+84fx+70gx^2+60hx^3\right)\right)\right)+\frac{1}{840}ab^2x^5\left(504c+x\left(420d+x\left(360e+315fx+280gx^2+252hx^3\right)\right)\right)+\frac{b^3x^8\left(6435c+5720dx+6x^2\left(858e+780fx+715gx^2+660hx^3\right)\right)}{51480}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] a^3*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) + (b^3*x^8*
(6435*c + 5720*d*x + 6*x^2*(858*e + 780*f*x + 715*g*x^2 + 660*h*x
^3)))/51480 + (a^2*b*x^2*(210*c + x*(140*d + x*(105*e + 84*f*x +
70*g*x^2 + 60*h*x^3))))/140 + (a*b^2*x^5*(504*c + x*(420*d + x*(3
60*e + 315*f*x + 280*g*x^2 + 252*h*x^3))))/840 + a^3*d*Log[x]
```

Maple [A] time = 0.01, size = 224, normalized size = 1.1

$$\frac{b^3hx^{13}}{13}+\frac{b^3gx^{12}}{12}+\frac{b^3fx^{11}}{11}+\frac{3x^{10}ab^2h}{10}+\frac{x^{10}b^3e}{10}+\frac{x^9ab^2g}{3}+\frac{b^3dx^9}{9}+\frac{3x^8ab^2f}{8}+\frac{x^8b^3c}{8}+\frac{3x^7a^2bh}{7}+\frac{3x^7ab^2e}{7}+\frac{x^6a^2bg}{2}+\frac{ab^2dx^6}{2}+\frac{3x^5a^2bf}{5}+\frac{3x^5ab^2c}{5}+\frac{x^4a^3h}{4}+\frac{3x^4a^2be}{4}+\frac{x^3a^3g}{3}+a^2bdx^3+\frac{x^2a^3f}{2}+\frac{3x^2a^2bc}{2}+a^3ex+a^3d\ln(x)-\frac{a^3c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 1/13*b^3*h*x^13+1/12*b^3*g*x^12+1/11*b^3*f*x^11+3/10*x^10*a*b^2*h
+1/10*x^10*b^3*e+1/3*x^9*a*b^2*g+1/9*b^3*d*x^9+3/8*x^8*a*b^2*f+1/
8*x^8*b^3*c+3/7*x^7*a^2*b*h+3/7*x^7*a*b^2*e+1/2*x^6*a^2*b*g+1/2*a
*b^2*d*x^6+3/5*x^5*a^2*b*f+3/5*x^5*a*b^2*c+1/4*x^4*a^3*h+3/4*x^4*
a^2*b*e+1/3*x^3*a^3*g+a^2*b*d*x^3+1/2*x^2*a^3*f+3/2*x^2*a^2*b*c+a
^3*e*x+a^3*d*ln(x)-a^3*c/x
```

Maxima [A] time = 1.38434, size = 286, normalized size = 1.44

$$\frac{1}{13}b^3hx^{13}+\frac{1}{12}b^3gx^{12}+\frac{1}{11}b^3fx^{11}+\frac{1}{10}\left(b^3e+3ab^2h\right)x^{10}+\frac{1}{9}\left(b^3d+3ab^2g\right)x^9+\frac{1}{8}\left(b^3c+3ab^2f\right)x^8+\frac{3}{7}\left(ab^2e+a^2bh\right)x^7+\frac{1}{2}\left(ab^2d+a^2bg\right)x^6+\frac{3}{5}\left(ab^2c+a^2bf\right)x^5+a^3ex+\frac{1}{4}\left(3a^2be+a^3h\right)x^4+a^3d\log(x)+\frac{1}{3}\left(3a^2bd+a^3g\right)x^3-\frac{a^3c}{x}+\frac{1}{2}\left(3a^2bc+a^3f\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^2,x, algorithm
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 1/10*(b^3*e
 + 3*a*b^2*h)*x^10 + 1/9*(b^3*d + 3*a*b^2*g)*x^9 + 1/8*(b^3*c + 3
*a*b^2*f)*x^8 + 3/7*(a*b^2*e + a^2*b*h)*x^7 + 1/2*(a*b^2*d + a^2*
b*g)*x^6 + 3/5*(a*b^2*c + a^2*b*f)*x^5 + a^3*e*x + 1/4*(3*a^2*b*e
 + a^3*h)*x^4 + a^3*d*log(x) + 1/3*(3*a^2*b*d + a^3*g)*x^3 - a^3*
c/x + 1/2*(3*a^2*b*c + a^3*f)*x^2
```

Fricas [A] time = 0.23267, size = 296, normalized size = 1.49

$$\frac{27720\,b^3hx^{14} + 30030\,b^3gx^{13} + 32760\,b^3fx^{12} + 36036\,(b^3e + 3\,ab^2h)\,x^{11} + 40040\,(b^3d + 3\,ab^2g)\,x^{10} + 45045\,(b^3c + 3\,ab^2f)\,x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^2,x, algorithm
```

```
[Out] 1/360360*(27720*b^3*h*x^14 + 30030*b^3*g*x^13 + 32760*b^3*f*x^12
+ 36036*(b^3*e + 3*a*b^2*h)*x^11 + 40040*(b^3*d + 3*a*b^2*g)*x^10
 + 45045*(b^3*c + 3*a*b^2*f)*x^9 + 154440*(a*b^2*e + a^2*b*h)*x^8
 + 180180*(a*b^2*d + a^2*b*g)*x^7 + 216216*(a*b^2*c + a^2*b*f)*x^
6 + 360360*a^3*e*x^2 + 90090*(3*a^2*b*e + a^3*h)*x^5 + 360360*a^3
*d*x*log(x) + 120120*(3*a^2*b*d + a^3*g)*x^4 - 360360*a^3*c + 180
180*(3*a^2*b*c + a^3*f)*x^3)/x
```

Sympy [A] time = 1.32554, size = 236, normalized size = 1.19

$$-\frac{a^3c}{x} + a^3d\log(x) + a^3ex + \frac{b^3fx^{11}}{11} + \frac{b^3gx^{12}}{12} + \frac{b^3hx^{13}}{13} + x^{10}\left(\frac{3ab^2h}{10} + \frac{b^3e}{10}\right) + x^9\left(\frac{ab^2g}{3} + \frac{b^3d}{9}\right) + x^8\left(\frac{3ab^2f}{8} + \frac{b^3c}{8}\right) + x^7\left(\frac{3a^2bh}{7} + \frac{3ab^2e}{7}\right) + x^6\left(\frac{a^2bg}{2} + \frac{ab^2d}{2}\right) + x^5\left(\frac{3a^2bf}{5} + \frac{3ab^2c}{5}\right) + x^4\left(\frac{a^3h}{4} + \frac{3a^2be}{4}\right) + x^3\left(\frac{a^3g}{3} + a^2bd\right) + x^2\left(\frac{a^3f}{2} + \frac{3a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + b**3*f*x**11/11 + b**3*g*x
**12/12 + b**3*h*x**13/13 + x**10*(3*a*b**2*h/10 + b**3*e/10) + x
**9*(a*b**2*g/3 + b**3*d/9) + x**8*(3*a*b**2*f/8 + b**3*c/8) + x*
*7*(3*a**2*b*h/7 + 3*a*b**2*e/7) + x**6*(a**2*b*g/2 + a*b**2*d/2)
 + x**5*(3*a**2*b*f/5 + 3*a*b**2*c/5) + x**4*(a**3*h/4 + 3*a**2*b
*e/4) + x**3*(a**3*g/3 + a**2*b*d) + x**2*(a**3*f/2 + 3*a**2*b*c/
2)
```

GIAC/XCAS [A] time = 0.218127, size = 308, normalized size = 1.56

$$\frac{1}{13}b^3hx^{13} + \frac{1}{12}b^3gx^{12} + \frac{1}{11}b^3fx^{11} + \frac{3}{10}ab^2hx^{10} + \frac{1}{10}b^3x^{10}e + \frac{1}{9}b^3dx^9 + \frac{1}{3}ab^2gx^9 + \frac{1}{8}b^3cx^8 + \frac{3}{8}ab^2fx^8 + \frac{3}{7}a^2bhx^7 + \frac{3}{7}ab^2x^7e + \frac{1}{2}ab^2dx^6 + \frac{1}{2}a^2bgx^6 + \frac{3}{5}ab^2cx^5 + \frac{3}{5}a^2bfx^5 + \frac{1}{4}a^3hx^4 + \frac{3}{4}a^2bx^4e + a^2bdx^3 + \frac{1}{3}a^3gx^3 + \frac{3}{2}a^2bcx^2 + \frac{1}{2}a^3fx^2 + a^3xe + a^3d\ln(|x|) - \frac{a^3c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^2,x, algorithm
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 3/10*a*b^2*
h*x^10 + 1/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/3*a*b^2*g*x^9 + 1/8*
b^3*c*x^8 + 3/8*a*b^2*f*x^8 + 3/7*a^2*b*h*x^7 + 3/7*a*b^2*x^7*e +
 1/2*a*b^2*d*x^6 + 1/2*a^2*b*g*x^6 + 3/5*a*b^2*c*x^5 + 3/5*a^2*b*
f*x^5 + 1/4*a^3*h*x^4 + 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 1/3*a^3*g
*x^3 + 3/2*a^2*b*c*x^2 + 1/2*a^3*f*x^2 + a^3*x*e + a^3*d*ln(abs(x
)) - a^3*c/x
```

3.388 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=198

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+a^2x(af+3bc)+\frac{1}{2}a^2x^2(ag+3bd)+a^2bex^3+\frac{1}{7}b^2x^7(3af+bc)+\frac{1}{8}b^2x^8(3ag+bd)+\frac{1}{2}ab^2ex^6+\frac{3}{4}abx^4(af+bc)+\frac{3}{5}abx^5(ag+bd)+\frac{h\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3ex^9+\frac{1}{10}b^3fx^{10}+\frac{1}{11}b^3gx^{11}$$

```
[Out] -(a^3*c)/(2*x^2) - (a^3*d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d
+ a*g)*x^2)/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f)*x^4)/4 + (3*a*b*
(b*d + a*g)*x^5)/5 + (a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f)*x^7)/7
+ (b^2*(b*d + 3*a*g)*x^8)/8 + (b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (
b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(12*b) + a^3*e*Log[x]
```

Rubi [A] time = 0.465261, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+a^2x(af+3bc)+\frac{1}{2}a^2x^2(ag+3bd)+a^2bex^3+\frac{1}{7}b^2x^7(3af+bc)+\frac{1}{8}b^2x^8(3ag+bd)+\frac{1}{2}ab^2ex^6+\frac{3}{4}abx^4(af+bc)+\frac{3}{5}abx^5(ag+bd)+\frac{h\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3ex^9+\frac{1}{10}b^3fx^{10}+\frac{1}{11}b^3gx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -(a^3*c)/(2*x^2) - (a^3*d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d
+ a*g)*x^2)/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f)*x^4)/4 + (3*a*b*
(b*d + a*g)*x^5)/5 + (a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f)*x^7)/7
+ (b^2*(b*d + 3*a*g)*x^8)/8 + (b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (
b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(12*b) + a^3*e*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+\frac{a^2x^3(ah+3be)}{3}+a^2(ag+3bd)\int x\,dx+\frac{a^2(af+3bc)\int f\,dx}{f}+\frac{abx^6(ah+be)}{2}+\frac{3abx^5(ag+bd)}{5}+\frac{3abx^4(af+bc)}{4}+\frac{b^3fx^{10}}{10}+\frac{b^3gx^{11}}{11}+\frac{b^3hx^{12}}{12}+\frac{b^2x^9(3ah+be)}{9}+\frac{b^2x^8(3ag+bd)}{8}+\frac{b^2x^7(3af+bc)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] -a**3*c/(2*x**2) - a**3*d/x + a**3*e*log(x) + a**2*x**3*(a*h + 3*
b*e)/3 + a**2*(a*g + 3*b*d)*Integral(x, x) + a**2*(a*f + 3*b*c)*I
ntegral(f, x)/f + a*b*x**6*(a*h + b*e)/2 + 3*a*b*x**5*(a*g + b*d)
/5 + 3*a*b*x**4*(a*f + b*c)/4 + b**3*f*x**10/10 + b**3*g*x**11/11
 + b**3*h*x**12/12 + b**2*x**9*(3*a*h + b*e)/9 + b**2*x**8*(3*a*g
 + b*d)/8 + b**2*x**7*(3*a*f + b*c)/7
```

Mathematica [A] time = 0.499923, size = 174, normalized size = 0.88

$$\frac{a^3\left(-3c-6dx+x^3\left(6f+3gx+2hx^2\right)\right)}{6x^2}+a^3e\log(x)+\frac{1}{20}a^2bx\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+\frac{1}{840}ab^2x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))+\frac{b^3x^7\left(3960c+7x\left(495d+440ex+6x^2\left(66f+60gx+55hx^2\right)\right)\right)}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^3*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (b^3
*x^7*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h
*x^2))))/27720 + (a^2*b*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12
*g*x^2 + 10*h*x^3))))/20 + (a*b^2*x^4*(630*c + x*(504*d + 5*x*(84
*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/840 + a^3*e*Log[x]
```

Maple [A] time = 0.011, size = 222, normalized size = 1.1

$$\frac{b^3hx^{12}}{12}+\frac{b^3gx^{11}}{11}+\frac{b^3fx^{10}}{10}+\frac{x^9ab^2h}{3}+\frac{b^3ex^9}{9}+\frac{3\,x^8ab^2g}{8}+\frac{x^8b^3d}{8}+\frac{3\,x^7ab^2f}{7}+\frac{x^7b^3c}{7}+\frac{x^6a^2bh}{2}+\frac{ab^2ex^6}{2}+\frac{3\,x^5a^2bg}{5}+\frac{3\,x^5ab^2d}{5}+\frac{3\,x^4a^2bf}{4}+\frac{3\,x^4ab^2c}{4}+\frac{x^3a^3h}{3}+a^2bex^3+\frac{x^2a^3g}{2}+\frac{3\,x^2a^2bd}{2}+a^3fx+3\,xa^2bc+a^3e\ln(x)-\frac{a^3c}{2\,x^2}-\frac{a^3d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] 1/12*b^3*h*x^12+1/11*b^3*g*x^11+1/10*b^3*f*x^10+1/3*x^9*a*b^2*h+1
/9*b^3*e*x^9+3/8*x^8*a*b^2*g+1/8*x^8*b^3*d+3/7*x^7*a*b^2*f+1/7*x^
7*b^3*c+1/2*x^6*a^2*b*h+1/2*a*b^2*e*x^6+3/5*x^5*a^2*b*g+3/5*x^5*a
*b^2*d+3/4*x^4*a^2*b*f+3/4*x^4*a*b^2*c+1/3*x^3*a^3*h+a^2*b*e*x^3+
1/2*x^2*a^3*g+3/2*x^2*a^2*b*d+a^3*f*x+3*x*a^2*b*c+a^3*e*ln(x)-1/2
*a^3*c/x^2-a^3*d/x
```

Maxima [A] time = 1.38317, size = 286, normalized size = 1.44

$$\frac{1}{12}\,b^3hx^{12}+\frac{1}{11}\,b^3gx^{11}+\frac{1}{10}\,b^3fx^{10}+\frac{1}{9}\left(b^3e+3\,ab^2h\right)x^9+\frac{1}{8}\left(b^3d+3\,ab^2g\right)x^8+\frac{1}{7}\left(b^3c+3\,ab^2f\right)x^7+\frac{1}{2}\left(ab^2e+a^2bh\right)x^6+\frac{3}{5}\left(ab^2d+a^2bg\right)x^5+\frac{3}{4}\left(ab^2c+a^2bf\right)x^4+a^3e\log(x)+\frac{1}{3}\left(3\,a^2be+a^3h\right)x^3+\frac{1}{2}\left(3\,a^2bd+a^3g\right)x^2+\left(3\,a^2bc+a^3f\right)x-\frac{2\,a^3dx+a^3c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^3,x, algorithm
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/9*(b^3*e
+ 3*a*b^2*h)*x^9 + 1/8*(b^3*d + 3*a*b^2*g)*x^8 + 1/7*(b^3*c + 3*a
*b^2*f)*x^7 + 1/2*(a*b^2*e + a^2*b*h)*x^6 + 3/5*(a*b^2*d + a^2*b*
g)*x^5 + 3/4*(a*b^2*c + a^2*b*f)*x^4 + a^3*e*log(x) + 1/3*(3*a^2*
b*e + a^3*h)*x^3 + 1/2*(3*a^2*b*d + a^3*g)*x^2 + (3*a^2*b*c + a^3
*f)*x - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Fricas [A] time = 0.233693, size = 296, normalized size = 1.49

$$\frac{2310\,b^3hx^{14} + 2520\,b^3gx^{13} + 2772\,b^3fx^{12} + 3080\left(b^3e + 3\,ab^2h\right)x^{11} + 3465\left(b^3d + 3\,ab^2g\right)x^{10} + 3960\left(b^3c + 3\,ab^2f\right)x^9 + 138}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^3,x, algorithm
```

```
[Out] 1/27720*(2310*b^3*h*x^14 + 2520*b^3*g*x^13 + 2772*b^3*f*x^12 + 30
80*(b^3*e + 3*a*b^2*h)*x^11 + 3465*(b^3*d + 3*a*b^2*g)*x^10 + 396
0*(b^3*c + 3*a*b^2*f)*x^9 + 13860*(a*b^2*e + a^2*b*h)*x^8 + 16632
*(a*b^2*d + a^2*b*g)*x^7 + 20790*(a*b^2*c + a^2*b*f)*x^6 + 27720*
a^3*e*x^2*log(x) + 9240*(3*a^2*b*e + a^3*h)*x^5 - 27720*a^3*d*x +
 13860*(3*a^2*b*d + a^3*g)*x^4 - 13860*a^3*c + 27720*(3*a^2*b*c +
 a^3*f)*x^3)/x^2
```

Sympy [A] time = 1.42572, size = 236, normalized size = 1.19

$$a^3e\log(x) + \frac{b^3fx^{10}}{10} + \frac{b^3gx^{11}}{11} + \frac{b^3hx^{12}}{12} + x^9\left(\frac{ab^2h}{3} + \frac{b^3e}{9}\right) + x^8\left(\frac{3ab^2g}{8} + \frac{b^3d}{8}\right) + x^7\left(\frac{3ab^2f}{7} + \frac{b^3c}{7}\right) + x^6\left(\frac{a^2bh}{2} + \frac{ab^2e}{2}\right) + x^5\left(\frac{3a^2bg}{5} + \frac{3ab^2d}{5}\right) + x^4\left(\frac{3a^2bf}{4} + \frac{3ab^2c}{4}\right) + x^3\left(\frac{a^3h}{3} + a^2be\right) + x^2\left(\frac{a^3g}{2} + \frac{3a^2bd}{2}\right) + x\left(a^3f + 3a^2bc\right) - \frac{a^3c + 2a^3dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**3*e*log(x) + b**3*f*x**10/10 + b**3*g*x**11/11 + b**3*h*x**12/
12 + x**9*(a*b**2*h/3 + b**3*e/9) + x**8*(3*a*b**2*g/8 + b**3*d/8
) + x**7*(3*a*b**2*f/7 + b**3*c/7) + x**6*(a**2*b*h/2 + a*b**2*e/
2) + x**5*(3*a**2*b*g/5 + 3*a*b**2*d/5) + x**4*(3*a**2*b*f/4 + 3*
a*b**2*c/4) + x**3*(a**3*h/3 + a**2*b*e) + x**2*(a**3*g/2 + 3*a**
2*b*d/2) + x*(a**3*f + 3*a**2*b*c) - (a**3*c + 2*a**3*d*x)/(2*x**
2)
```

GIAC/XCAS [A] time = 0.218421, size = 305, normalized size = 1.54

$$\frac{1}{12}\,b^3hx^{12} + \frac{1}{11}\,b^3gx^{11} + \frac{1}{10}\,b^3fx^{10} + \frac{1}{3}\,ab^2hx^9 + \frac{1}{9}\,b^3x^9e + \frac{1}{8}\,b^3dx^8 + \frac{3}{8}\,ab^2gx^8 + \frac{1}{7}\,b^3cx^7 + \frac{3}{7}\,ab^2fx^7 + \frac{1}{2}\,a^2bhx^6 + \frac{1}{2}\,ab^2x^6e + \frac{3}{5}\,ab^2dx^5 + \frac{3}{5}\,a^2bgx^5 + \frac{3}{4}\,ab^2cx^4 + \frac{3}{4}\,a^2bfx^4 + \frac{1}{3}\,a^3hx^3 + a^2bx^3e + \frac{3}{2}\,a^2bdx^2 + \frac{1}{2}\,a^3gx^2 + 3\,a^2bcx + a^3fx + a^3e\ln\left(|x|\right) - \frac{2\,a^3dx + a^3c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^3,x, algorithm
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/3*a*b^2*h
*x^9 + 1/9*b^3*x^9*e + 1/8*b^3*d*x^8 + 3/8*a*b^2*g*x^8 + 1/7*b^3*
c*x^7 + 3/7*a*b^2*f*x^7 + 1/2*a^2*b*h*x^6 + 1/2*a*b^2*x^6*e + 3/5
*a*b^2*d*x^5 + 3/5*a^2*b*g*x^5 + 3/4*a*b^2*c*x^4 + 3/4*a^2*b*f*x^
4 + 1/3*a^3*h*x^3 + a^2*b*x^3*e + 3/2*a^2*b*d*x^2 + 1/2*a^3*g*x^2
 + 3*a^2*b*c*x + a^3*f*x + a^3*e*ln(abs(x)) - 1/2*(2*a^3*d*x + a^
3*c)/x^2
```

3.389 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

Optimal. Leaf size=209

$$-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2\log(x)(af+3bc)+a^2x(ag+3bd)+\frac{1}{2}a^2x^2(ah+3be)+\frac{1}{6}b^2x^6(3af+bc)+\frac{1}{7}b^2x^7(3ag+bd)+\frac{1}{8}b^2x^8(3ah+be)+abx^3(af+bc)+\frac{3}{4}abx^4(ag+bd)+\frac{3}{5}abx^5(ah+be)+\frac{1}{9}b^3fx^9+\frac{1}{10}b^3gx^{10}+\frac{1}{11}b^3hx^{11}$$

```
[Out] -(a^3*c)/(3*x^3) - (a^3*d)/(2*x^2) - (a^3*e)/x + a^2*(3*b*d + a*g
)*x + (a^2*(3*b*e + a*h)*x^2)/2 + a*b*(b*c + a*f)*x^3 + (3*a*b*(b
*d + a*g)*x^4)/4 + (3*a*b*(b*e + a*h)*x^5)/5 + (b^2*(b*c + 3*a*f)
*x^6)/6 + (b^2*(b*d + 3*a*g)*x^7)/7 + (b^2*(b*e + 3*a*h)*x^8)/8 +
 (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*h*x^11)/11 + a^2*(3*b*c +
 a*f)*Log[x]
```

Rubi [A] time = 0.485404, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026

$$-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2\log(x)(af+3bc)+a^2x(ag+3bd)+\frac{1}{2}a^2x^2(ah+3be)+\frac{1}{6}b^2x^6(3af+bc)+\frac{1}{7}b^2x^7(3ag+bd)+\frac{1}{8}b^2x^8(3ah+be)+abx^3(af+bc)+\frac{3}{4}abx^4(ag+bd)+\frac{3}{5}abx^5(ah+be)+\frac{1}{9}b^3fx^9+\frac{1}{10}b^3gx^{10}+\frac{1}{11}b^3hx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^3*c)/(3*x^3) - (a^3*d)/(2*x^2) - (a^3*e)/x + a^2*(3*b*d + a*g
)*x + (a^2*(3*b*e + a*h)*x^2)/2 + a*b*(b*c + a*f)*x^3 + (3*a*b*(b
*d + a*g)*x^4)/4 + (3*a*b*(b*e + a*h)*x^5)/5 + (b^2*(b*c + 3*a*f)
*x^6)/6 + (b^2*(b*d + 3*a*g)*x^7)/7 + (b^2*(b*e + 3*a*h)*x^8)/8 +
 (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*h*x^11)/11 + a^2*(3*b*c +
 a*f)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2\left(af+3bc\right)\log\left(x\right)+a^2\left(ah+3be\right)\int x\,dx+\frac{a^2\left(ag+3bd\right)\int g\,dx}{g}+\frac{3abx^5\left(ah+be\right)}{5}+\frac{3abx^4\left(ag+bd\right)}{4}+abx^3\left(af+bc\right)+\frac{b^3fx^9}{9}+\frac{b^3gx^{10}}{10}+\frac{b^3hx^{11}}{11}+\frac{b^2x^8\left(3ah+be\right)}{8}+\frac{b^2x^7\left(3ag+bd\right)}{7}+\frac{b^2x^6\left(3af+bc\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] -a**3*c/(3*x**3) - a**3*d/(2*x**2) - a**3*e/x + a**2*(a*f + 3*b*c
)*log(x) + a**2*(a*h + 3*b*e)*Integral(x, x) + a**2*(a*g + 3*b*d)
*Integral(g, x)/g + 3*a*b*x**5*(a*h + b*e)/5 + 3*a*b*x**4*(a*g +
b*d)/4 + a*b*x**3*(a*f + b*c) + b**3*f*x**9/9 + b**3*g*x**10/10 +
 b**3*h*x**11/11 + b**2*x**8*(3*a*h + b*e)/8 + b**2*x**7*(3*a*g +
 b*d)/7 + b**2*x**6*(3*a*f + b*c)/6
```

Mathematica [A] time = 0.238397, size = 172, normalized size = 0.82

$$-\frac{a^3\left(2c+3x\left(d+2ex+x^3(-(2g+hx))\right)\right)}{6x^3}+a^2\log(x)(af+3bc)+\frac{1}{20}a^2bx\left(60d+x\left(30e+x\left(20f+15gx+12hx^2\right)\right)\right)+\frac{1}{280}ab^2x^3\left(280c+x\left(210d+x\left(168e+140fx+120gx^2+105hx^3\right)\right)\right)+\frac{b^3x^6\left(4620c+x\left(3960d+7x\left(495e+4x\left(110f+99gx+90hx^2\right)\right)\right)\right)}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^3*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/(6*x^3) + (a^2*b
*x*(60*d + x*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/20 + (a*b^2*
x^3*(280*c + x*(210*d + x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^
3))))/280 + (b^3*x^6*(4620*c + x*(3960*d + 7*x*(495*e + 4*x*(110*
f + 99*g*x + 90*h*x^2)))))/27720 + a^2*(3*b*c + a*f)*Log[x]
```

Maple [A] time = 0.01, size = 220, normalized size = 1.1

$$\frac{b^3hx^{11}}{11}+\frac{b^3gx^{10}}{10}+\frac{b^3fx^9}{9}+\frac{3x^8ab^2h}{8}+\frac{x^8b^3e}{8}+\frac{3x^7ab^2g}{7}+\frac{x^7b^3d}{7}+\frac{x^6ab^2f}{2}+\frac{x^6b^3c}{6}+\frac{3x^5a^2bh}{5}+\frac{3x^5ab^2e}{5}+\frac{3x^4a^2bg}{4}+\frac{3x^4ab^2d}{4}+x^3a^2bf+x^3ab^2c+\frac{x^2a^3h}{2}+\frac{3x^2a^2be}{2}+xa^3g+3xa^2bd+\ln(x)a^3f+3\ln(x)a^2bc-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] 1/11*b^3*h*x^11+1/10*b^3*g*x^10+1/9*b^3*f*x^9+3/8*x^8*a*b^2*h+1/8
*x^8*b^3*e+3/7*x^7*a*b^2*g+1/7*x^7*b^3*d+1/2*x^6*a*b^2*f+1/6*x^6*
b^3*c+3/5*x^5*a^2*b*h+3/5*x^5*a*b^2*e+3/4*x^4*a^2*b*g+3/4*x^4*a*b
^2*d+x^3*a^2*b*f+x^3*a*b^2*c+1/2*x^2*a^3*h+3/2*x^2*a^2*b*e+x*a^3*
g+3*x*a^2*b*d+ln(x)*a^3*f+3*ln(x)*a^2*b*c-1/3*a^3*c/x^3-1/2*a^3*d
/x^2-a^3*e/x
```

Maxima [A] time = 1.38419, size = 286, normalized size = 1.37

$$\frac{1}{11}b^3hx^{11}+\frac{1}{10}b^3gx^{10}+\frac{1}{9}b^3fx^9+\frac{1}{8}\left(b^3e+3ab^2h\right)x^8+\frac{1}{7}\left(b^3d+3ab^2g\right)x^7+\frac{1}{6}\left(b^3c+3ab^2f\right)x^6+\frac{3}{5}\left(ab^2e+a^2bh\right)x^5+\frac{3}{4}\left(ab^2d+a^2bg\right)x^4+\left(ab^2c+a^2bf\right)x^3+\frac{1}{2}\left(3a^2be+a^3h\right)x^2+\left(3a^2bd+a^3g\right)x+\left(3a^2bc+a^3f\right)\log(x)-\frac{6a^3ex^2+3a^3dx+2a^3c}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^4,x, algorithm
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 1/8*(b^3*e +
3*a*b^2*h)*x^8 + 1/7*(b^3*d + 3*a*b^2*g)*x^7 + 1/6*(b^3*c + 3*a*b
^2*f)*x^6 + 3/5*(a*b^2*e + a^2*b*h)*x^5 + 3/4*(a*b^2*d + a^2*b*g)
*x^4 + (a*b^2*c + a^2*b*f)*x^3 + 1/2*(3*a^2*b*e + a^3*h)*x^2 + (3
*a^2*b*d + a^3*g)*x + (3*a^2*b*c + a^3*f)*log(x) - 1/6*(6*a^3*e*x
^2 + 3*a^3*d*x + 2*a^3*c)/x^3
```

Fricas [A] time = 0.246329, size = 296, normalized size = 1.42

$$\frac{2520\,b^3hx^{14} + 2772\,b^3gx^{13} + 3080\,b^3fx^{12} + 3465\left(b^3e + 3\,ab^2h\right)x^{11} + 3960\left(b^3d + 3\,ab^2g\right)x^{10} + 4620\left(b^3c + 3\,ab^2f\right)x^9 + 166}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^4,x, algorithm
```

```
[Out] 1/27720*(2520*b^3*h*x^14 + 2772*b^3*g*x^13 + 3080*b^3*f*x^12 + 34
65*(b^3*e + 3*a*b^2*h)*x^11 + 3960*(b^3*d + 3*a*b^2*g)*x^10 + 462
0*(b^3*c + 3*a*b^2*f)*x^9 + 16632*(a*b^2*e + a^2*b*h)*x^8 + 20790
*(a*b^2*d + a^2*b*g)*x^7 + 27720*(a*b^2*c + a^2*b*f)*x^6 - 27720*
a^3*e*x^2 + 13860*(3*a^2*b*e + a^3*h)*x^5 - 13860*a^3*d*x + 27720
*(3*a^2*b*d + a^3*g)*x^4 + 27720*(3*a^2*b*c + a^3*f)*x^3*log(x) -
 9240*a^3*c)/x^3
```

Sympy [A] time = 2.19441, size = 235, normalized size = 1.12

$$a^2\left(af + 3bc\right)\log\left(x\right) + \frac{b^3fx^9}{9} + \frac{b^3gx^{10}}{10} + \frac{b^3hx^{11}}{11} + x^8\left(\frac{3ab^2h}{8} + \frac{b^3e}{8}\right) + x^7\left(\frac{3ab^2g}{7} + \frac{b^3d}{7}\right) + x^6\left(\frac{ab^2f}{2} + \frac{b^3c}{6}\right) + x^5\left(\frac{3a^2bh}{5} + \frac{3ab^2e}{5}\right) + x^4\left(\frac{3a^2bg}{4} + \frac{3ab^2d}{4}\right) + x^3\left(a^2bf + ab^2c\right) + x^2\left(\frac{a^3h}{2} + \frac{3a^2be}{2}\right) + x\left(a^3g + 3a^2bd\right) - \frac{2a^3c + 3a^3dx + 6a^3ex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a**2*(a*f + 3*b*c)*log(x) + b**3*f*x**9/9 + b**3*g*x**10/10 + b**
3*h*x**11/11 + x**8*(3*a*b**2*h/8 + b**3*e/8) + x**7*(3*a*b**2*g/
7 + b**3*d/7) + x**6*(a*b**2*f/2 + b**3*c/6) + x**5*(3*a**2*b*h/5
 + 3*a*b**2*e/5) + x**4*(3*a**2*b*g/4 + 3*a*b**2*d/4) + x**3*(a**
2*b*f + a*b**2*c) + x**2*(a**3*h/2 + 3*a**2*b*e/2) + x*(a**3*g +
3*a**2*b*d) - (2*a**3*c + 3*a**3*d*x + 6*a**3*e*x**2)/(6*x**3)
```

GIAC/XCAS [A] time = 0.218201, size = 304, normalized size = 1.45

$$\frac{1}{11}\,b^3hx^{11} + \frac{1}{10}\,b^3gx^{10} + \frac{1}{9}\,b^3fx^9 + \frac{3}{8}\,ab^2hx^8 + \frac{1}{8}\,b^3x^8e + \frac{1}{7}\,b^3dx^7 + \frac{3}{7}\,ab^2gx^7 + \frac{1}{6}\,b^3cx^6 + \frac{1}{2}\,ab^2fx^6 + \frac{3}{5}\,a^2bhx^5 + \frac{3}{5}\,ab^2x^5e + \frac{3}{4}\,ab^2dx^4 + \frac{3}{4}\,a^2bgx^4 + ab^2cx^3 + a^2bfx^3 + \frac{1}{2}\,a^3hx^2 + \frac{3}{2}\,a^2bx^2e + 3\,a^2bdx + a^3gx + \left(3\,a^2bc + a^3f\right)\ln\left(|x|\right) - \frac{6\,a^3x^2e + 3\,a^3dx + 2\,a^3c}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^4,x, algorithm
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 3/8*a*b^2*h*x
^8 + 1/8*b^3*x^8*e + 1/7*b^3*d*x^7 + 3/7*a*b^2*g*x^7 + 1/6*b^3*c*
x^6 + 1/2*a*b^2*f*x^6 + 3/5*a^2*b*h*x^5 + 3/5*a*b^2*x^5*e + 3/4*a
*b^2*d*x^4 + 3/4*a^2*b*g*x^4 + a*b^2*c*x^3 + a^2*b*f*x^3 + 1/2*a^
3*h*x^2 + 3/2*a^2*b*x^2*e + 3*a^2*b*d*x + a^3*g*x + (3*a^2*b*c +
a^3*f)*ln(abs(x)) - 1/6*(6*a^3*x^2*e + 3*a^3*d*x + 2*a^3*c)/x^3
```

3.390 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(af+3bc)}{x}+a^2\log(x)(ag+3bd)+a^2x(ah+3be)+\frac{1}{5}b^2x^5(3af+bc)+\frac{1}{6}b^2x^6(3ag+bd)+\frac{1}{7}b^2x^7(3ah+be)+\frac{3}{2}abx^2(af+bc)+abx^3(ag+bd)+\frac{3}{4}abx^4(ah+be)+\frac{1}{8}b^3fx^8+\frac{1}{9}b^3gx^9+\frac{1}{10}b^3hx^{10}$$

```
[Out] -(a^3*c)/(4*x^4) - (a^3*d)/(3*x^3) - (a^3*e)/(2*x^2) - (a^2*(3*b*
c + a*f))/x + a^2*(3*b*e + a*h)*x + (3*a*b*(b*c + a*f)*x^2)/2 + a
*b*(b*d + a*g)*x^3 + (3*a*b*(b*e + a*h)*x^4)/4 + (b^2*(b*c + 3*a*
f)*x^5)/5 + (b^2*(b*d + 3*a*g)*x^6)/6 + (b^2*(b*e + 3*a*h)*x^7)/7
 + (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*h*x^10)/10 + a^2*(3*b*d +
 a*g)*Log[x]
```

Rubi [A] time = 0.479195, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026

$$-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(af+3bc)}{x}+a^2\log(x)(ag+3bd)+a^2x(ah+3be)+\frac{1}{5}b^2x^5(3af+bc)+\frac{1}{6}b^2x^6(3ag+bd)+\frac{1}{7}b^2x^7(3ah+be)+\frac{3}{2}abx^2(af+bc)+abx^3(ag+bd)+\frac{3}{4}abx^4(ah+be)+\frac{1}{8}b^3fx^8+\frac{1}{9}b^3gx^9+\frac{1}{10}b^3hx^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -(a^3*c)/(4*x^4) - (a^3*d)/(3*x^3) - (a^3*e)/(2*x^2) - (a^2*(3*b*
c + a*f))/x + a^2*(3*b*e + a*h)*x + (3*a*b*(b*c + a*f)*x^2)/2 + a
*b*(b*d + a*g)*x^3 + (3*a*b*(b*e + a*h)*x^4)/4 + (b^2*(b*c + 3*a*
f)*x^5)/5 + (b^2*(b*d + 3*a*g)*x^6)/6 + (b^2*(b*e + 3*a*h)*x^7)/7
 + (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*h*x^10)/10 + a^2*(3*b*d +
 a*g)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}+a^2(ag+3bd)\log(x)-\frac{a^2(af+3bc)}{x}+\frac{a^2(ah+3be)\int h\,dx}{h}+\frac{3abx^4(ah+be)}{4}+abx^3(ag+bd)+3ab(af+bc)\int x\,dx+\frac{b^3fx^8}{8}+\frac{b^3gx^9}{9}+\frac{b^3hx^{10}}{10}+\frac{b^2x^7(3ah+be)}{7}+\frac{b^2x^6(3ag+bd)}{6}+\frac{b^2x^5(3af+bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] -a**3*c/(4*x**4) - a**3*d/(3*x**3) - a**3*e/(2*x**2) + a**2*(a*g
+ 3*b*d)*log(x) - a**2*(a*f + 3*b*c)/x + a**2*(a*h + 3*b*e)*Integ
ral(h, x)/h + 3*a*b*x**4*(a*h + b*e)/4 + a*b*x**3*(a*g + b*d) + 3
*a*b*(a*f + b*c)*Integral(x, x) + b**3*f*x**8/8 + b**3*g*x**9/9 +
 b**3*h*x**10/10 + b**2*x**7*(3*a*h + b*e)/7 + b**2*x**6*(3*a*g +
 b*d)/6 + b**2*x**5*(3*a*f + b*c)/5
```

Mathematica [A] time = 0.328199, size = 170, normalized size = 0.81

$$a^2\log(x)(ag+3bd)+\frac{-210a^3\left(3c+4dx+6x^2\left(e+2fx-2hx^3\right)\right)+630a^2bx^3\left(x^2\left(12e+6fx+4gx^2+3hx^3\right)-12c\right)+18ab^2x^6\left(210c+x\left(140d\right.\right.}{2520x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] (-210*a^3*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)) + 630*a^2*b
*x^3*(-12*c + x^2*(12*e + 6*f*x + 4*g*x^2 + 3*h*x^3)) + 18*a*b^2*
x^6*(210*c + x*(140*d + 105*e*x + 84*f*x^2 + 70*g*x^3 + 60*h*x^4)
) + b^3*x^9*(504*c + x*(420*d + 360*e*x + 315*f*x^2 + 280*g*x^3 +
 252*h*x^4)))/(2520*x^4) + a^2*(3*b*d + a*g)*Log[x]
```

Maple [A] time = 0.011, size = 220, normalized size = 1.1

$$\frac{b^3hx^{10}}{10}+\frac{b^3gx^9}{9}+\frac{b^3fx^8}{8}+\frac{3x^7ab^2h}{7}+\frac{x^7b^3e}{7}+\frac{x^6ab^2g}{2}+\frac{x^6b^3d}{6}+\frac{3x^5ab^2f}{5}+\frac{x^5b^3c}{5}+\frac{3x^4a^2bh}{4}+\frac{3x^4ab^2e}{4}+x^3a^2bg+ab^2dx^3+\frac{3x^2a^2bf}{2}+\frac{3cx^2ab^2}{2}+xa^3h+3a^2bex+\ln(x)a^3g+3\ln(x)a^2bd-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^3f}{x}-3\frac{a^2bc}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] 1/10*b^3*h*x^10+1/9*b^3*g*x^9+1/8*b^3*f*x^8+3/7*x^7*a*b^2*h+1/7*x
^7*b^3*e+1/2*x^6*a*b^2*g+1/6*x^6*b^3*d+3/5*x^5*a*b^2*f+1/5*x^5*b^
3*c+3/4*x^4*a^2*b*h+3/4*x^4*a*b^2*e+x^3*a^2*b*g+a*b^2*d*x^3+3/2*x
^2*a^2*b*f+3/2*c*x^2*a*b^2+x*a^3*h+3*a^2*b*e*x+ln(x)*a^3*g+3*ln(x
)*a^2*b*d-1/4*a^3*c/x^4-1/3*a^3*d/x^3-1/2*a^3*e/x^2-a^3/x*f-3*a^2
/x*b*c
```

Maxima [A] time = 1.38383, size = 286, normalized size = 1.37

$$\frac{1}{10}b^3hx^{10}+\frac{1}{9}b^3gx^9+\frac{1}{8}b^3fx^8+\frac{1}{7}\left(b^3e+3ab^2h\right)x^7+\frac{1}{6}\left(b^3d+3ab^2g\right)x^6+\frac{1}{5}\left(b^3c+3ab^2f\right)x^5+\frac{3}{4}\left(ab^2e+a^2bh\right)x^4+\left(ab^2d+a^2bg\right)x^3+\frac{3}{2}\left(ab^2c+a^2bf\right)x^2+\left(3a^2be+a^3h\right)x+\left(3a^2bd+a^3g\right)\log(x)-\frac{6a^3ex^2+4a^3dx+3a^3c+12\left(3a^2bc+a^3f\right)x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^5,x, algorithm
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 1/7*(b^3*e + 3*
a*b^2*h)*x^7 + 1/6*(b^3*d + 3*a*b^2*g)*x^6 + 1/5*(b^3*c + 3*a*b^2
*f)*x^5 + 3/4*(a*b^2*e + a^2*b*h)*x^4 + (a*b^2*d + a^2*b*g)*x^3 +
 3/2*(a*b^2*c + a^2*b*f)*x^2 + (3*a^2*b*e + a^3*h)*x + (3*a^2*b*d
 + a^3*g)*log(x) - 1/12*(6*a^3*e*x^2 + 4*a^3*d*x + 3*a^3*c + 12*(
3*a^2*b*c + a^3*f)*x^3)/x^4
```

Fricas [A] time = 0.246749, size = 296, normalized size = 1.42

$$\frac{252b^3hx^{14}+280b^3gx^{13}+315b^3fx^{12}+360\left(b^3e+3ab^2h\right)x^{11}+420\left(b^3d+3ab^2g\right)x^{10}+504\left(b^3c+3ab^2f\right)x^9+1890\left(ab^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^5,x, algorithm
```

```
[Out] 1/2520*(252*b^3*h*x^14 + 280*b^3*g*x^13 + 315*b^3*f*x^12 + 360*(b
^3*e + 3*a*b^2*h)*x^11 + 420*(b^3*d + 3*a*b^2*g)*x^10 + 504*(b^3*
c + 3*a*b^2*f)*x^9 + 1890*(a*b^2*e + a^2*b*h)*x^8 + 2520*(a*b^2*d
 + a^2*b*g)*x^7 + 3780*(a*b^2*c + a^2*b*f)*x^6 - 1260*a^3*e*x^2 +
 2520*(3*a^2*b*e + a^3*h)*x^5 + 2520*(3*a^2*b*d + a^3*g)*x^4*log(
x) - 840*a^3*d*x - 630*a^3*c - 2520*(3*a^2*b*c + a^3*f)*x^3)/x^4
```

Sympy [A] time = 6.36861, size = 233, normalized size = 1.11

$$a^2\left(ag+3bd\right)\log\left(x\right)+\frac{b^3fx^8}{8}+\frac{b^3gx^9}{9}+\frac{b^3hx^{10}}{10}+x^7\left(\frac{3ab^2h}{7}+\frac{b^3e}{7}\right)+x^6\left(\frac{ab^2g}{2}+\frac{b^3d}{6}\right)+x^5\left(\frac{3ab^2f}{5}+\frac{b^3c}{5}\right)+x^4\left(\frac{3a^2bh}{4}+\frac{3ab^2e}{4}\right)+x^3\left(a^2bg+ab^2d\right)+x^2\left(\frac{3a^2bf}{2}+\frac{3ab^2c}{2}\right)+x\left(a^3h+3a^2be\right)-\frac{3a^3c+4a^3dx+6a^3ex^2+x^3\left(12a^3f+36a^2bc\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a**2*(a*g + 3*b*d)*log(x) + b**3*f*x**8/8 + b**3*g*x**9/9 + b**3*
h*x**10/10 + x**7*(3*a*b**2*h/7 + b**3*e/7) + x**6*(a*b**2*g/2 +
b**3*d/6) + x**5*(3*a*b**2*f/5 + b**3*c/5) + x**4*(3*a**2*b*h/4 +
 3*a*b**2*e/4) + x**3*(a**2*b*g + a*b**2*d) + x**2*(3*a**2*b*f/2
+ 3*a*b**2*c/2) + x*(a**3*h + 3*a**2*b*e) - (3*a**3*c + 4*a**3*d*
x + 6*a**3*e*x**2 + x**3*(12*a**3*f + 36*a**2*b*c))/(12*x**4)
```

GIAC/XCAS [A] time = 0.220889, size = 302, normalized size = 1.44

$$\frac{1}{10}b^3hx^{10}+\frac{1}{9}b^3gx^9+\frac{1}{8}b^3fx^8+\frac{3}{7}ab^2hx^7+\frac{1}{7}b^3x^7e+\frac{1}{6}b^3dx^6+\frac{1}{2}ab^2gx^6+\frac{1}{5}b^3cx^5+\frac{3}{5}ab^2fx^5+\frac{3}{4}a^2bhx^4+\frac{3}{4}ab^2x^4e+ab^2dx^3+a^2bgx^3+\frac{3}{2}ab^2cx^2+\frac{3}{2}a^2bfx^2+a^3hx+3\,a^2bxe+\left(3\,a^2bd+a^3g\right)\ln\left(|x|\right)-\frac{6\,a^3x^2e+4\,a^3dx+3\,a^3c+12\left(3\,a^2bc+a^3f\right)x^3}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^3/x^5,x, algorithm
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 3/7*a*b^2*h*x^7
 + 1/7*b^3*x^7*e + 1/6*b^3*d*x^6 + 1/2*a*b^2*g*x^6 + 1/5*b^3*c*x^
5 + 3/5*a*b^2*f*x^5 + 3/4*a^2*b*h*x^4 + 3/4*a*b^2*x^4*e + a*b^2*d
*x^3 + a^2*b*g*x^3 + 3/2*a*b^2*c*x^2 + 3/2*a^2*b*f*x^2 + a^3*h*x
+ 3*a^2*b*x*e + (3*a^2*b*d + a^3*g)*ln(abs(x)) - 1/12*(6*a^3*x^2*
e + 4*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*x^3)/x^4
```

3.391 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=331

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6b^{10/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3b^{10/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}b^{10/3}}-\frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3}-\frac{ax(be-ah)}{b^3}+\frac{x^2(bc-af)}{2b^2}+\frac{x^3(bd-ag)}{3b^2}+\frac{x^4(be-ah)}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}$$

```
[Out] -((a*(b*e - a*h)*x)/b^3) + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*
g)*x^3)/(3*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*
x^6)/(6*b) + (h*x^7)/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e -
a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*b^(10/3)) + (a^(2/3)*(b^(2/3)*(b*c - a*f) + a
^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) - (a^(
2/3)*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) - (a*(b*d - a*g)*Log
[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 2.02567, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6b^{10/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3b^{10/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}b^{10/3}}-\frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3}-\frac{ax(be-ah)}{b^3}+\frac{x^2(bc-af)}{2b^2}+\frac{x^3(bd-ag)}{3b^2}+\frac{x^4(be-ah)}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```


```
[Out] -((a*(b*e - a*h)*x)/b^3) + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*
g)*x^3)/(3*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*
x^6)/(6*b) + (h*x^7)/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e -
a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*b^(10/3)) + (a^(2/3)*(b^(2/3)*(b*c - a*f) + a
^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) - (a^(
2/3)*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) - (a*(b*d - a*g)*Log
[a + b*x^3])/(3*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{3}a^{\frac{2}{3}}\left(a^{\frac{2}{3}}(ah-be)-b^{\frac{2}{3}}(af-bc)\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{10}{3}}}-\frac{a^{\frac{2}{3}}\left(a^{\frac{2}{3}}(ah-be)+b^{\frac{2}{3}}(af-bc)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{10}{3}}}+\frac{a^{\frac{2}{3}}\left(a^{\frac{2}{3}}(ah-be)+b^{\frac{2}{3}}(af-bc)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{10}{3}}}+\frac{a(ag-bd)\log\left(a+bx^3\right)}{3b^3}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}-\frac{x^4(ah-be)}{4b^2}-\frac{x^3(ag-bd)}{3b^2}-\frac{(af-bc)\int x\,dx}{b^2}+\frac{(ah-be)\int a\,dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] sqrt(3)*a**(2/3)*(a**(2/3)*(a*h - b*e) - b**(2/3)*(a*f - b*c))*at
an(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*b**(10/3))
- a**(2/3)*(a**(2/3)*(a*h - b*e) + b**(2/3)*(a*f - b*c))*log(a**(
1/3) + b**(1/3)*x)/(3*b**(10/3)) + a**(2/3)*(a**(2/3)*(a*h - b*e)
 + b**(2/3)*(a*f - b*c))*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**
(2/3)*x**2)/(6*b**(10/3)) + a*(a*g - b*d)*log(a + b*x**3)/(3*b**3
) + f*x**5/(5*b) + g*x**6/(6*b) + h*x**7/(7*b) - x**4*(a*h - b*e)
/(4*b**2) - x**3*(a*g - b*d)/(3*b**2) - (a*f - b*c)*Integral(x, x
)/b**2 + (a*h - b*e)*Integral(a, x)/b**3
```

Mathematica [A] time = 1.23964, size = 334, normalized size = 1.01

$$\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}be+a^{5/3}h+ab^{2/3}f-b^{5/3}c\right)}{6b^{10/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+b^{5/3}c\right)}{3b^{10/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}b^{10/3}}+\frac{a(ag-bd)\log\left(a+bx^3\right)}{3b^3}+\frac{ax(ah-be)}{b^3}+\frac{x^2(bc-af)}{2b^2}+\frac{x^3(bd-ag)}{3b^2}+\frac{x^4(be-ah)}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (a*(-(b*e) + a*h)*x)/b^3 + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*
g)*x^3)/(3*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*
x^6)/(6*b) + (h*x^7)/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e -
a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/(Sqrt[3]*b^(10/3)) + (a^(2/3)*(b^(5/3)*c + a^(2/3)*b*e - a*b
^(2/3)*f - a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) + (a
^(2/3)*(-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) + (a*(-
(b*d) + a*g)*Log[a + b*x^3])/(3*b^3)
```

Maple [B] time = 0.009, size = 533, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/b^2*a*e*x+1/6*g*x^6/b+1/7*h*x^7/b-1/2/b^2*x^2*a*f+1/3*a/b^2/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/3/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*a^2*e-1/6*a/b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*
c+1/4/b*x^4*e-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-1/4/b^2
*x^4*a*h-1/3/b^2*x^3*a*g+1/3/b^3*a^2*ln(b*x^3+a)*g-1/3*a/b^2*3^(1
/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/2/b*x
^2*c+1/3/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))*a^2*e+1/3*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))*f+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/
3)+(a/b)^(2/3))*f-1/3/b^2*ln(b*x^3+a)*a*d+1/3/b*d*x^3-1/3/b^4*a^3
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h-1/
6/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^2*e-1/3/b^4
*a^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*h+1/5*f*x^5/b+1/b^3*x*a^2*h+1/
6/b^4*a^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*h
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^3 + a),x, algorithm="
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^3 + a),x, algorithm="
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 50.4846, size = 874, normalized size = 2.64

$$\operatorname{RootSum}\left(27t^3b^{10} + t^2\left(-27a^2b^7g + 27ab^8d\right) + t\left(-9a^4b^4fh + 9a^4b^4g^2 + 9a^3b^5ch - 18a^3b^5dg + 9a^3b^5ef - 9a^2b^6ce + 9a^2b^6\right.\right.$$

$$+ \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} - \frac{x^4(ah-be)}{4b^2} - \frac{x^3(ag-bd)}{3b^2} - \frac{x^2(af-bc)}{2b^2} + \frac{x(a^2h-abe)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**10 + _t**2*(-27*a**2*b**7*g + 27*a*b**8*d) +
_t*(-9*a**4*b**4*f*h + 9*a**4*b**4*g**2 + 9*a**3*b**5*c*h - 18*a*
*3*b**5*d*g + 9*a**3*b**5*e*f - 9*a**2*b**6*c*e + 9*a**2*b**6*d**
2) + a**7*h**3 - 3*a**6*b*e*h**2 + 3*a**6*b*f*g*h - a**6*b*g**3 -
 3*a**5*b**2*c*g*h - 3*a**5*b**2*d*f*h + 3*a**5*b**2*d*g**2 + 3*a
**5*b**2*e**2*h - 3*a**5*b**2*e*f*g + a**5*b**2*f**3 + 3*a**4*b**
3*c*d*h + 3*a**4*b**3*c*e*g - 3*a**4*b**3*c*f**2 - 3*a**4*b**3*d*
*2*g + 3*a**4*b**3*d*e*f - a**4*b**3*e**3 + 3*a**3*b**4*c**2*f -
3*a**3*b**4*c*d*e + a**3*b**4*d**3 - a**2*b**5*c**3, Lambda(_t, _
```

```
t*log(x + (-9*_t**2*a*b**7*f + 9*_t**2*b**8*c - 3*_t*a**4*b**3*h*
*2 + 6*_t*a**3*b**4*e*h + 6*_t*a**3*b**4*f*g - 6*_t*a**2*b**5*c*g
 - 6*_t*a**2*b**5*d*f - 3*_t*a**2*b**5*e**2 + 6*_t*a*b**6*c*d + a
**6*g*h**2 - a**5*b*d*h**2 - 2*a**5*b*e*g*h + 2*a**5*b*f**2*h - a
**5*b*f*g**2 - 4*a**4*b**2*c*f*h + a**4*b**2*c*g**2 + 2*a**4*b**2
*d*e*h + 2*a**4*b**2*d*f*g + a**4*b**2*e**2*g - 2*a**4*b**2*e*f**
2 + 2*a**3*b**3*c**2*h - 2*a**3*b**3*c*d*g + 4*a**3*b**3*c*e*f -
a**3*b**3*d**2*f - a**3*b**3*d*e**2 - 2*a**2*b**4*c**2*e + a**2*b
**4*c*d**2)/(a**6*h**3 - 3*a**5*b*e*h**2 + 3*a**4*b**2*e**2*h - a
**4*b**2*f**3 + 3*a**3*b**3*c*f**2 - a**3*b**3*e**3 - 3*a**2*b**4
*c**2*f + a*b**5*c**3)))) + f*x**5/(5*b) + g*x**6/(6*b) + h*x**7/
(7*b) - x**4*(a*h - b*e)/(4*b**2) - x**3*(a*g - b*d)/(3*b**2) - x
**2*(a*f - b*c)/(2*b**2) + x*(a**2*h - a*b*e)/b**3
```

GIAC/XCAS [A] time = 0.225604, size = 513, normalized size = 1.55

$$
\begin{aligned}
&-\frac{\left(abd-a^2g\right)\ln\left(\left|bx^3+a\right|\right)}{3\,b^3}\\
&-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h-\left(-ab^2\right)^{\frac{1}{3}}abe-\left(-ab^2\right)^{\frac{2}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}\\
&-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h-\left(-ab^2\right)^{\frac{1}{3}}abe+\left(-ab^2\right)^{\frac{2}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}af\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4}\\
&+\frac{60\,b^6hx^7+70\,b^6gx^6+84\,b^6fx^5-105\,ab^5hx^4+105\,b^6x^4e+140\,b^6dx^3-140\,ab^5gx^3+210\,b^6cx^2-210\,ab^5fx^2+420\,a^2b^4h}{420\,b^7}\\
&+\frac{\left(ab^{14}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^{13}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^{12}h-a^2b^{13}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{15}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^3 + a),x, algorithm="
```

```
[Out] -1/3*(a*b*d - a^2*g)*ln(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^
2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c + (-a*
b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/b^4 - 1/6*((-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a
*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*ln(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/b^4 + 1/420*(60*b^6*h*x^7 + 70*b^6*g*x^6 + 84*b^6*f*
x^5 - 105*a*b^5*h*x^4 + 105*b^6*x^4*e + 140*b^6*d*x^3 - 140*a*b^5
*g*x^3 + 210*b^6*c*x^2 - 210*a*b^5*f*x^2 + 420*a^2*b^4*h*x - 420*
a*b^5*x*e)/b^7 + 1/3*(a*b^14*c*(-a/b)^(1/3) - a^2*b^13*f*(-a/b)^(
1/3) + a^3*b^12*h - a^2*b^13*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1
/3)))/(a*b^15)
```

3.392 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=313

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}b^{8/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3b^{8/3}}-\frac{a(be-ah)\log\left(a+bx^3\right)}{3b^3}+\frac{x(bc-af)}{b^2}+\frac{x^2(bd-ag)}{2b^2}+\frac{x^3(be-ah)}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}$$

```
[Out] ((b*c - a*f)*x)/b^2 + ((b*d - a*g)*x^2)/(2*b^2) + ((b*e - a*h)*x^
3)/(3*b^2) + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6*b) + (a^(
1/3)*(b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a
^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(1/3) +
b^(1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)
*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*
b^(8/3)) - (a*(b*e - a*h)*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 1.97049, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}b^{8/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3b^{8/3}}-\frac{a(be-ah)\log\left(a+bx^3\right)}{3b^3}+\frac{x(bc-af)}{b^2}+\frac{x^2(bd-ag)}{2b^2}+\frac{x^3(be-ah)}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*c - a*f)*x)/b^2 + ((b*d - a*g)*x^2)/(2*b^2) + ((b*e - a*h)*x^
3)/(3*b^2) + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6*b) + (a^(
1/3)*(b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a
^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(1/3) +
b^(1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)
*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*
b^(8/3)) - (a*(b*e - a*h)*Log[a + b*x^3])/(3*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\,(ag-bd)-\sqrt[3]{b}\,(af-bc)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{8}{3}}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\,(ag-bd)-\sqrt[3]{b}\,(af-bc)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{8}{3}}}-\frac{\sqrt{3}\sqrt[3]{a}\left(a^{\frac{4}{3}}g-\sqrt[3]{a}bd+a\sqrt[3]{b}f-b^{\frac{4}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{8}{3}}}+\frac{a\,(ah-be)\log\left(a+bx^3\right)}{3b^3}-(af-bc)\int\frac{1}{b^2}\,dx+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}-\frac{x^3\,(ah-be)}{3b^2}-\frac{(ag-bd)\int x\,dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] -a**(1/3)*(a**(1/3)*(a*g - b*d) - b**(1/3)*(a*f - b*c))*log(a**(1
/3) + b**(1/3)*x)/(3*b**(8/3)) + a**(1/3)*(a**(1/3)*(a*g - b*d) -
 b**(1/3)*(a*f - b*c))*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(6*b**(8/3)) - sqrt(3)*a**(1/3)*(a**(4/3)*g - a**(1/3)*
b*d + a*b**(1/3)*f - b**(4/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**
(1/3)*x/3)/a**(1/3))/(3*b**(8/3)) + a*(a*h - b*e)*log(a + b*x**3)
/(3*b**3) - (a*f - b*c)*Integral(b**(-2), x) + f*x**4/(4*b) + g*x
**5/(5*b) + h*x**6/(6*b) - x**3*(a*h - b*e)/(3*b**2) - (a*g - b*d
)*Integral(x, x)/b**2
```

Mathematica [A] time = 0.514915, size = 299, normalized size = 0.96

$$\frac{10\sqrt[3]{a}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)-20\sqrt[3]{a}\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (60*b*(b*c - a*f)*x + 30*b*(b*d - a*g)*x^2 + 20*b*(b*e - a*h)*x^3
 + 15*b^2*f*x^4 + 12*b^2*g*x^5 + 10*b^2*h*x^6 - 20*Sqrt[3]*a^(1/3
)*b^(1/3)*(-(b^(4/3)*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 20*a^(1/3)*b^(1/3)*
(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(1/3) +
 b^(1/3)*x] + 10*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1/3)*b*d - a*b^(
1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
] + 20*a*(-(b*e) + a*h)*Log[a + b*x^3])/(60*b^3)
```

Maple [B] time = 0.007, size = 505, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/6*h*x^6/b+1/5*g*x^5/b+1/4*f*x^4/b-1/3/b^2*x^3*a*h+1/3*e*x^3/b-1
/2/b^2*x^2*a*g+1/2*d*x^2/b-1/b^2*a*f*x+c*x/b+1/3*a^2/b^3/(a/b)^(2
```

```
/3)*ln(x+(a/b)^(1/3))*f-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c
-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/6*
a/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+1/3*a^2/b^3
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/
3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))*c-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*g+1/3/b^2/(a/b)^(1
/3)*ln(x+(a/b)^(1/3))*a*d+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))*g-1/6/b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/
b)^(2/3))*a*d+1/3*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))*g-1/3/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*a*d+1/3*a^2/b^3*ln(b*x^3+a)*h-1/3*a*e*l
n(b*x^3+a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^3 + a),x, algorithm="
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^3 + a),x, algorithm="
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 47.2096, size = 842, normalized size = 2.69

$$\operatorname{RootSum}\left(27t^3b^9 + t^2\left(-27a^2b^6h + 27ab^7e\right) + t\left(9a^4b^3h^2 - 18a^3b^4eh + 9a^3b^4fg - 9a^2b^5cg - 9a^2b^5df + 9a^2b^5e^2 + 9ab^6cd\right)\right.$$
$$+ \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{x^3\left(ah - be\right)}{3b^2} - \frac{x^2\left(ag - bd\right)}{2b^2} - \frac{x\left(af - bc\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**9 + _t**2*(-27*a**2*b**6*h + 27*a*b**7*e) + _
t*(9*a**4*b**3*h**2 - 18*a**3*b**4*e*h + 9*a**3*b**4*f*g - 9*a**2
*b**5*c*g - 9*a**2*b**5*d*f + 9*a**2*b**5*e**2 + 9*a*b**6*c*d) -
a**6*h**3 + 3*a**5*b*e*h**2 - 3*a**5*b*f*g*h + a**5*b*g**3 + 3*a*
*4*b**2*c*g*h + 3*a**4*b**2*d*f*h - 3*a**4*b**2*d*g**2 - 3*a**4*b
**2*e**2*h + 3*a**4*b**2*e*f*g - a**4*b**2*f**3 - 3*a**3*b**3*c*d
*h - 3*a**3*b**3*c*e*g + 3*a**3*b**3*c*f**2 + 3*a**3*b**3*d**2*g
- 3*a**3*b**3*d*e*f + a**3*b**3*e**3 - 3*a**2*b**4*c**2*f + 3*a**
2*b**4*c*d*e - a**2*b**4*d**3 + a*b**5*c**3, Lambda(_t, _t*log(x
+ (9*_t**2*a*b**6*g - 9*_t**2*b**7*d - 6*_t*a**3*b**3*g*h + 6*_t*
a**2*b**4*d*h + 6*_t*a**2*b**4*e*g + 3*_t*a**2*b**4*f**2 - 6*_t*a
*b**5*c*f - 6*_t*a*b**5*d*e + 3*_t*b**6*c**2 + a**5*g*h**2 - a**4
*b*d*h**2 - 2*a**4*b*e*g*h - a**4*b*f**2*h + 2*a**4*b*f*g**2 + 2*
a**3*b**2*c*f*h - 2*a**3*b**2*c*g**2 + 2*a**3*b**2*d*e*h - 4*a**3
*b**2*d*f*g + a**3*b**2*e**2*g + a**3*b**2*e*f**2 - a**2*b**3*c**
2*h + 4*a**2*b**3*c*d*g - 2*a**2*b**3*c*e*f + 2*a**2*b**3*d**2*f
```

```
- a**2*b**3*d*e**2 + a*b**4*c**2*e - 2*a*b**4*c*d**2)/(a**4*b*g**
3 - 3*a**3*b**2*d*g**2 + a**3*b**2*f**3 - 3*a**2*b**3*c*f**2 + 3*
a**2*b**3*d**2*g + 3*a*b**4*c**2*f - a*b**4*d**3 - b**5*c**3))))
+ f*x**4/(4*b) + g*x**5/(5*b) + h*x**6/(6*b) - x**3*(a*h - b*e)/(
3*b**2) - x**2*(a*g - b*d)/(2*b**2) - x*(a*f - b*c)/b**2
```

GIAC/XCAS [A] time = 0.223959, size = 477, normalized size = 1.52

$$\frac{\left(a^{2} h-a b e\right) \ln \left(\left|b x^{3}+a\right|\right)}{3 b^{3}}-\frac{\sqrt{3}\left(\left(-a b^{2}\right)^{\frac{1}{3}} b^{2} c-\left(-a b^{2}\right)^{\frac{1}{3}} a b f-\left(-a b^{2}\right)^{\frac{2}{3}} b d+\left(-a b^{2}\right)^{\frac{2}{3}} a g\right) \arctan \left(\frac{\sqrt{3}\left(2 x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 b^{4}}-\frac{\left(\left(-a b^{2}\right)^{\frac{1}{3}} b^{2} c-\left(-a b^{2}\right)^{\frac{1}{3}} a b f+\left(-a b^{2}\right)^{\frac{2}{3}} b d-\left(-a b^{2}\right)^{\frac{2}{3}} a g\right) \ln \left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b^{4}}+\frac{10 b^{5} h x^{6}+12 b^{5} g x^{5}+15 b^{5} f x^{4}-20 a b^{4} h x^{3}+20 b^{5} x^{3} e+30 b^{5} d x^{2}-30 a b^{4} g x^{2}+60 b^{5} c x-60 a b^{4} f x}{60 b^{6}}+\frac{\left(a b^{12} d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{2} b^{11} g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a b^{12} c-a^{2} b^{11} f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln \left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 a b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^3 + a),x, algorithm="
```

```
[Out] 1/3*(a^2*h - a*b*e)*ln(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2
)^(1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b
^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/b^4 - 1/6*((-a*b^2)^(1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*
b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*ln(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/b^4 + 1/60*(10*b^5*h*x^6 + 12*b^5*g*x^5 + 15*b^5*f*x^
4 - 20*a*b^4*h*x^3 + 20*b^5*x^3*e + 30*b^5*d*x^2 - 30*a*b^4*g*x^2
 + 60*b^5*c*x - 60*a*b^4*f*x)/b^6 + 1/3*(a*b^12*d*(-a/b)^(1/3) -
a^2*b^11*g*(-a/b)^(1/3) + a*b^12*c - a^2*b^11*f)*(-a/b)^(1/3)*ln(
abs(x - (-a/b)^(1/3)))/(a*b^13)
```

3.393 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=294

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}b^{8/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3b^{8/3}}+\frac{(bc-af)\log\left(a+bx^3\right)}{3b^2}+\frac{x(bd-ag)}{b^2}+\frac{x^2(be-ah)}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}$$

```
[Out] ((b*d - a*g)*x)/b^2 + ((b*e - a*h)*x^2)/(2*b^2) + (f*x^3)/(3*b) +
 (g*x^4)/(4*b) + (h*x^5)/(5*b) + (a^(1/3)*(b^(4/3)*d + a^(1/3)*b*
e - a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(1/3)*(b^(1/3)*(b*d - a*g)
- a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) + (a
^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3)) + ((b*c - a*f)*Log[
a + b*x^3])/(3*b^2)
```

Rubi [A] time = 1.85241, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}b^{8/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3b^{8/3}}+\frac{(bc-af)\log\left(a+bx^3\right)}{3b^2}+\frac{x(bd-ag)}{b^2}+\frac{x^2(be-ah)}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*d - a*g)*x)/b^2 + ((b*e - a*h)*x^2)/(2*b^2) + (f*x^3)/(3*b) +
 (g*x^4)/(4*b) + (h*x^5)/(5*b) + (a^(1/3)*(b^(4/3)*d + a^(1/3)*b*
e - a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(1/3)*(b^(1/3)*(b*d - a*g)
- a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) + (a
^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3)) + ((b*c - a*f)*Log[
a + b*x^3])/(3*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\left(ah-be\right)-\sqrt[3]{b}\left(ag-bd\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{8}{3}}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\left(ah-be\right)-\sqrt[3]{b}\left(ag-bd\right)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}\right)}{6b^{\frac{8}{3}}}-\frac{\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}\left(ah-be\right)+\sqrt[3]{b}\left(ag-bd\right)\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{8}{3}}}-\left(ag-bd\right)\int\frac{1}{b^{2}}\,dx+\frac{fx^{3}}{3b}+\frac{gx^{4}}{4b}+\frac{hx^{5}}{5b}-\frac{\left(af-bc\right)\log\left(a+bx^{3}\right)}{3b^{2}}-\frac{\left(ah-be\right)\int x\,dx}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] -a**(1/3)*(a**(1/3)*(a*h - b*e) - b**(1/3)*(a*g - b*d))*log(a**(1
/3) + b**(1/3)*x)/(3*b**(8/3)) + a**(1/3)*(a**(1/3)*(a*h - b*e) -
 b**(1/3)*(a*g - b*d))*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(6*b**(8/3)) - sqrt(3)*a**(1/3)*(a**(1/3)*(a*h - b*e) +
 b**(1/3)*(a*g - b*d))*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)
/a**(1/3))/(3*b**(8/3)) - (a*g - b*d)*Integral(b**(-2), x) + f*x*
*3/(3*b) + g*x**4/(4*b) + h*x**5/(5*b) - (a*f - b*c)*log(a + b*x*
*3)/(3*b**2) - (a*h - b*e)*Integral(x, x)/b**2
```

Mathematica [A] time = 0.442584, size = 290, normalized size = 0.99

$$10\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^{2}\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)+20\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be+a\sqrt[3]{b}g-b^{4}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (60*b^(2/3)*(b*d - a*g)*x + 30*b^(2/3)*(b*e - a*h)*x^2 + 20*b^(5/
3)*f*x^3 + 15*b^(5/3)*g*x^4 + 12*b^(5/3)*h*x^5 - 20*Sqrt[3]*a^(1/
3)*(-(b^(4/3)*d) - a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 20*a^(1/3)*(-(b^(4/3)*d) +
 a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x]
+ 10*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(2/3)*(b*c
- a*f)*Log[a + b*x^3])/(60*b^(8/3))
```

Maple [B] time = 0.007, size = 483, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/5*h*x^5/b+1/4*g*x^4/b+1/3*f*x^3/b-1/2/b^2*x^2*a*h+1/2*e*x^2/b-1
/b^2*x*a*g+d*x/b+1/3/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^2*g-1/3/
b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*d-1/6/b^3/(a/b)^(2/3)*ln(x^2-
```

```
x*(a/b)^(1/3)+(a/b)^(2/3))*a^2*g+1/6/b^2/(a/b)^(2/3)*ln(x^2-x*(a/
b)^(1/3)+(a/b)^(2/3))*a*d+1/3/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*g-1/3/b^2/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*d-1/3/b^3/(a/b)^(1/3)*ln(
x+(a/b)^(1/3))*a^2*h+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e+1/
6/b^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^2*h-1/6*a/b
^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/3/b^3*3^(1/2
)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*h-1/3*a
/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*
e-1/3*a/b^2*ln(b*x^3+a)*f+1/3*c*ln(b*x^3+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^3 + a),x, algorithm="
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^3 + a),x, algorithm="
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 95.2032, size = 789, normalized size = 2.68

$$\text{RootSum}\Big(27t^3b^8 + t^2\left(27ab^6f - 27b^7c\right) + t\left(9a^3b^3gh - 9a^2b^4dh - 9a^2b^4eg + 9a^2b^4f^2 - 18ab^5cf + 9ab^5de + 9b^6c^2\right) + a^5h^3$$

$$+ \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{x^2\left(ah - be\right)}{2b^2} - \frac{x\left(ag - bd\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**8 + _t**2*(27*a*b**6*f - 27*b**7*c) + _t*(9*a
**3*b**3*g*h - 9*a**2*b**4*d*h - 9*a**2*b**4*e*g + 9*a**2*b**4*f*
*2 - 18*a*b**5*c*f + 9*a*b**5*d*e + 9*b**6*c**2) + a**5*h**3 - 3*
a**4*b*e*h**2 + 3*a**4*b*f*g*h - a**4*b*g**3 - 3*a**3*b**2*c*g*h
- 3*a**3*b**2*d*f*h + 3*a**3*b**2*d*g**2 + 3*a**3*b**2*e**2*h - 3
*a**3*b**2*e*f*g + a**3*b**2*f**3 + 3*a**2*b**3*c*d*h + 3*a**2*b*
*3*c*e*g - 3*a**2*b**3*c*f**2 - 3*a**2*b**3*d**2*g + 3*a**2*b**3*
d*e*f - a**2*b**3*e**3 + 3*a*b**4*c**2*f - 3*a*b**4*c*d*e + a*b**
4*d**3 - b**5*c**3, Lambda(_t, _t*log(x + (9*_t**2*a*b**5*h - 9*_
t**2*b**6*e + 6*_t*a**2*b**3*f*h + 3*_t*a**2*b**3*g**2 - 6*_t*a*b
**4*c*h - 6*_t*a*b**4*d*g - 6*_t*a*b**4*e*f + 6*_t*b**5*c*e + 3*_
t*b**5*d**2 + 2*a**4*g*h**2 - 2*a**3*b*d*h**2 - 4*a**3*b*e*g*h +
a**3*b*f**2*h + a**3*b*f*g**2 - 2*a**2*b**2*c*f*h - a**2*b**2*c*g
**2 + 4*a**2*b**2*d*e*h - 2*a**2*b**2*d*f*g + 2*a**2*b**2*e**2*g
- a**2*b**2*e*f**2 + a*b**3*c**2*h + 2*a*b**3*c*d*g + 2*a*b**3*c*
e*f + a*b**3*d**2*f - 2*a*b**3*d*e**2 - b**4*c**2*e - b**4*c*d**2
)/(a**4*h**3 - 3*a**3*b*e*h**2 + a**3*b*g**3 - 3*a**2*b**2*d*g**2
 + 3*a**2*b**2*e**2*h + 3*a*b**3*d**2*g - a*b**3*e**3 - b**4*d**3
```

```
)))) + f*x**3/(3*b) + g*x**4/(4*b) + h*x**5/(5*b) - x**2*(a*h - b
*e)/(2*b**2) - x*(a*g - b*d)/b**2
```

GIAC/XCAS [A] time = 0.224048, size = 450, normalized size = 1.53

$$\frac{(bc-af)\ln\left(\left|bx^3+a\right|\right)}{3\,b^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d - \left(-ab^2\right)^{\frac{1}{3}}abg + \left(-ab^2\right)^{\frac{2}{3}}ah - \left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d - \left(-ab^2\right)^{\frac{1}{3}}abg - \left(-ab^2\right)^{\frac{2}{3}}ah + \left(-ab^2\right)^{\frac{2}{3}}be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} + \frac{12\,b^4hx^5+15\,b^4gx^4+20\,b^4fx^3-30\,ab^3hx^2+30\,b^4x^2e+60\,b^4dx-60\,ab^3gx}{60\,b^5} - \frac{\left(a^2b^9h\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^{10}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e - ab^{10}d + a^2b^9g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^3 + a),x, algorithm="
```

```
[Out] 1/3*(b*c - a*f)*ln(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1
/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h - (-a*b^2)^
(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
b^4 - 1/6*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)
^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/b^4 + 1/60*(12*b^4*h*x^5 + 15*b^4*g*x^4 + 20*b^4*f*x^3 -
30*a*b^3*h*x^2 + 30*b^4*x^2*e + 60*b^4*d*x - 60*a*b^3*g*x)/b^5 -
1/3*(a^2*b^9*h*(-a/b)^(1/3) - a*b^10*(-a/b)^(1/3)*e - a*b^10*d +
a^2*b^9*g)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^11)
```

3.394 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=275

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{a}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3\sqrt[3]{a}b^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}\sqrt[3]{a}b^{7/3}}+\frac{(bd-ag)\log\left(a+bx^3\right)}{3b^2}+\frac{x(be-ah)}{b^2}+\frac{fx^2}{2b}+\frac{gx^3}{3b}+\frac{hx^4}{4b}$$

```
[Out] ((b*e - a*h)*x)/b^2 + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*
b) - ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(7
/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) +
 b^(1/3)*x])/(3*a^(1/3)*b^(7/3)) + ((b^(2/3)*(b*c - a*f) + a^(2/3
)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6
*a^(1/3)*b^(7/3)) + ((b*d - a*g)*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 1.82005, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{a}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3\sqrt[3]{a}b^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}\sqrt[3]{a}b^{7/3}}+\frac{(bd-ag)\log\left(a+bx^3\right)}{3b^2}+\frac{x(be-ah)}{b^2}+\frac{fx^2}{2b}+\frac{gx^3}{3b}+\frac{hx^4}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```


```
[Out] ((b*e - a*h)*x)/b^2 + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*
b) - ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(7
/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) +
 b^(1/3)*x])/(3*a^(1/3)*b^(7/3)) + ((b^(2/3)*(b*c - a*f) + a^(2/3
)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6
*a^(1/3)*b^(7/3)) + ((b*d - a*g)*Log[a + b*x^3])/(3*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-(ah-be)\int \frac{1}{b^2}\,dx+\frac{f\int x\,dx}{b}+\frac{gx^3}{3b}+\frac{hx^4}{4b}-\frac{(ag-bd)\log\left(a+bx^3\right)}{3b^2}-\frac{\sqrt{3}\left(a^{\frac{2}{3}}(ah-be)-b^{\frac{2}{3}}(af-bc)\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}b^{\frac{7}{3}}}+\frac{\left(a^{\frac{2}{3}}(ah-be)+b^{\frac{2}{3}}(af-bc)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{\frac{7}{3}}}-\frac{\left(a^{\frac{2}{3}}(ah-be)+b^{\frac{2}{3}}(af-bc)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6\sqrt[3]{a}b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] -(a*h - b*e)*Integral(b**(-2), x) + f*Integral(x, x)/b + g*x**3/(
3*b) + h*x**4/(4*b) - (a*g - b*d)*log(a + b*x**3)/(3*b**2) - sqrt
(3)*(a**(2/3)*(a*h - b*e) - b**(2/3)*(a*f - b*c))*atan(sqrt(3)*(a
**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(1/3)*b**(7/3)) + (a*
*(2/3)*(a*h - b*e) + b**(2/3)*(a*f - b*c))*log(a**(1/3) + b**(1/3
)*x)/(3*a**(1/3)*b**(7/3)) - (a**(2/3)*(a*h - b*e) + b**(2/3)*(a*
f - b*c))*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*
a**(1/3)*b**(7/3))
```

Mathematica [A] time = 1.26289, size = 272, normalized size = 0.99

$$\frac{\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+b^{5/3}c\right)}{\sqrt[3]{a}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^{2/3}be+a^{5/3}h+ab^{2/3}f-b^{5/3}c\right)}{\sqrt[3]{a}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-a^{2/3}be+\right.}{\sqrt[3]{a}}}{12b^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (12*b^(1/3)*(b*e - a*h)*x + 6*b^(4/3)*f*x^2 + 4*b^(4/3)*g*x^3 + 3
*b^(4/3)*h*x^4 - (4*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*
f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/
3) + (4*(-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h)*Lo
g[a^(1/3) + b^(1/3)*x])/a^(1/3) + (2*(b^(5/3)*c + a^(2/3)*b*e - a
*b^(2/3)*f - a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/a^(1/3) + 4*b^(1/3)*(b*d - a*g)*Log[a + b*x^3])/(12*b^(7/3
))
```

Maple [B] time = 0.006, size = 455, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/4*h*x^4/b+1/3*g*x^3/b+1/2*f*x^2/b-1/b^2*x*a*h+e*x/b+1/3/b^3/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))*a^2*h-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)
```

```
^(1/3))*e-1/6/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a
^2*h+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/
3/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
*a^2*h-1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))*e+1/3/b^2*a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-1/3/b/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))*c-1/6/b^2*a/(a/b)^(1/3)*ln(x^2-x*(a/b)
^(1/3)+(a/b)^(2/3))*f+1/6/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b
)^(2/3))*c-1/3/b^2*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))*f+1/3/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*c-1/3/b^2*ln(b*x^3+a)*a*g+1/3*d*ln(b*x^3+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x/(b*x^3 + a),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x/(b*x^3 + a),x, algorithm="fr
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 56.4232, size = 811, normalized size = 2.95

$$\operatorname{RootSum}\left(27t^3ab^7 + t^2\left(27a^2b^5g - 27ab^6d\right) + t\left(-9a^3b^3fh + 9a^3b^3g^2 + 9a^2b^4ch - 18a^2b^4dg + 9a^2b^4ef - 9ab^5ce + 9ab^5d^2\right)\right.$$
$$+ \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{x\left(ah - be\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**7 + _t**2*(27*a**2*b**5*g - 27*a*b**6*d) +
_t*(-9*a**3*b**3*f*h + 9*a**3*b**3*g**2 + 9*a**2*b**4*c*h - 18*a*
*2*b**4*d*g + 9*a**2*b**4*e*f - 9*a*b**5*c*e + 9*a*b**5*d**2) - a
**5*h**3 + 3*a**4*b*e*h**2 - 3*a**4*b*f*g*h + a**4*b*g**3 + 3*a**
3*b**2*c*g*h + 3*a**3*b**2*d*f*h - 3*a**3*b**2*d*g**2 - 3*a**3*b*
*2*e**2*h + 3*a**3*b**2*e*f*g - a**3*b**2*f**3 - 3*a**2*b**3*c*d*
h - 3*a**2*b**3*c*e*g + 3*a**2*b**3*c*f**2 + 3*a**2*b**3*d**2*g -
 3*a**2*b**3*d*e*f + a**2*b**3*e**3 - 3*a*b**4*c**2*f + 3*a*b**4*
c*d*e - a*b**4*d**3 + b**5*c**3, Lambda(_t, _t*log(x + (-9*_t**2*
a**2*b**5*f + 9*_t**2*a*b**6*c + 3*_t*a**4*b**2*h**2 - 6*_t*a**3*
b**3*e*h - 6*_t*a**3*b**3*f*g + 6*_t*a**2*b**4*c*g + 6*_t*a**2*b*
*4*d*f + 3*_t*a**2*b**4*e**2 - 6*_t*a*b**5*c*d + a**5*g*h**2 - a*
*4*b*d*h**2 - 2*a**4*b*e*g*h + 2*a**4*b*f**2*h - a**4*b*f*g**2 -
4*a**3*b**2*c*f*h + a**3*b**2*c*g**2 + 2*a**3*b**2*d*e*h + 2*a**3
*b**2*d*f*g + a**3*b**2*e**2*g - 2*a**3*b**2*e*f**2 + 2*a**2*b**3
*c**2*h - 2*a**2*b**3*c*d*g + 4*a**2*b**3*c*e*f - a**2*b**3*d**2*
f - a**2*b**3*d*e**2 - 2*a*b**4*c**2*e + a*b**4*c*d**2)/(a**5*h**
3 - 3*a**4*b*e*h**2 + 3*a**3*b**2*e**2*h - a**3*b**2*f**3 + 3*a**
```

```
2*b**3*c*f**2 - a**2*b**3*e**3 - 3*a*b**4*c**2*f + b**5*c**3))))
+ f*x**2/(2*b) + g*x**3/(3*b) + h*x**4/(4*b) - x*(a*h - b*e)/b**2
```

GIAC/XCAS [A] time = 0.227269, size = 428, normalized size = 1.56

$$\frac{(bd-ag)\ln\left(\left|bx^3+a\right|\right)}{3\,b^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h-\left(-ab^2\right)^{\frac{1}{3}}abe-\left(-ab^2\right)^{\frac{2}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h-\left(-ab^2\right)^{\frac{1}{3}}abe+\left(-ab^2\right)^{\frac{2}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}af\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3}+\frac{3\,b^3hx^4+4\,b^3gx^3+6\,b^3fx^2-12\,ab^2hx+12\,b^3xe}{12\,b^4}-\frac{\left(b^9c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^8f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^7h-ab^8e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x/(b*x^3 + a),x, algorithm="gi
```

```
[Out] 1/3*(b*d - a*g)*ln(abs(b*x^3 + a))/b^2 + 1/3*sqrt(3)*((-a*b^2)^(1
/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c + (-a*b^2)^
(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a*b^3) + 1/6*((-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a*
b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*ln(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a*b^3) + 1/12*(3*b^3*h*x^4 + 4*b^3*g*x^3 + 6*b^3*f*x
^2 - 12*a*b^2*h*x + 12*b^3*x*e)/b^4 - 1/3*(b^9*c*(-a/b)^(1/3) - a
*b^8*f*(-a/b)^(1/3) + a^2*b^7*h - a*b^8*e)*(-a/b)^(1/3)*ln(abs(x
- (-a/b)^(1/3)))/(a*b^9)
```

3.395 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$

Optimal. Leaf size=259

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6a^{2/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{2/3}b^{5/3}}+\frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}+\frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}$$

```
[Out] (f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b
*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f)
 - a^(1/3)*(b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5
/3)) - ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(5/3)) + ((b*e -
a*h)*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.801398, antiderivative size = 257, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{2/3}b^{5/3}}+\frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}+\frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b
*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f)
 - a^(1/3)*(b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5
/3)) - ((b*c - a*f - (a^(1/3)*(b*d - a*g))/b^(1/3))*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3)) + ((b*e -
a*h)*Log[a + b*x^3])/(3*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{g\int x\,dx}{b}+\frac{hx^3}{3b}+\frac{\int f\,dx}{b}-\frac{(ah-be)\log\left(a+bx^3\right)}{3b^2}+\frac{\left(\sqrt[3]{a}\,(ag-bd)-\sqrt[3]{b}\,(af-bc)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{5}{3}}}-\frac{\left(\sqrt[3]{a}\,(ag-bd)-\sqrt[3]{b}\,(af-bc)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{5}{3}}}+\frac{\sqrt{3}\left(a^{\frac{4}{3}}g-\sqrt[3]{a}bd+a\sqrt[3]{b}f-b^{\frac{4}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] g*Integral(x, x)/b + h*x**3/(3*b) + Integral(f, x)/b - (a*h - b*e
)*log(a + b*x**3)/(3*b**2) + (a**(1/3)*(a*g - b*d) - b**(1/3)*(a*
f - b*c))*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(5/3)) - (a**
(1/3)*(a*g - b*d) - b**(1/3)*(a*f - b*c))*log(a**(2/3) - a**(1/3)
*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*b**(5/3)) + sqrt(3)*(a**
(4/3)*g - a**(1/3)*b*d + a*b**(1/3)*f - b**(4/3)*c)*atan(sqrt(3)*
(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*b**(5/3))
```

Mathematica [A] time = 0.740207, size = 254, normalized size = 0.98

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}g-\sqrt[3]{a}ba\cdots\right.}{a^{2/3}}\cdots}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*f*x + 3*b^(2/3)*g*x^2 + 2*b^(2/3)*h*x^3 + (2*Sqrt[3]*(
-(b^(4/3)*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(b^(4/3)*c - a^(1/
3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/
3) - ((b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) + (2*(b*e - a*h)
*Log[a + b*x^3])/b^(1/3))/(6*b^(5/3))
```

Maple [B] time = 0.006, size = 429, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*h*x^3/b+1/2*g*x^2/b+f*x/b-1/3/b^2*a/(a/b)^(2/3)*ln(x+(a/b)^(1
/3))*f+1/3*c/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/b^2*a/(a/b)^(2/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f-1/6*c/b/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))-1/3/b^2*a/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+1/3*c/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b^2/(a/b)^(1/3)*ln(x+(a/
b)^(1/3))*a*g-1/3*d/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6/b^2/(a/b)
```

```
^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a*g+1/6*d/b/(a/b)^(1/3)*
ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1/3/b^2*3^(1/2)/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*g+1/3*d*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/b^2*ln(b*x^3+a)*a
*h+1/3*e*ln(b*x^3+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a),x, algorithm="frica
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 49.168, size = 804, normalized size = 3.1

$$\operatorname{RootSum}\left(27t^3a^2b^6 + t^2\left(27a^3b^4h - 27a^2b^5e\right) + t\left(9a^4b^2h^2 - 18a^3b^3eh + 9a^3b^3fg - 9a^2b^4cg - 9a^2b^4df + 9a^2b^4e^2 + 9ab^5cd\right.\right.$$

$$+ \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**6 + _t**2*(27*a**3*b**4*h - 27*a**2*b**5
*e) + _t*(9*a**4*b**2*h**2 - 18*a**3*b**3*e*h + 9*a**3*b**3*f*g -
 9*a**2*b**4*c*g - 9*a**2*b**4*d*f + 9*a**2*b**4*e**2 + 9*a*b**5*
c*d) + a**5*h**3 - 3*a**4*b*e*h**2 + 3*a**4*b*f*g*h - a**4*b*g**3
 - 3*a**3*b**2*c*g*h - 3*a**3*b**2*d*f*h + 3*a**3*b**2*d*g**2 + 3
*a**3*b**2*e**2*h - 3*a**3*b**2*e*f*g + a**3*b**2*f**3 + 3*a**2*b
**3*c*d*h + 3*a**2*b**3*c*e*g - 3*a**2*b**3*c*f**2 - 3*a**2*b**3*
d**2*g + 3*a**2*b**3*d*e*f - a**2*b**3*e**3 + 3*a*b**4*c**2*f - 3
*a*b**4*c*d*e + a*b**4*d**3 - b**5*c**3, Lambda(_t, _t*log(x + (9
*_t**2*a**3*b**4*g - 9*_t**2*a**2*b**5*d + 6*_t*a**4*b**2*g*h - 6
*_t*a**3*b**3*d*h - 6*_t*a**3*b**3*e*g - 3*_t*a**3*b**3*f**2 + 6*
_t*a**2*b**4*c*f + 6*_t*a**2*b**4*d*e - 3*_t*a*b**5*c**2 + a**5*g
*h**2 - a**4*b*d*h**2 - 2*a**4*b*e*g*h - a**4*b*f**2*h + 2*a**4*b
*f*g**2 + 2*a**3*b**2*c*f*h - 2*a**3*b**2*c*g**2 + 2*a**3*b**2*d*
e*h - 4*a**3*b**2*d*f*g + a**3*b**2*e**2*g + a**3*b**2*e*f**2 - a
**2*b**3*c**2*h + 4*a**2*b**3*c*d*g - 2*a**2*b**3*c*e*f + 2*a**2*
b**3*d**2*f - a**2*b**3*d*e**2 + a*b**4*c**2*e - 2*a*b**4*c*d**2)
/(a**4*b*g**3 - 3*a**3*b**2*d*g**2 + a**3*b**2*f**3 - 3*a**2*b**3
*c*f**2 + 3*a**2*b**3*d**2*g + 3*a*b**4*c**2*f - a*b**4*d**3 - b*
*5*c**3)))) + f*x/b + g*x**2/(2*b) + h*x**3/(3*b)
```

GIAC/XCAS [A] time = 0.223974, size = 397, normalized size = 1.53

$$-\frac{(ah-be)\ln\left(\left|bx^3+a\right|\right)}{3\,b^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-\left(-ab^2\right)^{\frac{1}{3}}abf-\left(-ab^2\right)^{\frac{2}{3}}bd+\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}+\frac{2\,b^2hx^3+3\,b^2gx^2+6\,b^2fx}{6\,b^3}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-\left(-ab^2\right)^{\frac{1}{3}}abf+\left(-ab^2\right)^{\frac{2}{3}}bd-\left(-ab^2\right)^{\frac{2}{3}}ag\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3}-\frac{\left(b^7d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^6g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^7c-ab^6f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a),x, algorithm="giac
```

```
[Out] -1/3*(a*h - b*e)*ln(abs(b*x^3 + a))/b^2 + 1/3*sqrt(3)*((-a*b^2)^(
1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)
^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))
/(a*b^3) + 1/6*(2*b^2*h*x^3 + 3*b^2*g*x^2 + 6*b^2*f*x)/b^3 + 1/6*
((-a*b^2)^(1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d
 - (-a*b^2)^(2/3)*a*g)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a
*b^3) - 1/3*(b^7*d*(-a/b)^(1/3) - a*b^6*g*(-a/b)^(1/3) + b^7*c -
a*b^6*f)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^7)
```

3.396 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$

Optimal. Leaf size=258

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6a^{2/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{2/3}b^{5/3}}-\frac{(bc-af)\log\left(a+bx^3\right)}{3ab}+\frac{c\log(x)}{a}+\frac{gx}{b}+\frac{hx^2}{2b}$$

```
[Out] (g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g
 - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(2/3)*b^(5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g)
- a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/
3)) - ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(5/3)) - ((b*c - a
*f)*Log[a + b*x^3])/(3*a*b)
```

Rubi [A] time = 0.967689, antiderivative size = 256, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{2/3}b^{5/3}}-\frac{(bc-af)\log\left(a+bx^3\right)}{3ab}+\frac{c\log(x)}{a}+\frac{gx}{b}+\frac{hx^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g
 - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(2/3)*b^(5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g)
- a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/
3)) - ((b*d - a*g - (a^(1/3)*(b*e - a*h))/b^(1/3))*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3)) - ((b*c - a
*f)*Log[a + b*x^3])/(3*a*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{h\int x\,dx}{b}+\frac{\int g\,dx}{b}+\frac{c\log(x)}{a}+\frac{(af-bc)\log\left(a+bx^3\right)}{3ab}+\frac{\left(\sqrt[3]{a}\,(ah-be)-\sqrt[3]{b}\,(ag-bd)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{5}{3}}}-\frac{\left(\sqrt[3]{a}\,(ah-be)-\sqrt[3]{b}\,(ag-bd)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{5}{3}}}+\frac{\sqrt{3}\left(a^{\frac{4}{3}}h-\sqrt[3]{a}be+a\sqrt[3]{b}g-b^{\frac{4}{3}}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] h*Integral(x, x)/b + Integral(g, x)/b + c*log(x)/a + (a*f - b*c)*
log(a + b*x**3)/(3*a*b) + (a**(1/3)*(a*h - b*e) - b**(1/3)*(a*g -
 b*d))*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(5/3)) - (a**(1/
3)*(a*h - b*e) - b**(1/3)*(a*g - b*d))*log(a**(2/3) - a**(1/3)*b*
*(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*b**(5/3)) + sqrt(3)*(a**(4/
3)*h - a**(1/3)*b*e + a*b**(1/3)*g - b**(4/3)*d)*atan(sqrt(3)*(a*
*(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*b**(5/3))
```

Mathematica [A] time = 0.443865, size = 258, normalized size = 1.

$$\frac{-\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)+2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (6*a*b^(2/3)*g*x + 3*a*b^(2/3)*h*x^2 + 2*Sqrt[3]*a^(1/3)*(-(b^(4/
3)*d) - a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(
1/3)*x)/a^(1/3))/Sqrt[3]] + 6*b^(5/3)*c*Log[x] + 2*a^(1/3)*(b^(4/
3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/
3)*x] - a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*
h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(2/3)*(b*
c - a*f)*Log[a + b*x^3])/(6*a*b^(5/3))
```

Maple [B] time = 0.01, size = 426, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x)
```

```
[Out] 1/2*h*x^2/b+g*x/b+c*ln(x)/a-1/3/b^2*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*g+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+1/6/b^2*a/(a/b)^(2/3)*
ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*g-1/6/b/(a/b)^(2/3)*ln(x^2-x*(a
/b)^(1/3)+(a/b)^(2/3))*d-1/3/b^2*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*g+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(
```

```
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/3/b^2*a/(a/b)^(1/3)*ln(x+(a/
b)^(1/3))*h-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-1/6/b^2*a/(a/b)
^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*h+1/6/b/(a/b)^(1/3)*ln(x
^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e-1/3/b^2*a*3^(1/2)/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h+1/3/b*3^(1/2)/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3/b*ln(b*x^3+a)*f-1/3
*c*ln(b*x^3+a)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x),x, algorithm="
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x),x, algorithm="
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225447, size = 409, normalized size = 1.59

$$\frac{c\ln\left(|x|\right)}{a} - \frac{(bc - af)\ln\left(\left|bx^3 + a\right|\right)}{3\,ab} + \frac{bhx^2 + 2\,bgx}{2\,b^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d - \left(-ab^2\right)^{\frac{1}{3}}abg + \left(-ab^2\right)^{\frac{2}{3}}ah - \left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d - \left(-ab^2\right)^{\frac{1}{3}}abg - \left(-ab^2\right)^{\frac{2}{3}}ah + \left(-ab^2\right)^{\frac{2}{3}}be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3} + \frac{\left(a^3b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e - a^2b^3d + a^3b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x),x, algorithm="
```

```
[Out] c*ln(abs(x))/a - 1/3*(b*c - a*f)*ln(abs(b*x^3 + a))/(a*b) + 1/2*(
b*h*x^2 + 2*b*g*x)/b^2 + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d - (-a*
b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/6*((
-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h +
 (-a*b^2)^(2/3)*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b
^3) + 1/3*(a^3*b^2*h*(-a/b)^(1/3) - a^2*b^3*(-a/b)^(1/3)*e - a^2*
b^3*d + a^3*b^2*g)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b^
3)
```

3.397 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=253

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}a^{4/3}b^{4/3}}-\frac{(bd-ag)\log\left(a+bx^3\right)}{3ab}-\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{hx}{b}$$

```
[Out] -(c/(a*x)) + (h*x)/b + ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f +
a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(4/3)*b^(4/3)) + (d*Log[x])/a + ((b^(2/3)*(b*c - a*f) + a
^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(4/3))
 - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(4/3)) - ((b*d - a*g)
*Log[a + b*x^3])/(3*a*b)
```

Rubi [A] time = 0.985783, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}a^{4/3}b^{4/3}}-\frac{(bd-ag)\log\left(a+bx^3\right)}{3ab}-\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{hx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```


```
[Out] -(c/(a*x)) + (h*x)/b + ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f +
a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(4/3)*b^(4/3)) + (d*Log[x])/a + ((b^(2/3)*(b*c - a*f) + a
^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(4/3))
 - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(4/3)) - ((b*d - a*g)
*Log[a + b*x^3])/(3*a*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int h\,dx}{b} - \frac{c}{ax} + \frac{d\log(x)}{a} + \frac{(ag-bd)\log\left(a+bx^3\right)}{3ab} - \frac{\left(a^{\frac{2}{3}}(ah-be)+b^{\frac{2}{3}}(af-bc)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{4}{3}}b^{\frac{4}{3}}}$$
$$+\frac{\left(a^{\frac{2}{3}}(ah-be)+b^{\frac{2}{3}}(af-bc)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{4}{3}}b^{\frac{4}{3}}}$$
$$+\frac{\sqrt{3}\left(a^{\frac{5}{3}}h-a^{\frac{2}{3}}be-ab^{\frac{2}{3}}f+b^{\frac{5}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{4}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Integral(h, x)/b - c/(a*x) + d*log(x)/a + (a*g - b*d)*log(a + b*x
**3)/(3*a*b) - (a**(2/3)*(a*h - b*e) + b**(2/3)*(a*f - b*c))*log(
a**(1/3) + b**(1/3)*x)/(3*a**(4/3)*b**(4/3)) + (a**(2/3)*(a*h - b
*e) + b**(2/3)*(a*f - b*c))*log(a**(2/3) - a**(1/3)*b**(1/3)*x +
b**(2/3)*x**2)/(6*a**(4/3)*b**(4/3)) + sqrt(3)*(a**(5/3)*h - a**(
2/3)*b*e - a*b**(2/3)*f + b**(5/3)*c)*atan(sqrt(3)*(a**(1/3)/3 -
2*b**(1/3)*x/3)/a**(1/3))/(3*a**(4/3)*b**(4/3))
```

Mathematica [A] time = 0.588497, size = 257, normalized size = 1.02

$$\frac{1}{6}\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}be+a^{5/3}h+ab^{2/3}f-b^{5/3}c\right)}{a^{4/3}b^{4/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+b^{5/3}c\right)}{a^{4/3}b^{4/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{a^{4/3}b^{4/3}}+\frac{2(ag-bd)\log\left(a+bx^3\right)}{ab}-\frac{6c}{ax}+\frac{6d\log(x)}{a}+\frac{6hx}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

```
[Out] ((-6*c)/(a*x) + (6*h*x)/b + (2*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e -
 a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/(a^(4/3)*b^(4/3)) + (6*d*Log[x])/a + (2*(b^(5/3)*c + a^(2/3
)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/(a^(4/
3)*b^(4/3)) + ((-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(5/3
)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(4/3)*b^(
4/3)) + (2*(-(b*d) + a*g)*Log[a + b*x^3])/(a*b))/6
```

Maple [B] time = 0.009, size = 423, normalized size = 1.7

$$\frac{hx}{b}+\frac{d\ln(x)}{a}-\frac{c}{ax}-\frac{ah}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{e}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ah}{6b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{a\sqrt{3}h}{3b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}e}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{f}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{c}{3a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{f}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c}{6a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\sqrt{3}f}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c\sqrt{3}}{3a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\ln\left(bx^3+a\right)g}{3b}-\frac{d\ln\left(bx^3+a\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x)
```

```
[Out] h*x/b+d*ln(x)/a-c/a/x-1/3/b^2*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*h+1
/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1/6/b^2*a/(a/b)^(2/3)*ln(x^2
-x*(a/b)^(1/3)+(a/b)^(2/3))*h-1/6/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))*e-1/3/b^2*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))*h+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*
f+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/6/b/(a/b)^(1/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))*f-1/6/a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/
3)+(a/b)^(2/3))*c+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*f-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*c+1/3/b*ln(b*x^3+a)*g-1/3*d*ln(b*x^3+a)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x^2),x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x^2),x, algorithm
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225894, size = 404, normalized size = 1.6

$$\frac{hx}{b} + \frac{d\ln\left(|x|\right)}{a} - \frac{(bd - ag)\ln\left(\left|bx^3 + a\right|\right)}{3\,ab} - \frac{c}{ax} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}} a^2h - \left(-ab^2\right)^{\frac{1}{3}} abe - \left(-ab^2\right)^{\frac{2}{3}} bc + \left(-ab^2\right)^{\frac{2}{3}} af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}} a^2h - \left(-ab^2\right)^{\frac{1}{3}} abe + \left(-ab^2\right)^{\frac{2}{3}} bc - \left(-ab^2\right)^{\frac{2}{3}} af\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2} + \frac{\left(ab^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^3b^2h - a^2b^3e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x^2),x, algorithm
```

```
[Out] h*x/b + d*ln(abs(x))/a - 1/3*(b*d - a*g)*ln(abs(b*x^3 + a))/(a*b)
 - c/(a*x) - 1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a
*b*e - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) - 1/6*((-a*b^2)^(1
/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c - (-a*b^2)^
(2/3)*a*f)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2) + 1/
3*(a*b^4*c*(-a/b)^(1/3) - a^2*b^3*f*(-a/b)^(1/3) + a^3*b^2*h - a^
2*b^3*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

3.398 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=260

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6a^{5/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{5/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{5/3}b^{2/3}}-\frac{(be-ah)\log\left(a+bx^3\right)}{3ab}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

```
[Out] -c/(2*a*x^2) - d/(a*x) + ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f
- a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(5/3)*b^(2/3)) + (e*Log[x])/a - ((b^(1/3)*(b*c - a*f) -
 a^(1/3)*(b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3
)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(2/3)) - ((b*e - a*
h)*Log[a + b*x^3])/(3*a*b)
```

Rubi [A] time = 0.813767, antiderivative size = 258, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{5/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{5/3}b^{2/3}}-\frac{(be-ah)\log\left(a+bx^3\right)}{3ab}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

```
[Out] -c/(2*a*x^2) - d/(a*x) + ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f
- a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(5/3)*b^(2/3)) + (e*Log[x])/a - ((b^(1/3)*(b*c - a*f) -
 a^(1/3)*(b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3
)) + ((b*c - a*f - (a^(1/3)*(b*d - a*g))/b^(1/3))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(1/3)) - ((b*e - a*
h)*Log[a + b*x^3])/(3*a*b)
```

Rubi in Sympy [A] time = 106.028, size = 228, normalized size = 0.88

$$-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log{(x)}}{a}+\frac{(ah-be)\log{\left(a+bx^3\right)}}{3ab}-\frac{\left(\sqrt[3]{a}\,(ag-bd)-\sqrt[3]{b}\,(af-bc)\right)\log{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}$$
$$+\frac{\left(\sqrt[3]{a}\,(ag-bd)-\sqrt[3]{b}\,(af-bc)\right)\log{\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}}{6a^{\frac{5}{3}}b^{\frac{2}{3}}}$$
$$-\frac{\sqrt{3}\left(a^{\frac{4}{3}}g-\sqrt[3]{a}bd+a\sqrt[3]{b}f-b^{\frac{4}{3}}c\right)\operatorname{atan}{\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] -c/(2*a*x**2) - d/(a*x) + e*log(x)/a + (a*h - b*e)*log(a + b*x**3
)/(3*a*b) - (a**(1/3)*(a*g - b*d) - b**(1/3)*(a*f - b*c))*log(a**
(1/3) + b**(1/3)*x)/(3*a**(5/3)*b**(2/3)) + (a**(1/3)*(a*g - b*d)
 - b**(1/3)*(a*f - b*c))*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**
(2/3)*x**2)/(6*a**(5/3)*b**(2/3)) - sqrt(3)*(a**(4/3)*g - a**(1/3
)*b*d + a*b**(1/3)*f - b**(4/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b
**(1/3)*x/3)/a**(1/3))/(3*a**(5/3)*b**(2/3))
```

Mathematica [A] time = 0.781863, size = 257, normalized size = 0.99

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{b^{2/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}b\right.}{b^{2/3}}}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(2/3)*c)/x^2 - (6*a^(2/3)*d)/x + (2*Sqrt[3]*(b^(4/3)*c + a
^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]])/b^(2/3) + 6*a^(2/3)*e*Log[x] - (2*(b^(4/3)*c -
a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/
b^(2/3) + ((b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + (2*a^(2/3
)*(-(b*e) + a*h)*Log[a + b*x^3])/b)/(6*a^(5/3))
```

Maple [B] time = 0.008, size = 423, normalized size = 1.6

$$-\frac{d}{ax}+\frac{e\ln(x)}{a}-\frac{c}{2\,ax^2}+\frac{f}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{3\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{f}{6\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c}{6\,a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{\sqrt{3}f}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c\sqrt{3}}{3\,a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{g}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d}{3\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{g}{6\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{d}{6\,a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\sqrt{3}g}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$-\frac{d\sqrt{3}}{3\,a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\ln\left(bx^3+a\right)h}{3\,b}-\frac{e\ln\left(bx^3+a\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x)
```

```
[Out] -d/a/x+e*ln(x)/a-1/2*c/a/x^2+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*
f-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/6/b/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/6/a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/
3)+(a/b)^(2/3))*c+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*f-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*c-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*g+1/3
/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d+1/6/b/(a/b)^(1/3)*ln(x^2-x*(a/
b)^(1/3)+(a/b)^(2/3))*g-1/6/a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a
/b)^(2/3))*d+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*g-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*d+1/3/b*ln(b*x^3+a)*h-1/3*e*ln(b*x^3+a)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x^3),x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x^3),x, algorithm
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225554, size = 393, normalized size = 1.51

$$\frac{e\ln\left(|x|\right)}{a}+\frac{(ah-be)\ln\left(\left|bx^3+a\right|\right)}{3\,ab}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-\left(-ab^2\right)^{\frac{1}{3}}abf-\left(-ab^2\right)^{\frac{2}{3}}bd+\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2}+\frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c-a^2bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}-\frac{2\,dx+c}{2\,ax^2}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-\left(-ab^2\right)^{\frac{1}{3}}abf+\left(-ab^2\right)^{\frac{2}{3}}bd-\left(-ab^2\right)^{\frac{2}{3}}ag\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x^3),x, algorithm
```

```
[Out] e*ln(abs(x))/a + 1/3*(a*h - b*e)*ln(abs(b*x^3 + a))/(a*b) - 1/3*s
qrt(3)*((-a*b^2)^(1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2
/3)*b*d + (-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/3*(a*b^2*d*(-a/b)^(1/3) - a^2*b*
g*(-a/b)^(1/3) + a*b^2*c - a^2*b*f)*(-a/b)^(1/3)*ln(abs(x - (-a/b
)^(1/3)))/(a^3*b) - 1/2*(2*d*x + c)/(a*x^2) - 1/6*((-a*b^2)^(1/3)
*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/
3)*a*g)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

3.399 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=276

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6a^{5/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{5/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{5/3}b^{2/3}}+\frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(bc-af)}{a^2}-\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}$$

```
[Out] -c/(3*a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^(4/3)*d + a^(1/3)*b*e
- a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(2/3)) - ((b*c - a*f)*Log[x])/a^2
 - ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b
*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(
5/3)*b^(2/3)) + ((b*c - a*f)*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 0.959213, antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{5/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{5/3}b^{2/3}}+\frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(bc-af)}{a^2}-\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -c/(3*a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^(4/3)*d + a^(1/3)*b*e
- a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(2/3)) - ((b*c - a*f)*Log[x])/a^2
 - ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*d - a*g - (a^(1/3)*(b*e - a*h)
)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(
5/3)*b^(1/3)) + ((b*c - a*f)*Log[a + b*x^3])/(3*a^2)
```

Rubi in Sympy [A] time = 121.797, size = 243, normalized size = 0.88

$$-\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}+\frac{(af-bc)\log(x)}{a^2}-\frac{(af-bc)\log\left(a+bx^3\right)}{3a^2}-\frac{\left(\sqrt[3]{a}(ah-be)-\sqrt[3]{b}(ag-bd)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}+\frac{\left(\sqrt[3]{a}(ah-be)-\sqrt[3]{b}(ag-bd)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\sqrt{3}\left(a^{\frac{4}{3}}h-\sqrt[3]{a}be+a\sqrt[3]{b}g-b^{\frac{4}{3}}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a),x)
```

```
[Out] -c/(3*a*x**3) - d/(2*a*x**2) - e/(a*x) + (a*f - b*c)*log(x)/a**2
- (a*f - b*c)*log(a + b*x**3)/(3*a**2) - (a**(1/3)*(a*h - b*e) -
b**(1/3)*(a*g - b*d))*log(a**(1/3) + b**(1/3)*x)/(3*a**(5/3)*b**(
2/3)) + (a**(1/3)*(a*h - b*e) - b**(1/3)*(a*g - b*d))*log(a**(2/3
) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(5/3)*b**(2/3)) -
sqrt(3)*(a**(4/3)*h - a**(1/3)*b*e + a*b**(1/3)*g - b**(4/3)*d)*a
tan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(5/3)*b
**(2/3))
```

Mathematica [A] time = 1.08363, size = 264, normalized size = 0.96

$$-\frac{-\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{b^{2/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{b^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\cdots}{\cdots}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -((2*a*c)/x^3 + (3*a*d)/x^2 + (6*a*e)/x + (2*Sqrt[3]*a^(1/3)*(-(b
^(4/3)*d) - a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 6*(b*c - a*f)*Log[x] + (
2*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log
[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)*(b^(4/3)*d - a^(1/3)*b*
e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/b^(2/3) - 2*(b*c - a*f)*Log[a + b*x^3])/(6*a^2)
```

Maple [B] time = 0.01, size = 442, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x)
```

```
[Out] -1/2*d/a/x^2-e/a/x-1/3*c/a/x^3+1/a*ln(x)*f-1/a^2*ln(x)*b*c+1/3/b/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))*g-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*d-1/6/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*g+1/6/a/
(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/3/b/(a/b)^(2/3)
```

```
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*g-1/3/a/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3/b/(a/b)
^(1/3)*ln(x+(a/b)^(1/3))*h+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e+
1/6/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*h-1/6/a/(a/b)
^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/3*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h-1/3/a*3^(1/2)/(a/b)
^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/3/a*ln(b*x^3+a
)*f+1/3/a^2*b*ln(b*x^3+a)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x^4),x, algorithm

[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x^4),x, algorithm

[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.226677, size = 423, normalized size = 1.53

$$\frac{(bc-af)\ln\left(\left|bx^3+a\right|\right)}{3\,a^2}-\frac{(bc-af)\ln\left(|x|\right)}{a^2}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-\left(-ab^2\right)^{\frac{1}{3}}abg+\left(-ab^2\right)^{\frac{2}{3}}ah-\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-\left(-ab^2\right)^{\frac{1}{3}}abg-\left(-ab^2\right)^{\frac{2}{3}}ah+\left(-ab^2\right)^{\frac{2}{3}}be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2}-\frac{\left(a^4bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-a^3b^2d+a^4bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^5b}-\frac{6\,ax^2e+3\,adx+2\,ac}{6\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)*x^4),x, algorithm
```

```
[Out] 1/3*(b*c - a*f)*ln(abs(b*x^3 + a))/a^2 - (b*c - a*f)*ln(abs(x))/a
^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g + (
-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) - 1/6*((-a*b^2)^(1/3)*b^2*
d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*
e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2) - 1/3*(a^4*b
*h*(-a/b)^(1/3) - a^3*b^2*(-a/b)^(1/3)*e - a^3*b^2*d + a^4*b*g)*(
-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/6*(6*a*x^2*e +
3*a*d*x + 2*a*c)/(a^2*x^3)
```

3.400 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=337

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{18\sqrt[3]{a}b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{9\sqrt[3]{a}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^{2/3}be+7a^{5/3}h-5ab^{2/3}f+2b^{5/3}c\right)}{3\sqrt{3}\sqrt[3]{a}b^{10/3}}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3b^3\left(a+bx^3\right)}+\frac{(bd-2ag)\log\left(a+bx^3\right)}{3b^3}+\frac{x(be-2ah)}{b^3}+\frac{fx^2}{2b^2}+\frac{gx^3}{3b^2}+\frac{hx^4}{4b^2}$$

```
[Out] ((b*e - 2*a*h)*x)/b^3 + (f*x^2)/(2*b^2) + (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*x^3)) - ((2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(10/3)) - ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(10/3)) + ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(10/3)) + ((b*d - 2*a*g)*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 1.47564, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{18\sqrt[3]{a}b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{9\sqrt[3]{a}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^{2/3}be+7a^{5/3}h-5ab^{2/3}f+2b^{5/3}c\right)}{3\sqrt{3}\sqrt[3]{a}b^{10/3}}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3b^3\left(a+bx^3\right)}+\frac{(bd-2ag)\log\left(a+bx^3\right)}{3b^3}+\frac{x(be-2ah)}{b^3}+\frac{fx^2}{2b^2}+\frac{gx^3}{3b^2}+\frac{hx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```


```
[Out] ((b*e - 2*a*h)*x)/b^3 + (f*x^2)/(2*b^2) + (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*x^3)) - ((2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(10/3)) - ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(10/3)) + ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(10/3)) + ((b*d - 2*a*g)*Log[a + b*x^3])/(3*b^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.27878, size = 334, normalized size = 0.99

$$\frac{\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(4a^{2/3}b^{4/3}e-7a^{5/3}\sqrt[3]{b}h-5abf+2b^2c\right)}{\sqrt[3]{a}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-4a^{2/3}b^{4/3}e+7a^{5/3}\sqrt[3]{b}h+5abf-2b^2c\right)}{\sqrt[3]{a}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\cdots}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(2/3)*(b*e - 2*a*h)*x + 18*b^(5/3)*f*x^2 + 12*b^(5/3)*g*x^3
 + 9*b^(5/3)*h*x^4 - (12*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b
*(d + x*(e + f*x))))/(a + b*x^3) - (4*Sqrt[3]*(2*b^2*c - 4*a^(2/3
)*b^(4/3)*e - 5*a*b*f + 7*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/
3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-2*b^2*c - 4*a^(2/3)*b^(4/
3)*e + 5*a*b*f + 7*a^(5/3)*b^(1/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a
^(1/3) + (2*(2*b^2*c + 4*a^(2/3)*b^(4/3)*e - 5*a*b*f - 7*a^(5/3)*
b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3
) + 12*b^(2/3)*(b*d - 2*a*g)*Log[a + b*x^3])/(36*b^(11/3))
```

Maple [B] time = 0.015, size = 562, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] -4/9/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))*e-5/9*a/b^3*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))+7/9/b^4*a^2*h/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))+1/2*f*x^2/b^2+1/b^2*e*x+1/3/b^2/(b*x^3+a)
*a*d-1/3/b*x^2/(b*x^3+a)*c-2/9/b^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3)
)+1/9/b^2*c/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+5/9*a/b
^3*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18*a/b^3*f/(a/b)^(1/3)*ln(x^
2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9/b^2*c*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b^2*ln(b*x^3+a)*d-2/b^3*x*a
*h-1/3/b^3/(b*x^3+a)*a^2*g-2/3/b^3*ln(b*x^3+a)*a*g+1/3/b^2*x*a/(b
*x^3+a)*e+7/9/b^4*a^2*h/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-7/18/b^4*a^
2*h/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*a/b^2*x^2/(
b*x^3+a)*f+1/3*g*x^3/b^2+1/4*h*x^4/b^2-1/3/b^3/(b*x^3+a)*x*a^2*h-
4/9/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+2/9/b^3*a/(a/b)^(2/3)*l
n(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^3 + a)^2,x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^3 + a)^2,x, algorithm
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.230377, size = 512, normalized size = 1.52

$$\frac{(bd-2\,ag)\ln\left(\left|bx^3+a\right|\right)}{3\,b^3}+\frac{\sqrt{3}\left(7\,\left(-ab^2\right)^{\frac{1}{3}}a^2h-4\,\left(-ab^2\right)^{\frac{1}{3}}abe-2\,\left(-ab^2\right)^{\frac{2}{3}}bc+5\,\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4}+\frac{abd-a^2g-\left(b^2c-abf\right)x^2-\left(a^2h-abe\right)x}{3\,(bx^3+a)b^3}+\frac{\left(7\,\left(-ab^2\right)^{\frac{1}{3}}a^2h-4\,\left(-ab^2\right)^{\frac{1}{3}}abe+2\,\left(-ab^2\right)^{\frac{2}{3}}bc-5\,\left(-ab^2\right)^{\frac{2}{3}}af\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4}-\frac{\left(2\,b^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^2b^4h-4\,ab^5e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^7}+\frac{3\,b^6hx^4+4\,b^6gx^3+6\,b^6fx^2-24\,ab^5hx+12\,b^6xe}{12\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^3 + a)^2,x, algorithm
```

```
[Out] 1/3*(b*d - 2*a*g)*ln(abs(b*x^3 + a))/b^3 + 1/9*sqrt(3)*(7*(-a*b^2
)^(1/3)*a^2*h - 4*(-a*b^2)^(1/3)*a*b*e - 2*(-a*b^2)^(2/3)*b*c + 5
*(-a*b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a*b^4) + 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 - (a
^2*h - a*b*e)*x)/((b*x^3 + a)*b^3) + 1/18*(7*(-a*b^2)^(1/3)*a^2*h
 - 4*(-a*b^2)^(1/3)*a*b*e + 2*(-a*b^2)^(2/3)*b*c - 5*(-a*b^2)^(2/
3)*a*f)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) - 1/9*(2*
b^6*c*(-a/b)^(1/3) - 5*a*b^5*f*(-a/b)^(1/3) + 7*a^2*b^4*h - 4*a*b
^5*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/12*(3*b^
6*h*x^4 + 4*b^6*g*x^3 + 6*b^6*f*x^2 - 24*a*b^5*h*x + 12*b^6*x*e)/
b^8
```

3.401 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=311

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{9a^{2/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}g+2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{3\sqrt{3}a^{2/3}b^{8/3}}+\frac{(be-2ah)\log\left(a+bx^3\right)}{3b^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3b^2\left(a+bx^3\right)}+\frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}$$

```
[Out] (f*x)/b^2 + (g*x^2)/(2*b^2) + (h*x^3)/(3*b^2) - (x*(b*c - a*f + (
b*d - a*g)*x + (b*e - a*h)*x^2))/(3*b^2*(a + b*x^3)) - ((b^(4/3)*
c + 2*a^(1/3)*b*d - 4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(8/3)) +
((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(1/3) +
b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*c - 4*a*f) - a^(1/
3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(2/3)*b^(8/3)) + ((b*e - 2*a*h)*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 1.29019, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{9a^{2/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}g+2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{3\sqrt{3}a^{2/3}b^{8/3}}+\frac{(be-2ah)\log\left(a+bx^3\right)}{3b^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3b^2\left(a+bx^3\right)}+\frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```


```
[Out] (f*x)/b^2 + (g*x^2)/(2*b^2) + (h*x^3)/(3*b^2) - (x*(b*c - a*f + (
b*d - a*g)*x + (b*e - a*h)*x^2))/(3*b^2*(a + b*x^3)) - ((b^(4/3)*
c + 2*a^(1/3)*b*d - 4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(8/3)) +
((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(1/3) +
b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*c - 4*a*f) - a^(1/
3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(2/3)*b^(8/3)) + ((b*e - 2*a*h)*Log[a + b*x^3])/(3*b^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.39672, size = 294, normalized size = 0.95

$$\frac{-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}g-2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^{4/3}g-2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\ }}{\sqrt[3]{\ }}}{\sqrt{3}}\right.}{}}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b*f*x + 9*b*g*x^2 + 6*b*h*x^3 - (6*(a^2*h + b^2*x*(c + d*x) -
 a*b*(e + x*(f + g*x))))/(a + b*x^3) + (2*Sqrt[3]*b^(1/3)*(-(b^(4
/3)*c) - 2*a^(1/3)*b*d + 4*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*b^(1/3)*(b^(4/3)*c
 - 2*a^(1/3)*b*d - 4*a*b^(1/3)*f + 5*a^(4/3)*g)*Log[a^(1/3) + b^(
1/3)*x])/a^(2/3) - (b^(1/3)*(b^(4/3)*c - 2*a^(1/3)*b*d - 4*a*b^(1
/3)*f + 5*a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(2/3) + 6*(b*e - 2*a*h)*Log[a + b*x^3])/(18*b^3)
```

Maple [B] time = 0.015, size = 533, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*h*x^3/b^2+1/2*g*x^2/b^2+f*x/b^2+1/3/b^2/(b*x^3+a)*x^2*a*g-1/3
*x^2*d/(b*x^3+a)/b+1/3*a/b^2*x/(b*x^3+a)*f-1/3/b*x/(b*x^3+a)*c-1/
3/b^3/(b*x^3+a)*a^2*h+1/3*a/b^2/(b*x^3+a)*e-4/9*a/b^3*f/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))+2/9*a/b^3*f/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)
+(a/b)^(2/3))-4/9*a/b^3*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))+1/9/b^2*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18
/b^2*c/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/9/b^2*c/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+5/9/b^3
*a*g/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18/b^3*a*g/(a/b)^(1/3)*ln(x^
2-x*(a/b)^(1/3)+(a/b)^(2/3))-5/9/b^3*a*g*3^(1/2)/(a/b)^(1/3)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/9/b^2/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))*d+1/9/b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*
d+2/9/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*d-2/3/b^3*ln(b*x^3+a)*a*h+1/3/b^2*ln(b*x^3+a)*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^3 + a)^2,x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^3 + a)^2,x, algorithm
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22687, size = 475, normalized size = 1.53

$$-\frac{(2\,ah-be)\ln\left(\left|bx^3+a\right|\right)}{3\,b^3}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-4\left(-ab^2\right)^{\frac{1}{3}}abf-2\left(-ab^2\right)^{\frac{2}{3}}bd+5\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4}-\frac{a^2h+\left(b^2d-abg\right)x^2-abe+\left(b^2c-abf\right)x}{3\,(bx^3+a)b^3}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-4\left(-ab^2\right)^{\frac{1}{3}}abf+2\left(-ab^2\right)^{\frac{2}{3}}bd-5\left(-ab^2\right)^{\frac{2}{3}}ag\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4}-\frac{\left(2\,b^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^3g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^4c-4\,ab^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}+\frac{2\,b^4hx^3+3\,b^4gx^2+6\,b^4fx}{6\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^3 + a)^2,x, algorithm
```

```
[Out] -1/3*(2*a*h - b*e)*ln(abs(b*x^3 + a))/b^3 + 1/9*sqrt(3)*((-a*b^2)
^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*f - 2*(-a*b^2)^(2/3)*b*d + 5*
(-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b
)^(1/3))/(a*b^4) - 1/3*(a^2*h + (b^2*d - a*b*g)*x^2 - a*b*e + (b^
2*c - a*b*f)*x)/((b*x^3 + a)*b^3) + 1/18*((-a*b^2)^(1/3)*b^2*c -
4*(-a*b^2)^(1/3)*a*b*f + 2*(-a*b^2)^(2/3)*b*d - 5*(-a*b^2)^(2/3)*
a*g)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) - 1/9*(2*b^4
*d*(-a/b)^(1/3) - 5*a*b^3*g*(-a/b)^(1/3) + b^4*c - 4*a*b^3*f)*(-a
/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^5) + 1/6*(2*b^4*h*x^3 +
3*b^4*g*x^2 + 6*b^4*f*x)/b^6
```

3.402 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=290

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)}{9a^{2/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}h+2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{3\sqrt{3}a^{2/3}b^{8/3}}+\frac{f\log\left(a+bx^3\right)}{3b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}$$

```
[Out] (4*g*x)/(3*b^2) + (5*h*x^2)/(6*b^2) - (c + d*x + e*x^2 + f*x^3 +
g*x^4 + h*x^5)/(3*b*(a + b*x^3)) - ((b^(4/3)*d + 2*a^(1/3)*b*e -
4*a*b^(1/3)*g - 5*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(8/3)) + ((b^(1/3)*(b*d - 4*a
*g) - a^(1/3)*(2*b*e - 5*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/
3)*b^(8/3)) - ((b^(1/3)*(b*d - 4*a*g) - a^(1/3)*(2*b*e - 5*a*h))*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*b^(8/
3)) + (f*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.994918, antiderivative size = 288, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}-4ag+bd\right)}{18a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)}{9a^{2/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}h+2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{3\sqrt{3}a^{2/3}b^{8/3}}+\frac{f\log\left(a+bx^3\right)}{3b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (4*g*x)/(3*b^2) + (5*h*x^2)/(6*b^2) - (c + d*x + e*x^2 + f*x^3 +
g*x^4 + h*x^5)/(3*b*(a + b*x^3)) - ((b^(4/3)*d + 2*a^(1/3)*b*e -
4*a*b^(1/3)*g - 5*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(8/3)) + ((b^(1/3)*(b*d - 4*a
*g) - a^(1/3)*(2*b*e - 5*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/
3)*b^(8/3)) - ((b*d - 4*a*g - (a^(1/3)*(2*b*e - 5*a*h))/b^(1/3))*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*b^(7/
3)) + (f*Log[a + b*x^3])/(3*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)}+\frac{f\log\left(a+bx^3\right)}{3b^2}+\frac{4gx}{3b^2}+\frac{5h\int x\,dx}{3b^2}+\frac{\left(\sqrt[3]{a}\,(5ah-2be)-\sqrt[3]{b}\,(4ag-bd)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{2}{3}}b^{\frac{8}{3}}}-\frac{\left(\sqrt[3]{a}\,(5ah-2be)-\sqrt[3]{b}\,(4ag-bd)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{2}{3}}b^{\frac{8}{3}}}+\frac{\sqrt{3}\left(5a^{\frac{4}{3}}h-2\sqrt[3]{a}be+4a\sqrt[3]{b}g-b^{\frac{4}{3}}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{2}{3}}b^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] -(c + d*x + e*x**2 + f*x**3 + g*x**4 + h*x**5)/(3*b*(a + b*x**3))
 + f*log(a + b*x**3)/(3*b**2) + 4*g*x/(3*b**2) + 5*h*Integral(x,
x)/(3*b**2) + (a**(1/3)*(5*a*h - 2*b*e) - b**(1/3)*(4*a*g - b*d))
*log(a**(1/3) + b**(1/3)*x)/(9*a**(2/3)*b**(8/3)) - (a**(1/3)*(5*
a*h - 2*b*e) - b**(1/3)*(4*a*g - b*d))*log(a**(2/3) - a**(1/3)*b*
*(1/3)*x + b**(2/3)*x**2)/(18*a**(2/3)*b**(8/3)) + sqrt(3)*(5*a**
(4/3)*h - 2*a**(1/3)*b*e + 4*a*b**(1/3)*g - b**(4/3)*d)*atan(sqrt
(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(2/3)*b**(8/3))
```

Mathematica [A] time = 0.368352, size = 280, normalized size = 0.97

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}h-2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{2/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^{4/3}h-2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{2/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}h\right.}{a^{2/}}}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*g*x + 9*b^(2/3)*h*x^2 - (6*b^(2/3)*(b*(c + x*(d + e*x
)) - a*(f + x*(g + h*x))))/(a + b*x^3) + (2*Sqrt[3]*(-(b^(4/3)*d)
 - 2*a^(1/3)*b*e + 4*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(b^(4/3)*d - 2*a^(1/3)*b
*e - 4*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(2/
3) - ((b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g + 5*a^(4/3)*h)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) + 6*b^(2/3
)*f*Log[a + b*x^3])/(18*b^(8/3))
```

Maple [B] time = 0.014, size = 506, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/2*h*x^2/b^2+g*x/b^2+1/3/b^2/(b*x^3+a)*x^2*a*h-1/3/b*x^2/(b*x^3+
a)*e+1/3/b^2/(b*x^3+a)*x*a*g-1/3/b*x/(b*x^3+a)*d+1/3*a/b^2/(b*x^3
```

```
+a)*f-1/3/b/(b*x^3+a)*c-4/9/b^3*a*g/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
+2/9/b^3*a*g/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-4/9/b^
3*a*g/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-1/18/b^2/(a/b)^(2/3)*ln(
x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+5/9/b^3*a*h/(a/b)^(1/3)*ln(
x+(a/b)^(1/3))-5/18/b^3*a*h/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b
)^(2/3))-5/9/b^3*a*h*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))-2/9/b^2*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*e
/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9/b^2*e*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*f*ln(b*x^
3+a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^3 + a)^2,x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^3 + a)^2,x, algorithm
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.227333, size = 444, normalized size = 1.53

$$\frac{f\ln\left(\left|bx^3+a\right|\right)}{3\,b^2}+\frac{(ah-be)x^2-bc+af-(bd-ag)x}{3\,(bx^3+a)b^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-4\,\left(-ab^2\right)^{\frac{1}{3}}abg+5\,\left(-ab^2\right)^{\frac{2}{3}}ah-2\,\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4}+\frac{b^2hx^2+2\,b^2gx}{2\,b^4}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-4\,\left(-ab^2\right)^{\frac{1}{3}}abg-5\,\left(-ab^2\right)^{\frac{2}{3}}ah+2\,\left(-ab^2\right)^{\frac{2}{3}}be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4}+\frac{\left(5\,ab^3h\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-b^4d+4\,ab^3g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^3 + a)^2,x, algorithm
```

```
[Out] 1/3*f*ln(abs(b*x^3 + a))/b^2 + 1/3*((a*h - b*e)*x^2 - b*c + a*f -
 (b*d - a*g)*x)/((b*x^3 + a)*b^2) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b
^2*d - 4*(-a*b^2)^(1/3)*a*b*g + 5*(-a*b^2)^(2/3)*a*h - 2*(-a*b^2)
^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))
/(a*b^4) + 1/2*(b^2*h*x^2 + 2*b^2*g*x)/b^4 + 1/18*((-a*b^2)^(1/3)
*b^2*d - 4*(-a*b^2)^(1/3)*a*b*g - 5*(-a*b^2)^(2/3)*a*h + 2*(-a*b^
2)^(2/3)*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) + 1
/9*(5*a*b^3*h*(-a/b)^(1/3) - 2*b^4*(-a/b)^(1/3)*e - b^4*d + 4*a*b
^3*g)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

3.403 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=289

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{18a^{4/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{9a^{4/3}b^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}be-4a^{5/3}h+2ab^{2/3}f+b^{5/3}c\right)}{3\sqrt{3}a^{4/3}b^{7/3}}-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3ab^2\left(a+bx^3\right)}+\frac{g\log\left(a+bx^3\right)}{3b^2}+\frac{hx}{b^2}$$

```
[Out] (h*x)/b^2 - (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x
^2))/(3*a*b^2*(a + b*x^3)) - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2
/3)*f - 4*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(3*Sqrt[3]*a^(4/3)*b^(7/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^
(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(7/3)
) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(7/3)) + (g*Log
[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 1.04753, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{18a^{4/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{9a^{4/3}b^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}be-4a^{5/3}h+2ab^{2/3}f+b^{5/3}c\right)}{3\sqrt{3}a^{4/3}b^{7/3}}-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3ab^2\left(a+bx^3\right)}+\frac{g\log\left(a+bx^3\right)}{3b^2}+\frac{hx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```


```
[Out] (h*x)/b^2 - (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x
^2))/(3*a*b^2*(a + b*x^3)) - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2
/3)*f - 4*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(3*Sqrt[3]*a^(4/3)*b^(7/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^
(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(7/3)
) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(7/3)) + (g*Log
[a + b*x^3])/(3*b^2)
```

Rubi in Sympy [A] time = 160.671, size = 265, normalized size = 0.92

$$\frac{g\log\left(a+bx^3\right)}{3b^2}+\frac{hx}{b^2}+\frac{x\left(a\left(ah-be\right)-bx^2\left(ag-bd\right)-bx\left(af-bc\right)\right)}{3ab^2\left(a+bx^3\right)} - \frac{\left(a^{\frac{2}{3}}\left(4ah-be\right)+b^{\frac{2}{3}}\left(2af+bc\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{4}{3}}b^{\frac{7}{3}}} + \frac{\left(a^{\frac{2}{3}}\left(4ah-be\right)+b^{\frac{2}{3}}\left(2af+bc\right)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{4}{3}}b^{\frac{7}{3}}} + \frac{\sqrt{3}\left(4a^{\frac{5}{3}}h-a^{\frac{2}{3}}be-2ab^{\frac{2}{3}}f-b^{\frac{5}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{4}{3}}b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] g*log(a + b*x**3)/(3*b**2) + h*x/b**2 + x*(a*(a*h - b*e) - b*x**2
*(a*g - b*d) - b*x*(a*f - b*c))/(3*a*b**2*(a + b*x**3)) - (a**(2/
3)*(4*a*h - b*e) + b**(2/3)*(2*a*f + b*c))*log(a**(1/3) + b**(1/3
)*x)/(9*a**(4/3)*b**(7/3)) + (a**(2/3)*(4*a*h - b*e) + b**(2/3)*(
2*a*f + b*c))*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)
/(18*a**(4/3)*b**(7/3)) + sqrt(3)*(4*a**(5/3)*h - a**(2/3)*b*e -
2*a*b**(2/3)*f - b**(5/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3
)*x/3)/a**(1/3))/(9*a**(4/3)*b**(7/3))
```

Mathematica [A] time = 0.385777, size = 285, normalized size = 0.99

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}b^{4/3}e+4a^{5/3}\sqrt[3]{b}h+2abf+b^2c\right)}{a^{4/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^{2/3}b^{4/3}e+4a^{5/3}\sqrt[3]{b}h+2abf+b^2c\right)}{a^{4/3}}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{2/3}\cdots\right.}{\cdots}}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*h*x + (6*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d
+ x*(e + f*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + a^(2/3)*b^
(4/3)*e + 2*a*b*f - 4*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/a^(4/3) - (2*(b^2*c - a^(2/3)*b^(4/3)*e + 2*
a*b*f + 4*a^(5/3)*b^(1/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) +
((b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*b^(1/3)*h)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) + 6*b^(2/3)*g
*Log[a + b*x^3])/(18*b^(8/3))
```

Maple [B] time = 0.014, size = 504, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] h*x/b^2-1/3/b*x^2/(b*x^3+a)*f+1/3*x^2/a/(b*x^3+a)*c+1/3/b^2/(b*x^
3+a)*x*a*h-1/3/b*x/(b*x^3+a)*e+1/3/b^2/(b*x^3+a)*a*g-1/3/b/(b*x^3
```

```
+a)*d-4/9/b^3*a*h/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9/b^3*a*h/(a/b)
^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-4/9/b^3*a*h/(a/b)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9/b^2*e/(a/b)^(
2/3)*ln(x+(a/b)^(1/3))-1/18/b^2*e/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3
)+(a/b)^(2/3))+1/9/b^2*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))-2/9/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+1/9/b
^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+2/9/b^2*3^(1/2
)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/9/b/a/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/18/b/a/(a/b)^(1/3)*ln(x^2-x*(a/b
)^(1/3)+(a/b)^(2/3))*c+1/9/b/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*c+1/3/b^2*g*ln(a*(b*x^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x/(b*x^3 + a)^2,x, algorithm="
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x/(b*x^3 + a)^2,x, algorithm="
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.227123, size = 451, normalized size = 1.56

$$\frac{hx}{b^2} + \frac{g\ln\left(\left|bx^3 + a\right|\right)}{3\,b^2} - \frac{\sqrt{3}\left(4\,\left(-ab^2\right)^{\frac{1}{3}}a^2h - \left(-ab^2\right)^{\frac{1}{3}}abe + \left(-ab^2\right)^{\frac{2}{3}}bc + 2\,\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3} - \frac{abd - a^2g - \left(b^2c - abf\right)x^2 - \left(a^2h - abe\right)x}{3\,(bx^3 + a)ab^2} - \frac{\left(4\,\left(-ab^2\right)^{\frac{1}{3}}a^2h - \left(-ab^2\right)^{\frac{1}{3}}abe - \left(-ab^2\right)^{\frac{2}{3}}bc - 2\,\left(-ab^2\right)^{\frac{2}{3}}af\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^3} - \frac{\left(ab^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,a^2b^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4\,a^3b^3h + a^2b^4e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x/(b*x^3 + a)^2,x, algorithm="
```

```
[Out] h*x/b^2 + 1/3*g*ln(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(4*(-a*b^2)^
(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c + 2*(-a*b
^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(a^2*b^3) - 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 - (a^2*h
 - a*b*e)*x)/((b*x^3 + a)*a*b^2) - 1/18*(4*(-a*b^2)^(1/3)*a^2*h -
 (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c - 2*(-a*b^2)^(2/3)*a*f
)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3) - 1/9*(a*b^5*
c*(-a/b)^(1/3) + 2*a^2*b^4*f*(-a/b)^(1/3) - 4*a^3*b^3*h + a^2*b^4
*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b^5)
```

3.404 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=276

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{9a^{5/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{3\sqrt{3}a^{5/3}b^{5/3}}+\frac{h\log\left(a+bx^3\right)}{3b^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x
^3)) - ((2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5
/3)*b^(5/3)) + ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(5/3)) - ((b^(1/3)*(2*b*c +
 a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(18*a^(5/3)*b^(5/3)) + (h*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.807343, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{9a^{5/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{3\sqrt{3}a^{5/3}b^{5/3}}+\frac{h\log\left(a+bx^3\right)}{3b^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```


```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x
^3)) - ((2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5
/3)*b^(5/3)) + ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(5/3)) - ((b^(1/3)*(2*b*c +
 a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(18*a^(5/3)*b^(5/3)) + (h*Log[a + b*x^3])/(3*b^2)
```

Rubi in Sympy [A] time = 118.282, size = 252, normalized size = 0.91

$$\frac{h\log\left(a+bx^3\right)}{3b^2}-\frac{x\left(af-bc+x^2\left(ah-be\right)+x\left(ag-bd\right)\right)}{3ab\left(a+bx^3\right)}-\frac{\left(\sqrt[3]{a}\left(2ag+bd\right)-\sqrt[3]{b}\left(af+2bc\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{5}{3}}}+\frac{\left(\sqrt[3]{a}\left(2ag+bd\right)-\sqrt[3]{b}\left(af+2bc\right)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{5}{3}}b^{\frac{5}{3}}}-\frac{\sqrt{3}\left(2a^{\frac{4}{3}}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{\frac{4}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] h*log(a + b*x**3)/(3*b**2) - x*(a*f - b*c + x**2*(a*h - b*e) + x*
(a*g - b*d))/(3*a*b*(a + b*x**3)) - (a**(1/3)*(2*a*g + b*d) - b**
(1/3)*(a*f + 2*b*c))*log(a**(1/3) + b**(1/3)*x)/(9*a**(5/3)*b**(5
/3)) + (a**(1/3)*(2*a*g + b*d) - b**(1/3)*(a*f + 2*b*c))*log(a**(
2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*a**(5/3)*b**(5/3)
) - sqrt(3)*(2*a**(4/3)*g + a**(1/3)*b*d + a*b**(1/3)*f + 2*b**(4
/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a*
*(5/3)*b**(5/3))
```

Mathematica [A] time = 0.339598, size = 268, normalized size = 0.97

$$\frac{\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}g+\sqrt[3]{a}bd-a\sqrt[3]{b}f-2b^{4/3}c\right)}{a^{5/3}}+\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^{4/3}g-\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{a^{5/3}}-\frac{2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{}}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

```
[Out] ((6*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a*(a + b*
x^3)) - (2*Sqrt[3]*b^(1/3)*(2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)
*f + 2*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^
(5/3) + (2*b^(1/3)*(2*b^(4/3)*c - a^(1/3)*b*d + a*b^(1/3)*f - 2*a
^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + (b^(1/3)*(-2*b^(4/3
)*c + a^(1/3)*b*d - a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 6*h*Log[a + b*x^3])/(18*b
^2)
```

Maple [B] time = 0.013, size = 465, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (-1/3*(a*g-b*d)/a/b*x^2-1/3*(a*f-b*c)/a/b*x+1/3*(a*h-b*e)/b^2)/(b
*x^3+a)+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1/18/b^2/(a/b)^(2
```

```
/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/9/b^2/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+2/9*c/a/b/(a/b)^(2/3
)*ln(x+(a/b)^(1/3))-1/9*c/a/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a
/b)^(2/3))+2/9*c/a/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))-2/9*g/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9*g/b^2
/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9*g*3^(1/2)/b^2/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9*d/a/b/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))+1/18*d/a/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))+1/9*d/a*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))+1/3*h/b^2*ln(a*b*(b*x^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^2,x, algorithm="ma

[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^2,x, algorithm="fr

[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.224916, size = 429, normalized size = 1.55

$$\frac{h\ln\left(\left|bx^3+a\right|\right)}{3\,b^2}+\frac{(bd-ag)x^2+(bc-af)x+\frac{a^2h-abe}{b}}{3\,(bx^3+a)ab}+\frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}b^2c+\left(-ab^2\right)^{\frac{1}{3}}abf-\left(-ab^2\right)^{\frac{2}{3}}bd-2\,\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3}+\frac{\left(2\,\left(-ab^2\right)^{\frac{1}{3}}b^2c+\left(-ab^2\right)^{\frac{1}{3}}abf+\left(-ab^2\right)^{\frac{2}{3}}bd+2\,\left(-ab^2\right)^{\frac{2}{3}}ag\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^3}-\frac{\left(ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b^2g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^3c+a^2b^2f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^2,x, algorithm="gi
```

```
[Out] 1/3*h*ln(abs(b*x^3 + a))/b^2 + 1/3*((b*d - a*g)*x^2 + (b*c - a*f)
*x + (a^2*h - a*b*e)/b)/((b*x^3 + a)*a*b) + 1/9*sqrt(3)*(2*(-a*b^
2)^(1/3)*b^2*c + (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d - 2*(-
a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/(a^2*b^3) + 1/18*(2*(-a*b^2)^(1/3)*b^2*c + (-a*b^2)^(1/3)*
a*b*f + (-a*b^2)^(2/3)*b*d + 2*(-a*b^2)^(2/3)*a*g)*ln(x^2 + x*(-a
/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3) - 1/9*(a*b^3*d*(-a/b)^(1/3) +
 2*a^2*b^2*g*(-a/b)^(1/3) + 2*a*b^3*c + a^2*b^2*f)*(-a/b)^(1/3)*l
n(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

3.405 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2} \, dx$

Optimal. Leaf size=289

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah+be)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah+be)\right)}{9a^{5/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{4/3}h+\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{4/3}d\right)}{3\sqrt{3}a^{5/3}b^{5/3}}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{3a^2b\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^2}+\frac{c\log(x)}{a^2}$$

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*
b*(a + b*x^3)) - ((2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(5/3)*b^(5/3)) + (c*Log[x])/a^2 + ((b^(1/3)*(2*b*d + a*g)
 - a^(1/3)*(b*e + 2*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^
(5/3)) - ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*h))*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(5/3)) - (
c*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 1.10344, antiderivative size = 287, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2ah+be)}{\sqrt[3]{b}}+ag+2bd\right)}{18a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah+be)\right)}{9a^{5/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{4/3}h+\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{4/3}d\right)}{3\sqrt{3}a^{5/3}b^{5/3}}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{3a^2b\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^2}+\frac{c\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*
b*(a + b*x^3)) - ((2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(5/3)*b^(5/3)) + (c*Log[x])/a^2 + ((b^(1/3)*(2*b*d + a*g)
 - a^(1/3)*(b*e + 2*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^
(5/3)) - ((2*b*d + a*g - (a^(1/3)*(b*e + 2*a*h))/b^(1/3))*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3)) - (
c*Log[a + b*x^3])/(3*a^2)
```

Rubi in Sympy [A] time = 149.517, size = 262, normalized size = 0.91

$$\frac{f\log(x)}{ab}-\frac{f\log\left(a+bx^{3}\right)}{3ab}-\frac{x\left(\frac{af}{x}+ag+ahx-\frac{bc}{x}-bd-bex\right)}{3ab\left(a+bx^{3}\right)}-\frac{\left(\sqrt[3]{a}\left(2ah+be\right)-\sqrt[3]{b}\left(ag+2bd\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{5}{3}}}+\frac{\left(\sqrt[3]{a}\left(2ah+be\right)-\sqrt[3]{b}\left(ag+2bd\right)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}\right)}{18a^{\frac{5}{3}}b^{\frac{5}{3}}}-\frac{\sqrt{3}\left(2a^{\frac{4}{3}}h+\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{\frac{4}{3}}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] f*log(x)/(a*b) - f*log(a + b*x**3)/(3*a*b) - x*(a*f/x + a*g + a*h
*x - b*c/x - b*d - b*e*x)/(3*a*b*(a + b*x**3)) - (a**(1/3)*(2*a*h
 + b*e) - b**(1/3)*(a*g + 2*b*d))*log(a**(1/3) + b**(1/3)*x)/(9*a
**(5/3)*b**(5/3)) + (a**(1/3)*(2*a*h + b*e) - b**(1/3)*(a*g + 2*b
*d))*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*a**(
5/3)*b**(5/3)) - sqrt(3)*(2*a**(4/3)*h + a**(1/3)*b*e + a*b**(1/3
)*g + 2*b**(4/3)*d)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a*
*(1/3))/(9*a**(5/3)*b**(5/3))
```

Mathematica [A] time = 0.373485, size = 269, normalized size = 0.93

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}h+\sqrt[3]{a}be-a\sqrt[3]{b}g-2b^{4/3}d\right)}{b^{5/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^{4/3}h-\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{4/3}d\right)}{b^{5/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{}}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*
x^3)) - (2*Sqrt[3]*a^(1/3)*(2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)
*g + 2*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(5/3) + 18*c*Log[x] + (2*a^(1/3)*(2*b^(4/3)*d - a^(1/3)*b*e + a*b
^(1/3)*g - 2*a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) + (a^(1
/3)*(-2*b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g + 2*a^(4/3)*h)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 6*c*Log[a +
 b*x^3])/(18*a^2)
```

Maple [B] time = 0.019, size = 509, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x)
```

```
[Out] c*ln(x)/a^2-1/3/(b*x^3+a)*x^2/b*h+1/3/a*x^2/(b*x^3+a)*e-1/3/(b*x^
3+a)*x/b*g+1/3/a*x/(b*x^3+a)*d-1/3/b/(b*x^3+a)*f+1/3/a/(b*x^3+a)*
```

```
c+1/9*g/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18*g/b^2/(a/b)^(2/3)*
ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/9*g/b^2/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/a/b*d/(a/b)^(2/3)*ln(x
+(a/b)^(1/3))-1/9/a/b*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2
/3))+2/9/a/b*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-2/9*h/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9*h/b^2/(a/b)
^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9*h*3^(1/2)/b^2/(a/b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a/b/(a/b)^(1/3)
*ln(x+(a/b)^(1/3))*e+1/18/a/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a
/b)^(2/3))*e+1/9/a/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))*e-1/3/a^2*c*ln(b*(b*x^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x),x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x),x, algorithm
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.226775, size = 452, normalized size = 1.56

$$-\frac{c\ln\left(\left|bx^{3}+a\right|\right)}{3\,a^{2}}+\frac{c\ln\left(\left|x\right|\right)}{a^{2}}+\frac{\sqrt{3}\left(2\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{2}d+\left(-ab^{2}\right)^{\frac{1}{3}}abg-2\,\left(-ab^{2}\right)^{\frac{2}{3}}ah-\left(-ab^{2}\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b^{3}}+\frac{abc-a^{2}f-\left(a^{2}h-abe\right)x^{2}+\left(abd-a^{2}g\right)x}{3\,(bx^{3}+a)a^{2}b}+\frac{\left(2\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{2}d+\left(-ab^{2}\right)^{\frac{1}{3}}abg+2\,\left(-ab^{2}\right)^{\frac{2}{3}}ah+\left(-ab^{2}\right)^{\frac{2}{3}}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b^{3}}-\frac{\left(2\,a^{4}b^{2}h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{3}b^{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+2\,a^{3}b^{3}d+a^{4}b^{2}g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{5}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x),x, algorithm
```

```
[Out] -1/3*c*ln(abs(b*x^3 + a))/a^2 + c*ln(abs(x))/a^2 + 1/9*sqrt(3)*(2
*(-a*b^2)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g - 2*(-a*b^2)^(2/3)*a
*h - (-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/(a^2*b^3) + 1/3*(a*b*c - a^2*f - (a^2*h - a*b*e)*x^
2 + (a*b*d - a^2*g)*x)/((b*x^3 + a)*a^2*b) + 1/18*(2*(-a*b^2)^(1/
3)*b^2*d + (-a*b^2)^(1/3)*a*b*g + 2*(-a*b^2)^(2/3)*a*h + (-a*b^2)
^(2/3)*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3) - 1
/9*(2*a^4*b^2*h*(-a/b)^(1/3) + a^3*b^3*(-a/b)^(1/3)*e + 2*a^3*b^3
*d + a^4*b^2*g)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^5*b^3)
```

3.406 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=301

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{18a^{7/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{9a^{7/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+4b^{5/3}c\right)}{3\sqrt{3}a^{7/3}b^{4/3}}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3a^2b\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^2}-\frac{c}{a^2x}+\frac{d\log(x)}{a^2}$$

```
[Out] -(c/(a^2*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g
)*x^2))/(3*a^2*b*(a + b*x^3)) + ((4*b^(5/3)*c - 2*a^(2/3)*b*e - a
*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a
^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(4/3)) + (d*Log[x])/a^2 + ((b^(2/3
)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x]
)/(9*a^(7/3)*b^(4/3)) - ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e
+ a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/
3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 1.18186, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{18a^{7/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{9a^{7/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+4b^{5/3}c\right)}{3\sqrt{3}a^{7/3}b^{4/3}}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3a^2b\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^2}-\frac{c}{a^2x}+\frac{d\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```


```
[Out] -(c/(a^2*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g
)*x^2))/(3*a^2*b*(a + b*x^3)) + ((4*b^(5/3)*c - 2*a^(2/3)*b*e - a
*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a
^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(4/3)) + (d*Log[x])/a^2 + ((b^(2/3
)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x]
)/(9*a^(7/3)*b^(4/3)) - ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e
+ a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/
3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^2)
```

Rubi in Sympy [A] time = 138.149, size = 250, normalized size = 0.83

$$-\frac{f}{abx}+\frac{g\log(x)}{ab}-\frac{g\log\left(a+bx^3\right)}{3ab}-\frac{x\left(\frac{af}{x^2}+\frac{ag}{x}+ah-\frac{bc}{x^2}-\frac{bd}{x}-be\right)}{3ab\left(a+bx^3\right)}-\frac{\sqrt{3}\left(-3\sqrt[3]{a}b^{\frac{2}{3}}f+ah+2be\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{4}{3}}}+\frac{\left(3\sqrt[3]{a}b^{\frac{2}{3}}f+ah+2be\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{4}{3}}}-\frac{\left(3\sqrt[3]{a}b^{\frac{2}{3}}f+ah+2be\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{5}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] -f/(a*b*x) + g*log(x)/(a*b) - g*log(a + b*x**3)/(3*a*b) - x*(a*f/
x**2 + a*g/x + a*h - b*c/x**2 - b*d/x - b*e)/(3*a*b*(a + b*x**3))
 - sqrt(3)*(-3*a**(1/3)*b**(2/3)*f + a*h + 2*b*e)*atan(sqrt(3)*(a
**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*b**(4/3)) + (3*
a**(1/3)*b**(2/3)*f + a*h + 2*b*e)*log(a**(1/3) + b**(1/3)*x)/(9*
a**(5/3)*b**(4/3)) - (3*a**(1/3)*b**(2/3)*f + a*h + 2*b*e)*log(a*
*(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*a**(5/3)*b**(4/
3))
```

Mathematica [A] time = 0.557731, size = 285, normalized size = 0.95

$$-\frac{\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{2/3}be+a^{5/3}h-ab^{2/3}f+4b^{5/3}c\right)}{b^{4/3}}-\frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{2/3}be+a^{5/3}h-ab^{2/3}f+4b^{5/3}c\right)}{b^{4/3}}+\frac{2\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{}}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((18*a*c)/x + (6*a*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e +
f*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-4*b^(5/3)*c + 2*a^
(2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]])/b^(4/3) - 18*a*d*Log[x] - (2*a^(2/3)*(4*b^(5/3)*
c + 2*a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3
)*x])/b^(4/3) + (a^(2/3)*(4*b^(5/3)*c + 2*a^(2/3)*b*e - a*b^(2/3)
*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b
^(4/3) + 6*a*d*Log[a + b*x^3])/(18*a^3)
```

Maple [B] time = 0.019, size = 519, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] d*ln(x)/a^2-1/a^2*c/x+1/3/a*x^2/(b*x^3+a)*f-1/3/a^2*b*x^2/(b*x^3+
a)*c-1/3/(b*x^3+a)*x/b*h+1/3/a*x/(b*x^3+a)*e-1/3/(b*x^3+a)/b*g+1/
3/a/(b*x^3+a)*d+1/9*h/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18*h/b^
2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/9*h/b^2/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/a/b/(a/b
```

```
)^(2/3)*ln(x+(a/b)^(1/3))*e-1/9/a/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))*e+2/9/a/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*e-1/9/a*f/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1
/18/a*f/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/9/a*f*3
^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+4/9/
a^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*c/(a/b)^(1/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))-4/9/a^2*c*3^(1/2)/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/a^2*d*ln(b*(b*x^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x^2),x, algorit
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x^2),x, algorit
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.226488, size = 473, normalized size = 1.57

$$
\begin{aligned}
&-\frac{d\ln\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{d\ln\left(|x|\right)}{a^2} \\
&+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h+2\,\left(-ab^2\right)^{\frac{1}{3}}abe+4\,\left(-ab^2\right)^{\frac{2}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2} \\
&-\frac{4\,b^2cx^3-abfx^3+a^2hx^2-abx^2e-abdx+a^2gx+3\,abc}{3\,(bx^4+ax)a^2b} \\
&+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h+2\,\left(-ab^2\right)^{\frac{1}{3}}abe-4\,\left(-ab^2\right)^{\frac{2}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}af\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^2} \\
&+\frac{\left(4\,a^2b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^4b^2h-2\,a^3b^3e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b^3}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x^2),x, algorit
```

```
[Out] -1/3*d*ln(abs(b*x^3 + a))/a^2 + d*ln(abs(x))/a^2 + 1/9*sqrt(3)*((
-a*b^2)^(1/3)*a^2*h + 2*(-a*b^2)^(1/3)*a*b*e + 4*(-a*b^2)^(2/3)*b
*c - (-a*b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/(a^3*b^2) - 1/3*(4*b^2*c*x^3 - a*b*f*x^3 + a^2*h*x^
2 - a*b*x^2*e - a*b*d*x + a^2*g*x + 3*a*b*c)/((b*x^4 + a*x)*a^2*b
) + 1/18*((-a*b^2)^(1/3)*a^2*h + 2*(-a*b^2)^(1/3)*a*b*e - 4*(-a*b
^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*ln(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/(a^3*b^2) + 1/9*(4*a^2*b^4*c*(-a/b)^(1/3) - a^3*b^3*f*
(-a/b)^(1/3) - a^4*b^2*h - 2*a^3*b^3*e)*(-a/b)^(1/3)*ln(abs(x - (
-a/b)^(1/3)))/(a^5*b^3)
```

3.407 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=306

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)}{18a^{8/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)}{9a^{8/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{3\sqrt{3}a^{8/3}b^{2/3}}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3a^2\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^2}-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\log(x)}{a^2}$$

```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e
 - a*h)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*c + 4*a^(1/3)*b*d
 - 2*a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(2/3)) + (e*Log[x])/a^2 - ((
b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b*d - a*g))*Log[a^(1/3) + b^
(1/3)*x])/(9*a^(8/3)*b^(2/3)) + ((b^(1/3)*(5*b*c - 2*a*f) - a^(1/
3)*(4*b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(18*a^(8/3)*b^(2/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 1.14383, antiderivative size = 304, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}-2af+5bc\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)}{9a^{8/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{3\sqrt{3}a^{8/3}b^{2/3}}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3a^2\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^2}-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e
 - a*h)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*c + 4*a^(1/3)*b*d
 - 2*a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(2/3)) + (e*Log[x])/a^2 - ((
b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b*d - a*g))*Log[a^(1/3) + b^
(1/3)*x])/(9*a^(8/3)*b^(2/3)) + ((5*b*c - 2*a*f - (a^(1/3)*(4*b*d
 - a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(18*a^(8/3)*b^(1/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rubi in Sympy [A] time = 110.265, size = 241, normalized size = 0.79

$$-\frac{f}{2abx^2}-\frac{g}{abx}+\frac{h\log(x)}{ab}-\frac{h\log\left(a+bx^3\right)}{3ab}-\frac{x\left(\frac{af}{x^3}+\frac{ag}{x^2}+\frac{ah}{x}-\frac{bc}{x^3}-\frac{bd}{x^2}-\frac{be}{x}\right)}{3ab\left(a+bx^3\right)}+\frac{\left(\sqrt[3]{a}g-\sqrt[3]{b}f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\left(\sqrt[3]{a}g-\sqrt[3]{b}f\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{5}{3}}b^{\frac{2}{3}}}+\frac{\sqrt{3}\left(\sqrt[3]{a}g+\sqrt[3]{b}f\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] -f/(2*a*b*x**2) - g/(a*b*x) + h*log(x)/(a*b) - h*log(a + b*x**3)/
(3*a*b) - x*(a*f/x**3 + a*g/x**2 + a*h/x - b*c/x**3 - b*d/x**2 -
b*e/x)/(3*a*b*(a + b*x**3)) + (a**(1/3)*g - b**(1/3)*f)*log(a**(1
/3) + b**(1/3)*x)/(3*a**(5/3)*b**(2/3)) - (a**(1/3)*g - b**(1/3)*
f)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(5/3
)*b**(2/3)) + sqrt(3)*(a**(1/3)*g + b**(1/3)*f)*atan(sqrt(3)*(a**
(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(5/3)*b**(2/3))
```

Mathematica [A] time = 1.08419, size = 292, normalized size = 0.95

$$-\frac{-\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(1-\cdots\right)}{\cdots}}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -((9*a*c)/x^2 + (18*a*d)/x + (6*a*(a^2*h + b^2*x*(c + d*x) - a*b*
(e + x*(f + g*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(1/3)*(-5*b^(4
/3)*c - 4*a^(1/3)*b*d + 2*a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) - 18*a*e*Log[x] + (2*a^(1/
3)*(5*b^(4/3)*c - 4*a^(1/3)*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[
a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)*(5*b^(4/3)*c - 4*a^(1/3)
*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/b^(2/3) + 6*a*e*Log[a + b*x^3])/(18*a^3)
```

Maple [B] time = 0.021, size = 527, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] -d/a^2/x+e*ln(x)/a^2-1/2*c/a^2/x^2+1/3/a/(b*x^3+a)*x^2*g-1/3/a^2*
b*x^2/(b*x^3+a)*d+1/3/a*x/(b*x^3+a)*f-1/3/a^2*b*x/(b*x^3+a)*c-1/3
/(b*x^3+a)/b*h+1/3/a/(b*x^3+a)*e-5/9/a^2*c/(a/b)^(2/3)*ln(x+(a/b)
^(1/3))+5/18/a^2*c/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-
5/9/a^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))+2/9/a*f/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9/a*f/b/(a/b)^(2/3
```

```
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9/a*f/b/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+4/9/a^2/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))*d-2/9/a^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2
/3))*d-4/9/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))*d-1/9/a*g/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18/a*g/b/(a
/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/9/a*g*3^(1/2)/b/(a/
b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*e*ln(b*x^3+a
)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x^3),x, algorit
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x^3),x, algorit
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.226211, size = 483, normalized size = 1.58

$$-\frac{e\ln\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{e\ln\left(|x|\right)}{a^2}-\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^2c-2\,\left(-ab^2\right)^{\frac{1}{3}}abf-4\,\left(-ab^2\right)^{\frac{2}{3}}bd+\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2}-\frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^2c-2\,\left(-ab^2\right)^{\frac{1}{3}}abf+4\,\left(-ab^2\right)^{\frac{2}{3}}bd-\left(-ab^2\right)^{\frac{2}{3}}ag\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^2}+\frac{\left(4\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^2c-2\,a^3bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}-\frac{2\left(4\,b^2d-abg\right)x^4+6\,abdx+\left(5\,b^2c-2\,abf\right)x^3+3\,abc+2\left(a^2h-abe\right)x^2}{6\left(bx^3+a\right)a^2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x^3),x, algorit
```

```
[Out] -1/3*e*ln(abs(b*x^3 + a))/a^2 + e*ln(abs(x))/a^2 - 1/9*sqrt(3)*(5
*(-a*b^2)^(1/3)*b^2*c - 2*(-a*b^2)^(1/3)*a*b*f - 4*(-a*b^2)^(2/3)
*b*d + (-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a^3*b^2) - 1/18*(5*(-a*b^2)^(1/3)*b^2*c - 2*(-a*
b^2)^(1/3)*a*b*f + 4*(-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*ln(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2) + 1/9*(4*a^2*b^2*d
*(-a/b)^(1/3) - a^3*b*g*(-a/b)^(1/3) + 5*a^2*b^2*c - 2*a^3*b*f)*(
-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/6*(2*(4*b^2*d -
 a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*a*b*f)*x^3 + 3*a*b*c + 2*(
a^2*h - a*b*e)*x^2)/((b*x^3 + a)*a^2*b*x^2)
```

3.408 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$

Optimal. Leaf size=338

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)}{18a^{8/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)}{9a^{8/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{3\sqrt{3}a^{8/3}b^{2/3}}+\frac{(2bc-af)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2bc-af)}{a^3}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{3a^2\left(a+bx^3\right)}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}$$

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d - a*g + (b*e
 - a*h)*x - b*((b*c)/a - f)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/
3)*d + 4*a^(1/3)*b*e - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(2/3)) -
 ((2*b*c - a*f)*Log[x])/a^3 - ((b^(1/3)*(5*b*d - 2*a*g) - a^(1/3)
*(4*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^(2/3)) + (
(b^(1/3)*(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(2/3)) + ((2*b*c -
 a*f)*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 1.45272, antiderivative size = 336, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4be-ah)}{\sqrt[3]{b}}-2ag+5bd\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)}{9a^{8/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{3\sqrt{3}a^{8/3}b^{2/3}}+\frac{(2bc-af)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2bc-af)}{a^3}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{3a^2\left(a+bx^3\right)}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d - a*g + (b*e
 - a*h)*x - b*((b*c)/a - f)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/
3)*d + 4*a^(1/3)*b*e - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(2/3)) -
 ((2*b*c - a*f)*Log[x])/a^3 - ((b^(1/3)*(5*b*d - 2*a*g) - a^(1/3)
*(4*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^(2/3)) + (
(5*b*d - 2*a*g - (a^(1/3)*(4*b*e - a*h))/b^(1/3))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(1/3)) + ((2*b*c -
 a*f)*Log[a + b*x^3])/(3*a^3)
```

Rubi in Sympy [A] time = 128.198, size = 255, normalized size = 0.75

$$-\frac{f}{3abx^3}-\frac{g}{2abx^2}-\frac{h}{abx}-\frac{x\left(\frac{af}{x^4}+\frac{ag}{x^3}+\frac{ah}{x^2}-\frac{bc}{x^4}-\frac{bd}{x^3}-\frac{be}{x^2}\right)}{3ab\,(a+bx^3)}-\frac{f\log{(x)}}{a^2}+\frac{f\log{(a+bx^3)}}{3a^2}+\frac{\left(\sqrt[3]{a}h-\sqrt[3]{b}g\right)\log{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}-\frac{\left(\sqrt[3]{a}h-\sqrt[3]{b}g\right)\log{\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}}{6a^{\frac{5}{3}}b^{\frac{2}{3}}}+\frac{\sqrt{3}\left(\sqrt[3]{a}h+\sqrt[3]{b}g\right)\operatorname{atan}{\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] -f/(3*a*b*x**3) - g/(2*a*b*x**2) - h/(a*b*x) - x*(a*f/x**4 + a*g/
x**3 + a*h/x**2 - b*c/x**4 - b*d/x**3 - b*e/x**2)/(3*a*b*(a + b*x
**3)) - f*log(x)/a**2 + f*log(a + b*x**3)/(3*a**2) + (a**(1/3)*h
- b**(1/3)*g)*log(a**(1/3) + b**(1/3)*x)/(3*a**(5/3)*b**(2/3)) -
(a**(1/3)*h - b**(1/3)*g)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b*
*(2/3)*x**2)/(6*a**(5/3)*b**(2/3)) + sqrt(3)*(a**(1/3)*h + b**(1/
3)*g)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**
(5/3)*b**(2/3))
```

Mathematica [A] time = 1.1456, size = 303, normalized size = 0.9

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{2/3}}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}h-4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{2/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x + (a*(-6*b*(c + x*(d + e
*x)) + 6*a*(f + x*(g + h*x))))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(
-5*b^(4/3)*d - 4*a^(1/3)*b*e + 2*a*b^(1/3)*g + a^(4/3)*h)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 18*(-2*b*c + a*f)
*Log[x] - (2*a^(1/3)*(5*b^(4/3)*d - 4*a^(1/3)*b*e - 2*a*b^(1/3)*g
 + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (a^(1/3)*(5*b^(
4/3)*d - 4*a^(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*(2*b*c - a*f)*Log[
a + b*x^3])/(18*a^3)
```

Maple [B] time = 0.022, size = 561, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 1/9/a*h*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))-2*b*c*ln(x)/a^3+2/3*b*c*ln(b*x^3+a)/a^3-1/9/a*h/b/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))+1/18/a*h/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(
a/b)^(2/3))+2/9/a*g/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
```

```
a/b)^(1/3)*x-1))+1/a^2*ln(x)*f-1/3/a^2*ln(b*x^3+a)*f+1/3/a/(b*x^3
+a)*f-1/3/a^2*b/(b*x^3+a)*c-1/3*c/a^2/x^3-1/3/a^2*x^2/(b*x^3+a)*b
*e-5/9/a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+5/18/a^2/(a/b)^(2/3)*l
n(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-1/2*d/a^2/x^2-e/a^2/x+1/3/a/(b
*x^3+a)*x*g+2/9/a*g/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-4/9/a^2*e*3^(
1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a*g/
b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-5/9/a^2/(a/b)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/3/a/(b*x^3
+a)*x^2*h-1/3/a^2*x/(b*x^3+a)*b*d+4/9/a^2*e/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))-2/9/a^2*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x^4),x, algorit
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x^4),x, algorit
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.226904, size = 520, normalized size = 1.54

$$\frac{(2\,bc-af)\ln\left(\left|bx^3+a\right|\right)}{3\,a^3}-\frac{(2\,bc-af)\ln\left(|x|\right)}{a^3}-\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^2d-2\,\left(-ab^2\right)^{\frac{1}{3}}abg+\left(-ab^2\right)^{\frac{2}{3}}ah-4\,\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2}-\frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^2d-2\,\left(-ab^2\right)^{\frac{1}{3}}abg-\left(-ab^2\right)^{\frac{2}{3}}ah+4\,\left(-ab^2\right)^{\frac{2}{3}}be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^2}-\frac{\left(a^5bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^4b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-5\,a^4b^2d+2\,a^5bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7b}+\frac{2\left(a^2h-4\,abe\right)x^5-\left(5\,abd-2\,a^2g\right)x^4-6\,a^2x^2e-3\,a^2dx-2\left(2\,abc-a^2f\right)x^3-2\,a^2c}{6\left(bx^3+a\right)a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^2*x^4),x, algorit
```

```
[Out] 1/3*(2*b*c - a*f)*ln(abs(b*x^3 + a))/a^3 - (2*b*c - a*f)*ln(abs(x
))/a^3 - 1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*b^2*d - 2*(-a*b^2)^(1/3)*a
*b*g + (-a*b^2)^(2/3)*a*h - 4*(-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) - 1/18*(5*(-a*b^
2)^(1/3)*b^2*d - 2*(-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h + 4*
(-a*b^2)^(2/3)*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*
b^2) - 1/9*(a^5*b*h*(-a/b)^(1/3) - 4*a^4*b^2*(-a/b)^(1/3)*e - 5*a
^4*b^2*d + 2*a^5*b*g)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^7
*b) + 1/6*(2*(a^2*h - 4*a*b*e)*x^5 - (5*a*b*d - 2*a^2*g)*x^4 - 6*
a^2*x^2*e - 3*a^2*d*x - 2*(2*a*b*c - a^2*f)*x^3 - 2*a^2*c)/((b*x^
3 + a)*a^3*x^3)
```

3.409 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=345

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{27a^{4/3}b^{10/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{2/3}be-14a^{5/3}h+5ab^{2/3}f+b^{5/3}c\right)}{9\sqrt{3}a^{4/3}b^{10/3}} - \frac{x\left(-2bx(bc-4af)-3bx^2(bd-3ag)+a(7be-13ah)\right)}{18ab^3\left(a+bx^3\right)} + \frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6b^3\left(a+bx^3\right)^2} + \frac{g\log\left(a+bx^3\right)}{3b^3} + \frac{hx}{b^3}$$

```
[Out] (h*x)/b^3 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x
^2))/(6*b^3*(a + b*x^3)^2) - (x*(a*(7*b*e - 13*a*h) - 2*b*(b*c -
4*a*f)*x - 3*b*(b*d - 3*a*g)*x^2))/(18*a*b^3*(a + b*x^3)) - ((b^(
5/3)*c + 2*a^(2/3)*b*e + 5*a*b^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(10
/3)) - ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(
1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(10/3)) + ((b^(2/3)*(b*c + 5*a*f
) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(54*a^(4/3)*b^(10/3)) + (g*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 1.71711, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{27a^{4/3}b^{10/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{2/3}be-14a^{5/3}h+5ab^{2/3}f+b^{5/3}c\right)}{9\sqrt{3}a^{4/3}b^{10/3}} - \frac{x\left(-2bx(bc-4af)-3bx^2(bd-3ag)+a(7be-13ah)\right)}{18ab^3\left(a+bx^3\right)} + \frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6b^3\left(a+bx^3\right)^2} + \frac{g\log\left(a+bx^3\right)}{3b^3} + \frac{hx}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```


```
[Out] (h*x)/b^3 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x
^2))/(6*b^3*(a + b*x^3)^2) - (x*(a*(7*b*e - 13*a*h) - 2*b*(b*c -
4*a*f)*x - 3*b*(b*d - 3*a*g)*x^2))/(18*a*b^3*(a + b*x^3)) - ((b^(
5/3)*c + 2*a^(2/3)*b*e + 5*a*b^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(10
/3)) - ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(
1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(10/3)) + ((b^(2/3)*(b*c + 5*a*f
) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(54*a^(4/3)*b^(10/3)) + (g*Log[a + b*x^3])/(3*b^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.594297, size = 342, normalized size = 0.99

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^{2/3}b^{4/3}e+14a^{5/3}\sqrt[3]{b}h+5abf+b^2c\right)}{a^{4/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^{2/3}b^{4/3}e+14a^{5/3}\sqrt[3]{b}h+5abf+b^2c\right)}{a^{4/3}}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\cdots}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(2/3)*h*x - (9*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d
+ x*(e + f*x))))/(a + b*x^3)^2 + (3*b^(2/3)*(2*b^2*c*x^2 + a^2*(1
2*g + 13*h*x) - a*b*(6*d + x*(7*e + 8*f*x))))/(a*(a + b*x^3)) - (
2*Sqrt[3]*(b^2*c + 2*a^(2/3)*b^(4/3)*e + 5*a*b*f - 14*a^(5/3)*b^(
1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) - (2
*(b^2*c - 2*a^(2/3)*b^(4/3)*e + 5*a*b*f + 14*a^(5/3)*b^(1/3)*h)*L
og[a^(1/3) + b^(1/3)*x])/a^(4/3) + ((b^2*c - 2*a^(2/3)*b^(4/3)*e
+ 5*a*b*f + 14*a^(5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/a^(4/3) + 18*b^(2/3)*g*Log[a + b*x^3])/(54*b^(11
/3))
```

Maple [B] time = 0.019, size = 621, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/27/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))*c-5/18*a/(b*x^3+a)^2/b^2*x^2*f+1/9/(b*x^3+a)^2/a*x^5*c-1/18
/b/(b*x^3+a)^2*x^2*c-4/9/(b*x^3+a)^2/b*x^5*f-5/27/b^3/(a/b)^(1/3)
*ln(x+(a/b)^(1/3))*f+5/54/b^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a
/b)^(2/3))*f+1/3/b^3*g*ln(a*(b*x^3+a))-1/27/b^2/a/(a/b)^(1/3)*ln(
x+(a/b)^(1/3))*c+h*x/b^3+2/3/b^2/(b*x^3+a)^2*x^3*a*g-1/3/b/(b*x^3
+a)^2*d*x^3+1/2/b^3/(b*x^3+a)^2*a^2*g-1/6/b^2/(b*x^3+a)^2*a*d+1/5
4/b^2/a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-2/9/b^2/(
b*x^3+a)^2*a*e*x+5/9/b^3/(b*x^3+a)^2*x*a^2*h-14/27/b^4*a*h/(a/b)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/27/b^3/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+7/27/
b^4*a*h/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-14/27/b^4*a
*h/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+13/18/b^2/(b*x^3+a)^2*x^4*a*h+5/
27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*f-7/18/b/(b*x^3+a)^2*x^4*e+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*e-1/27/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^3 + a)^3,x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^3 + a)^3,x, algorithm
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.228512, size = 541, normalized size = 1.57

$$\frac{hx}{b^3} + \frac{g\ln\left(\left|bx^3+a\right|\right)}{3\,b^3} - \frac{\sqrt{3}\left(14\,\left(-ab^2\right)^{\frac{1}{3}}a^2h - 2\,\left(-ab^2\right)^{\frac{1}{3}}abe + \left(-ab^2\right)^{\frac{2}{3}}bc + 5\,\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4} - \frac{\left(14\,\left(-ab^2\right)^{\frac{1}{3}}a^2h - 2\,\left(-ab^2\right)^{\frac{1}{3}}abe - \left(-ab^2\right)^{\frac{2}{3}}bc - 5\,\left(-ab^2\right)^{\frac{2}{3}}af\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^4} + \frac{2\left(b^3c - 4\,ab^2f\right)x^5 + \left(13\,a^2bh - 7\,ab^2e\right)x^4 - 3\,a^2bd + 9\,a^3g - 6\left(ab^2d - 2\,a^2bg\right)x^3 - \left(ab^2c + 5\,a^2bf\right)x^2 + 2\left(5\,a^3h - 2\,a^2b\right.}{18\left(bx^3+a\right)^2ab^3} - \frac{\left(ab^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^2b^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14\,a^3b^4h + 2\,a^2b^5e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^3 + a)^3,x, algorithm
```

```
[Out] h*x/b^3 + 1/3*g*ln(abs(b*x^3 + a))/b^3 - 1/27*sqrt(3)*(14*(-a*b^2
)^(1/3)*a^2*h - 2*(-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c + 5*(
-a*b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(a^2*b^4) - 1/54*(14*(-a*b^2)^(1/3)*a^2*h - 2*(-a*b^2)^(1
/3)*a*b*e - (-a*b^2)^(2/3)*b*c - 5*(-a*b^2)^(2/3)*a*f)*ln(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4) + 1/18*(2*(b^3*c - 4*a*b^
2*f)*x^5 + (13*a^2*b*h - 7*a*b^2*e)*x^4 - 3*a^2*b*d + 9*a^3*g - 6
*(a*b^2*d - 2*a^2*b*g)*x^3 - (a*b^2*c + 5*a^2*b*f)*x^2 + 2*(5*a^3
*h - 2*a^2*b*e)*x)/((b*x^3 + a)^2*a*b^3) - 1/27*(a*b^6*c*(-a/b)^(
1/3) + 5*a^2*b^5*f*(-a/b)^(1/3) - 14*a^3*b^4*h + 2*a^2*b^5*e)*(-a
/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b^7)
```

3.410 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=325

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)}{9\sqrt{3}a^{5/3}b^{8/3}}+\frac{h\log\left(a+bx^3\right)}{3b^3}+\frac{x\left(2x(bd-4ag)+3x^2(be-3ah)-7af+bc\right)}{18ab^2\left(a+bx^3\right)}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

```
[Out] -(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*b^2*(a + b*
x^3)^2) + (x*(b*c - 7*a*f + 2*(b*d - 4*a*g)*x + 3*(b*e - 3*a*h)*x
^2))/(18*a*b^2*(a + b*x^3)) - ((b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(
1/3)*f + 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((b^(1/3)*(b*c + 2*a*f) - a
^(1/3)*(b*d + 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/
3)) - ((b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3)) + (h*L
og[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 1.32227, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)}{9\sqrt{3}a^{5/3}b^{8/3}}+\frac{h\log\left(a+bx^3\right)}{3b^3}+\frac{x\left(2x(bd-4ag)+3x^2(be-3ah)-7af+bc\right)}{18ab^2\left(a+bx^3\right)}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```


```
[Out] -(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*b^2*(a + b*
x^3)^2) + (x*(b*c - 7*a*f + 2*(b*d - 4*a*g)*x + 3*(b*e - 3*a*h)*x
^2))/(18*a*b^2*(a + b*x^3)) - ((b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(
1/3)*f + 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((b^(1/3)*(b*c + 2*a*f) - a
^(1/3)*(b*d + 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/
3)) - ((b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3)) + (h*L
og[a + b*x^3])/(3*b^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.539033, size = 315, normalized size = 0.97

$$\frac{\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}g+\sqrt[3]{a}bd-2a\sqrt[3]{b}f-b^{4/3}c\right)}{a^{5/3}}+\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^{4/3}g-\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{5/3}}-\frac{2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{}}{54b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x^
3)^2 + (36*a^2*h + 3*b^2*x*(c + 2*d*x) - 3*a*b*(6*e + x*(7*f + 8*
g*x)))/(a*(a + b*x^3)) - (2*Sqrt[3]*b^(1/3)*(b^(4/3)*c + a^(1/3)*
b*d + 2*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(b^(4/3)*c - a^(1/3)*b*d + 2*
a*b^(1/3)*f - 5*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + (b
^(1/3)*(-(b^(4/3)*c) + a^(1/3)*b*d - 2*a*b^(1/3)*f + 5*a^(4/3)*g)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 18*h*L
og[a + b*x^3])/(54*b^3)
```

Maple [A] time = 0.017, size = 520, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (-1/9*(4*a*g-b*d)/a/b*x^5-1/18*(7*a*f-b*c)/a/b*x^4+1/3*(2*a*h-b*e
)/b^2*x^3-1/18*(5*a*g+b*d)/b^2*x^2-1/9*(2*a*f+b*c)/b^2*x+1/6*a*(3
*a*h-b*e)/b^3)/(b*x^3+a)^2+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
*f-1/27/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+2/27/
b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f
+1/27/b^2/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/54/b^2/a/(a/b)^(2/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+1/27/b^2/a/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-5/27*g/b^3/(a/b)^(1
/3)*ln(x+(a/b)^(1/3))+5/54*g/b^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)
+(a/b)^(2/3))+5/27*g*3^(1/2)/b^3/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))-1/27/b^2/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d+1/
54/b^2/a/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/27/b^2
/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+
1/3*h/b^3*ln(a*b^2*(b*x^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^3 + a)^3,x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^3 + a)^3,x, algorithm
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.226425, size = 512, normalized size = 1.58

$$\frac{h\ln\left(\left|bx^3+a\right|\right)}{3\,b^3} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c + 2\,\left(-ab^2\right)^{\frac{1}{3}}abf - \left(-ab^2\right)^{\frac{2}{3}}bd - 5\,\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4} + \frac{2\left(b^2d-4\,abg\right)x^5 + \left(b^2c-7\,abf\right)x^4 + 6\left(2\,a^2h-abe\right)x^3 - \left(abd+5\,a^2g\right)x^2 - 2\left(abc+2\,a^2f\right)x + \frac{3\left(3\,a^3h-a^2be\right)}{b}}{18\left(bx^3+a\right)^2ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c + 2\,\left(-ab^2\right)^{\frac{1}{3}}abf + \left(-ab^2\right)^{\frac{2}{3}}bd + 5\,\left(-ab^2\right)^{\frac{2}{3}}ag\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^4} - \frac{\left(ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^2b^3g\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ab^4c + 2\,a^2b^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^3 + a)^3,x, algorithm
```

```
[Out] 1/3*h*ln(abs(b*x^3 + a))/b^3 + 1/27*sqrt(3)*((-a*b^2)^(1/3)*b^2*c
 + 2*(-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d - 5*(-a*b^2)^(2/3)
*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*
b^4) + 1/18*(2*(b^2*d - 4*a*b*g)*x^5 + (b^2*c - 7*a*b*f)*x^4 + 6*
(2*a^2*h - a*b*e)*x^3 - (a*b*d + 5*a^2*g)*x^2 - 2*(a*b*c + 2*a^2*
f)*x + 3*(3*a^3*h - a^2*b*e)/b)/((b*x^3 + a)^2*a*b^2) + 1/54*((-a
*b^2)^(1/3)*b^2*c + 2*(-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d +
 5*(-a*b^2)^(2/3)*a*g)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a
^2*b^4) - 1/27*(a*b^4*d*(-a/b)^(1/3) + 5*a^2*b^3*g*(-a/b)^(1/3) +
 a*b^4*c + 2*a^2*b^3*f)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a
^3*b^5)
```

3.411 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=297

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{9\sqrt{3}a^{5/3}b^{8/3}}+\frac{x\left(x(2be-5ah)-4ag+bd+3bfx^2\right)}{18ab^2\left(a+bx^3\right)}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

```
[Out] (x*(b*d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(18*a*b^2*(a +
b*x^3)) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(6*b*(a + b*x
^3)^2) - ((b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^
(5/3)*b^(8/3)) + ((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))
*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/3)) - ((b^(1/3)*(b*d
+ 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3))
```

Rubi [A] time = 0.901279, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{9\sqrt{3}a^{5/3}b^{8/3}}+\frac{x\left(x(2be-5ah)-4ag+bd+3bfx^2\right)}{18ab^2\left(a+bx^3\right)}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (x*(b*d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(18*a*b^2*(a +
b*x^3)) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(6*b*(a + b*x
^3)^2) - ((b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^
(5/3)*b^(8/3)) + ((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))
*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/3)) - ((b^(1/3)*(b*d
+ 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3))
```

Rubi in Sympy [A] time = 114.252, size = 279, normalized size = 0.94

$$-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2}-\frac{x\left(4ag-bd-3bfx^2+x(5ah-2be)\right)}{18ab^2(a+bx^3)}-\frac{\left(\sqrt[3]{a}(5ah+be)-\sqrt[3]{b}(2ag+bd)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{5}{3}}b^{\frac{8}{3}}}+\frac{\left(\sqrt[3]{a}(5ah+be)-\sqrt[3]{b}(2ag+bd)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{5}{3}}b^{\frac{8}{3}}}-\frac{\sqrt{3}\left(5a^{\frac{4}{3}}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{\frac{4}{3}}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{5}{3}}b^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] -(c + d*x + e*x**2 + f*x**3 + g*x**4 + h*x**5)/(6*b*(a + b*x**3)*
*2) - x*(4*a*g - b*d - 3*b*f*x**2 + x*(5*a*h - 2*b*e))/(18*a*b**2
*(a + b*x**3)) - (a**(1/3)*(5*a*h + b*e) - b**(1/3)*(2*a*g + b*d)
)*log(a**(1/3) + b**(1/3)*x)/(27*a**(5/3)*b**(8/3)) + (a**(1/3)*(
5*a*h + b*e) - b**(1/3)*(2*a*g + b*d))*log(a**(2/3) - a**(1/3)*b*
*(1/3)*x + b**(2/3)*x**2)/(54*a**(5/3)*b**(8/3)) - sqrt(3)*(5*a**
(4/3)*h + a**(1/3)*b*e + 2*a*b**(1/3)*g + b**(4/3)*d)*atan(sqrt(3
)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a**(5/3)*b**(8/3))
```

Mathematica [A] time = 0.461016, size = 287, normalized size = 0.97

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}h+\sqrt[3]{a}be-2a\sqrt[3]{b}g-b^{4/3}d\right)}{a^{5/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^{4/3}h-\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{5/3}}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}h+\sqrt[3]{\ }\right.}{a^{5/3}}}{54b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f + x*(g + h*x))))/(a + b*
x^3)^2 + (3*b^(2/3)*(b*x*(d + 2*e*x) - a*(6*f + x*(7*g + 8*h*x)))
)/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(
1/3)*g + 5*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
)/a^(5/3) + (2*(b^(4/3)*d - a^(1/3)*b*e + 2*a*b^(1/3)*g - 5*a^(4/
3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + ((-(b^(4/3)*d) + a^(1/3
)*b*e - 2*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(8/3))
```

Maple [A] time = 0.016, size = 490, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (-1/9*(4*a*h-b*e)/a/b*x^5-1/18*(7*a*g-b*d)/a/b*x^4-1/3*f*x^3/b-1/
18*(5*a*h+b*e)/b^2*x^2-1/9*(2*a*g+b*d)/b^2*x-1/6*(a*f+b*c)/b^2)/(
```

```
b*x^3+a)^2+2/27*g/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/27*g/b^3/(a
/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/27*g/b^3/(a/b)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/27/b^2/a/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))*d-1/54/b^2/a/(a/b)^(2/3)*ln(x^2-x*(a/b)
^(1/3)+(a/b)^(2/3))*d+1/27/b^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*d-5/27*h/b^3/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))+5/54*h/b^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+5/
27*h*3^(1/2)/b^3/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e+1/54/a/b^2/(a/b)^(
1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/27/a/b^2*3^(1/2)/(a/b)
^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^3 + a)^3,x, algorithm

[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^3 + a)^3,x, algorithm

[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.227394, size = 454, normalized size = 1.53

$$-\frac{\left(5\,ah\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+bd+2\,ag\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{2}b^{2}}+\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{2}d+2\,\left(-ab^{2}\right)^{\frac{1}{3}}abg-5\,\left(-ab^{2}\right)^{\frac{2}{3}}ah-\left(-ab^{2}\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b^{4}}-\frac{8\,abhx^{5}-2\,b^{2}x^{5}e-b^{2}dx^{4}+7\,abgx^{4}+6\,abfx^{3}+5\,a^{2}hx^{2}+abx^{2}e+2\,abdx+4\,a^{2}gx+3\,abc+3\,a^{2}f}{18\,(bx^{3}+a)^{2}ab^{2}}+\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{2}d+2\,\left(-ab^{2}\right)^{\frac{1}{3}}abg+5\,\left(-ab^{2}\right)^{\frac{2}{3}}ah+\left(-ab^{2}\right)^{\frac{2}{3}}be\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{2}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^3 + a)^3,x, algorithm
```

```
[Out] -1/27*(5*a*h*(-a/b)^(1/3) + b*(-a/b)^(1/3)*e + b*d + 2*a*g)*(-a/b
)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^2*b^2) + 1/27*sqrt(3)*((-a*b
^2)^(1/3)*b^2*d + 2*(-a*b^2)^(1/3)*a*b*g - 5*(-a*b^2)^(2/3)*a*h -
 (-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a^2*b^4) - 1/18*(8*a*b*h*x^5 - 2*b^2*x^5*e - b^2*d*x^4
 + 7*a*b*g*x^4 + 6*a*b*f*x^3 + 5*a^2*h*x^2 + a*b*x^2*e + 2*a*b*d*
x + 4*a^2*g*x + 3*a*b*c + 3*a^2*f)/((b*x^3 + a)^2*a*b^2) + 1/54*(
(-a*b^2)^(1/3)*b^2*d + 2*(-a*b^2)^(1/3)*a*b*g + 5*(-a*b^2)^(2/3)*
a*h + (-a*b^2)^(2/3)*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^2*b^4)
```

3.412 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=323

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{54a^{7/3}b^{7/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{27a^{7/3}b^{7/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)}{9\sqrt{3}a^{7/3}b^{7/3}} + \frac{x\left(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah)\right)}{18a^2b^2\left(a+bx^3\right)} - \frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

```
[Out] -(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a*b
^2*(a + b*x^3)^2) + (x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3
*b*(b*d + a*g)*x^2))/(18*a^2*b^2*(a + b*x^3)) - ((2*b^(5/3)*c + a
^(2/3)*b*e + a*b^(2/3)*f + 2*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(7/3)) - ((b^(2/3)
*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(1/3) + b^(1/3)*x])
/(27*a^(7/3)*b^(7/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e +
2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/
3)*b^(7/3))
```

Rubi [A] time = 1.01775, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{54a^{7/3}b^{7/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{27a^{7/3}b^{7/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)}{9\sqrt{3}a^{7/3}b^{7/3}} + \frac{x\left(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah)\right)}{18a^2b^2\left(a+bx^3\right)} - \frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```


```
[Out] -(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a*b
^2*(a + b*x^3)^2) + (x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3
*b*(b*d + a*g)*x^2))/(18*a^2*b^2*(a + b*x^3)) - ((2*b^(5/3)*c + a
^(2/3)*b*e + a*b^(2/3)*f + 2*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(7/3)) - ((b^(2/3)
*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(1/3) + b^(1/3)*x])
/(27*a^(7/3)*b^(7/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e +
2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/
3)*b^(7/3))
```

Rubi in Sympy [A] time = 144.401, size = 299, normalized size = 0.93

$$\frac{x\left(a\left(ah-be\right)-bx^{2}\left(ag-bd\right)-bx\left(af-bc\right)\right)}{6ab^{2}\left(a+bx^{3}\right)^{2}}-\frac{x\left(a\left(7ah-be\right)-3bx^{2}\left(ag+bd\right)-2bx\left(af+2bc\right)\right)}{18a^{2}b^{2}\left(a+bx^{3}\right)}+\frac{\left(a^{\frac{2}{3}}\left(2ah+be\right)-b^{\frac{2}{3}}\left(af+2bc\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{7}{3}}b^{\frac{7}{3}}}-\frac{\left(a^{\frac{2}{3}}\left(2ah+be\right)-b^{\frac{2}{3}}\left(af+2bc\right)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}\right)}{54a^{\frac{7}{3}}b^{\frac{7}{3}}}-\frac{\sqrt{3}\left(2a^{\frac{5}{3}}h+a^{\frac{2}{3}}be+ab^{\frac{2}{3}}f+2b^{\frac{5}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{7}{3}}b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] x*(a*(a*h - b*e) - b*x**2*(a*g - b*d) - b*x*(a*f - b*c))/(6*a*b**
2*(a + b*x**3)**2) - x*(a*(7*a*h - b*e) - 3*b*x**2*(a*g + b*d) -
2*b*x*(a*f + 2*b*c))/(18*a**2*b**2*(a + b*x**3)) + (a**(2/3)*(2*a
*h + b*e) - b**(2/3)*(a*f + 2*b*c))*log(a**(1/3) + b**(1/3)*x)/(2
7*a**(7/3)*b**(7/3)) - (a**(2/3)*(2*a*h + b*e) - b**(2/3)*(a*f +
2*b*c))*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(54*a
**(7/3)*b**(7/3)) - sqrt(3)*(2*a**(5/3)*h + a**(2/3)*b*e + a*b**(
2/3)*f + 2*b**(5/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)
/a**(1/3))/(27*a**(7/3)*b**(7/3))
```

Mathematica [A] time = 0.576882, size = 297, normalized size = 0.92

$$\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^{2}\right)\left(-a^{2/3}be-2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}be+2a^{5/3}h-ab^{2/3}f-2b^{5/3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-3*a^(1/3)*b^(1/3)*(-4*b^2*c*x^2 - a*b*x*(e + 2*f*x) + a^2*(6*g
 + 7*h*x)))/(a + b*x^3) + (9*a^(4/3)*b^(1/3)*(b^2*c*x^2 + a^2*(g
+ h*x) - a*b*(d + x*(e + f*x))))/(a + b*x^3)^2 - 2*Sqrt[3]*(2*b^(
5/3)*c + a^(2/3)*b*e + a*b^(2/3)*f + 2*a^(5/3)*h)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*b^(5/3)*c + a^(2/3)*b*e - a*
b^(2/3)*f + 2*a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x] + (2*b^(5/3)*c
- a^(2/3)*b*e + a*b^(2/3)*f - 2*a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(7/3))
```

Maple [A] time = 0.015, size = 498, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(a*f+2*b*c)/a^2*x^5-1/18*(7*a*h-b*e)/a/b*x^4-1/3*g*x^3/b-1/1
8*(a*f-7*b*c)/a/b*x^2-1/9*(2*a*h+b*e)/b^2*x-1/6*(a*g+b*d)/b^2)/(b
*x^3+a)^2+2/27*h/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/27*h/b^3/(a/
b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/27*h/b^3/(a/b)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/27/a/b^2/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))*e-1/54/a/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^
(1/3)+(a/b)^(2/3))*e+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))*f+1/54/a/b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f
+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))*f-2/27/b/a^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/27/b/a^2/(a
/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+2/27/b/a^2*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x/(b*x^3 + a)^3,x, algorithm="
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x/(b*x^3 + a)^3,x, algorithm="
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22797, size = 481, normalized size = 1.49

$$-\frac{\left(2\,b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2h+abe\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^2}+\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}a^2h+\left(-ab^2\right)^{\frac{1}{3}}abe-2\left(-ab^2\right)^{\frac{2}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3}+\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}a^2h+\left(-ab^2\right)^{\frac{1}{3}}abe+2\left(-ab^2\right)^{\frac{2}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}af\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^3}+\frac{4\,b^3cx^5+2\,ab^2fx^5-7\,a^2bhx^4+ab^2x^4e-6\,a^2bgx^3+7\,ab^2cx^2-a^2bfx^2-4\,a^3hx-2\,a^2bxe-3\,a^2bd-3\,a^3g}{18\left(bx^3+a\right)^2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)*x/(b*x^3 + a)^3,x, algorithm="
```

```
[Out] -1/27*(2*b^2*c*(-a/b)^(1/3) + a*b*f*(-a/b)^(1/3) + 2*a^2*h + a*b*
e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b^2) + 1/27*sqrt(3
)*(2*(-a*b^2)^(1/3)*a^2*h + (-a*b^2)^(1/3)*a*b*e - 2*(-a*b^2)^(2/
3)*b*c - (-a*b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^3*b^3) + 1/54*(2*(-a*b^2)^(1/3)*a^2*h + (-a*
b^2)^(1/3)*a*b*e + 2*(-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*ln(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^3) + 1/18*(4*b^3*c*x^
5 + 2*a*b^2*f*x^5 - 7*a^2*b*h*x^4 + a*b^2*x^4*e - 6*a^2*b*g*x^3 +
 7*a*b^2*c*x^2 - a^2*b*f*x^2 - 4*a^3*h*x - 2*a^2*b*x*e - 3*a^2*b*
d - 3*a^3*g)/((b*x^3 + a)^2*a^2*b^2)
```

3.413 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3} dx$

Optimal. Leaf size=313

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{27a^{8/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)}{9\sqrt{3}a^{8/3}b^{5/3}}-\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{18a^2b^2(a+bx^3)}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab(a+bx^3)^2}$$

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x
^3)^2) - (3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x)
)/(18*a^2*b^2*(a + b*x^3)) - ((5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^
(1/3)*f + a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + ((b^(1/3)*(5*b*c + a*f) - a^
(1/3)*(2*b*d + a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3
)) - ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3))
```

Rubi [A] time = 0.916225, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{27a^{8/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)}{9\sqrt{3}a^{8/3}b^{5/3}}-\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{18a^2b^2(a+bx^3)}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```


```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x
^3)^2) - (3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x)
)/(18*a^2*b^2*(a + b*x^3)) - ((5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^
(1/3)*f + a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + ((b^(1/3)*(5*b*c + a*f) - a^
(1/3)*(2*b*d + a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3
)) - ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3))
```

Rubi in Sympy [A] time = 120.495, size = 287, normalized size = 0.92

$$-\frac{x\left(af-bc+x^{2}\left(ah-be\right)+x\left(ag-bd\right)\right)}{6ab\left(a+bx^{3}\right)^{2}}-\frac{3a\left(ah+be\right)-bx\left(af+5bc+x\left(2ag+4bd\right)\right)}{18a^{2}b^{2}\left(a+bx^{3}\right)}-\frac{\left(\sqrt[3]{a}\left(ag+2bd\right)-\sqrt[3]{b}\left(af+5bc\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{8}{3}}b^{\frac{5}{3}}}+\frac{\left(\sqrt[3]{a}\left(ag+2bd\right)-\sqrt[3]{b}\left(af+5bc\right)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}\right)}{54a^{\frac{8}{3}}b^{\frac{5}{3}}}-\frac{\sqrt{3}\left(a^{\frac{4}{3}}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{\frac{4}{3}}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{8}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] -x*(a*f - b*c + x**2*(a*h - b*e) + x*(a*g - b*d))/(6*a*b*(a + b*x
**3)**2) - (3*a*(a*h + b*e) - b*x*(a*f + 5*b*c + x*(2*a*g + 4*b*d
)))/(18*a**2*b**2*(a + b*x**3)) - (a**(1/3)*(a*g + 2*b*d) - b**(1
/3)*(a*f + 5*b*c))*log(a**(1/3) + b**(1/3)*x)/(27*a**(8/3)*b**(5/
3)) + (a**(1/3)*(a*g + 2*b*d) - b**(1/3)*(a*f + 5*b*c))*log(a**(2
/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(54*a**(8/3)*b**(5/3))
 - sqrt(3)*(a**(4/3)*g + 2*a**(1/3)*b*d + a*b**(1/3)*f + 5*b**(4/
3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a*
*(8/3)*b**(5/3))
```

Mathematica [A] time = 0.45834, size = 295, normalized size = 0.94

$$\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g+2\sqrt[3]{a}bd-a\sqrt[3]{b}f-5b^{4/3}c\right)+2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}(-g)-2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

```
[Out] ((3*a^(2/3)*(-6*a^2*h + b^2*x*(5*c + 4*d*x) + a*b*x*(f + 2*g*x)))
/(a + b*x^3) + (9*a^(5/3)*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(
f + g*x))))/(a + b*x^3)^2 - 2*Sqrt[3]*b^(1/3)*(5*b^(4/3)*c + 2*a^
(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*b^(4/3)*c - 2*a^(1/3)*b*d + a*b^(1
/3)*f - a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(-5*b^(4/3)
*c + 2*a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^2)
```

Maple [A] time = 0.014, size = 506, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(a*g+2*b*d)/a^2*x^5+1/18*(a*f+5*b*c)/a^2*x^4-1/3*h*x^3/b-1/1
8*(a*g-7*b*d)/a/b*x^2-1/9*(a*f-4*b*c)/a/b*x-1/6*(a*h+b*e)/b^2)/(b
```

```
*x^3+a)^2+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1/54/a/b^2/(
a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*f+1/27/a/b^2/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+5/27*c/a^2
/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-5/54*c/a^2/b/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))+5/27*c/a^2/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/27/a*g/b^2/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))+1/54/a*g/b^2/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^
(2/3))+1/27/a*g*3^(1/2)/b^2/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))-2/27*d/a^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27*d/
a^2/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/27*d/a^2*3^
(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^3,x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^3,x, algorithm="fr
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.225924, size = 467, normalized size = 1.49

$$-\frac{\left(2\,bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{3}b}+\frac{\sqrt{3}\left(5\left(-ab^{2}\right)^{\frac{1}{3}}b^{2}c+\left(-ab^{2}\right)^{\frac{1}{3}}abf-2\left(-ab^{2}\right)^{\frac{2}{3}}bd-\left(-ab^{2}\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{3}b^{3}}+\frac{\left(5\left(-ab^{2}\right)^{\frac{1}{3}}b^{2}c+\left(-ab^{2}\right)^{\frac{1}{3}}abf+2\left(-ab^{2}\right)^{\frac{2}{3}}bd+\left(-ab^{2}\right)^{\frac{2}{3}}ag\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{3}b^{3}}+\frac{4\,b^{3}dx^{5}+2\,ab^{2}gx^{5}+5\,b^{3}cx^{4}+ab^{2}fx^{4}-6\,a^{2}bhx^{3}+7\,ab^{2}dx^{2}-a^{2}bgx^{2}+8\,ab^{2}cx-2\,a^{2}bfx-3\,a^{3}h-3\,a^{2}be}{18\left(bx^{3}+a\right)^{2}a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^3,x, algorithm="gi
```

```
[Out] -1/27*(2*b*d*(-a/b)^(1/3) + a*g*(-a/b)^(1/3) + 5*b*c + a*f)*(-a/b
)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/27*sqrt(3)*(5*(-a*b
^2)^(1/3)*b^2*c + (-a*b^2)^(1/3)*a*b*f - 2*(-a*b^2)^(2/3)*b*d - (
-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(a^3*b^3) + 1/54*(5*(-a*b^2)^(1/3)*b^2*c + (-a*b^2)^(1/3)
*a*b*f + 2*(-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*ln(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^3) + 1/18*(4*b^3*d*x^5 + 2*a*b^
2*g*x^5 + 5*b^3*c*x^4 + a*b^2*f*x^4 - 6*a^2*b*h*x^3 + 7*a*b^2*d*x
^2 - a^2*b*g*x^2 + 8*a*b^2*c*x - 2*a^2*b*f*x - 3*a^3*h - 3*a^2*b*
e)/((b*x^3 + a)^2*a^2*b^2)
```

3.414 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^3}\,dx$

Optimal. Leaf size=347

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2be)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2be)\right)}{27a^{8/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}h+2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{4/3}d\right)}{9\sqrt{3}a^{8/3}b^{5/3}}+\frac{x\left(-3bx^2(3bc-af)+a(ag+5bd)+2ax(ah+2be)\right)}{18a^3b\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{c\log(x)}{a^3}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*
b*(a + b*x^3)^2) + (x*(a*(5*b*d + a*g) + 2*a*(2*b*e + a*h)*x - 3*
b*(3*b*c - a*f)*x^2))/(18*a^3*b*(a + b*x^3)) - ((5*b^(4/3)*d + 2*
a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3
)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + (c*Log[x])
/a^3 + ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e + a*h))*Log[a^(1/
3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*d + a*g) -
 a^(1/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(8/3)*b^(5/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 1.40924, antiderivative size = 345, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(ah+2be)}{\sqrt[3]{b}}+ag+5bd\right)}{54a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2be)\right)}{27a^{8/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}h+2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{4/3}d\right)}{9\sqrt{3}a^{8/3}b^{5/3}}+\frac{x\left(-3bx^2(3bc-af)+a(ag+5bd)+2ax(ah+2be)\right)}{18a^3b\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{c\log(x)}{a^3}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*
b*(a + b*x^3)^2) + (x*(a*(5*b*d + a*g) + 2*a*(2*b*e + a*h)*x - 3*
b*(3*b*c - a*f)*x^2))/(18*a^3*b*(a + b*x^3)) - ((5*b^(4/3)*d + 2*
a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3
)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + (c*Log[x])
/a^3 + ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e + a*h))*Log[a^(1/
3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3)) - ((5*b*d + a*g - (a^(1/3)*
(2*b*e + a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(8/3)*b^(4/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rubi in Sympy [A] time = 136.863, size = 284, normalized size = 0.82

$$-\frac{x\left(\frac{af}{x}+ag+ahx-\frac{bc}{x}-bd-bex\right)}{6ab\left(a+bx^3\right)^2}+\frac{x\left(\frac{6af}{x}+ag+2ahx+5bd+4bex\right)}{18a^2b\left(a+bx^3\right)} -\frac{\left(\sqrt[3]{a}\left(ah+2be\right)-\sqrt[3]{b}\left(ag+5bd\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{8}{3}}b^{\frac{5}{3}}} +\frac{\left(\sqrt[3]{a}\left(ah+2be\right)-\sqrt[3]{b}\left(ag+5bd\right)\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{8}{3}}b^{\frac{5}{3}}} -\frac{\sqrt{3}\left(a^{\frac{4}{3}}h+2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{\frac{4}{3}}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{8}{3}}b^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] -x*(a*f/x + a*g + a*h*x - b*c/x - b*d - b*e*x)/(6*a*b*(a + b*x**3
)**2) + x*(6*a*f/x + a*g + 2*a*h*x + 5*b*d + 4*b*e*x)/(18*a**2*b*
(a + b*x**3)) - (a**(1/3)*(a*h + 2*b*e) - b**(1/3)*(a*g + 5*b*d))
*log(a**(1/3) + b**(1/3)*x)/(27*a**(8/3)*b**(5/3)) + (a**(1/3)*(a
*h + 2*b*e) - b**(1/3)*(a*g + 5*b*d))*log(a**(2/3) - a**(1/3)*b**
(1/3)*x + b**(2/3)*x**2)/(54*a**(8/3)*b**(5/3)) - sqrt(3)*(a**(4/
3)*h + 2*a**(1/3)*b*e + a*b**(1/3)*g + 5*b**(4/3)*d)*atan(sqrt(3)
*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a**(8/3)*b**(5/3))
```

Mathematica [A] time = 0.50999, size = 311, normalized size = 0.9

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h+2\sqrt[3]{a}be-a\sqrt[3]{b}g-5b^{4/3}d\right)}{b^{5/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}(-h)-2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{5/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{}}{54a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((3*a*(6*b*c + b*x*(5*d + 4*e*x) + a*x*(g + 2*h*x)))/(b*(a + b*x^
3)) - (9*a^2*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(
a + b*x^3)^2) - (2*Sqrt[3]*a^(1/3)*(5*b^(4/3)*d + 2*a^(1/3)*b*e +
 a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/b^(5/3) + 54*c*Log[x] + (2*a^(1/3)*(5*b^(4/3)*d - 2*a^(1/3)
*b*e + a*b^(1/3)*g - a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3)
 + (a^(1/3)*(-5*b^(4/3)*d + 2*a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)
*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 18*
c*Log[a + b*x^3])/(54*a^3)
```

Maple [B] time = 0.029, size = 620, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x)
```

```
[Out] 1/2/a/(b*x^3+a)^2*c+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*d+2/27/a^2/b*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/27/a^2/b/(a/b)^(1/3)*ln(x^2
-x*(a/b)^(1/3)+(a/b)^(2/3))*e+4/9/a/(b*x^3+a)^2*x*d+7/18/a/(b*x^3
+a)^2*x^2*e-2/27/a^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e+5/27/a^2/b
/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-5/54/a^2/b/(a/b)^(2/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))*d+c*ln(x)/a^3+5/18/a^2/(b*x^3+a)^2*x^4*b
*d-1/3/a^3*c*ln(b*(b*x^3+a))+1/27/a*g/b^2/(a/b)^(2/3)*ln(x+(a/b)^
(1/3))-1/54/a*g/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
-1/27/a*h/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/54/a*h/b^2/(a/b)^(1
/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+2/9/a^2/(b*x^3+a)^2*x^5*b*e
+1/9/a/(b*x^3+a)^2*x^5*h+1/18/a/(b*x^3+a)^2*x^4*g-1/18/(b*x^3+a)^
2/b*x^2*h-1/9/(b*x^3+a)^2/b*x*g-1/6/b/(b*x^3+a)^2*f+1/3/a^2/(b*x^
3+a)^2*x^3*c*b+1/27/a*g/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))+1/27/a*h*3^(1/2)/b^2/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x),x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x),x, algorithm
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.232004, size = 529, normalized size = 1.52

$$-\frac{c\ln\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{c\ln\left(|x|\right)}{a^3}+\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^2d+\left(-ab^2\right)^{\frac{1}{3}}abg-\left(-ab^2\right)^{\frac{2}{3}}ah-2\,\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3}+\frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^2d+\left(-ab^2\right)^{\frac{1}{3}}abg+\left(-ab^2\right)^{\frac{2}{3}}ah+2\,\left(-ab^2\right)^{\frac{2}{3}}be\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^3}+\frac{6\,ab^2cx^3+2\left(a^2bh+2\,ab^2e\right)x^5+\left(5\,ab^2d+a^2bg\right)x^4+9\,a^2bc-3\,a^3f-\left(a^3h-7\,a^2be\right)x^2+2\left(4\,a^2bd-a^3g\right)x}{18\left(bx^3+a\right)^2a^3b}-\frac{\left(a^5b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^4b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5\,a^4b^3d+a^5b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x),x, algorithm
```

```
[Out] -1/3*c*ln(abs(b*x^3 + a))/a^3 + c*ln(abs(x))/a^3 + 1/27*sqrt(3)*(
5*(-a*b^2)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*
h - 2*(-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^3*b^3) + 1/54*(5*(-a*b^2)^(1/3)*b^2*d + (-a*b^2
)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h + 2*(-a*b^2)^(2/3)*b*e)*ln(x^2
 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^3) + 1/18*(6*a*b^2*c*x^3
 + 2*(a^2*b*h + 2*a*b^2*e)*x^5 + (5*a*b^2*d + a^2*b*g)*x^4 + 9*a^
2*b*c - 3*a^3*f - (a^3*h - 7*a^2*b*e)*x^2 + 2*(4*a^2*b*d - a^3*g)
*x)/((b*x^3 + a)^2*a^3*b) - 1/27*(a^5*b^2*h*(-a/b)^(1/3) + 2*a^4*
b^3*(-a/b)^(1/3)*e + 5*a^4*b^3*d + a^5*b^2*g)*(-a/b)^(1/3)*ln(abs
(x - (-a/b)^(1/3)))/(a^7*b^3)
```

3.415 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=362

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{27a^{10/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{2/3}be+a^{5/3}(-h)-2ab^{2/3}f+14b^{5/3}c\right)}{9\sqrt{3}a^{10/3}b^{4/3}}+\frac{x\left(-2bx(5bc-2af)-3bx^2(3bd-ag)+a(ah+5be)\right)}{18a^3b\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^3}-\frac{c}{a^3x}+\frac{d\log(x)}{a^3}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

```
[Out] -(c/(a^3*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g
)*x^2))/(6*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*e + a*h) - 2*b*(5*b*
c - 2*a*f)*x - 3*b*(3*b*d - a*g)*x^2))/(18*a^3*b*(a + b*x^3)) + (
(14*b^(5/3)*c - 5*a^(2/3)*b*e - 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*
b^(4/3)) + (d*Log[x])/a^3 + ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(
5*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(4/3)) - (
(2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(4/3)) - (d*Log[a
 + b*x^3])/(3*a^3)
```

Rubi [A] time = 1.63075, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{27a^{10/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{2/3}be+a^{5/3}(-h)-2ab^{2/3}f+14b^{5/3}c\right)}{9\sqrt{3}a^{10/3}b^{4/3}}+\frac{x\left(-2bx(5bc-2af)-3bx^2(3bd-ag)+a(ah+5be)\right)}{18a^3b\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^3}-\frac{c}{a^3x}+\frac{d\log(x)}{a^3}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```


```
[Out] -(c/(a^3*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g
)*x^2))/(6*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*e + a*h) - 2*b*(5*b*
c - 2*a*f)*x - 3*b*(3*b*d - a*g)*x^2))/(18*a^3*b*(a + b*x^3)) + (
(14*b^(5/3)*c - 5*a^(2/3)*b*e - 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*
b^(4/3)) + (d*Log[x])/a^3 + ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(
5*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(4/3)) - (
(2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(4/3)) - (d*Log[a
 + b*x^3])/(3*a^3)
```

Rubi in Sympy [A] time = 96.0268, size = 219, normalized size = 0.6

$$-\frac{x\left(\frac{af}{x^2}+\frac{ag}{x}+ah-\frac{bc}{x^2}-\frac{bd}{x}-be\right)}{6ab\left(a+bx^3\right)^2}+\frac{x\left(\frac{6af}{x^2}+\frac{6ag}{x}+ah+5be\right)}{18a^2b\left(a+bx^3\right)}+\frac{(ah+5be)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{\frac{8}{3}}b^{\frac{4}{3}}}-\frac{(ah+5be)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{8}{3}}b^{\frac{4}{3}}}-\frac{\sqrt{3}(ah+5be)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{8}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] -x*(a*f/x**2 + a*g/x + a*h - b*c/x**2 - b*d/x - b*e)/(6*a*b*(a +
b*x**3)**2) + x*(6*a*f/x**2 + 6*a*g/x + a*h + 5*b*e)/(18*a**2*b*(
a + b*x**3)) + (a*h + 5*b*e)*log(a**(1/3) + b**(1/3)*x)/(27*a**(8
/3)*b**(4/3)) - (a*h + 5*b*e)*log(a**(2/3) - a**(1/3)*b**(1/3)*x
+ b**(2/3)*x**2)/(54*a**(8/3)*b**(4/3)) - sqrt(3)*(a*h + 5*b*e)*a
tan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(27*a**(8/3)*
b**(4/3))
```

Mathematica [A] time = 1.15837, size = 336, normalized size = 0.93

$$-\frac{\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{2/3}be+a^{5/3}h-2ab^{2/3}f+14b^{5/3}c\right)}{b^{4/3}}-\frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^{2/3}be+a^{5/3}h-2ab^{2/3}f+14b^{5/3}c\right)}{b^{4/3}}+\frac{2\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-2\ldots}{\ldots}\right)\ldots}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -((54*a*c)/x + (9*a^2*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e
+ f*x))))/(b*(a + b*x^3)^2) - (3*a*(a^2*h*x - 10*b^2*c*x^2 + a*b*
(6*d + x*(5*e + 4*f*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-
14*b^(5/3)*c + 5*a^(2/3)*b*e + 2*a*b^(2/3)*f + a^(5/3)*h)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3) - 54*a*d*Log[x] - (
2*a^(2/3)*(14*b^(5/3)*c + 5*a^(2/3)*b*e - 2*a*b^(2/3)*f + a^(5/3)
*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(14*b^(5/3)*c +
5*a^(2/3)*b*e - 2*a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + 18*a*d*Log[a + b*x^3])/(54*a^
4)
```

Maple [B] time = 0.025, size = 624, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x)
```

```
[Out] 5/27/a^2/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e-5/54/a^2/b/(a/b)^(2/3)
*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*e+1/27/a*h/b^2/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))+7/18/a/(b*x^3+a)^2*x^2*f-13/18/a^2/(b*x^3+a)^2*x^2
*b*c+1/3/a^2/(b*x^3+a)^2*x^3*b*d+1/27/a^2*f/b/(a/b)^(1/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))+14/27/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3)
)*c-7/27/a^3/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-c/a^
3/x+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
```

```
/3)*x-1))*e+2/27/a^2*f*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))+4/9/a/(b*x^3+a)^2*x*e+1/2/a/(b*x^3+a)^2*d-5/9
/a^3/(b*x^3+a)^2*b^2*x^5*c+1/18/a/(b*x^3+a)^2*x^4*h-1/9/(b*x^3+a)
^2/b*x*h-2/27/a^2*f/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+2/9/a^2/(b*x^
3+a)^2*x^5*f*b-1/54/a*h/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b
)^(2/3))-1/6/(b*x^3+a)^2/b*g-1/3/a^3*d*ln(b*(b*x^3+a))-14/27/a^3*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/2
7/a*h/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))+5/18/a^2/(b*x^3+a)^2*x^4*b*e+d*ln(x)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x^2),x, algorit
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x^2),x, algorit
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.230017, size = 556, normalized size = 1.54

$$
\begin{aligned}
&-\frac{d\ln\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{d\ln\left(\left|x\right|\right)}{a^3}\\
&+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h+5\left(-ab^2\right)^{\frac{1}{3}}abe+14\left(-ab^2\right)^{\frac{2}{3}}bc-2\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2}\\
&+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h+5\left(-ab^2\right)^{\frac{1}{3}}abe-14\left(-ab^2\right)^{\frac{2}{3}}bc+2\left(-ab^2\right)^{\frac{2}{3}}af\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2}\\
&+\frac{6\,ab^2dx^4-4\left(7\,b^3c-ab^2f\right)x^6+\left(a^2bh+5\,ab^2e\right)x^5-18\,a^2bc-7\left(7\,ab^2c-a^2bf\right)x^3-2\left(a^3h-4\,a^2be\right)x^2+3\left(3\,a^2bd-a\right.}{18\left(bx^3+a\right)^2a^3bx}\\
&+\frac{\left(14\,a^3b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^4b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^5b^2h-5\,a^4b^3e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b^3}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x^2),x, algorit
```

```
[Out] -1/3*d*ln(abs(b*x^3 + a))/a^3 + d*ln(abs(x))/a^3 + 1/27*sqrt(3)*(
(-a*b^2)^(1/3)*a^2*h + 5*(-a*b^2)^(1/3)*a*b*e + 14*(-a*b^2)^(2/3)
*b*c - 2*(-a*b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^4*b^2) + 1/54*((-a*b^2)^(1/3)*a^2*h + 5*(-a*
b^2)^(1/3)*a*b*e - 14*(-a*b^2)^(2/3)*b*c + 2*(-a*b^2)^(2/3)*a*f)*
ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^2) + 1/18*(6*a*b^2
*d*x^4 - 4*(7*b^3*c - a*b^2*f)*x^6 + (a^2*b*h + 5*a*b^2*e)*x^5 -
18*a^2*b*c - 7*(7*a*b^2*c - a^2*b*f)*x^3 - 2*(a^3*h - 4*a^2*b*e)*
x^2 + 3*(3*a^2*b*d - a^3*g)*x)/((b*x^3 + a)^2*a^3*b*x) + 1/27*(14
*a^3*b^4*c*(-a/b)^(1/3) - 2*a^4*b^3*f*(-a/b)^(1/3) - a^5*b^2*h -
5*a^4*b^3*e)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^7*b^3)
```

3.416 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx$

Optimal. Leaf size=360

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{54a^{11/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{27a^{11/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}g+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{9\sqrt{3}a^{11/3}b^{2/3}}-\frac{x\left(2x(5bd-2ag)+3x^2(3be-ah)-5af+11bc\right)}{18a^3\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^3}-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\log(x)}{a^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}$$

```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e
 - a*h)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c - 5*a*f + 2*(5*b
*d - 2*a*g)*x + 3*(3*b*e - a*h)*x^2))/(18*a^3*(a + b*x^3)) + ((20
*b^(4/3)*c + 14*a^(1/3)*b*d - 5*a*b^(1/3)*f - 2*a^(4/3)*g)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*
b^(2/3)) + (e*Log[x])/a^3 - ((5*b^(1/3)*(4*b*c - a*f) - 2*a^(1/3)
*(7*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) +
 ((5*b^(1/3)*(4*b*c - a*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(2/3)) - (e*L
og[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 1.56156, antiderivative size = 357, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}-5af+20bc\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{27a^{11/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}g+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{9\sqrt{3}a^{11/3}b^{2/3}}-\frac{x\left(2x(5bd-2ag)+3x^2(3be-ah)-5af+11bc\right)}{18a^3\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^3}-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\log(x)}{a^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e
 - a*h)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c - 5*a*f + 2*(5*b
*d - 2*a*g)*x + 3*(3*b*e - a*h)*x^2))/(18*a^3*(a + b*x^3)) + ((20
*b^(4/3)*c + 14*a^(1/3)*b*d - 5*a*b^(1/3)*f - 2*a^(4/3)*g)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*
b^(2/3)) + (e*Log[x])/a^3 - ((5*b^(1/3)*(4*b*c - a*f) - 2*a^(1/3)
*(7*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) +
 ((20*b*c - 5*a*f - (2*a^(1/3)*(7*b*d - a*g))/b^(1/3))*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3)) - (e*
Log[a + b*x^3])/(3*a^3)
```

Rubi in Sympy [A] time = 51.3888, size = 83, normalized size = 0.23

$$\frac{x\left(\frac{6f}{x^3}+\frac{6g}{x^2}+\frac{6h}{x}\right)}{18ab\left(a+bx^3\right)}-\frac{x\left(\frac{af}{x^3}+\frac{ag}{x^2}+\frac{ah}{x}-\frac{bc}{x^3}-\frac{bd}{x^2}-\frac{be}{x}\right)}{6ab\left(a+bx^3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] x*(6*f/x**3 + 6*g/x**2 + 6*h/x)/(18*a*b*(a + b*x**3)) - x*(a*f/x*
*3 + a*g/x**2 + a*h/x - b*c/x**3 - b*d/x**2 - b*e/x)/(6*a*b*(a +
b*x**3)**2)
```

Mathematica [A] time = 1.29957, size = 337, normalized size = 0.94

$$-\frac{-\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}g-14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{4/3}g-14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -((27*a*c)/x^2 + (54*a*d)/x - (3*a*(6*a*e - b*x*(11*c + 10*d*x) +
 a*x*(5*f + 4*g*x)))/(a + b*x^3) + (9*a^2*(a^2*h + b^2*x*(c + d*x
) - a*b*(e + x*(f + g*x))))/(b*(a + b*x^3)^2) + (2*Sqrt[3]*a^(1/3
)*(-20*b^(4/3)*c - 14*a^(1/3)*b*d + 5*a*b^(1/3)*f + 2*a^(4/3)*g)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) - 54*a*e*Log
[x] + (2*a^(1/3)*(20*b^(4/3)*c - 14*a^(1/3)*b*d - 5*a*b^(1/3)*f +
 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)*(20*b^
(4/3)*c - 14*a^(1/3)*b*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*a*e*Log[a + b
*x^3])/(54*a^4)
```

Maple [B] time = 0.027, size = 626, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x)
```

```
[Out] 1/27/a^2*g/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+5/27/a
^2*f/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)+4/9/a/(b*x^3+a)^2*f*x-13/18/a^2/(b*x^3+a)^2*x^2*b*d+5/27/a^2*f/
b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-5/54/a^2*f/b/(a/b)^(2/3)*ln(x^2-x
*(a/b)^(1/3)+(a/b)^(2/3))-1/2*c/a^3/x^2-20/27/a^3/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))*c+10/27/a^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)
^(2/3))*c+14/27/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-7/27/a^3/(a/b
)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-1/6/(b*x^3+a)^2/b*h-d
/a^3/x+2/9/a^2/(b*x^3+a)^2*x^5*b*g-7/9/a^2/(b*x^3+a)^2*x*b*c-20/2
7/a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
*c-2/27/a^2*g/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+2/27/a^2*g*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-11/18/a^3/(
b*x^3+a)^2*x^4*b^2*c-14/27/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*d+7/18/a/(b*x^3+a)^2*x^2*g-5/9/a^3/(b*x
^3+a)^2*x^5*b^2*d+5/18/a^2/(b*x^3+a)^2*x^4*b*f+1/2/a/(b*x^3+a)^2*
e+1/3*b/a^2/(b*x^3+a)^2*e*x^3+e*ln(x)/a^3-1/3*e*ln(b*x^3+a)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x^3),x, algorit
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x^3),x, algorit
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.228878, size = 568, normalized size = 1.58

$$-\frac{e\ln\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{e\ln\left(\left|x\right|\right)}{a^3}$$
$$-\frac{\sqrt{3}\left(20\,\left(-ab^2\right)^{\frac{1}{3}}b^2c-5\,\left(-ab^2\right)^{\frac{1}{3}}abf-14\,\left(-ab^2\right)^{\frac{2}{3}}bd+2\,\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2}$$
$$-\frac{\left(20\,\left(-ab^2\right)^{\frac{1}{3}}b^2c-5\,\left(-ab^2\right)^{\frac{1}{3}}abf+14\,\left(-ab^2\right)^{\frac{2}{3}}bd-2\,\left(-ab^2\right)^{\frac{2}{3}}ag\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2}$$
$$-\frac{28\,b^3dx^7-4\,ab^2gx^7+20\,b^3cx^6-5\,ab^2fx^6-6\,ab^2x^5e+49\,ab^2dx^4-7\,a^2bgx^4+32\,ab^2cx^3-8\,a^2bfx^3+3\,a^3hx^2-9\,a^2bx}{18\left(bx^4+ax\right)^2a^3b}$$
$$+\frac{\left(14\,a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^4bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,a^3b^2c-5\,a^4bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x^3),x, algorit
```

```
[Out] -1/3*e*ln(abs(b*x^3 + a))/a^3 + e*ln(abs(x))/a^3 - 1/27*sqrt(3)*(
20*(-a*b^2)^(1/3)*b^2*c - 5*(-a*b^2)^(1/3)*a*b*f - 14*(-a*b^2)^(2
/3)*b*d + 2*(-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
```

```
(1/3))/(-a/b)^(1/3))/(a^4*b^2) - 1/54*(20*(-a*b^2)^(1/3)*b^2*c -
5*(-a*b^2)^(1/3)*a*b*f + 14*(-a*b^2)^(2/3)*b*d - 2*(-a*b^2)^(2/3)
*a*g)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^2) - 1/18*(2
8*b^3*d*x^7 - 4*a*b^2*g*x^7 + 20*b^3*c*x^6 - 5*a*b^2*f*x^6 - 6*a*
b^2*x^5*e + 49*a*b^2*d*x^4 - 7*a^2*b*g*x^4 + 32*a*b^2*c*x^3 - 8*a
^2*b*f*x^3 + 3*a^3*h*x^2 - 9*a^2*b*x^2*e + 18*a^2*b*d*x + 9*a^2*b
*c)/((b*x^4 + a*x)^2*a^3*b) + 1/27*(14*a^3*b^2*d*(-a/b)^(1/3) - 2
*a^4*b*g*(-a/b)^(1/3) + 20*a^3*b^2*c - 5*a^4*b*f)*(-a/b)^(1/3)*ln
(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

3.417 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=395

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{54a^{11/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{27a^{11/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}h+14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{9\sqrt{3}a^{11/3}b^{2/3}}+\frac{(3bc-af)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-af)}{a^4}-\frac{x\left(-3bx^2\left(\frac{5bc}{a}-3f\right)+2x(5be-2ah)-5ag+11bd\right)}{18a^3\left(a+bx^3\right)}-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2\left(a+bx^3\right)^2}$$

```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d - a*g + (b*e
 - a*h)*x - b*((b*c)/a - f)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*
b*d - 5*a*g + 2*(5*b*e - 2*a*h)*x - 3*b*((5*b*c)/a - 3*f)*x^2))/(
18*a^3*(a + b*x^3)) + ((20*b^(4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/
3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) - ((3*b*c - a*f)*Log[x])/a^4 -
 ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[a^(1/3)
 + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) + ((5*b^(1/3)*(4*b*d - a*g)
- 2*a^(1/3)*(7*b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(54*a^(11/3)*b^(2/3)) + ((3*b*c - a*f)*Log[a + b*x^3])/
(3*a^4)
```

Rubi [A] time = 1.97627, antiderivative size = 392, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7be-ah)}{\sqrt[3]{b}}-5ag+20bd\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{27a^{11/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}h+14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{9\sqrt{3}a^{11/3}b^{2/3}}+\frac{(3bc-af)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-af)}{a^4}-\frac{x\left(-3bx^2\left(\frac{5bc}{a}-3f\right)+2x(5be-2ah)-5ag+11bd\right)}{18a^3\left(a+bx^3\right)}-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d - a*g + (b*e
 - a*h)*x - b*((b*c)/a - f)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*
b*d - 5*a*g + 2*(5*b*e - 2*a*h)*x - 3*b*((5*b*c)/a - 3*f)*x^2))/(
18*a^3*(a + b*x^3)) + ((20*b^(4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/
3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) - ((3*b*c - a*f)*Log[x])/a^4 -
 ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[a^(1/3)
 + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) + ((20*b*d - 5*a*g - (2*a^(1
/3)*(7*b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
```

```
2/3)*x^2])/(54*a^(11/3)*b^(1/3)) + ((3*b*c - a*f)*Log[a + b*x^3])
/(3*a^4)
```

Rubi in Sympy [A] time = 49.5221, size = 88, normalized size = 0.22

$$\frac{x\left(\frac{6f}{x^4}+\frac{6g}{x^3}+\frac{6h}{x^2}\right)}{18ab\,(a+bx^3)}-\frac{x\left(\frac{af}{x^4}+\frac{ag}{x^3}+\frac{ah}{x^2}-\frac{bc}{x^4}-\frac{bd}{x^3}-\frac{be}{x^2}\right)}{6ab\,(a+bx^3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] x*(6*f/x**4 + 6*g/x**3 + 6*h/x**2)/(18*a*b*(a + b*x**3)) - x*(a*f
/x**4 + a*g/x**3 + a*h/x**2 - b*c/x**4 - b*d/x**3 - b*e/x**2)/(6*
a*b*(a + b*x**3)**2)
```

Mathematica [A] time = 1.87989, size = 352, normalized size = 0.89

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}h-14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{b^{2/3}}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{4/3}h-14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{b^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-18*a*c)/x^3 - (27*a*d)/x^2 - (54*a*e)/x + (3*a*(-12*b*c + 6*a*
f - b*x*(11*d + 10*e*x) + a*x*(5*g + 4*h*x)))/(a + b*x^3) + (a^2*
(-9*b*(c + x*(d + e*x)) + 9*a*(f + x*(g + h*x))))/(a + b*x^3)^2 +
 (2*Sqrt[3]*a^(1/3)*(20*b^(4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*
g - 2*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(
2/3) + 54*(-3*b*c + a*f)*Log[x] - (2*a^(1/3)*(20*b^(4/3)*d - 14*a
^(1/3)*b*e - 5*a*b^(1/3)*g + 2*a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x
])/b^(2/3) + (a^(1/3)*(20*b^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3
)*g + 2*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/b^(2/3) + 18*(3*b*c - a*f)*Log[a + b*x^3])/(54*a^4)
```

Maple [B] time = 0.028, size = 680, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x)
```

```
[Out] -5/54/a^2*g/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-2/27/
a^2*h/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27/a^2*h/b/(a/b)^(1/3)*ln
(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-2/3/a^3*b^2/(b*x^3+a)^2*x^3*c+5/1
8/a^2/(b*x^3+a)^2*x^4*b*g-7/9/a^2/(b*x^3+a)^2*x*b*d-20/27/a^3/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-3*b*c*
ln(x)/a^4+b*c*ln(b*x^3+a)/a^4-1/3*c/a^3/x^3+14/27/a^3*e/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))-7/27/a^3*e/(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+
(a/b)^(2/3))-13/18/a^2/(b*x^3+a)^2*x^2*b*e-14/27/a^3*e*3^(1/2)/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-5/6/a^2*b/(b*x^
3+a)^2*c+5/27/a^2*g/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))-20/27/a^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+10/27/
```

```
a^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/3/a^2/(b*x^
3+a)^2*x^3*b*f+2/9/a^2/(b*x^3+a)^2*x^5*b*h-1/2*d/a^3/x^2-e/a^3/x+
2/27/a^2*h*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))+5/27/a^2*g/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-5/9/a^3/(b*x^3
+a)^2*x^5*e*b^2-11/18/a^3/(b*x^3+a)^2*x^4*b^2*d+4/9/a/(b*x^3+a)^2
*x*g+7/18/a/(b*x^3+a)^2*x^2*h+1/a^3*ln(x)*f-1/3/a^3*ln(b*x^3+a)*f
+1/2/a/(b*x^3+a)^2*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x^4),x, algorit
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x^4),x, algorit
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.229167, size = 612, normalized size = 1.55

$$\frac{(3\,bc - af)\ln\left(\left|bx^3 + a\right|\right)}{3\,a^4} - \frac{(3\,bc - af)\ln\left(|x|\right)}{a^4} - \frac{\sqrt{3}\left(20\,\left(-ab^2\right)^{\frac{1}{3}}b^2d - 5\,\left(-ab^2\right)^{\frac{1}{3}}abg + 2\,\left(-ab^2\right)^{\frac{2}{3}}ah - 14\,\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2} - \frac{\left(20\,\left(-ab^2\right)^{\frac{1}{3}}b^2d - 5\,\left(-ab^2\right)^{\frac{1}{3}}abg - 2\,\left(-ab^2\right)^{\frac{2}{3}}ah + 14\,\left(-ab^2\right)^{\frac{2}{3}}be\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2} - \frac{\left(2\,a^6bh\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14\,a^5b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e - 20\,a^5b^2d + 5\,a^6bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^9b} + \frac{4\left(a^2bh - 7\,ab^2e\right)x^8 - 5\left(4\,ab^2d - a^2bg\right)x^7 - 6\left(3\,ab^2c - a^2bf\right)x^6 + 7\left(a^3h - 7\,a^2be\right)x^5 - 18\,a^3x^2e - 9\,a^3dx - 8\left(4\,a^2bd - \right.}{18\left(bx^3 + a\right)^2a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/((b*x^3 + a)^3*x^4),x, algorit
```

```
[Out] 1/3*(3*b*c - a*f)*ln(abs(b*x^3 + a))/a^4 - (3*b*c - a*f)*ln(abs(x
))/a^4 - 1/27*sqrt(3)*(20*(-a*b^2)^(1/3)*b^2*d - 5*(-a*b^2)^(1/3)
*a*b*g + 2*(-a*b^2)^(2/3)*a*h - 14*(-a*b^2)^(2/3)*b*e)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2) - 1/54*(20*
(-a*b^2)^(1/3)*b^2*d - 5*(-a*b^2)^(1/3)*a*b*g - 2*(-a*b^2)^(2/3)*
a*h + 14*(-a*b^2)^(2/3)*b*e)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/(a^4*b^2) - 1/27*(2*a^6*b*h*(-a/b)^(1/3) - 14*a^5*b^2*(-a/b)^
(1/3)*e - 20*a^5*b^2*d + 5*a^6*b*g)*(-a/b)^(1/3)*ln(abs(x - (-a/b
)^(1/3)))/(a^9*b) + 1/18*(4*(a^2*b*h - 7*a*b^2*e)*x^8 - 5*(4*a*b^
2*d - a^2*b*g)*x^7 - 6*(3*a*b^2*c - a^2*b*f)*x^6 + 7*(a^3*h - 7*a
^2*b*e)*x^5 - 18*a^3*x^2*e - 9*a^3*d*x - 8*(4*a^2*b*d - a^3*g)*x^
4 - 6*a^3*c - 9*(3*a^2*b*c - a^3*f)*x^3)/((b*x^3 + a)^2*a^4*x^3)
```

3.418 $\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=583

$$-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{8ad\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4ae\sqrt{a+bx^3}}{9b^2}+\frac{2cx\sqrt{a+bx^3}}{5b}+\frac{2dx^2\sqrt{a+bx^3}}{7b}+\frac{2ex^3\sqrt{a+bx^3}}{9b}$$

```
[Out] (-4*a*e*Sqrt[a + b*x^3])/(9*b^2) + (2*c*x*Sqrt[a + b*x^3])/(5*b)
+ (2*d*x^2*Sqrt[a + b*x^3])/(7*b) + (2*e*x^3*Sqrt[a + b*x^3])/(9*
b) - (8*a*d*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*Sqrt[2 + S
qrt[3]]*a*(7*b^(1/3)*c - 10*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.33722, antiderivative size = 583, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{8ad\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4ae\sqrt{a+bx^3}}{9b^2}+\frac{2cx\sqrt{a+bx^3}}{5b}+\frac{2dx^2\sqrt{a+bx^3}}{7b}+\frac{2ex^3\sqrt{a+bx^3}}{9b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (-4*a*e*Sqrt[a + b*x^3])/(9*b^2) + (2*c*x*Sqrt[a + b*x^3])/(5*b)
+ (2*d*x^2*Sqrt[a + b*x^3])/(7*b) + (2*e*x^3*Sqrt[a + b*x^3])/(9*
```

```
b) - (8*a*d*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*Sqrt[2 + S
qrt[3]]*a*(7*b^(1/3)*c - 10*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 163.345, size = 522, normalized size = 0.9

$$\frac{4\sqrt[4]{3}a^{\frac{4}{3}}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{7b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}$$

$$-\frac{4ae\sqrt{a+bx^{3}}}{9b^{2}}-\frac{8ad\sqrt{a+bx^{3}}}{7b^{\frac{5}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

$$-\frac{4\cdot 3^{\frac{3}{4}}a\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-10\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+7\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{105b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}$$

$$+\frac{2cx\sqrt{a+bx^{3}}}{5b}+\frac{2dx^{2}\sqrt{a+bx^{3}}}{7b}+\frac{2ex^{3}\sqrt{a+bx^{3}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] 4*3**(1/4)*a**(4/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(
2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3
) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 + s
qrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
- 4*sqrt(3))/(7*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a
**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 4*a*e
*sqrt(a + b*x**3)/(9*b**2) - 8*a*d*sqrt(a + b*x**3)/(7*b**(5/3)*(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) - 4*3**(3/4)*a*sqrt((a**(2/
3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3))
 + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-10
*a**(1/3)*d*(-sqrt(3) + 1) + 7*b**(1/3)*c)*elliptic_f(asin((-a**(
1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1
/3)*x)), -7 - 4*sqrt(3))/(105*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) +
b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*
x**3)) + 2*c*x*sqrt(a + b*x**3)/(5*b) + 2*d*x**2*sqrt(a + b*x**3)
/(7*b) + 2*e*x**3*sqrt(a + b*x**3)/(9*b)
```

Mathematica [C] time = 2.89881, size = 329, normalized size = 0.56

$$-12i3^{3/4}a^{4/3}b\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^{2}}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(10\sqrt[3]{a}d+7\sqrt[3]{-b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+360(-1)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(-b)^(2/3)*(a + b*x^3)*(-70*a*e + b*x*(63*c + 5*x*(9*d + 7*e*x
))) + 360*(-1)^(2/3)*3^(1/4)*a^(5/3)*b*d*Sqrt[(-1)^(5/6)*(-1 + ((
-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2
/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/
3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] - (12*I)*3^(3/4)*a^(4/3)*b*(
7*(-b)^(1/3)*c + 10*a^(1/3)*d)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^
(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*
x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x
)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(315*(-b)^(8/3)*Sqrt[a + b*x^3]
)
```

Maple [A] time = 0.027, size = 793, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] d*(2/7/b*x^2*(b*x^3+a)^(1/2)+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(2/5/b*x
*(b*x^3+a)^(1/2)+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))+e*(2/9/b*x^3*(b*x^3+a)^(1/2)-4/9*a/b^2*(b*
x^3+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + dx + c\right)x^3}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^5 + dx^4 + cx^3}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^5 + d*x^4 + c*x^3)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 4.15227, size = 129, normalized size = 0.22

$$e\left(\begin{cases} -\frac{4a\sqrt{a+bx^3}}{9b^2} + \frac{2x^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise} \end{cases}\right) + \frac{cx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)} + \frac{dx^5\,\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((-4*a*sqrt(a + b*x**3)/(9*b**2) + 2*x**3*sqrt(a + b*x
**3)/(9*b), Ne(b, 0)), (x**6/(6*sqrt(a)), True)) + c*x**4*gamma(4
/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*gamma(7/3)) + d*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**
3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + dx + c\right)x^3}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

3.419 $\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=560

$$-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2c\sqrt{a+bx^3}}{3b}+\frac{2dx\sqrt{a+bx^3}}{5b}+\frac{2ex^2\sqrt{a+bx^3}}{7b}$$

```
[Out] (2*c*Sqrt[a + b*x^3])/(3*b) + (2*d*x*Sqrt[a + b*x^3])/(5*b) + (2*
e*x^2*Sqrt[a + b*x^3])/(7*b) - (8*a*e*Sqrt[a + b*x^3])/(7*b^(5/3)
*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*d - 10*(1 - Sqrt[3
])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.964226, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2c\sqrt{a+bx^3}}{3b}+\frac{2dx\sqrt{a+bx^3}}{5b}+\frac{2ex^2\sqrt{a+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```



```
[Out] (2*c*Sqrt[a + b*x^3])/(3*b) + (2*d*x*Sqrt[a + b*x^3])/(5*b) + (2*
e*x^2*Sqrt[a + b*x^3])/(7*b) - (8*a*e*Sqrt[a + b*x^3])/(7*b^(5/3)
```

```
*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*d - 10*(1 - Sqrt[3
])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 117.422, size = 498, normalized size = 0.89

$$\frac{4\sqrt[4]{3}a^{\frac{4}{3}}e\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{7b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{8ae\sqrt{a+bx^3}}{7b^{\frac{5}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$
$$-\frac{4\cdot 3^{\frac{3}{4}}a\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-10\sqrt[3]{a}e\left(-\sqrt{3}+1\right)+7\sqrt[3]{b}d\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{105b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2c\sqrt{a+bx^3}}{3b}+\frac{2dx\sqrt{a+bx^3}}{5b}+\frac{2ex^2\sqrt{a+bx^3}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] 4*3**(1/4)*a**(4/3)*e*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(
2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3
) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 + s
qrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
- 4*sqrt(3))/(7*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a
**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 8*a*e
*sqrt(a + b*x**3)/(7*b**(5/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*
x)) - 4*3**(3/4)*a*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3
)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) +
2)*(a**(1/3) + b**(1/3)*x)*(-10*a**(1/3)*e*(-sqrt(3) + 1) + 7*b**
(1/3)*d)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/
(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(105*b**(
5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)
) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 2*c*sqrt(a + b*x**3)/(3*b
) + 2*d*x*sqrt(a + b*x**3)/(5*b) + 2*e*x**2*sqrt(a + b*x**3)/(7*b
)
```

Mathematica [C] time = 1.4202, size = 319, normalized size = 0.57

$$-\frac{-4i3^{3/4}a^{4/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(10\sqrt[3]{a}e+7\sqrt[3]{-b}d\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+120(-1)^{2}\cdots}{105\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] -(2*(-b)^(2/3)*(a + b*x^3)*(35*c + 3*x*(7*d + 5*e*x)) + 120*(-1)^
(2/3)*3^(1/4)*a^(5/3)*e*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1
/3))]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]
*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^
(1/4)], (-1)^(1/3)] - (4*I)*3^(3/4)*a^(4/3)*(7*(-b)^(1/3)*d + 10*
a^(1/3)*e)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*S
qrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*Ellipt
icF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)],
 (-1)^(1/3)])/(105*(-b)^(5/3)*Sqrt[a + b*x^3])
```

Maple [A] time = 0.01, size = 773, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] d*(2/5/b*x*(b*x^3+a)^(1/2)+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/3*c*(b*x^3+a)^(1/2)/b+e*(2/7/b
*x^2*(b*x^3+a)^(1/2)+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{bx^3+a}c}{3b}+\int\frac{ex^4+dx^3}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/sqrt(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*c/b + integrate((e*x^4 + d*x^3)/sqrt(b*x^3 +
a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^4+dx^3+cx^2}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/sqrt(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3 + c*x^2)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.30835, size = 107, normalized size = 0.19

$$c\left(\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for } b = 0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases}\right) + \frac{dx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)} + \frac{ex^5\,\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] c*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*
b), True)) + d*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + e*x**5*gamma(5/3)*hype
r((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(
8/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + dx + c\right)x^2}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^2/sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^2/sqrt(b*x^3 + a), x)
```

3.420 $\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=537

$$-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2d\sqrt{a+bx^3}}{3b} + \frac{2ex\sqrt{a+bx^3}}{5b}$$

```
[Out] (2*d*Sqrt[a + b*x^3])/(3*b) + (2*e*x*Sqrt[a + b*x^3])/(5*b) + (2*
c*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*a^(1/3)*
(5*(1 - Sqrt[3])*b^(2/3)*c + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.668483, antiderivative size = 537, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2d\sqrt{a+bx^3}}{3b} + \frac{2ex\sqrt{a+bx^3}}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(3*b) + (2*e*x*Sqrt[a + b*x^3])/(5*b) + (2*
c*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
```

```
- (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*a^(1/3)*
(5*(1 - Sqrt[3])*b^(2/3)*c + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 70.8356, size = 474, normalized size = 0.88

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}-\frac{4\cdot 3^{\frac{3}{4}}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{\frac{2}{3}}e+\frac{5b^{\frac{2}{3}}c\left(-\sqrt{3}+1\right)}{2}\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{15b^{\frac{4}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}+\frac{2d\sqrt{a+bx^{3}}}{3b}+\frac{2ex\sqrt{a+bx^{3}}}{5b}+\frac{2c\sqrt{a+bx^{3}}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*c*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3)
 + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 + sq
rt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 -
 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 4*3**(3/
4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)
/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**
(1/3) + b**(1/3)*x)*(a**(2/3)*e + 5*b**(2/3)*c*(-sqrt(3) + 1)/2)*
elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)
*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(15*b**(4/3)*sqrt(
a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/
3)*x)**2)*sqrt(a + b*x**3)) + 2*d*sqrt(a + b*x**3)/(3*b) + 2*e*x*
sqrt(a + b*x**3)/(5*b) + 2*c*sqrt(a + b*x**3)/(b**(2/3)*(a**(1/3)
*(1 + sqrt(3)) + b**(1/3)*x))
```

Mathematica [C] time = 2.75785, size = 314, normalized size = 0.58

$$\frac{2i3^{3/4}a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^{2}}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(2a^{2/3}\sqrt[3]{-b}e-5bc\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+30(-1)^{2/3}\cdots}{15(-b)^{5/3}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*(-b)^(2/3)*(5*d + 3*e*x)*(a + b*x^3) + 30*(-1)^(2/3)*3^(1/4)*
a^(2/3)*b*c*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1
 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[A
rcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)
^(1/3)] + (2*I)*3^(3/4)*a^(2/3)*(-5*b*c + 2*a^(2/3)*(-b)^(1/3)*e)
*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((
-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin
[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3
)])/(15*(-b)^(5/3)*Sqrt[a + b*x^3])
```

Maple [A] time = 0.01, size = 753, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/3*d*(b*x^3+a)^(1/2)/b-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(2/5/b*x*(b*x^3+a
)^(1/2)+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)\,x}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^3 + dx^2 + cx}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x^2 + c*x)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.08724, size = 107, normalized size = 0.2

$$d\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{cx^2\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{ex^4\,\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] d*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*
b), True)) + c*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + e*x**4*gamma(4/3)*hype
r((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(
7/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+dx+c\right)x}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)
```

3.421 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=509

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2e\sqrt{a+bx^3}}{3b}$$

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.430257, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2e\sqrt{a+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
```

```
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 39.0994, size = 447, normalized size = 0.88

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2e\sqrt{a+bx^3}}{3b}+\frac{2d\sqrt{a+bx^3}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3)
 + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 + sq
rt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 -
 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 2*e*sqrt
(a + b*x**3)/(3*b) + 2*d*sqrt(a + b*x**3)/(b**(2/3)*(a**(1/3)*(1
+ sqrt(3)) + b**(1/3)*x)) + 2*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*
b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)
**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-a**(1/3)*d*(-sqr
t(3) + 1) + b**(1/3)*c)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3))
 + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqr
t(3))/(3*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)
*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 1.61493, size = 305, normalized size = 0.6

$$-\frac{2i3^{3/4}\sqrt[3]{a}b\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(\sqrt[3]{a}d+\sqrt[3]{-b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-6(-1)^{2/3}\sqrt[4]{3}a^2}{3(-b)^{5/3}\sqrt{a+b}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

```
[Out] -(2*(-b)^(2/3)*e*(a + b*x^3) - 6*(-1)^(2/3)*3^(1/4)*a^(2/3)*b*d*S
qrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-b)^(1/3
```

```
)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(
-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + (2*
I)*3^(3/4)*a^(1/3)*b*((-b)^(1/3)*c + a^(1/3)*d)*Sqrt[((-1)^(5/6)*
(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/
3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6)
- (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(3*(-b)^(5/3)*
Sqrt[a + b*x^3])
```

Maple [A] time = 0.006, size = 735, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*
(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)))+2/3*e*(b*x^3+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + dx + c}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.61992, size = 105, normalized size = 0.21

$$e\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{cx\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*
b), True)) + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((
1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

3.422 $\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=518

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] (2*e*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - (1 -
 Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.494028, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4
```

```
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - (1 -
 Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 45.6999, size = 457, normalized size = 0.88

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}+\frac{2e\sqrt{a+bx^{3}}}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}e\left(-\sqrt{3}+1\right)+\sqrt[3]{b}d\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}-\frac{2c\operatorname{atanh}\left(\frac{\sqrt{a+bx^{3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*a**(1/3)*e*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3)
 + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 + sq
rt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 -
 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 2*e*sqrt
(a + b*x**3)/(b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 2
*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1
/3) + b**(1/3)*x)*(-a**(1/3)*e*(-sqrt(3) + 1) + b**(1/3)*d)*ellip
tic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 +
 sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/
3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)*
*2)*sqrt(a + b*x**3)) - 2*c*atanh(sqrt(a + b*x**3)/sqrt(a))/(3*sq
rt(a))
```

Mathematica [C] time = 1.69806, size = 493, normalized size = 0.95

$$-\frac{2\sqrt{2}\sqrt[3]{a}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}}\left(F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+\left((-1)^{2/3}-1\right)E\left(\sin^{-}\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

$$-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$-\frac{2d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (2*d*((-1)^
(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(b^
(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3))]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*a^(1/3)*e*((-1)^(1/3)*a^(1/3
) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*
I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6)
 - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]
 + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1
/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(b^(2/3)*Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [A] time = 0.01, size = 740, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))-2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-2/3*I*e*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
```

```
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+dx+c}{\sqrt{bx^3+ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

Sympy [A] time = 3.52212, size = 105, normalized size = 0.2

$$-\frac{2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}+\frac{dx\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{ex^2\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + d*x*gamma(1/
3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)
*gamma(4/3)) + e*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3
*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

3.423 $\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=547

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{c\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -((c*Sqrt[a + b*x^3])/(a*x)) + (b^(1/3)*c*Sqrt[a + b*x^3])/(a*((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/
Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (Sqrt
[2 + Sqrt[3]]*((1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.727663, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{c\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((c*Sqrt[a + b*x^3])/(a*x)) + (b^(1/3)*c*Sqrt[a + b*x^3])/(a*((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/
```

```
Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (Sqrt
[2 + Sqrt[3]]*((1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 78.0417, size = 479, normalized size = 0.88

$$\frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}-\frac{c\sqrt{a+bx^3}}{ax}-\frac{2d\,\operatorname{atanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt[4]{3}\sqrt[3]{b}c\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{2a^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{\frac{2}{3}}e-b^{\frac{2}{3}}c\left(-\sqrt{3}+1\right)\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3a^{\frac{2}{3}}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*c*sqrt(a + b*x**3)/(a*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)
*x)) - c*sqrt(a + b*x**3)/(a*x) - 2*d*atanh(sqrt(a + b*x**3)/sqrt
(a))/(3*sqrt(a)) - 3**(1/4)*b**(1/3)*c*sqrt((a**(2/3) - a**(1/3)*
b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)
**2)*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((
-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) +
b**(1/3)*x)), -7 - 4*sqrt(3))/(2*a**(2/3)*sqrt(a**(1/3)*(a**(1/3)
 + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a +
 b*x**3)) + 3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3)
+ 2)*(a**(1/3) + b**(1/3)*x)*(2*a**(2/3)*e - b**(2/3)*c*(-sqrt(3)
 + 1))*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a
**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(3*a**(2/3)
*b**(1/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sq
rt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 1.9029, size = 513, normalized size = 0.94

$$-\frac{\sqrt{2}\sqrt[3]{b}c\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}}\left(F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+\left((-1)^{2/3}-1\right)E\left(\sin^{-1}\right.\right.}{a^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

$$-\frac{c\sqrt{a+bx^3}}{ax}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$-\frac{2e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((c*Sqrt[a + b*x^3])/(a*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[
a]])/(3*Sqrt[a]) - (2*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^
(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a
^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/(b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (Sqrt[2]*
b^(1/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/
3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1
 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*Elli
pticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (
-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) -
 (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))
/(a^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [A] time = 0.011, size = 759, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))+c*(-1/a*(b*x^3+a)^(1/2)/x-1/3*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
```

```
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))-2/3*d*arctanh((
b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + dx + c}{\sqrt{bx^3 + ax^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

Sympy [A] time = 3.72053, size = 107, normalized size = 0.2

$$\frac{c\,\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\,\Gamma\left(\frac{2}{3}\right)} - \frac{2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} + \frac{ex\,\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(1/2),x)
```

```
[Out] c*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*x*gamma(2/3)) - 2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2))
)/(3*sqrt(a)) + e*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

3.424 $\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=569

$$-\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{c\sqrt{a+bx^3}}{2ax^2}-\frac{d\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -(c*Sqrt[a + b*x^3])/(2*a*x^2) - (d*Sqrt[a + b*x^3])/(a*x) + (b^(
1/3)*d*Sqrt[a + b*x^3])/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sq
rt[2 - Sqrt[3]]*b^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) - (Sqrt[2 + Sqrt[3]]*b^(1/3)*(b^(1/3)*c +
2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.949988, antiderivative size = 569, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$-\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{c\sqrt{a+bx^3}}{2ax^2}-\frac{d\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -(c*Sqrt[a + b*x^3])/(2*a*x^2) - (d*Sqrt[a + b*x^3])/(a*x) + (b^(
1/3)*d*Sqrt[a + b*x^3])/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sq
rt[2 - Sqrt[3]]*b^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) - (Sqrt[2 + Sqrt[3]]*b^(1/3)*(b^(1/3)*c +
2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 111.172, size = 495, normalized size = 0.87

$$\frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)} - \frac{3^{\frac{3}{4}}\sqrt[3]{b}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{6a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} - \frac{2e\operatorname{atanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$-\frac{\sqrt[4]{3}\sqrt[3]{b}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{2a^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*d*sqrt(a + b*x**3)/(a*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)
*x)) - 3**(3/4)*b**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b
**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt
(3) + 2)*(a**(1/3) + b**(1/3)*x)*(2*a**(1/3)*d*(-sqrt(3) + 1) + b
**(1/3)*c)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x
)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(6*a*sq
rt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**
(1/3)*x)**2)*sqrt(a + b*x**3)) - c*sqrt(a + b*x**3)/(2*a*x**2) -
d*sqrt(a + b*x**3)/(a*x) - 2*e*atanh(sqrt(a + b*x**3)/sqrt(a))/(3
*sqrt(a)) - 3**(1/4)*b**(1/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3
)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sq
rt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/
3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3
)*x)), -7 - 4*sqrt(3))/(2*a**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(
1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3
))
```

Mathematica [C] time = 3.4901, size = 525, normalized size = 0.92

$$-\frac{\sqrt{2}\sqrt[3]{b}d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}}\Bigg(F\Bigg(\sin^{-1}\Bigg(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\Bigg)\Bigg|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\Bigg)+\left((-1)^{2/3}-1\right)E\Bigg(\sin^{-1}}{a^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

$$+\frac{b^{2/3}c\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{2a\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt{a+bx^3}(c+2dx)}{2ax^2}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -((c + 2*d*x)*Sqrt[a + b*x^3])/(2*a*x^2) - (2*e*ArcTanh[Sqrt[a +
b*x^3]/Sqrt[a]])/(3*Sqrt[a]) + (b^(2/3)*c*((-1)^(1/3)*a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*
Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(2*a*Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3
]) - (Sqrt[2]*b^(1/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-
1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-
1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3
)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqr
t[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 +
(-1)^(1/3))]))/(a^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [A] time = 0.012, size = 778, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x)
```

```
[Out] c*(-1/2/a*(b*x^3+a)^(1/2)/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1/a*(b*x^3+a)^(1/2)/x-1/3*I/a*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
```

```
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))))-2/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + dx + c}{\sqrt{bx^3 + a}x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

Sympy [A] time = 4.03287, size = 112, normalized size = 0.2

$$\frac{c\,\Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\,\Gamma\left(\frac{1}{3}\right)} + \frac{d\,\Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}x\,\Gamma\left(\frac{2}{3}\right)} - \frac{2e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] c*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*x**2*gamma(1/3)) + d*gamma(-1/3)*hyper((-1/3, 1/2),
(2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3)) - 2*e*
asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

3.425 $\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=594

$$-\frac{16\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{40\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\,3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{80ae\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}+\frac{4c\sqrt{a+bx^3}}{3b^2}+\frac{2dx\sqrt{a+bx^3}}{5b^2}+\frac{2ex^2\sqrt{a+bx^3}}{7b^2}$$

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*c*Sqrt
[a + b*x^3])/(3*b^2) + (2*d*x*Sqrt[a + b*x^3])/(5*b^2) + (2*e*x^2
*Sqrt[a + b*x^3])/(7*b^2) - (80*a*e*Sqrt[a + b*x^3])/(21*b^(8/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (40*Sqrt[2 - Sqrt[3]]*a^(4
/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (16*Sqrt[2 + Sqrt[3]]*a*(14*b^(1/3)*d - 25*(1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(105*3^(1/4)*b^(8/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.2364, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$-\frac{16\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{40\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\,3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{80ae\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}+\frac{4c\sqrt{a+bx^3}}{3b^2}+\frac{2dx\sqrt{a+bx^3}}{5b^2}+\frac{2ex^2\sqrt{a+bx^3}}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*c*Sqrt
[a + b*x^3])/(3*b^2) + (2*d*x*Sqrt[a + b*x^3])/(5*b^2) + (2*e*x^2
*Sqrt[a + b*x^3])/(7*b^2) - (80*a*e*Sqrt[a + b*x^3])/(21*b^(8/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (40*Sqrt[2 - Sqrt[3]]*a^(4
/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (16*Sqrt[2 + Sqrt[3]]*a*(14*b^(1/3)*d - 25*(1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(105*3^(1/4)*b^(8/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.79225, size = 334, normalized size = 0.56

$$-16i3^{3/4}a^{4/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(25\sqrt[3]{a}e+14\sqrt[3]{-b}d\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)+1200(-$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (6*(-b)^(2/3)*(a*(70*c + 56*d*x + 50*e*x^2) + b*x^3*(35*c + 3*x*(
7*d + 5*e*x))) + 1200*(-1)^(2/3)*3^(1/4)*a^(5/3)*e*Sqrt[(-1)^(5/6
)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3)
+ ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (
I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] - (16*I)*3^(3/4)*a
^(4/3)*(14*(-b)^(1/3)*d + 25*a^(1/3)*e)*Sqrt[((-1)^(5/6)*(-a^(1/3
) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-
b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b
)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(315*(-b)^(8/3)*Sqrt[a
 + b*x^3])
```

Maple [A] time = 0.031, size = 836, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] d*(2/3/b^2*a*x/((x^3+a/b)*b)^(1/2)+2/5/b^2*x*(b*x^3+a)^(1/2)+32/4
5*I*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
```

```
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))+c*(2/3/b^2*a/((x^3+a/b)*b)^(1/2)+2/3/b^2*(b*x^3+a)^(1/2))+e*(2
/3/b^2*a*x^2/((x^3+a/b)*b)^(1/2)+2/7/b^2*x^2*(b*x^3+a)^(1/2)+80/6
3*I*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{3}\,c\left(\frac{\sqrt{bx^3+a}}{b^2}+\frac{a}{\sqrt{bx^3+a}\,b^2}\right)+\int\frac{\left(ex^7+dx^6\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^5/(b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) + integrate
((e*x^7 + d*x^6)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^7+dx^6+cx^5}{\left(bx^3+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^5/(b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^7 + d*x^6 + c*x^5)/(b*x^3 + a)^(3/2), x)
```

Sympy [A] time = 60.0994, size = 129, normalized size = 0.22

$$c\left(\begin{cases}\frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for}\ b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{dx^7\,\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{10}{3}\right)}+\frac{ex^8\,\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a +
 b*x**3)), Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + d*x**7*gamma(7
```

```
/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*gamma(10/3)) + e*x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b
*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(11/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)\,x^5}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^5/(b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^5/(b*x^3 + a)^(3/2), x)
```

3.426 $\int \frac{x^4\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=574

$$-\frac{8\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x\left(ae-bcx-bdx^2\right)}{3b^2\sqrt{a+bx^3}}+\frac{4d\sqrt{a+bx^3}}{3b^2}+\frac{2ex\sqrt{a+bx^3}}{5b^2}$$

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*d*Sqrt
[a + b*x^3])/(3*b^2) + (2*e*x*Sqrt[a + b*x^3])/(5*b^2) + (8*c*Sqr
t[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (
4*Sqrt[2 - Sqrt[3]]*a^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(
5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
5*(1 - Sqrt[3])*b^(2/3)*c + 4*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.969283, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$-\frac{8\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x\left(ae-bcx-bdx^2\right)}{3b^2\sqrt{a+bx^3}}+\frac{4d\sqrt{a+bx^3}}{3b^2}+\frac{2ex\sqrt{a+bx^3}}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*d*Sqrt
[a + b*x^3])/(3*b^2) + (2*e*x*Sqrt[a + b*x^3])/(5*b^2) + (8*c*Sqr
```

```
t[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (
4*Sqrt[2 - Sqrt[3]]*a^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(
5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
5*(1 - Sqrt[3])*b^(2/3)*c + 4*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 134.209, size = 517, normalized size = 0.9

$$-\frac{4\sqrt[4]{3}\sqrt[3]{a}c\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}} - \frac{8\cdot 3^{\frac{3}{4}}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(4a^{\frac{2}{3}}e+5b^{\frac{2}{3}}c\left(-\sqrt{3}+1\right)\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{45b^{\frac{7}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}} + \frac{4d\sqrt{a+bx^{3}}}{3b^{2}} + \frac{2ex\sqrt{a+bx^{3}}}{5b^{2}} + \frac{2x\left(ae-bcx-bdx^{2}\right)}{3b^{2}\sqrt{a+bx^{3}}} + \frac{8c\sqrt{a+bx^{3}}}{3b^{\frac{5}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] -4*3**(1/4)*a**(1/3)*c*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**
(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(
3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 +
sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
 - 4*sqrt(3))/(3*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 8*3*
*(3/4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x
**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*
(a**(1/3) + b**(1/3)*x)*(4*a**(2/3)*e + 5*b**(2/3)*c*(-sqrt(3) +
1))*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(45*b**(7/3)*s
qrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b*
*(1/3)*x)**2)*sqrt(a + b*x**3)) + 4*d*sqrt(a + b*x**3)/(3*b**2) +
 2*e*x*sqrt(a + b*x**3)/(5*b**2) + 2*x*(a*e - b*c*x - b*d*x**2)/(
3*b**2*sqrt(a + b*x**3)) + 8*c*sqrt(a + b*x**3)/(3*b**(5/3)*(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x))
```

Mathematica [C] time = 2.38195, size = 330, normalized size = 0.57

$$-8i3^{3/4}a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^{2}}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(4a^{2/3}\sqrt[3]{-b}e-5bc\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-120(-1)^{2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (6*(-b)^(2/3)*(2*a*(5*d + 4*e*x) + b*x^2*(-5*c + 5*d*x + 3*e*x^2)
) - 120*(-1)^(2/3)*3^(1/4)*a^(2/3)*b*c*Sqrt[(-1)^(5/6)*(-1 + ((-b
)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3
)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)
*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] - (8*I)*3^(3/4)*a^(2/3)*(-5*b*
c + 4*a^(2/3)*(-b)^(1/3)*e)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/
3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2
)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a
^(1/3)]/3^(1/4)], (-1)^(1/3)])/(45*(-b)^(8/3)*Sqrt[a + b*x^3])
```

Maple [A] time = 0.012, size = 817, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] c*(-2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+d*(2/3/b
^2*a/((x^3+a/b)*b)^(1/2)+2/3/b^2*(b*x^3+a)^(1/2))+e*(2/3/b^2*a*x/
((x^3+a/b)*b)^(1/2)+2/5/b^2*x*(b*x^3+a)^(1/2)+32/45*I*a/b^3*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^4}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^6 + dx^5 + cx^4}{(bx^3 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^6 + d*x^5 + c*x^4)/(b*x^3 + a)^(3/2), x)
```

Sympy [A] time = 39.837, size = 129, normalized size = 0.22

$$d\left(\begin{cases}\frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^5\,\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{8}{3}\right)}+\frac{ex^7\,\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] d*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a +
 b*x**3)), Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**5*gamma(5
/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*gamma(8/3)) + e*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x
**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+dx+c)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

3.427 $\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=542

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{4e\sqrt{a+bx^3}}{3b^2}-\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

```
[Out] (-2*x*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (4*e*Sqrt[a + b*
x^3])/(3*b^2) + (8*d*Sqrt[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqr
t[2 + Sqrt[3]]*(b^(1/3)*c - 2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.73039, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{4e\sqrt{a+bx^3}}{3b^2}-\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```


```
[Out] (-2*x*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (4*e*Sqrt[a + b*
x^3])/(3*b^2) + (8*d*Sqrt[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a
```

```
^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqr
t[2 + Sqrt[3]]*(b^(1/3)*c - 2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 76.853, size = 483, normalized size = 0.89

$$-\frac{4\sqrt[4]{3}\sqrt[3]{a}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}} + \frac{4e\sqrt{a+bx^3}}{3b^2} + \frac{8d\sqrt{a+bx^3}}{3b^{\frac{5}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)} + \frac{4\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{9b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] -4*3**(1/4)*a**(1/3)*d*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**
(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(
3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 +
sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
 - 4*sqrt(3))/(3*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 2*x*
(c + d*x + e*x**2)/(3*b*sqrt(a + b*x**3)) + 4*e*sqrt(a + b*x**3)/
(3*b**2) + 8*d*sqrt(a + b*x**3)/(3*b**(5/3)*(a**(1/3)*(1 + sqrt(3
)) + b**(1/3)*x)) + 4*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)
*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqr
t(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(-2*a**(1/3)*d*(-sqrt(3) +
 1) + b**(1/3)*c)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**
(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/
(9*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 +
sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 1.2744, size = 319, normalized size = 0.59

$$\frac{4i3^{3/4}\sqrt[3]{a}b\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(2\sqrt[3]{a}d+\sqrt[3]{-b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)-24(-1)^{2/3}\sqrt[4]{3}a}{9(-b)^{8/}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (6*(-b)^(2/3)*(2*a*e + b*x*(-c - d*x + e*x^2)) - 24*(-1)^(2/3)*3^
(1/4)*a^(2/3)*b*d*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*
Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*Ellip
ticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)]
, (-1)^(1/3)] + (4*I)*3^(3/4)*a^(1/3)*b*((-b)^(1/3)*c + 2*a^(1/3)
*d)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 +
 ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[Arc
Sin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(
1/3)])/(9*(-b)^(8/3)*Sqrt[a + b*x^3])
```

Maple [A] time = 0.011, size = 800, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] d*(-2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(-2/3/
b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)))+e*(2/3/b^2*a/((x^3+a/b)*b)^(1/2)+2/3/b
^2*(b*x^3+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^3}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^5 + dx^4 + cx^3}{(bx^3 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^5 + d*x^4 + c*x^3)/(b*x^3 + a)^(3/2), x)
```

Sympy [A] time = 30.5744, size = 129, normalized size = 0.24

$$e\left(\begin{cases}\frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{7}{3}\right)}+\frac{dx^5\,\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a +
 b*x**3)), Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**4*gamma(4
/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*gamma(7/3)) + d*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+dx+c\right)x^3}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

3.428 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=522

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-2\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

```
[Out] (-2*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (8*e*Sqrt[a + b*x^
3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 -
 Sqrt[3]]*a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - 2*(1 -
 Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.599532, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-2\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (8*e*Sqrt[a + b*x^
3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 -
```

```
 Sqrt[3]]*a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - 2*(1 -
 Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 50.4199, size = 466, normalized size = 0.89

$$-\frac{4\sqrt[4]{3}\sqrt[3]{a}e\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}+\frac{8e\sqrt{a+bx^3}}{3b^{\frac{5}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{4\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2\sqrt[3]{a}e\left(-\sqrt{3}+1\right)+\sqrt[3]{b}d\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{9b^{\frac{5}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] -4*3**(1/4)*a**(1/3)*e*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**
(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(
3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 +
sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
 - 4*sqrt(3))/(3*b**(5/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 2*(c
 + d*x + e*x**2)/(3*b*sqrt(a + b*x**3)) + 8*e*sqrt(a + b*x**3)/(3
*b**(5/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 4*3**(3/4)*sqr
t((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 +
 sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3
)*x)*(-2*a**(1/3)*e*(-sqrt(3) + 1) + b**(1/3)*d)*elliptic_f(asin(
(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) +
 b**(1/3)*x)), -7 - 4*sqrt(3))/(9*b**(5/3)*sqrt(a**(1/3)*(a**(1/3
) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a
+ b*x**3))
```

Mathematica [C] time = 2.27536, size = 305, normalized size = 0.58

$$\frac{-4i3^{3/4}\sqrt[3]{a}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(2\sqrt[3]{a}e+\sqrt[3]{-b}d\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+24(-1)^{2/3}\sqrt[4]{3}a}{9(-b)^{5/3}\sqrt{a+b}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (6*(-b)^(2/3)*(c + x*(d + e*x)) + 24*(-1)^(2/3)*3^(1/4)*a^(2/3)*e
*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-b)^(1
/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[
-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] - (
4*I)*3^(3/4)*a^(1/3)*((-b)^(1/3)*d + 2*a^(1/3)*e)*Sqrt[((-1)^(5/6
)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(
1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6
) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(9*(-b)^(5/3
)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.01, size = 779, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] d*(-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*c/b/(b*x^3+a)^(1/2)+e*(-2/
3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,c}{3\,\sqrt{bx^3+ab}}+\int\frac{\left(ex^4+dx^3\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*c/(sqrt(b*x^3 + a)*b) + integrate((e*x^4 + d*x^3)*sqrt(b*x^3
 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^4+dx^3+cx^2}{(bx^3+a)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3 + c*x^2)/(b*x^3 + a)^(3/2), x)
```

Sympy [A] time = 25.4716, size = 109, normalized size = 0.21

$$c\left(\begin{cases}-\frac{2}{3b\sqrt{a+bx^3}} & \text{for}\ b\neq 0\\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{dx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{7}{3}\right)}+\frac{ex^5\,\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3
/2)), True)) + d*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + e*x**5*gamma(5/3)*h
yper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*ga
mma(8/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+dx+c\right)x^2}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^(3/2), x)
```

3.429 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=561

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+b^{2/3}\left(c-\sqrt{3}c\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2x\left(ae-bcx-bdx^2\right)}{3ab\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab}$$

```
[Out] (-2*x*(a*e - b*c*x - b*d*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*d*Sqr
t[a + b*x^3])/(3*a*b) - (2*c*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*c*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
+ (2*Sqrt[2 + Sqrt[3]]*(b^(2/3)*(c - Sqrt[3]*c) + 2*a^(2/3)*e)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(2/3)*b^(4/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])
```

Rubi [A] time = 0.703661, antiderivative size = 561, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+b^{2/3}\left(c-\sqrt{3}c\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2x\left(ae-bcx-bdx^2\right)}{3ab\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x*(a*e - b*c*x - b*d*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*d*Sqr
t[a + b*x^3])/(3*a*b) - (2*c*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 +
```

```
Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*c*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
+ (2*Sqrt[2 + Sqrt[3]]*(b^(2/3)*(c - Sqrt[3]*c) + 2*a^(2/3)*e)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(2/3)*b^(4/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])
```

Rubi in Sympy [A] time = 71.4795, size = 495, normalized size = 0.88

$$-\frac{2d\sqrt{a+bx^3}}{3ab}-\frac{2x\left(ae-bcx-bdx^2\right)}{3ab\sqrt{a+bx^3}}-\frac{2c\sqrt{a+bx^3}}{3ab^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}c\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{\frac{2}{3}}e+\frac{b^{\frac{2}{3}}c\left(-\sqrt{3}+1\right)}{2}\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{9a^{\frac{2}{3}}b^{\frac{4}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] -2*d*sqrt(a + b*x**3)/(3*a*b) - 2*x*(a*e - b*c*x - b*d*x**2)/(3*a
*b*sqrt(a + b*x**3)) - 2*c*sqrt(a + b*x**3)/(3*a*b**(2/3)*(a**(1/
3)*(1 + sqrt(3)) + b**(1/3)*x)) + 3**(1/4)*c*sqrt((a**(2/3) - a**
(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1
/3)*x)**2)*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(
asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(
3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(3*a**(2/3)*b**(2/3)*sqrt(a**
(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*
x)**2)*sqrt(a + b*x**3)) + 4*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b
**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)*
*2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(a**(2/3)*e + b**(2
/3)*c*(-sqrt(3) + 1)/2)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3))
 + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqr
t(3))/(9*a**(2/3)*b**(4/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/
(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 1.59112, size = 317, normalized size = 0.57

$$-\frac{2i3^{3/4}a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(2a^{2/3}\sqrt[3]{-b}e-bc\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+6(-1)^{2/3}\sqrt[4]{\cdots}}{9a(-b)^{5/3}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] -(6*(-b)^(2/3)*(b*c*x^2 - a*(d + e*x)) + 6*(-1)^(2/3)*3^(1/4)*a^(
2/3)*b*c*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 +
((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[ArcS
in[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1
/3)] + (2*I)*3^(3/4)*a^(2/3)*(-(b*c) + 2*a^(2/3)*(-b)^(1/3)*e)*Sq
rt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)
^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sq
rt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])
/(9*a*(-b)^(5/3)*Sqrt[a + b*x^3])
```

Maple [A] time = 0.008, size = 782, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*d/b/(b*x^3+a)^(1/2)+c*(2/3/a*x^2/((x^3+a/b)*b)^(1/2)+2/9*I/a
*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2))))+e*(-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + dx + c\right)x}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^3 + dx^2 + cx}{\left(bx^3 + a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x^2 + c*x)/(b*x^3 + a)^(3/2), x)
```

Sympy [A] time = 25.3657, size = 109, normalized size = 0.19

$$d\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for}\ b\neq 0\\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{5}{3}\right)}+\frac{ex^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] d*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3
/2)), True)) + c*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + e*x**4*gamma(4/3)*h
yper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*ga
mma(7/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+dx+c\right)x}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

3.430 $\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=532

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

```
[Out] (-2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)) - (2*(a*e - b*x*(c + d*x)))/(3*a*b*Sqrt[a + b*x^3]) + (Sq
rt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c +
(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.589241, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```



```
[Out] (-2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)) - (2*(a*e - b*x*(c + d*x)))/(3*a*b*Sqrt[a + b*x^3]) + (Sq
```

```
rt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c +
(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 48.2634, size = 468, normalized size = 0.88

$$-\frac{2\left(ae-bx\left(c+dx\right)\right)}{3ab\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{a}d\left(-\sqrt{3}+1\right)+\sqrt[3]{b}c\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{9ab^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt[4]{3}d\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] -2*(a*e - b*x*(c + d*x))/(3*a*b*sqrt(a + b*x**3)) - 2*d*sqrt(a +
b*x**3)/(3*a*b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 2*
3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a
**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/
3) + b**(1/3)*x)*(a**(1/3)*d*(-sqrt(3) + 1) + b**(1/3)*c)*ellipti
c_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + s
qrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(9*a*b**(2/3)*sqrt(a**(1/
3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)*
*2)*sqrt(a + b*x**3)) + 3**(1/4)*d*sqrt((a**(2/3) - a**(1/3)*b**(
1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)
*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**
(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(
1/3)*x)), -7 - 4*sqrt(3))/(3*a**(2/3)*b**(2/3)*sqrt(a**(1/3)*(a**
(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqr
t(a + b*x**3))
```

Mathematica [C] time = 1.56967, size = 314, normalized size = 0.59

$$-\frac{2i3^{3/4}\sqrt[3]{a}b\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(\sqrt[3]{-b}c-\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+6(-1)^{2/3}\sqrt[4]{3}a^2}{9a(-b)^{5/3}\sqrt{a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```

```
[Out] -(6*(-b)^(2/3)*(-(a*e) + b*x*(c + d*x)) + 6*(-1)^(2/3)*3^(1/4)*a^
(2/3)*b*d*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 +
 ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[Arc
Sin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(
1/3)] + (2*I)*3^(3/4)*a^(1/3)*b*((-b)^(1/3)*c - a^(1/3)*d)*Sqrt[(
(-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/
3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-
(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(9*
a*(-b)^(5/3)*Sqrt[a + b*x^3])
```

Maple [A] time = 0.006, size = 785, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] c*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(2/3/a*x^2/((x^3+a/b)*b)^(1/2
)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))))-2/3*e/b/(b*x^3+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Sympy [A] time = 24.9724, size = 107, normalized size = 0.2

$$e\left(\begin{cases}-\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3
/2)), True)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hype
r((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma
(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

3.431 $\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=579

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}e+\sqrt[3]{b}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}+\frac{2c\sqrt{a+bx^3}}{3a^2}-\frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*c*Sqrt
[a + b*x^3])/(3*a^2) - (2*e*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2)) + (Sqrt[2 - Sqrt[3]]*e*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 +
 Sqrt[3]]*(b^(1/3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.839432, antiderivative size = 579, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}e+\sqrt[3]{b}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}+\frac{2c\sqrt{a+bx^3}}{3a^2}-\frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*c*Sqrt
[a + b*x^3])/(3*a^2) - (2*e*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2)) + (Sqrt[2 - Sqrt[3]]*e*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 +
 Sqrt[3]]*(b^(1/3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 57.6426, size = 462, normalized size = 0.8

$$\frac{2x\left(\frac{c}{x}+d+ex\right)}{3a\sqrt{a+bx^3}}-\frac{2e\sqrt{a+bx^3}}{3ab^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$
$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{a}e\left(-\sqrt{3}+1\right)+\sqrt[3]{b}d\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{9ab^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{\sqrt[4]{3}e\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*x*(c/x + d + e*x)/(3*a*sqrt(a + b*x**3)) - 2*e*sqrt(a + b*x**3)
/(3*a*b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) + 2*3**(3/4
)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)
*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b*
*(1/3)*x)*(a**(1/3)*e*(-sqrt(3) + 1) + b**(1/3)*d)*elliptic_f(asi
n((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3))
 + b**(1/3)*x)), -7 - 4*sqrt(3))/(9*a*b**(2/3)*sqrt(a**(1/3)*(a**
(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqr
t(a + b*x**3)) + 3**(1/4)*e*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x
+ b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-
sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(
-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)
), -7 - 4*sqrt(3))/(3*a**(2/3)*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) +
 b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b
*x**3))
```

Mathematica [C] time = 3.50248, size = 518, normalized size = 0.89

$$2\left(\frac{\sqrt{2}\sqrt[3]{a}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}}\left(F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+\left((-1)^{2/3}-1\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-}}{-1+\sqrt[3]{}}\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}\right.$$

$$3a$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*((c + x*(d + e*x))/Sqrt[a + b*x^3] - (c*ArcTanh[Sqrt[a + b*x^3
]/Sqrt[a]])/Sqrt[a] - (d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a
^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*
a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Ellip
ticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]], (-1)^(1/3)])/(b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b
^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) + (Sqrt[2]
*a^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(
1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*Ell
ipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)],
(-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6)
- (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])
)/(b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[a + b*x^3])))/(3*a)
```

Maple [A] time = 0.009, size = 810, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x)
```

```
[Out] d*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+c*(2/3/a/((x^3+a/b)*b)^(1/2)-2/
3/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+e*(2/3/a*x^2/((x^3+a/
b)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + dx + c}{(bx^4 + ax)\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/((b*x^4 + a*x)*sqrt(b*x^3 + a)), x)
```

Sympy [A] time = 40.6825, size = 265, normalized size = 0.46

$$c\left(\frac{2a^3\sqrt{1 + \frac{bx^3}{a}}}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} + \frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} - \frac{2a^3\log\left(\sqrt{1 + \frac{bx^3}{a}} + 1\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} + \frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} - \frac{2a^2bx^3\log\left(\sqrt{1 + \frac{bx^3}{a}} + 1\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3}\right) + \frac{dx\,\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{ex^2\,\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{3}{2} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] c*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a
**3*log(b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(s
qrt(1 + b*x**3/a) + 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*
x**3*log(b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**2*b*x*
*3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3))
+ d*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)
/a)/(3*a**(3/2)*gamma(4/3)) + e*x**2*gamma(2/3)*hyper((2/3, 3/2),
 (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

3.432 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=607

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}-\frac{c\sqrt{a+bx^3}}{a^2x}+\frac{5\sqrt[3]{b}c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2d\sqrt{a+bx^3}}{3a^2}$$

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*d*Sqrt
[a + b*x^3])/(3*a^2) - (c*Sqrt[a + b*x^3])/(a^2*x) + (5*b^(1/3)*c
*Sqrt[a + b*x^3])/(3*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (
2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) - (5*Sqrt[2 - S
qrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) - (Sqrt[2 + Sqrt[3]]*(5*(1 - Sqrt[3])*b^(2
/3)*c - 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(5/3)*b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.09321, antiderivative size = 607, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}-\frac{c\sqrt{a+bx^3}}{a^2x}+\frac{5\sqrt[3]{b}c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2d\sqrt{a+bx^3}}{3a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*d*Sqrt
[a + b*x^3])/(3*a^2) - (c*Sqrt[a + b*x^3])/(a^2*x) + (5*b^(1/3)*c
*Sqrt[a + b*x^3])/(3*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (
2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) - (5*Sqrt[2 - S
qrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) - (Sqrt[2 + Sqrt[3]]*(5*(1 - Sqrt[3])*b^(2
/3)*c - 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(5/3)*b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 31.6695, size = 214, normalized size = 0.35

$$\frac{2x\left(\frac{c}{x^2}+\frac{d}{x}+e\right)}{3a\sqrt{a+bx^3}}+\frac{2\cdot 3^{\frac{3}{4}}e\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{9a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*x*(c/x**2 + d/x + e)/(3*a*sqrt(a + b*x**3)) + 2*3**(3/4)*e*sqrt
((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3)*(1 +
sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b**(1/3)
*x)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(9*a*b**(1/3)*
sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b
**(1/3)*x)**2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 4.75116, size = 542, normalized size = 0.89

$$\frac{-3ac+2ax(d+ex)-5bcx^3}{3a^2x\sqrt{a+bx^3}}-\frac{\frac{10\sqrt{2}\sqrt[3]{a}\sqrt[3]{b}c\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}}\left(F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+\left((-1)^{2/3}-1\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\right|}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}}{6a}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-3*a*c - 5*b*c*x^3 + 2*a*x*(d + e*x))/(3*a^2*x*Sqrt[a + b*x^3])
- (4*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + (4*a*e*((-1)^(1
/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1
/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(b^(1
/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]*Sqrt[a + b*x^3]) + (10*Sqrt[2]*a^(1/3)*b^(1/3)*c*((-1)^(1/3)
```

```
*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3
)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1
)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^
(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3
)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(Sqrt[(a^(1/3) + (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]))/
(6*a^2)
```

Maple [A] time = 0.012, size = 825, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x)
```

```
[Out] e*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+c*(-2/3*b/a^2*x^2/((x^3+a/b)*b)
^(1/2)-1/a^2*(b*x^3+a)^(1/2)/x-5/9*I/a^2*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+d*(2/3/a/((
x^3+a/b)*b)^(1/2)-2/3/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^2 + dx + c}{(bx^5 + ax^2)\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/((b*x^5 + a*x^2)*sqrt(b*x^3 + a)), x)
```

Sympy [A] time = 56.8838, size = 267, normalized size = 0.44

$$d\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)+\frac{c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}+\frac{ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(3/2),x)
```

```
[Out] d*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a
**3*log(b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(s
qrt(1 + b*x**3/a) + 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*
x**3*log(b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**2*b*x*
*3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3))
+ c*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)
/a)/(3*a**(3/2)*x*gamma(2/3)) + e*x*gamma(1/3)*hyper((1/3, 3/2),
(4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)
```

3.433 $\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=733

$$\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bd-10ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{24a^2\sqrt{a+bx^3}(19bd-10ag)}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4a^2e\sqrt{a+bx^3}}{45b^2}$$

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-8af)-1870\right.}{1616615b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6ax\sqrt{a+bx^3}(17bc-8af)}{935b^2}+\frac{6ax^2\sqrt{a+bx^3}(19bd-10ag)}{1729b^2}$$

$$+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$+\frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b}$$

```
[Out] (-4*a^2*e*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*c - 8*a*f)*x*Sqr
t[a + b*x^3])/(935*b^2) + (6*a*(19*b*d - 10*a*g)*x^2*Sqrt[a + b*x
^3])/(1729*b^2) + (2*a*e*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*f*x^4
*Sqrt[a + b*x^3])/(187*b) + (6*a*g*x^5*Sqrt[a + b*x^3])/(247*b) -
 (24*a^2*(19*b*d - 10*a*g)*Sqrt[a + b*x^3])/(1729*b^(8/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^3*Sqrt[a + b*x^3]*(62985*c*x
 + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/692835
 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*d - 10*a*g)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(1729*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*3
^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1729*b^(1/3)*(17*b*c - 8*a*f) - 187
0*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 10*a*g))*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(1616615*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 3.31923, antiderivative size = 733, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257

$$\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bd-10ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{24a^2\sqrt{a+bx^3}(19bd-10ag)}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4a^2e\sqrt{a+bx^3}}{45b^2}$$

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-8af)-1870\right.}{1616615b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6ax\sqrt{a+bx^3}(17bc-8af)}{935b^2}+\frac{6ax^2\sqrt{a+bx^3}(19bd-10ag)}{1729b^2}$$

$$+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$+\frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (-4*a^2*e*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*c - 8*a*f)*x*Sqr
t[a + b*x^3])/(935*b^2) + (6*a*(19*b*d - 10*a*g)*x^2*Sqrt[a + b*x
^3])/(1729*b^2) + (2*a*e*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*f*x^4
*Sqrt[a + b*x^3])/(187*b) + (6*a*g*x^5*Sqrt[a + b*x^3])/(247*b) -
 (24*a^2*(19*b*d - 10*a*g)*Sqrt[a + b*x^3])/(1729*b^(8/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^3*Sqrt[a + b*x^3]*(62985*c*x
 + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/692835
 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*d - 10*a*g)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(1729*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*3
^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1729*b^(1/3)*(17*b*c - 8*a*f) - 187
0*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 10*a*g))*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(1616615*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.78934, size = 433, normalized size = 0.59

$$-36i3^{3/4}a^{7/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\left(323b\left(110\sqrt[3]{a}d+91\sqrt[3]{-b}c\right)-\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*(-b)^(2/3)*(a + b*x^3)*(-2*a^2*(323323*e + 27*x*(6916*f + 4675
*g*x)) + 21*b^2*x^4*(62985*c + 11*x*(4845*d + 13*x*(323*e + 285*f
*x + 255*g*x^2))) + a*b*x*(793611*c + x*(479655*d + 7*x*(46189*e
+ 135*x*(247*f + 187*g*x))))) + 201960*(-1)^(2/3)*3^(1/4)*a^(8/3)
*(19*b*d - 10*a*g)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^
(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)
]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3
^(1/4)], (-1)^(1/3)] - (36*I)*3^(3/4)*a^(7/3)*(323*b*(91*(-b)^(1/
3)*c + 110*a^(1/3)*d) - 4*(3458*a*(-b)^(1/3)*f + 4675*a^(4/3)*g))
*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((
-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin
[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3
)])/(14549535*(-b)^(8/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.028, size = 1674, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)
```

```
[Out] d*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a/b*x^2*(b*x^3+a)^(1/2)+8/91*I/b
^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^
2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))))+c*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a/b*x*(b*x^3
+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)))+e*(2/15*x^6*(b*x^3+a)^(1/2)+2/45*a/b*x^3*(b*x^3
+a)^(1/2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))+f*(2/17*x^7*(b*x^3+a)^(1/
2)+6/187*a/b*x^4*(b*x^3+a)^(1/2)-48/935*a^2/b^2*x*(b*x^3+a)^(1/2)
-32/935*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
```

```
))^(1/2)))+g*(2/19*x^8*(b*x^3+a)^(1/2)+6/247*a/b*x^5*(b*x^3+a)^(1
/2)-60/1729*a^2/b^2*x^2*(b*x^3+a)^(1/2)-80/1729*I*a^3/b^3*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^7 + fx^6 + ex^5 + dx^4 + cx^3\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3,x, algorithm="fric
```

```
[Out] integral((g*x^7 + f*x^6 + e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^3 + a),
 x)
```

Sympy [A] time = 5.98812, size = 238, normalized size = 0.32

$$\frac{\sqrt{a}cx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}dx^5\,\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}fx^7\,\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}gx^8\,\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{11}{3}\right)} + e\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2} + \frac{2ax^3\sqrt{a+bx^3}}{45b} + \frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b \neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*d*x**5*gamma(5/3)*hyper((-
1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqr
t(a)*f*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*g*x**8*gamma(8/3)*hyper((-1
```

```
/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + e*
Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a +
 b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)
*x**6/6, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3,
x)
```

3.434 $\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=681

$$\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{24a^2e\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag+1547bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{85085b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2a\sqrt{a+bx^3}(5bc-2af)}{45b^2}+\frac{6ax\sqrt{a+bx^3}(17bd-8ag)}{935b^2}$$
$$+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$
$$+\frac{6aex^2\sqrt{a+bx^3}}{91b}+\frac{2afx^3\sqrt{a+bx^3}}{45b}+\frac{6agx^4\sqrt{a+bx^3}}{187b}$$

```
[Out] (2*a*(5*b*c - 2*a*f)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*d - 8
*a*g)*x*Sqrt[a + b*x^3])/(935*b^2) + (6*a*e*x^2*Sqrt[a + b*x^3])/
(91*b) + (2*a*f*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*g*x^4*Sqrt[a +
 b*x^3])/(187*b) - (24*a^2*e*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*Sqrt[a + b*x^3]*(12155*c*x
 + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x^5))/109395 + (
12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91
*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*3^(3/4)*Sqrt[2 + Sqrt[3
]]*a^2*(1547*b*d - 1870*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 728*a*g
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(85085*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rubi [A] time = 2.49199, antiderivative size = 681, normalized size of antiderivative = 1., number of

steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$

$$\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{24a^2e\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag+1547bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{85085b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2a\sqrt{a+bx^3}(5bc-2af)}{45b^2}+\frac{6ax\sqrt{a+bx^3}(17bd-8ag)}{935b^2}$$
$$+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$
$$+\frac{6aex^2\sqrt{a+bx^3}}{91b}+\frac{2afx^3\sqrt{a+bx^3}}{45b}+\frac{6agx^4\sqrt{a+bx^3}}{187b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*(5*b*c - 2*a*f)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*d - 8
*a*g)*x*Sqrt[a + b*x^3])/(935*b^2) + (6*a*e*x^2*Sqrt[a + b*x^3])/
(91*b) + (2*a*f*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*g*x^4*Sqrt[a +
 b*x^3])/(187*b) - (24*a^2*e*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*Sqrt[a + b*x^3]*(12155*c*x
 + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x^5))/109395 + (
12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91
*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*3^(3/4)*Sqrt[2 + Sqrt[3
]]*a^2*(1547*b*d - 1870*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 728*a*g
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(85085*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.08717, size = 399, normalized size = 0.59

$$-36i3^{3/4}a^{7/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(17b\left(110\sqrt[3]{a}e+91\sqrt[3]{-b}d\right)-728a\sqrt[3]{-b}g\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1}}{\sqrt[4]{3}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*(-b)^(2/3)*(a + b*x^3)*(-182*a^2*(187*f + 108*g*x) + 7*b^2*x^3
*(12155*c + 9945*d*x + 33*x^2*(255*e + 13*x*(17*f + 15*g*x))) + a
*b*(85085*c + x*(41769*d + x*(25245*e + 17017*f*x + 12285*g*x^2))
)) + 201960*(-1)^(2/3)*3^(1/4)*a^(8/3)*b*e*Sqrt[(-1)^(5/6)*(-1 +
((-b)^(1/3)*x)/a^(1/3))]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^
(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(
1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] - (36*I)*3^(3/4)*a^(7/3)*(
17*b*(91*(-b)^(1/3)*d + 110*a^(1/3)*e) - 728*a*(-b)^(1/3)*g)*Sqrt
[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(
1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt
[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(
765765*(-b)^(8/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.012, size = 1197, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)
```

```
[Out] d*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a/b*x*(b*x^3+a)^(1/2)+4/55*I/b^2
*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+2/
9*c/b*(b*x^3+a)^(3/2)+e*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a/b*x^2*(b
*x^3+a)^(1/2)+8/91*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(2/15*x^6*(b*x^3+a)^(
1/2)+2/45*a/b*x^3*(b*x^3+a)^(1/2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))+g
*(2/17*x^7*(b*x^3+a)^(1/2)+6/187*a/b*x^4*(b*x^3+a)^(1/2)-48/935*a
^2/b^2*x*(b*x^3+a)^(1/2)-32/935*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}c}{9\,b}+\int\left(gx^6+fx^5+ex^4+dx^3\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^2,x, algorithm="maxi
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*c/b + integrate((g*x^6 + f*x^5 + e*x^4 + d*
x^3)*sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^6+fx^5+ex^4+dx^3+cx^2\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^2,x, algorithm="fric
```

```
[Out] integral((g*x^6 + f*x^5 + e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^3 + a),
 x)
```

Sympy [A] time = 5.41305, size = 223, normalized size = 0.33

$$\frac{\sqrt{a}dx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}ex^5\,\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}gx^7\,\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{10}{3}\right)}+c\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+f\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*e*x**5*gamma(5/3)*hyper((-
1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqr
t(a)*g*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(10/3)) + c*Piecewise((sqrt(a)*x**3/3, Eq(b,
0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + f*Piecewise((-4*a**2*
sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2
*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^2,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^2,
x)
```

3.435 $\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=667

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13bc-4af)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6a\sqrt{a+bx^3}(13bc-4af)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a\sqrt{a+bx^3}(5bd-2ag)}{45b^2}$$

$$+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$$+\frac{6aex\sqrt{a+bx^3}}{55b}+\frac{6afx^2\sqrt{a+bx^3}}{91b}+\frac{2agx^3\sqrt{a+bx^3}}{45b}$$

```
[Out] (2*a*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*e*x*Sqrt[a
+ b*x^3])/(55*b) + (6*a*f*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*g*x^
3*Sqrt[a + b*x^3])/(45*b) + (6*a*(13*b*c - 4*a*f)*Sqrt[a + b*x^3]
)/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x*Sqrt[a
+ b*x^3]*(6435*c*x + 5005*d*x^2 + 4095*e*x^3 + 3465*f*x^4 + 3003*
g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*c - 4*
a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(182*a^(2/3)*b^(1/3)*e
+ 55*(1 - Sqrt[3])*(13*b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005
*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.92648, antiderivative size = 667, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13bc-4af)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6a\sqrt{a+bx^3}(13bc-4af)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a\sqrt{a+bx^3}(5bd-2ag)}{45b^2}$$

$$+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$$+\frac{6aex\sqrt{a+bx^3}}{55b}+\frac{6afx^2\sqrt{a+bx^3}}{91b}+\frac{2agx^3\sqrt{a+bx^3}}{45b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*e*x*Sqrt[a
+ b*x^3])/(55*b) + (6*a*f*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*g*x^
3*Sqrt[a + b*x^3])/(45*b) + (6*a*(13*b*c - 4*a*f)*Sqrt[a + b*x^3]
)/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x*Sqrt[a
+ b*x^3]*(6435*c*x + 5005*d*x^2 + 4095*e*x^3 + 3465*f*x^4 + 3003*
g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*c - 4*
a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(182*a^(2/3)*b^(1/3)*e
+ 55*(1 - Sqrt[3])*(13*b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005
*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.23078, size = 390, normalized size = 0.58

$$-18i3^{3/4}a^{5/3}b\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(182a^{2/3}\sqrt[3]{-b}e+220af-715bc\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*(-b)^(2/3)*(a + b*x^3)*(-2002*a^2*g + b^2*x^2*(6435*c + 7*x*(7
15*d + 585*e*x + 495*f*x^2 + 429*g*x^3)) + a*b*(5005*d + x*(2457*
e + 11*x*(135*f + 91*g*x)))) - 2970*(-1)^(2/3)*3^(1/4)*a^(5/3)*b*
(13*b*c - 4*a*f)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1
/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*
EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(
1/4)], (-1)^(1/3)] - (18*I)*3^(3/4)*a^(5/3)*b*(-715*b*c + 182*a^(
2/3)*(-b)^(1/3)*e + 220*a*f)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1
/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^
2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/
a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(45045*(-b)^(8/3)*Sqrt[a + b*x^3]
)
```

Maple [B] time = 0.01, size = 1311, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)
```

```
[Out] 2/9*d/b*(b*x^3+a)^(3/2)+c*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2
)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))))+e*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a/b*x*(b*x^3+a)^(1/2)+4
/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)))+f*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a/b*x^2*(b*x^3+a)^(1/2)+8
/91*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))))+g*(2/15*x^6*(b*x^3+a)^(1/2)+2/45*a/b*x
^3*(b*x^3+a)^(1/2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x,x, algorithm="maxima
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^5+fx^4+ex^3+dx^2+cx\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x,x, algorithm="fricas
```

```
[Out] integral((g*x^5 + f*x^4 + e*x^3 + d*x^2 + c*x)*sqrt(b*x^3 + a), x
)
```

Sympy [A] time = 4.89791, size = 223, normalized size = 0.33

$$\frac{\sqrt{a}cx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}ex^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}fx^5\,\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{8}{3}\right)}+d\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for}\ b=0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+g\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*e*x**4*gamma(4/3)*hyper((-
1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqr
t(a)*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(8/3)) + d*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)
), (2*(a + b*x**3)**(3/2)/(9*b), True)) + g*Piecewise((-4*a**2*sq
rt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x
**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

3.436 $\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=639

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}(11bc-2af)-55\left(1-\sqrt{3}\right.\right.}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13bd-4ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6a\sqrt{a+bx^3}(13bd-4ag)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$+\frac{2ae\sqrt{a+bx^3}}{9b}+\frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}$$

```
[Out] (2*a*e*Sqrt[a + b*x^3])/(9*b) + (6*a*f*x*Sqrt[a + b*x^3])/(55*b)
+ (6*a*g*x^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*b*d - 4*a*g)*Sqrt
[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*Sqrt[a + b*x^3]*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*f*x^
4 + 3465*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13
*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c
- 2*a*f) - 55*(1 - Sqrt[3])*a^(1/3)*(13*b*d - 4*a*g))*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.44081, antiderivative size = 639, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}(11bc-2af)-55\left(1-\sqrt{3}\right.\right.}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13bd-4ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6a\sqrt{a+bx^3}(13bd-4ag)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$+\frac{2ae\sqrt{a+bx^3}}{9b}+\frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*e*Sqrt[a + b*x^3])/(9*b) + (6*a*f*x*Sqrt[a + b*x^3])/(55*b)
+ (6*a*g*x^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*b*d - 4*a*g)*Sqrt
[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*Sqrt[a + b*x^3]*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*f*x^
4 + 3465*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13
*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c
- 2*a*f) - 55*(1 - Sqrt[3])*a^(1/3)*(13*b*d - 4*a*g))*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.35775, size = 393, normalized size = 0.62

$$-\frac{18i3^{3/4}a^{4/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\left(143b\left(5\sqrt[3]{a}d+7\sqrt[3]{-b}c\right)-2a\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] -(2*(-b)^(2/3)*(a + b*x^3)*(a*(5005*e + 27*x*(91*f + 55*g*x)) + b
*x*(9009*c + 5*x*(1287*d + 7*x*(143*e + 117*f*x + 99*g*x^2)))) -
2970*(-1)^(2/3)*3^(1/4)*a^(5/3)*(13*b*d - 4*a*g)*Sqrt[((-1)^(5/6)
*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1
/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6)
 - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] + (18*I)*3^(3/
4)*a^(4/3)*(143*b*(7*(-b)^(1/3)*c + 5*a^(1/3)*d) - 2*a*(91*(-b)^(
1/3)*f + 110*a^(1/3)*g))*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*
x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a
^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1
/3)]/3^(1/4)], (-1)^(1/3)])/(45045*(-b)^(5/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.009, size = 1557, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)
```

```
[Out] c*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))+d*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))))+2/9*e/b*(b*x^3+a)^(3/2)+f*(2/11*x^4*(b*x^3+a)^(1/2)+
6/55*a/b*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g*(2/13*x^5*(b*x^3+a)^(1/2)+6/
91*a/b*x^2*(b*x^3+a)^(1/2)+8/91*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 4.50598, size = 194, normalized size = 0.3

$$\frac{\sqrt{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+e\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((-1/2
, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a
)*f*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I
*pi)/a)/(3*gamma(7/3)) + sqrt(a)*g*x**5*gamma(5/3)*hyper((-1/2, 5
/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + e*Piecewi
se((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

3.437 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx$

Optimal. Leaf size=620

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag+77bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{385b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

$$-\frac{2}{3}\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2af\sqrt{a+bx^3}}{9b}+\frac{6agx\sqrt{a+bx^3}}{55b}$$

```
[Out] (2*a*f*Sqrt[a + b*x^3])/(9*b) + (6*a*g*x*Sqrt[a + b*x^3])/(55*b)
+ (6*a*e*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) + (2*Sqrt[a + b*x^3]*(1155*c*x + 693*d*x^2 + 495*e*x^3 +
 385*f*x^4 + 315*g*x^5))/(3465*x) - (2*Sqrt[a]*c*ArcTanh[Sqrt[a +
 b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^
(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*b*d - 55*(1 - Sqrt[3])*a^(1/3)*b^(2
/3)*e - 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(385*b^(4/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3])
```

Rubi [A] time = 1.11399, antiderivative size = 620, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag+77bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{385b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

$$-\frac{2}{3}\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2af\sqrt{a+bx^3}}{9b}+\frac{6agx\sqrt{a+bx^3}}{55b}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (2*a*f*Sqrt[a + b*x^3])/(9*b) + (6*a*g*x*Sqrt[a + b*x^3])/(55*b)
+ (6*a*e*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) + (2*Sqrt[a + b*x^3]*(1155*c*x + 693*d*x^2 + 495*e*x^3 +
 385*f*x^4 + 315*g*x^5))/(3465*x) - (2*Sqrt[a]*c*ArcTanh[Sqrt[a +
 b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^
(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*b*d - 55*(1 - Sqrt[3])*a^(1/3)*b^(2
/3)*e - 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(385*b^(4/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3])
```

Rubi in Sympy [A] time = 142.471, size = 568, normalized size = 0.92

$$-\frac{3\sqrt[4]{3}a^{\frac{4}{3}}e\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{7b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a}c\operatorname{atanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3}+\frac{2af\sqrt{a+bx^3}}{9b}+\frac{6agx\sqrt{a+bx^3}}{55b}+\frac{6ae\sqrt{a+bx^3}}{7b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

$$-\frac{2\cdot 3^{\frac{3}{4}}a\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(55\sqrt[3]{a}b^{\frac{2}{3}}e\left(-\sqrt{3}+1\right)+14ag-77bd\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-\right.}{385b^{\frac{4}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{a+bx^3}\left(\frac{2cx}{3}+\frac{2dx^2}{5}+\frac{2ex^3}{7}+\frac{2fx^4}{9}+\frac{2gx^5}{11}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] -3*3**(1/4)*a**(4/3)*e*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**
(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(
3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_e(asin((-a**(1/3)*(-1 +
sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
 - 4*sqrt(3))/(7*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 2*sq
rt(a)*c*atanh(sqrt(a + b*x**3)/sqrt(a))/3 + 2*a*f*sqrt(a + b*x**3
)/(9*b) + 6*a*g*x*sqrt(a + b*x**3)/(55*b) + 6*a*e*sqrt(a + b*x**3
)/(7*b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)) - 2*3**(3/4)
*a*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(1/3
)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) + b
**(1/3)*x)*(55*a**(1/3)*b**(2/3)*e*(-sqrt(3) + 1) + 14*a*g - 77*b
*d)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(385*b**(4/3)*
sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b
**(1/3)*x)**2)*sqrt(a + b*x**3)) + sqrt(a + b*x**3)*(2*c*x/3 + 2*
d*x**2/5 + 2*e*x**3/7 + 2*f*x**4/9 + 2*g*x**5/11)/x
```

Mathematica [C] time = 3.46682, size = 714, normalized size = 1.15

$$\frac{2\sqrt{a+bx^3}(7a(55f+27gx)+1155bc+bx(693d+5x(99e+7x(11f+9gx))))}{3465b}$$

$$-\frac{2\sqrt{a}\left(-495\sqrt{2}a^{5/6}b^{2/3}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}}\left(-F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)-((-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(1155*b*c + 7*a*(55*f + 27*g*x) + b*x*(693*d +
 5*x*(99*e + 7*x*(11*f + 9*g*x)))))/(3465*b) - (2*Sqrt[a]*(385*b^
(4/3)*c*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a
^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 693*S
qrt[a]*b*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1
)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[
Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
], (-1)^(1/3)] - 126*a^(3/2)*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*S
qrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^
(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3)
)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 495*Sqrt[2]*a^(5/6)*b^(2/3)*
e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-
1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^
(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*Elliptic
E[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^
(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I
*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(
1155*b^(4/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.01, size = 1118, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x)
```

```
[Out] d*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+2/9*f/b*(b*x^3+a)^(3/2)+c*(2/3*(b*x^3
+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+e*(2/7*x^
2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
```

```
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+g*(2/11*x^4*(b*x^3+a)^(1
/2)+6/55*a/b*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x,x, algorithm="maxima
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x,x, algorithm="fricas
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

Sympy [A] time = 6.66672, size = 235, normalized size = 0.38

$$-\frac{2\sqrt{a}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}dx\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}ex^2\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}gx^4\,\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2ac}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2\sqrt{b}cx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + f\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b = 0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*d*x*ga
mma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(4/3)) + sqrt(a)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*g*x**4*gamma
(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(7/3)) + 2*a*c/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*sq
```

```
rt(b)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + f*Piecewise((sqrt(a)*
x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

3.438 $\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

Optimal. Leaf size=638

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(2af+7bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2af+7bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\sqrt{a+bx^3}(2af+7bc)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$-\frac{3c\sqrt{a+bx^3}}{x}-\frac{2}{3}\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2ag\sqrt{a+bx^3}}{9b}$$

```
[Out] (2*a*g*Sqrt[a + b*x^3])/(9*b) - (3*c*Sqrt[a + b*x^3])/x + (3*(7*b
*c + 2*a*f)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(315*c*x + 105*d*x^2 + 63*e*x^3
+ 45*f*x^4 + 35*g*x^5))/(315*x^2) - (2*Sqrt[a]*d*ArcTanh[Sqrt[a +
 b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*c
 + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(14*a^(2/3)*b^(1/3)*
e - 5*(1 - Sqrt[3])*(7*b*c + 2*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*b
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.29417, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(2af+7bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2af+7bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\sqrt{a+bx^3}(2af+7bc)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$-\frac{3c\sqrt{a+bx^3}}{x}-\frac{2}{3}\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2ag\sqrt{a+bx^3}}{9b}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*a*g*Sqrt[a + b*x^3])/(9*b) - (3*c*Sqrt[a + b*x^3])/x + (3*(7*b
*c + 2*a*f)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(315*c*x + 105*d*x^2 + 63*e*x^3
+ 45*f*x^4 + 35*g*x^5))/(315*x^2) - (2*Sqrt[a]*d*ArcTanh[Sqrt[a +
 b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*c
 + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(14*a^(2/3)*b^(1/3)*
e - 5*(1 - Sqrt[3])*(7*b*c + 2*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*b
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 168.208, size = 580, normalized size = 0.91

$$-\frac{3\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{2af}{7}+bc\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{2b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}$$

$$+\frac{3^{\frac{3}{4}}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(14a^{\frac{2}{3}}\sqrt[3]{b}e-\left(-5\sqrt{3}+5\right)(2af+7bc)\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\right.}{35b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}$$

$$-\frac{2\sqrt{a}d\operatorname{atanh}\left(\frac{\sqrt{a+bx^{3}}}{\sqrt{a}}\right)}{3}+\frac{2ag\sqrt{a+bx^{3}}}{9b}-\frac{3c\sqrt{a+bx^{3}}}{x}+\frac{\sqrt{a+bx^{3}}\left(2cx+\frac{2dx^{2}}{3}+\frac{2ex^{3}}{5}+\frac{2fx^{4}}{7}+\frac{2gx^{5}}{9}\right)}{x^{2}}+\frac{3\sqrt{a+bx^{3}}\left(\frac{2af}{7}+bc\right)}{b^{\frac{2}{3}}\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] -3*3**(1/4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2
/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3)
 + 2)*(a**(1/3) + b**(1/3)*x)*(2*a*f/7 + b*c)*elliptic_e(asin((-a
**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b*
*(1/3)*x)), -7 - 4*sqrt(3))/(2*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) +
 b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b
*x**3)) + 3**(3/4)*a**(1/3)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x
+ b**(2/3)*x**2)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(s
qrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(14*a**(2/3)*b**(1/3)*e - (-5
*sqrt(3) + 5)*(2*a*f + 7*b*c))*elliptic_f(asin((-a**(1/3)*(-1 + s
qrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
- 4*sqrt(3))/(35*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 2*sq
rt(a)*d*atanh(sqrt(a + b*x**3)/sqrt(a))/3 + 2*a*g*sqrt(a + b*x**3
)/(9*b) - 3*c*sqrt(a + b*x**3)/x + sqrt(a + b*x**3)*(2*c*x + 2*d*
x**2/3 + 2*e*x**3/5 + 2*f*x**4/7 + 2*g*x**5/9)/x**2 + 3*sqrt(a +
b*x**3)*(2*a*f/7 + b*c)/(b**(2/3)*(a**(1/3)*(1 + sqrt(3)) + b**(1
/3)*x))
```

Mathematica [C] time = 2.70796, size = 810, normalized size = 1.27

$$270\sqrt{2}\sqrt[3]{b}fx\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)-$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]*(a + b*x^3)*(-315*b*c + 70*a*g*x + 2*b*x*(105*d + x*(63*e + 5*x
*(9*f + 7*g*x)))) - 210*Sqrt[a]*b*d*x*Sqrt[(a^(1/3) + (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]] - 378*a*b^(2/3)*e*x*((-1)^(1/3)*a^(1/3) -
b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1
/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + 945*Sqrt[2]*a^(1
/3)*b^(4/3)*c*x*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)
*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(
2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3
^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(
-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1
)^(1/3))]) + 270*Sqrt[2]*a^(4/3)*b^(1/3)*f*x*((-1)^(1/3)*a^(1/3)
- b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*
I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/
6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3)
)]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3
^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(315*b*x*Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3
])
```

Maple [B] time = 0.014, size = 1248, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x)
```

```
[Out] e*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+f*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
```

```
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))))+c*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+d*(2/3*(b*x^3+a
)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+2/9*g/b*(b*
x^3+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2,x, algorithm="fric
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x
)
```

Sympy [A] time = 6.68745, size = 236, normalized size = 0.37

$$\frac{\sqrt{a}c\,\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\,\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}ex\,\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}fx^2\,\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{5}{3}\right)} + \frac{2ad}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2\sqrt{b}dx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + g\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for}\ b = 0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*
x**(3/2)))/3 + sqrt(a)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*f*x**2*gamma(2
/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma
(5/3)) + 2*a*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*sqrt
(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + g*Piecewise((sqrt(a)*x*
*3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2,
x)
```

3.439 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx$

Optimal. Leaf size=640

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(4af+5bc)-10\left(1-\sqrt{3}\right)\sqrt[3]{a}(\right.}{70b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2ag+7bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\sqrt{a+bx^3}(2ag+7bd)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$+\frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}-\frac{2}{3}\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[Out] (3*c*Sqrt[a + b*x^3])/(2*x^2) - (3*d*Sqrt[a + b*x^3])/x + (3*(7*b
*d + 2*a*g)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)) - (2*Sqrt[a + b*x^3]*(105*c*x - 105*d*x^2 - 35*e*x^3
- 21*f*x^4 - 15*g*x^5))/(105*x^3) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a +
 b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*d
 + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(5*b*c + 4*a*f) -
 10*(1 - Sqrt[3])*a^(1/3)*(7*b*d + 2*a*g))*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(70*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.4719, antiderivative size = 640, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(4af+5bc)-10\left(1-\sqrt{3}\right)\sqrt[3]{a}(\right.}{70b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2ag+7bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\sqrt{a+bx^3}(2ag+7bd)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$+\frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}-\frac{2}{3}\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (3*c*Sqrt[a + b*x^3])/(2*x^2) - (3*d*Sqrt[a + b*x^3])/x + (3*(7*b
*d + 2*a*g)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)) - (2*Sqrt[a + b*x^3]*(105*c*x - 105*d*x^2 - 35*e*x^3
- 21*f*x^4 - 15*g*x^5))/(105*x^3) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a +
 b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*d
 + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(5*b*c + 4*a*f) -
 10*(1 - Sqrt[3])*a^(1/3)*(7*b*d + 2*a*g))*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(70*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Mathematica [C]   time = 3.09217, size = 962, normalized size = 1.5

$$\frac{-140\sqrt{a}b^{2/3}e\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)x^2-315b^{4/3}c\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{\cdots}\right.}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]*(a + b*x^3)*(-105*c + 2*x*(-105*d + 70*e*x + 42*f*x^2 +
 30*g*x^3)) - 140*Sqrt[a]*b^(2/3)*e*x^2*Sqrt[(a^(1/3) + (-1)^(2/3
)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]] - 315*b^(4/3)*c*x^2*((-1)^(1/3)*a^(1/3)
- b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^
(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 252*a*b^(1/3)*
f*x^2*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/
3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[
(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-
1)^(1/3)] + 630*Sqrt[2]*a^(1/3)*b*d*x^2*((-1)^(1/3)*a^(1/3) - b^(
1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + S
qrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) -
```

```
(I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) -
 EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4
)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) + 180*Sqrt[2]*a^(4/3)*g*x^2*((
-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3
)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[Ar
cSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3
)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(
1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(210*b
^(2/3)*x^2*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.014, size = 1529, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x)
```

```
[Out] f*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+c*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-(b*x^3
+a)^(1/2)/x-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))))+e*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arc
tanh((b*x^3+a)^(1/2)/a^(1/2)))+g*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a
*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3,x, algorithm="fric
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x
)
```

Sympy [A] time = 7.29367, size = 255, normalized size = 0.4

$$\frac{\sqrt{a}c\,\Gamma\left(-\frac{2}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\,\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}d\,\Gamma\left(-\frac{1}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\,\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}e\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}fx\,\Gamma\left(\frac{1}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}gx^2\,\Gamma\left(\frac{2}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{5}{3}\right)} + \frac{2ae}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2\sqrt{b}ex^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] sqrt(a)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*d*gamma(-1/3)*hyper((-1/
2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*
sqrt(a)*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*f*x*gamma
(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3)) + sqrt(a)*g*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*e/(3*sqrt(b)*x**(3/
2)*sqrt(a/(b*x**3) + 1)) + 2*sqrt(b)*e*x**(3/2)/(3*sqrt(a/(b*x**3
) + 1))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3,
x)
```

3.440 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}\,dx$

Optimal. Leaf size=637

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+5bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}-\frac{(2af+bc)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$+\frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}$$

```
[Out] (c*Sqrt[a + b*x^3])/(3*x^3) + (3*d*Sqrt[a + b*x^3])/(2*x^2) - (3*
e*Sqrt[a + b*x^3])/x + (3*b^(1/3)*e*Sqrt[a + b*x^3])/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x) - (2*Sqrt[a + b*x^3]*(5*c*x + 15*d*x^2 -
15*e*x^3 - 5*f*x^4 - 3*g*x^5))/(15*x^4) - ((b*c + 2*a*f)*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(1/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(5*b*d - 10*(1 - Sqrt[3])
*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.59714, antiderivative size = 637, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+5bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}-\frac{(2af+bc)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$+\frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] (c*Sqrt[a + b*x^3])/(3*x^3) + (3*d*Sqrt[a + b*x^3])/(2*x^2) - (3*
e*Sqrt[a + b*x^3])/x + (3*b^(1/3)*e*Sqrt[a + b*x^3])/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x) - (2*Sqrt[a + b*x^3]*(5*c*x + 15*d*x^2 -
15*e*x^3 - 5*f*x^4 - 3*g*x^5))/(15*x^4) - ((b*c + 2*a*f)*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(1/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(5*b*d - 10*(1 - Sqrt[3])
*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.64752, size = 769, normalized size = 1.21

$$-\frac{3b^{2/3}d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{2\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

$$+\sqrt{a+bx^3}\left(\frac{2f}{3}-\frac{10c+3x\left(5d+10ex-4gx^3\right)}{30x^3}\right)-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{3\sqrt{2}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] Sqrt[a + b*x^3]*((2*f)/3 - (10*c + 3*x*(5*d + 10*e*x - 4*g*x^3))/
(30*x^3)) - (b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) -
(2*Sqrt[a]*f*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*b^(2/3)*d*(
(-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
+ (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]
)/(2*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]*Sqrt[a + b*x^3]) - (6*a*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)
```

```
^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-
1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(5*b^(1/3)*Sqrt[(a^(1/3) + (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) -
 (3*Sqrt[2]*a^(1/3)*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-
1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/
a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcS
in[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/
(-1 + (-1)^(1/3))]))/(Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.015, size = 1114, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] g*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+c*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arc
tanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+d*(-1/2*(b*x^3+a)^(1/2)/x^
2-1/2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
+e*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(2/3*(b*x^3+a)^(1/2)-2/3*
a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4,x, algorithm="fric
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x
)
```

Sympy [A] time = 9.14721, size = 265, normalized size = 0.42

$$\frac{\sqrt{a}d\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\left(\frac{1}{3}\right)} + \frac{\sqrt{a}e\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}f\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}gx\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\left(\frac{4}{3}\right)} + \frac{2af}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3} + 1}} - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^3} + 1}}{3x^{\frac{3}{2}}} + \frac{2\sqrt{b}fx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3} + 1}} - \frac{bc\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] sqrt(a)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*e*gamma(-1/3)*hyper((-1/
2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*
sqrt(a)*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*g*x*gamma
(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3)) + 2*a*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - sqrt
(b)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*f*x**(3/2)/(3
*sqrt(a/(b*x**3) + 1)) - b*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3
*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4,
x)
```

3.441 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

Optimal. Leaf size=694

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}\sqrt[3]{b}e-\left(1-\sqrt{3}\right)(8af+bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{}\right.}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8af+bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}-\frac{3\sqrt{a+bx^3}(8af+bc)}{8ax}$$

$$+\frac{3\sqrt[3]{b}\sqrt{a+bx^3}(8af+bc)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3c\sqrt{a+bx^3}}{20x^4}-\frac{(2ag+bd)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}$$

```
[Out] (3*c*Sqrt[a + b*x^3])/(20*x^4) + (d*Sqrt[a + b*x^3])/(3*x^3) + (3
*e*Sqrt[a + b*x^3])/(2*x^2) - (3*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(
8*a*x) + (3*b^(1/3)*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqrt[a + b*x^3]*(3*c*x + 5*d*x^2
 + 15*e*x^3 - 15*f*x^4 - 5*g*x^5))/(15*x^5) - ((b*d + 2*a*g)*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - S
qrt[3]]*b^(1/3)*(b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*
(4*a^(2/3)*b^(1/3)*e - (1 - Sqrt[3])*(b*c + 8*a*f))*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(8*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.06006, antiderivative size = 694, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}\sqrt[3]{b}e-\left(1-\sqrt{3}\right)(8af+bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8af+bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}-\frac{3\sqrt{a+bx^3}(8af+bc)}{8ax}$$
$$+\frac{3\sqrt[3]{b}\sqrt{a+bx^3}(8af+bc)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3c\sqrt{a+bx^3}}{20x^4}-\frac{(2ag+bd)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (3*c*Sqrt[a + b*x^3])/(20*x^4) + (d*Sqrt[a + b*x^3])/(3*x^3) + (3
*e*Sqrt[a + b*x^3])/(2*x^2) - (3*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(
8*a*x) + (3*b^(1/3)*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqrt[a + b*x^3]*(3*c*x + 5*d*x^2
 + 15*e*x^3 - 15*f*x^4 - 5*g*x^5))/(15*x^5) - ((b*d + 2*a*g)*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - S
qrt[3]]*b^(1/3)*(b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*
(4*a^(2/3)*b^(1/3)*e - (1 - Sqrt[3])*(b*c + 8*a*f))*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(8*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.70394, size = 855, normalized size = 1.23

$$\frac{\sqrt{bx^3+a}\left(-9bcx^3-4a\left(2d+x\left(-4gx^2+6fx+3e\right)\right)x-6ac\right)}{24ax^4}$$

$$-\frac{16g\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)a^{3/2}-72\sqrt{2}\sqrt[3]{b}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\Bigg(-}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-6*a*c - 9*b*c*x^3 - 4*a*x*(2*d + x*(3*e + 6*f*
x - 4*g*x^2))))/(24*a*x^4) - (8*Sqrt[a]*b*d*Sqrt[(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]] + 16*a^(3/2)*g*Sqrt[(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]] + 36*a*b^(2/3)*e*((-1)^(1/3)*a^(1/3)
 - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)
^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 9*Sqrt[2]*a^(
1/3)*b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*
(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqr
t[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2
/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^
(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-
1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)
^(1/3))]) - 72*Sqrt[2]*a^(4/3)*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^
(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 +
 (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I +
Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) -
 (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])
- EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/
4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(24*a*Sqrt[(a^(1/3) + (-1)^(
2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.014, size = 1286, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x)
```

```
[Out] c*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))))+d*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(
1/2)/a^(1/2))/a^(1/2))+e*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/
```

```
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+f*(-(b*x^3+a)^(
1/2)/x-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*
b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))))+g*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh(
(b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5,x, algorithm="fric
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x
)
```

Sympy [A] time = 9.38185, size = 274, normalized size = 0.39

$$\frac{\sqrt{a}c\,\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3}, -\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\,\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}e\,\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3}, -\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\,\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}f\,\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\,\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}g\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{2ag}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2\sqrt{b}gx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*e*gamma(-2/3)*hyper((-
2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
 + sqrt(a)*f*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*g*asinh(sqrt(a)/(sqrt(
b)*x**(3/2)))/3 + 2*a*g/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1))
 - sqrt(b)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*g*x**(
3/2)/(3*sqrt(a/(b*x**3) + 1)) - b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/
2)))/(3*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5,
x)
```

3.442 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx$

Optimal. Leaf size=652

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(2\sqrt[3]{b}(bc-10af)+5\left(1-\sqrt{3}\right)\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8ag+bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)-\frac{3bc\sqrt{a+bx^3}}{20ax^2}+\frac{3\sqrt[3]{b}\sqrt{a+bx^3}(8ag+bd)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{3bd\sqrt{a+bx^3}}{8ax}-\frac{be\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*
Sqrt[a + b*x^3])/60 - (3*b*c*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*d
*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(b*d + 8*a*g)*Sqrt[a + b*x
^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (b*e*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]
*b^(1/3)*(b*d + 8*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(2*b^(1
/3)*(b*c - 10*a*f) + 5*(1 - Sqrt[3])*a^(1/3)*(b*d + 8*a*g))*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.55837, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(2\sqrt[3]{b}(bc-10af)+5\left(1-\sqrt{3}\right)\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8ag+bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)-\frac{3bc\sqrt{a+bx^3}}{20ax^2}+\frac{3\sqrt[3]{b}\sqrt{a+bx^3}(8ag+bd)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{3bd\sqrt{a+bx^3}}{8ax}-\frac{be\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*
Sqrt[a + b*x^3])/60 - (3*b*c*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*d
*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(b*d + 8*a*g)*Sqrt[a + b*x
^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (b*e*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]
*b^(1/3)*(b*d + 8*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(2*b^(1
/3)*(b*c - 10*a*f) + 5*(1 - Sqrt[3])*a^(1/3)*(b*d + 8*a*g))*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.9211, size = 934, normalized size = 1.43

$$-\frac{\sqrt{bx^3+a}\left(9b(2c+5dx)x^3+10a\left(6(f+2gx)x^2+4ex+3d\right)x+24ac\right)}{120ax^5}-\frac{\sqrt[3]{b}\left(-360\sqrt{2}g\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(24*a*c + 9*b*x^3*(2*c + 5*d*x) + 10*a*x*(3*d +
 4*e*x + 6*x^2*(f + 2*g*x))))/(120*a*x^5) - (b^(1/3)*(40*Sqrt[a]*
b^(2/3)*e*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 18*
b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[S
qrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]]
, (-1)^(1/3)] + 180*a*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)
^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3
))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 45*Sqrt[2]*a^(1/3)*b*d*((-1
)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/
3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*
x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcS
in[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/
(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/
3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - 360*Sqr
```

```
t[2]*a^(4/3)*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*
(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqr
t[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2
/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^
(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-
1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)
^(1/3))])))/(120*a*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-
1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.014, size = 1571, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x)
```

```
[Out] c*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x^2+1/20*I/a
*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-
1/4*(b*x^3+a)^(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))))+e*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)
/a^(1/2))/a^(1/2))+f*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g*(-(b*x^3+a)^(1/2)
/x-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6,x, algorithm="fric
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x
)
```

Sympy [A] time = 8.93337, size = 240, normalized size = 0.37

$$\frac{\sqrt{a}c\,\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\,\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}d\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\,\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt{a}f\,\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\,\Gamma\left(\frac{1}{3}\right)}+\frac{\sqrt{a}g\,\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\,\Gamma\left(\frac{2}{3}\right)}-\frac{\sqrt{b}e\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{be\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*d*gamma(-4/3)*hyper((-
4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3
)) + sqrt(a)*f*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp
_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*g*gamma(-1/3)*hyper
((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
 - sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b*e*asinh(sqrt(a
)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6,
x)
```

3.443 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$

Optimal. Leaf size=659

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag+2bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\Bigg|-7-\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\Bigg|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b(bc-4af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{3b^{4/3}e\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+bx^3}}{20ax^2}-\frac{3be\sqrt{a+bx^3}}{8ax}$$

```
[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2
)*Sqrt[a + b*x^3])/60 - (b*c*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*d
*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*e*Sqrt[a + b*x^3])/(8*a*x) +
(3*b^(4/3)*e*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (b*(b*c - 4*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a
^(3/2)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*e*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2
+ Sqrt[3]]*b^(2/3)*(2*b*d + 5*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 2
0*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.86534, antiderivative size = 659, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag+2bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\Bigg|-7-\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\Bigg|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b(bc-4af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{3b^{4/3}e\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+bx^3}}{20ax^2}-\frac{3be\sqrt{a+bx^3}}{8ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2
)*Sqrt[a + b*x^3])/60 - (b*c*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*d
*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*e*Sqrt[a + b*x^3])/(8*a*x) +
(3*b^(4/3)*e*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (b*(b*c - 4*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a
^(3/2)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*e*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2
+ Sqrt[3]]*b^(2/3)*(2*b*d + 5*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 2
0*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.47184, size = 800, normalized size = 1.21

$$b\left(\frac{20bc\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{80}{3}\sqrt{a}f\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)+\frac{12b^{2/3}d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}\cdots}{\left(1+\sqrt[3]{-1}\cdots\right.}}\right.\right.}{\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}}\cdots\right.$$

$$-\frac{\sqrt{bx^3+a}\left(b(10c+9x(2d+5ex))x^3+a\left(20c+2x\left(12d+5x\left(6gx^2+4fx+3e\right)\right)\right)\right)}{120ax^6}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(b*x^3*(10*c + 9*x*(2*d + 5*e*x)) + a*(20*c + 2
*x*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)))))/(120*a*x^6) + (b*((20*
b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (80*Sqrt[a]*f
*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + (12*b^(2/3)*d*((-1)^(1/3)*
a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3
)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(Sqrt[(a^
(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a
+ b*x^3]) - (120*a*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/
3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF
[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a
```

```
^(1/3))]], (-1)^(1/3)])/(b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (30*Sqrt[2]*
a^(1/3)*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/
3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/
3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(
1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)
^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(
1/3))]))/(Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]*Sqrt[a + b*x^3])))/(80*a)
```

Maple [B] time = 0.014, size = 1180, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] c*(-1/6*(b*x^3+a)^(1/2)/x^6-1/12*b/a*(b*x^3+a)^(1/2)/x^3+1/12/a^(
3/2)*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+d*(-1/5*(b*x^3+a)^(1/2
)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x^2+1/20*I/a*b*3^(1/2)*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(-1/4*(b*x^3+a)^(1/2)/x^
4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(-1/3*(b*x^3
+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+g*(
-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7,x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7,x, algorithm="fric
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7, x
)
```

Sympy [A] time = 13.9158, size = 304, normalized size = 0.46

$$\frac{\sqrt{a}d\,\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\,\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}e\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\,\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt{a}g\,\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\,\Gamma\left(\frac{1}{3}\right)}-\frac{ac}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}c}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}f\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{b^{\frac{3}{2}}c}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}+\frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] sqrt(a)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*e*gamma(-4/3)*hyper((-
4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3
)) + sqrt(a)*g*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp
_polar(I*pi)/a)/(3*x**2*gamma(1/3)) - a*c/(6*sqrt(b)*x**(15/2)*sq
rt(a/(b*x**3) + 1)) - sqrt(b)*c/(4*x**(9/2)*sqrt(a/(b*x**3) + 1))
 - sqrt(b)*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/(12*a
*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*f*asinh(sqrt(a)/(sqrt(b)*x**(
3/2)))/(3*sqrt(a)) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12
*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7,
x)
```

3.444 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}\,dx$

Optimal. Leaf size=711

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-\right.}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bc-14af)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b(bd-4ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}-\frac{3b^{4/3}\sqrt{a+bx^3}(5bc-14af)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3b\sqrt{a+bx^3}(5bc-14af)}{112a^2x}$$

$$-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{3be\sqrt{a+bx^3}}{20ax^2}$$

```
[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x
^3)*Sqrt[a + b*x^3])/420 - (3*b*c*Sqrt[a + b*x^3])/(56*a*x^4) - (
b*d*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*e*Sqrt[a + b*x^3])/(20*a*x
^2) + (3*b*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(112*a^2*x) - (3*b^(
4/3)*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(112*a^2*((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)) + (b*(b*d - 4*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*
c - 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(28*a^(2/3)*b^(1/
3)*e - 5*(1 - Sqrt[3])*(5*b*c - 14*a*f))*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
560*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.14917, antiderivative size = 711, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bc-14af)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b(bd-4ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}-\frac{3b^{4/3}\sqrt{a+bx^3}(5bc-14af)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3b\sqrt{a+bx^3}(5bc-14af)}{112a^2x}$$

$$-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{3be\sqrt{a+bx^3}}{20ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x
^3)*Sqrt[a + b*x^3])/420 - (3*b*c*Sqrt[a + b*x^3])/(56*a*x^4) - (
b*d*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*e*Sqrt[a + b*x^3])/(20*a*x
^2) + (3*b*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(112*a^2*x) - (3*b^(
4/3)*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(112*a^2*((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)) + (b*(b*d - 4*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*
c - 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(28*a^(2/3)*b^(1/
3)*e - 5*(1 - Sqrt[3])*(5*b*c - 14*a*f))*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
560*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**8,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.20824, size = 892, normalized size = 1.25

$$\frac{\sqrt{bx^3+a}\left(225b^2cx^6-2ab(45c+7x(10d+9x(2e+5fx)))x^3-4a^2(60c+7x(10d+x(12e+5x(3f+4gx))))\right)}{1680a^2x^7}$$
$$+\frac{b\left(-560g\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)a^{3/2}+630\sqrt{2}\sqrt[3]{b}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right.}{3i+\sqrt{3}}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*(225*b^2*c*x^6 - 2*a*b*x^3*(45*c + 7*x*(10*d + 9
*x*(2*e + 5*f*x))) - 4*a^2*(60*c + 7*x*(10*d + x*(12*e + 5*x*(3*f
 + 4*g*x))))))/(1680*a^2*x^7) + (b*(140*Sqrt[a]*b*d*Sqrt[(a^(1/3)
 + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x
^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 560*a^(3/2)*g*Sqrt[(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*
x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 252*a*b^(2/3)*e*((-1)^(1/
3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/
((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)
^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 225
*Sqrt[2]*a^(1/3)*b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[
((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a
^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((
-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)
/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[Ar
cSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3
)/(-1 + (-1)^(1/3))]) + 630*Sqrt[2]*a^(4/3)*b^(1/3)*f*((-1)^(1/3)
*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1
/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1
/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt
[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (
-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a
^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(1680*a^2*Sqrt
[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqr
t[a + b*x^3])
```

Maple [B] time = 0.013, size = 1376, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x)
```

```
[Out] c*(-1/7*(b*x^3+a)^(1/2)/x^7-3/56*b/a*(b*x^3+a)^(1/2)/x^4+15/112/a
^2*b^2*(b*x^3+a)^(1/2)/x+5/112*I/a^2*b^2*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+d*(-1/6*(b*
x^3+a)^(1/2)/x^6-1/12*b/a*(b*x^3+a)^(1/2)/x^3+1/12/a^(3/2)*b^2*ar
```

```
ctanh((b*x^3+a)^(1/2)/a^(1/2)))+e*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*
b/a*(b*x^3+a)^(1/2)/x^2+1/20*I/a*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+f*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b/a*(
b*x^3+a)^(1/2)/x-1/8*I/a*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+g*(-1/3*(b*x^3+a)^(1/2)/x
^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8,x, algorithm="fric
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x
)
```

Sympy [A] time = 14.7553, size = 308, normalized size = 0.43

$$\frac{\sqrt{a}c\,\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\,\Gamma\left(-\frac{4}{3}\right)}+\frac{\sqrt{a}e\,\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\,\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}f\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\,\Gamma\left(-\frac{1}{3}\right)}-\frac{ad}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}d}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}g\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{b^{\frac{3}{2}}d}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{bg\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}+\frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**8,x)
```

```
[Out] sqrt(a)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*e*gamma(-5/3)*hyper((-
5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3
)) + sqrt(a)*f*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) - a*d/(6*sqrt(b)*x**(15/2)*
sqrt(a/(b*x**3) + 1)) - sqrt(b)*d/(4*x**(9/2)*sqrt(a/(b*x**3) + 1
)) - sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*d/(12
*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*g*asinh(sqrt(a)/(sqrt(b)*x*
*(3/2)))/(3*sqrt(a)) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(
12*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8,
x)
```

3.445 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

Optimal. Leaf size=743

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bd-14ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}-\frac{3b^{4/3}\sqrt{a+bx^3}(5bd-14ag)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{3b\sqrt{a+bx^3}(7bc-16af)}{320a^2x^2}+\frac{3b\sqrt{a+bx^3}(5bd-14ag)}{112a^2x}$$

$$+\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{}\right.\right.}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)-\frac{3bc\sqrt{a+bx^3}}{80ax^5}-\frac{3bd\sqrt{a+bx^3}}{56ax^4}-\frac{be\sqrt{a+bx^3}}{12ax^3}$$

```
[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g
)/x^4)*Sqrt[a + b*x^3])/840 - (3*b*c*Sqrt[a + b*x^3])/(80*a*x^5)
- (3*b*d*Sqrt[a + b*x^3])/(56*a*x^4) - (b*e*Sqrt[a + b*x^3])/(12*
a*x^3) + (3*b*(7*b*c - 16*a*f)*Sqrt[a + b*x^3])/(320*a^2*x^2) + (
3*b*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5
*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) + (b^2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/
2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*d - 14*a*g)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(
3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(7*b*c - 16*a*f) + 20*(
1 - Sqrt[3])*a^(1/3)*(5*b*d - 14*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(22
40*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.46992, antiderivative size = 743, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bd-14ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}-\frac{3b^{4/3}\sqrt{a+bx^3}(5bd-14ag)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{3b\sqrt{a+bx^3}(7bc-16af)}{320a^2x^2}+\frac{3b\sqrt{a+bx^3}(5bd-14ag)}{112a^2x}$$
$$+\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{}\right.\right.}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)-\frac{3bc\sqrt{a+bx^3}}{80ax^5}-\frac{3bd\sqrt{a+bx^3}}{56ax^4}-\frac{be\sqrt{a+bx^3}}{12ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g
)/x^4)*Sqrt[a + b*x^3])/840 - (3*b*c*Sqrt[a + b*x^3])/(80*a*x^5)
- (3*b*d*Sqrt[a + b*x^3])/(56*a*x^4) - (b*e*Sqrt[a + b*x^3])/(12*
a*x^3) + (3*b*(7*b*c - 16*a*f)*Sqrt[a + b*x^3])/(320*a^2*x^2) + (
3*b*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5
*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) + (b^2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/
2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*d - 14*a*g)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(
3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(7*b*c - 16*a*f) + 20*(
1 - Sqrt[3])*a^(1/3)*(5*b*d - 14*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(22
40*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**9,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.14912, size = 979, normalized size = 1.32

$$\left(2520\sqrt{2}g\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)-F\right.\right.$$

$$+\frac{\sqrt{bx^3+a}\left(9b^2(49c+100dx)x^6-4ab(63c+2x(45d+7x(10e+9x(2f+5gx))))x^3-8a^2(105c+2x(60d+7x(10e+3x(4f+\right.}{6720a^2x^8}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^3]*(9*b^2*x^6*(49*c + 100*d*x) - 4*a*b*x^3*(63*c +
2*x*(45*d + 7*x*(10*e + 9*x*(2*f + 5*g*x)))) - 8*a^2*(105*c + 2*x
*(60*d + 7*x*(10*e + 3*x*(4*f + 5*g*x))))))/(6720*a^2*x^8) + (b^(
4/3)*(560*Sqrt[a]*b^(2/3)*e*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x
^3]/Sqrt[a]] - 441*b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqr
t[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1
/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]
*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]], (-1)^(1/3)] + 1008*a*b^(1/3)*f*((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1
+ (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 900*Sqr
t[2]*a^(1/3)*b*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3
)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*S
qrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^
(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/
3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[
(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-
1)^(1/3))]) + 2520*Sqrt[2]*a^(4/3)*g*((-1)^(1/3)*a^(1/3) - b^(1/3
)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt
[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*
b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - El
lipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)],
 (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(6720*a^2*Sqrt[(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.016, size = 1679, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x)
```

```
[Out] c*(-1/8*(b*x^3+a)^(1/2)/x^8-3/80*b/a*(b*x^3+a)^(1/2)/x^5+21/320/a
^2*b^2*(b*x^3+a)^(1/2)/x^2-7/320*I/a^2*b^2*3^(1/2)*(-a*b^2)^(1/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1/7*(b*x^3+a)^(1/2)/x^7-3
/56*b/a*(b*x^3+a)^(1/2)/x^4+15/112/a^2*b^2*(b*x^3+a)^(1/2)/x+5/11
2*I/a^2*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
```

```
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))))+e*(-1/6*(b*x^3+a)^(1/2)/x^6-1/12*b/a*(b*
x^3+a)^(1/2)/x^3+1/12/a^(3/2)*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2)
))+f*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x^2+1/20*
I/a*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g
*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9,x, algorithm="fric
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x
)
```

Sympy [A] time = 14.3074, size = 304, normalized size = 0.41

$$\frac{\sqrt{a}c\,\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\,\Gamma\left(-\frac{5}{3}\right)}+\frac{\sqrt{a}d\,\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\,\Gamma\left(-\frac{4}{3}\right)}+\frac{\sqrt{a}f\,\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\,\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}g\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\,\Gamma\left(-\frac{1}{3}\right)}-\frac{ae}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}e}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{b^{\frac{3}{2}}e}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**9,x)
```

```
[Out] sqrt(a)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**8*gamma(-5/3)) + sqrt(a)*d*gamma(-7/3)*hyper((-
7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3
)) + sqrt(a)*f*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*g*gamma(-4/3)*hyp
er((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma
(-1/3)) - a*e/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b
)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*e/(12*a*x**(3/2)
*sqrt(a/(b*x**3) + 1)) + b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))
/(12*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9,
x)
```

3.446 $\int x^3\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=791

$$\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bd-2ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{216a^3\sqrt{a+bx^3}(5bd-2ag)}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4a^3e\sqrt{a+bx^3}}{105b^2}+\frac{54a^2x\sqrt{a+bx^3}(23bc-8af)}{21505b^2}$$

$$+\frac{54a^2x^2\sqrt{a+bx^3}(5bd-2ag)}{8645b^2}+\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}+\frac{54a^2fx^4\sqrt{a+bx^3}}{4301b}+\frac{54a^2gx^5\sqrt{a+bx^3}}{6175b}$$

$$-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(23bc-8af)-8602\right.}{37182145b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$$+\frac{2ax^3\sqrt{a+bx^3}\left(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5\right)}{185910725}$$

```
[Out] (-4*a^3*e*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*c - 8*a*f)*x
*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*(5*b*d - 2*a*g)*x^2*Sqrt[
a + b*x^3])/(8645*b^2) + (2*a^2*e*x^3*Sqrt[a + b*x^3])/(105*b) +
(54*a^2*f*x^4*Sqrt[a + b*x^3])/(4301*b) + (54*a^2*g*x^5*Sqrt[a +
b*x^3])/(6175*b) - (216*a^3*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(864
5*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^3*(a + b*x^
3)^(3/2)*(229425*c*x + 205275*d*x^2 + 185725*e*x^3 + 169575*f*x^4
 + 156009*g*x^5))/3900225 + (2*a*x^3*Sqrt[a + b*x^3]*(8947575*c*x
 + 6774075*d*x^2 + 5311735*e*x^3 + 4279275*f*x^4 + 3522519*g*x^5)
)/185910725 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*b*d - 2*
a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]) - (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*(23*b*c -
8*a*f) - 8602*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 2*a*g))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(37182145*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 3.64592, antiderivative size = 791, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$

$$\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bd-2ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{216a^3\sqrt{a+bx^3}(5bd-2ag)}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4a^3e\sqrt{a+bx^3}}{105b^2}+\frac{54a^2x\sqrt{a+bx^3}(23bc-8af)}{21505b^2}$$

$$+\frac{54a^2x^2\sqrt{a+bx^3}(5bd-2ag)}{8645b^2}+\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}+\frac{54a^2fx^4\sqrt{a+bx^3}}{4301b}+\frac{54a^2gx^5\sqrt{a+bx^3}}{6175b}$$

$$-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(23bc-8af)-8602\right.}{37182145b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$$+\frac{2ax^3\sqrt{a+bx^3}\left(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5\right)}{185910725}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (-4*a^3*e*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*c - 8*a*f)*x
*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*(5*b*d - 2*a*g)*x^2*Sqrt[
a + b*x^3])/(8645*b^2) + (2*a^2*e*x^3*Sqrt[a + b*x^3])/(105*b) +
(54*a^2*f*x^4*Sqrt[a + b*x^3])/(4301*b) + (54*a^2*g*x^5*Sqrt[a +
b*x^3])/(6175*b) - (216*a^3*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(864
5*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^3*(a + b*x^
3)^(3/2)*(229425*c*x + 205275*d*x^2 + 185725*e*x^3 + 169575*f*x^4
 + 156009*g*x^5))/3900225 + (2*a*x^3*Sqrt[a + b*x^3]*(8947575*c*x
 + 6774075*d*x^2 + 5311735*e*x^3 + 4279275*f*x^4 + 3522519*g*x^5)
)/185910725 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*b*d - 2*
a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]) - (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*(23*b*c -
8*a*f) - 8602*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 2*a*g))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(37182145*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.47341, size = 466, normalized size = 0.59

$$-540i3^{3/4}a^{10/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\left(-17204a^{4/3}g+43010\sqrt[3]{a}bd\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*(-b)^(2/3)*(a + b*x^3)*(-10*a^3*(1062347*e + 81*x*(6916*f + 43
01*g*x)) + a^2*b*x*(16105635*c + x*(8709525*d + 5311735*e*x + 350
1225*f*x^2 + 2438667*g*x^3)) + 143*b^3*x^7*(229425*c + 17*x*(1207
5*d + 19*x*(575*e + 525*f*x + 483*g*x^2))) + 2*a*b^2*x^4*(2982525
0*c + 11*x*(2258025*d + 13*x*(148580*e + 21*x*(6175*f + 5474*g*x)
)))) + 13935240*(-1)^(2/3)*3^(1/4)*a^(11/3)*(5*b*d - 2*a*g)*Sqrt[
((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1
/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[
-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] - (
540*I)*3^(3/4)*a^(10/3)*(39767*(-b)^(1/3)*b*c + 43010*a^(1/3)*b*d
 - 13832*a*(-b)^(1/3)*f - 17204*a^(4/3)*g)*Sqrt[((-1)^(5/6)*(-a^(
1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) +
((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*
(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(557732175*(-b)^(8/
3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.029, size = 1764, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] d*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/172
9/b*a^2*x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+c*(2/1
7*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*a^2
*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105
*a*x^6*(b*x^3+a)^(1/2)+2/105/b*a^2*x^3*(b*x^3+a)^(1/2)-4/105*a^3/
b^2*(b*x^3+a)^(1/2))+f*(2/23*b*x^10*(b*x^3+a)^(1/2)+52/391*a*x^7*
(b*x^3+a)^(1/2)+54/4301/b*a^2*x^4*(b*x^3+a)^(1/2)-432/21505*a^3/b
^2*x*(b*x^3+a)^(1/2)-288/21505*I*a^4/b^3*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
```

```
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g*(2/25*b*x^11*(b*x^3+a)^(1/2)+
56/475*a*x^8*(b*x^3+a)^(1/2)+54/6175/b*a^2*x^5*(b*x^3+a)^(1/2)-10
8/8645*a^3/b^2*x^2*(b*x^3+a)^(1/2)-144/8645*I*a^4/b^3*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3,x, algorithm="ma
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bgx^{10} + bfx^9 + bex^8 + (bd + ag)x^7 + aex^5 + (bc + af)x^6 + adx^4 + acx^3\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3,x, algorithm="fr
```

```
[Out] integral((b*g*x^10 + b*f*x^9 + b*e*x^8 + (b*d + a*g)*x^7 + a*e*x^
5 + (b*c + a*f)*x^6 + a*d*x^4 + a*c*x^3)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 14.6178, size = 512, normalized size = 0.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*d*x**5*gamma(5/3)*hyper(
(-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a
**(3/2)*f*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(10/3)) + a**(3/2)*g*x**8*gamma(8/3)*hyper
((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
+ sqrt(a)*b*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*d*x**8*gamma(8/3)*
```

```
hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11
/3)) + sqrt(a)*b*f*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/3)) + sqrt(a)*b*g*x**11*ga
mma(11/3)*hyper((-1/2, 11/3), (14/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(14/3)) + a*e*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**
2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/1
5, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*e*Piecewise((16*a**3*sq
rt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x**3)/(315*b**
2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**3)/
21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3,x, algorithm="gi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3
, x)
```

3.447 $\int x^2 \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=742

$$\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{216a^3e\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2\sqrt{a+bx^3}(7bc-2af)}{105b^2}$$
$$+\frac{54a^2x\sqrt{a+bx^3}(23bd-8ag)}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}$$
$$+\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-1729(23bd-8ag)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{37182145b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$
$$+\frac{2ax^2\sqrt{a+bx^3}\left(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5\right)}{111546435}$$

```
[Out] (2*a^2*(7*b*c - 2*a*f)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b
*d - 8*a*g)*x*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*e*x^2*Sqrt[a
 + b*x^3])/(1729*b) + (2*a^2*f*x^3*Sqrt[a + b*x^3])/(105*b) + (54
*a^2*g*x^4*Sqrt[a + b*x^3])/(4301*b) - (216*a^3*e*Sqrt[a + b*x^3]
)/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*(a
+ b*x^3)^(3/2)*(52003*c*x + 45885*d*x^2 + 41055*e*x^3 + 37145*f*x
^4 + 33915*g*x^5))/780045 + (2*a*x^2*Sqrt[a + b*x^3]*(7436429*c*x
 + 5368545*d*x^2 + 4064445*e*x^3 + 3187041*f*x^4 + 2567565*g*x^5)
)/111546435 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*e*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (36*3^(
3/4)*Sqrt[2 + Sqrt[3]]*a^3*(43010*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e
 - 1729*(23*b*d - 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(37182145*b^(7/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.77342, antiderivative size = 742, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$

$$\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{216a^3e\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2\sqrt{a+bx^3}(7bc-2af)}{105b^2}$$
$$+\frac{54a^2x\sqrt{a+bx^3}(23bd-8ag)}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}$$
$$+\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-1729(23bd-8ag)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{37182145b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$
$$+\frac{2ax^2\sqrt{a+bx^3}\left(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5\right)}{111546435}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a^2*(7*b*c - 2*a*f)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b
*d - 8*a*g)*x*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*e*x^2*Sqrt[a
 + b*x^3])/(1729*b) + (2*a^2*f*x^3*Sqrt[a + b*x^3])/(105*b) + (54
*a^2*g*x^4*Sqrt[a + b*x^3])/(4301*b) - (216*a^3*e*Sqrt[a + b*x^3]
)/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*(a
+ b*x^3)^(3/2)*(52003*c*x + 45885*d*x^2 + 41055*e*x^3 + 37145*f*x
^4 + 33915*g*x^5))/780045 + (2*a*x^2*Sqrt[a + b*x^3]*(7436429*c*x
 + 5368545*d*x^2 + 4064445*e*x^3 + 3187041*f*x^4 + 2567565*g*x^5)
)/111546435 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*e*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (36*3^(
3/4)*Sqrt[2 + Sqrt[3]]*a^3*(43010*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e
 - 1729*(23*b*d - 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(37182145*b^(7/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.09175, size = 436, normalized size = 0.59

$$-108i3^{3/4}a^{10/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(43010\sqrt[3]{a}be-13832a\sqrt[3]{-b}g+39767\sqrt[3]{-b}bd\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}}}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*(-b)^(2/3)*(a + b*x^3)*(-494*a^3*(4301*f + 2268*g*x) + 143*b^3
*x^6*(52003*c + 5*x*(9177*d + 17*x*(483*e + 437*f*x + 399*g*x^2))
) + a^2*b*(7436429*c + x*(3221127*d + x*(1741905*e + 1062347*f*x
+ 700245*g*x^2))) + 2*a*b^2*x^3*(7436429*c + x*(5965050*d + 11*x*
(451605*e + 247*x*(1564*f + 1365*g*x))))) + 13935240*(-1)^(2/3)*3
^(1/4)*a^(11/3)*b*e*Sqrt[(-1)^(5/6)*(-1 + ((-b)^(1/3)*x)/a^(1/3))
]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*Ell
ipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4
)], (-1)^(1/3)] - (108*I)*3^(3/4)*a^(10/3)*(39767*(-b)^(1/3)*b*d
+ 43010*a^(1/3)*b*e - 13832*a*(-b)^(1/3)*g)*Sqrt[((-1)^(5/6)*(-a^
(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) +
 ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I
*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(111546435*(-b)^(8
/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.011, size = 1269, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] d*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935
/b*a^2*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+2/15*c/b*(b*x^3+a)^(5/2)+e*(2/
19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729/b*a
^2*x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(2/21*b*x
^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/2)+2/105/b*a^2*x^3*(
b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))+g*(2/23*b*x^10*(b*x
^3+a)^(1/2)+52/391*a*x^7*(b*x^3+a)^(1/2)+54/4301/b*a^2*x^4*(b*x^3
+a)^(1/2)-432/21505*a^3/b^2*x*(b*x^3+a)^(1/2)-288/21505*I*a^4/b^3
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
```

```
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}c}{15\,b}+\int\left(bgx^9+bfx^8+bex^7+afx^5+(bd+ag)x^6+aex^4+adx^3\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^2,x, algorithm="ma
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*c/b + integrate((b*g*x^9 + b*f*x^8 + b*e*x
^7 + a*f*x^5 + (b*d + a*g)*x^6 + a*e*x^4 + a*d*x^3)*sqrt(b*x^3 +
a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bgx^9+bfx^8+bex^7+(bd+ag)x^6+aex^4+(bc+af)x^5+adx^3+acx^2\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^2,x, algorithm="fr
```

```
[Out] integral((b*g*x^9 + b*f*x^8 + b*e*x^7 + (b*d + a*g)*x^6 + a*e*x^4
 + (b*c + a*f)*x^5 + a*d*x^3 + a*c*x^2)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 12.8972, size = 525, normalized size = 0.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*e*x**5*gamma(5/3)*hyper(
(-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a
**(3/2)*g*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*d*x**7*gamma(7/3)*hype
r((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
 + sqrt(a)*b*e*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*b*g*x**10*gamma(10/
3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(13/3)) + a*c*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x*
*3)**(3/2)/(9*b), True)) + a*f*Piecewise((-4*a**2*sqrt(a + b*x**3
)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a +
b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*c*Piecewise((-
4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45
*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, Tru
e)) + b*f*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2
*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(10
5*b) + 2*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, Tr
ue))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^2,x, algorithm="gi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^2
, x)
```

3.448 $\int x \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=723

$$-\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bc-4af)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{54a^2\sqrt{a+bx^3}(19bc-4af)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2\sqrt{a+bx^3}(7bd-2ag)}{105b^2}+\frac{54a^2ex\sqrt{a+bx^3}}{935b}+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b}$$

$$+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b}+\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$+\frac{2ax\sqrt{a+bx^3}\left(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5\right)}{4849845}$$

```
[Out] (2*a^2*(7*b*d - 2*a*g)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*e*x*S
qrt[a + b*x^3])/(935*b) + (54*a^2*f*x^2*Sqrt[a + b*x^3])/(1729*b)
 + (2*a^2*g*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*(19*b*c - 4*a*
f)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)) + (2*x*(a + b*x^3)^(3/2)*(33915*c*x + 29393*d*x^2 + 25935*e
*x^3 + 23205*f*x^4 + 20995*g*x^5))/440895 + (2*a*x*Sqrt[a + b*x^3
]*(479655*c*x + 323323*d*x^2 + 233415*e*x^3 + 176715*f*x^4 + 1385
67*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*
c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(3458*a^(2/3)*
b^(1/3)*e + 935*(1 - Sqrt[3])*(19*b*c - 4*a*f))*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.35221, antiderivative size = 723, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$

$$-\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bc-4af)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{54a^2\sqrt{a+bx^3}(19bc-4af)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2\sqrt{a+bx^3}(7bd-2ag)}{105b^2}+\frac{54a^2ex\sqrt{a+bx^3}}{935b}+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b}$$

$$+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b}+\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$+\frac{2ax\sqrt{a+bx^3}\left(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5\right)}{4849845}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a^2*(7*b*d - 2*a*g)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*e*x*S
qrt[a + b*x^3])/(935*b) + (54*a^2*f*x^2*Sqrt[a + b*x^3])/(1729*b)
 + (2*a^2*g*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*(19*b*c - 4*a*
f)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)) + (2*x*(a + b*x^3)^(3/2)*(33915*c*x + 29393*d*x^2 + 25935*e
*x^3 + 23205*f*x^4 + 20995*g*x^5))/440895 + (2*a*x*Sqrt[a + b*x^3
]*(479655*c*x + 323323*d*x^2 + 233415*e*x^3 + 176715*f*x^4 + 1385
67*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*
c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(3458*a^(2/3)*
b^(1/3)*e + 935*(1 - Sqrt[3])*(19*b*c - 4*a*f))*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.17908, size = 429, normalized size = 0.59

$$-54i3^{3/4}a^{8/3}b\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}\left(3458a^{2/3}\sqrt[3]{-b}e+3740af-17765bc\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/}}}{\sqrt[4]{3}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*(-b)^(2/3)*(a + b*x^3)*(-92378*a^3*g + a^2*b*(323323*d + x*(14
0049*e + 187*x*(405*f + 247*g*x))) + 11*b^3*x^5*(33915*c + 13*x*(
2261*d + 5*x*(399*e + 357*f*x + 323*g*x^2))) + 2*a*b^2*x^2*(42636
0*c + x*(323323*d + x*(259350*e + 215985*f*x + 184756*g*x^2)))) -
 151470*(-1)^(2/3)*3^(1/4)*a^(8/3)*b*(19*b*c - 4*a*f)*Sqrt[((-1)^
(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3)*x)
/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticE[ArcSin[Sqrt[-(-1)^
(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)] - (54*I)*
3^(3/4)*a^(8/3)*b*(-17765*b*c + 3458*a^(2/3)*(-b)^(1/3)*e + 3740*
a*f)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1
+ ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[Ar
cSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^
(1/3)])/(4849845*(-b)^(8/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.01, size = 1383, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 2/15*d/b*(b*x^3+a)^(5/2)+c*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^
2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(
b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+e*(2/17*b*x^7*(b*x^3
+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*a^2*x*(b*x^3+a)^(
1/2)+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)))+f*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+
a)^(1/2)+54/1729/b*a^2*x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
```

```
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))))+g*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/
2)+2/105/b*a^2*x^3*(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bgx^8 + bfx^7 + bex^6 + (bd + ag)x^5 + aex^3 + (bc + af)x^4 + adx^2 + acx\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x,x, algorithm="fric
```

```
[Out] integral((b*g*x^8 + b*f*x^7 + b*e*x^6 + (b*d + a*g)*x^5 + a*e*x^3
 + (b*c + a*f)*x^4 + a*d*x^2 + a*c*x)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 11.3712, size = 525, normalized size = 0.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(5/3)) + a**(3/2)*e*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a
**(3/2)*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b*c*x**5*gamma(5/3)*hyper(
(-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + s
qrt(a)*b*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*f*x**8*gamma(8/3)*hyp
er((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)
) + a*d*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3
/2)/(9*b), True)) + a*g*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b
**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)
/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*d*Piecewise((-4*a**2*
sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2
*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b
*g*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*s
qrt(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) +
2*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x,
x)
```

3.449 $\int \left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=694

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bd-4ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{54a^2\sqrt{a+bx^3}(19bd-4ag)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2e\sqrt{a+bx^3}}{15b}+\frac{54a^2fx\sqrt{a+bx^3}}{935b}+\frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}$$

$$+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-2af)-935\left(\right.\right.}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2a\sqrt{a+bx^3}\left(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5\right)}{4849845}$$

$$+\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

```
[Out] (2*a^2*e*Sqrt[a + b*x^3])/(15*b) + (54*a^2*f*x*Sqrt[a + b*x^3])/(
935*b) + (54*a^2*g*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*b*
d - 4*a*g)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)) + (2*(a + b*x^3)^(3/2)*(62985*c*x + 53295*d*x^2 + 4
6189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/692835 + (2*a*Sqrt[a + b
*x^3]*(793611*c*x + 479655*d*x^2 + 323323*e*x^3 + 233415*f*x^4 +
176715*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(1
9*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1729*b^(1/3)*
(17*b*c - 2*a*f) - 935*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 4*a*g))*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.75716, antiderivative size = 694, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bd-4ag)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{54a^2\sqrt{a+bx^3}(19bd-4ag)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2e\sqrt{a+bx^3}}{15b}+\frac{54a^2fx\sqrt{a+bx^3}}{935b}+\frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}$$
$$+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-2af)-935\left(\right.\right.}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2a\sqrt{a+bx^3}\left(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5\right)}{4849845}$$
$$+\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a^2*e*Sqrt[a + b*x^3])/(15*b) + (54*a^2*f*x*Sqrt[a + b*x^3])/(
935*b) + (54*a^2*g*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*b*
d - 4*a*g)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)) + (2*(a + b*x^3)^(3/2)*(62985*c*x + 53295*d*x^2 + 4
6189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/692835 + (2*a*Sqrt[a + b
*x^3]*(793611*c*x + 479655*d*x^2 + 323323*e*x^3 + 233415*f*x^4 +
176715*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(1
9*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1729*b^(1/3)*
(17*b*c - 2*a*f) - 935*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 4*a*g))*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.4873, size = 429, normalized size = 0.62

$$-\frac{54i3^{3/4}a^{7/3}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-b}x-\sqrt[3]{a}\right)}{\sqrt[3]{a}}}\sqrt{\frac{(-b)^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-b}x}{\sqrt[3]{a}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)\left(-3740a^{4/3}g+323b\left(55\sqrt[3]{a}d+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] -(2*(-b)^(2/3)*(a + b*x^3)*(a^2*(323323*e + 81*x*(1729*f + 935*g*
x)) + 7*b^2*x^4*(62985*c + 11*x*(4845*d + 13*x*(323*e + 285*f*x +
 255*g*x^2))) + 2*a*b*x*(617253*c + x*(426360*d + 7*x*(46189*e +
37050*f*x + 30855*g*x^2)))) - 151470*(-1)^(2/3)*3^(1/4)*a^(8/3)*(
19*b*d - 4*a*g)*Sqrt[((-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/
3)]*Sqrt[1 + ((-b)^(1/3)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*E
llipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1
/4)], (-1)^(1/3)] + (54*I)*3^(3/4)*a^(7/3)*(323*b*(91*(-b)^(1/3)*
c + 55*a^(1/3)*d) - 3458*a*(-b)^(1/3)*f - 3740*a^(4/3)*g)*Sqrt[((
-1)^(5/6)*(-a^(1/3) + (-b)^(1/3)*x))/a^(1/3)]*Sqrt[1 + ((-b)^(1/3
)*x)/a^(1/3) + ((-b)^(2/3)*x^2)/a^(2/3)]*EllipticF[ArcSin[Sqrt[-(
-1)^(5/6) - (I*(-b)^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)])/(484
9845*(-b)^(5/3)*Sqrt[a + b*x^3])
```

Maple [B] time = 0.008, size = 1629, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] c*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*
(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2))))+2/15*e/b*(b*x^3+a)^(5/2)+f*(2/17*b*x^7*(b*x^3+a
)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*a^2*x*(b*x^3+a)^(1/
2)+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)))+g*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)
```

```
^(1/2)+54/1729/b*a^2*x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2
)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2),x, algorithm="maxima
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2),x, algorithm="fricas
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 9.16609, size = 444, normalized size = 0.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1
/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + a**(
3/2)*f*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*g*x**5*gamma(5/3)*hyper((-1/
2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(
a)*b*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**5*gamma(5/3)*hyper((-1
/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt
(a)*b*f*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*g*x**8*gamma(8/3)*hyper(
(-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) +
 a*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)
/(9*b), True)) + b*e*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2
) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15
```

```
, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)
```

3.450 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx$

Optimal. Leaf size=676

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$18\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\right.$

$$-\frac{2}{3}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{54a^2e\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2f\sqrt{a+bx^3}}{15b}+\frac{54a^2gx\sqrt{a+bx^3}}{935b}+$$

```
[Out] (2*a^2*f*Sqrt[a + b*x^3])/(15*b) + (54*a^2*g*x*Sqrt[a + b*x^3])/(
935*b) + (54*a^2*e*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)) + (2*(a + b*x^3)^(3/2)*(12155*c*x + 9945*d*x^
2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x^5))/(109395*x) + (2*a*Sqrt
[a + b*x^3]*(85085*c*x + 41769*d*x^2 + 25245*e*x^3 + 17017*f*x^4
+ 12285*g*x^5))/(255255*x) - (2*a^(3/2)*c*ArcTanh[Sqrt[a + b*x^3]
/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (18*3^(3/4
)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 935*(1 - Sqrt[3])*a^(1/3)*b^(
2/3)*e - 182*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(85085*b^(4/3)*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.33436, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$18\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\right.$

$$-\frac{2}{3}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{54a^2e\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2f\sqrt{a+bx^3}}{15b}+\frac{54a^2gx\sqrt{a+bx^3}}{935b}+$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (2*a^2*f*Sqrt[a + b*x^3])/(15*b) + (54*a^2*g*x*Sqrt[a + b*x^3])/(
935*b) + (54*a^2*e*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)) + (2*(a + b*x^3)^(3/2)*(12155*c*x + 9945*d*x^
2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x^5))/(109395*x) + (2*a*Sqrt
```

```
[a + b*x^3]*(85085*c*x + 41769*d*x^2 + 25245*e*x^3 + 17017*f*x^4
+ 12285*g*x^5))/(255255*x) - (2*a^(3/2)*c*ArcTanh[Sqrt[a + b*x^3]
/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (18*3^(3/4
)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 935*(1 - Sqrt[3])*a^(1/3)*b^(
2/3)*e - 182*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(85085*b^(4/3)*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.4264, size = 753, normalized size = 1.11

$$\frac{2\sqrt{a+bx^3}\left(273a^2(187f+81gx)+2ab\left(170170c+97461dx+67320ex^2+51051fx^3+40950gx^4\right)+7b^2x^3\left(12155c+9945dx\right.\right.}{765765b}$$

$$-\frac{2a^{3/2}\left(-75735\sqrt{2}a^{5/6}b^{2/3}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{bx}\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{bx}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{bx}}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}}\left(-F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{bx}}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(273*a^2*(187*f + 81*g*x) + 2*a*b*(170170*c +
97461*d*x + 67320*e*x^2 + 51051*f*x^3 + 40950*g*x^4) + 7*b^2*x^3*
(12155*c + 9945*d*x + 33*x^2*(255*e + 13*x*(17*f + 15*g*x)))))/(7
65765*b) - (2*a^(3/2)*(85085*b^(4/3)*c*Sqrt[(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]] + 125307*Sqrt[a]*b*d*((-1)^(1/3)*a^(1/3)
- b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^
(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 14742*a^(3/2)*
g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(
1/3)] - 75735*Sqrt[2]*a^(5/6)*b^(2/3)*e*((-1)^(1/3)*a^(1/3) - b^(
1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + S
qrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) -
(I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) -
 EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4
)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(255255*b^(4/3)*Sqrt[(a^(1/3
```

```
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*
x^3])
```

Maple [B] time = 0.013, size = 1188, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] d*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+2/
15*f/b*(b*x^3+a)^(5/2)+c*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+
a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+e*(2/13*b*
x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))))+g*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1
/2)+54/935/b*a^2*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-a*b
^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x,x, algorithm="maxi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x,x, algorithm="fric
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x, x)
```

Sympy [A] time = 14.6858, size = 473, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] -2*a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*d*x*
gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) + a**(3/2)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + a**(3/2)*g*x**4*g
amma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(7/3)) + sqrt(a)*b*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*e*x**5*
gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(8/3)) + sqrt(a)*b*g*x**7*gamma(7/3)*hyper((-1/2, 7/3), (1
0/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**2*c/(3*sq
rt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*c*x**(3/2)/(3*
sqrt(a/(b*x**3) + 1)) + a*f*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)),
 (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*c*Piecewise((sqrt(a)*x*
*3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*f*Piece
wise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x*
*3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6
/6, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x,x, algorithm="giac
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x,
x)
```

3.451 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx$

Optimal. Leaf size=692

$$\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(182a^{2/3}\sqrt[3]{b}e-55\left(1-\sqrt{3}\right)(2af+13bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2af+13bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2}{3}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2a^2g\sqrt{a+bx^3}}{15b}+\frac{27a\sqrt{a+bx^3}(2af+13bc)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+\right.}{15015x^2}$$

```
[Out] (2*a^2*g*Sqrt[a + b*x^3])/(15*b) - (27*a*c*Sqrt[a + b*x^3])/(7*x)
 + (27*a*(13*b*c + 2*a*f)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)) + (2*a*Sqrt[a + b*x^3]*(19305*c*x + 50
05*d*x^2 + 2457*e*x^3 + 1485*f*x^4 + 1001*g*x^5))/(15015*x^2) + (
2*(a + b*x^3)^(3/2)*(6435*c*x + 5005*d*x^2 + 4095*e*x^3 + 3465*f*
x^4 + 3003*g*x^5))/(45045*x^2) - (2*a^(3/2)*d*ArcTanh[Sqrt[a + b*
x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*c
+ 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(182*a^(2/3)*b^(1/
3)*e - 55*(1 - Sqrt[3])*(13*b*c + 2*a*f))*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.52162, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$

$$\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(182a^{2/3}\sqrt[3]{b}e-55\left(1-\sqrt{3}\right)(2af+13bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2af+13bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2}{3}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2a^2g\sqrt{a+bx^3}}{15b}+\frac{27a\sqrt{a+bx^3}(2af+13bc)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+\right.}{15015x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*a^2*g*Sqrt[a + b*x^3])/(15*b) - (27*a*c*Sqrt[a + b*x^3])/(7*x)
 + (27*a*(13*b*c + 2*a*f)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)) + (2*a*Sqrt[a + b*x^3]*(19305*c*x + 50
05*d*x^2 + 2457*e*x^3 + 1485*f*x^4 + 1001*g*x^5))/(15015*x^2) + (
2*(a + b*x^3)^(3/2)*(6435*c*x + 5005*d*x^2 + 4095*e*x^3 + 3465*f*
x^4 + 3003*g*x^5))/(45045*x^2) - (2*a^(3/2)*d*ArcTanh[Sqrt[a + b*
x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*c
+ 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(182*a^(2/3)*b^(1/
3)*e - 55*(1 - Sqrt[3])*(13*b*c + 2*a*f))*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.72676, size = 817, normalized size = 1.18

$$\frac{\sqrt{bx^3+a}\left(2b^2\left(6435c+7x\left(429gx^3+495fx^2+585ex+715d\right)\right)x^3+6006a^2gx+ab\left(4x\left(33(120f+91gx)x^2+5733ex+10\right.\right.\right.}{45045bx}$$

$$-\frac{a\left(-8910\sqrt{2}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{bx}\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{bx}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{bx}}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{bx}}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^3]*(6006*a^2*g*x + 2*b^2*x^3*(6435*c + 7*x*(715*d +
 585*e*x + 495*f*x^2 + 429*g*x^3)) + a*b*(-45045*c + 4*x*(10010*d
 + 5733*e*x + 33*x^2*(120*f + 91*g*x)))))/(45045*b*x) - (a*(10010
*Sqrt[a]*b^(2/3)*d*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-
1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[
a]] + 14742*a*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^
(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(
a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Elli
pticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]], (-1)^(1/3)] - 57915*Sqrt[2]*a^(1/3)*b*c*((-1)^(1/
3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^
(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^
(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sq
rt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 +
```

```
(-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)
/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - 8910*Sqrt[2]
*a^(4/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(
1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I
*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))
*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4
)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(
1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/
3))])))/(15015*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.015, size = 1317, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] e*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+f*
(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2))))+c*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/
2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-
a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))))+d*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3
+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+2/15*g/b*
(b*x^3+a)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2,x, algorithm="ma
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2,x, algorithm="fr
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^2, x)
```

Sympy [A] time = 14.3169, size = 474, normalized size = 0.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] a**(3/2)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*d*asinh(sqrt(a)/(sqrt(b
)*x**(3/2)))/3 + a**(3/2)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*f*x**2*gam
ma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(5/3)) + sqrt(a)*b*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*e*x**4*ga
mma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(7/3)) + sqrt(a)*b*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + 2*a**2*d/(3*sqrt(b
)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*d*x**(3/2)/(3*sqrt
(a/(b*x**3) + 1)) + a*g*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*
(a + b*x**3)**(3/2)/(9*b), True)) + b*d*Piecewise((sqrt(a)*x**3/3
, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*g*Piecewise
((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/
(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6,
True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2,x, algorithm="gi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2
, x)
```

3.452 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

Optimal. Leaf size=694

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}(4af+11bc)-110\left(1-\sqrt{\right.\right.}{10010b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2ag+13bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2}{3}a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{27a\sqrt{a+bx^3}(2ag+13bd)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2a\sqrt{a+bx^3}\left(27027cx-19305dx^2-5005ex^3-2457fx^4-14\right.}{15015x^3}$$

```
[Out] (27*a*c*Sqrt[a + b*x^3])/(10*x^2) - (27*a*d*Sqrt[a + b*x^3])/(7*x
) + (27*a*(13*b*d + 2*a*g)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(27027*c*x - 1
9305*d*x^2 - 5005*e*x^3 - 2457*f*x^4 - 1485*g*x^5))/(15015*x^3) +
 (2*(a + b*x^3)^(3/2)*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*
f*x^4 + 3465*g*x^5))/(45045*x^3) - (2*a^(3/2)*e*ArcTanh[Sqrt[a +
b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*
d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c +
4*a*f) - 110*(1 - Sqrt[3])*a^(1/3)*(13*b*d + 2*a*g))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(10010*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.68613, antiderivative size = 694, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}(4af+11bc)-110\left(1-\sqrt{\right.\right.}{10010b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2ag+13bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2}{3}a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{27a\sqrt{a+bx^3}(2ag+13bd)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2a\sqrt{a+bx^3}\left(27027cx-19305dx^2-5005ex^3-2457fx^4-14\right.}{15015x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (27*a*c*Sqrt[a + b*x^3])/(10*x^2) - (27*a*d*Sqrt[a + b*x^3])/(7*x
) + (27*a*(13*b*d + 2*a*g)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(27027*c*x - 1
9305*d*x^2 - 5005*e*x^3 - 2457*f*x^4 - 1485*g*x^5))/(15015*x^3) +
 (2*(a + b*x^3)^(3/2)*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*
f*x^4 + 3465*g*x^5))/(45045*x^3) - (2*a^(3/2)*e*ArcTanh[Sqrt[a +
b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*
d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c +
4*a*f) - 110*(1 - Sqrt[3])*a^(1/3)*(13*b*d + 2*a*g))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(10010*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.09528, size = 952, normalized size = 1.37

$$\frac{\sqrt{bx^3+a}\left(4b\left(9009c+5x\left(1287d+7x\left(99gx^2+117fx+143e\right)\right)\right)x^3+a\left(8(10010e+9x(637f+440gx))x^2-90090dx-45\right.\right.}{90090x^2}$$

$$-\frac{a\left(-17820\sqrt{2}g\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^3]*(a*(-45045*c - 90090*d*x + 8*x^2*(10010*e + 9*x*
(637*f + 440*g*x))) + 4*b*x^3*(9009*c + 5*x*(1287*d + 7*x*(143*e
+ 117*f*x + 99*g*x^2)))))/(90090*x^2) - (a*(20020*Sqrt[a]*b^(2/3)
*e*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 81081*b^(4
/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/
3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[
(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-
1)^(1/3)] + 29484*a*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(
1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))
]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-
1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 115830*Sqrt[2]*a^(1/3)*b*d*((
```

```
-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3
)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[Ar
cSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3
)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(
1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - 17820
*Sqrt[2]*a^(4/3)*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1
/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1
)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)
]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqr
t[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 +
(-1)^(1/3))])))/(30030*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.012, size = 1613, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] f*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+c*
(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-a*(b*x^
3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(
1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+e*(2/
9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh
((b*x^3+a)^(1/2)/a^(1/2)))+g*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*
x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3,x, algorithm="ma
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3,x, algorithm="fr
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^3, x)
```

Sympy [A] time = 14.4792, size = 462, normalized size = 0.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**(3/2)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*d*gamma(-1/3)*hyper((-
1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) -
2*a**(3/2)*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*f*x*g
amma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(4/3)) + a**(3/2)*g*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x*gamm
a(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(4/3)) + sqrt(a)*b*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*f*x**4*gam
ma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(7/3)) + sqrt(a)*b*g*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + 2*a**2*e/(3*sqrt(b)
*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*e*x**(3/2)/(3*sqrt(
a/(b*x**3) + 1)) + b*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(
a + b*x**3)**(3/2)/(9*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3,x, algorithm="gi

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3
, x)
```

3.453 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

Optimal. Leaf size=692

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-110\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+28ag+77bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{770\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$$-\frac{2a\sqrt{a+bx^3}\left(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5\right)}{1155x^4}$$

$$-\frac{1}{3}\sqrt{a}(2af+3bc)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{27ae\sqrt{a+bx^3}}{7x}+\frac{27a\sqrt[3]{b}e\sqrt{a+bx^3}}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (a*c*Sqrt[a + b*x^3])/x^3 + (27*a*d*Sqrt[a + b*x^3])/(10*x^2) - (
27*a*e*Sqrt[a + b*x^3])/(7*x) + (27*a*b^(1/3)*e*Sqrt[a + b*x^3])/
(7*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(1
155*c*x + 2079*d*x^2 - 1485*e*x^3 - 385*f*x^4 - 189*g*x^5))/(1155
*x^4) + (2*(a + b*x^3)^(3/2)*(1155*c*x + 693*d*x^2 + 495*e*x^3 +
385*f*x^4 + 315*g*x^5))/(3465*x^4) - (Sqrt[a]*(3*b*c + 2*a*f)*Arc
Tanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*
a^(4/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*b*d - 110*(1 - Sqrt[
3])*a^(1/3)*b^(2/3)*e + 28*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(770*b^(1/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.80453, antiderivative size = 692, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-110\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+28ag+77bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{770\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$$-\frac{2a\sqrt{a+bx^3}\left(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5\right)}{1155x^4}$$

$$-\frac{1}{3}\sqrt{a}(2af+3bc)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{27ae\sqrt{a+bx^3}}{7x}+\frac{27a\sqrt[3]{b}e\sqrt{a+bx^3}}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] (a*c*Sqrt[a + b*x^3])/x^3 + (27*a*d*Sqrt[a + b*x^3])/(10*x^2) - (
27*a*e*Sqrt[a + b*x^3])/(7*x) + (27*a*b^(1/3)*e*Sqrt[a + b*x^3])/
(7*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(1
155*c*x + 2079*d*x^2 - 1485*e*x^3 - 385*f*x^4 - 189*g*x^5))/(1155
*x^4) + (2*(a + b*x^3)^(3/2)*(1155*c*x + 693*d*x^2 + 495*e*x^3 +
385*f*x^4 + 315*g*x^5))/(3465*x^4) - (Sqrt[a]*(3*b*c + 2*a*f)*Arc
Tanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*
a^(4/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*b*d - 110*(1 - Sqrt[
3])*a^(1/3)*b^(2/3)*e + 28*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(770*b^(1/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.86644, size = 813, normalized size = 1.17

$$-\frac{54g\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)a^2}{55\sqrt[3]{b}\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}}$$

$$-\frac{2}{3}f\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)a^{3/2}-\frac{27\sqrt{2}\sqrt[3]{b}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\sqrt{\cdots}\right)\right)\right)}{7\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] Sqrt[a + b*x^3]*(a*((8*f)/9 - c/(3*x^3) - d/(2*x^2) - e/x + (28*g
*x)/55) + b*((2*c)/3 + (2*d*x)/5 + (2*e*x^2)/7 + (2*f*x^3)/9 + (2
*g*x^4)/11)) - Sqrt[a]*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - (2*
a^(3/2)*f*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*a*b^(2/3)*d*(
(-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
+ (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]
)/(10*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3))]*Sqrt[a + b*x^3]) - (54*a^2*g*((-1)^(1/3)*a^(1/3) - b^(1/3)
*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(
(-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(55*b^(1/3)*Sqrt[(a^(1/3)
+ (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^
3]) - (27*Sqrt[2]*a^(4/3)*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)
*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(
1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]
)]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/
3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + Elliptic
F[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^
(1/3)/(-1 + (-1)^(1/3))]))/(7*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*
x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.014, size = 1193, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] g*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+c*
(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arcta
nh((b*x^3+a)^(1/2)/a^(1/2)))+d*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*
x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
```

```
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)))+e*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(
1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*
(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))))+f*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x
^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4,x, algorithm="ma
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4,x, algorithm="fr
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^4, x)
```

Sympy [A] time = 17.1531, size = 484, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a**(3/2)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*e*gamma(-1/3)*hyper((-
1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) -
2*a**(3/2)*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*g*x*g
amma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3
```

```
*gamma(4/3)) - sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sq
rt(a)*b*d*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*e*x**2*gamma(2/3)*hyper((-1
/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt
(a)*b*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(7/3)) + 2*a**2*f/(3*sqrt(b)*x**(3/2)*sqrt(a/
(b*x**3) + 1)) - a*sqrt(b)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) +
2*a*sqrt(b)*c/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*f*x
**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*c*x**(3/2)/(3*sqrt(
a/(b*x**3) + 1)) + b*f*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(
a + b*x**3)**(3/2)/(9*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4,x, algorithm="gi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4
, x)
```

3.454 $\quad \int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

Optimal. Leaf size=741

$$\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(8af+7bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right|}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8af+7bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$$-\frac{2a\sqrt{a+bx^3}\left(189cx+105dx^2+189ex^3-135fx^4-35gx^5\right)}{105x^5}$$

$$-\frac{27\sqrt{a+bx^3}(8af+7bc)}{56x}+\frac{27\sqrt[3]{b}\sqrt{a+bx^3}(8af+7bc)}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{27ac\sqrt{a+bx^3}}{20x^4}$$

$$-\frac{1}{3}\sqrt{a}(2ag+3bd)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}$$

```
[Out] (27*a*c*Sqrt[a + b*x^3])/(20*x^4) + (a*d*Sqrt[a + b*x^3])/x^3 + (
27*a*e*Sqrt[a + b*x^3])/(10*x^2) - (27*(7*b*c + 8*a*f)*Sqrt[a + b
*x^3])/(56*x) + (27*b^(1/3)*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(189*
c*x + 105*d*x^2 + 189*e*x^3 - 135*f*x^4 - 35*g*x^5))/(105*x^5) +
(2*(a + b*x^3)^(3/2)*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 +
 35*g*x^5))/(315*x^5) - (Sqrt[a]*(3*b*d + 2*a*g)*ArcTanh[Sqrt[a +
 b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/
3)*(7*b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(1/3)*(28*a^(2
/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 8*a*f))*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(280*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.27812, antiderivative size = 741, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(8af+7bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|\right.}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8af+7bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$$-\frac{2a\sqrt{a+bx^3}\left(189cx+105dx^2+189ex^3-135fx^4-35gx^5\right)}{105x^5}$$

$$-\frac{27\sqrt{a+bx^3}(8af+7bc)}{56x}+\frac{27\sqrt[3]{b}\sqrt{a+bx^3}(8af+7bc)}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{27ac\sqrt{a+bx^3}}{20x^4}$$

$$-\frac{1}{3}\sqrt{a}(2ag+3bd)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (27*a*c*Sqrt[a + b*x^3])/(20*x^4) + (a*d*Sqrt[a + b*x^3])/x^3 + (
27*a*e*Sqrt[a + b*x^3])/(10*x^2) - (27*(7*b*c + 8*a*f)*Sqrt[a + b
*x^3])/(56*x) + (27*b^(1/3)*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(189*
c*x + 105*d*x^2 + 189*e*x^3 - 135*f*x^4 - 35*g*x^5))/(105*x^5) +
(2*(a + b*x^3)^(3/2)*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 +
 35*g*x^5))/(315*x^5) - (Sqrt[a]*(3*b*d + 2*a*g)*ArcTanh[Sqrt[a +
 b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/
3)*(7*b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(1/3)*(28*a^(2
/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 8*a*f))*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(280*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.24058, size = 878, normalized size = 1.18

$$-\frac{2}{3}g\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)a^{3/2}$$

$$-\frac{27\sqrt{2}\sqrt[3]{b}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+F\left(s\right.\right.}{7\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}}$$

$$-\frac{27b^{2/3}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)a}{10\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}}$$

$$-bd\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)\sqrt{a}-\frac{27b^{4/3}c\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}\ldots}}{\sqrt[4]{\ldots}}\right.\right.\right.}{4\sqrt{2}\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-70*a*(9*c + 2*x*(6*d + x*(9*e + 2*x*(9*f - 8*g
*x)))) + b*x^3*(-3465*c + 16*x*(105*d + x*(63*e + 5*x*(9*f + 7*g*
x))))))/(2520*x^4) - Sqrt[a]*b*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]
 - (2*a^(3/2)*g*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*a*b^(2/
3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^
(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^
(1/3)])/(10*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]*Sqrt[a + b*x^3]) - (27*a^(1/3)*b^(4/3)*c*((-1)^(1/3)
*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3
)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1
)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^
(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3
)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(4*Sqrt[2]*Sqrt[(a^(
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a +
 b*x^3]) - (27*Sqrt[2]*a^(4/3)*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^
(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sq
rt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*
b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + Ell
ipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)],
(-1)^(1/3)/(-1 + (-1)^(1/3))]))/(7*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.014, size = 1342, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] c*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))+d*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^
(1/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+e*(-1/2*a*(b*x^3+a)^(1/
2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+f*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^
2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+g*(2/9*b*x^3*(b*x^3+a)^(1/
2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1
/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5,x, algorithm="ma
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5,x, algorithm="fr
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^5, x)
```

Sympy [A] time = 17.5428, size = 495, normalized size = 0.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a**(3/2)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*e*gamma(-2/3)*hyper(
(-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3
)) + a**(3/2)*f*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*g*asinh(sqrt(a)/(s
qrt(b)*x**(3/2)))/3 + sqrt(a)*b*c*gamma(-1/3)*hyper((-1/2, -1/3),
 (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*d
*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*e*x*gamma(1/3)*hyp
er((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))
+ sqrt(a)*b*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a**2*g/(3*sqrt(b)*x**(3/2)*s
qrt(a/(b*x**3) + 1)) - a*sqrt(b)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/
2)) + 2*a*sqrt(b)*d/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(
b)*g*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*d*x**(3/2)/(3
*sqrt(a/(b*x**3) + 1)) + b*g*Piecewise((sqrt(a)*x**3/3, Eq(b, 0))
, (2*(a + b*x**3)**(3/2)/(9*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5,x, algorithm="gi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5
, x)
```

3.455 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx$

Optimal. Leaf size=689

$$\frac{9\, 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\left(14\sqrt[3]{b}(2af+bc)-5\left(1-\sqrt{3}\right)\right.}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8ag+7bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)-\frac{b\sqrt{a+bx^3}\left(252cx-315dx^2-140ex^3-126fx^4-180gx^5\right)}{140x^3}+\frac{27bc\sqrt{a+b\cdots}}{20x^2}$$

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(20*x^2) - (27*b*d*Sqrt[a + b*x^3])/(8*x
) + (27*b^(1/3)*(7*b*d + 8*a*g)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)) - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3
 + (30*f)/x^2 + (60*g)/x)*(a + b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x
^3]*(252*c*x - 315*d*x^2 - 140*e*x^3 - 126*f*x^4 - 180*g*x^5))/(1
40*x^3) - Sqrt[a]*b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - (27*3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*b*d + 8*a*g)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2
+ Sqrt[3]]*b^(1/3)*(14*b^(1/3)*(b*c + 2*a*f) - 5*(1 - Sqrt[3])*a^
(1/3)*(7*b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rubi [A] time = 1.76498, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314

$$\frac{9\, 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\left(14\sqrt[3]{b}(2af+bc)-5\left(1-\sqrt{3}\right)\right.}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8ag+7bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)-\frac{b\sqrt{a+bx^3}\left(252cx-315dx^2-140ex^3-126fx^4-180gx^5\right)}{140x^3}+\frac{27bc\sqrt{a+b\cdots}}{20x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(20*x^2) - (27*b*d*Sqrt[a + b*x^3])/(8*x
) + (27*b^(1/3)*(7*b*d + 8*a*g)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)) - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3
 + (30*f)/x^2 + (60*g)/x)*(a + b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x
^3]*(252*c*x - 315*d*x^2 - 140*e*x^3 - 126*f*x^4 - 180*g*x^5))/(1
40*x^3) - Sqrt[a]*b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - (27*3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*b*d + 8*a*g)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2
+ Sqrt[3]]*b^(1/3)*(14*b^(1/3)*(b*c + 2*a*f) - 5*(1 - Sqrt[3])*a^
(1/3)*(7*b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**6,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.65425, size = 949, normalized size = 1.38

$$-\frac{\sqrt{bx^3+a}\left(b(546c+x(1155d-16x(35e+3x(7f+5gx))))x^3+14a\left(12c+5x\left(6(f+2gx)x^2+4ex+3d\right)\right)\right)}{840x^5}-\frac{\sqrt[3]{b}\left(-1080\sqrt{2}g\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(14*a*(12*c + 5*x*(3*d + 4*e*x + 6*x^2*(f + 2*g
*x))) + b*x^3*(546*c + x*(1155*d - 16*x*(35*e + 3*x*(7*f + 5*g*x)
)))))/(840*x^5) - (b^(1/3)*(280*Sqrt[a]*b^(2/3)*e*Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3
]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 378*b^(4/3)*c*((-1)^(1/3)*a^
(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a
^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 +
 (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3
)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + 756*a*b^
(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*
x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqr
t[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
(-1)^(1/3)] - 945*Sqrt[2]*a^(1/3)*b*d*((-1)^(1/3)*a^(1/3) - b^(1/
```

```
3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-
1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqr
t[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I
*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - E
llipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)]
, (-1)^(1/3)/(-1 + (-1)^(1/3))]) - 1080*Sqrt[2]*a^(4/3)*g*((-1)^(
1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*
b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/
a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[
Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1
 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*
x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(280*Sqrt[
(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt
[a + b*x^3])
```

Maple [B] time = 0.015, size = 1606, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x)
```

```
[Out] c*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*
b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1
/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))))+e*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)
*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+f*(-1/2*a*(b*x^3+a)^(1/2)/x^
2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+g*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*
x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6,x, algorithm="ma
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6,x, algorithm="fr
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^6, x)
```

Sympy [A] time = 17.3745, size = 476, normalized size = 0.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**6,x)
```

```
[Out] a**(3/2)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*d*gamma(-4/3)*hyper(
(-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1
/3)) + a**(3/2)*f*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*
exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*g*gamma(-1/3)*h
yper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2
/3)) + sqrt(a)*b*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3
*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*d*gamma(-1/3)
*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma
(2/3)) - sqrt(a)*b*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*
b*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + sqrt(a)*b*g*x**2*gamma(2/3)*hyper((-1/2, 2/
3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) - a*sqrt(b)*
e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*e/(3*x**(3/2)*s
qrt(a/(b*x**3) + 1)) + 2*b**(3/2)*e*x**(3/2)/(3*sqrt(a/(b*x**3) +
 1))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6,x, algorithm="gi

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6
, x)
```

3.456 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

Optimal. Leaf size=692

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+2bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\ldots\right)}{40\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{b\sqrt{a+bx^3}\left(10cx+36dx^2-45ex^3-20fx^4-18gx^5\right)}{20x^4}$$

$$-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)-\frac{b(4af+bc)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+b\ldots}}{8x}$$

```
[Out] (b*c*Sqrt[a + b*x^3])/(4*x^3) + (27*b*d*Sqrt[a + b*x^3])/(20*x^2)
 - (27*b*e*Sqrt[a + b*x^3])/(8*x) + (27*b^(4/3)*e*Sqrt[a + b*x^3]
)/(8*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((10*c)/x^6 + (12*d)
/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*(a + b*x^3)^(3/2))/6
0 - (b*Sqrt[a + b*x^3]*(10*c*x + 36*d*x^2 - 45*e*x^3 - 20*f*x^4 -
 18*g*x^5))/(20*x^4) - (b*(b*c + 4*a*f)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(4
/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9
*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d - 5*(1 - Sqrt[3])*a^(1/
3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rubi [A] time = 1.90298, antiderivative size = 692, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+2bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{40\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{b\sqrt{a+bx^3}\left(10cx+36dx^2-45ex^3-20fx^4-18gx^5\right)}{20x^4}$$

$$-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)-\frac{b(4af+bc)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+b}}{8x}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] (b*c*Sqrt[a + b*x^3])/(4*x^3) + (27*b*d*Sqrt[a + b*x^3])/(20*x^2)
 - (27*b*e*Sqrt[a + b*x^3])/(8*x) + (27*b^(4/3)*e*Sqrt[a + b*x^3]
)/(8*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((10*c)/x^6 + (12*d)
/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*(a + b*x^3)^(3/2))/6
0 - (b*Sqrt[a + b*x^3]*(10*c*x + 36*d*x^2 - 45*e*x^3 - 20*f*x^4 -
 18*g*x^5))/(20*x^4) - (b*(b*c + 4*a*f)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(4
/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9
*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d - 5*(1 - Sqrt[3])*a^(1/
3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**7,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.58663, size = 805, normalized size = 1.16

$$\frac{3}{80}b\left(-\frac{20bc\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{80}{3}\sqrt{a}f\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)\right.$$

$$-\frac{36b^{2/3}d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}}$$

$$-\frac{72ag\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{b}\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}}$$

$$-\frac{90\sqrt{2}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)+F\right.}{\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(b*x^3*(50*c + x*(78*d + x*(165*e - 80*f*x - 48
*g*x^2))) + a*(20*c + 2*x*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)))))
/(120*x^6) + (3*b*((-20*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*
Sqrt[a]) - (80*Sqrt[a]*f*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3
6*b^(2/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[S
qrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]]
, (-1)^(1/3)])/(Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))]*Sqrt[a + b*x^3]) - (72*a*g*((-1)^(1/3)*a^(1/3) -
 b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(b^(1/3)*Sqrt[(a^(
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a +
 b*x^3]) - (90*Sqrt[2]*a^(1/3)*b^(1/3)*e*((-1)^(1/3)*a^(1/3) - b^
(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sq
rt[3])]*((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*
b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + Ell
ipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)],
(-1)^(1/3)/(-1 + (-1)^(1/3))]))/(Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])))/80
```

Maple [B] time = 0.014, size = 1196, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x)
```

```
[Out] c*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*
arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+d*(-1/5*a*(b*x^3+a)^(1/
2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(-1/4*a*(b*x^3+a)^(1/2)/x^
4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+f*(-1/3*a*(b*x^3+
a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^3+a)^(1
/2)/a^(1/2)))+g*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/
2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^7,x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^7,x, algorithm="fr
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^7, x)
```

Sympy [A] time = 24.1275, size = 524, normalized size = 0.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**7,x)
```

```
[Out] a**(3/2)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*e*gamma(-4/3)*hyper(
(-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1
/3)) + a**(3/2)*g*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*
exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*d*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gam
ma(1/3)) + sqrt(a)*b*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*
x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*f*asinh(sqrt
(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*g*x*gamma(1/3)*hyper((-1/2, 1
/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - a**2*c/(6
*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*c/(4*x**(9/2
)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*f*sqrt(a/(b*x**3) + 1)/(3*x**
(3/2)) + 2*a*sqrt(b)*f/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(3/
2)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/(12*x**(3/2)*
sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*f*x**(3/2)/(3*sqrt(a/(b*x**3)
+ 1)) - b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^7,x, algorithm="gi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^7
, x)
```

3.457 $\quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

Optimal. Leaf size=746

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(14af+bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-\right.}{560a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14af+bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27b^{4/3}\sqrt{a+bx^3}(14af+bc)}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{b\sqrt{a+bx^3}\left(36cx+70dx^2+252ex^3-315fx^4-140gx^5\right)}{140x^5}$$

$$-\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)-\frac{27b\sqrt{a+bx^3}(14af+bc)}{112ax}+\frac{27bc\sqrt{a+bx^3}}{280x^4}-\frac{b(4ag+bd)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right.}{4\sqrt{a}}$$

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(280*x^4) + (b*d*Sqrt[a + b*x^3])/(4*x^3
) + (27*b*e*Sqrt[a + b*x^3])/(20*x^2) - (27*b*(b*c + 14*a*f)*Sqrt
[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*c + 14*a*f)*Sqrt[a + b*x^
3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((60*c)/x^7 +
(70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*(a + b*x^3)^
(3/2))/420 - (b*Sqrt[a + b*x^3]*(36*c*x + 70*d*x^2 + 252*e*x^3 -
315*f*x^4 - 140*g*x^5))/(140*x^5) - (b*(b*d + 4*a*g)*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]
*b^(4/3)*(b*c + 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(28
*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(b*c + 14*a*f))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(560*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.37261, antiderivative size = 746, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(14af+bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-}{560a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14af+bc)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27b^{4/3}\sqrt{a+bx^3}(14af+bc)}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{b\sqrt{a+bx^3}\left(36cx+70dx^2+252ex^3-315fx^4-140gx^5\right)}{140x^5}$$

$$-\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)-\frac{27b\sqrt{a+bx^3}(14af+bc)}{112ax}+\frac{27bc\sqrt{a+bx^3}}{280x^4}-\frac{b(4ag+bd)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right.}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(280*x^4) + (b*d*Sqrt[a + b*x^3])/(4*x^3
) + (27*b*e*Sqrt[a + b*x^3])/(20*x^2) - (27*b*(b*c + 14*a*f)*Sqrt
[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*c + 14*a*f)*Sqrt[a + b*x^
3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((60*c)/x^7 +
(70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*(a + b*x^3)^
(3/2))/420 - (b*Sqrt[a + b*x^3]*(36*c*x + 70*d*x^2 + 252*e*x^3 -
315*f*x^4 - 140*g*x^5))/(140*x^5) - (b*(b*d + 4*a*g)*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]
*b^(4/3)*(b*c + 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(28
*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(b*c + 14*a*f))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(560*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**8,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.16604, size = 897, normalized size = 1.2

$$-\frac{\sqrt{bx^3+a}\left(405b^2cx^6+2ab\left(255c+7x\left(50d+x\left(-80gx^2+165fx+78e\right)\right)\right)x^3+4a^2(60c+7x(10d+x(12e+5x(3f+4gx))\right.}{1680ax^7}$$

$$-\frac{b\left(560g\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)a^{3/2}-1890\sqrt{2}\sqrt[3]{b}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right.}{3i+\sqrt{}}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(405*b^2*c*x^6 + 2*a*b*x^3*(255*c + 7*x*(50*d +
 x*(78*e + 165*f*x - 80*g*x^2))) + 4*a^2*(60*c + 7*x*(10*d + x*(1
2*e + 5*x*(3*f + 4*g*x))))))/(1680*a*x^7) - (b*(140*Sqrt[a]*b*d*S
qrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*
Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 560*a^(3/2)*g*
Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 756*a*b^(2/3)
*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*
b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^
(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^
(1/3)] - 135*Sqrt[2]*a^(1/3)*b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1
/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sq
rt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (
I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) -
EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)
], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - 1890*Sqrt[2]*a^(4/3)*b^(1/3)*
f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-
1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^
(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*Elliptic
E[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^
(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I
*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(
560*a*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.015, size = 1375, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x)
```

```
[Out] c*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/112/
a*b^2*(b*x^3+a)^(1/2)/x-9/112*I/a*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+d*(-1/6*a*(b*x
^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b*x^3
```

```
+a)^(1/2)/a^(1/2))/a^(1/2))+e*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b
*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))+f*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^
3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+g*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2
/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8,x, algorithm="ma
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8,x, algorithm="fr
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^8, x)
```

Sympy [A] time = 25.2408, size = 536, normalized size = 0.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**8,x)
```

```
[Out] a**(3/2)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*e*gamma(-5/3)*hyper(
```

```
(-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2
/3)) + a**(3/2)*f*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3
*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*c*gamma(-4/3
)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*
gamma(-1/3)) + sqrt(a)*b*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*f*gam
ma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3
*x*gamma(2/3)) - sqrt(a)*b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) -
a**2*d/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d/(
4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*g*sqrt(a/(b*x**3) +
1)/(3*x**(3/2)) + 2*a*sqrt(b)*g/(3*x**(3/2)*sqrt(a/(b*x**3) + 1))
 - b**(3/2)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*d/(12*
x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*g*x**(3/2)/(3*sqrt(a/
(b*x**3) + 1)) - b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt
(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8,x, algorithm="gi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8
, x)
```

3.458 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

Optimal. Leaf size=705

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(bc-16af)+20\left(1-\sqrt{3}\right.\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14ag+bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27b^{4/3}\sqrt{a+bx^3}(14ag+bd)}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2d\sqrt{a+bx^3}}{112ax}-\frac{b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

$$-\frac{1}{560}b\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)-\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

```
[Out] -(b*((63*c)/x^5 + (90*d)/x^4 + (140*e)/x^3 + (252*f)/x^2 + (630*g
)/x)*Sqrt[a + b*x^3])/560 - (27*b^2*c*Sqrt[a + b*x^3])/(320*a*x^2
) - (27*b^2*d*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*d + 14*
a*g)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210
*g)/x^4)*(a + b*x^3)^(3/2))/840 - (b^2*e*ArcTanh[Sqrt[a + b*x^3]/
Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(b*
d + 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(b*c
 - 16*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(b*d + 14*a*g))*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(2240*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.85532, antiderivative size = 705, normalized size of antiderivative = 1., number of

steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(bc-16af)+20\left(1-\right.\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14ag+bd)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27b^{4/3}\sqrt{a+bx^3}(14ag+bd)}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2d\sqrt{a+bx^3}}{112ax}-\frac{b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

$$-\frac{1}{560}b\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)-\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -(b*((63*c)/x^5 + (90*d)/x^4 + (140*e)/x^3 + (252*f)/x^2 + (630*g
)/x)*Sqrt[a + b*x^3])/560 - (27*b^2*c*Sqrt[a + b*x^3])/(320*a*x^2
) - (27*b^2*d*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*d + 14*
a*g)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210
*g)/x^4)*(a + b*x^3)^(3/2))/840 - (b^2*e*ArcTanh[Sqrt[a + b*x^3]/
Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(b*
d + 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(b*c
 - 16*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(b*d + 14*a*g))*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(2240*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**9,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.47552, size = 978, normalized size = 1.39

$$-\frac{\left(-7560\sqrt{2}g\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)-\right.\right.}{}$$

$$-\frac{\sqrt{bx^3+a}\left(81b^2(7c+20dx)x^6+4ab\left(399c+2x\left(255d+7x\left(165gx^2+78fx+50e\right)\right)\right)x^3+8a^2(105c+2x(60d+7x(10e+3\right.}{6720ax^8}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(81*b^2*x^6*(7*c + 20*d*x) + 4*a*b*x^3*(399*c +
 2*x*(255*d + 7*x*(50*e + 78*f*x + 165*g*x^2))) + 8*a^2*(105*c +
2*x*(60*d + 7*x*(10*e + 3*x*(4*f + 5*g*x))))))/(6720*a*x^8) - (b^
(4/3)*(560*Sqrt[a]*b^(2/3)*e*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*
x^3]/Sqrt[a]] - 189*b^(4/3)*c*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(
1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))
]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-
1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + 3024*a*b^(1/3)*f*((-1)^(1/3)*
a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1
 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2
/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 540*Sq
rt[2]*a^(1/3)*b*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/
3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*
Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)
^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]
/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt
[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (
-1)^(1/3))]) - 7560*Sqrt[2]*a^(4/3)*g*((-1)^(1/3)*a^(1/3) - b^(1/
3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-
1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqr
t[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I
*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) - E
llipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)]
, (-1)^(1/3)/(-1 + (-1)^(1/3))])))/(2240*a*Sqrt[(a^(1/3) + (-1)^(
2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.015, size = 1663, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x)
```

```
[Out] c*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320/
a*b^2*(b*x^3+a)^(1/2)/x^2+9/320*I/a*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+d*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17
/56*b*(b*x^3+a)^(1/2)/x^4-27/112/a*b^2*(b*x^3+a)^(1/2)/x-9/112*I/
a*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
```

```
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))))+e*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)
^(1/2)/x^3-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+f*(-
1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+g*(-1/4*a
*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9,x, algorithm="ma
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bgx^7+bfx^6+bex^5+(bd+ag)x^4+aex^2+(bc+af)x^3+adx+ac\right)\sqrt{bx^3+a}}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9,x, algorithm="fr
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^9, x)
```

Sympy [A] time = 24.4418, size = 527, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**9,x)
```

```
[Out] a**(3/2)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*d*gamma(-7/3)*hyper(
(-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4
/3)) + a**(3/2)*f*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3
*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*g*gamma(-4/3)
*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*g
amma(-1/3)) + sqrt(a)*b*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*d*ga
mma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x**4*gamma(-1/3)) + sqrt(a)*b*f*gamma(-2/3)*hyper((-2/3, -1/2)
, (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)
*b*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x*gamma(2/3)) - a**2*e/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x*
*3) + 1)) - a*sqrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3
/2)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*e/(12*x**(3/2)
*sqrt(a/(b*x**3) + 1)) - b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))
/(4*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9,x, algorithm="gi
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9
, x)
```

3.459 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx$

Optimal. Leaf size=714

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag+7bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b^2(bc-6af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}+\frac{27b^{7/3}e\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{b^2c\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2d\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2e\sqrt{a+bx^3}}{112ax}$$

$$-\frac{b\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)}{1680}-\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

```
[Out] -(b*((140*c)/x^6 + (189*d)/x^5 + (270*e)/x^4 + (420*f)/x^3 + (756
*g)/x^2)*Sqrt[a + b*x^3])/1680 - (b^2*c*Sqrt[a + b*x^3])/(24*a*x^
3) - (27*b^2*d*Sqrt[a + b*x^3])/(320*a*x^2) - (27*b^2*e*Sqrt[a +
b*x^3])/(112*a*x) + (27*b^(7/3)*e*Sqrt[a + b*x^3])/(112*a*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (((280*c)/x^9 + (315*d)/x^8 + (36
0*e)/x^7 + (420*f)/x^6 + (504*g)/x^5)*(a + b*x^3)^(3/2))/2520 + (
b^2*(b*c - 6*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2))
- (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*e*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*Sqrt[2 + Sq
rt[3]]*b^(5/3)*(7*b*d + 20*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 112*
a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.03138, antiderivative size = 714, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag+7bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b^2(bc-6af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}+\frac{27b^{7/3}e\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{b^2c\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2d\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2e\sqrt{a+bx^3}}{112ax}$$

$$-\frac{b\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)}{1680}-\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```

```
[Out] -(b*((140*c)/x^6 + (189*d)/x^5 + (270*e)/x^4 + (420*f)/x^3 + (756
*g)/x^2)*Sqrt[a + b*x^3])/1680 - (b^2*c*Sqrt[a + b*x^3])/(24*a*x^
3) - (27*b^2*d*Sqrt[a + b*x^3])/(320*a*x^2) - (27*b^2*e*Sqrt[a +
b*x^3])/(112*a*x) + (27*b^(7/3)*e*Sqrt[a + b*x^3])/(112*a*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (((280*c)/x^9 + (315*d)/x^8 + (36
0*e)/x^7 + (420*f)/x^6 + (504*g)/x^5)*(a + b*x^3)^(3/2))/2520 + (
b^2*(b*c - 6*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2))
- (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*e*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*Sqrt[2 + Sq
rt[3]]*b^(5/3)*(7*b*d + 20*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 112*
a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**10,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 5.32637, size = 844, normalized size = 1.18

$$\frac{280c\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)b^3+567\sqrt{a}d\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\Bigg(\sin^{-}}{}$$

$$-\frac{\sqrt{bx^3+a}\left(3b^2(280c+81x(7d+20ex))x^6+4ab\left(980c+3x\left(28(25f+39gx)x^2+510ex+399d\right)\right)x^3+8a^2(280c+3x(105d}{20160ax^9}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(3*b^2*x^6*(280*c + 81*x*(7*d + 20*e*x)) + 4*a*
b*x^3*(980*c + 3*x*(399*d + 510*e*x + 28*x^2*(25*f + 39*g*x))) +
8*a^2*(280*c + 3*x*(105*d + 4*x*(30*e + 7*x*(5*f + 6*g*x))))))/(2
0160*a*x^9) + (280*b^3*c*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]
/Sqrt[a]] - 1680*a*b^2*f*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]
/Sqrt[a]] + 567*Sqrt[a]*b^(8/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3)*x
)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-
1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - 9072*a^(3/2)*b^(5/3)*g*((-1
)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*
x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (
-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] -
1620*Sqrt[2]*a^(5/6)*b^(7/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*S
qrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*((
-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)
/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[Arc
Sin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)
/(-1 + (-1)^(1/3))]))/(6720*a^(3/2)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.041, size = 1273, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x)
```

```
[Out] c*(-1/9*a*(b*x^3+a)^(1/2)/x^9-7/36*b*(b*x^3+a)^(1/2)/x^6-1/24/a*b
^2*(b*x^3+a)^(1/2)/x^3+1/24/a^(3/2)*b^3*arctanh((b*x^3+a)^(1/2)/a
^(1/2)))+d*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^
5-27/320/a*b^2*(b*x^3+a)^(1/2)/x^2+9/320*I/a*b^2*3^(1/2)*(-a*b^2)
^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+e*(-1/7*a*(b*x^3+a)^(1/
2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/112/a*b^2*(b*x^3+a)^(1/2)/x
-9/112*I/a*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
```

```
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/
b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))))+f*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*
(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1
/2))+g*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/
20*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^10,x, algorithm="m
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^{7}+bfx^{6}+bex^{5}+(bd+ag)x^{4}+aex^{2}+(bc+af)x^{3}+adx+ac\right)\sqrt{bx^{3}+a}}{x^{10}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^10,x, algorithm="f
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^10, x)
```

Sympy [A] time = 38.8684, size = 573, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**10,x)
```

```
[Out] a**(3/2)*d*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*e*gamma(-7/3)*hyper(
(-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4
/3)) + a**(3/2)*g*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3
*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*d*gamma(-5/3
)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*
gamma(-2/3)) + sqrt(a)*b*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*g*g
amma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x**2*gamma(1/3)) - a**2*c/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3)
```

```
 + 1)) - a**2*f/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a
*sqrt(b)*c/(36*x**(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*f/(4*x
**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3/2)*c/(72*x**(9/2)*sqrt(a
/(b*x**3) + 1)) - b**(3/2)*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) -
b**(3/2)*f/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*c/(24*a*
x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*f*asinh(sqrt(a)/(sqrt(b)*x*
*(3/2)))/(4*sqrt(a)) + b**3*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(
24*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^10,x, algorithm="g
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^1
0, x)
```

3.460 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx$

Optimal. Leaf size=764

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\ldots}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(bc-4af)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{b^2(bd-6ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}-\frac{27b^{7/3}\sqrt{a+bx^3}(bc-4af)}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{27b^2\sqrt{a+bx^3}(bc-4af)}{448a^2x}-\frac{27b^2c\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2d\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2e\sqrt{a+bx^3}}{320ax^2}$$
$$-\frac{b\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)}{1680}-\frac{\left(a+bx^3\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

```
[Out] -(b*((108*c)/x^7 + (140*d)/x^6 + (189*e)/x^5 + (270*f)/x^4 + (420
*g)/x^3)*Sqrt[a + b*x^3])/1680 - (27*b^2*c*Sqrt[a + b*x^3])/(1120
*a*x^4) - (b^2*d*Sqrt[a + b*x^3])/(24*a*x^3) - (27*b^2*e*Sqrt[a +
 b*x^3])/(320*a*x^2) + (27*b^2*(b*c - 4*a*f)*Sqrt[a + b*x^3])/(44
8*a^2*x) - (27*b^(7/3)*(b*c - 4*a*f)*Sqrt[a + b*x^3])/(448*a^2*((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((252*c)/x^10 + (280*d)/x^9
 + (315*e)/x^8 + (360*f)/x^7 + (420*g)/x^6)*(a + b*x^3)^(3/2))/25
20 + (b^2*(b*d - 6*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(
3/2)) + (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*c - 4*a*f)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3
^(3/4)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*a^(2/3)*b^(1/3)*e - 5*(1 - Sq
rt[3])*(b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a^(5/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.48437, antiderivative size = 764, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{9\, 3^{3/4}\sqrt{2+\sqrt{3}}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(bc-4af)E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b^2(bd-6ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}-\frac{27b^{7/3}\sqrt{a+bx^3}(bc-4af)}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{27b^2\sqrt{a+bx^3}(bc-4af)}{448a^2x}-\frac{27b^2c\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2d\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2e\sqrt{a+bx^3}}{320ax^2}$$

$$-\frac{b\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)}{1680}-\frac{\left(a+bx^3\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

```
[Out] -(b*((108*c)/x^7 + (140*d)/x^6 + (189*e)/x^5 + (270*f)/x^4 + (420
*g)/x^3)*Sqrt[a + b*x^3])/1680 - (27*b^2*c*Sqrt[a + b*x^3])/(1120
*a*x^4) - (b^2*d*Sqrt[a + b*x^3])/(24*a*x^3) - (27*b^2*e*Sqrt[a +
 b*x^3])/(320*a*x^2) + (27*b^2*(b*c - 4*a*f)*Sqrt[a + b*x^3])/(44
8*a^2*x) - (27*b^(7/3)*(b*c - 4*a*f)*Sqrt[a + b*x^3])/(448*a^2*((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((252*c)/x^10 + (280*d)/x^9
 + (315*e)/x^8 + (360*f)/x^7 + (420*g)/x^6)*(a + b*x^3)^(3/2))/25
20 + (b^2*(b*d - 6*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(
3/2)) + (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*c - 4*a*f)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3
^(3/4)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*a^(2/3)*b^(1/3)*e - 5*(1 - Sq
rt[3])*(b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a^(5/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**11,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.58854, size = 930, normalized size = 1.22

$$b^2\left(-1680g\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3+a}\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)a^{3/2}+1620\sqrt{2}\sqrt[3]{b}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\dots\right.}{3i+\dots}}\right.$$

$$-\frac{\sqrt{bx^3+a}\left(-1215b^3cx^9+3ab^2(162c+x(280d+81x(7e+20fx)))x^6+4a^2b\left(828c+x\left(980d+3x\left(700gx^2+510fx+399e\right)\right.\right.\right.}{20160a^2x^{10}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(-1215*b^3*c*x^9 + 8*a^3*(252*c + 5*x*(56*d + 6
3*e*x + 72*f*x^2 + 84*g*x^3)) + 3*a*b^2*x^6*(162*c + x*(280*d + 8
1*x*(7*e + 20*f*x))) + 4*a^2*b*x^3*(828*c + x*(980*d + 3*x*(399*e
 + 510*f*x + 700*g*x^2))))))/(20160*a^2*x^10) + (b^2*(280*Sqrt[a]*
b*d*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 1680*a^(3
/2)*g*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3))]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 567*a*b
^(2/3)*e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^
(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sq
rt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
 (-1)^(1/3)] - 405*Sqrt[2]*a^(1/3)*b^(4/3)*c*((-1)^(1/3)*a^(1/3)
- b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*
I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^(1/
6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3)
)]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3
^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) + 1620*Sqrt[2]*a^(4/3)*b^
(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3
) - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1
 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*El
lipticE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)],
 (-1)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6
) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))
])))/(6720*a^2*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(
1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.015, size = 1470, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x)
```

```
[Out] c*(-1/10*a*(b*x^3+a)^(1/2)/x^10-23/140*b*(b*x^3+a)^(1/2)/x^7-27/1
120/a*b^2*(b*x^3+a)^(1/2)/x^4+27/448/a^2*b^3*(b*x^3+a)^(1/2)/x+9/
448*I*b^3/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
```

```
*(-a*b^2)^(1/3)))^(1/2))))+d*(-1/9*a*(b*x^3+a)^(1/2)/x^9-7/36*b*(
b*x^3+a)^(1/2)/x^6-1/24/a*b^2*(b*x^3+a)^(1/2)/x^3+1/24/a^(3/2)*b^
3*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+e*(-1/8*a*(b*x^3+a)^(1/2)/x^8
-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320/a*b^2*(b*x^3+a)^(1/2)/x^2+9/3
20*I/a*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)))+f*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/
112/a*b^2*(b*x^3+a)^(1/2)/x-9/112*I/a*b^2*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+g*(-1/6*a*
(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b
*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11,x, algorithm="m
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^1
1, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11,x, algorithm="f
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^11, x)
```

Sympy [A] time = 41.5186, size = 576, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**11,x)
```

```
[Out] a**(3/2)*c*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*e*gamma(-8/3)*hyp
er((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma
(-5/3)) + a**(3/2)*f*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*c*gamma(-
7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*
*7*gamma(-4/3)) + sqrt(a)*b*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*
f*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**4*gamma(-1/3)) - a**2*d/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*
x**3) + 1)) - a**2*g/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) -
 11*a*sqrt(b)*d/(36*x**(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*g
/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3/2)*d/(72*x**(9/2)*s
qrt(a/(b*x**3) + 1)) - b**(3/2)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2
)) - b**(3/2)*g/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*d/(
24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*g*asinh(sqrt(a)/(sqrt(
b)*x**(3/2)))/(4*sqrt(a)) + b**3*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(24*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11,x, algorithm="g
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^1
1, x)
```

3.461 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx$

Optimal. Leaf size=796

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)b^3}{24a^{3/2}}$$
$$+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(bd-4ag)\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)b^{7/3}}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$
$$+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(7bc-22af)+110\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{49280a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$
$$-\frac{27(bd-4ag)\sqrt{bx^3+a}b^{7/3}}{448a^2\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}+\frac{27(bd-4ag)\sqrt{bx^3+a}b^2}{448a^2x}+\frac{27(7bc-22af)\sqrt{bx^3+a}b^2}{7040a^2x^2}-\frac{e\sqrt{bx^3+a}b^2}{24ax^3}$$
$$-\frac{27d\sqrt{bx^3+a}b^2}{1120ax^4}-\frac{27c\sqrt{bx^3+a}b^2}{1760ax^5}-\frac{\left(\frac{945c}{x^8}+\frac{2970g}{x^4}+\frac{2079f}{x^5}+\frac{1540e}{x^6}+\frac{1188d}{x^7}\right)\sqrt{bx^3+a}b}{18480}$$
$$-\frac{\left(\frac{2520c}{x^{11}}+\frac{3960g}{x^7}+\frac{3465f}{x^8}+\frac{3080e}{x^9}+\frac{2772d}{x^{10}}\right)\left(bx^3+a\right)^{3/2}}{27720}$$

```
[Out] -(b*((945*c)/x^8 + (1188*d)/x^7 + (1540*e)/x^6 + (2079*f)/x^5 + (
2970*g)/x^4)*Sqrt[a + b*x^3])/18480 - (27*b^2*c*Sqrt[a + b*x^3])/
(1760*a*x^5) - (27*b^2*d*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*e*S
qrt[a + b*x^3])/(24*a*x^3) + (27*b^2*(7*b*c - 22*a*f)*Sqrt[a + b*
x^3])/(7040*a^2*x^2) + (27*b^2*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(44
8*a^2*x) - (27*b^(7/3)*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(448*a^2*((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((2520*c)/x^11 + (2772*d)/x
^10 + (3080*e)/x^9 + (3465*f)/x^8 + (3960*g)/x^7)*(a + b*x^3)^(3/
2))/27720 + (b^3*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2))
 + (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*d - 4*a*g)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4
)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*b^(1/3)*(7*b*c - 22*a*f) + 110*(1
- Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(49280*a
^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.83545, antiderivative size = 796, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)b^3}{24a^{3/2}} + \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(bd-4ag)\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)b^{7/3}}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(7bc-22af)+110\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{49280a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}} - \frac{27(bd-4ag)\sqrt{bx^3+a}b^{7/3}}{448a^2\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)} + \frac{27(bd-4ag)\sqrt{bx^3+a}b^2}{448a^2x} + \frac{27(7bc-22af)\sqrt{bx^3+a}b^2}{7040a^2x^2} - \frac{e\sqrt{bx^3+a}b^2}{24ax^3} - \frac{27d\sqrt{bx^3+a}b^2}{1120ax^4} - \frac{27c\sqrt{bx^3+a}b^2}{1760ax^5} - \frac{\left(\frac{945c}{x^8}+\frac{2970g}{x^4}+\frac{2079f}{x^5}+\frac{1540e}{x^6}+\frac{1188d}{x^7}\right)\sqrt{bx^3+a}b}{18480} - \frac{\left(\frac{2520c}{x^{11}}+\frac{3960g}{x^7}+\frac{3465f}{x^8}+\frac{3080e}{x^9}+\frac{2772d}{x^{10}}\right)\left(bx^3+a\right)^{3/2}}{27720}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

```
[Out] -(b*((945*c)/x^8 + (1188*d)/x^7 + (1540*e)/x^6 + (2079*f)/x^5 + (
2970*g)/x^4)*Sqrt[a + b*x^3])/18480 - (27*b^2*c*Sqrt[a + b*x^3])/
(1760*a*x^5) - (27*b^2*d*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*e*S
qrt[a + b*x^3])/(24*a*x^3) + (27*b^2*(7*b*c - 22*a*f)*Sqrt[a + b*
x^3])/(7040*a^2*x^2) + (27*b^2*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(44
8*a^2*x) - (27*b^(7/3)*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(448*a^2*((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((2520*c)/x^11 + (2772*d)/x
^10 + (3080*e)/x^9 + (3465*f)/x^8 + (3960*g)/x^7)*(a + b*x^3)^(3/
2))/27720 + (b^3*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2))
 + (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*d - 4*a*g)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4
)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*b^(1/3)*(7*b*c - 22*a*f) + 110*(1
- Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(49280*a
^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**12,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.69395, size = 1017, normalized size = 1.28

$$\frac{b^{7/3}\left(35640\sqrt{2}g\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{3i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}}{\sqrt[4]{3}}\right)\middle|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right.\right.\right.}{}$$

$$-\frac{\sqrt{bx^3+a}\left(-243b^3(49c+110dx)x^9+6ab^2(1134c+11x(162d+x(280e+81x(7f+20gx))))x^6+8a^2b(7875c+11x(828d+x\right.}{443520a^2x^{11}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(-243*b^3*x^9*(49*c + 110*d*x) + 16*a^3*(2520*c
 + 11*x*(252*d + 5*x*(56*e + 9*x*(7*f + 8*g*x)))) + 6*a*b^2*x^6*(
1134*c + 11*x*(162*d + x*(280*e + 81*x*(7*f + 20*g*x)))) + 8*a^2*
b*x^3*(7875*c + 11*x*(828*d + x*(980*e + 9*x*(133*f + 170*g*x))))
))/(443520*a^2*x^11) + (b^(7/3)*(6160*Sqrt[a]*b^(2/3)*e*Sqrt[(a^(
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a +
 b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 3969*b^(4/3)*c*((-1)^(
1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(
1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x)
)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + 1
2474*a*b^(1/3)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(a^(1/3) +
 b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[((-1)^(1/3)*(a^(1/3)
 - (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[A
rcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3))]], (-1)^(1/3)] - 8910*Sqrt[2]*a^(1/3)*b*d*((-1)^(1/3)*a^(1/
3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x)
)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (b^(1/3)*x)/a^(1/3)))/
(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*EllipticE[ArcSin[Sqrt[(-1)^
(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1
/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)
]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]) + 35640*Sqrt[2]*a^(4/3
)*g*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) -
(-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[(I*(1 + (
b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3])]*(-((-1 + (-1)^(2/3))*Ellipt
icE[ArcSin[Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1
)^(1/3)/(-1 + (-1)^(1/3))]) - EllipticF[ArcSin[Sqrt[(-1)^(1/6) -
(I*b^(1/3)*x)/a^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))])))
/(147840*a^2*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [B] time = 0.046, size = 1773, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x)
```

```
[Out] c*(-1/11*a*(b*x^3+a)^(1/2)/x^11-25/176*b*(b*x^3+a)^(1/2)/x^8-27/1
760/a*b^2*(b*x^3+a)^(1/2)/x^5+189/7040/a^2*b^3*(b*x^3+a)^(1/2)/x^
2-63/7040*I*b^3/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
```

```
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)))+d*(-1/10*a*(b*x^3+a)^(1/2)/x^10-23/140*b*(b*x^3+a)^(
1/2)/x^7-27/1120/a*b^2*(b*x^3+a)^(1/2)/x^4+27/448/a^2*b^3*(b*x^3+
a)^(1/2)/x+9/448*I*b^3/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+e*(-1/9*a*(b*x^3+a)^(1/2)
/x^9-7/36*b*(b*x^3+a)^(1/2)/x^6-1/24/a*b^2*(b*x^3+a)^(1/2)/x^3+1/
24/a^(3/2)*b^3*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+f*(-1/8*a*(b*x^3
+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320/a*b^2*(b*x^3+a)^
(1/2)/x^2+9/320*I/a*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)))+g*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^
(1/2)/x^4-27/112/a*b^2*(b*x^3+a)^(1/2)/x-9/112*I/a*b^2*3^(1/2)*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12,x, algorithm="m
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^1
2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^{12}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12,x, algorithm="f
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2
 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^12, x)
```

Sympy [A] time = 38.6223, size = 541, normalized size = 0.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**12,x)
```

```
[Out] a**(3/2)*c*gamma(-11/3)*hyper((-11/3, -1/2), (-8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*x**11*gamma(-8/3)) + a**(3/2)*d*gamma(-10/3)*hy
per((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**10*ga
mma(-7/3)) + a**(3/2)*f*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*g*gamma
(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
x**7*gamma(-4/3)) + sqrt(a)*b*c*gamma(-8/3)*hyper((-8/3, -1/2), (
-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + sqrt(a)*
b*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*f*gamma(-5/3)*hyper((-5/3,
 -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) +
 sqrt(a)*b*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_
polar(I*pi)/a)/(3*x**4*gamma(-1/3)) - a**2*e/(9*sqrt(b)*x**(21/2)
*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*e/(36*x**(15/2)*sqrt(a/(b*x
**3) + 1)) - 17*b**(3/2)*e/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b
**(5/2)*e/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + b**3*e*asinh(sqr
t(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12,x, algorithm="g
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^1
2, x)
```

3.462 $\int (c+dx+ex^2)\,(a+bx^3)^p\,dx$

Optimal. Leaf size=102

$$\frac{cx\,(a+bx^3)^{p+1}\,{}_2F_1\left(1,p+\frac{4}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{a}+\frac{dx^2\,(a+bx^3)^{p+1}\,{}_2F_1\left(1,p+\frac{5}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}+\frac{e\,(a+bx^3)^{p+1}}{3b(p+1)}$$

```
[Out] (e*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x*(a + b*x^3)^(1 + p)*
Hypergeometric2F1[1, 4/3 + p, 4/3, -((b*x^3)/a)])/a + (d*x^2*(a +
 b*x^3)^(1 + p)*Hypergeometric2F1[1, 5/3 + p, 5/3, -((b*x^3)/a)])
/(2*a)
```

Rubi [A] time = 0.160852, antiderivative size = 120, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$cx\,(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-\frac{bx^3}{a}\right)+\frac{1}{2}dx^2\,(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+\frac{e\,(a+bx^3)^{p+1}}{3b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (e*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x*(a + b*x^3)^p*Hyperg
eometric2F1[1/3, -p, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p + (d*x
^2*(a + b*x^3)^p*Hypergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(
2*(1 + (b*x^3)/a)^p)
```

Rubi in Sympy [A] time = 23.0659, size = 95, normalized size = 0.93

$$cx\left(1+\frac{bx^3}{a}\right)^{-p}(a+bx^3)^p\,{}_2F_1\left(\left.\begin{matrix}-p,\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|-\frac{bx^3}{a}\right)+\frac{dx^2\left(1+\frac{bx^3}{a}\right)^{-p}(a+bx^3)^p\,{}_2F_1\left(\left.\begin{matrix}-p,\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|-\frac{bx^3}{a}\right)}{2}+\frac{e\,(a+bx^3)^{p+1}}{3b\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] c*x*(1 + b*x**3/a)**(-p)*(a + b*x**3)**p*hyper((-p, 1/3), (4/3,),
 -b*x**3/a) + d*x**2*(1 + b*x**3/a)**(-p)*(a + b*x**3)**p*hyper((
-p, 2/3), (5/3,), -b*x**3/a)/2 + e*(a + b*x**3)**(p + 1)/(3*b*(p
+ 1))
```

Mathematica [A] time = 0.156256, size = 129, normalized size = 1.26

$$\frac{(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\Bigl(6bc(p+1)x\,{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-\frac{bx^3}{a}\right)+3bd(p+1)x^2\,{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+2e\left(bx^3\left(\frac{bx^3}{a}+1\right)^p+a\Bigl(\Bigl(\cdots}{6b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(2*e*(b*x^3*(1 + (b*x^3)/a)^p + a*(-1 + (1 + (b*x^
3)/a)^p)) + 6*b*c*(1 + p)*x*Hypergeometric2F1[1/3, -p, 4/3, -((b*
x^3)/a)] + 3*b*d*(1 + p)*x^2*Hypergeometric2F1[2/3, -p, 5/3, -((b
*x^3)/a)]))/(6*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(e x^2+d x+c\right) \left(b x^3+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e x^2+d x+c\right) \left(b x^3+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e x^2+d x+c\right) \left(b x^3+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e x^2+d x+c\right) \left(b x^3+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

3.463 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx$

Optimal. Leaf size=107

$$\frac{cx^2\left(a+bx^3\right)^{p+1}{}_2F_1\left(1,p+\frac{5}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}+\frac{d\left(a+bx^3\right)^{p+1}}{3b(p+1)}+\frac{ex^4\left(a+bx^3\right)^{p+1}{}_2F_1\left(1,p+\frac{7}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4a}$$

```
[Out] (d*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x^2*(a + b*x^3)^(1 + p
)*Hypergeometric2F1[1, 5/3 + p, 5/3, -((b*x^3)/a)])/(2*a) + (e*x^
4*(a + b*x^3)^(1 + p)*Hypergeometric2F1[1, 7/3 + p, 7/3, -((b*x^3
)/a)])/(4*a)
```

Rubi [A] time = 0.191935, antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.19$

$$\frac{1}{2}cx^2\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+\frac{d\left(a+bx^3\right)^{p+1}}{3b(p+1)}+\frac{1}{4}ex^4\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (d*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x^2*(a + b*x^3)^p*Hype
rgeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p)
+ (e*x^4*(a + b*x^3)^p*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/
a)])/(4*(1 + (b*x^3)/a)^p)
```

Rubi in Sympy [A] time = 23.8972, size = 99, normalized size = 0.93

$$\frac{cx^2\left(1+\frac{bx^3}{a}\right)^{-p}\left(a+bx^3\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{2}+\frac{ex^4\left(1+\frac{bx^3}{a}\right)^{-p}\left(a+bx^3\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{4}+\frac{d\left(a+bx^3\right)^{p+1}}{3b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] c*x**2*(1 + b*x**3/a)**(-p)*(a + b*x**3)**p*hyper((-p, 2/3), (5/3
,), -b*x**3/a)/2 + e*x**4*(1 + b*x**3/a)**(-p)*(a + b*x**3)**p*hy
per((-p, 4/3), (7/3,), -b*x**3/a)/4 + d*(a + b*x**3)**(p + 1)/(3*
b*(p + 1))
```

Mathematica [A] time = 0.124217, size = 131, normalized size = 1.22

$$\frac{\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(6bc(p+1)x^2{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+4d\left(bx^3\left(\frac{bx^3}{a}+1\right)^p+a\left(\left(\frac{bx^3}{a}+1\right)^p-1\right)\right)+3be(p+1)x^4{}_2F_1\left(\right.\right.}{12b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(4*d*(b*x^3*(1 + (b*x^3)/a)^p + a*(-1 + (1 + (b*x^
3)/a)^p)) + 6*b*c*(1 + p)*x^2*Hypergeometric2F1[2/3, -p, 5/3, -((
b*x^3)/a)] + 3*b*e*(1 + p)*x^4*Hypergeometric2F1[4/3, -p, 7/3, -(
(b*x^3)/a)]))/(12*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int x\left(ex^{2}+dx+c\right)\left(bx^{3}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex^{2}+dx+c\right)\left(bx^{3}+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ex^{3}+dx^{2}+cx\right)\left(bx^{3}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x,x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x^2 + c*x)*(b*x^3 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex^{2}+dx+c\right)\left(bx^{3}+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x,x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)
```

3.464 $\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx$

Optimal. Leaf size=107

$$\frac{c\left(a+bx^3\right)^{p+1}}{3b(p+1)}+\frac{dx^4\left(a+bx^3\right)^{p+1}\,{}_2F_1\left(1,p+\frac{7}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4a}+\frac{ex^5\left(a+bx^3\right)^{p+1}\,{}_2F_1\left(1,p+\frac{8}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5a}$$

```
[Out] (c*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (d*x^4*(a + b*x^3)^(1 + p
)*Hypergeometric2F1[1, 7/3 + p, 7/3, -((b*x^3)/a)])/(4*a) + (e*x^
5*(a + b*x^3)^(1 + p)*Hypergeometric2F1[1, 8/3 + p, 8/3, -((b*x^3
)/a)])/(5*a)
```

Rubi [A] time = 0.225218, antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$

$$\frac{c\left(a+bx^3\right)^{p+1}}{3b(p+1)}+\frac{1}{4}dx^4\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)+\frac{1}{5}ex^5\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{5}{3},-p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (c*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (d*x^4*(a + b*x^3)^p*Hype
rgeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^p)
+ (e*x^5*(a + b*x^3)^p*Hypergeometric2F1[5/3, -p, 8/3, -((b*x^3)/
a)])/(5*(1 + (b*x^3)/a)^p)
```

Rubi in Sympy [A] time = 26.8163, size = 99, normalized size = 0.93

$$\frac{dx^4\left(1+\frac{bx^3}{a}\right)^{-p}\left(a+bx^3\right)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{4}+\frac{ex^5\left(1+\frac{bx^3}{a}\right)^{-p}\left(a+bx^3\right)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{5}+\frac{c\left(a+bx^3\right)^{p+1}}{3b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] d*x**4*(1 + b*x**3/a)**(-p)*(a + b*x**3)**p*hyper((-p, 4/3), (7/3
,), -b*x**3/a)/4 + e*x**5*(1 + b*x**3/a)**(-p)*(a + b*x**3)**p*hy
per((-p, 5/3), (8/3,), -b*x**3/a)/5 + c*(a + b*x**3)**(p + 1)/(3*
b*(p + 1))
```

Mathematica [A] time = 0.149455, size = 131, normalized size = 1.22

$$\frac{\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(20c\left(bx^3\left(\frac{bx^3}{a}+1\right)^p+a\left(\left(\frac{bx^3}{a}+1\right)^p-1\right)\right)+15bd(p+1)x^4\,{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)+12be(p+1)x^5\,{}_2\right.}{60b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(20*c*(b*x^3*(1 + (b*x^3)/a)^p + a*(-1 + (1 + (b*x
^3)/a)^p)) + 15*b*d*(1 + p)*x^4*Hypergeometric2F1[4/3, -p, 7/3, -
((b*x^3)/a)] + 12*b*e*(1 + p)*x^5*Hypergeometric2F1[5/3, -p, 8/3,
 -((b*x^3)/a)]))/(60*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int x^{2} \left(e x^{2}+d x +c \right) \left(b \,x^{3}+a \right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e x^{4} + d x^{3} + c x^{2}\right) \left(b x^{3} + a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3 + c*x^2)*(b*x^3 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e x^{2} + d x + c\right) \left(b x^{3} + a\right)^{p} x^{2}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x^2, x)
```

3.465 $\int (c + dx + ex^2 + fx^3) (a + bx^4) \, dx$

Optimal. Leaf size=68

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8$$

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (
b*d*x^6)/6 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Rubi [A] time = 0.0876593, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (
b*d*x^6)/6 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ad \int x \, dx + \frac{aex^3}{3} + \frac{afx^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7} + \frac{bfx^8}{8} + c \int a \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*d*Integral(x, x) + a*e*x**3/3 + a*f*x**4/4 + b*c*x**5/5 + b*d*x
**6/6 + b*e*x**7/7 + b*f*x**8/8 + c*Integral(a, x)
```

Mathematica [A] time = 0.00835988, size = 68, normalized size = 1.

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (
b*d*x^6)/6 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Maple [A] time = 0.001, size = 55, normalized size = 0.8

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{afx^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7} + \frac{bfx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6
+1/7*b*e*x^7+1/8*b*f*x^8
```

Maxima [A] time = 1.38915, size = 73, normalized size = 1.07

$$\frac{1}{8}bfx^8 + \frac{1}{7}bex^7 + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}afx^4 + \frac{1}{3}aex^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x
^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Fricas [A] time = 0.186994, size = 1, normalized size = 0.01

$$\frac{1}{8}x^8fb + \frac{1}{7}x^7eb + \frac{1}{6}x^6db + \frac{1}{5}x^5cb + \frac{1}{4}x^4fa + \frac{1}{3}x^3ea + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*f*b + 1/7*x^7*e*b + 1/6*x^6*d*b + 1/5*x^5*c*b + 1/4*x^4*f
*a + 1/3*x^3*e*a + 1/2*x^2*d*a + x*c*a
```

Sympy [A] time = 0.05271, size = 63, normalized size = 0.93

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{afx^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7} + \frac{bfx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + a*f*x**4/4 + b*c*x**5/5 + b*d*x
**6/6 + b*e*x**7/7 + b*f*x**8/8
```

GIAC/XCAS [A] time = 0.214752, size = 76, normalized size = 1.12

$$\frac{1}{8}bfx^8 + \frac{1}{7}bx^7e + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}afx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x
^4 + 1/3*a*x^3*e + 1/2*a*d*x^2 + a*c*x
```

3.466 $\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right) \, dx$

Optimal. Leaf size=73

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)
/8 + (b*d*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Rubi [A] time = 0.125718, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)
/8 + (b*d*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Rubi in Sympy [A] time = 17.9902, size = 66, normalized size = 0.9

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{afx^7}{7} + \frac{bcx^8}{8} + \frac{bdx^9}{9} + \frac{bex^{10}}{10} + \frac{bfx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + a*f*x**7/7 + b*c*x**8/8 +
b*d*x**9/9 + b*e*x**10/10 + b*f*x**11/11
```

Mathematica [A] time = 0.0045108, size = 73, normalized size = 1.

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)
/8 + (b*d*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{afx^7}{7} + \frac{bcx^8}{8} + \frac{bdx^9}{9} + \frac{bex^{10}}{10} + \frac{bfx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)
```

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b
*d*x^9+1/10*b*e*x^10+1/11*b*f*x^11
```

Maxima [A] time = 1.37417, size = 77, normalized size = 1.05

$$\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{9}\,bdx^{9}+\frac{1}{8}\,bcx^{8}+\frac{1}{7}\,afx^{7}+\frac{1}{6}\,aex^{6}+\frac{1}{5}\,adx^{5}+\frac{1}{4}\,acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="maxima")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a
*f*x^7 + 1/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Fricas [A] time = 0.191768, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}fb+\frac{1}{10}x^{10}eb+\frac{1}{9}x^{9}db+\frac{1}{8}x^{8}cb+\frac{1}{7}x^{7}fa+\frac{1}{6}x^{6}ea+\frac{1}{5}x^{5}da+\frac{1}{4}x^{4}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f*b + 1/10*x^10*e*b + 1/9*x^9*d*b + 1/8*x^8*c*b + 1/7*x
^7*f*a + 1/6*x^6*e*a + 1/5*x^5*d*a + 1/4*x^4*c*a
```

Sympy [A] time = 0.059197, size = 66, normalized size = 0.9

$$\frac{acx^{4}}{4}+\frac{adx^{5}}{5}+\frac{aex^{6}}{6}+\frac{afx^{7}}{7}+\frac{bcx^{8}}{8}+\frac{bdx^{9}}{9}+\frac{bex^{10}}{10}+\frac{bfx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + a*f*x**7/7 + b*c*x**8/8 +
b*d*x**9/9 + b*e*x**10/10 + b*f*x**11/11
```

GIAC/XCAS [A] time = 0.222835, size = 80, normalized size = 1.1

$$\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bx^{10}e+\frac{1}{9}\,bdx^{9}+\frac{1}{8}\,bcx^{8}+\frac{1}{7}\,afx^{7}+\frac{1}{6}\,ax^{6}e+\frac{1}{5}\,adx^{5}+\frac{1}{4}\,acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="giac")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*x^10*e + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a
*f*x^7 + 1/6*a*x^6*e + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

3.467 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx$

Optimal. Leaf size=109

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*
d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b
^2*e*x^11)/11 + (f*(a + b*x^4)^3)/(12*b)
```

Rubi [A] time = 0.144531, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*
d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b
^2*e*x^11)/11 + (f*(a + b*x^4)^3)/(12*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2d\int x\,dx+\frac{a^2ex^3}{3}+a^2\int c\,dx+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{f\left(a+bx^4\right)^3}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*d*Integral(x, x) + a**2*e*x**3/3 + a**2*Integral(c, x) + 2*a
*b*c*x**5/5 + a*b*d*x**6/3 + 2*a*b*e*x**7/7 + b**2*c*x**9/9 + b**
2*d*x**10/10 + b**2*e*x**11/11 + f*(a + b*x**4)**3/(12*b)
```

Mathematica [A] time = 0.00625311, size = 124, normalized size = 1.14

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*
c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2
*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Maple [A] time = 0., size = 103, normalized size = 0.9

$$\frac{b^2fx^{12}}{12}+\frac{b^2ex^{11}}{11}+\frac{b^2dx^{10}}{10}+\frac{b^2cx^9}{9}+\frac{fabx^8}{4}+\frac{2\,abex^7}{7}+\frac{abdx^6}{3}+\frac{2\,abcx^5}{5}+\frac{fa^2x^4}{4}+\frac{a^2ex^3}{3}+\frac{a^2dx^2}{2}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

```
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4
*f*a*b*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*f*a^2*x^
4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x
```

Maxima [A] time = 1.36978, size = 138, normalized size = 1.27

$$\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7 \\ +\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(f*x^3 + e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x
^9 + 1/4*a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^
5 + 1/4*a^2*f*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Fricas [A] time = 0.187411, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}fb^2+\frac{1}{11}x^{11}eb^2+\frac{1}{10}x^{10}db^2+\frac{1}{9}x^9cb^2+\frac{1}{4}x^8fba+\frac{2}{7}x^7eba \\ +\frac{1}{3}x^6dba+\frac{2}{5}x^5cba+\frac{1}{4}x^4fa^2+\frac{1}{3}x^3ea^2+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(f*x^3 + e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b
^2 + 1/4*x^8*f*b*a + 2/7*x^7*e*b*a + 1/3*x^6*d*b*a + 2/5*x^5*c*b*
a + 1/4*x^4*f*a^2 + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.078685, size = 121, normalized size = 1.11

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*
c*x**5/5 + a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*
x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

GIAC/XCAS [A] time = 0.228603, size = 142, normalized size = 1.3

$$\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2x^{11}e+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abx^7e \\ +\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2x^3e+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^2*(f*x^3 + e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x
^9 + 1/4*a*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^
5 + 1/4*a^2*f*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

3.468 $\int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^2 \, dx$

Optimal. Leaf size=114

$$\frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{7}a^2fx^7 + \frac{c(a+bx^4)^3}{12b} + \frac{2}{9}abdx^9 + \frac{1}{5}abex^{10} + \frac{2}{11}abfx^{11} + \frac{1}{13}b^2dx^{13} + \frac{1}{14}b^2ex^{14} + \frac{1}{15}b^2fx^{15}$$

```
[Out] (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 +
 (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^
14)/14 + (b^2*f*x^15)/15 + (c*(a + b*x^4)^3)/(12*b)
```

Rubi [A] time = 0.318637, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{7}a^2fx^7 + \frac{c(a+bx^4)^3}{12b} + \frac{2}{9}abdx^9 + \frac{1}{5}abex^{10} + \frac{2}{11}abfx^{11} + \frac{1}{13}b^2dx^{13} + \frac{1}{14}b^2ex^{14} + \frac{1}{15}b^2fx^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 +
 (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^
14)/14 + (b^2*f*x^15)/15 + (c*(a + b*x^4)^3)/(12*b)
```

Rubi in Sympy [A] time = 42.3562, size = 107, normalized size = 0.94

$$\frac{a^2dx^5}{5} + \frac{a^2ex^6}{6} + \frac{a^2fx^7}{7} + \frac{2abdx^9}{9} + \frac{abex^{10}}{5} + \frac{2abfx^{11}}{11} + \frac{b^2dx^{13}}{13} + \frac{b^2ex^{14}}{14} + \frac{b^2fx^{15}}{15} + \frac{c(a+bx^4)^3}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*d*x**5/5 + a**2*e*x**6/6 + a**2*f*x**7/7 + 2*a*b*d*x**9/9 +
a*b*e*x**10/5 + 2*a*b*f*x**11/11 + b**2*d*x**13/13 + b**2*e*x**14
/14 + b**2*f*x**15/15 + c*(a + b*x**4)**3/(12*b)
```

Mathematica [A] time = 0.00889073, size = 129, normalized size = 1.13

$$\frac{1}{4}a^2cx^4 + \frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{7}a^2fx^7 + \frac{1}{4}abcx^8 + \frac{2}{9}abdx^9 + \frac{1}{5}abex^{10} + \frac{2}{11}abfx^{11} + \frac{1}{12}b^2cx^{12} + \frac{1}{13}b^2dx^{13} + \frac{1}{14}b^2ex^{14} + \frac{1}{15}b^2fx^{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (
a*b*c*x^8)/4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/
11 + (b^2*c*x^12)/12 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f
*x^15)/15
```

Maple [A] time = 0.001, size = 106, normalized size = 0.9

$$\frac{b^2fx^{15}}{15}+\frac{b^2ex^{14}}{14}+\frac{b^2dx^{13}}{13}+\frac{b^2cx^{12}}{12}+\frac{2\,abfx^{11}}{11}+\frac{abex^{10}}{5}+\frac{2\,abdx^9}{9}+\frac{abcx^8}{4}+\frac{a^2fx^7}{7}+\frac{a^2ex^6}{6}+\frac{a^2dx^5}{5}+\frac{a^2cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

```
[Out] 1/15*b^2*f*x^15+1/14*b^2*e*x^14+1/13*b^2*d*x^13+1/12*b^2*c*x^12+2
/11*a*b*f*x^11+1/5*a*b*e*x^10+2/9*a*b*d*x^9+1/4*a*b*c*x^8+1/7*a^2
*f*x^7+1/6*a^2*e*x^6+1/5*a^2*d*x^5+1/4*a^2*c*x^4
```

Maxima [A] time = 1.37863, size = 142, normalized size = 1.25

$$\frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2ex^{14}+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*
x^12 + 2/11*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b
*c*x^8 + 1/7*a^2*f*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*
c*x^4
```

Fricas [A] time = 0.189671, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}fb^2+\frac{1}{14}x^{14}eb^2+\frac{1}{13}x^{13}db^2+\frac{1}{12}x^{12}cb^2+\frac{2}{11}x^{11}fba+\frac{1}{5}x^{10}eba+\frac{2}{9}x^9dba+\frac{1}{4}x^8cba+\frac{1}{7}x^7fa^2+\frac{1}{6}x^6ea^2+\frac{1}{5}x^5da^2+\frac{1}{4}x^4ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*f*b^2 + 1/14*x^14*e*b^2 + 1/13*x^13*d*b^2 + 1/12*x^12*c
*b^2 + 2/11*x^11*f*b*a + 1/5*x^10*e*b*a + 2/9*x^9*d*b*a + 1/4*x^8
*c*b*a + 1/7*x^7*f*a^2 + 1/6*x^6*e*a^2 + 1/5*x^5*d*a^2 + 1/4*x^4*
c*a^2
```

Sympy [A] time = 0.080523, size = 124, normalized size = 1.09

$$\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{a^2fx^7}{7}+\frac{abcx^8}{4}+\frac{2abdx^9}{9}+\frac{abex^{10}}{5}+\frac{2abfx^{11}}{11}+\frac{b^2cx^{12}}{12}+\frac{b^2dx^{13}}{13}+\frac{b^2ex^{14}}{14}+\frac{b^2fx^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + a**2*f*x**7/7 + a
*b*c*x**8/4 + 2*a*b*d*x**9/9 + a*b*e*x**10/5 + 2*a*b*f*x**11/11 +
 b**2*c*x**12/12 + b**2*d*x**13/13 + b**2*e*x**14/14 + b**2*f*x**
15/15
```

GIAC/XCAS [A] time = 0.222893, size = 146, normalized size = 1.28

$$\frac{1}{15}\,b^2fx^{15} + \frac{1}{14}\,b^2x^{14}e + \frac{1}{13}\,b^2dx^{13} + \frac{1}{12}\,b^2cx^{12} + \frac{2}{11}\,abfx^{11} + \frac{1}{5}\,abx^{10}e + \frac{2}{9}\,abdx^9 + \frac{1}{4}\,abcx^8 + \frac{1}{7}\,a^2fx^7 + \frac{1}{6}\,a^2x^6e + \frac{1}{5}\,a^2dx^5 + \frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^2*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="giac")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*x^14*e + 1/13*b^2*d*x^13 + 1/12*b^2*c*
x^12 + 2/11*a*b*f*x^11 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/4*a*b
*c*x^8 + 1/7*a^2*f*x^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*
c*x^4
```

3.469 $\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx$

Optimal. Leaf size=151

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^
2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x
^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14
 + (b^3*e*x^15)/15 + (f*(a + b*x^4)^4)/(16*b)
```

Rubi [A] time = 0.218757, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^
2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x
^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14
 + (b^3*e*x^15)/15 + (f*(a + b*x^4)^4)/(16*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3d\int x\,dx+\frac{a^3ex^3}{3}+a^3\int c\,dx+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{b^3cx^{13}}{13}+\frac{b^3dx^{14}}{14}+\frac{b^3ex^{15}}{15}+\frac{f\left(a+bx^4\right)^4}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*d*Integral(x, x) + a**3*e*x**3/3 + a**3*Integral(c, x) + 3*a
**2*b*c*x**5/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + a*b**2*c*x
**9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + b**3*c*x**13/
13 + b**3*d*x**14/14 + b**3*e*x**15/15 + f*(a + b*x**4)**4/(16*b)
```

Mathematica [A] time = 0.0074924, size = 180, normalized size = 1.19

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*
b*c*x^5)/5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8
)/8 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11
 + (a*b^2*f*x^12)/4 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*
x^15)/15 + (b^3*f*x^16)/16
```

Maple [A] time = 0., size = 151, normalized size = 1.

$$\frac{b^3fx^{16}}{16}+\frac{b^3ex^{15}}{15}+\frac{b^3dx^{14}}{14}+\frac{b^3cx^{13}}{13}+\frac{ab^2fx^{12}}{4}+\frac{3\,ab^2ex^{11}}{11}+\frac{3\,ab^2dx^{10}}{10}+\frac{ab^2cx^9}{3}+\frac{3\,fa^2bx^8}{8}+\frac{3\,a^2bex^7}{7}+\frac{a^2bdx^6}{2}+\frac{3\,a^2bcx^5}{5}+\frac{a^3fx^4}{4}+\frac{a^3ex^3}{3}+\frac{a^3dx^2}{2}+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/16*b^3*f*x^16+1/15*b^3*e*x^15+1/14*b^3*d*x^14+1/13*b^3*c*x^13+1
/4*a*b^2*f*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x
^9+3/8*f*a^2*b*x^8+3/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^
5+1/4*a^3*f*x^4+1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

Maxima [A] time = 7.00687, size = 203, normalized size = 1.34

$$\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^3*(f*x^3 + e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*
x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 +
 1/3*a*b^2*c*x^9 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*
d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3
*d*x^2 + a^3*c*x
```

Fricas [A] time = 0.189283, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}fb^3+\frac{1}{15}x^{15}eb^3+\frac{1}{14}x^{14}db^3+\frac{1}{13}x^{13}cb^3+\frac{1}{4}x^{12}fb^2a+\frac{3}{11}x^{11}eb^2a+\frac{3}{10}x^{10}db^2a+\frac{1}{3}x^9cb^2a+\frac{3}{8}x^8fba^2+\frac{3}{7}x^7eba^2+\frac{1}{2}x^6dba^2+\frac{3}{5}x^5cba^2+\frac{1}{4}x^4fa^3+\frac{1}{3}x^3ea^3+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^3*(f*x^3 + e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/14*x^14*d*b^3 + 1/13*x^13*c
*b^3 + 1/4*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a +
 1/3*x^9*c*b^2*a + 3/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*
b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*x^4*f*a^3 + 1/3*x^3*e*a^3 + 1/2*x^2
*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.096298, size = 180, normalized size = 1.19

$$a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{a^3fx^4}{4} + \frac{3a^2bcx^5}{5} + \frac{a^2bdx^6}{2} + \frac{3a^2bex^7}{7} + \frac{3a^2bfx^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{ab^2fx^{12}}{4} + \frac{b^3cx^{13}}{13} + \frac{b^3dx^{14}}{14} + \frac{b^3ex^{15}}{15} + \frac{b^3fx^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2
*b*c*x**5/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x*
*8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/1
1 + a*b**2*f*x**12/4 + b**3*c*x**13/13 + b**3*d*x**14/14 + b**3*e
*x**15/15 + b**3*f*x**16/16
```

GIAC/XCAS [A] time = 0.222526, size = 208, normalized size = 1.38

$$\frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3x^{15}e + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2x^{11}e + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bx^7e + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3x^3e + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^3*(f*x^3 + e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*
x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 +
 1/3*a*b^2*c*x^9 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*
d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3
*d*x^2 + a^3*c*x
```

3.470 $\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^3 \, dx$

Optimal. Leaf size=156

$$\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{c\left(a+bx^4\right)^4}{16b}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

```
[Out] (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 +
 (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13
+ (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3
*e*x^18)/18 + (b^3*f*x^19)/19 + (c*(a + b*x^4)^4)/(16*b)
```

Rubi [A] time = 0.406915, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{c\left(a+bx^4\right)^4}{16b}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 +
 (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13
+ (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3
*e*x^18)/18 + (b^3*f*x^19)/19 + (c*(a + b*x^4)^4)/(16*b)
```

Rubi in Sympy [A] time = 51.0076, size = 151, normalized size = 0.97

$$\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{a^3fx^7}{7}+\frac{a^2bdx^9}{3}+\frac{3a^2bex^{10}}{10}+\frac{3a^2bfx^{11}}{11}+\frac{3ab^2dx^{13}}{13}+\frac{3ab^2ex^{14}}{14}+\frac{ab^2fx^{15}}{5}+\frac{b^3dx^{17}}{17}+\frac{b^3ex^{18}}{18}+\frac{b^3fx^{19}}{19}+\frac{c\left(a+bx^4\right)^4}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*d*x**5/5 + a**3*e*x**6/6 + a**3*f*x**7/7 + a**2*b*d*x**9/3 +
 3*a**2*b*e*x**10/10 + 3*a**2*b*f*x**11/11 + 3*a*b**2*d*x**13/13
+ 3*a*b**2*e*x**14/14 + a*b**2*f*x**15/5 + b**3*d*x**17/17 + b**3
*e*x**18/18 + b**3*f*x**19/19 + c*(a + b*x**4)**4/(16*b)
```

Mathematica [A] time = 0.0085941, size = 185, normalized size = 1.19

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{3}{8}a^2bcx^8+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{1}{4}ab^2cx^{12}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{1}{16}b^3cx^{16}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (
3*a^2*b*c*x^8)/8 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2
*b*f*x^11)/11 + (a*b^2*c*x^12)/4 + (3*a*b^2*d*x^13)/13 + (3*a*b^2
*e*x^14)/14 + (a*b^2*f*x^15)/5 + (b^3*c*x^16)/16 + (b^3*d*x^17)/1
7 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
```

Maple [A] time = 0.002, size = 154, normalized size = 1.

$$\frac{b^3fx^{19}}{19}+\frac{b^3ex^{18}}{18}+\frac{b^3dx^{17}}{17}+\frac{b^3cx^{16}}{16}+\frac{ab^2fx^{15}}{5}+\frac{3\,ab^2ex^{14}}{14}+\frac{3\,ab^2dx^{13}}{13}+\frac{acb^2x^{12}}{4}+\frac{3\,a^2bfx^{11}}{11}+\frac{3\,a^2bex^{10}}{10}+\frac{a^2bdx^9}{3}+\frac{3\,a^2bcx^8}{8}+\frac{a^3fx^7}{7}+\frac{a^3ex^6}{6}+\frac{a^3dx^5}{5}+\frac{a^3cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/19*b^3*f*x^19+1/18*b^3*e*x^18+1/17*b^3*d*x^17+1/16*b^3*c*x^16+1
/5*a*b^2*f*x^15+3/14*a*b^2*e*x^14+3/13*a*b^2*d*x^13+1/4*a*c*b^2*x
^12+3/11*a^2*b*f*x^11+3/10*a^2*b*e*x^10+1/3*a^2*b*d*x^9+3/8*a^2*b
*c*x^8+1/7*a^3*f*x^7+1/6*a^3*e*x^6+1/5*a^3*d*x^5+1/4*a^3*c*x^4
```

Maxima [A] time = 1.3704, size = 207, normalized size = 1.33

$$\frac{1}{19}\,b^3fx^{19}+\frac{1}{18}\,b^3ex^{18}+\frac{1}{17}\,b^3dx^{17}+\frac{1}{16}\,b^3cx^{16}+\frac{1}{5}\,ab^2fx^{15}+\frac{3}{14}\,ab^2ex^{14}+\frac{3}{13}\,ab^2dx^{13}+\frac{1}{4}\,ab^2cx^{12}+\frac{3}{11}\,a^2bfx^{11}+\frac{3}{10}\,a^2bex^{10}+\frac{1}{3}\,a^2bdx^9+\frac{3}{8}\,a^2bcx^8+\frac{1}{7}\,a^3fx^7+\frac{1}{6}\,a^3ex^6+\frac{1}{5}\,a^3dx^5+\frac{1}{4}\,a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^3*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*
x^16 + 1/5*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 +
 1/4*a*b^2*c*x^12 + 3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a
^2*b*d*x^9 + 3/8*a^2*b*c*x^8 + 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/
5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Fricas [A] time = 0.19155, size = 1, normalized size = 0.01

$$\frac{1}{19}x^{19}fb^3+\frac{1}{18}x^{18}eb^3+\frac{1}{17}x^{17}db^3+\frac{1}{16}x^{16}cb^3+\frac{1}{5}x^{15}fb^2a+\frac{3}{14}x^{14}eb^2a+\frac{3}{13}x^{13}db^2a+\frac{1}{4}x^{12}cb^2a+\frac{3}{11}x^{11}fba^2+\frac{3}{10}x^{10}eba^2+\frac{1}{3}x^9dba^2+\frac{3}{8}x^8cba^2+\frac{1}{7}x^7fa^3+\frac{1}{6}x^6ea^3+\frac{1}{5}x^5da^3+\frac{1}{4}x^4ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^3*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*f*b^3 + 1/18*x^18*e*b^3 + 1/17*x^17*d*b^3 + 1/16*x^16*c
*b^3 + 1/5*x^15*f*b^2*a + 3/14*x^14*e*b^2*a + 3/13*x^13*d*b^2*a +
 1/4*x^12*c*b^2*a + 3/11*x^11*f*b*a^2 + 3/10*x^10*e*b*a^2 + 1/3*x
^9*d*b*a^2 + 3/8*x^8*c*b*a^2 + 1/7*x^7*f*a^3 + 1/6*x^6*e*a^3 + 1/
5*x^5*d*a^3 + 1/4*x^4*c*a^3
```

Sympy [A] time = 0.101382, size = 184, normalized size = 1.18

$$\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{a^3fx^7}{7}+\frac{3a^2bcx^8}{8}+\frac{a^2bdx^9}{3}+\frac{3a^2bex^{10}}{10}+\frac{3a^2bfx^{11}}{11}+\frac{ab^2cx^{12}}{4}+\frac{3ab^2dx^{13}}{13}+\frac{3ab^2ex^{14}}{14}+\frac{ab^2fx^{15}}{5}+\frac{b^3cx^{16}}{16}+\frac{b^3dx^{17}}{17}+\frac{b^3ex^{18}}{18}+\frac{b^3fx^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + a**3*f*x**7/7 + 3
*a**2*b*c*x**8/8 + a**2*b*d*x**9/3 + 3*a**2*b*e*x**10/10 + 3*a**2
*b*f*x**11/11 + a*b**2*c*x**12/4 + 3*a*b**2*d*x**13/13 + 3*a*b**2
*e*x**14/14 + a*b**2*f*x**15/5 + b**3*c*x**16/16 + b**3*d*x**17/1
7 + b**3*e*x**18/18 + b**3*f*x**19/19
```

GIAC/XCAS [A] time = 0.228499, size = 212, normalized size = 1.36

$$\frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3x^{18}e+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2x^{14}e+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bx^{10}e+\frac{1}{3}a^2bdx^9+\frac{3}{8}a^2bcx^8+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3x^6e+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^3*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="giac")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*x^18*e + 1/17*b^3*d*x^17 + 1/16*b^3*c*
x^16 + 1/5*a*b^2*f*x^15 + 3/14*a*b^2*x^14*e + 3/13*a*b^2*d*x^13 +
 1/4*a*b^2*c*x^12 + 3/11*a^2*b*f*x^11 + 3/10*a^2*b*x^10*e + 1/3*a
^2*b*d*x^9 + 3/8*a^2*b*c*x^8 + 1/7*a^3*f*x^7 + 1/6*a^3*x^6*e + 1/
5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

3.471 $\int (c + dx + ex^2 + fx^3) (a + bx^4)^4 dx$

Optimal. Leaf size=193

$$a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{4}{13}ab^3cx^{13} + \frac{2}{7}ab^3dx^{14} + \frac{4}{15}ab^3ex^{15} + \frac{f(a+bx^4)^5}{20b} + \frac{1}{17}b^4cx^{17} + \frac{1}{18}b^4dx^{18} + \frac{1}{19}b^4ex^{19}$$

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*
a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2
*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (4*a*b^3*c*x^13)/13 + (2
*a*b^3*d*x^14)/7 + (4*a*b^3*e*x^15)/15 + (b^4*c*x^17)/17 + (b^4*d
*x^18)/18 + (b^4*e*x^19)/19 + (f*(a + b*x^4)^5)/(20*b)
```

Rubi [A] time = 0.29869, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{4}{13}ab^3cx^{13} + \frac{2}{7}ab^3dx^{14} + \frac{4}{15}ab^3ex^{15} + \frac{f(a+bx^4)^5}{20b} + \frac{1}{17}b^4cx^{17} + \frac{1}{18}b^4dx^{18} + \frac{1}{19}b^4ex^{19}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*
a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2
*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (4*a*b^3*c*x^13)/13 + (2
*a*b^3*d*x^14)/7 + (4*a*b^3*e*x^15)/15 + (b^4*c*x^17)/17 + (b^4*d
*x^18)/18 + (b^4*e*x^19)/19 + (f*(a + b*x^4)^5)/(20*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4d\int x\,dx + \frac{a^4ex^3}{3} + a^4\int c\,dx + \frac{4a^3bcx^5}{5} + \frac{2a^3bdx^6}{3} + \frac{4a^3bex^7}{7} + \frac{2a^2b^2cx^9}{3} + \frac{3a^2b^2dx^{10}}{5} + \frac{6a^2b^2ex^{11}}{11} + \frac{4ab^3cx^{13}}{13} + \frac{2ab^3dx^{14}}{7} + \frac{4ab^3ex^{15}}{15} + \frac{b^4cx^{17}}{17} + \frac{b^4dx^{18}}{18} + \frac{b^4ex^{19}}{19} + \frac{f(a+bx^4)^5}{20b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*d*Integral(x, x) + a**4*e*x**3/3 + a**4*Integral(c, x) + 4*a
**3*b*c*x**5/5 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + 2*a**2*b
**2*c*x**9/3 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + 4
*a*b**3*c*x**13/13 + 2*a*b**3*d*x**14/7 + 4*a*b**3*e*x**15/15 + b
**4*c*x**17/17 + b**4*d*x**18/18 + b**4*e*x**19/19 + f*(a + b*x**
4)**5/(20*b)
```

Mathematica [A] time = 0.00941198, size = 236, normalized size = 1.22

$$a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + \frac{1}{4}a^4fx^4 + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{1}{2}a^3bfx^8 + \frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{1}{2}a^2b^2fx^{12} + \frac{4}{13}ab^3cx^{13} + \frac{2}{7}ab^3dx^{14} + \frac{4}{15}ab^3ex^{15} + \frac{1}{4}ab^3fx^{16} + \frac{1}{17}b^4cx^{17} + \frac{1}{18}b^4dx^{18} + \frac{1}{19}b^4ex^{19} + \frac{1}{20}b^4fx^{20}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (a^4*f*x^4)/4 + (4*a^3*
b*c*x^5)/5 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (a^3*b*f*x^8
)/2 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x
^11)/11 + (a^2*b^2*f*x^12)/2 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x
^14)/7 + (4*a*b^3*e*x^15)/15 + (a*b^3*f*x^16)/4 + (b^4*c*x^17)/17
 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (b^4*f*x^20)/20
```

Maple [A]   time = 0.002, size = 199, normalized size = 1.

$$\frac{fb^4x^{20}}{20}+\frac{b^4ex^{19}}{19}+\frac{b^4dx^{18}}{18}+\frac{b^4cx^{17}}{17}+\frac{fab^3x^{16}}{4}+\frac{4\,ab^3ex^{15}}{15}+\frac{2\,ab^3dx^{14}}{7}+\frac{4\,ab^3cx^{13}}{13}+\frac{fa^2b^2x^{12}}{2}+\frac{6\,a^2b^2ex^{11}}{11}+\frac{3\,a^2b^2dx^{10}}{5}+\frac{2\,a^2b^2cx^9}{3}+\frac{fba^3x^8}{2}+\frac{4\,a^3bex^7}{7}+\frac{2\,a^3bdx^6}{3}+\frac{4\,a^3bcx^5}{5}+\frac{a^4fx^4}{4}+\frac{a^4ex^3}{3}+\frac{a^4dx^2}{2}+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

```
[Out] 1/20*f*b^4*x^20+1/19*b^4*e*x^19+1/18*b^4*d*x^18+1/17*b^4*c*x^17+1
/4*f*a*b^3*x^16+4/15*a*b^3*e*x^15+2/7*a*b^3*d*x^14+4/13*a*b^3*c*x
^13+1/2*f*a^2*b^2*x^12+6/11*a^2*b^2*e*x^11+3/5*a^2*b^2*d*x^10+2/3
*a^2*b^2*c*x^9+1/2*f*b*a^3*x^8+4/7*a^3*b*e*x^7+2/3*a^3*b*d*x^6+4/
5*a^3*b*c*x^5+1/4*a^4*f*x^4+1/3*a^4*e*x^3+1/2*a^4*d*x^2+a^4*c*x
```

Maxima [A]   time = 1.3726, size = 267, normalized size = 1.38

$$\frac{1}{20}\,b^4fx^{20}+\frac{1}{19}\,b^4ex^{19}+\frac{1}{18}\,b^4dx^{18}+\frac{1}{17}\,b^4cx^{17}+\frac{1}{4}\,ab^3fx^{16}+\frac{4}{15}\,ab^3ex^{15}+\frac{2}{7}\,ab^3dx^{14}+\frac{4}{13}\,ab^3cx^{13}+\frac{1}{2}\,a^2b^2fx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bfx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4fx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^4*(f*x^3 + e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*
x^17 + 1/4*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 +
4/13*a*b^3*c*x^13 + 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/
5*a^2*b^2*d*x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*
b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5 + 1/4*a^4*f*x^4 + 1/3
*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Fricas [A]   time = 0.185216, size = 1, normalized size = 0.01

$$\frac{1}{20}x^{20}fb^4+\frac{1}{19}x^{19}eb^4+\frac{1}{18}x^{18}db^4+\frac{1}{17}x^{17}cb^4+\frac{1}{4}x^{16}fb^3a+\frac{4}{15}x^{15}eb^3a+\frac{2}{7}x^{14}db^3a+\frac{4}{13}x^{13}cb^3a+\frac{1}{2}x^{12}fb^2a^2+\frac{6}{11}x^{11}eb^2a^2+\frac{3}{5}x^{10}db^2a^2+\frac{2}{3}x^9cb^2a^2+\frac{1}{2}x^8fba^3+\frac{4}{7}x^7eba^3+\frac{2}{3}x^6dba^3+\frac{4}{5}x^5cba^3+\frac{1}{4}x^4fa^4+\frac{1}{3}x^3ea^4+\frac{1}{2}x^2da^4+xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^4*(f*x^3 + e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] 1/20*x^20*f*b^4 + 1/19*x^19*e*b^4 + 1/18*x^18*d*b^4 + 1/17*x^17*c
*b^4 + 1/4*x^16*f*b^3*a + 4/15*x^15*e*b^3*a + 2/7*x^14*d*b^3*a +
4/13*x^13*c*b^3*a + 1/2*x^12*f*b^2*a^2 + 6/11*x^11*e*b^2*a^2 + 3/
5*x^10*d*b^2*a^2 + 2/3*x^9*c*b^2*a^2 + 1/2*x^8*f*b*a^3 + 4/7*x^7*
e*b*a^3 + 2/3*x^6*d*b*a^3 + 4/5*x^5*c*b*a^3 + 1/4*x^4*f*a^4 + 1/3
*x^3*e*a^4 + 1/2*x^2*d*a^4 + x*c*a^4
```

Sympy [A] time = 0.115817, size = 241, normalized size = 1.25

$$a^{4}cx + \frac{a^{4}dx^{2}}{2} + \frac{a^{4}ex^{3}}{3} + \frac{a^{4}fx^{4}}{4} + \frac{4a^{3}bcx^{5}}{5} + \frac{2a^{3}bdx^{6}}{3} + \frac{4a^{3}bex^{7}}{7} + \frac{a^{3}bfx^{8}}{2} + \frac{2a^{2}b^{2}cx^{9}}{3} + \frac{3a^{2}b^{2}dx^{10}}{5} + \frac{6a^{2}b^{2}ex^{11}}{11} + \frac{a^{2}b^{2}fx^{12}}{2} + \frac{4ab^{3}cx^{13}}{13} + \frac{2ab^{3}dx^{14}}{7} + \frac{4ab^{3}ex^{15}}{15} + \frac{ab^{3}fx^{16}}{4} + \frac{b^{4}cx^{17}}{17} + \frac{b^{4}dx^{18}}{18} + \frac{b^{4}ex^{19}}{19} + \frac{b^{4}fx^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**4*f*x**4/4 + 4*a**3
*b*c*x**5/5 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + a**3*b*f*x*
*8/2 + 2*a**2*b**2*c*x**9/3 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2
*e*x**11/11 + a**2*b**2*f*x**12/2 + 4*a*b**3*c*x**13/13 + 2*a*b**
3*d*x**14/7 + 4*a*b**3*e*x**15/15 + a*b**3*f*x**16/4 + b**4*c*x**
17/17 + b**4*d*x**18/18 + b**4*e*x**19/19 + b**4*f*x**20/20
```

GIAC/XCAS [A] time = 0.228286, size = 274, normalized size = 1.42

$$\frac{1}{20}\,b^{4}fx^{20} + \frac{1}{19}\,b^{4}x^{19}e + \frac{1}{18}\,b^{4}dx^{18} + \frac{1}{17}\,b^{4}cx^{17} + \frac{1}{4}\,ab^{3}fx^{16} + \frac{4}{15}\,ab^{3}x^{15}e + \frac{2}{7}\,ab^{3}dx^{14} + \frac{4}{13}\,ab^{3}cx^{13} + \frac{1}{2}\,a^{2}b^{2}fx^{12} + \frac{6}{11}\,a^{2}b^{2}x^{11}e + \frac{3}{5}\,a^{2}b^{2}dx^{10} + \frac{2}{3}\,a^{2}b^{2}cx^{9} + \frac{1}{2}\,a^{3}bfx^{8} + \frac{4}{7}\,a^{3}bx^{7}e + \frac{2}{3}\,a^{3}bdx^{6} + \frac{4}{5}\,a^{3}bcx^{5} + \frac{1}{4}\,a^{4}fx^{4} + \frac{1}{3}\,a^{4}x^{3}e + \frac{1}{2}\,a^{4}dx^{2} + a^{4}cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^4*(f*x^3 + e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*x^19*e + 1/18*b^4*d*x^18 + 1/17*b^4*c*
x^17 + 1/4*a*b^3*f*x^16 + 4/15*a*b^3*x^15*e + 2/7*a*b^3*d*x^14 +
4/13*a*b^3*c*x^13 + 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*x^11*e + 3/
5*a^2*b^2*d*x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*
b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5 + 1/4*a^4*f*x^4 + 1/3
*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x
```

3.472 $\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^4 \, dx$

Optimal. Leaf size=198

$$\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{c\left(a+bx^4\right)^5}{20b}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

```
[Out] (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9
 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/
13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^1
7)/17 + (2*a*b^3*e*x^18)/9 + (4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/2
1 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 + (c*(a + b*x^4)^5)/(20*b)
```

Rubi [A] time = 0.495861, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{c\left(a+bx^4\right)^5}{20b}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9
 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/
13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^1
7)/17 + (2*a*b^3*e*x^18)/9 + (4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/2
1 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 + (c*(a + b*x^4)^5)/(20*b)
```

Rubi in Sympy [A] time = 64.7355, size = 201, normalized size = 1.02

$$\frac{a^4dx^5}{5}+\frac{a^4ex^6}{6}+\frac{a^4fx^7}{7}+\frac{4a^3bdx^9}{9}+\frac{2a^3bex^{10}}{5}+\frac{4a^3bfx^{11}}{11}+\frac{6a^2b^2dx^{13}}{13}+\frac{3a^2b^2ex^{14}}{7}+\frac{2a^2b^2fx^{15}}{5}+\frac{4ab^3dx^{17}}{17}+\frac{2ab^3ex^{18}}{9}+\frac{4ab^3fx^{19}}{19}+\frac{b^4dx^{21}}{21}+\frac{b^4ex^{22}}{22}+\frac{b^4fx^{23}}{23}+\frac{c\left(a+bx^4\right)^5}{20b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*d*x**5/5 + a**4*e*x**6/6 + a**4*f*x**7/7 + 4*a**3*b*d*x**9/9
 + 2*a**3*b*e*x**10/5 + 4*a**3*b*f*x**11/11 + 6*a**2*b**2*d*x**13
/13 + 3*a**2*b**2*e*x**14/7 + 2*a**2*b**2*f*x**15/5 + 4*a*b**3*d*
x**17/17 + 2*a*b**3*e*x**18/9 + 4*a*b**3*f*x**19/19 + b**4*d*x**2
1/21 + b**4*e*x**22/22 + b**4*f*x**23/23 + c*(a + b*x**4)**5/(20*
b)
```

Mathematica [A] time = 0.0101972, size = 241, normalized size = 1.22

$$\frac{1}{4}a^4cx^4+\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{1}{2}a^3bcx^8+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{1}{2}a^2b^2cx^{12}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{1}{4}ab^3cx^{16}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{1}{20}b^4cx^{20}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (
a^3*b*c*x^8)/2 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*
b*f*x^11)/11 + (a^2*b^2*c*x^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^
2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)/5 + (a*b^3*c*x^16)/4 + (4*a*
b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (4*a*b^3*f*x^19)/19 + (b^4*
c*x^20)/20 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23
```

Maple [A] time = 0.003, size = 202, normalized size = 1.

$$\frac{b^4fx^{23}}{23}+\frac{b^4ex^{22}}{22}+\frac{b^4dx^{21}}{21}+\frac{b^4cx^{20}}{20}+\frac{4\,ab^3fx^{19}}{19}+\frac{2\,ab^3ex^{18}}{9}+\frac{4\,ab^3dx^{17}}{17}+\frac{acb^3x^{16}}{4}+\frac{2\,a^2b^2fx^{15}}{5}+\frac{3\,a^2b^2ex^{14}}{7}+\frac{6\,a^2b^2dx^{13}}{13}+\frac{a^2b^2cx^{12}}{2}+\frac{4\,a^3bfx^{11}}{11}+\frac{2\,a^3bex^{10}}{5}+\frac{4\,a^3bdx^9}{9}+\frac{ca^3bx^8}{2}+\frac{a^4fx^7}{7}+\frac{a^4ex^6}{6}+\frac{a^4dx^5}{5}+\frac{a^4cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

```
[Out] 1/23*b^4*f*x^23+1/22*b^4*e*x^22+1/21*b^4*d*x^21+1/20*b^4*c*x^20+4
/19*a*b^3*f*x^19+2/9*a*b^3*e*x^18+4/17*a*b^3*d*x^17+1/4*a*c*b^3*x
^16+2/5*a^2*b^2*f*x^15+3/7*a^2*b^2*e*x^14+6/13*a^2*b^2*d*x^13+1/2
*a^2*b^2*c*x^12+4/11*a^3*b*f*x^11+2/5*a^3*b*e*x^10+4/9*a^3*b*d*x^
9+1/2*c*a^3*b*x^8+1/7*a^4*f*x^7+1/6*a^4*e*x^6+1/5*a^4*d*x^5+1/4*a
^4*c*x^4
```

Maxima [A] time = 1.37761, size = 271, normalized size = 1.37

$$\frac{1}{23}\,b^4fx^{23}+\frac{1}{22}\,b^4ex^{22}+\frac{1}{21}\,b^4dx^{21}+\frac{1}{20}\,b^4cx^{20}+\frac{4}{19}\,ab^3fx^{19}+\frac{2}{9}\,ab^3ex^{18}+\frac{4}{17}\,ab^3dx^{17}+\frac{1}{4}\,ab^3cx^{16}+\frac{2}{5}\,a^2b^2fx^{15}+\frac{3}{7}\,a^2b^2ex^{14}+\frac{6}{13}\,a^2b^2dx^{13}+\frac{1}{2}\,a^2b^2cx^{12}+\frac{4}{11}\,a^3bfx^{11}+\frac{2}{5}\,a^3bex^{10}+\frac{4}{9}\,a^3bdx^9+\frac{1}{2}\,a^3bcx^8+\frac{1}{7}\,a^4fx^7+\frac{1}{6}\,a^4ex^6+\frac{1}{5}\,a^4dx^5+\frac{1}{4}\,a^4cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^4*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="maxima")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*
x^20 + 4/19*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 +
 1/4*a*b^3*c*x^16 + 2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/1
3*a^2*b^2*d*x^13 + 1/2*a^2*b^2*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a
^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*a^3*b*c*x^8 + 1/7*a^4*f*x^7 +
 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

Fricas [A] time = 0.187893, size = 1, normalized size = 0.01

$$\frac{1}{23}x^{23}fb^4+\frac{1}{22}x^{22}eb^4+\frac{1}{21}x^{21}db^4+\frac{1}{20}x^{20}cb^4+\frac{4}{19}x^{19}fb^3a+\frac{2}{9}x^{18}eb^3a+\frac{4}{17}x^{17}db^3a+\frac{1}{4}x^{16}cb^3a+\frac{2}{5}x^{15}fb^2a^2+\frac{3}{7}x^{14}eb^2a^2+\frac{6}{13}x^{13}db^2a^2+\frac{1}{2}x^{12}cb^2a^2+\frac{4}{11}x^{11}fba^3+\frac{2}{5}x^{10}eba^3+\frac{4}{9}x^9dba^3+\frac{1}{2}x^8cba^3+\frac{1}{7}x^7fa^4+\frac{1}{6}x^6ea^4+\frac{1}{5}x^5da^4+\frac{1}{4}x^4ca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^4*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*f*b^4 + 1/22*x^22*e*b^4 + 1/21*x^21*d*b^4 + 1/20*x^20*c
*b^4 + 4/19*x^19*f*b^3*a + 2/9*x^18*e*b^3*a + 4/17*x^17*d*b^3*a +
 1/4*x^16*c*b^3*a + 2/5*x^15*f*b^2*a^2 + 3/7*x^14*e*b^2*a^2 + 6/1
3*x^13*d*b^2*a^2 + 1/2*x^12*c*b^2*a^2 + 4/11*x^11*f*b*a^3 + 2/5*x
^10*e*b*a^3 + 4/9*x^9*d*b*a^3 + 1/2*x^8*c*b*a^3 + 1/7*x^7*f*a^4 +
 1/6*x^6*e*a^4 + 1/5*x^5*d*a^4 + 1/4*x^4*c*a^4
```

Sympy [A] time = 0.114462, size = 245, normalized size = 1.24

$$\frac{a^4cx^4}{4}+\frac{a^4dx^5}{5}+\frac{a^4ex^6}{6}+\frac{a^4fx^7}{7}+\frac{a^3bcx^8}{2}+\frac{4a^3bdx^9}{9}+\frac{2a^3bex^{10}}{5}+\frac{4a^3bfx^{11}}{11}+\frac{a^2b^2cx^{12}}{2}+\frac{6a^2b^2dx^{13}}{13}+\frac{3a^2b^2ex^{14}}{7}+\frac{2a^2b^2fx^{15}}{5}+\frac{ab^3cx^{16}}{4}+\frac{4ab^3dx^{17}}{17}+\frac{2ab^3ex^{18}}{9}+\frac{4ab^3fx^{19}}{19}+\frac{b^4cx^{20}}{20}+\frac{b^4dx^{21}}{21}+\frac{b^4ex^{22}}{22}+\frac{b^4fx^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x**4/4 + a**4*d*x**5/5 + a**4*e*x**6/6 + a**4*f*x**7/7 + a
**3*b*c*x**8/2 + 4*a**3*b*d*x**9/9 + 2*a**3*b*e*x**10/5 + 4*a**3*
b*f*x**11/11 + a**2*b**2*c*x**12/2 + 6*a**2*b**2*d*x**13/13 + 3*a
**2*b**2*e*x**14/7 + 2*a**2*b**2*f*x**15/5 + a*b**3*c*x**16/4 + 4
*a*b**3*d*x**17/17 + 2*a*b**3*e*x**18/9 + 4*a*b**3*f*x**19/19 + b
**4*c*x**20/20 + b**4*d*x**21/21 + b**4*e*x**22/22 + b**4*f*x**23
/23
```

GIAC/XCAS [A] time = 0.22603, size = 278, normalized size = 1.4

$$\frac{1}{23}b^4fx^{23}+\frac{1}{22}b^4x^{22}e+\frac{1}{21}b^4dx^{21}+\frac{1}{20}b^4cx^{20}+\frac{4}{19}ab^3fx^{19}+\frac{2}{9}ab^3x^{18}e+\frac{4}{17}ab^3dx^{17}+\frac{1}{4}ab^3cx^{16}+\frac{2}{5}a^2b^2fx^{15}+\frac{3}{7}a^2b^2x^{14}e+\frac{6}{13}a^2b^2dx^{13}+\frac{1}{2}a^2b^2cx^{12}+\frac{4}{11}a^3bfx^{11}+\frac{2}{5}a^3bx^{10}e+\frac{4}{9}a^3bdx^9+\frac{1}{2}a^3bcx^8+\frac{1}{7}a^4fx^7+\frac{1}{6}a^4x^6e+\frac{1}{5}a^4dx^5+\frac{1}{4}a^4cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^4*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="giac")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*x^22*e + 1/21*b^4*d*x^21 + 1/20*b^4*c*
x^20 + 4/19*a*b^3*f*x^19 + 2/9*a*b^3*x^18*e + 4/17*a*b^3*d*x^17 +
 1/4*a*b^3*c*x^16 + 2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*x^14*e + 6/1
3*a^2*b^2*d*x^13 + 1/2*a^2*b^2*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a
^3*b*x^10*e + 4/9*a^3*b*d*x^9 + 1/2*a^3*b*c*x^8 + 1/7*a^4*f*x^7 +
 1/6*a^4*x^6*e + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

3.473 $\int \frac{c+dx+ex^2+fx^3}{a-bx^4} dx$

Optimal. Leaf size=133

$$\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*
b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Rubi [A] time = 0.282279, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*
b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Rubi in Sympy [A] time = 38.7046, size = 119, normalized size = 0.89

$$-\frac{f\log\left(a-bx^4\right)}{4b}+\frac{d\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{a}e-\sqrt{b}c\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -f*log(a - b*x**4)/(4*b) + d*atanh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(
a)*sqrt(b)) - (sqrt(a)*e - sqrt(b)*c)*atan(b**(1/4)*x/a**(1/4))/(
2*a**(3/4)*b**(3/4)) + (sqrt(a)*e + sqrt(b)*c)*atanh(b**(1/4)*x/a
**(1/4))/(2*a**(3/4)*b**(3/4))
```

Mathematica [A] time = 0.110883, size = 214, normalized size = 1.61

$$-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{3/4}e+\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d\right)}{4ab^{3/4}}-\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-a^{3/4}e-\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d\right)}{4ab^{3/4}}+\frac{\left(\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}}+\frac{d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] ((a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*
a*b^(3/4)) - ((a^(1/4)*Sqrt[b]*c + Sqrt[a]*b^(1/4)*d + a^(3/4)*e)
*Log[a^(1/4) - b^(1/4)*x])/(4*a*b^(3/4)) - ((-(a^(1/4)*Sqrt[b]*c)
 + Sqrt[a]*b^(1/4)*d - a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/(4*a*
b^(3/4)) + (d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*Sqrt[a]*Sqrt[b]) - (
f*Log[a - b*x^4])/(4*b)
```

Maple [A] time = 0.006, size = 177, normalized size = 1.3

$$\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{d}{4}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{e}{2b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e}{4b}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{f\ln\left(bx^4-a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*c*(a/
b)^(1/4)/a*arctan(x/(a/b)^(1/4))-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*
b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/2*e/b/(a/b)^(1/4)*arctan(x/(a/b
)^(1/4))+1/4*e/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-
1/4/b*f*ln(b*x^4-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 25.6382, size = 952, normalized size = 7.16

$$-\operatorname{RootSum}\Bigg(256t^4a^3b^4-256t^3a^3b^3f+t^2\left(96a^3b^2f^2-64a^2b^3ce-32a^2b^3d^2\right)+t\left(-16a^3bf^3+32a^2b^2cef+16a^2b^2d^2f-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3*f + _t**2*(96*
a**3*b**2*f**2 - 64*a**2*b**3*c*e - 32*a**2*b**3*d**2) + _t*(-16*
a**3*b*f**3 + 32*a**2*b**2*c*e*f + 16*a**2*b**2*d**2*f - 16*a**2*
b**2*d*e**2 - 16*a*b**3*c**2*d) + a**3*f**4 - 4*a**2*b*c*e*f**2 -
 2*a**2*b*d**2*f**2 + 4*a**2*b*d*e**2*f - a**2*b*e**4 + 4*a*b**2*
c**2*d*f + 2*a*b**2*c**2*e**2 - 4*a*b**2*c*d**2*e + a*b**2*d**4 -
 b**3*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b**3*e**3 - 64*
_t**3*a**3*b**4*c**2*e + 128*_t**3*a**3*b**4*c*d**2 + 48*_t**2*a*
*4*b**2*e**3*f + 48*_t**2*a**3*b**3*c**2*e*f - 96*_t**2*a**3*b**3
*c*d**2*f + 48*_t**2*a**3*b**3*c*d*e**2 - 32*_t**2*a**3*b**3*d**3
*e - 16*_t**2*a**2*b**4*c**3*d - 12*_t*a**4*b*e**3*f**2 - 12*_t*a
**3*b**2*c**2*e*f**2 + 24*_t*a**3*b**2*c*d**2*f**2 - 24*_t*a**3*b
**2*c*d*e**2*f + 12*_t*a**3*b**2*c*e**4 + 16*_t*a**3*b**2*d**3*e*
f + 12*_t*a**3*b**2*d**2*e**3 + 8*_t*a**2*b**3*c**3*d*f + 16*_t*a
**2*b**3*c**3*e**2 - 36*_t*a**2*b**3*c**2*d**2*e - 8*_t*a**2*b**3
*c*d**4 + 4*_t*a*b**4*c**5 + a**4*e**3*f**3 + a**3*b*c**2*e*f**3
- 2*a**3*b*c*d**2*f**3 + 3*a**3*b*c*d*e**2*f**2 - 3*a**3*b*c*e**4
*f - 2*a**3*b*d**3*e*f**2 - 3*a**3*b*d**2*e**3*f + 3*a**3*b*d*e**
5 - a**2*b**2*c**3*d*f**2 - 4*a**2*b**2*c**3*e**2*f + 9*a**2*b**2
*c**2*d**2*e*f + 2*a**2*b**2*c*d**4*f - 5*a**2*b**2*c*d**3*e**2 +
 2*a**2*b**2*d**5*e - a*b**3*c**5*f + 5*a*b**3*c**4*d*e - 5*a*b**
3*c**3*d**3)/(a**3*b*e**6 + a**2*b**2*c**2*e**4 - 8*a**2*b**2*c*d
**2*e**3 + 4*a**2*b**2*d**4*e**2 - a*b**3*c**4*e**2 + 8*a*b**3*c*
*3*d**2*e - 4*a*b**3*c**2*d**4 - b**4*c**6))))
```

GIAC/XCAS [A] time = 0.242442, size = 419, normalized size = 3.15

$$-\frac{f\ln\left(\left|bx^4-a\right|\right)}{4\,b}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x^3 + e*x^2 + d*x + c)/(b*x^4 - a),x, algorithm="giac")
```

```
[Out] -1/4*f*ln(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2
*d - (-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(-a*b)*b^2*d - (-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))
/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*
ln(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(
2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(a*b^3)
```

3.474 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}$$

```
[Out] -((d*x)/b) - (e*x^2)/(2*b) - (f*x^3)/(3*b) + (a^(1/4)*(Sqrt[b]*d
- Sqrt[a]*f)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (a^(1/4)*
(Sqrt[b]*d + Sqrt[a]*f)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4))
 + (Sqrt[a]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)) - (c*Lo
g[a - b*x^4])/(4*b)
```

Rubi [A] time = 0.483711, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31

$$\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] -((d*x)/b) - (e*x^2)/(2*b) - (f*x^3)/(3*b) + (a^(1/4)*(Sqrt[b]*d
- Sqrt[a]*f)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (a^(1/4)*
(Sqrt[b]*d + Sqrt[a]*f)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4))
 + (Sqrt[a]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)) - (c*Lo
g[a - b*x^4])/(4*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}f-\sqrt{b}d\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{\frac{7}{4}}}+\frac{\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{\frac{7}{4}}}+\frac{\sqrt{a}e\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-\frac{fx^3}{3b}-\frac{\int^{x^2}e\,dx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -a**(1/4)*(sqrt(a)*f - sqrt(b)*d)*atan(b**(1/4)*x/a**(1/4))/(2*b*
*(7/4)) + a**(1/4)*(sqrt(a)*f + sqrt(b)*d)*atanh(b**(1/4)*x/a**(1
/4))/(2*b**(7/4)) + sqrt(a)*e*atanh(sqrt(b)*x**2/sqrt(a))/(2*b**(
3/2)) - c*log(a - b*x**4)/(4*b) - d*x/b - f*x**3/(3*b) - Integral
(e, (x, x**2))/(2*b)
```

Mathematica [A] time = 0.149653, size = 221, normalized size = 1.36

$$\frac{-3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{3/4}f+\sqrt[4]{a}\sqrt{b}d+\sqrt{a}\sqrt[4]{b}e\right)+3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{3/4}f+\sqrt[4]{a}\sqrt{b}d-\sqrt{a}\sqrt[4]{b}e\right)+6\left(\sqrt[4]{a}\sqrt{b}d-a^{3/4}f\right)\tan}{12b^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*d*x - 6*b^(3/4)*e*x^2 - 4*b^(3/4)*f*x^3 + 6*(a^(1/4)
*Sqrt[b]*d - a^(3/4)*f)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(a^(1/4)*
Sqrt[b]*d + Sqrt[a]*b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) - b^(1/4)*
x] + 3*(a^(1/4)*Sqrt[b]*d - Sqrt[a]*b^(1/4)*e + a^(3/4)*f)*Log[a^
(1/4) + b^(1/4)*x] + 3*Sqrt[a]*b^(1/4)*e*Log[Sqrt[a] + Sqrt[b]*x^
2] - 3*b^(3/4)*c*Log[a - b*x^4])/(12*b^(7/4))
```

Maple [A] time = 0.006, size = 208, normalized size = 1.3

$$-\frac{fx^3}{3b}-\frac{ex^2}{2b}-\frac{dx}{b}+\frac{d}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{d}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{ae}{4b}\ln\left(1\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{af}{2b^2}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{af}{4b^2}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{c\ln\left(bx^4-a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/3*f*x^3/b-1/2*e*x^2/b-d*x/b+1/2/b*d*(a/b)^(1/4)*arctan(x/(a/b)
^(1/4))+1/4/b*d*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1
/4/b*a*e/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2))
)-1/2/b^2*a*f/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))+1/4/b^2*a*f/(a/b)
^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b*c*ln(b*x^4-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 - a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 24.244, size = 887, normalized size = 5.48

$$-\operatorname{RootSum}\left(256t^4b^7 - 256t^3b^6c + t^2\left(-64ab^4df - 32ab^4e^2 + 96b^5c^2\right) + t\left(-16a^2b^2ef^2 + 32ab^3cdf + 16ab^3ce^2 - 16ab^3d^2e\right.\right.$$
$$-\frac{dx}{b} - \frac{ex^2}{2b} - \frac{fx^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*b**7 - 256*_t**3*b**6*c + _t**2*(-64*a*b**4*d*
f - 32*a*b**4*e**2 + 96*b**5*c**2) + _t*(-16*a**2*b**2*e*f**2 + 3
2*a*b**3*c*d*f + 16*a*b**3*c*e**2 - 16*a*b**3*d**2*e - 16*b**4*c*
*3) - a**3*f**4 + 4*a**2*b*c*e*f**2 + 2*a**2*b*d**2*f**2 - 4*a**2
*b*d*e**2*f + a**2*b*e**4 - 4*a*b**2*c**2*d*f - 2*a*b**2*c**2*e**
2 + 4*a*b**2*c*d**2*e - a*b**2*d**4 + b**3*c**4, Lambda(_t, _t*lo
g(x + (-64*_t**3*a*b**5*f**3 - 64*_t**3*b**6*d**2*f + 128*_t**3*b
**6*d*e**2 + 48*_t**2*a*b**4*c*f**3 + 48*_t**2*a*b**4*d*e*f**2 -
32*_t**2*a*b**4*e**3*f + 48*_t**2*b**5*c*d**2*f - 96*_t**2*b**5*c
*d*e**2 - 16*_t**2*b**5*d**3*e + 12*_t*a**2*b**2*d*f**4 + 12*_t*a
**2*b**2*e**2*f**3 - 12*_t*a*b**3*c**2*f**3 - 24*_t*a*b**3*c*d*e*
f**2 + 16*_t*a*b**3*c*e**3*f + 16*_t*a*b**3*d**3*f**2 - 36*_t*a*b
**3*d**2*e**2*f - 8*_t*a*b**3*d*e**4 - 12*_t*b**4*c**2*d**2*f + 2
4*_t*b**4*c**2*d*e**2 + 8*_t*b**4*c*d**3*e + 4*_t*b**4*d**5 + 3*a
**3*e*f**5 - 3*a**2*b*c*d*f**4 - 3*a**2*b*c*e**2*f**3 - 5*a**2*b*
d*e**3*f**2 + 2*a**2*b*e**5*f + a*b**2*c**3*f**3 + 3*a*b**2*c**2*
d*e*f**2 - 2*a*b**2*c**2*e**3*f - 4*a*b**2*c*d**3*f**2 + 9*a*b**2
*c*d**2*e**2*f + 2*a*b**2*c*d*e**4 + 5*a*b**2*d**4*e*f - 5*a*b**2
*d**3*e**3 + b**3*c**3*d**2*f - 2*b**3*c**3*d*e**2 - b**3*c**2*d*
*3*e - b**3*c*d**5)/(a**3*f**6 + a**2*b*d**2*f**4 - 8*a**2*b*d*e*
*2*f**3 + 4*a**2*b*e**4*f**2 - a*b**2*d**4*f**2 + 8*a*b**2*d**3*e
**2*f - 4*a*b**2*d**2*e**4 - b**3*d**6)))) - d*x/b - e*x**2/(2*b)
 - f*x**3/(3*b)
```

GIAC/XCAS [A] time = 0.241904, size = 443, normalized size = 2.73

$$-\frac{c\ln\left(\left|bx^4-a\right|\right)}{4b} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e - \left(-ab^3\right)^{\frac{1}{4}}b^2d - \left(-ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b^4}$$
$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e - \left(-ab^3\right)^{\frac{1}{4}}b^2d - \left(-ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b^4}$$
$$+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2d - \left(-ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8b^4}$$
$$-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2d - \left(-ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8b^4} - \frac{2b^2fx^3+3b^2x^2e+6b^2dx}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 - a),x, algorithm="giac")
```

```
[Out] -1/4*c*ln(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2
*e - (-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/4*sqrt(2)*(sqr
t(2)*sqrt(-a*b)*b^2*e - (-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4
 + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*ln(x^2 +
 sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/8*sqrt(2)*((-a*b^3)
^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*ln(x^2 - sqrt(2)*x*(-a/b)^(1/4)
```

```
+ sqrt(-a/b))/b^4 - 1/6*(2*b^2*f*x^3 + 3*b^2*x^2*e + 6*b^2*d*x)/b
^3
```

3.475 $\int \frac{c+dx+ex^2+fx^3}{a+bx^4} \, dx$

Optimal. Leaf size=293

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]
*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c
- Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3
/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rubi [A] time = 0.519734, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]
*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c
- Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3
/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rubi in Sympy [A] time = 82.7339, size = 272, normalized size = 0.93

$$\frac{f\log\left(a+bx^4\right)}{4b}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{b}c\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{b}c\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] f*log(a + b*x**4)/(4*b) + d*atan(sqrt(b)*x**2/sqrt(a))/(2*sqrt(a)
*sqrt(b)) + sqrt(2)*(sqrt(a)*e - sqrt(b)*c)*log(-sqrt(2)*a**(1/4)
*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a**(3/4)*b**(3/4)) - s
qrt(2)*(sqrt(a)*e - sqrt(b)*c)*log(sqrt(2)*a**(1/4)*b**(3/4)*x +
sqrt(a)*sqrt(b) + b*x**2)/(8*a**(3/4)*b**(3/4)) - sqrt(2)*(sqrt(a
)*e + sqrt(b)*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4
)*b**(3/4)) + sqrt(2)*(sqrt(a)*e + sqrt(b)*c)*atan(1 + sqrt(2)*b*
*(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(3/4))
```

Mathematica [A] time = 0.570866, size = 296, normalized size = 1.01

$$-\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2\sqrt[4]{a}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*b^(1/4)*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sq
rt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1
/4)*b^(1/4)*(Sqrt[2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sq
rt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*b^(1/4
)*(a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(a^(1/4)*Sqrt[b]*c - a^
(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] +
 2*a*f*Log[a + b*x^4])/(8*a*b)
```

Maple [A] time = 0.005, size = 294, normalized size = 1.

$$\begin{aligned}
&\frac{c\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\\
&+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{d}{2}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}\\
&+\frac{e\sqrt{2}}{8b}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\\
&+\frac{e\sqrt{2}}{4b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e\sqrt{2}}{4b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{f\ln\left(bx^4+a\right)}{4b}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*c*(a/b)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*c*(a/b)^(1/4)/a*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/2*d/(a*b)^(1/2)*arctan(x^2
*(b/a)^(1/2))+1/8*e/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*
e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*e/b/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*f*ln(b*x^4
```

+a)/b

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 24.9786, size = 950, normalized size = 3.24

$$\text{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3f + t^2\left(96a^3b^2f^2 + 64a^2b^3ce + 32a^2b^3d^2\right) + t\left(-16a^3bf^3 - 32a^2b^2cef - 16a^2b^2d^2f + 16a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3*f + _t**2*(96*a
**3*b**2*f**2 + 64*a**2*b**3*c*e + 32*a**2*b**3*d**2) + _t*(-16*a
**3*b*f**3 - 32*a**2*b**2*c*e*f - 16*a**2*b**2*d**2*f + 16*a**2*b
**2*d*e**2 - 16*a*b**3*c**2*d) + a**3*f**4 + 4*a**2*b*c*e*f**2 +
2*a**2*b*d**2*f**2 - 4*a**2*b*d*e**2*f + a**2*b*e**4 + 4*a*b**2*c
**2*d*f + 2*a*b**2*c**2*e**2 - 4*a*b**2*c*d**2*e + a*b**2*d**4 +
b**3*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**4*b**3*e**3 - 64*_t
**3*a**3*b**4*c**2*e + 128*_t**3*a**3*b**4*c*d**2 - 48*_t**2*a**4
*b**2*e**3*f + 48*_t**2*a**3*b**3*c**2*e*f - 96*_t**2*a**3*b**3*c
*d**2*f + 48*_t**2*a**3*b**3*c*d*e**2 - 32*_t**2*a**3*b**3*d**3*e
 + 16*_t**2*a**2*b**4*c**3*d + 12*_t*a**4*b*e**3*f**2 - 12*_t*a**
3*b**2*c**2*e*f**2 + 24*_t*a**3*b**2*c*d**2*f**2 - 24*_t*a**3*b**
2*c*d*e**2*f + 12*_t*a**3*b**2*c*e**4 + 16*_t*a**3*b**2*d**3*e*f
+ 12*_t*a**3*b**2*d**2*e**3 - 8*_t*a**2*b**3*c**3*d*f - 16*_t*a**
2*b**3*c**3*e**2 + 36*_t*a**2*b**3*c**2*d**2*e + 8*_t*a**2*b**3*c
*d**4 + 4*_t*a*b**4*c**5 - a**4*e**3*f**3 + a**3*b*c**2*e*f**3 -
2*a**3*b*c*d**2*f**3 + 3*a**3*b*c*d*e**2*f**2 - 3*a**3*b*c*e**4*f
 - 2*a**3*b*d**3*e*f**2 - 3*a**3*b*d**2*e**3*f + 3*a**3*b*d*e**5
+ a**2*b**2*c**3*d*f**2 + 4*a**2*b**2*c**3*e**2*f - 9*a**2*b**2*c
**2*d**2*e*f - 2*a**2*b**2*c*d**4*f + 5*a**2*b**2*c*d**3*e**2 - 2
*a**2*b**2*d**5*e - a*b**3*c**5*f + 5*a*b**3*c**4*d*e - 5*a*b**3*
c**3*d**3)/(a**3*b*e**6 - a**2*b**2*c**2*e**4 + 8*a**2*b**2*c*d**
2*e**3 - 4*a**2*b**2*d**4*e**2 - a*b**3*c**4*e**2 + 8*a*b**3*c**3
*d**2*e - 4*a*b**3*c**2*d**4 + b**4*c**6))))
```

GIAC/XCAS [A] time = 0.23119, size = 392, normalized size = 1.34

$$\frac{f\ln\left(\left|bx^4+a\right|\right)}{4\,b}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*f*ln(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d
 - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(
2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) +
1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*ln(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^2*c - (a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt
(a/b))/(a*b^3)
```

3.476 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$

Optimal. Leaf size=321

$$\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}b^{7/4}}-\frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{c\log\left(a+bx^4\right)}{4b}+\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}$$

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt
[b]*x^2)/Sqrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (
a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*
b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log[a
 + b*x^4])/(4*b)
```

Rubi [A] time = 0.779693, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}b^{7/4}}-\frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{c\log\left(a+bx^4\right)}{4b}+\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt
[b]*x^2)/Sqrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (
a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*
b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log[a
 + b*x^4])/(4*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}f-\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8b^{\frac{7}{4}}}+\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}f-\sqrt{b}d\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{8b^{\frac{7}{4}}}+\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4b^{\frac{7}{4}}}-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4b^{\frac{7}{4}}}-\frac{\sqrt{a}e\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+\frac{c\log\left(a+bx^4\right)}{4b}+\frac{dx}{b}+\frac{fx^3}{3b}+\frac{\int^{x^2}e\,dx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*a**(1/4)*(sqrt(a)*f - sqrt(b)*d)*log(-sqrt(2)*a**(1/4)*b
**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*b**(7/4)) + sqrt(2)*a**(
1/4)*(sqrt(a)*f - sqrt(b)*d)*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sq
rt(a)*sqrt(b) + b*x**2)/(8*b**(7/4)) + sqrt(2)*a**(1/4)*(sqrt(a)*
f + sqrt(b)*d)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*b**(7/4))
 - sqrt(2)*a**(1/4)*(sqrt(a)*f + sqrt(b)*d)*atan(1 + sqrt(2)*b**(
1/4)*x/a**(1/4))/(4*b**(7/4)) - sqrt(a)*e*atan(sqrt(b)*x**2/sqrt(
a))/(2*b**(3/2)) + c*log(a + b*x**4)/(4*b) + d*x/b + f*x**3/(3*b)
 + Integral(e, (x, x**2))/(2*b)
```

Mathematica [A] time = 0.309194, size = 311, normalized size = 0.97

$$\frac{-3\sqrt{2}\left(a^{3/4}f-\sqrt[4]{a}\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+3\sqrt{2}\left(a^{3/4}f-\sqrt[4]{a}\sqrt{b}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+6b^{3/4}c\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*d*x + 12*b^(3/4)*e*x^2 + 8*b^(3/4)*f*x^3 + 6*a^(1/4)*
(Sqrt[2]*Sqrt[b]*d + 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 6*a^(1/4)*(Sqrt[2]*Sqrt[b]
*d - 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*
f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqr
t[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 6*b^(3/4)*c*Log[a + b*x^4])/(24*b
^(7/4))
```

Maple [A] time = 0.009, size = 325, normalized size = 1.

$$\frac{f x^3}{3 b}+\frac{e x^2}{2 b}+\frac{d x}{b}-\frac{d\sqrt{2}}{4 b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{d\sqrt{2}}{4 b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{d\sqrt{2}}{8 b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)-\frac{ae}{2 b}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}$$
$$-\frac{af\sqrt{2}}{8 b^2}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{af\sqrt{2}}{4 b^2}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{af\sqrt{2}}{4 b^2}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{c\ln\left(bx^4+a\right)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/3*f*x^3/b+1/2*e*x^2/b+d*x/b-1/4/b*d*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x+1)-1/4/b*d*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x-1)-1/8/b*d*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/
4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))
)-1/2/b*a*e/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))-1/8/b^2*a*f/(a/b)
^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a
/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4/b^2*a*f/(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/b^2*a*f/(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*c*ln(b*x^4+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 24.1428, size = 886, normalized size = 2.76

$$\operatorname{RootSum}\left(256t^4b^7-256t^3b^6c+t^2\left(64ab^4df+32ab^4e^2+96b^5c^2\right)+t\left(-16a^2b^2ef^2-32ab^3cdf-16ab^3ce^2+16ab^3d^2e-1\right.\right.$$
$$+\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*b**7 - 256*_t**3*b**6*c + _t**2*(64*a*b**4*d*f
+ 32*a*b**4*e**2 + 96*b**5*c**2) + _t*(-16*a**2*b**2*e*f**2 - 32*
a*b**3*c*d*f - 16*a*b**3*c*e**2 + 16*a*b**3*d**2*e - 16*b**4*c**3
) + a**3*f**4 + 4*a**2*b*c*e*f**2 + 2*a**2*b*d**2*f**2 - 4*a**2*b
*d*e**2*f + a**2*b*e**4 + 4*a*b**2*c**2*d*f + 2*a*b**2*c**2*e**2
- 4*a*b**2*c*d**2*e + a*b**2*d**4 + b**3*c**4, Lambda(_t, _t*log(
x + (-64*_t**3*a*b**5*f**3 + 64*_t**3*b**6*d**2*f - 128*_t**3*b**
6*d*e**2 + 48*_t**2*a*b**4*c*f**3 + 48*_t**2*a*b**4*d*e*f**2 - 32
*_t**2*a*b**4*e**3*f - 48*_t**2*b**5*c*d**2*f + 96*_t**2*b**5*c*d
*e**2 + 16*_t**2*b**5*d**3*e - 12*_t*a**2*b**2*d*f**4 - 12*_t*a**
2*b**2*e**2*f**3 - 12*_t*a*b**3*c**2*f**3 - 24*_t*a*b**3*c*d*e*f*
*2 + 16*_t*a*b**3*c*e**3*f + 16*_t*a*b**3*d**3*f**2 - 36*_t*a*b**
3*d**2*e**2*f - 8*_t*a*b**3*d*e**4 + 12*_t*b**4*c**2*d**2*f - 24*
_t*b**4*c**2*d*e**2 - 8*_t*b**4*c*d**3*e - 4*_t*b**4*d**5 + 3*a**
3*e*f**5 + 3*a**2*b*c*d*f**4 + 3*a**2*b*c*e**2*f**3 + 5*a**2*b*d*
e**3*f**2 - 2*a**2*b*e**5*f + a*b**2*c**3*f**3 + 3*a*b**2*c**2*d*
e*f**2 - 2*a*b**2*c**2*e**3*f - 4*a*b**2*c*d**3*f**2 + 9*a*b**2*c
*d**2*e**2*f + 2*a*b**2*c*d*e**4 + 5*a*b**2*d**4*e*f - 5*a*b**2*d
**3*e**3 - b**3*c**3*d**2*f + 2*b**3*c**3*d*e**2 + b**3*c**2*d**3
*e + b**3*c*d**5)/(a**3*f**6 - a**2*b*d**2*f**4 + 8*a**2*b*d*e**2
*f**3 - 4*a**2*b*e**4*f**2 - a*b**2*d**4*f**2 + 8*a*b**2*d**3*e**
2*f - 4*a*b**2*d**2*e**4 + b**3*d**6)))) + d*x/b + e*x**2/(2*b) +
 f*x**3/(3*b)
```

GIAC/XCAS [A] time = 0.230516, size = 416, normalized size = 1.3

$$\frac{c\ln\left(\left|bx^4+a\right|\right)}{4\,b}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}+\frac{2\,b^2fx^3+3\,b^2x^2e+6\,b^2dx}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*c*ln(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e
 - (a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*(sqrt(2)*s
qrt(a*b)*b^2*e - (a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 - 1/8*sqrt
(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*ln(x^2 + sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/b^4 + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (
a*b^3)^(3/4)*f)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 +
 1/6*(2*b^2*f*x^3 + 3*b^2*x^2*e + 6*b^2*d*x)/b^3
```

3.477 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2} dx$

Optimal. Leaf size=318

$$-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}$$

```
[Out] -(a*f - b*x*(c + d*x + e*x^2))/(4*a*b*(a + b*x^4)) + (d*ArcTan[(S
qrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)
*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sqrt[b]*c - Sqrt
[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(1
6*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4
)*b^(3/4))
```

Rubi [A] time = 0.582672, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] -(a*f - b*x*(c + d*x + e*x^2))/(4*a*b*(a + b*x^4)) + (d*ArcTan[(S
qrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)
*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sqrt[b]*c - Sqrt
[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(1
6*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4
)*b^(3/4))
```

Rubi in Sympy [A] time = 94.1335, size = 294, normalized size = 0.92

$$-\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}+\frac{d\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{a}e-3\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{7}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e-3\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{7}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e+3\sqrt{b}c\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}e+3\sqrt{b}c\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] -(a*f - b*x*(c + d*x + e*x**2))/(4*a*b*(a + b*x**4)) + d*atan(sqr
t(b)*x**2/sqrt(a))/(4*a**(3/2)*sqrt(b)) + sqrt(2)*(sqrt(a)*e - 3*
sqrt(b)*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b
*x**2)/(32*a**(7/4)*b**(3/4)) - sqrt(2)*(sqrt(a)*e - 3*sqrt(b)*c)
*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(32*
a**(7/4)*b**(3/4)) - sqrt(2)*(sqrt(a)*e + 3*sqrt(b)*c)*atan(1 - s
qrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(3/4)) + sqrt(2)*(sqr
t(a)*e + 3*sqrt(b)*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a
**(7/4)*b**(3/4))
```

Mathematica [A] time = 0.634726, size = 315, normalized size = 0.99

$$\sqrt{2}\sqrt[4]{b}\left(a^{3/4}e-3\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}\sqrt[4]{b}\left(3\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\frac{8a(af}{}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a*(a*f - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2*a^(1/4)*b^(
1/4)*(3*Sqrt[2]*Sqrt[b]*c + 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]
*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(
3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-3*a^(1/
4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*a^(1/4)*Sqrt[b]*c - a^(3/4)*
e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^
2*b)
```

Maple [A] time = 0.011, size = 362, normalized size = 1.1

$$\begin{aligned}
&\frac{cx}{4a(bx^4+a)}+\frac{3c\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\\
&+\frac{3c\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3c\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\\
&+\frac{dx^2}{4a(bx^4+a)}+\frac{d}{4a}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}+\frac{ex^3}{4a(bx^4+a)}\\
&+\frac{e\sqrt{2}}{32ab}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\\
&+\frac{e\sqrt{2}}{16ab}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{e\sqrt{2}}{16ab}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{fx^4}{4a(bx^4+a)}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] 1/4*c*x/a/(b*x^4+a)+3/32*c/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^
(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2)))+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+
```

```
1)+3/16*c/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
+1/4*d*x^2/a/(b*x^4+a)+1/4*d/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2)
)+1/4*e*x^3/a/(b*x^4+a)+1/32*e/a/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a
/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2)))+1/16*e/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x+1)+1/16*e/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x-1)+1/4*f*x^4/a/(b*x^4+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 32.1885, size = 517, normalized size = 1.63

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2\left(3072a^4b^2ce + 2048a^4b^2d^2\right) + t\left(128a^3bde^2 - 1152a^2b^2c^2d\right) + a^2e^4 + 18abc^2e^2 - 48abcd^2e + 16\right.$$
$$+ \frac{-af + bcx + bdx^2 + bex^3}{4a^2b + 4ab^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*
a**4*b**2*d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d)
+ a**2*e**4 + 18*a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 +
81*b**2*c**4, Lambda(_t, _t*log(x + (4096*_t**3*a**7*b**2*e**3 -
36864*_t**3*a**6*b**3*c**2*e + 98304*_t**3*a**6*b**3*c*d**2 + 460
8*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*d**3*e + 13824*
_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*b*d**
2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**
2*e + 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*
d*e**5 + 120*a**2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c
**4*d*e - 1080*a*b**2*c**3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4
+ 96*a**2*b*c*d**2*e**3 - 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e*
*2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2*d**4 + 729*b**3*c**
6)))) + (-a*f + b*c*x + b*d*x**2 + b*e*x**3)/(4*a**2*b + 4*a*b**2
*x**4)
```

GIAC/XCAS [A] time = 0.229595, size = 427, normalized size = 1.34

$$\frac{bx^3e + bdx^2 + bcx - af}{4\,(bx^4 + a)ab} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sq
rt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3
)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*
b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3
)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*
b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^
3)
```

3.478 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}-\frac{\left(3\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}}+\frac{\left(3\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{3/4}b^{7/4}}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}}-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(4*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]
*x^2)/Sqrt[a]])/(4*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*d + 3*Sqrt[a]*f)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/
4)) + ((Sqrt[b]*d + 3*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*d - 3*Sqrt[a]*f)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt
[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7
/4))
```

Rubi [A] time = 0.581243, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}-\frac{\left(3\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}}+\frac{\left(3\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{3/4}b^{7/4}}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}}-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```


```
[Out] -(c + d*x + e*x^2 + f*x^3)/(4*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]
*x^2)/Sqrt[a]])/(4*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*d + 3*Sqrt[a]*f)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/
4)) + ((Sqrt[b]*d + 3*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*d - 3*Sqrt[a]*f)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt
[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7
/4))
```

Rubi in Sympy [A] time = 100.513, size = 291, normalized size = 0.94

$$-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}+\frac{e\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{\frac{3}{2}}}+\frac{\sqrt{2}\left(3\sqrt{a}f-\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{1}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{3}{4}}b^{\frac{7}{4}}}-\frac{\sqrt{2}\left(3\sqrt{a}f-\sqrt{b}d\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{1}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{32a^{\frac{3}{4}}b^{\frac{7}{4}}}-\frac{\sqrt{2}\left(3\sqrt{a}f+\sqrt{b}d\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{3}{4}}b^{\frac{7}{4}}}+\frac{\sqrt{2}\left(3\sqrt{a}f+\sqrt{b}d\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{3}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] -(c + d*x + e*x**2 + f*x**3)/(4*b*(a + b*x**4)) + e*atan(sqrt(b)*
x**2/sqrt(a))/(4*sqrt(a)*b**(3/2)) + sqrt(2)*(3*sqrt(a)*f - sqrt(
b)*d)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2
)/(32*a**(3/4)*b**(7/4)) - sqrt(2)*(3*sqrt(a)*f - sqrt(b)*d)*log(
sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(32*a**(3
/4)*b**(7/4)) - sqrt(2)*(3*sqrt(a)*f + sqrt(b)*d)*atan(1 - sqrt(2
)*b**(1/4)*x/a**(1/4))/(16*a**(3/4)*b**(7/4)) + sqrt(2)*(3*sqrt(a
)*f + sqrt(b)*d)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(3/
4)*b**(7/4))
```

Mathematica [A] time = 0.531374, size = 294, normalized size = 0.95

$$-\frac{\frac{2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+\sqrt{2}\sqrt{b}d\right)}{a^{3/4}}+\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+\sqrt{2}\sqrt{b}d\right)}{a^{3/4}}+\frac{\sqrt{2}\left(3\sqrt{a}f-\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\cdots\right.}{a^{3/4}}}{32b^{7/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] ((-8*b^(3/4)*(c + x*(d + x*(e + f*x))))/(a + b*x^4) - (2*(Sqrt[2]
*Sqrt[b]*d + 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1
- (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (2*(Sqrt[2]*Sqrt[b]*d -
 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*(-(Sqrt[b]*d) + 3*Sqrt[a]*
f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4
) + (Sqrt[2]*(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4))/(32*b^(7/4))
```

Maple [A] time = 0.021, size = 333, normalized size = 1.1

$$\begin{aligned}
&\frac{1}{bx^4+a}\left(-\frac{fx^3}{4b}-\frac{ex^2}{4b}-\frac{dx}{4b}-\frac{c}{4b}\right) \\
&+\frac{d\sqrt{2}}{32ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) \\
&+\frac{d\sqrt{2}}{16ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{d\sqrt{2}}{16ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{e}{4}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab^3}} \\
&+\frac{3f\sqrt{2}}{32b^2}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} \\
&+\frac{3f\sqrt{2}}{16b^2}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{3f\sqrt{2}}{16b^2}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (-1/4*f*x^3/b-1/4*e*x^2/b-1/4*d*x/b-1/4*c/b)/(b*x^4+a)+1/32*d/b*(
a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(
x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/16*d/b*(a/b)^(1/4)/a*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/16*d/b*(a/b)^(1/4)/a*2^(1
```

```
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*e/(a*b^3)^(1/2)*arctan(x^
2*(b/a)^(1/2))+3/32*f/b^2/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)
*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+
3/16*f/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/
16*f/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 64.074, size = 508, normalized size = 1.64

$$\operatorname{RootSum}\left(65536t^4a^3b^7 + t^2\left(3072a^2b^4df + 2048a^2b^4e^2\right) + t\left(1152a^2b^2ef^2 - 128ab^3d^2e\right) + 81a^2f^4 + 18abd^2f^2 - 48abde^2f\right.$$
$$- \frac{c + dx + ex^2 + fx^3}{4ab + 4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**3*b**7 + _t**2*(3072*a**2*b**4*d*f + 2048*
a**2*b**4*e**2) + _t*(1152*a**2*b**2*e*f**2 - 128*a*b**3*d**2*e)
+ 81*a**2*f**4 + 18*a*b*d**2*f**2 - 48*a*b*d*e**2*f + 16*a*b*e**4
 + b**2*d**4, Lambda(_t, _t*log(x + (110592*_t**3*a**4*b**5*f**3
- 12288*_t**3*a**3*b**6*d**2*f + 32768*_t**3*a**3*b**6*d*e**2 + 1
3824*_t**2*a**3*b**4*d*e*f**2 - 12288*_t**2*a**3*b**4*e**3*f + 51
2*_t**2*a**2*b**5*d**3*e + 3888*_t*a**3*b**2*d*f**4 + 5184*_t*a**
3*b**2*e**2*f**3 - 576*_t*a**2*b**3*d**3*f**2 + 1728*_t*a**2*b**3
*d**2*e**2*f + 512*_t*a**2*b**3*d*e**4 + 16*_t*a*b**4*d**5 + 1458
*a**3*e*f**5 + 360*a**2*b*d*e**3*f**2 - 192*a**2*b*e**5*f + 30*a*
b**2*d**4*e*f - 40*a*b**2*d**3*e**3)/(729*a**3*f**6 - 81*a**2*b*d
**2*f**4 + 864*a**2*b*d*e**2*f**3 - 576*a**2*b*e**4*f**2 - 9*a*b*
*2*d**4*f**2 + 96*a*b**2*d**3*e**2*f - 64*a*b**2*d**2*e**4 + b**3
*d**6)))) - (c + d*x + e*x**2 + f*x**3)/(4*a*b + 4*b**2*x**4)
```

GIAC/XCAS [A] time = 0.232053, size = 409, normalized size = 1.32

$$-\frac{fx^3 + x^2e + dx + c}{4\,(bx^4 + a)b} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2e + \left(ab^3\right)^{\frac{1}{4}}b^2d + 3\,\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2e + \left(ab^3\right)^{\frac{1}{4}}b^2d + 3\,\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d - 3\,\left(ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,ab^4} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d - 3\,\left(ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)*b) + 1/16*sqrt(2)*(2*
sqrt(2)*sqrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*
b^4) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^
2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a*b^4) + 1/32*sqrt(2)*((a*b^3)^(1/4)*b^2*d -
 3*(a*b^3)^(3/4)*f)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a*b^4) - 1/32*sqrt(2)*((a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*l
n(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
```

3.479 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=351

$$-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}b^{3/4}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)}-\frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2}$$

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) - (a*f - b*x*(c
+ d*x + e*x^2))/(8*a*b*(a + b*x^4)^2) + (3*d*ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3
/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5*S
qrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[
2]*a^(11/4)*b^(3/4))
```

Rubi [A] time = 0.688333, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44

$$-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}b^{3/4}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)}-\frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```


```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) - (a*f - b*x*(c
+ d*x + e*x^2))/(8*a*b*(a + b*x^4)^2) + (3*d*ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3
/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5*S
qrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[
2]*a^(11/4)*b^(3/4))
```

Rubi in Sympy [A] time = 112.38, size = 333, normalized size = 0.95

$$-\frac{af-bx\left(c+dx+ex^{2}\right)}{8ab\left(a+bx^{4}\right)^{2}}+\frac{x\left(7c+6dx+5ex^{2}\right)}{32a^{2}\left(a+bx^{4}\right)}+\frac{3d\operatorname{atan}\left(\frac{\sqrt{b}x^{2}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}+\frac{\sqrt{2}\left(5\sqrt{a}e-21\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^{2}\right)}{256a^{\frac{11}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(5\sqrt{a}e-21\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^{2}\right)}{256a^{\frac{11}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(5\sqrt{a}e+21\sqrt{b}c\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(5\sqrt{a}e+21\sqrt{b}c\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] -(a*f - b*x*(c + d*x + e*x**2))/(8*a*b*(a + b*x**4)**2) + x*(7*c
+ 6*d*x + 5*e*x**2)/(32*a**2*(a + b*x**4)) + 3*d*atan(sqrt(b)*x**
2/sqrt(a))/(16*a**(5/2)*sqrt(b)) + sqrt(2)*(5*sqrt(a)*e - 21*sqrt
(b)*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**
2)/(256*a**(11/4)*b**(3/4)) - sqrt(2)*(5*sqrt(a)*e - 21*sqrt(b)*c
)*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(25
6*a**(11/4)*b**(3/4)) - sqrt(2)*(5*sqrt(a)*e + 21*sqrt(b)*c)*atan
(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(128*a**(11/4)*b**(3/4)) + sqrt
(2)*(5*sqrt(a)*e + 21*sqrt(b)*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(
1/4))/(128*a**(11/4)*b**(3/4))
```

Mathematica [A] time = 0.586528, size = 347, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(5a^{3/4}e-21\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^{2}\right)}{b^{3/4}}+\frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{b}c-5a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^{2}\right)}{b^{3/4}}-\frac{32a^{2}(af-bx(c+x(d+ex)))}{b\left(a+bx^{4}\right)^{2}}-\frac{2\sqrt[4]{a}\tan^{-}}{}}{256a^{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a*x*(7*c + x*(6*d + 5*e*x)))/(a + b*x^4) - (32*a^2*(a*f - b*x
*(c + x*(d + e*x))))/(b*(a + b*x^4)^2) - (2*a^(1/4)*(21*Sqrt[2]*S
qrt[b]*c + 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*S
qrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-21*a^(1/4)*Sq
rt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4
)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3
/4))/(256*a^3)
```

Maple [A] time = 0.01, size = 432, normalized size = 1.2

$$\frac{cx}{8a\left(bx^{4}+a\right)^{2}}+\frac{7cx}{32a^{2}\left(bx^{4}+a\right)}+\frac{21c\sqrt{2}}{256a^{3}}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^{2}+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^{2}-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{21c\sqrt{2}}{128a^{3}}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{21c\sqrt{2}}{128a^{3}}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{dx^{2}}{8a\left(bx^{4}+a\right)^{2}}+\frac{3dx^{2}}{16a^{2}\left(bx^{4}+a\right)}+\frac{3d}{16a^{2}}\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}+\frac{ex^{3}}{8a\left(bx^{4}+a\right)^{2}}+\frac{5ex^{3}}{32a^{2}\left(bx^{4}+a\right)}+\frac{5e\sqrt{2}}{256a^{2}b}\ln\left(1\left(x^{2}-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^{2}+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5e\sqrt{2}}{128a^{2}b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5e\sqrt{2}}{128a^{2}b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{fx^{4}}{8a\left(bx^{4}+a\right)^{2}}+\frac{fx^{4}}{8a^{2}\left(bx^{4}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(a/b)^(
1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/8*d*x^2/a/(b*x^4+a)^2+3/16*d/a^
2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/
8*e*x^3/a/(b*x^4+a)^2+5/32*e/a^2*x^3/(b*x^4+a)+5/256*e/a^2/b/(a/b
)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+5/128*e/a^2/b/(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+5/128*e/a^2/b/(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/8*f*x^4/a/(b*x^4+a)^2+1/8*
f/a^2*x^4/(b*x^4+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 130.201, size = 578, normalized size = 1.65

$$\mathrm{RootSum}\left(268435456t^4a^{11}b^3 + t^2\left(6881280a^6b^2ce + 4718592a^6b^2d^2\right) + t\left(153600a^4bde^2 - 2709504a^3b^2c^2d\right) + 625a^2e^4 + 22\right.$$
$$+\frac{-4a^2f + 11abcx + 10abdx^2 + 9abex^3 + 7b^2cx^5 + 6b^2dx^6 + 5b^2ex^7}{32a^4b + 64a^3b^2x^4 + 32a^2b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e
 + 4718592*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a
**3*b**2*c**2*d) + 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*
b*c*d**2*e + 20736*a*b*d**4 + 194481*b**2*c**4, Lambda(_t, _t*log
(x + (262144000*_t**3*a**10*b**2*e**3 - 4624220160*_t**3*a**9*b**
3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 309657600*_t**2*a
**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e + 1820786688
*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a
**5*b*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t
*a**4*b**2*c**2*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 52276492
8*_t*a**3*b**3*c**5 + 112500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*
e**2 - 2488320*a**2*b*d**5*e + 58344300*a*b**2*c**4*d*e - 8001504
0*a*b**2*c**3*d**3)/(15625*a**3*e**6 - 275625*a**2*b*c**2*e**4 +
3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b*d**4*e**2 - 4862025*a
*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c
**2*d**4 + 85766121*b**3*c**6)))) + (-4*a**2*f + 11*a*b*c*x + 10*
a*b*d*x**2 + 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5*b**
2*e*x**7)/(32*a**4*b + 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)
```

GIAC/XCAS [A] time = 0.231672, size = 478, normalized size = 1.36

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d + 21\left(ab^3\right)^{\frac{1}{4}}b^2c + 5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d + 21\left(ab^3\right)^{\frac{1}{4}}b^2c + 5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c - 5\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{256\,a^3b^3} - \frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c - 5\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{256\,a^3b^3} + \frac{5\,b^2x^7e + 6\,b^2dx^6 + 7\,b^2cx^5 + 9\,abx^3e + 10\,abdx^2 + 11\,abcx - 4\,a^2f}{32\left(bx^4 + a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*
c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)
```

```
*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256
*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*ln(x^2 + sq
rt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a
*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7
*b^2*c*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 + 11*a*b*c*x - 4*a^2*f)/(
(b*x^4 + a)^2*a^2*b)
```

3.480 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=340

$$-\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}-\frac{3\left(\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{7/4}b^{7/4}}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}$$

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(8*b*(a + b*x^4)^2) + (x*(d + 2*e*x +
3*f*x^2))/(32*a*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]
)/(16*a^(3/2)*b^(3/2)) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(7/4)*b^(7/4)) + (3*(Sq
rt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64
*Sqrt[2]*a^(7/4)*b^(7/4)) - (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(7/4
)*b^(7/4)) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rubi [A] time = 0.689006, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393

$$-\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}-\frac{3\left(\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{a}f+\sqrt{b}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{7/4}b^{7/4}}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```


```
[Out] -(c + d*x + e*x^2 + f*x^3)/(8*b*(a + b*x^4)^2) + (x*(d + 2*e*x +
3*f*x^2))/(32*a*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]
)/(16*a^(3/2)*b^(3/2)) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(7/4)*b^(7/4)) + (3*(Sq
rt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64
*Sqrt[2]*a^(7/4)*b^(7/4)) - (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(7/4
)*b^(7/4)) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rubi in Sympy [A] time = 116.304, size = 320, normalized size = 0.94

$$-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}+\frac{e\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}b^{\frac{3}{2}}}+\frac{3\sqrt{2}\left(\sqrt{a}f-\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{256a^{\frac{7}{4}}b^{\frac{7}{4}}}-\frac{3\sqrt{2}\left(\sqrt{a}f-\sqrt{b}d\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{256a^{\frac{7}{4}}b^{\frac{7}{4}}}-\frac{3\sqrt{2}\left(\sqrt{a}f+\sqrt{b}d\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{7}{4}}b^{\frac{7}{4}}}+\frac{3\sqrt{2}\left(\sqrt{a}f+\sqrt{b}d\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{128a^{\frac{7}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] -(c + d*x + e*x**2 + f*x**3)/(8*b*(a + b*x**4)**2) + x*(d + 2*e*x
 + 3*f*x**2)/(32*a*b*(a + b*x**4)) + e*atan(sqrt(b)*x**2/sqrt(a))
/(16*a**(3/2)*b**(3/2)) + 3*sqrt(2)*(sqrt(a)*f - sqrt(b)*d)*log(-
sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(256*a**(
7/4)*b**(7/4)) - 3*sqrt(2)*(sqrt(a)*f - sqrt(b)*d)*log(sqrt(2)*a*
*(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(256*a**(7/4)*b**(7
/4)) - 3*sqrt(2)*(sqrt(a)*f + sqrt(b)*d)*atan(1 - sqrt(2)*b**(1/4
)*x/a**(1/4))/(128*a**(7/4)*b**(7/4)) + 3*sqrt(2)*(sqrt(a)*f + sq
rt(b)*d)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(128*a**(7/4)*b**(
7/4))
```

Mathematica [A] time = 0.79978, size = 329, normalized size = 0.97

$$\frac{-\frac{2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+3\sqrt{2}\sqrt{b}d\right)}{a^{7/4}}+\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-8\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+3\sqrt{2}\sqrt{b}d\right)}{a^{7/4}}+\frac{3\sqrt{2}\left(\sqrt{a}f-\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}\right.}{a^{7/4}}}{256b^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

```
[Out] ((8*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)) - (32*b^(3/4
)*(c + x*(d + x*(e + f*x))))/(a + b*x^4)^2 - (2*(3*Sqrt[2]*Sqrt[b
]*d + 8*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*(3*Sqrt[2]*Sqrt[b]*d - 8*a^
(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/a^(7/4) + (3*Sqrt[2]*(-(Sqrt[b]*d) + Sqrt[a]*f)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (
3*Sqrt[2]*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(256*b^(7/4))
```

Maple [A] time = 0.018, size = 371, normalized size = 1.1

$$\frac{1}{(bx^4+a)^2}\left(\frac{3fx^7}{32a}+\frac{ex^6}{16a}+\frac{dx^5}{32a}-\frac{fx^3}{32b}-\frac{ex^2}{16b}-\frac{3dx}{32b}-\frac{c}{8b}\right)$$
$$+\frac{3d\sqrt{2}}{256a^2b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{3d\sqrt{2}}{128a^2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$
$$+\frac{3d\sqrt{2}}{128a^2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{e}{16}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{a^3b^3}}$$
$$+\frac{3f\sqrt{2}}{256ab^2}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{3f\sqrt{2}}{128ab^2}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{3f\sqrt{2}}{128ab^2}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (3/32*f/a*x^7+1/16/a*e*x^6+1/32*d/a*x^5-1/32*f*x^3/b-1/16*e*x^2/b
-3/32*d*x/b-1/8*c/b)/(b*x^4+a)^2+3/256*d/b/a^2*(a/b)^(1/4)*2^(1/2
)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2)))+3/128*d/b/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)+3/128*d/b/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x-1)+1/16*e/(a^3*b^3)^(1/2)*arctan(x^2*(b/a)^(1
/2))+3/256*f/b^2/a/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1
/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+3/128*f
/b^2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/128*
f/b^2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.233291, size = 456, normalized size = 1.34

$$\frac{3\,bfx^7+2\,bx^6e+bdx^5-afx^3-2\,ax^2e-3\,adx-4\,ac}{32\,(bx^4+a)^2ab}+\frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ab}b^2e+3\,\left(ab^3\right)^{\frac{1}{4}}b^2d+3\,\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}+\frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ab}b^2e+3\,\left(ab^3\right)^{\frac{1}{4}}b^2d+3\,\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}+\frac{3\,\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}-\frac{3\,\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*b*f*x^7 + 2*b*x^6*e + b*d*x^5 - a*f*x^3 - 2*a*x^2*e - 3*a
*d*x - 4*a*c)/((b*x^4 + a)^2*a*b) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt
(a*b)*b^2*e + 3*(a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1
/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e + 3*(a*b^3)^(1/4)*b^2*d +
 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)
)/(a/b)^(1/4))/(a^2*b^4) + 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (
a*b^3)^(3/4)*f)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*
b^4) - 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*ln(x
^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
```

3.481 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=382

$$-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}a^{15/4}b^{3/4}}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2}-\frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3}$$

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c +
 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4)) - (a*f - b*x*(c + d*x
+ e*x^2))/(12*a*b*(a + b*x^4)^3) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt
[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)
) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*S
qrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(512*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt
[2]*a^(15/4)*b^(3/4))
```

Rubi [A] time = 0.822525, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44

$$-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}a^{15/4}b^{3/4}}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2}-\frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```


```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c +
 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4)) - (a*f - b*x*(c + d*x
+ e*x^2))/(12*a*b*(a + b*x^4)^3) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt
[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)
) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*S
qrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(512*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt
[2]*a^(15/4)*b^(3/4))
```

Rubi in Sympy [A] time = 137.466, size = 364, normalized size = 0.95

$$-\frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)}+\frac{5d\operatorname{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}}+\frac{\sqrt{2}\left(15\sqrt{a}e-77\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(15\sqrt{a}e-77\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(15\sqrt{a}e+77\sqrt{b}c\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{15}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(15\sqrt{a}e+77\sqrt{b}c\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{15}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] -(a*f - b*x*(c + d*x + e*x**2))/(12*a*b*(a + b*x**4)**3) + x*(11*
c + 10*d*x + 9*e*x**2)/(96*a**2*(a + b*x**4)**2) + x*(77*c + 60*d
*x + 45*e*x**2)/(384*a**3*(a + b*x**4)) + 5*d*atan(sqrt(b)*x**2/s
qrt(a))/(32*a**(7/2)*sqrt(b)) + sqrt(2)*(15*sqrt(a)*e - 77*sqrt(b
)*c)*log(-sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)
/(1024*a**(15/4)*b**(3/4)) - sqrt(2)*(15*sqrt(a)*e - 77*sqrt(b)*c
)*log(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(10
24*a**(15/4)*b**(3/4)) - sqrt(2)*(15*sqrt(a)*e + 77*sqrt(b)*c)*at
an(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(512*a**(15/4)*b**(3/4)) + sq
rt(2)*(15*sqrt(a)*e + 77*sqrt(b)*c)*atan(1 + sqrt(2)*b**(1/4)*x/a
**(1/4))/(512*a**(15/4)*b**(3/4))
```

Mathematica [A] time = 0.929587, size = 379, normalized size = 0.99

$$\frac{\frac{3\sqrt{2}\left(15a^{3/4}e-77\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}+\frac{3\sqrt{2}\left(77\sqrt[4]{a}\sqrt{b}c-15a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}-\frac{256a^3(af-bx(c+x(d+ex)))}{b\left(a+bx^4\right)^3}+32a^2\ldots}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*
c + x*(10*d + 9*e*x)))/(a + b*x^4)^2 - (256*a^3*(a*f - b*x*(c + x
*(d + e*x))))/(b*(a + b*x^4)^3) - (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*
c + 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]
*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*Sqrt[
b]*c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/
4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(
3/4))/(3072*a^4)
```

Maple [A] time = 0.022, size = 399, normalized size = 1.

$$\frac{1}{\left(bx^4+a\right)^3}\left(\frac{15\,b^2ex^{11}}{128\,a^3}+\frac{5\,b^2dx^{10}}{32\,a^3}+\frac{77\,b^2cx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}+\frac{113\,ex^3}{384\,a}+\frac{11\,dx^2}{32\,a}+\frac{51\,cx}{128\,a}-\frac{f}{12\,b}\right)$$
$$+\frac{77\,c\sqrt{2}}{1024\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{77\,c\sqrt{2}}{512\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{77\,c\sqrt{2}}{512\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{5\,d}{32}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ba^7}}+\frac{15\,e\sqrt{2}}{1024\,a^3b}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{15\,e\sqrt{2}}{512\,a^3b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{15\,e\sqrt{2}}{512\,a^3b}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+2
1/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5+113/384/a*e*x
^3+11/32*d/a*x^2+51/128/a*c*x-1/12*f/b)/(b*x^4+a)^3+77/1024*c*(a/
b)^(1/4)/a^4*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(
x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+77/512*c*(a/b)^(1/4)/a^4*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+77/512*c*(a/b)^(1/4)/a^4*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+5/32*d/(b*a^7)^(1/2)*arct
an(x^2*(b/a)^(1/2))+15/1024*e/a^3/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2)))+15/512*e/a^3/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x+1)+15/512*e/a^3/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.237505, size = 528, normalized size = 1.38

$$\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d+77\,\left(ab^3\right)^{\frac{1}{4}}b^2c+15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d+77\,\left(ab^3\right)^{\frac{1}{4}}b^2c+15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(77\,\left(ab^3\right)^{\frac{1}{4}}b^2c-15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}-\frac{\sqrt{2}\left(77\,\left(ab^3\right)^{\frac{1}{4}}b^2c-15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}+\frac{45\,b^3x^{11}e+60\,b^3dx^{10}+77\,b^3cx^9+126\,ab^2x^7e+160\,ab^2dx^6+198\,ab^2cx^5+113\,a^2bx^3e+132\,a^2bdx^2+153\,a^2bcx-32\,a^3f}{384\,(bx^4+a)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*
c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b
)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1
024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*ln(x^2
+ sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(
77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3
*d*x^10 + 77*b^3*c*x^9 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 198*
a*b^2*c*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x -
 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

3.482 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=380

$$-\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}+\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}-\frac{\left(5\sqrt{a}f+7\sqrt{b}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}+\frac{\left(5\sqrt{a}f+7\sqrt{b}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}a^{11/4}b^{7/4}}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}$$

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(12*b*(a + b*x^4)^3) + (x*(d + 2*e*x +
 3*f*x^2))/(96*a*b*(a + b*x^4)^2) + (x*(7*d + 12*e*x + 15*f*x^2))
/(384*a^2*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*
a^(5/2)*b^(3/2)) - ((7*Sqrt[b]*d + 5*Sqrt[a]*f)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(11/4)*b^(7/4)) + ((7*Sqrt
[b]*d + 5*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(25
6*Sqrt[2]*a^(11/4)*b^(7/4)) - ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^
(11/4)*b^(7/4)) + ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4)*b^(7/
4))
```

Rubi [A] time = 0.845255, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393

$$-\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}+\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}-\frac{\left(5\sqrt{a}f+7\sqrt{b}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}+\frac{\left(5\sqrt{a}f+7\sqrt{b}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}a^{11/4}b^{7/4}}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(12*b*(a + b*x^4)^3) + (x*(d + 2*e*x +
 3*f*x^2))/(96*a*b*(a + b*x^4)^2) + (x*(7*d + 12*e*x + 15*f*x^2))
/(384*a^2*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*
a^(5/2)*b^(3/2)) - ((7*Sqrt[b]*d + 5*Sqrt[a]*f)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(11/4)*b^(7/4)) + ((7*Sqrt
[b]*d + 5*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(25
6*Sqrt[2]*a^(11/4)*b^(7/4)) - ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^
(11/4)*b^(7/4)) + ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4)*b^(7/
4))
```

Rubi in Sympy [A] time = 137.143, size = 359, normalized size = 0.94

$$-\frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3}+\frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2}+\frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)}+\frac{e\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{\frac{5}{2}}b^{\frac{3}{2}}}+\frac{\sqrt{2}\left(5\sqrt{a}f-7\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{1024a^{\frac{11}{4}}b^{\frac{7}{4}}}-\frac{\sqrt{2}\left(5\sqrt{a}f-7\sqrt{b}d\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^2\right)}{1024a^{\frac{11}{4}}b^{\frac{7}{4}}}-\frac{\sqrt{2}\left(5\sqrt{a}f+7\sqrt{b}d\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{11}{4}}b^{\frac{7}{4}}}+\frac{\sqrt{2}\left(5\sqrt{a}f+7\sqrt{b}d\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{512a^{\frac{11}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] -(c + d*x + e*x**2 + f*x**3)/(12*b*(a + b*x**4)**3) + x*(d + 2*e*
x + 3*f*x**2)/(96*a*b*(a + b*x**4)**2) + x*(7*d + 12*e*x + 15*f*x
**2)/(384*a**2*b*(a + b*x**4)) + e*atan(sqrt(b)*x**2/sqrt(a))/(32
*a**(5/2)*b**(3/2)) + sqrt(2)*(5*sqrt(a)*f - 7*sqrt(b)*d)*log(-sq
rt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(1024*a**(1
1/4)*b**(7/4)) - sqrt(2)*(5*sqrt(a)*f - 7*sqrt(b)*d)*log(sqrt(2)*
a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(1024*a**(11/4)*b
**(7/4)) - sqrt(2)*(5*sqrt(a)*f + 7*sqrt(b)*d)*atan(1 - sqrt(2)*b
**(1/4)*x/a**(1/4))/(512*a**(11/4)*b**(7/4)) + sqrt(2)*(5*sqrt(a)
*f + 7*sqrt(b)*d)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(512*a**(
11/4)*b**(7/4))
```

Mathematica [A] time = 0.789085, size = 366, normalized size = 0.96

$$\frac{-\frac{6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(16\sqrt[4]{a}\sqrt[4]{b}e+5\sqrt{2}\sqrt{a}f+7\sqrt{2}\sqrt{b}d\right)}{a^{11/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-16\sqrt[4]{a}\sqrt[4]{b}e+5\sqrt{2}\sqrt{a}f+7\sqrt{2}\sqrt{b}d\right)}{a^{11/4}}+\frac{3\sqrt{2}\left(5\sqrt{a}f-7\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x\right.}{a^{11/4}}}{3072b^{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] ((32*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)^2) + (8*b^(3
/4)*x*(7*d + 3*x*(4*e + 5*f*x)))/(a^2*(a + b*x^4)) - (256*b^(3/4)
*(c + x*(d + x*(e + f*x))))/(a + b*x^4)^3 - (6*(7*Sqrt[2]*Sqrt[b]
*d + 16*a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*(7*Sqrt[2]*Sqrt[b]*d - 16*
a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/a^(11/4) + (3*Sqrt[2]*(-7*Sqrt[b]*d + 5*Sqrt[a]*
f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/
4) + (3*Sqrt[2]*(7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/4))/(3072*b^(7/4))
```

Maple [A] time = 0.022, size = 401, normalized size = 1.1

$$\frac{1}{(bx^4+a)^3}\left(\frac{5\,bfx^{11}}{128\,a^2}+\frac{bex^{10}}{32\,a^2}+\frac{7\,bdx^9}{384\,a^2}+\frac{7\,fx^7}{64\,a}+\frac{ex^6}{12\,a}+\frac{3\,dx^5}{64\,a}-\frac{5\,fx^3}{384\,b}-\frac{ex^2}{32\,b}-\frac{7\,dx}{128\,b}-\frac{c}{12\,b}\right)$$
$$+\frac{7\,d\sqrt{2}}{1024\,a^3b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{7\,d\sqrt{2}}{512\,a^3b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{7\,d\sqrt{2}}{512\,a^3b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{e}{32}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{a^5b^3}}+\frac{5\,f\sqrt{2}}{1024\,a^2b^2}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{5\,f\sqrt{2}}{512\,a^2b^2}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5\,f\sqrt{2}}{512\,a^2b^2}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/128*f/a^2*b*x^11+1/32/a^2*b*e*x^10+7/384/a^2*d*b*x^9+7/64*f/a*
x^7+1/12/a*e*x^6+3/64*d/a*x^5-5/384*f*x^3/b-1/32*e*x^2/b-7/128*d*
x/b-1/12*c/b)/(b*x^4+a)^3+7/1024*d*(a/b)^(1/4)/a^3/b*2^(1/2)*ln((
x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2)))+7/512*d*(a/b)^(1/4)/a^3/b*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)+7/512*d*(a/b)^(1/4)/a^3/b*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)+1/32*e/(a^5*b^3)^(1/2)*arctan(x^2*(b/a)^(1/2))+5
/1024*f/a^2/b^2/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+5/512*f/a^
2/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+5/512*f
/a^2/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.232425, size = 513, normalized size = 1.35

$$\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\left(ab^3\right)^{\frac{1}{4}}b^2d+5\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}+\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\left(ab^3\right)^{\frac{1}{4}}b^2d+5\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}+\frac{\sqrt{2}\left(7\left(ab^3\right)^{\frac{1}{4}}b^2d-5\left(ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^3b^4}-\frac{\sqrt{2}\left(7\left(ab^3\right)^{\frac{1}{4}}b^2d-5\left(ab^3\right)^{\frac{3}{4}}f\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^3b^4}+\frac{15\,b^2fx^{11}+12\,b^2x^{10}e+7\,b^2dx^9+42\,abfx^7+32\,abx^6e+18\,abdx^5-5\,a^2fx^3-12\,a^2x^2e-21\,a^2dx-32\,a^2c}{384\left(bx^4+a\right)^3a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d
+ 5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4
))/(a/b)^(1/4))/(a^3*b^4) + 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^
2*e + 7*(a*b^3)^(1/4)*b^2*d + 5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + 1/1024*sq
rt(2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*ln(x^2 + sqrt(2
)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/1024*sqrt(2)*(7*(a*b^3
)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(a^3*b^4) + 1/384*(15*b^2*f*x^11 + 12*b^2*x^10*e +
7*b^2*d*x^9 + 42*a*b*f*x^7 + 32*a*b*x^6*e + 18*a*b*d*x^5 - 5*a^2*
f*x^3 - 12*a^2*x^2*e - 21*a^2*d*x - 32*a^2*c)/((b*x^4 + a)^3*a^2*
b)
```

3.483 $\int x^4\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=418

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}-\frac{a^2d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\left(a+bx^4\right)^{3/2}\left(8af-15bdx^2\right)}{120b^2}+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9c+7ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{fx^4\left(a+bx^4\right)^{3/2}}{10b}$$

```
[Out] (2*a*c*x*Sqrt[a + b*x^4])/(21*b) - (a*d*x^2*Sqrt[a + b*x^4])/(16*
b) + (2*a*e*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*e*x*Sqrt[a + b*x
^4])/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (x^5*(9*c + 7*e*x^2)*
Sqrt[a + b*x^4])/63 + (f*x^4*(a + b*x^4)^(3/2))/(10*b) - ((8*a*f
- 15*b*d*x^2)*(a + b*x^4)^(3/2))/(120*b^2) - (a^2*d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (2*a^(9/4)*e*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^
4]) - (a^(7/4)*(5*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 1.07942, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}-\frac{a^2d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\left(a+bx^4\right)^{3/2}\left(8af-15bdx^2\right)}{120b^2}+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9c+7ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{fx^4\left(a+bx^4\right)^{3/2}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```


```
[Out] (2*a*c*x*Sqrt[a + b*x^4])/(21*b) - (a*d*x^2*Sqrt[a + b*x^4])/(16*
b) + (2*a*e*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*e*x*Sqrt[a + b*x
^4])/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (x^5*(9*c + 7*e*x^2)*
Sqrt[a + b*x^4])/63 + (f*x^4*(a + b*x^4)^(3/2))/(10*b) - ((8*a*f
- 15*b*d*x^2)*(a + b*x^4)^(3/2))/(120*b^2) - (a^2*d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (2*a^(9/4)*e*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^
4]) - (a^(7/4)*(5*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 102.131, size = 386, normalized size = 0.92

$$\frac{2a^{\frac{9}{4}}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{a^{\frac{7}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{a}e+5\sqrt{b}c\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{105b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{a^2d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{\frac{3}{2}}} - \frac{2a^2ex\sqrt{a+bx^4}}{15b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2acx\sqrt{a+bx^4}}{21b} - \frac{adx^2\sqrt{a+bx^4}}{16b} + \frac{2aex^3\sqrt{a+bx^4}}{45b} + \frac{x^5\sqrt{a+bx^4}\left(9c+7ex^2\right)}{63} + \frac{fx^4\left(a+bx^4\right)^{\frac{3}{2}}}{10b} - \frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(8af-15bdx^2\right)}{120b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] 2*a**(9/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt
(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/
(15*b**(7/4)*sqrt(a + b*x**4)) - a**(7/4)*sqrt((a + b*x**4)/(sqrt
(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(7*sqrt(a)*e + 5
*sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(105*b**
(7/4)*sqrt(a + b*x**4)) - a**2*d*atanh(sqrt(b)*x**2/sqrt(a + b*x*
*4))/(16*b**(3/2)) - 2*a**2*e*x*sqrt(a + b*x**4)/(15*b**(3/2)*(sq
rt(a) + sqrt(b)*x**2)) + 2*a*c*x*sqrt(a + b*x**4)/(21*b) - a*d*x*
*2*sqrt(a + b*x**4)/(16*b) + 2*a*e*x**3*sqrt(a + b*x**4)/(45*b) +
 x**5*sqrt(a + b*x**4)*(9*c + 7*e*x**2)/63 + f*x**4*(a + b*x**4)*
*(3/2)/(10*b) - (a + b*x**4)**(3/2)*(8*a*f - 15*b*d*x**2)/(120*b*
*2)
```

Mathematica [C] time = 0.859589, size = 296, normalized size = 0.71

$$\frac{-672a^{5/2}\sqrt{b}e\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left(a+bx^4\right)\left(336a^2f-abx(480c+7x(45d+8x(4e+3fx)))-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(336*a^2*f - 2*b^2*x^5*
(360*c + 7*x*(45*d + 40*e*x + 36*f*x^2)) - a*b*x*(480*c + 7*x*(45
*d + 8*x*(4*e + 3*f*x))))) - 315*a^2*Sqrt[b]*d*Sqrt[a + b*x^4]*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) - 672*a^(5/2)*Sqrt[b]*e*Sqr
t[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x]
, -1] + 96*a^2*Sqrt[b]*((5*I)*Sqrt[b]*c + 7*Sqrt[a]*e)*Sqrt[1 + (
b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/
(5040*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.041, size = 390, normalized size = 0.9

$$
\begin{aligned}
&\frac{cx^5}{7}\sqrt{bx^4+a}+\frac{2\,acx}{21\,b}\sqrt{bx^4+a}\\
&-\frac{2\,a^2c}{21\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{dx^2}{8\,b}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{adx^2}{16\,b}\sqrt{bx^4+a}\\
&-\frac{a^2d}{16}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{ex^7}{9}\sqrt{bx^4+a}+\frac{2\,aex^3}{45\,b}\sqrt{bx^4+a}\\
&-\frac{2\,i}{15}ea^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{2\,i}{15}ea^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{f\left(-3\,bx^4+2\,a\right)}{30\,b^2}\left(bx^4+a\right)^{\frac{3}{2}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] 1/7*c*x^5*(b*x^4+a)^(1/2)+2/21*a*c*x*(b*x^4+a)^(1/2)/b-2/21*c/b*a
^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)+1/8*d*x^2*(b*x^4+a)^(3/2)/b-1/16*a*d*x^2*(b*x
^4+a)^(1/2)/b-1/16*d*a^2/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))+
1/9*e*x^7*(b*x^4+a)^(1/2)+2/45*a*e*x^3*(b*x^4+a)^(1/2)/b-2/15*I*e
/b^(3/2)*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+2/15*I*e/b^(3/2)*a^(5/2)/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-1/30*f*(b*x^4+a)^(3/2)*(-3*b*x^4+2*a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(fx^7+ex^6+dx^5+cx^4\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4,x, algorithm="fricas")
```

```
[Out] integral((f*x^7 + e*x^6 + d*x^5 + c*x^4)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 10.2662, size = 252, normalized size = 0.6

$$\frac{a^{\frac{3}{2}}dx^2}{16b\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}dx^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}-\frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+f\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+\frac{bdx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*d*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**5*gamma(
5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamm
a(9/4)) + 3*sqrt(a)*d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*e*x*
*7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(11/4)) - a**2*d*asinh(sqrt(b)*x**2/sqrt(a))/(16*b**(3
/2)) + f*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqr
t(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt
(a)*x**8/8, True)) + b*d*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

3.484 $\int x^3 \left(c+dx+ex^2+fx^3\right) \sqrt{a+bx^4}\,dx$

Optimal. Leaf size=394

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}-\frac{a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\left(a+bx^4\right)^{3/2}\left(4c+3ex^2\right)}{24b}+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9d+7fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}$$

```
[Out] (2*a*d*x*Sqrt[a + b*x^4])/(21*b) - (a*e*x^2*Sqrt[a + b*x^4])/(16*
b) + (2*a*f*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*f*x*Sqrt[a + b*x
^4])/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*
Sqrt[a + b*x^4])/63 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/(24*b)
- (a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (
2*a^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15
*b^(7/4)*Sqrt[a + b*x^4]) - (a^(7/4)*(5*Sqrt[b]*d + 7*Sqrt[a]*f)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)*Sq
rt[a + b*x^4])
```

Rubi [A] time = 0.978458, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}-\frac{a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\left(a+bx^4\right)^{3/2}\left(4c+3ex^2\right)}{24b}+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9d+7fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```


```
[Out] (2*a*d*x*Sqrt[a + b*x^4])/(21*b) - (a*e*x^2*Sqrt[a + b*x^4])/(16*
b) + (2*a*f*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*f*x*Sqrt[a + b*x
^4])/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*
Sqrt[a + b*x^4])/63 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/(24*b)
- (a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (
2*a^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15
*b^(7/4)*Sqrt[a + b*x^4]) - (a^(7/4)*(5*Sqrt[b]*d + 7*Sqrt[a]*f)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)*Sq
rt[a + b*x^4])
```

Rubi in Sympy [A] time = 94.692, size = 362, normalized size = 0.92

$$\frac{2a^{\frac{9}{4}}f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{a^{\frac{7}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{a}f+5\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{105b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{a^2e\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{\frac{3}{2}}} - \frac{2a^2fx\sqrt{a+bx^4}}{15b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2adx\sqrt{a+bx^4}}{21b} - \frac{aex^2\sqrt{a+bx^4}}{16b} + \frac{2afx^3\sqrt{a+bx^4}}{45b} + \frac{x^5\sqrt{a+bx^4}\left(9d+7fx^2\right)}{63} + \frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(4c+3ex^2\right)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] 2*a**(9/4)*f*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt
(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/
(15*b**(7/4)*sqrt(a + b*x**4)) - a**(7/4)*sqrt((a + b*x**4)/(sqrt
(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(7*sqrt(a)*f + 5
*sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(105*b**
(7/4)*sqrt(a + b*x**4)) - a**2*e*atanh(sqrt(b)*x**2/sqrt(a + b*x*
*4))/(16*b**(3/2)) - 2*a**2*f*x*sqrt(a + b*x**4)/(15*b**(3/2)*(sq
rt(a) + sqrt(b)*x**2)) + 2*a*d*x*sqrt(a + b*x**4)/(21*b) - a*e*x*
*2*sqrt(a + b*x**4)/(16*b) + 2*a*f*x**3*sqrt(a + b*x**4)/(45*b) +
 x**5*sqrt(a + b*x**4)*(9*d + 7*f*x**2)/63 + (a + b*x**4)**(3/2)*
(4*c + 3*e*x**2)/(24*b)
```

Mathematica [C] time = 0.833208, size = 275, normalized size = 0.7

$$\frac{-672a^{5/2}f\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\left(a+bx^4\right)\left(a(840c+x(480d+7x(45e+32fx)))+10bx^4(84c+\right.\right.}{5040b^{3/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[b]*(a + b*x^4)*(10*b*x^4*(84*c +
 x*(72*d + 7*x*(9*e + 8*f*x))) + a*(840*c + x*(480*d + 7*x*(45*e
+ 32*f*x)))) - 315*a^2*e*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a + b*x^4]]) - 672*a^(5/2)*f*Sqrt[1 + (b*x^4)/a]*EllipticE[I*A
rcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + 96*a^2*((5*I)*Sqrt[b]*
d + 7*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[b])/Sqrt[a]]*x], -1])/(5040*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^(3/2
)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.013, size = 380, normalized size = 1.

$$\frac{x^5 d}{7}\sqrt{bx^4+a}+\frac{2\,adx}{21\,b}\sqrt{bx^4+a}$$
$$-\frac{2\,a^2 d}{21\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{c}{6\,b}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{ex^2}{8\,b}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{aex^2}{16\,b}\sqrt{bx^4+a}$$
$$-\frac{ea^2}{16}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{fx^7}{9}\sqrt{bx^4+a}+\frac{2\,x^3 af}{45\,b}\sqrt{bx^4+a}$$
$$-\frac{2\,i}{15}fa^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{2\,i}{15}fa^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] 1/7*x^5*d*(b*x^4+a)^(1/2)+2/21*a*d*x*(b*x^4+a)^(1/2)/b-2/21*d/b*a
^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)+1/6*c/b*(b*x^4+a)^(3/2)+1/8*e*x^2*(b*x^4+a)^(
3/2)/b-1/16*a*e*x^2*(b*x^4+a)^(1/2)/b-1/16*e*a^2/b^(3/2)*ln(b^(1/
2)*x^2+(b*x^4+a)^(1/2))+1/9*f*x^7*(b*x^4+a)^(1/2)+2/45*a*f*x^3*(b
*x^4+a)^(1/2)/b-2/15*I*f/b^(3/2)*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2
)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+2/15*I*
f/b^(3/2)*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipt
icE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bx^4+a\right)^{\frac{3}{2}}c}{6\,b}+\int\left(fx^6+ex^5+dx^4\right)\sqrt{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^4 + a)^(3/2)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)*sqr
t(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(fx^6+ex^5+dx^4+cx^3\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 8.80098, size = 212, normalized size = 0.54

$$\frac{a^{\frac{3}{2}}ex^2}{16b\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}dx^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}ex^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}fx^7\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{11}{4}\right)}-\frac{a^2e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+\frac{bex^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*e*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*d*x**5*gamma(
5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamm
a(9/4)) + 3*sqrt(a)*e*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*f*x*
*7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(11/4)) - a**2*e*asinh(sqrt(b)*x**2/sqrt(a))/(16*b**(3
/2)) + c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/
2)/(6*b), True)) + b*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

3.485 $\int x^2 \left(c+dx+ex^2+fx^3\right) \sqrt{a+bx^4}\,dx$

Optimal. Leaf size=369

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}c-5\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}-\frac{a^2f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7c+5ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\left(a+bx^4\right)^{3/2}\left(4d+3fx^2\right)}{24b}+\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}$$

```
[Out] (2*a*e*x*Sqrt[a + b*x^4])/(21*b) - (a*f*x^2*Sqrt[a + b*x^4])/(16*
b) + (2*a*c*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)
) + (x^3*(7*c + 5*e*x^2)*Sqrt[a + b*x^4])/35 + ((4*d + 3*f*x^2)*(
a + b*x^4)^(3/2))/(24*b) - (a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(16*b^(3/2)) - (2*a^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*
Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(105*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.819695, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}c-5\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}-\frac{a^2f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7c+5ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\left(a+bx^4\right)^{3/2}\left(4d+3fx^2\right)}{24b}+\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```


```
[Out] (2*a*e*x*Sqrt[a + b*x^4])/(21*b) - (a*f*x^2*Sqrt[a + b*x^4])/(16*
b) + (2*a*c*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)
) + (x^3*(7*c + 5*e*x^2)*Sqrt[a + b*x^4])/35 + ((4*d + 3*f*x^2)*(
a + b*x^4)^(3/2))/(24*b) - (a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(16*b^(3/2)) - (2*a^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*
Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(105*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 75.3073, size = 338, normalized size = 0.92

$$-\frac{2a^{\frac{5}{4}}c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{a^{\frac{5}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(5\sqrt{a}e-21\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{105b^{\frac{5}{4}}\sqrt{a+bx^4}}-\frac{a^2f\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{\frac{3}{2}}}+\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{x^3\sqrt{a+bx^4}\left(7c+5ex^2\right)}{35}+\frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(4d+3fx^2\right)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] -2*a**(5/4)*c*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(5*b**(3/4)*sqrt(a + b*x**4)) - a**(5/4)*sqrt((a + b*x**4)/(sqrt
(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(5*sqrt(a)*e - 2
1*sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(105*b*
*(5/4)*sqrt(a + b*x**4)) - a**2*f*atanh(sqrt(b)*x**2/sqrt(a + b*x
**4))/(16*b**(3/2)) + 2*a*e*x*sqrt(a + b*x**4)/(21*b) - a*f*x**2*
sqrt(a + b*x**4)/(16*b) + 2*a*c*x*sqrt(a + b*x**4)/(5*sqrt(b)*(sq
rt(a) + sqrt(b)*x**2)) + x**3*sqrt(a + b*x**4)*(7*c + 5*e*x**2)/3
5 + (a + b*x**4)**(3/2)*(4*d + 3*f*x**2)/(24*b)
```

Mathematica [C] time = 0.72535, size = 280, normalized size = 0.76

$$\frac{32ia^{3/2}\sqrt{b}\sqrt{\frac{bx^4}{a}+1}\left(5\sqrt{a}e+21i\sqrt{b}c\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+672a^{3/2}bc\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\cdots}{1680b^{3/2}\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\cdots}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[b]*(a + b*x^4)*(5*a*(56*d + x*(3
2*e + 21*f*x)) + 2*b*x^3*(168*c + 5*x*(28*d + 3*x*(8*e + 7*f*x)))
) - 105*a^2*f*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^
4]]) + 672*a^(3/2)*b*c*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sq
rt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + (32*I)*a^(3/2)*Sqrt[b]*((21*I)*
Sqrt[b]*c + 5*Sqrt[a]*e)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[
Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(1680*Sqrt[(I*Sqrt[b])/Sqrt[a]
]*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.015, size = 361, normalized size = 1.

$$
\frac{d}{6b}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{cx^3}{5}\sqrt{bx^4+a}
+\frac{2i}{5}ca^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}
-\frac{2i}{5}ca^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}
+\frac{ex^5}{7}\sqrt{bx^4+a}+\frac{2aex}{21b}\sqrt{bx^4+a}
-\frac{2ea^2}{21b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}
+\frac{fx^2}{8b}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{x^2af}{16b}\sqrt{bx^4+a}-\frac{a^2f}{16}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] 1/6*d/b*(b*x^4+a)^(3/2)+1/5*c*x^3*(b*x^4+a)^(1/2)+2/5*I*c*a^(3/2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*c*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)+1/7*e*x^5*(b*x^4+a)^(1/2)+2/21*a*e*x*(b*x^4+a)^(1/2)/b-2/21*e/b
*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)+1/8*f*x^2*(b*x^4+a)^(3/2)/b-1/16*a*f*x^2*(b
*x^4+a)^(1/2)/b-1/16*f*a^2/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^5+ex^4+dx^3+cx^2\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2,x, algorithm="fricas")
```

```
[Out] integral((f*x^5 + e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 8.43041, size = 212, normalized size = 0.57

$$\frac{a^{\frac{3}{2}}fx^2}{16b\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}cx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}ex^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}fx^6}{16\sqrt{1+\frac{bx^4}{a}}}-\frac{a^2f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+\frac{bfx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*f*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**3*gamma(
3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamm
a(7/4)) + sqrt(a)*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*f*x**6/(16*sqr
t(1 + b*x**4/a)) - a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*b**(3/2
)) + d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)
/(6*b), True)) + b*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

3.486 $\int x\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=354

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}d-5\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{ac\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7d+5fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\left(a+bx^4\right)^{3/2}}{6b}+\frac{2afx\sqrt{a+bx^4}}{21b}$$

```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(21*b) + (c*x^2*Sqrt[a + b*x^4])/4 + (2
*a*d*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^
3*(7*d + 5*f*x^2)*Sqrt[a + b*x^4])/35 + (e*(a + b*x^4)^(3/2))/(6*
b) + (a*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (
2*a^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*
b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*d - 5*Sqrt[a]*f)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sq
rt[a + b*x^4])
```

Rubi [A] time = 0.735569, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}d-5\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{ac\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7d+5fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\left(a+bx^4\right)^{3/2}}{6b}+\frac{2afx\sqrt{a+bx^4}}{21b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```



```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(21*b) + (c*x^2*Sqrt[a + b*x^4])/4 + (2
*a*d*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^
3*(7*d + 5*f*x^2)*Sqrt[a + b*x^4])/35 + (e*(a + b*x^4)^(3/2))/(6*
b) + (a*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (
2*a^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*
b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*d - 5*Sqrt[a]*f)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sq
rt[a + b*x^4])
```

Rubi in Sympy [A] time = 69.5724, size = 325, normalized size = 0.92

$$-\frac{2a^{\frac{5}{4}}d\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{a^{\frac{5}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(5\sqrt{a}f-21\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{105b^{\frac{5}{4}}\sqrt{a+bx^4}}+\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{ac\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{cx^2\sqrt{a+bx^4}}{4}+\frac{x^3\sqrt{a+bx^4}\left(7d+5fx^2\right)}{35}+\frac{e\left(a+bx^4\right)^{\frac{3}{2}}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] -2*a**(5/4)*d*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(5*b**(3/4)*sqrt(a + b*x**4)) - a**(5/4)*sqrt((a + b*x**4)/(sqrt
(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(5*sqrt(a)*f - 2
1*sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(105*b*
*(5/4)*sqrt(a + b*x**4)) + 2*a*f*x*sqrt(a + b*x**4)/(21*b) + a*c*
atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(4*sqrt(b)) + 2*a*d*x*sqrt(a
 + b*x**4)/(5*sqrt(b)*(sqrt(a) + sqrt(b)*x**2)) + c*x**2*sqrt(a +
 b*x**4)/4 + x**3*sqrt(a + b*x**4)*(7*d + 5*f*x**2)/35 + e*(a + b
*x**4)**(3/2)/(6*b)
```

Mathematica [C] time = 0.719077, size = 266, normalized size = 0.75

$$\frac{8ia^{3/2}\sqrt{\frac{bx^4}{a}+1}\left(5\sqrt{a}f+21i\sqrt{b}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+168a^{3/2}\sqrt{b}d\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}{420b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(10*a*(7*e + 4*f*x) + b*x
^2*(105*c + 84*d*x + 70*e*x^2 + 60*f*x^3)) + 105*a*Sqrt[b]*c*Sqrt
[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) + 168*a^(3/2)
*Sqrt[b]*d*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b
])/Sqrt[a]]*x], -1] + (8*I)*a^(3/2)*((21*I)*Sqrt[b]*d + 5*Sqrt[a]
*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt
[a]]*x], -1])/(420*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.013, size = 337, normalized size = 1.

$$\frac{dx^{3}}{5}\sqrt{bx^{4}+a}
+\frac{2i}{5}da^{\frac{3}{2}}\sqrt{1-ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^{4}+a}}\frac{1}{\sqrt{b}}
-\frac{2i}{5}da^{\frac{3}{2}}\sqrt{1-ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^{4}+a}}\frac{1}{\sqrt{b}}
+\frac{cx^{2}}{4}\sqrt{bx^{4}+a}+\frac{ac}{4}\ln\left(\sqrt{b}x^{2}+\sqrt{bx^{4}+a}\right)\frac{1}{\sqrt{b}}+\frac{e}{6b}\left(bx^{4}+a\right)^{\frac{3}{2}}+\frac{fx^{5}}{7}\sqrt{bx^{4}+a}+\frac{2afx}{21b}\sqrt{bx^{4}+a}
-\frac{2a^{2}f}{21b}\sqrt{1-ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^{4}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] 1/5*x^3*d*(b*x^4+a)^(1/2)+2/5*I*d*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)-2/5*I*d*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2
)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*c*x^2*(b*x^4+a)^(1
/2)+1/4*c*a/b^(1/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))+1/6*e*(b*x^4+
a)^(3/2)/b+1/7*f*x^5*(b*x^4+a)^(1/2)+2/21*a*f*x*(b*x^4+a)^(1/2)/b
-2/21*f/b*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^{4}+a}\left(fx^{4}+ex^{3}+dx^{2}+cx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^4 + e*x^3 + d*x^2 + c*x), x)
```

Sympy [A] time = 5.44284, size = 158, normalized size = 0.45

$$\frac{\sqrt{a}cx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+e\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*d*x**3*gamma(3/4)*h
yper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)
) + sqrt(a)*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*gamma(9/4)) + a*c*asinh(sqrt(b)*x**2/sqrt(a)
)/(4*sqrt(b)) + e*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x
**4)**(3/2)/(6*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{bx^4+a}\,(fx^3+ex^2+dx+c)\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x, x)
```

3.487 $\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=331

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}} + \frac{1}{15}x\sqrt{a+bx^4}\left(5c+3ex^2\right) + \frac{1}{4}dx^2\sqrt{a+bx^4} + \frac{ad\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} + \frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\left(a+bx^4\right)^{3/2}}{6b}$$

```
[Out] (d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15
 + (f*(a + b*x^4)^(3/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[
a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(
5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.517732, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.37$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}} + \frac{1}{15}x\sqrt{a+bx^4}\left(5c+3ex^2\right) + \frac{1}{4}dx^2\sqrt{a+bx^4} + \frac{ad\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} + \frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\left(a+bx^4\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```


```
[Out] (d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15
 + (f*(a + b*x^4)^(3/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[
a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(
5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 50.8368, size = 303, normalized size = 0.92

$$-\frac{2a^{\frac{5}{4}}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{a^{\frac{3}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{a}e+5\sqrt{b}c\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{ad\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{dx^2\sqrt{a+bx^4}}{4}+\frac{x\sqrt{a+bx^4}\left(5c+3ex^2\right)}{15}+\frac{f\left(a+bx^4\right)^{\frac{3}{2}}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] -2*a**(5/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(5*b**(3/4)*sqrt(a + b*x**4)) + a**(3/4)*sqrt((a + b*x**4)/(sqrt
(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(3*sqrt(a)*e + 5
*sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(15*b**(
3/4)*sqrt(a + b*x**4)) + a*d*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))
/(4*sqrt(b)) + 2*a*e*x*sqrt(a + b*x**4)/(5*sqrt(b)*(sqrt(a) + sqr
t(b)*x**2)) + d*x**2*sqrt(a + b*x**4)/4 + x*sqrt(a + b*x**4)*(5*c
 + 3*e*x**2)/15 + f*(a + b*x**4)**(3/2)/(6*b)
```

Mathematica [C] time = 0.763197, size = 257, normalized size = 0.78

$$\frac{24a^{3/2}\sqrt{b}e\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(10af+bx(20c+x(15d+2x(6e+5fx)))\right)+15a\sqrt{b}d\sqrt{a}\right.}{60b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(10*a*f + b*x*(20*c + x*(
15*d + 2*x*(6*e + 5*f*x)))) + 15*a*Sqrt[b]*d*Sqrt[a + b*x^4]*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) + 24*a^(3/2)*Sqrt[b]*e*Sqrt[1
 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -
1] - 8*a*Sqrt[b]*((5*I)*Sqrt[b]*c + 3*Sqrt[a]*e)*Sqrt[1 + (b*x^4)
/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sq
rt[(I*Sqrt[b])/Sqrt[a]]*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.011, size = 313, normalized size = 1.

$$\frac{cx}{3}\sqrt{bx^4+a}+\frac{2\,ac}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{dx^2}{4}\sqrt{bx^4+a}+\frac{ad}{4}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}+\frac{ex^3}{5}\sqrt{bx^4+a}+\frac{2\,i}{5}ea^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}-\frac{2\,i}{5}ea^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}+\frac{f}{6\,b}\left(bx^4+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] 1/3*c*x*(b*x^4+a)^(1/2)+2/3*c*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*d*x^2*(b*x^4
+a)^(1/2)+1/4*d*a/b^(1/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))+1/5*e*x
^3*(b*x^4+a)^(1/2)+2/5*I*e*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I
*e*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*Ellip
ticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/6*f*(b*x^4+a)^(3/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

Sympy [A] time = 5.15827, size = 156, normalized size = 0.47

$$\frac{\sqrt{a}cx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}ex^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+f\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/4 +
 sqrt(a)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_
polar(I*pi)/a)/(4*gamma(7/4)) + a*d*asinh(sqrt(b)*x**2/sqrt(a))/(
4*sqrt(b)) + f*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4
)**(3/2)/(6*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

3.488 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx$

Optimal. Leaf size=345

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}} + \frac{1}{4}\sqrt{a+bx^4}\left(2c+ex^2\right) - \frac{1}{2}\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) + \frac{1}{15}x\sqrt{a+bx^4}\left(5d+3fx^2\right) + \frac{ae\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} + \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (
(2*c + e*x^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*
x^4])/15 + (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b
]) - (Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*
f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sq
rt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4
])
```

Rubi [A] time = 0.6867, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}} + \frac{1}{4}\sqrt{a+bx^4}\left(2c+ex^2\right) - \frac{1}{2}\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) + \frac{1}{15}x\sqrt{a+bx^4}\left(5d+3fx^2\right) + \frac{ae\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} + \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```


```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (
(2*c + e*x^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*
x^4])/15 + (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b
]) - (Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*
f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sq
rt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4
])
```

Rubi in Sympy [A] time = 69.0769, size = 316, normalized size = 0.92

$$-\frac{2a^{\frac{5}{4}}f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{a^{\frac{3}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{a}f+5\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{\sqrt{a}c\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2}+\frac{ae\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{x\sqrt{a+bx^4}\left(5d+3fx^2\right)}{15}+\frac{\sqrt{a+bx^4}\left(2c+ex^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x,x)
```

```
[Out] -2*a**(5/4)*f*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(5*b**(3/4)*sqrt(a + b*x**4)) + a**(3/4)*sqrt((a + b*x**4)/(sqrt
(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(3*sqrt(a)*f + 5
*sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(15*b**(
3/4)*sqrt(a + b*x**4)) - sqrt(a)*c*atanh(sqrt(a + b*x**4)/sqrt(a)
)/2 + a*e*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(4*sqrt(b)) + 2*a*
f*x*sqrt(a + b*x**4)/(5*sqrt(b)*(sqrt(a) + sqrt(b)*x**2)) + x*sqr
t(a + b*x**4)*(5*d + 3*f*x**2)/15 + sqrt(a + b*x**4)*(2*c + e*x**
2)/4
```

Mathematica [C] time = 2.35789, size = 280, normalized size = 0.81

$$\frac{24a^{3/2}f\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\left(\left(a+bx^4\right)\left(30c+x(20d+3x(5e+4fx))\right)-30\sqrt{a}c\sqrt{a+bx^4}\tanh\right.\right.}{60\sqrt{b}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(15*a*e*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]] + Sqrt[b]*((a + b*x^4)*(30*c + x*(20*d +
 3*x*(5*e + 4*f*x))) - 30*Sqrt[a]*c*Sqrt[a + b*x^4]*ArcTanh[Sqrt[
a + b*x^4]/Sqrt[a]])) + 24*a^(3/2)*f*Sqrt[1 + (b*x^4)/a]*Elliptic
E[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 8*a*((5*I)*Sqrt[b
]*d + 3*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(
I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[b
]*Sqrt[a + b*x^4])
```

Maple [C] time = 0.019, size = 339, normalized size = 1.

$$\frac{dx}{3}\sqrt{bx^4+a}+\frac{2\,ad}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{fx^3}{5}\sqrt{bx^4+a}$$
$$+\frac{2\,i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$-\frac{2\,i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$+\frac{c}{2}\sqrt{bx^4+a}-\frac{c}{2}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)+\frac{ex^2}{4}\sqrt{bx^4+a}+\frac{ae}{4}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x)
```

```
[Out] 1/3*x*d*(b*x^4+a)^(1/2)+2/3*d*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/5*f*x^3*(b*x^4
+a)^(1/2)+2/5*I*f*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*f*a^(3/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)+1/2*c*(b*x^4+a)^(1/2)-1/2*c*a^(1/2)*ln
((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*e*x^2*(b*x^4+a)^(1/2)+1
/4*e*a/b^(1/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Sympy [A] time = 7.37201, size = 204, normalized size = 0.59

$$-\frac{\sqrt{a}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ac}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+\frac{\sqrt{b}cx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*d*x*gamma(1/
4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(
5/4)) + sqrt(a)*e*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*f*x**3*gamm
a(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(7/4)) + a*c/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)) + a*e*asinh
(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*c*x**2/(2*sqrt(a/(b*
x**4) + 1))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

3.489 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx$

Optimal. Leaf size=341

$$-\frac{\sqrt{a+bx^4}\left(3c-ex^2\right)}{3x}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}+\frac{1}{4}\sqrt{a+bx^4}\left(2d+fx^2\right)-\frac{1}{2}\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}$$

```
[Out] (2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c -
 e*x^2)*Sqrt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/
4 + (a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (S
qrt[a]*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)
*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x
^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.718692, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{\sqrt{a+bx^4}\left(3c-ex^2\right)}{3x}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}+\frac{1}{4}\sqrt{a+bx^4}\left(2d+fx^2\right)-\frac{1}{2}\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

```
[Out] (2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c -
 e*x^2)*Sqrt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/
4 + (a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (S
qrt[a]*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)
*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x
^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 73.2619, size = 308, normalized size = 0.9

$$-\frac{2\sqrt[4]{a}\sqrt[4]{b}c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}e+3\sqrt{b}c\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt{a}d\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2}+\frac{af\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}+\frac{\sqrt{a+bx^4}\left(2d+fx^2\right)}{4}-\frac{\sqrt{a+bx^4}\left(3c-ex^2\right)}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**2,x)
```

```
[Out] -2*a**(1/4)*b**(1/4)*c*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)
**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/
4)), 1/2)/sqrt(a + b*x**4) + a**(1/4)*sqrt((a + b*x**4)/(sqrt(a)
+ sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*e + 3*sqrt(
b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(3*b**(1/4)*sq
rt(a + b*x**4)) - sqrt(a)*d*atanh(sqrt(a + b*x**4)/sqrt(a))/2 + a
*f*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(4*sqrt(b)) + 2*sqrt(b)*c
*x*sqrt(a + b*x**4)/(sqrt(a) + sqrt(b)*x**2) + sqrt(a + b*x**4)*(
2*d + f*x**2)/4 - sqrt(a + b*x**4)*(3*c - e*x**2)/(3*x)
```

Mathematica [C] time = 6.18026, size = 355, normalized size = 1.04

$$\sqrt{a+bx^4}\left(-\frac{c}{x}+\frac{d}{2}+\frac{ex}{3}+\frac{fx^2}{4}\right)+\frac{1}{6}\left(\frac{12\sqrt{a}\sqrt{b}c\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}x^2}{\sqrt{a}}}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}-3\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{4iae\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}x^2}{\sqrt{a}}}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}+\frac{3af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

```
[Out] (d/2 - c/x + (e*x)/3 + (f*x^2)/4)*Sqrt[a + b*x^4] + ((3*a*f*ArcTa
nh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - 3*Sqrt[a]*d*ArcT
anh[Sqrt[a + b*x^4]/Sqrt[a]] + (12*Sqrt[a]*Sqrt[b]*c*Sqrt[1 - (I*
Sqrt[b]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[b]*x^2)/Sqrt[a]]*(Elliptic
E[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - EllipticF[I*ArcSi
nh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[b])/Sqrt[a]]
*Sqrt[a + b*x^4]) - ((4*I)*a*e*Sqrt[1 - (I*Sqrt[b]*x^2)/Sqrt[a]]*
Sqrt[1 + (I*Sqrt[b]*x^2)/Sqrt[a]]*EllipticF[I*ArcSinh[Sqrt[(I*Sqr
t[b])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*x^4
]))/6
```

Maple [C] time = 0.02, size = 339, normalized size = 1.

$$\frac{ex}{3}\sqrt{bx^4+a}+\frac{2\,ae}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{fx^2}{4}\sqrt{bx^4+a}+\frac{af}{4}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}-\frac{c}{x}\sqrt{bx^4+a}$$
$$+2\,ic\sqrt{a}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-2\,ic\sqrt{a}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{d}{2}\sqrt{bx^4+a}-\frac{d}{2}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x)
```

```
[Out] 1/3*e*x*(b*x^4+a)^(1/2)+2/3*e*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*x^2*f*(b*x^4
+a)^(1/2)+1/4*f*a/b^(1/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))-c/x*(b*
x^4+a)^(1/2)+2*I*c*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2*I*c*b^(1/2)
*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)+1/2*d*(b*x^4+a)^(1/2)-1/2*d*a^(1/2)*ln(
(2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Sympy [A] time = 7.45978, size = 206, normalized size = 0.6

$$\frac{\sqrt{a}c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{a}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}fx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{ad}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+\frac{\sqrt{b}dx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x*
*2))/2 + sqrt(a)*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4
*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*f*x**2*sqrt(1 + b*x*
*4/a)/4 + a*d/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)) + a*f*asinh(s
qrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*d*x**2/(2*sqrt(a/(b*x*
*4) + 1))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

3.490 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$

Optimal. Leaf size=342

$$-\frac{\sqrt{a+bx^4}\,(c-ex^2)}{2x^2}+\frac{1}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{\sqrt{a+bx^4}\,(3d-fx^2)}{3x}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{1}{2}\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

```
[Out] (2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e
*x^2)*Sqrt[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/
(3*x) + (Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (S
qrt[a]*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)
*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x
^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.723866, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{\sqrt{a+bx^4}\,(c-ex^2)}{2x^2}+\frac{1}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{\sqrt{a+bx^4}\,(3d-fx^2)}{3x}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{1}{2}\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

```
[Out] (2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e
*x^2)*Sqrt[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/
(3*x) + (Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (S
qrt[a]*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)
*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x
^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 71.9011, size = 308, normalized size = 0.9

$$-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}f+3\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt{a}e\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2}+\frac{\sqrt{b}c\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2}+\frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{\sqrt{a+bx^4}\left(3d-fx^2\right)}{3x}-\frac{\sqrt{a+bx^4}\left(c-ex^2\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**3,x)
```

```
[Out] -2*a**(1/4)*b**(1/4)*d*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)
**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/
4)), 1/2)/sqrt(a + b*x**4) + a**(1/4)*sqrt((a + b*x**4)/(sqrt(a)
+ sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*f + 3*sqrt(
b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(3*b**(1/4)*sq
rt(a + b*x**4)) - sqrt(a)*e*atanh(sqrt(a + b*x**4)/sqrt(a))/2 + s
qrt(b)*c*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/2 + 2*sqrt(b)*d*x*s
qrt(a + b*x**4)/(sqrt(a) + sqrt(b)*x**2) - sqrt(a + b*x**4)*(3*d
- f*x**2)/(3*x) - sqrt(a + b*x**4)*(c - e*x**2)/(2*x**2)
```

Mathematica [C] time = 1.09468, size = 296, normalized size = 0.87

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(x\left(-6d+3ex+2fx^2\right)-3c\right)+3\sqrt{b}cx^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-3\sqrt{a}ex^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)}{6x^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(-3*c + x*(-6*d + 3*e*x +
 2*f*x^2)) + 3*Sqrt[b]*c*x^2*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]] - 3*Sqrt[a]*e*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt
[a + b*x^4]/Sqrt[a]]) + 12*Sqrt[a]*Sqrt[b]*d*x^2*Sqrt[1 + (b*x^4)
/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - (4*I)
*Sqrt[a]*((-3*I)*Sqrt[b]*d + Sqrt[a]*f)*x^2*Sqrt[1 + (b*x^4)/a]*E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(6*Sqrt[(I*
Sqrt[b])/Sqrt[a]]*x^2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.02, size = 360, normalized size = 1.1

$$\frac{fx}{3}\sqrt{bx^4+a}+\frac{2\,af}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{c}{2\,ax^2}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{x^2bc}{2\,a}\sqrt{bx^4+a}+\frac{c}{2}\sqrt{b}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{d}{x}\sqrt{bx^4+a}+2\,id\sqrt{a}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-2\,id\sqrt{a}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{e}{2}\sqrt{bx^4+a}-\frac{e}{2}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x)
```

```
[Out] 1/3*x*f*(b*x^4+a)^(1/2)+2/3*f*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*c/a/x^2*(b*x
^4+a)^(3/2)+1/2*c*b/a*x^2*(b*x^4+a)^(1/2)+1/2*c*b^(1/2)*ln(b^(1/2
)*x^2+(b*x^4+a)^(1/2))-d/x*(b*x^4+a)^(1/2)+2*I*d*b^(1/2)*a^(1/2)/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)-2*I*d*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*e*(b*
x^4+a)^(1/2)-1/2*e*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Sympy [A] time = 6.75743, size = 230, normalized size = 0.67

$$-\frac{\sqrt{a}c}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}d\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{a}e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}fx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}+\frac{ae}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{\sqrt{b}c\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}+\frac{\sqrt{b}ex^2}{2\sqrt{\frac{a}{bx^4}+1}}-\frac{bcx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(a)*c/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*d*gamma(-1/4)*hy
per((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/
```

```
4)) - sqrt(a)*e*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*f*x*gam
ma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(5/4)) + a*e/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)) + sqrt(b)*
c*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*e*x**2/(2*sqrt(a/(b*x**
4) + 1)) - b*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

3.491 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$

Optimal. Leaf size=357

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}\left(c-3ex^2\right)}{3x^3}-\frac{\sqrt{a+bx^4}\left(d-fx^2\right)}{2x^2}+\frac{1}{2}\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{1}{2}\sqrt{a}f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

```
[Out] (-2*e*Sqrt[a + b*x^4])/x + (2*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[
a] + Sqrt[b]*x^2) - ((c - 3*e*x^2)*Sqrt[a + b*x^4])/(3*x^3) - ((d
 - f*x^2)*Sqrt[a + b*x^4])/(2*x^2) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqr
t[a]])/2 - (2*a^(1/4)*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*c + 3*Sqrt[a
]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)
*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.823848, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}\left(c-3ex^2\right)}{3x^3}-\frac{\sqrt{a+bx^4}\left(d-fx^2\right)}{2x^2}+\frac{1}{2}\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{1}{2}\sqrt{a}f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```


```
[Out] (-2*e*Sqrt[a + b*x^4])/x + (2*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[
a] + Sqrt[b]*x^2) - ((c - 3*e*x^2)*Sqrt[a + b*x^4])/(3*x^3) - ((d
 - f*x^2)*Sqrt[a + b*x^4])/(2*x^2) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqr
t[a]])/2 - (2*a^(1/4)*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*c + 3*Sqrt[a
]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)
*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 81.2073, size = 325, normalized size = 0.91

$$-\frac{2\sqrt[4]{a}\sqrt[4]{b}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{\sqrt{a}f\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2}+\frac{\sqrt{b}d\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2e\sqrt{a+bx^4}}{x}-\frac{\sqrt{a+bx^4}\left(d-fx^2\right)}{2x^2}-\frac{\sqrt{a+bx^4}\left(c-3ex^2\right)}{3x^3}+\frac{\sqrt[4]{b}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{a}e+\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**4,x)
```

```
[Out] -2*a**(1/4)*b**(1/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)
**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/
4)), 1/2)/sqrt(a + b*x**4) - sqrt(a)*f*atanh(sqrt(a + b*x**4)/sqr
t(a))/2 + sqrt(b)*d*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/2 + 2*sq
rt(b)*e*x*sqrt(a + b*x**4)/(sqrt(a) + sqrt(b)*x**2) - 2*e*sqrt(a
+ b*x**4)/x - sqrt(a + b*x**4)*(d - f*x**2)/(2*x**2) - sqrt(a + b
*x**4)*(c - 3*e*x**2)/(3*x**3) + b**(1/4)*sqrt((a + b*x**4)/(sqrt
(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(3*sqrt(a)*e + s
qrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(3*a**(1/4
)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.891044, size = 295, normalized size = 0.83

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(2c+3x\left(d+2ex-fx^2\right)\right)-3\sqrt{b}dx^3\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+3\sqrt{a}fx^3\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)}{6x^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*c + 3*x*(d + 2*e*x -
 f*x^2)) - 3*Sqrt[b]*d*x^3*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/
Sqrt[a + b*x^4]] + 3*Sqrt[a]*f*x^3*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])) + 12*Sqrt[a]*Sqrt[b]*e*x^3*Sqrt[1 + (b*x^4)/
a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 4*Sqrt
[b]*(I*Sqrt[b]*c + 3*Sqrt[a]*e)*x^3*Sqrt[1 + (b*x^4)/a]*EllipticF
[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(6*Sqrt[(I*Sqrt[b])
/Sqrt[a]]*x^3*Sqrt[a + b*x^4])
```

Maple [C] time = 0.023, size = 362, normalized size = 1.

$$
\begin{aligned}
&-\frac{c}{3\,x^3}\sqrt{bx^4+a} \\
&+\frac{2\,bc}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
&-\frac{d}{2\,ax^2}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{x^2bd}{2\,a}\sqrt{bx^4+a}+\frac{d}{2}\sqrt{b}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{e}{x}\sqrt{bx^4+a} \\
&+2\,ie\sqrt{a}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
&-2\,ie\sqrt{a}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
&+\frac{f}{2}\sqrt{bx^4+a}-\frac{f}{2}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*c/x^3*(b*x^4+a)^(1/2)+2/3*c*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d/a/x^2*(
b*x^4+a)^(3/2)+1/2*d*b/a*x^2*(b*x^4+a)^(1/2)+1/2*d*b^(1/2)*ln(b^(
1/2)*x^2+(b*x^4+a)^(1/2))-e*(b*x^4+a)^(1/2)/x+2*I*e*b^(1/2)*a^(1/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)-2*I*e*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*f*
(b*x^4+a)^(1/2)-1/2*f*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/
x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Sympy [A] time = 7.04955, size = 235, normalized size = 0.66

$$\frac{\sqrt{a}c\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\left(\frac{1}{4}\right)}-\frac{\sqrt{a}d}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}e\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x\left(\frac{3}{4}\right)}$$
$$-\frac{\sqrt{a}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{af}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}+\frac{\sqrt{b}fx^2}{2\sqrt{\frac{a}{bx^4}+1}}-\frac{bdx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**4,x)
```

```
[Out] sqrt(a)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*d/(2*x**2*sqrt(1 + b*x**
4/a)) + sqrt(a)*e*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*f*asinh(sqrt(a)/(sq
rt(b)*x**2))/2 + a*f/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)) + sqrt
(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*f*x**2/(2*sqrt(a/(b
*x**4) + 1)) - b*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

3.492 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$

Optimal. Leaf size=329

$$-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}+\frac{1}{2}\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{2\sqrt{b}fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}$$

```
[Out] -(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4])
/12 + (2*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (
Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*c*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*a^(1/4)*b^(1/4)*f*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] +
 (b^(1/4)*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.596052, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}+\frac{1}{2}\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{2\sqrt{b}fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```


```
[Out] -(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4])
/12 + (2*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (
Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*c*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*a^(1/4)*b^(1/4)*f*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] +
 (b^(1/4)*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.39629, size = 267, normalized size = 0.81

$$\frac{1}{12}\left(-\frac{\sqrt{a+bx^4}\,(3c+4dx+6x^2(e+2fx))}{x^4}-\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{8\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{b}d-3i\sqrt{a}f\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{a+bx^4}}+6\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{24iaf\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{a+bx^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

```
[Out] (-((Sqrt[a + b*x^4]*(3*c + 4*d*x + 6*x^2*(e + 2*f*x)))/x^4) + 6*S
qrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - (3*b*c*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/Sqrt[a] - ((24*I)*a*Sqrt[(I*Sqrt[b])/Sq
rt[a]]*f*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])
/Sqrt[a]]*x], -1])/Sqrt[a + b*x^4] - (8*Sqrt[a]*Sqrt[(I*Sqrt[b])/
Sqrt[a]]*(Sqrt[b]*d - (3*I)*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*Ellipt
icF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/Sqrt[a + b*x^4])
/12
```

Maple [C] time = 0.02, size = 385, normalized size = 1.2

$$-\frac{c}{4\,ax^4}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{bc}{4}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{bc}{4\,a}\sqrt{bx^4+a}-\frac{d}{3\,x^3}\sqrt{bx^4+a}+\frac{2\,bd}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{e}{2\,ax^2}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{bex^2}{2\,a}\sqrt{bx^4+a}+\frac{e}{2}\sqrt{b}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{f}{x}\sqrt{bx^4+a}+2\,if\sqrt{a}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-2\,if\sqrt{a}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x)
```

```
[Out] -1/4*c/a/x^4*(b*x^4+a)^(3/2)-1/4*c*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b
*x^4+a)^(1/2))/x^2)+1/4*c*b/a*(b*x^4+a)^(1/2)-1/3*d/x^3*(b*x^4+a)
^(1/2)+2/3*d*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*e/a/x^2*(b*x^4+a)^(3/2)+1/2*e
*b/a*x^2*(b*x^4+a)^(1/2)+1/2*e*b^(1/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(
1/2))-f/x*(b*x^4+a)^(1/2)+2*I*f*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2
))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-
2*I*f*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*El
```

```
lipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

Sympy [A] time = 8.31159, size = 211, normalized size = 0.64

$$\frac{\sqrt{a}d\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\,\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}e}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}f\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4x^2}+\frac{\sqrt{b}e\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}-\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}-\frac{bex^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*e/(2*x**2*sqrt(1 + b*x**
4/a)) + sqrt(a)*f*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(b)*c*sqrt(a/(b*x**4) +
 1)/(4*x**2) + sqrt(b)*e*asinh(sqrt(b)*x**2/sqrt(a))/2 - b*c*asin
h(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*e*x**2/(2*sqrt(a)*sqrt(
1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

3.493 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx$

Optimal. Leaf size=360

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+3\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} + \frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right) - \frac{2bc\sqrt{a+bx^4}}{5ax} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} + \frac{1}{2}\sqrt{b}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)$$

```
[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*
x^4])/60 - (2*b*c*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*c*x*Sqrt[
a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (Sqrt[b]*f*ArcTanh[(S
qrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sq
rt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqr
t[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.818206, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+3\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} + \frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right) - \frac{2bc\sqrt{a+bx^4}}{5ax} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} + \frac{1}{2}\sqrt{b}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```


```
[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*
x^4])/60 - (2*b*c*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*c*x*Sqrt[
a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (Sqrt[b]*f*ArcTanh[(S
qrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sq
rt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqr
t[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**6,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.16428, size = 314, normalized size = 0.87

$$\frac{24\sqrt{a}b^{3/2}cx^5\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(12ac+5ax\left(3d+4ex+6fx^2\right)+24bcx^4\right)+15\sqrt{a}bd\right.}{60a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(12*a*c + 24*b*c*x^4 +
5*a*x*(3*d + 4*e*x + 6*f*x^2)) - 30*a*Sqrt[b]*f*x^5*Sqrt[a + b*x^
4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] + 15*Sqrt[a]*b*d*x^5*Sq
rt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 24*Sqrt[a]*b^(
3/2)*c*x^5*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b
])/Sqrt[a]]*x], -1] - (8*I)*Sqrt[a]*b*((-3*I)*Sqrt[b]*c + 5*Sqrt[
a]*e)*x^5*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]
)/Sqrt[a]]*x], -1])/(60*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^5*Sqrt[a +
b*x^4])
```

Maple [C] time = 0.027, size = 404, normalized size = 1.1

$$\begin{aligned}
&-\frac{c}{5x^5}\sqrt{bx^4+a}-\frac{2bc}{5ax}\sqrt{bx^4+a}\\
&+\frac{2i}{5}cb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{2i}{5}cb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{d}{4ax^4}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{bd}{4}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{bd}{4a}\sqrt{bx^4+a}-\frac{e}{3x^3}\sqrt{bx^4+a}\\
&+\frac{2be}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{f}{2ax^2}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{bfx^2}{2a}\sqrt{bx^4+a}+\frac{f}{2}\sqrt{b}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x)
```

```
[Out] -1/5*c/x^5*(b*x^4+a)^(1/2)-2/5*b*c*(b*x^4+a)^(1/2)/a/x+2/5*I*c/a^
(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*c/a^(1/2)*b^(3/2)/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1
/2),I)-1/4*d/a/x^4*(b*x^4+a)^(3/2)-1/4*d*b/a^(1/2)*ln((2*a+2*a^(1
/2)*(b*x^4+a)^(1/2))/x^2)+1/4*d*b/a*(b*x^4+a)^(1/2)-1/3*e/x^3*(b*
x^4+a)^(1/2)+2/3*e*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*f/a/x^2*(b*x^4+a)^(3/2)
+1/2*f*b/a*x^2*(b*x^4+a)^(1/2)+1/2*f*b^(1/2)*ln(b^(1/2)*x^2+(b*x^
4+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Sympy [A] time = 8.72219, size = 216, normalized size = 0.6

$$\frac{\sqrt{a}c\,\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}e\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\,\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}f}{2x^2\sqrt{1+\frac{bx^4}{a}}}-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4x^2}+\frac{\sqrt{b}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}-\frac{bfx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*e*gamma(-3/4)*hyper((-
3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4))
 - sqrt(a)*f/(2*x**2*sqrt(1 + b*x**4/a)) - sqrt(b)*d*sqrt(a/(b*x*
*4) + 1)/(4*x**2) + sqrt(b)*f*asinh(sqrt(b)*x**2/sqrt(a))/2 - b*d
*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*f*x**2/(2*sqrt(a)*
sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

3.494 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$

Optimal. Leaf size=352

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+3\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bd\sqrt{a+bx^4}}{5ax}-\frac{be\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*
x^4])/60 - (b*c*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*d*Sqrt[a + b*x^
4])/(5*a*x) + (2*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqr
t[b]*x^2)) - (b*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) -
 (2*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d + 5*Sqrt[a]*f)
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sq
rt[a + b*x^4])
```

Rubi [A] time = 0.802206, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+3\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bd\sqrt{a+bx^4}}{5ax}-\frac{be\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```


```
[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*
x^4])/60 - (b*c*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*d*Sqrt[a + b*x^
4])/(5*a*x) + (2*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqr
t[b]*x^2)) - (b*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) -
 (2*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d + 5*Sqrt[a]*f)
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sq
rt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**7,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.847566, size = 277, normalized size = 0.79

$$\frac{24\sqrt{a}b^{3/2}dx^6\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(10ac+ax(12d+5x(3e+4fx))+2bx^4(5c+12dx)\right)\right.}{60ax^6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(10*a*c + 2*b*x^4*(5*c
+ 12*d*x) + a*x*(12*d + 5*x*(3*e + 4*f*x))) + 15*Sqrt[a]*b*e*x^6*
Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 24*Sqrt[a]*b
^(3/2)*d*x^6*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt
[b])/Sqrt[a]]*x], -1] - (8*I)*Sqrt[a]*b*((-3*I)*Sqrt[b]*d + 5*Sqr
t[a]*f)*x^6*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[
b])/Sqrt[a]]*x], -1])/(60*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^6*Sqrt[a
+ b*x^4])
```

Maple [C] time = 0.022, size = 361, normalized size = 1.

$$-\frac{c}{6\,ax^6}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{d}{5\,x^5}\sqrt{bx^4+a}-\frac{2\,bd}{5\,ax}\sqrt{bx^4+a}$$
$$+\frac{2\,i}{5}db^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{2\,i}{5}db^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{e}{4\,ax^4}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{be}{4}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{be}{4\,a}\sqrt{bx^4+a}-\frac{f}{3\,x^3}\sqrt{bx^4+a}$$
$$+\frac{2\,fb}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x)
```

```
[Out] -1/6*c/a/x^6*(b*x^4+a)^(3/2)-1/5*d/x^5*(b*x^4+a)^(1/2)-2/5*b*d*(b
*x^4+a)^(1/2)/a/x+2/5*I*d/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*
d/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipt
icE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/4*e/a/x^4*(b*x^4+a)^(3/2)-1/
4*e*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*e*b/a*(
b*x^4+a)^(1/2)-1/3*f/x^3*(b*x^4+a)^(1/2)+2/3*f*b/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^
2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bx^4+a\right)^{\frac{3}{2}}c}{6\,ax^6}+\int\frac{\sqrt{bx^4+a}\left(fx^2+ex+d\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(b*x^4 + a)^(3/2)*c/(a*x^6) + integrate(sqrt(b*x^4 + a)*(f*x
^2 + e*x + d)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

Sympy [A] time = 7.98169, size = 189, normalized size = 0.54

$$\frac{\sqrt{a}d\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}f\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\,\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{6x^4}-\frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{4x^2}-\frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6a}-\frac{be\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**7,x)
```

```
[Out] sqrt(a)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*f*gamma(-3/4)*hyper((-
3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4))
 - sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(6*x**4) - sqrt(b)*e*sqrt(a/(b*
x**4) + 1)/(4*x**2) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(6*a) - b*e
*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

3.495 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx$

Optimal. Leaf size=375

$$-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-21\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{2b^{3/2}ex\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}-\frac{bf\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b
*x^4])/420 - (2*b*c*Sqrt[a + b*x^4])/(21*a*x^3) - (b*d*Sqrt[a + b
*x^4])/(6*a*x^2) - (2*b*e*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*e
*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*f*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*e*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4])
 - (b^(5/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.93181, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-21\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{2b^{3/2}ex\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}-\frac{bf\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```


```
[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b
*x^4])/420 - (2*b*c*Sqrt[a + b*x^4])/(21*a*x^3) - (b*d*Sqrt[a + b
*x^4])/(6*a*x^2) - (2*b*e*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*e
*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*f*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*e*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4])
 - (b^(5/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**8,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.809837, size = 283, normalized size = 0.75

$$\frac{-8b^{3/2}x^7\sqrt{\frac{bx^4}{a}+1}\left(21\sqrt{a}e-5i\sqrt{b}c\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+168\sqrt{a}b^{3/2}ex^7\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-\sqrt{\ldots}}{420ax^7\sqrt{\frac{i\sqrt{\ldots}}{\sqrt{\ldots}}}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*b*x^4*(20*c + 7*x*(5
*d + 12*e*x)) + a*(60*c + 7*x*(10*d + 3*x*(4*e + 5*f*x)))) + 105*
Sqrt[a]*b*f*x^7*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
) + 168*Sqrt[a]*b^(3/2)*e*x^7*Sqrt[1 + (b*x^4)/a]*EllipticE[I*Arc
Sinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 8*b^(3/2)*((-5*I)*Sqrt[b
]*c + 21*Sqrt[a]*e)*x^7*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[S
qrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(420*a*Sqrt[(I*Sqrt[b])/Sqrt[a]
]*x^7*Sqrt[a + b*x^4])
```

Maple [C] time = 0.027, size = 385, normalized size = 1.

$$\begin{aligned}
&-\frac{c}{7x^7}\sqrt{bx^4+a}-\frac{2bc}{21ax^3}\sqrt{bx^4+a}\\
&-\frac{2b^2c}{21a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{d}{6ax^6}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{e}{5x^5}\sqrt{bx^4+a}-\frac{2be}{5ax}\sqrt{bx^4+a}\\
&+\frac{2i}{5}eb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{2i}{5}eb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{f}{4ax^4}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{fb}{4}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{fb}{4a}\sqrt{bx^4+a}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x)
```

```
[Out] -1/7*c/x^7*(b*x^4+a)^(1/2)-2/21*b*c*(b*x^4+a)^(1/2)/a/x^3-2/21*c/
a*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(
1/2)*b^(1/2))^(1/2),I)-1/6*d/a/x^6*(b*x^4+a)^(3/2)-1/5*e/x^5*(b*x
^4+a)^(1/2)-2/5*b*e*(b*x^4+a)^(1/2)/a/x+2/5*I*e/a^(1/2)*b^(3/2)/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^
(1/2))^(1/2),I)-2/5*I*e/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/
(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/4*f/a/
x^4*(b*x^4+a)^(3/2)-1/4*f*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(
1/2))/x^2)+1/4*f*b/a*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Sympy [A] time = 8.68739, size = 192, normalized size = 0.51

$$\frac{\sqrt{a}c\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\,\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}e\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{6a} - \frac{bf\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**8,x)
```

```
[Out] sqrt(a)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*e*gamma(-5/4)*hyper((-
5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4
)) - sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(6*x**4) - sqrt(b)*f*sqrt(a/(
b*x**4) + 1)/(4*x**2) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(6*a) - b
*f*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

3.496 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx$

Optimal. Leaf size=400

$$-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}d-21\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{2b^{3/2}fx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}$$

```
[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a
+ b*x^4])/840 - (b*c*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*d*Sqrt[a
+ b*x^4])/(21*a*x^3) - (b*e*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*f*S
qrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*a*(S
qrt[a] + Sqrt[b]*x^2)) + (b^2*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/(16*a^(3/2)) - (2*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*d
 - 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(105*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 1.02869, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}d-21\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}+\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{2b^{3/2}fx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```


```
[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a
+ b*x^4])/840 - (b*c*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*d*Sqrt[a
+ b*x^4])/(21*a*x^3) - (b*e*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*f*S
qrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*a*(S
qrt[a] + Sqrt[b]*x^2)) + (b^2*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/(16*a^(3/2)) - (2*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*d
 - 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(105*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**9,x)
```

```
[Out] Timed out
```

Mathematica [C]   time = 0.93819, size = 293, normalized size = 0.73

$$\frac{-32\sqrt{a}b^{3/2}x^8\sqrt{\frac{bx^4}{a}+1}\left(21\sqrt{a}f-5i\sqrt{b}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+672ab^{3/2}fx^8\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{1680}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-(Sqrt[a]*(a + b*x^4)*(b*x^4*(105*c +
 8*x*(20*d + 35*e*x + 84*f*x^2)) + a*(210*c + 8*x*(30*d + 7*x*(5*
e + 6*f*x))))) + 105*b^2*c*x^8*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b
*x^4]/Sqrt[a]]) + 672*a*b^(3/2)*f*x^8*Sqrt[1 + (b*x^4)/a]*Ellipti
cE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 32*Sqrt[a]*b^(3/
2)*((-5*I)*Sqrt[b]*d + 21*Sqrt[a]*f)*x^8*Sqrt[1 + (b*x^4)/a]*Elli
pticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(1680*a^(3/2)*
Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^8*Sqrt[a + b*x^4])
```

Maple [C]   time = 0.023, size = 408, normalized size = 1.

$$-\frac{c}{8\,ax^8}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{bc}{16\,a^2x^4}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{b^2c}{16}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}$$
$$-\frac{b^2c}{16\,a^2}\sqrt{bx^4+a}-\frac{d}{7\,x^7}\sqrt{bx^4+a}-\frac{2\,bd}{21\,ax^3}\sqrt{bx^4+a}$$
$$-\frac{2\,b^2d}{21\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{e}{6\,ax^6}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{f}{5\,x^5}\sqrt{bx^4+a}-\frac{2\,fb}{5\,ax}\sqrt{bx^4+a}$$
$$+\frac{2\,i}{5}fb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{2\,i}{5}fb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x)
```

```
[Out] -1/8*c/a/x^8*(b*x^4+a)^(3/2)+1/16*c*b/a^2/x^4*(b*x^4+a)^(3/2)+1/1
6*c*b^2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/16*c*b^
2/a^2*(b*x^4+a)^(1/2)-1/7*d/x^7*(b*x^4+a)^(1/2)-2/21*b*d*(b*x^4+a
)^(1/2)/a/x^3-2/21*d/a*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/6*e/a/x^6*(b*x^4+a)
^(3/2)-1/5*f/x^5*(b*x^4+a)^(1/2)-2/5*b*f*(b*x^4+a)^(1/2)/a/x+2/5*
I*f/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elli
pticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*f/a^(1/2)*b^(3/2)/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/
2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

Sympy [A] time = 12.5247, size = 246, normalized size = 0.62

$$\frac{\sqrt{a}d\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\,\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}f\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)}-\frac{ac}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{3\sqrt{b}c}{16x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{6x^4}-\frac{b^{\frac{3}{2}}c}{16ax^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{6a}+\frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**9,x)
```

```
[Out] sqrt(a)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*f*gamma(-5/4)*hyper((-
5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4
)) - a*c/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*sqrt(b)*c/(16
*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(6*x
**4) - b**(3/2)*c/(16*a*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*e*s
qrt(a/(b*x**4) + 1)/(6*a) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x**2))/
(16*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

3.497 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx$

Optimal. Leaf size=425

$$-\frac{b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+7\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}}+\frac{2b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^4}}+\frac{b^2d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}-\frac{1}{504}\sqrt{a+bx^4}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-\frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}$$

```
[Out] -(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*Sqrt[a + b*
x^4])/504 - (2*b*c*Sqrt[a + b*x^4])/(45*a*x^5) - (b*d*Sqrt[a + b*
x^4])/(16*a*x^4) - (2*b*e*Sqrt[a + b*x^4])/(21*a*x^3) - (b*f*Sqrt
[a + b*x^4])/(6*a*x^2) + (2*b^2*c*Sqrt[a + b*x^4])/(15*a^2*x) - (
2*b^(5/2)*c*x*Sqrt[a + b*x^4])/(15*a^2*(Sqrt[a] + Sqrt[b]*x^2)) +
 (b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) + (2*b^(9/
4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(7/4
)*Sqrt[a + b*x^4]) - (b^(7/4)*(7*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a
] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(7/4)*Sqrt[a +
b*x^4])
```

Rubi [A] time = 1.18464, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$-\frac{b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+7\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}}+\frac{2b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^4}}+\frac{b^2d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}-\frac{1}{504}\sqrt{a+bx^4}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-\frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```


```
[Out] -(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*Sqrt[a + b*
x^4])/504 - (2*b*c*Sqrt[a + b*x^4])/(45*a*x^5) - (b*d*Sqrt[a + b*
x^4])/(16*a*x^4) - (2*b*e*Sqrt[a + b*x^4])/(21*a*x^3) - (b*f*Sqrt
[a + b*x^4])/(6*a*x^2) + (2*b^2*c*Sqrt[a + b*x^4])/(15*a^2*x) - (
2*b^(5/2)*c*x*Sqrt[a + b*x^4])/(15*a^2*(Sqrt[a] + Sqrt[b]*x^2)) +
 (b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) + (2*b^(9/
4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(7/4
)*Sqrt[a + b*x^4]) - (b^(7/4)*(7*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a
] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(7/4)*Sqrt[a +
b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**10,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.872723, size = 305, normalized size = 0.72

$$\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(315\sqrt{a}b^2dx^9\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\left(a+bx^4\right)\left(10a^2\left(56c+63dx+72ex^2+84fx^3\right)+abx^4(224c+15x(21d+8\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(-672*b^2*c*x^8 + 10*a^
2*(56*c + 63*d*x + 72*e*x^2 + 84*f*x^3) + a*b*x^4*(224*c + 15*x*(
21*d + 8*x*(4*e + 7*f*x))))) + 315*Sqrt[a]*b^2*d*x^9*Sqrt[a + b*x
^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) - 672*Sqrt[a]*b^(5/2)*c*x^9
*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]
]*x], -1] + 96*Sqrt[a]*b^2*(7*Sqrt[b]*c + (5*I)*Sqrt[a]*e)*x^9*Sq
rt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x
], -1])/(5040*a^2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^9*Sqrt[a + b*x^4])
```

Maple [C] time = 0.028, size = 429, normalized size = 1.

$$\begin{aligned}
&-\frac{c}{9x^9}\sqrt{bx^4+a}-\frac{2bc}{45ax^5}\sqrt{bx^4+a}+\frac{2b^2c}{15a^2x}\sqrt{bx^4+a}\\
&-\frac{2i}{15}cb^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{2i}{15}cb^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{d}{8ax^8}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{bd}{16a^2x^4}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{b^2d}{16}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}\\
&-\frac{b^2d}{16a^2}\sqrt{bx^4+a}-\frac{e}{7x^7}\sqrt{bx^4+a}-\frac{2be}{21ax^3}\sqrt{bx^4+a}\\
&-\frac{2b^2e}{21a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{f}{6ax^6}\left(bx^4+a\right)^{\frac{3}{2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x)
```

```
[Out] -1/9*c/x^9*(b*x^4+a)^(1/2)-2/45*b*c*(b*x^4+a)^(1/2)/a/x^5+2/15*b^
2*c*(b*x^4+a)^(1/2)/a^2/x-2/15*I*c/a^(3/2)*b^(5/2)/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
```

```
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)+2/15*I*c/a^(3/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/8*d/a/x^8*(b*x^4+a
)^(3/2)+1/16*d*b/a^2/x^4*(b*x^4+a)^(3/2)+1/16*d*b^2/a^(3/2)*ln((2
*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/16*d*b^2/a^2*(b*x^4+a)^(1/2)
-1/7*e/x^7*(b*x^4+a)^(1/2)-2/21*b*e*(b*x^4+a)^(1/2)/a/x^3-2/21*e/
a*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(
1/2)*b^(1/2))^(1/2),I)-1/6*f/a/x^6*(b*x^4+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Sympy [A] time = 14.0266, size = 246, normalized size = 0.58

$$\frac{\sqrt{a}c\left(-\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}e\left(-\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\left(-\frac{3}{4}\right)}-\frac{ad}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{3\sqrt{b}d}{16x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{6x^4}-\frac{b^{\frac{3}{2}}d}{16ax^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{6a}+\frac{b^2d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**10,x)
```

```
[Out] sqrt(a)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*e*gamma(-7/4)*hyper((-
7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4
)) - a*d/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*sqrt(b)*d/(16
*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(6*x
**4) - b**(3/2)*d/(16*a*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*f*s
qrt(a/(b*x**4) + 1)/(6*a) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x**2))/
(16*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

3.498 $\int x^4 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^{3/2} \, dx$

Optimal. Leaf size=476

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}e+65\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}+\frac{4a^{13/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}-\frac{a^3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{\left(a+bx^4\right)^{5/2}\left(12af-35bdx^2\right)}{420b^2}+\frac{1}{143}x^5\left(a+bx^4\right)^{3/2}\left(13c+11ex^2\right)+\frac{2ax^5\sqrt{a+bx^4}\left(117c+77ex^2\right)}{3003}-\frac{adx^2\left(a+bx^4\right)^{3/2}}{48b}+\frac{fx^4\left(a+bx^4\right)^{5/2}}{14b}$$

```
[Out] (4*a^2*c*x*Sqrt[a + b*x^4])/(77*b) - (a^2*d*x^2*Sqrt[a + b*x^4])/
(32*b) + (4*a^2*e*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*e*x*Sqrt[
a + b*x^4])/(65*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*
c + 77*e*x^2)*Sqrt[a + b*x^4])/3003 - (a*d*x^2*(a + b*x^4)^(3/2))
/(48*b) + (x^5*(13*c + 11*e*x^2)*(a + b*x^4)^(3/2))/143 + (f*x^4*
(a + b*x^4)^(5/2))/(14*b) - ((12*a*f - 35*b*d*x^2)*(a + b*x^4)^(5
/2))/(420*b^2) - (a^3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(
32*b^(3/2)) + (4*a^(13/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65*Sqrt[
b]*c + 77*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sq
rt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 1.31135, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}e+65\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}+\frac{4a^{13/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}-\frac{a^3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{\left(a+bx^4\right)^{5/2}\left(12af-35bdx^2\right)}{420b^2}+\frac{1}{143}x^5\left(a+bx^4\right)^{3/2}\left(13c+11ex^2\right)+\frac{2ax^5\sqrt{a+bx^4}\left(117c+77ex^2\right)}{3003}-\frac{adx^2\left(a+bx^4\right)^{3/2}}{48b}+\frac{fx^4\left(a+bx^4\right)^{5/2}}{14b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```


```
[Out] (4*a^2*c*x*Sqrt[a + b*x^4])/(77*b) - (a^2*d*x^2*Sqrt[a + b*x^4])/
(32*b) + (4*a^2*e*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*e*x*Sqrt[
a + b*x^4])/(65*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*
c + 77*e*x^2)*Sqrt[a + b*x^4])/3003 - (a*d*x^2*(a + b*x^4)^(3/2))
/(48*b) + (x^5*(13*c + 11*e*x^2)*(a + b*x^4)^(3/2))/143 + (f*x^4*
(a + b*x^4)^(5/2))/(14*b) - ((12*a*f - 35*b*d*x^2)*(a + b*x^4)^(5
/2))/(420*b^2) - (a^3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(
32*b^(3/2)) + (4*a^(13/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65*Sqrt[
b]*c + 77*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sq
rt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 116.903, size = 442, normalized size = 0.93

$$\frac{4a^{\frac{13}{4}}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{65b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{2a^{\frac{11}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(77\sqrt{a}e+65\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5005b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{a^3d\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{\frac{3}{2}}} - \frac{4a^3ex\sqrt{a+bx^4}}{65b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{4a^2cx\sqrt{a+bx^4}}{77b} - \frac{a^2dx^2\sqrt{a+bx^4}}{32b} + \frac{4a^2ex^3\sqrt{a+bx^4}}{195b} + \frac{2ax^5\sqrt{a+bx^4}\left(117c+77ex^2\right)}{3003} - \frac{adx^2\left(a+bx^4\right)^{\frac{3}{2}}}{48b} + \frac{x^5\left(a+bx^4\right)^{\frac{3}{2}}\left(13c+11ex^2\right)}{143} + \frac{fx^4\left(a+bx^4\right)^{\frac{5}{2}}}{14b} - \frac{\left(a+bx^4\right)^{\frac{5}{2}}\left(12af-35bdx^2\right)}{420b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] 4*a**(13/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(65*b**(7/4)*sqrt(a + b*x**4)) - 2*a**(11/4)*sqrt((a + b*x**4)/(
sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(77*sqrt(a)*
e + 65*sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(5
005*b**(7/4)*sqrt(a + b*x**4)) - a**3*d*atanh(sqrt(b)*x**2/sqrt(a
 + b*x**4))/(32*b**(3/2)) - 4*a**3*e*x*sqrt(a + b*x**4)/(65*b**(3
/2)*(sqrt(a) + sqrt(b)*x**2)) + 4*a**2*c*x*sqrt(a + b*x**4)/(77*b
) - a**2*d*x**2*sqrt(a + b*x**4)/(32*b) + 4*a**2*e*x**3*sqrt(a +
b*x**4)/(195*b) + 2*a*x**5*sqrt(a + b*x**4)*(117*c + 77*e*x**2)/3
003 - a*d*x**2*(a + b*x**4)**(3/2)/(48*b) + x**5*(a + b*x**4)**(3
/2)*(13*c + 11*e*x**2)/143 + f*x**4*(a + b*x**4)**(5/2)/(14*b) -
(a + b*x**4)**(5/2)*(12*a*f - 35*b*d*x**2)/(420*b**2)
```

Mathematica [C] time = 1.09568, size = 327, normalized size = 0.69

$$-29568a^{7/2}\sqrt{b}e\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+384a^3\sqrt{b}\sqrt{\frac{bx^4}{a}+1}\left(77\sqrt{a}e+65i\sqrt{b}c\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(13728*a^3*f - 40*b^3*x
^9*(1092*c + 11*x*(91*d + 84*e*x + 78*f*x^2)) - a^2*b*x*(24960*c
+ 11*x*(1365*d + 896*e*x + 624*f*x^2)) - 2*a*b^2*x^5*(40560*c + 1
1*x*(3185*d + 2800*e*x + 2496*f*x^2)))) - 15015*a^3*Sqrt[b]*d*Sqr
t[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) - 29568*a^(7
/2)*Sqrt[b]*e*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqr
t[b])/Sqrt[a]]*x], -1] + 384*a^3*Sqrt[b]*((65*I)*Sqrt[b]*c + 77*S
qrt[a]*e)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]
)/Sqrt[a]]*x], -1])/(480480*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^2*Sqrt[a
+ b*x^4])
```

Maple [C] time = 0.047, size = 462, normalized size = 1.

$$\frac{bcx^9}{11}\sqrt{bx^4+a}+\frac{13\,acx^5}{77}\sqrt{bx^4+a}+\frac{4\,a^2cx}{77\,b}\sqrt{bx^4+a}$$
$$-\frac{4\,a^3c}{77\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{a^2dx^2}{32\,b}\sqrt{bx^4+a}-\frac{a^3d}{32}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{bdx^{10}}{12}\sqrt{bx^4+a}$$
$$+\frac{7\,adx^6}{48}\sqrt{bx^4+a}+\frac{bex^{11}}{13}\sqrt{bx^4+a}+\frac{5\,aex^7}{39}\sqrt{bx^4+a}+\frac{4\,a^2ex^3}{195\,b}\sqrt{bx^4+a}$$
$$-\frac{4\,i}{65}ea^{\frac{7}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{4\,i}{65}ea^{\frac{7}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{f\left(-5\,bx^4+2\,a\right)\left(b^2x^8+2\,abx^4+a^2\right)}{70\,b^2}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] 1/11*c*b*x^9*(b*x^4+a)^(1/2)+13/77*c*a*x^5*(b*x^4+a)^(1/2)+4/77*a
^2*c*x*(b*x^4+a)^(1/2)/b-4/77*c/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/32*a^2*d
*x^2*(b*x^4+a)^(1/2)/b-1/32*d*a^3/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a
)^(1/2))+1/12*d*b*x^10*(b*x^4+a)^(1/2)+7/48*d*a*x^6*(b*x^4+a)^(1/
2)+1/13*e*b*x^11*(b*x^4+a)^(1/2)+5/39*e*a*x^7*(b*x^4+a)^(1/2)+4/1
95*a^2*e*x^3*(b*x^4+a)^(1/2)/b-4/65*I*e/b^(3/2)*a^(7/2)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)+4/65*I*e/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/70*f*(b*x^4+a
)^(1/2)*(-5*b*x^4+2*a)*(b^2*x^8+2*a*b*x^4+a^2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bfx^{11}+bex^{10}+bdx^9+bcx^8+afx^7+aex^6+adx^5+acx^4\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^11 + b*e*x^10 + b*d*x^9 + b*c*x^8 + a*f*x^7 + a*e
*x^6 + a*d*x^5 + a*c*x^4)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 28.9211, size = 462, normalized size = 0.97

$$\frac{a^{\frac{5}{2}}dx^{2}}{32b\sqrt{1+\frac{bx^{4}}{a}}}+\frac{a^{\frac{3}{2}}cx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}dx^{6}}{96\sqrt{1+\frac{bx^{4}}{a}}}+\frac{a^{\frac{3}{2}}ex^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bcx^{9}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{11\sqrt{a}bdx^{10}}{48\sqrt{1+\frac{bx^{4}}{a}}}+\frac{\sqrt{a}bex^{11}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}-\frac{a^{3}d\operatorname{asinh}\left(\frac{\sqrt{b}x^{2}}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+af\left(\begin{cases}-\frac{a^{2}\sqrt{a+bx^{4}}}{15b^{2}}+\frac{ax^{4}\sqrt{a+bx^{4}}}{30b}+\frac{x^{8}\sqrt{a+bx^{4}}}{10} & \text{for}\: b\neq 0\\ \frac{\sqrt{a}x^{8}}{8} & \text{otherwise}\end{cases}\right)+bf\left(\begin{cases}\frac{4a^{3}\sqrt{a+bx^{4}}}{105b^{3}}-\frac{2a^{2}x^{4}\sqrt{a+bx^{4}}}{105b^{2}}+\frac{ax^{8}\sqrt{a+bx^{4}}}{70b}+\frac{x^{12}\sqrt{a+bx^{4}}}{14} & \text{for}\: b\neq 0\\ \frac{\sqrt{a}x^{12}}{12} & \text{otherwise}\end{cases}\right)+\frac{b^{2}dx^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^{4}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*d*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**5*gamma
(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gam
ma(9/4)) + 17*a**(3/2)*d*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*
e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*c*x**9*gamma(9/4)*hyper((-1/2,
 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 11*sq
rt(a)*b*d*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**11*gamma
(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(15/4)) - a**3*d*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) +
 a*f*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a
+ b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*
x**8/8, True)) + b*f*Piecewise((4*a**3*sqrt(a + b*x**4)/(105*b**3
) - 2*a**2*x**4*sqrt(a + b*x**4)/(105*b**2) + a*x**8*sqrt(a + b*x
**4)/(70*b) + x**12*sqrt(a + b*x**4)/14, Ne(b, 0)), (sqrt(a)*x**1
2/12, True)) + b**2*d*x**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^{4}+a\right)^{\frac{3}{2}}\left(fx^{3}+ex^{2}+dx+c\right)x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

3.499 $\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^{3/2} \, dx$

Optimal. Leaf size=452

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}f+65\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}+\frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}-\frac{a^3e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{\left(a+bx^4\right)^{5/2}\left(6c+5ex^2\right)}{60b}+\frac{1}{143}x^5\left(a+bx^4\right)^{3/2}\left(13d+11fx^2\right)+\frac{2ax^5\sqrt{a+bx^4}\left(117d+77fx^2\right)}{3003}-\frac{aex^2\left(a+bx^4\right)^{3/2}}{48b}$$

```
[Out] (4*a^2*d*x*Sqrt[a + b*x^4])/(77*b) - (a^2*e*x^2*Sqrt[a + b*x^4])/
(32*b) + (4*a^2*f*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*f*x*Sqrt[
a + b*x^4])/(65*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*
d + 77*f*x^2)*Sqrt[a + b*x^4])/3003 - (a*e*x^2*(a + b*x^4)^(3/2))
/(48*b) + (x^5*(13*d + 11*f*x^2)*(a + b*x^4)^(3/2))/143 + ((6*c +
 5*e*x^2)*(a + b*x^4)^(5/2))/(60*b) - (a^3*e*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]])/(32*b^(3/2)) + (4*a^(13/4)*f*(Sqrt[a] + Sqrt[
b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^4]) -
(2*a^(11/4)*(65*Sqrt[b]*d + 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 1.19028, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}f+65\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}+\frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}-\frac{a^3e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{\left(a+bx^4\right)^{5/2}\left(6c+5ex^2\right)}{60b}+\frac{1}{143}x^5\left(a+bx^4\right)^{3/2}\left(13d+11fx^2\right)+\frac{2ax^5\sqrt{a+bx^4}\left(117d+77fx^2\right)}{3003}-\frac{aex^2\left(a+bx^4\right)^{3/2}}{48b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (4*a^2*d*x*Sqrt[a + b*x^4])/(77*b) - (a^2*e*x^2*Sqrt[a + b*x^4])/
(32*b) + (4*a^2*f*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*f*x*Sqrt[
a + b*x^4])/(65*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*
d + 77*f*x^2)*Sqrt[a + b*x^4])/3003 - (a*e*x^2*(a + b*x^4)^(3/2))
/(48*b) + (x^5*(13*d + 11*f*x^2)*(a + b*x^4)^(3/2))/143 + ((6*c +
 5*e*x^2)*(a + b*x^4)^(5/2))/(60*b) - (a^3*e*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]])/(32*b^(3/2)) + (4*a^(13/4)*f*(Sqrt[a] + Sqrt[
b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*A
```

```
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^4]) -
(2*a^(11/4)*(65*Sqrt[b]*d + 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 108.96, size = 418, normalized size = 0.92

$$\frac{4a^{\frac{13}{4}}f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{65b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{2a^{\frac{11}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(77\sqrt{a}f+65\sqrt{b}d\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5005b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{a^3e\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{\frac{3}{2}}} - \frac{4a^3fx\sqrt{a+bx^4}}{65b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{4a^2dx\sqrt{a+bx^4}}{77b} - \frac{a^2ex^2\sqrt{a+bx^4}}{32b} + \frac{4a^2fx^3\sqrt{a+bx^4}}{195b} + \frac{2ax^5\sqrt{a+bx^4}\left(117d+77fx^2\right)}{3003} - \frac{aex^2\left(a+bx^4\right)^{\frac{3}{2}}}{48b} + \frac{x^5\left(a+bx^4\right)^{\frac{3}{2}}\left(13d+11fx^2\right)}{143} + \frac{\left(a+bx^4\right)^{\frac{5}{2}}\left(6c+5ex^2\right)}{60b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] 4*a**(13/4)*f*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(65*b**(7/4)*sqrt(a + b*x**4)) - 2*a**(11/4)*sqrt((a + b*x**4)/(
sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(77*sqrt(a)*
f + 65*sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(5
005*b**(7/4)*sqrt(a + b*x**4)) - a**3*e*atanh(sqrt(b)*x**2/sqrt(a
 + b*x**4))/(32*b**(3/2)) - 4*a**3*f*x*sqrt(a + b*x**4)/(65*b**(3
/2)*(sqrt(a) + sqrt(b)*x**2)) + 4*a**2*d*x*sqrt(a + b*x**4)/(77*b
) - a**2*e*x**2*sqrt(a + b*x**4)/(32*b) + 4*a**2*f*x**3*sqrt(a +
b*x**4)/(195*b) + 2*a*x**5*sqrt(a + b*x**4)*(117*d + 77*f*x**2)/3
003 - a*e*x**2*(a + b*x**4)**(3/2)/(48*b) + x**5*(a + b*x**4)**(3
/2)*(13*d + 11*f*x**2)/143 + (a + b*x**4)**(5/2)*(6*c + 5*e*x**2)
/(60*b)
```

Mathematica [C] time = 6.13172, size = 431, normalized size = 0.95

$$\sqrt{a+bx^4}\left(\frac{a^2c}{10b}+\frac{4a^2dx}{77b}+\frac{a^2ex^2}{32b}+\frac{4a^2fx^3}{195b}+\frac{1}{5}acx^4+\frac{13}{77}adx^5+\frac{7}{48}aex^6+\frac{5}{39}afx^7+\frac{1}{10}bcx^8+\frac{1}{11}bdx^9+\frac{1}{12}bex^{10}+\frac{1}{13}bfx^{11}\right) - \frac{a^3\left(-\frac{4160id\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}x^2}{\sqrt{a}}}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}+\frac{5005e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{4928\sqrt{a}f\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}x^2}{\sqrt{a}}}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-F\left(\right.\right.}{\sqrt{b}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}\right.}{80080b}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] Sqrt[a + b*x^4]*((a^2*c)/(10*b) + (4*a^2*d*x)/(77*b) + (a^2*e*x^2
)/(32*b) + (4*a^2*f*x^3)/(195*b) + (a*c*x^4)/5 + (13*a*d*x^5)/77
+ (7*a*e*x^6)/48 + (5*a*f*x^7)/39 + (b*c*x^8)/10 + (b*d*x^9)/11 +
 (b*e*x^10)/12 + (b*f*x^11)/13) - (a^3*((5005*e*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (4928*Sqrt[a]*f*Sqrt[1 - (I*
```

```
Sqrt[b]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[b]*x^2)/Sqrt[a]]*(Elliptic
E[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - EllipticF[I*ArcSi
nh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[b])/Sqrt[a]]
*Sqrt[b]*Sqrt[a + b*x^4]) - ((4160*I)*d*Sqrt[1 - (I*Sqrt[b]*x^2)/
Sqrt[a]]*Sqrt[1 + (I*Sqrt[b]*x^2)/Sqrt[a]]*EllipticF[I*ArcSinh[Sq
rt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[
a + b*x^4])))/(80080*b)
```

Maple [C] time = 0.013, size = 434, normalized size = 1.

$$\frac{bdx^9}{11}\sqrt{bx^4+a}+\frac{13\,adx^5}{77}\sqrt{bx^4+a}+\frac{4\,a^2dx}{77\,b}\sqrt{bx^4+a}$$
$$-\frac{4\,a^3d}{77\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{c}{10\,b}\left(bx^4+a\right)^{\frac{5}{2}}+\frac{a^2ex^2}{32\,b}\sqrt{bx^4+a}-\frac{a^3e}{32}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{bex^{10}}{12}\sqrt{bx^4+a}$$
$$+\frac{7\,aex^6}{48}\sqrt{bx^4+a}+\frac{bfx^{11}}{13}\sqrt{bx^4+a}+\frac{5\,afx^7}{39}\sqrt{bx^4+a}+\frac{4\,a^2fx^3}{195\,b}\sqrt{bx^4+a}$$
$$-\frac{4\,i}{65}fa^{\frac{7}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{4\,i}{65}fa^{\frac{7}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] 1/11*d*b*x^9*(b*x^4+a)^(1/2)+13/77*d*a*x^5*(b*x^4+a)^(1/2)+4/77*a
^2*d*x*(b*x^4+a)^(1/2)/b-4/77*d/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/10*c/b*(
b*x^4+a)^(5/2)+1/32*a^2*e*x^2*(b*x^4+a)^(1/2)/b-1/32*e*a^3/b^(3/2
)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))+1/12*e*b*x^10*(b*x^4+a)^(1/2)+7
/48*e*a*x^6*(b*x^4+a)^(1/2)+1/13*f*b*x^11*(b*x^4+a)^(1/2)+5/39*f*
a*x^7*(b*x^4+a)^(1/2)+4/195*a^2*f*x^3*(b*x^4+a)^(1/2)/b-4/65*I*f/
b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4/65*I*f/b^(3/2)*a^(7/2)/(I/a^(1
/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bx^4+a\right)^{\frac{5}{2}}c}{10\,b}+\int\left(bfx^{10}+bex^9+bdx^8+afx^6+aex^5+adx^4\right)\sqrt{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^4 + a)^(5/2)*c/b + integrate((b*f*x^10 + b*e*x^9 + b*d*
x^8 + a*f*x^6 + a*e*x^5 + a*d*x^4)*sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bfx^{10}+bex^9+bdx^8+bcx^7+afx^6+aex^5+adx^4+acx^3\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^10 + b*e*x^9 + b*d*x^8 + b*c*x^7 + a*f*x^6 + a*e*
x^5 + a*d*x^4 + a*c*x^3)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 23.8878, size = 398, normalized size = 0.88

$$\frac{a^{\frac{5}{2}}ex^{2}}{32b\sqrt{1+\frac{bx^{4}}{a}}}+\frac{a^{\frac{3}{2}}dx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}ex^{6}}{96\sqrt{1+\frac{bx^{4}}{a}}}+\frac{a^{\frac{3}{2}}fx^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bdx^{9}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{11\sqrt{a}bex^{10}}{48\sqrt{1+\frac{bx^{4}}{a}}}+\frac{\sqrt{a}bfx^{11}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}-\frac{a^{3}e\operatorname{asinh}\left(\frac{\sqrt{b}x^{2}}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+ac\left(\begin{cases}\frac{\sqrt{a}x^{4}}{4} & \text{for}\: b=0\\ \frac{(a+bx^{4})^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+bc\left(\begin{cases}-\frac{a^{2}\sqrt{a+bx^{4}}}{15b^{2}}+\frac{ax^{4}\sqrt{a+bx^{4}}}{30b}+\frac{x^{8}\sqrt{a+bx^{4}}}{10} & \text{for}\: b\neq 0\\ \frac{\sqrt{a}x^{8}}{8} & \text{otherwise}\end{cases}\right)+\frac{b^{2}ex^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^{4}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*e*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*d*x**5*gamma
(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gam
ma(9/4)) + 17*a**(3/2)*e*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*
f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*d*x**9*gamma(9/4)*hyper((-1/2,
 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 11*sq
rt(a)*b*e*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*x**11*gamma
(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(15/4)) - a**3*e*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) +
 a*c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(
6*b), True)) + b*c*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) +
a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b,
0)), (sqrt(a)*x**8/8, True)) + b**2*e*x**14/(12*sqrt(a)*sqrt(1 +
b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^{4}+a\right)^{\frac{3}{2}}\left(fx^{3}+ex^{2}+dx+c\right)x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

3.500 $\int x^2 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^{3/2} \, dx$

Optimal. Leaf size=427

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}c-15\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{a^3 f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}} + \frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{4a^2ex\sqrt{a+bx^4}}{77b} - \frac{a^2fx^2\sqrt{a+bx^4}}{32b} + \frac{2ax^3\sqrt{a+bx^4}\left(77c+45ex^2\right)}{1155} + \frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11c+9ex^2\right) + \frac{\left(a+bx^4\right)^{5/2}\left(6d+5fx^2\right)}{60b} - \frac{afx^2\left(a+bx^4\right)^{3/2}}{48b}$$

```
[Out] (4*a^2*e*x*Sqrt[a + b*x^4])/(77*b) - (a^2*f*x^2*Sqrt[a + b*x^4])/
(32*b) + (4*a^2*c*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[
b]*x^2)) + (2*a*x^3*(77*c + 45*e*x^2)*Sqrt[a + b*x^4])/1155 - (a*
f*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^3*(11*c + 9*e*x^2)*(a + b*x^
4)^(3/2))/99 + ((6*d + 5*f*x^2)*(a + b*x^4)^(5/2))/(60*b) - (a^3*
f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2)) - (4*a^(9/
4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4
)*Sqrt[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt
[a + b*x^4])
```

Rubi [A] time = 1.02365, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}c-15\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{a^3 f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}} + \frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{4a^2ex\sqrt{a+bx^4}}{77b} - \frac{a^2fx^2\sqrt{a+bx^4}}{32b} + \frac{2ax^3\sqrt{a+bx^4}\left(77c+45ex^2\right)}{1155} + \frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11c+9ex^2\right) + \frac{\left(a+bx^4\right)^{5/2}\left(6d+5fx^2\right)}{60b} - \frac{afx^2\left(a+bx^4\right)^{3/2}}{48b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```


```
[Out] (4*a^2*e*x*Sqrt[a + b*x^4])/(77*b) - (a^2*f*x^2*Sqrt[a + b*x^4])/
(32*b) + (4*a^2*c*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[
b]*x^2)) + (2*a*x^3*(77*c + 45*e*x^2)*Sqrt[a + b*x^4])/1155 - (a*
f*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^3*(11*c + 9*e*x^2)*(a + b*x^
4)^(3/2))/99 + ((6*d + 5*f*x^2)*(a + b*x^4)^(5/2))/(60*b) - (a^3*
f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2)) - (4*a^(9/
4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4
)*Sqrt[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt
[a + b*x^4])
```

Rubi in Sympy [A] time = 91.5168, size = 394, normalized size = 0.92

$$-\frac{4a^{\frac{9}{4}}c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{2a^{\frac{9}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(15\sqrt{a}e-77\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{1155b^{\frac{5}{4}}\sqrt{a+bx^4}}-\frac{a^3f\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{\frac{3}{2}}}+\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^3\sqrt{a+bx^4}\left(77c+45ex^2\right)}{1155}-\frac{afx^2\left(a+bx^4\right)^{\frac{3}{2}}}{48b}+\frac{x^3\left(a+bx^4\right)^{\frac{3}{2}}\left(11c+9ex^2\right)}{99}+\frac{\left(a+bx^4\right)^{\frac{5}{2}}\left(6d+5fx^2\right)}{60b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] -4*a**(9/4)*c*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(15*b**(3/4)*sqrt(a + b*x**4)) - 2*a**(9/4)*sqrt((a + b*x**4)/(s
qrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(15*sqrt(a)*e
 - 77*sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(11
55*b**(5/4)*sqrt(a + b*x**4)) - a**3*f*atanh(sqrt(b)*x**2/sqrt(a
+ b*x**4))/(32*b**(3/2)) + 4*a**2*e*x*sqrt(a + b*x**4)/(77*b) - a
**2*f*x**2*sqrt(a + b*x**4)/(32*b) + 4*a**2*c*x*sqrt(a + b*x**4)/
(15*sqrt(b)*(sqrt(a) + sqrt(b)*x**2)) + 2*a*x**3*sqrt(a + b*x**4)
*(77*c + 45*e*x**2)/1155 - a*f*x**2*(a + b*x**4)**(3/2)/(48*b) +
x**3*(a + b*x**4)**(3/2)*(11*c + 9*e*x**2)/99 + (a + b*x**4)**(5/
2)*(6*d + 5*f*x**2)/(60*b)
```

Mathematica [C] time = 0.806616, size = 325, normalized size = 0.76

$$-\frac{a^3f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+\frac{4ia^3e\sqrt{\frac{bx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{77b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}+\frac{\sqrt{a+bx^4}\left(9a^2(1232d+5x(128e+77fx))+2abx^3(13552c+3x(3696d+5x(624e+539fx)))+56b^2x^7\left(220c+3x\left(66d+60\right.\right.\right.}{110880b}+\frac{4ia^2c\sqrt{\frac{bx^4}{a}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{15\left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*(9*a^2*(1232*d + 5*x*(128*e + 77*f*x)) + 56*b^2*
x^7*(220*c + 3*x*(66*d + 60*e*x + 55*f*x^2)) + 2*a*b*x^3*(13552*c
 + 3*x*(3696*d + 5*x*(624*e + 539*f*x)))))/(110880*b) - (a^3*f*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2)) + (((4*I)/15)*
a^2*c*Sqrt[1 + (b*x^4)/a]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/S
qrt[a]]*x], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x
], -1]))/(((I*Sqrt[b])/Sqrt[a])^(3/2)*Sqrt[a + b*x^4]) + (((4*I)/
77)*a^3*e*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]
)/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.017, size = 413, normalized size = 1.

$$\frac{d}{10b}\left(bx^4+a\right)^{\frac{5}{2}}+\frac{bcx^7}{9}\sqrt{bx^4+a}+\frac{11\,acx^3}{45}\sqrt{bx^4+a}$$
$$+\frac{4i}{15}ca^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$-\frac{4i}{15}ca^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$+\frac{bex^9}{11}\sqrt{bx^4+a}+\frac{13\,aex^5}{77}\sqrt{bx^4+a}+\frac{4\,a^2ex}{77\,b}\sqrt{bx^4+a}$$
$$-\frac{4\,a^3e}{77\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{x^2a^2f}{32\,b}\sqrt{bx^4+a}-\frac{a^3f}{32}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{bfx^{10}}{12}\sqrt{bx^4+a}+\frac{7\,afx^6}{48}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] 1/10*d/b*(b*x^4+a)^(5/2)+1/9*c*b*x^7*(b*x^4+a)^(1/2)+11/45*c*a*x^
3*(b*x^4+a)^(1/2)+4/15*I*c*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*
I*c*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*Elli
pticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/11*e*b*x^9*(b*x^4+a)^(1/2)
+13/77*e*a*x^5*(b*x^4+a)^(1/2)+4/77*a^2*e*x*(b*x^4+a)^(1/2)/b-4/7
7*e/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)+1/32*a^2*f*x^2*(b*x^4+a)^(1/2)/b-1/32*
f*a^3/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))+1/12*f*b*x^10*(b*x^
4+a)^(1/2)+7/48*f*a*x^6*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bfx^9+bex^8+bdx^7+bcx^6+afx^5+aex^4+adx^3+acx^2\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^9 + b*e*x^8 + b*d*x^7 + b*c*x^6 + a*f*x^5 + a*e*x
^4 + a*d*x^3 + a*c*x^2)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 22.1989, size = 398, normalized size = 0.93

$$\frac{a^{\frac{5}{2}}fx^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}cx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}ex^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{9}{4}\right)}$$
$$+\frac{17a^{\frac{3}{2}}fx^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bcx^7\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bex^9\,\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{13}{4}\right)}$$
$$+\frac{11\sqrt{a}bfx^{10}}{48\sqrt{1+\frac{bx^4}{a}}}-\frac{a^3f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+ad\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$
$$+bd\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+\frac{b^2fx^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*f*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**3*gamma
(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gam
ma(7/4)) + a**(3/2)*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,),
b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/2)*f*x**6/(96
*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*x**7*gamma(7/4)*hyper((-1/2, 7
/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)
*b*e*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar
(I*pi)/a)/(4*gamma(13/4)) + 11*sqrt(a)*b*f*x**10/(48*sqrt(1 + b*x
**4/a)) - a**3*f*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*d*
Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b),
 True)) + b*d*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**
4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)),
(sqrt(a)*x**8/8, True)) + b**2*f*x**14/(12*sqrt(a)*sqrt(1 + b*x**
4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

3.501 $\int x\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=409

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}d-15\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{3a^2c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2fx\sqrt{a+bx^4}}{77b}$$
$$+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11d+9fx^2\right)+\frac{2ax^3\sqrt{a+bx^4}\left(77d+45fx^2\right)}{1155}+\frac{e\left(a+bx^4\right)^{5/2}}{10b}$$

```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/
16 + (4*a^2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
^2)) + (2*a*x^3*(77*d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*
(a + b*x^4)^(3/2))/8 + (x^3*(11*d + 9*f*x^2)*(a + b*x^4)^(3/2))/9
9 + (e*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*c*ArcTanh[(Sqrt[b]*x^2)
/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*d*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2
*a^(9/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sq
rt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.909908, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}d-15\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{3a^2c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2fx\sqrt{a+bx^4}}{77b}$$
$$+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11d+9fx^2\right)+\frac{2ax^3\sqrt{a+bx^4}\left(77d+45fx^2\right)}{1155}+\frac{e\left(a+bx^4\right)^{5/2}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```


```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/
16 + (4*a^2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
^2)) + (2*a*x^3*(77*d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*
(a + b*x^4)^(3/2))/8 + (x^3*(11*d + 9*f*x^2)*(a + b*x^4)^(3/2))/9
9 + (e*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*c*ArcTanh[(Sqrt[b]*x^2)
/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*d*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2
*a^(9/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sq
rt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 85.4895, size = 382, normalized size = 0.93

$$-\frac{4a^{\frac{9}{4}}d\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{2a^{\frac{9}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(15\sqrt{a}f-77\sqrt{b}d\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{1155b^{\frac{5}{4}}\sqrt{a+bx^4}}+\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3a^2c\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3acx^2\sqrt{a+bx^4}}{16}+\frac{2ax^3\sqrt{a+bx^4}\left(77d+45fx^2\right)}{1155}+\frac{cx^2\left(a+bx^4\right)^{\frac{3}{2}}}{8}+\frac{x^3\left(a+bx^4\right)^{\frac{3}{2}}\left(11d+9fx^2\right)}{99}+\frac{e\left(a+bx^4\right)^{\frac{5}{2}}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] -4*a**(9/4)*d*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(15*b**(3/4)*sqrt(a + b*x**4)) - 2*a**(9/4)*sqrt((a + b*x**4)/(s
qrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(15*sqrt(a)*f
 - 77*sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(11
55*b**(5/4)*sqrt(a + b*x**4)) + 4*a**2*f*x*sqrt(a + b*x**4)/(77*b
) + 3*a**2*c*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(16*sqrt(b)) +
4*a**2*d*x*sqrt(a + b*x**4)/(15*sqrt(b)*(sqrt(a) + sqrt(b)*x**2))
 + 3*a*c*x**2*sqrt(a + b*x**4)/16 + 2*a*x**3*sqrt(a + b*x**4)*(77
*d + 45*f*x**2)/1155 + c*x**2*(a + b*x**4)**(3/2)/8 + x**3*(a + b
*x**4)**(3/2)*(11*d + 9*f*x**2)/99 + e*(a + b*x**4)**(5/2)/(10*b)
```

Mathematica [C] time = 0.836326, size = 302, normalized size = 0.74

$$\frac{192ia^{5/2}\sqrt{\frac{bx^4}{a}+1}\left(15\sqrt{a}f+77i\sqrt{b}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+14784a^{5/2}\sqrt{b}d\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\cdots}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(72*a^2*(77*e + 40*f*x) +
 14*b^2*x^6*(495*c + 4*x*(110*d + 99*e*x + 90*f*x^2)) + a*b*x^2*(
17325*c + 16*x*(847*d + 9*x*(77*e + 65*f*x)))) + 10395*a^2*Sqrt[b
]*c*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) + 147
84*a^(5/2)*Sqrt[b]*d*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt
[(I*Sqrt[b])/Sqrt[a]]*x], -1] + (192*I)*a^(5/2)*((77*I)*Sqrt[b]*d
 + 15*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[b])/Sqrt[a]]*x], -1])/(55440*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Sqr
t[a + b*x^4])
```

Maple [C] time = 0.019, size = 392, normalized size = 1.

$$\frac{bdx^7}{9}\sqrt{bx^4+a}+\frac{11\,adx^3}{45}\sqrt{bx^4+a}$$
$$+\frac{4\,i}{15}da^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$-\frac{4\,i}{15}da^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$+\frac{3\,a^2c}{16}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}+\frac{bcx^6}{8}\sqrt{bx^4+a}+\frac{5\,acx^2}{16}\sqrt{bx^4+a}$$
$$+\frac{e}{10\,b}\left(bx^4+a\right)^{\frac{5}{2}}+\frac{bfx^9}{11}\sqrt{bx^4+a}+\frac{13\,afx^5}{77}\sqrt{bx^4+a}+\frac{4\,a^2fx}{77\,b}\sqrt{bx^4+a}$$
$$-\frac{4\,a^3f}{77\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] 1/9*d*b*x^7*(b*x^4+a)^(1/2)+11/45*d*a*x^3*(b*x^4+a)^(1/2)+4/15*I*
d*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*d*a^(5/2)/(I/a^(1/2)*b^
(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2
))^(1/2),I)+3/16*c*a^2*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+1/
8*c*b*x^6*(b*x^4+a)^(1/2)+5/16*a*c*x^2*(b*x^4+a)^(1/2)+1/10*e*(b*
x^4+a)^(5/2)/b+1/11*f*b*x^9*(b*x^4+a)^(1/2)+13/77*f*a*x^5*(b*x^4+
a)^(1/2)+4/77*a^2*f*x*(b*x^4+a)^(1/2)/b-4/77*f/b*a^3/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bfx^8+bex^7+bdx^6+bcx^5+afx^4+aex^3+adx^2+acx\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^8 + b*e*x^7 + b*d*x^6 + b*c*x^5 + a*f*x^4 + a*e*x
^3 + a*d*x^2 + a*c*x)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 14.8972, size = 396, normalized size = 0.97

$$\frac{a^{\frac{3}{2}}cx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}cx^2}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}dx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}fx^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}bcx^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bdx^7\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bfx^9\,\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{13}{4}\right)}+\frac{3a^2c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}}+ae\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\\frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+be\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\\frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+\frac{b^2cx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*c*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*c*x**2/(16*sqrt(1
 + b*x**4/a)) + a**(3/2)*d*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/
4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*f*x**5*g
amma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4
*gamma(9/4)) + 3*sqrt(a)*b*c*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(
a)*b*d*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*f*x**9*gamma(9/4)*hyper((
-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) +
3*a**2*c*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*e*Piecewise
((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) +
 b*e*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a
+ b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*
x**8/8, True)) + b**2*c*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x, x)
```

3.502 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=382

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+15\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{3a^2d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{63}x\left(a+bx^4\right)^{3/2}\left(9c+7ex^2\right) + \frac{2}{105}ax\sqrt{a+bx^4}\left(15c+7ex^2\right) + \frac{1}{8}dx^2\left(a+bx^4\right)^{3/2} + \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{f\left(a+bx^4\right)^{5/2}}{10b}$$

```
[Out] (3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*
Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a
 + b*x^4])/105 + (d*x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)
*(a + b*x^4)^(3/2))/63 + (f*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*d*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)
*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*
Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a +
 b*x^4])
```

Rubi [A] time = 0.639411, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.37$

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+15\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{3a^2d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{63}x\left(a+bx^4\right)^{3/2}\left(9c+7ex^2\right) + \frac{2}{105}ax\sqrt{a+bx^4}\left(15c+7ex^2\right) + \frac{1}{8}dx^2\left(a+bx^4\right)^{3/2} + \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{f\left(a+bx^4\right)^{5/2}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```



```
[Out] (3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*
Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a
 + b*x^4])/105 + (d*x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)
*(a + b*x^4)^(3/2))/63 + (f*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*d*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)
*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*
Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a +
 b*x^4])
```

Rubi in Sympy [A] time = 61.3009, size = 357, normalized size = 0.93

$$-\frac{4a^{\frac{9}{4}}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{2a^{\frac{7}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{a}e+15\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{105b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{3a^2d\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3adx^2\sqrt{a+bx^4}}{16}+\frac{2ax\sqrt{a+bx^4}\left(45c+21ex^2\right)}{315}+\frac{dx^2\left(a+bx^4\right)^{\frac{3}{2}}}{8}+\frac{x\left(a+bx^4\right)^{\frac{3}{2}}\left(9c+7ex^2\right)}{63}+\frac{f\left(a+bx^4\right)^{\frac{5}{2}}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] -4*a**(9/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(15*b**(3/4)*sqrt(a + b*x**4)) + 2*a**(7/4)*sqrt((a + b*x**4)/(s
qrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(7*sqrt(a)*e
+ 15*sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(105
*b**(3/4)*sqrt(a + b*x**4)) + 3*a**2*d*atanh(sqrt(b)*x**2/sqrt(a
+ b*x**4))/(16*sqrt(b)) + 4*a**2*e*x*sqrt(a + b*x**4)/(15*sqrt(b)
*(sqrt(a) + sqrt(b)*x**2)) + 3*a*d*x**2*sqrt(a + b*x**4)/16 + 2*a
*x*sqrt(a + b*x**4)*(45*c + 21*e*x**2)/315 + d*x**2*(a + b*x**4)*
*(3/2)/8 + x*(a + b*x**4)**(3/2)*(9*c + 7*e*x**2)/63 + f*(a + b*x
**4)**(5/2)/(10*b)
```

Mathematica [C] time = 0.796908, size = 294, normalized size = 0.77

$$1344a^{5/2}\sqrt{b}e\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(504a^2f+abx(2160c+7x(225d+16x(11e+9fx)))\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(504*a^2*f + 2*b^2*x^5*(3
60*c + 7*x*(45*d + 40*e*x + 36*f*x^2)) + a*b*x*(2160*c + 7*x*(225
*d + 16*x*(11*e + 9*f*x)))) + 945*a^2*Sqrt[b]*d*Sqrt[a + b*x^4]*A
rcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) + 1344*a^(5/2)*Sqrt[b]*e*S
qrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*
x], -1] - 192*a^2*Sqrt[b]*((15*I)*Sqrt[b]*c + 7*Sqrt[a]*e)*Sqrt[1
 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -
1])/(5040*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.011, size = 368, normalized size = 1.

$$\frac{bcx^5}{7}\sqrt{bx^4+a}+\frac{3\,acx}{7}\sqrt{bx^4+a}$$
$$+\frac{4\,a^2c}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{3\,a^2d}{16}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}+\frac{bdx^6}{8}\sqrt{bx^4+a}$$
$$+\frac{5\,adx^2}{16}\sqrt{bx^4+a}+\frac{bex^7}{9}\sqrt{bx^4+a}+\frac{11\,aex^3}{45}\sqrt{bx^4+a}$$
$$+\frac{4\,i}{15}ea^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$-\frac{4\,i}{15}ea^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$+\frac{f}{10\,b}\left(bx^4+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] 1/7*c*b*x^5*(b*x^4+a)^(1/2)+3/7*c*a*x*(b*x^4+a)^(1/2)+4/7*c*a^2/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^
(1/2))^(1/2),I)+3/16*d*a^2*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2
)+1/8*d*b*x^6*(b*x^4+a)^(1/2)+5/16*a*d*x^2*(b*x^4+a)^(1/2)+1/9*e*
b*x^7*(b*x^4+a)^(1/2)+11/45*e*a*x^3*(b*x^4+a)^(1/2)+4/15*I*e*a^(5
/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*e*a^(5/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^
(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/
2),I)+1/10*f*(b*x^4+a)^(5/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 13.8238, size = 394, normalized size = 1.03

$$\frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}dx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}dx^2}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}bcx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}bdx^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{3a^2d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}}+af\left(\begin{cases}\frac{\sqrt{a}x^4}{4}&\text{for } b=0\\\frac{(a+bx^4)^{\frac{3}{2}}}{6b}&\text{otherwise}\end{cases}\right)+bf\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10}&\text{for } b\neq 0\\\frac{\sqrt{a}x^8}{8}&\text{otherwise}\end{cases}\right)+\frac{b^2dx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)*d*x**2*sqrt(1 + b*x**4/a)/4
 + a**(3/2)*d*x**2/(16*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**3*gamm
a(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(7/4)) + sqrt(a)*b*c*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*d*x**6/(
16*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**7*gamma(7/4)*hyper((-1/2,
 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + 3*a**
2*d*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*f*Piecewise((sqr
t(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*f*
Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x
**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/
8, True)) + b**2*d*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

3.503 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=403

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+15\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{16}a\sqrt{a+bx^4}\left(8c+3ex^2\right)+\frac{1}{24}\left(a+bx^4\right)^{3/2}(4c+$$

```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2))
+ (a*(8*c + 3*e*x^2)*Sqrt[a + b*x^4])/16 + (2*a*x*(15*d + 7*f*x^2
)*Sqrt[a + b*x^4])/105 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/24 +
 (x*(9*d + 7*f*x^2)*(a + b*x^4)^(3/2))/63 + (3*a^2*e*ArcTanh[(Sqr
t[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2)*c*ArcTanh[Sqr
t[a + b*x^4]/Sqrt[a]])/2 - (4*a^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4
)*(15*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.910761, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+15\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{16}a\sqrt{a+bx^4}\left(8c+3ex^2\right)+\frac{1}{24}\left(a+bx^4\right)^{3/2}(4c+$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```


```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2))
+ (a*(8*c + 3*e*x^2)*Sqrt[a + b*x^4])/16 + (2*a*x*(15*d + 7*f*x^2
)*Sqrt[a + b*x^4])/105 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/24 +
 (x*(9*d + 7*f*x^2)*(a + b*x^4)^(3/2))/63 + (3*a^2*e*ArcTanh[(Sqr
t[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2)*c*ArcTanh[Sqr
t[a + b*x^4]/Sqrt[a]])/2 - (4*a^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4
)*(15*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 90.4774, size = 376, normalized size = 0.93

$$-\frac{4a^{\frac{9}{4}}f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{2a^{\frac{7}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{a}f+15\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{105b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{a^{\frac{3}{2}}c\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2}+\frac{3a^2e\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax\sqrt{a+bx^4}\left(45d+21fx^2\right)}{315}+\frac{a\sqrt{a+bx^4}\left(8c+3ex^2\right)}{16}+\frac{x\left(a+bx^4\right)^{\frac{3}{2}}\left(9d+7fx^2\right)}{63}+\frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(4c+3ex^2\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x,x)
```

```
[Out] -4*a**(9/4)*f*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(15*b**(3/4)*sqrt(a + b*x**4)) + 2*a**(7/4)*sqrt((a + b*x**4)/(s
qrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(7*sqrt(a)*f
+ 15*sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(105
*b**(3/4)*sqrt(a + b*x**4)) - a**(3/2)*c*atanh(sqrt(a + b*x**4)/s
qrt(a))/2 + 3*a**2*e*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(16*sqr
t(b)) + 4*a**2*f*x*sqrt(a + b*x**4)/(15*sqrt(b)*(sqrt(a) + sqrt(b
)*x**2)) + 2*a*x*sqrt(a + b*x**4)*(45*d + 21*f*x**2)/315 + a*sqrt
(a + b*x**4)*(8*c + 3*e*x**2)/16 + x*(a + b*x**4)**(3/2)*(9*d + 7
*f*x**2)/63 + (a + b*x**4)**(3/2)*(4*c + 3*e*x**2)/24
```

Mathematica [C] time = 0.895006, size = 319, normalized size = 0.79

$$-\frac{1}{2}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{4ia^2d\sqrt{\frac{bx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}+\frac{3a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4ia^2f\sqrt{\frac{bx^4}{a}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{15\left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2}\sqrt{a+bx^4}}+\frac{\sqrt{a+bx^4}\left(a(3360c+x(2160d+7x(225e+176fx)))+10bx^4(84c+x(72d+7x(9e+8fx)))\right)}{5040}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^4]*(10*b*x^4*(84*c + x*(72*d + 7*x*(9*e + 8*f*x)))
+ a*(3360*c + x*(2160*d + 7*x*(225*e + 176*f*x)))))/5040 + (3*a^2
*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2
)*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 + (((4*I)/15)*a^2*f*Sqrt[
1 + (b*x^4)/a]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x],
 -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/((
(I*Sqrt[b])/Sqrt[a])^(3/2)*Sqrt[a + b*x^4]) - (((4*I)/7)*a^2*d*Sq
rt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x
], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*x^4])
```

Maple [C] time = 0.025, size = 411, normalized size = 1.

$$\frac{bdx^5}{7}\sqrt{bx^4+a}+\frac{3\,adx}{7}\sqrt{bx^4+a}$$
$$+\frac{4\,a^2d}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{bfx^7}{9}\sqrt{bx^4+a}+\frac{11\,x^3af}{45}\sqrt{bx^4+a}$$
$$+\frac{4\,i}{15}fa^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$-\frac{4\,i}{15}fa^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$-\frac{c}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)+\frac{bcx^4}{6}\sqrt{bx^4+a}+\frac{2\,ac}{3}\sqrt{bx^4+a}$$
$$+\frac{3\,ea^2}{16}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}+\frac{bex^6}{8}\sqrt{bx^4+a}+\frac{5\,aex^2}{16}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x)
```

```
[Out] 1/7*d*b*x^5*(b*x^4+a)^(1/2)+3/7*d*a*x*(b*x^4+a)^(1/2)+4/7*d*a^2/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^
(1/2))^(1/2),I)+1/9*f*b*x^7*(b*x^4+a)^(1/2)+11/45*f*a*x^3*(b*x^4+
a)^(1/2)+4/15*I*f*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*f*a^(5/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(
I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*c*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^
4+a)^(1/2))/x^2)+1/6*c*b*x^4*(b*x^4+a)^(1/2)+2/3*c*a*(b*x^4+a)^(1
/2)+3/16*e*a^2*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+1/8*e*b*x^
6*(b*x^4+a)^(1/2)+5/16*e*a*x^2*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x, x)
```

Sympy [A] time = 17.0931, size = 405, normalized size = 1.

$$-\frac{a^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{a^{\frac{3}{2}}dx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}ex^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}ex^2}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}fx^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}bdx^5\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}bex^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bfx^7\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{a^2c}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{3a^2e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{a\sqrt{b}cx^2}{2\sqrt{\frac{a}{bx^4}+1}}+bc\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+\frac{b^2ex^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x,x)
```

```
[Out] -a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*d*x*gamma(
1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamm
a(5/4)) + a**(3/2)*e*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*e*x**2/
(16*sqrt(1 + b*x**4/a)) + a**(3/2)*f*x**3*gamma(3/4)*hyper((-1/2,
 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)
*b*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(
I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*e*x**6/(16*sqrt(1 + b*x**4/
a)) + sqrt(a)*b*f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x
**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + a**2*c/(2*sqrt(b)*x**2*s
qrt(a/(b*x**4) + 1)) + 3*a**2*e*asinh(sqrt(b)*x**2/sqrt(a))/(16*s
qrt(b)) + a*sqrt(b)*c*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*c*Piecewi
se((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
 + b**2*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

3.504 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=404

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+21\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} - \frac{1}{2}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) + \frac{3a^2f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{\left(a+bx^4\right)^{3/2}\left(7c-ex^2\right)}{7x} + \frac{2}{35}x\sqrt{a+bx^4}\left(5ae+21bcx^2\right) + \frac{12a\sqrt{b}cx\sqrt{\cdots}}{5\left(\sqrt{a}+\sqrt{\cdots}\right.}$$

```
[Out] (12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) +
(2*x*(5*a*e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2
)*Sqrt[a + b*x^4])/16 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) +
 ((4*d + 3*f*x^2)*(a + b*x^4)^(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2)*d*ArcTanh[Sqrt[
a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*
(21*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(35*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.838123, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+21\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} - \frac{1}{2}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) + \frac{3a^2f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{\left(a+bx^4\right)^{3/2}\left(7c-ex^2\right)}{7x} + \frac{2}{35}x\sqrt{a+bx^4}\left(5ae+21bcx^2\right) + \frac{12a\sqrt{b}cx\sqrt{\cdots}}{5\left(\sqrt{a}+\sqrt{\cdots}\right.}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

```
[Out] (12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) +
(2*x*(5*a*e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2
)*Sqrt[a + b*x^4])/16 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) +
 ((4*d + 3*f*x^2)*(a + b*x^4)^(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2)*d*ArcTanh[Sqrt[
a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*
(21*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(35*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 96.5957, size = 376, normalized size = 0.93

$$-\frac{12a^{\frac{5}{4}}\sqrt[4]{b}c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{2a^{\frac{5}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(5\sqrt{a}e+21\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{a^{\frac{3}{2}}d\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2}+\frac{3a^2f\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{a\sqrt{a+bx^4}\left(8d+3fx^2\right)}{16}+\frac{2x\sqrt{a+bx^4}\left(5ae+21bcx^2\right)}{35}+\frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(4d+3fx^2\right)}{24}-\frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(7c-ex^2\right)}{7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**2,x)
```

```
[Out] -12*a**(5/4)*b**(1/4)*c*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2
)**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1
/4)), 1/2)/(5*sqrt(a + b*x**4)) + 2*a**(5/4)*sqrt((a + b*x**4)/(s
qrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(5*sqrt(a)*e
+ 21*sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(35*
b**(1/4)*sqrt(a + b*x**4)) - a**(3/2)*d*atanh(sqrt(a + b*x**4)/sq
rt(a))/2 + 3*a**2*f*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(16*sqrt
(b)) + 12*a*sqrt(b)*c*x*sqrt(a + b*x**4)/(5*(sqrt(a) + sqrt(b)*x*
*2)) + a*sqrt(a + b*x**4)*(8*d + 3*f*x**2)/16 + 2*x*sqrt(a + b*x*
*4)*(5*a*e + 21*b*c*x**2)/35 + (a + b*x**4)**(3/2)*(4*d + 3*f*x**
2)/24 - (a + b*x**4)**(3/2)*(7*c - e*x**2)/(7*x)
```

Mathematica [C] time = 0.972582, size = 328, normalized size = 0.81

$$-\frac{1}{2}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{4ia^2e\sqrt{\frac{bx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}+\frac{3a^2f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\sqrt{a+bx^4}\left(a\left(-\frac{c}{x}+\frac{2d}{3}+\frac{3ex}{7}+\frac{5fx^2}{16}\right)+b\left(\frac{cx^3}{5}+\frac{dx^4}{6}+\frac{ex^5}{7}+\frac{fx^6}{8}\right)\right)+\frac{12iabc\sqrt{\frac{bx^4}{a}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{5\left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

```
[Out] Sqrt[a + b*x^4]*(a*((2*d)/3 - c/x + (3*e*x)/7 + (5*f*x^2)/16) + b
*((c*x^3)/5 + (d*x^4)/6 + (e*x^5)/7 + (f*x^6)/8)) + (3*a^2*f*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2)*d*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 + (((12*I)/5)*a*b*c*Sqrt[1 + (b*
x^4)/a]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] -
EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/(((I*Sqrt
[b])/Sqrt[a])^(3/2)*Sqrt[a + b*x^4]) - (((4*I)/7)*a^2*e*Sqrt[1 +
(b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])
/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*x^4])
```

Maple [C] time = 0.02, size = 411, normalized size = 1.

$$\frac{x^5 be}{7}\sqrt{bx^4+a}+\frac{3\,aex}{7}\sqrt{bx^4+a}+\frac{4\,ea^2}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{3\,a^2 f}{16}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}+\frac{bfx^6}{8}\sqrt{bx^4+a}+\frac{5\,x^2 af}{16}\sqrt{bx^4+a}-\frac{ac}{x}\sqrt{bx^4+a}+\frac{x^3 bc}{5}\sqrt{bx^4+a}+\frac{12\,i}{5}ca^{\frac{3}{2}}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{12\,i}{5}ca^{\frac{3}{2}}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{d}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)+\frac{bdx^4}{6}\sqrt{bx^4+a}+\frac{2\,ad}{3}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x)
```

```
[Out] 1/7*e*b*x^5*(b*x^4+a)^(1/2)+3/7*e*a*x*(b*x^4+a)^(1/2)+4/7*e*a^2/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^
(1/2))^(1/2),I)+3/16*f*a^2*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2
)+1/8*f*b*x^6*(b*x^4+a)^(1/2)+5/16*f*a*x^2*(b*x^4+a)^(1/2)-c*a*(b
*x^4+a)^(1/2)/x+1/5*c*b*x^3*(b*x^4+a)^(1/2)+12/5*I*c*a^(3/2)*b^(1
/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*b^(1/2))^(1/2),I)-12/5*I*c*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^
(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/
2*d*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/6*d*b*x^4*(
b*x^4+a)^(1/2)+2/3*d*a*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^2, x)
```

Sympy [A] time = 16.8686, size = 406, normalized size = 1.

$$\frac{a^{\frac{3}{2}}c\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)}-\frac{a^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{a^{\frac{3}{2}}ex\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}fx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}fx^2}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bcx^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}bex^5\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}bfx^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{a^2d}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{3a^2f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{a\sqrt{b}dx^2}{2\sqrt{\frac{a}{bx^4}+1}}+bd\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for}\ b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+\frac{b^2fx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**2,x)
```

```
[Out] a**(3/2)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x*gamma(3/4)) - a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*
x**2))/2 + a**(3/2)*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x
**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)*f*x**2*sqrt(1 +
b*x**4/a)/4 + a**(3/2)*f*x**2/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*b
*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*e*x**5*gamma(5/4)*hyper((-1/2,
5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a
)*b*f*x**6/(16*sqrt(1 + b*x**4/a)) + a**2*d/(2*sqrt(b)*x**2*sqrt(
a/(b*x**4) + 1)) + 3*a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(
b)) + a*sqrt(b)*d*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*d*Piecewise((
sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b
**2*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

3.505 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3} dx$

Optimal. Leaf size=406

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+21\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} - \frac{1}{2}a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{(a+bx^4)^{3/2}(3c-ex^2)}{6x^2} + \frac{1}{4}\sqrt{a+bx^4}\left(2ae+3bcx^2\right) + \frac{3}{4}a\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{(a+bx^4)^3}{}$$

```
[Out] (12*a*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) +
((2*a*e + 3*b*c*x^2)*Sqrt[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^
2)*Sqrt[a + b*x^4])/35 - ((3*c - e*x^2)*(a + b*x^4)^(3/2))/(6*x^2
) - ((7*d - f*x^2)*(a + b*x^4)^(3/2))/(7*x) + (3*a*Sqrt[b]*c*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (a^(3/2)*e*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(
21*Sqrt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(35*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.826172, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+21\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} - \frac{1}{2}a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{(a+bx^4)^{3/2}(3c-ex^2)}{6x^2} + \frac{1}{4}\sqrt{a+bx^4}\left(2ae+3bcx^2\right) + \frac{3}{4}a\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{(a+bx^4)^3}{}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```


```
[Out] (12*a*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) +
((2*a*e + 3*b*c*x^2)*Sqrt[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^
2)*Sqrt[a + b*x^4])/35 - ((3*c - e*x^2)*(a + b*x^4)^(3/2))/(6*x^2
) - ((7*d - f*x^2)*(a + b*x^4)^(3/2))/(7*x) + (3*a*Sqrt[b]*c*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (a^(3/2)*e*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(
21*Sqrt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(35*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 93.4706, size = 377, normalized size = 0.93

$$-\frac{12a^{\frac{5}{4}}\sqrt[4]{b}d\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{2a^{\frac{5}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(5\sqrt{a}f+21\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{a^{\frac{3}{2}}e\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2}+\frac{3a\sqrt{b}c\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4}+\frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2x\sqrt{a+bx^4}\left(5af+21bdx^2\right)}{35}+\frac{\sqrt{a+bx^4}\left(4ae+6bcx^2\right)}{8}-\frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(7d-fx^2\right)}{7x}-\frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(3c-ex^2\right)}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**3,x)
```

```
[Out] -12*a**(5/4)*b**(1/4)*d*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2
)**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1
/4)), 1/2)/(5*sqrt(a + b*x**4)) + 2*a**(5/4)*sqrt((a + b*x**4)/(s
qrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(5*sqrt(a)*f
+ 21*sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(35*
b**(1/4)*sqrt(a + b*x**4)) - a**(3/2)*e*atanh(sqrt(a + b*x**4)/sq
rt(a))/2 + 3*a*sqrt(b)*c*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/4 +
 12*a*sqrt(b)*d*x*sqrt(a + b*x**4)/(5*(sqrt(a) + sqrt(b)*x**2)) +
 2*x*sqrt(a + b*x**4)*(5*a*f + 21*b*d*x**2)/35 + sqrt(a + b*x**4)
*(4*a*e + 6*b*c*x**2)/8 - (a + b*x**4)**(3/2)*(7*d - f*x**2)/(7*x
) - (a + b*x**4)**(3/2)*(3*c - e*x**2)/(6*x**2)
```

Mathematica [C] time = 1.15303, size = 326, normalized size = 0.8

$$\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-210a^{3/2}ex^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\left(a+bx^4\right)\left(-210ac+20ax(x(14e+9fx)-21d)+bx^4\left(105c+84dx+70e\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(-210*a*c + b*x^4*(105*c
+ 84*d*x + 70*e*x^2 + 60*f*x^3) + 20*a*x*(-21*d + x*(14*e + 9*f*x
))) + 315*a*Sqrt[b]*c*x^2*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a + b*x^4]] - 210*a^(3/2)*e*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[
a + b*x^4]/Sqrt[a]]) + 1008*a^(3/2)*Sqrt[b]*d*x^2*Sqrt[1 + (b*x^4
)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - (48*
I)*a^(3/2)*((-21*I)*Sqrt[b]*d + 5*Sqrt[a]*f)*x^2*Sqrt[1 + (b*x^4)
/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(420*S
qrt[(I*Sqrt[b])/Sqrt[a]]*x^2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.027, size = 409, normalized size = 1.

$$\frac{bfx^5}{7}\sqrt{bx^4+a}+\frac{3afx}{7}\sqrt{bx^4+a}+\frac{4a^2f}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{x^2bc}{4}\sqrt{bx^4+a}+\frac{3ac}{4}\sqrt{b}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{ac}{2x^2}\sqrt{bx^4+a}-\frac{ad}{x}\sqrt{bx^4+a}+\frac{x^3bd}{5}\sqrt{bx^4+a}+\frac{12i}{5}da^{\frac{3}{2}}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{12i}{5}da^{\frac{3}{2}}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{e}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)+\frac{x^4be}{6}\sqrt{bx^4+a}+\frac{2ae}{3}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x)
```

```
[Out] 1/7*f*b*x^5*(b*x^4+a)^(1/2)+3/7*f*a*x*(b*x^4+a)^(1/2)+4/7*f*a^2/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^
(1/2))^(1/2),I)+1/4*c*b*x^2*(b*x^4+a)^(1/2)+3/4*c*b^(1/2)*a*ln(b^
(1/2)*x^2+(b*x^4+a)^(1/2))-1/2*c*a/x^2*(b*x^4+a)^(1/2)-d*a*(b*x^4
+a)^(1/2)/x+1/5*d*b*x^3*(b*x^4+a)^(1/2)+12/5*I*d*a^(3/2)*b^(1/2)/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)-12/5*I*d*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*e*
a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/6*e*b*x^4*(b*x^
4+a)^(1/2)+2/3*e*a*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^3, x)
```

Sympy [A] time = 13.3853, size = 377, normalized size = 0.93

$$-\frac{a^{\frac{3}{2}}c}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}d\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)}-\frac{a^{\frac{3}{2}}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}$$
$$+\frac{a^{\frac{3}{2}}fx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}bcx^2\sqrt{1+\frac{bx^4}{a}}}{4}-\frac{\sqrt{a}bcx^2}{2\sqrt{1+\frac{bx^4}{a}}}$$
$$+\frac{\sqrt{a}bdx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}bfx^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{9}{4}\right)}$$
$$+\frac{a^2e}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{3a\sqrt{b}c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4}+\frac{a\sqrt{b}ex^2}{2\sqrt{\frac{a}{bx^4}+1}}+be\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**3,x)
```

```
[Out] -a**(3/2)*c/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*d*gamma(-1/4)*
hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(
3/4)) - a**(3/2)*e*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*f*x
*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4)) + sqrt(a)*b*c*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*
b*c*x**2/(2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d*x**3*gamma(3/4)*hyp
er((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4))
+ sqrt(a)*b*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*gamma(9/4)) + a**2*e/(2*sqrt(b)*x**2*sqrt(a/
(b*x**4) + 1)) + 3*a*sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*
sqrt(b)*e*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*e*Piecewise((sqrt(a)*
x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

3.506 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=408

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{2\sqrt{a+bx^4}\left(9ae-5bcx^2\right)}{15x}-\frac{\left(a+bx^4\right)^{3/2}\left(5c-3ex^2\right)}{15x^3}-\frac{\left(a+bx^4\right)^{3/2}\left(3d-fx^2\right)}{6x^2}+\frac{1}{4}\sqrt{a+bx^4}$$

```
[Out] (12*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) -
(2*(9*a*e - 5*b*c*x^2)*Sqrt[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*
x^2)*Sqrt[a + b*x^4])/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^(3/2))/(15
*x^3) - ((3*d - f*x^2)*(a + b*x^4)^(3/2))/(6*x^2) + (3*a*Sqrt[b]*
d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (a^(3/2)*f*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*e*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(
3/4)*b^(1/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.84761, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{2\sqrt{a+bx^4}\left(9ae-5bcx^2\right)}{15x}-\frac{\left(a+bx^4\right)^{3/2}\left(5c-3ex^2\right)}{15x^3}-\frac{\left(a+bx^4\right)^{3/2}\left(3d-fx^2\right)}{6x^2}+\frac{1}{4}\sqrt{a+bx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

```
[Out] (12*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) -
(2*(9*a*e - 5*b*c*x^2)*Sqrt[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*
x^2)*Sqrt[a + b*x^4])/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^(3/2))/(15
*x^3) - ((3*d - f*x^2)*(a + b*x^4)^(3/2))/(6*x^2) + (3*a*Sqrt[b]*
d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (a^(3/2)*f*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*e*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(
3/4)*b^(1/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 91.7594, size = 381, normalized size = 0.93

$$-\frac{12a^{\frac{5}{4}}\sqrt[4]{b}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{2a^{\frac{3}{4}}\sqrt[4]{b}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(9\sqrt{a}e+5\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{a^{\frac{3}{2}}f\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2}+\frac{3a\sqrt{b}d\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4}+\frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\sqrt{a+bx^4}\left(4af+6bdx^2\right)}{8}-\frac{2\sqrt{a+bx^4}\left(9ae-5bcx^2\right)}{15x}-\frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(3d-fx^2\right)}{6x^2}-\frac{\left(a+bx^4\right)^{\frac{3}{2}}\left(5c-3ex^2\right)}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**4,x)
```

```
[Out] -12*a**(5/4)*b**(1/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2
)**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1
/4)), 1/2)/(5*sqrt(a + b*x**4)) + 2*a**(3/4)*b**(1/4)*sqrt((a + b
*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(9*s
qrt(a)*e + 5*sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1
/2)/(15*sqrt(a + b*x**4)) - a**(3/2)*f*atanh(sqrt(a + b*x**4)/sqr
t(a))/2 + 3*a*sqrt(b)*d*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/4 +
12*a*sqrt(b)*e*x*sqrt(a + b*x**4)/(5*(sqrt(a) + sqrt(b)*x**2)) +
sqrt(a + b*x**4)*(4*a*f + 6*b*d*x**2)/8 - 2*sqrt(a + b*x**4)*(9*a
*e - 5*b*c*x**2)/(15*x) - (a + b*x**4)**(3/2)*(3*d - f*x**2)/(6*x
**2) - (a + b*x**4)**(3/2)*(5*c - 3*e*x**2)/(15*x**3)
```

Mathematica [C] time = 1.06983, size = 327, normalized size = 0.8

$$\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-30a^{3/2}fx^3\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\left(a+bx^4\right)\left(bx^4(20c+x(15d+2x(6e+5fx)))-10a\left(2c+x\left(3d+6ex-4f\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(-10*a*(2*c + x*(3*d + 6*
e*x - 4*f*x^2)) + b*x^4*(20*c + x*(15*d + 2*x*(6*e + 5*f*x)))) +
45*a*Sqrt[b]*d*x^3*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
 b*x^4]] - 30*a^(3/2)*f*x^3*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]]) + 144*a^(3/2)*Sqrt[b]*e*x^3*Sqrt[1 + (b*x^4)/a]*Elli
pticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 16*a*Sqrt[b]*
((5*I)*Sqrt[b]*c + 9*Sqrt[a]*e)*x^3*Sqrt[1 + (b*x^4)/a]*EllipticF
[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sqrt[b]
)/Sqrt[a]]*x^3*Sqrt[a + b*x^4])
```

Maple [C] time = 0.022, size = 408, normalized size = 1.

$$-\frac{ac}{3x^{3}}\sqrt{bx^{4}+a}+\frac{bcx}{3}\sqrt{bx^{4}+a}+\frac{4abc}{3}\sqrt{1-ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^{4}+a}}+\frac{x^{2}bd}{4}\sqrt{bx^{4}+a}+\frac{3ad}{4}\sqrt{b}\ln\left(\sqrt{b}x^{2}+\sqrt{bx^{4}+a}\right)-\frac{ad}{2x^{2}}\sqrt{bx^{4}+a}-\frac{ae}{x}\sqrt{bx^{4}+a}+\frac{x^{3}be}{5}\sqrt{bx^{4}+a}+\frac{12i}{5}ea^{\frac{3}{2}}\sqrt{b}\sqrt{1-ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^{4}+a}}-\frac{12i}{5}ea^{\frac{3}{2}}\sqrt{b}\sqrt{1-ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^{4}+a}}-\frac{f}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x^{2}}\left(2a+2\sqrt{a}\sqrt{bx^{4}+a}\right)\right)+\frac{bfx^{4}}{6}\sqrt{bx^{4}+a}+\frac{2af}{3}\sqrt{bx^{4}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x)
```

```
[Out] -1/3*c*a*(b*x^4+a)^(1/2)/x^3+1/3*c*b*x*(b*x^4+a)^(1/2)+4/3*c*a*b/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)+1/4*d*b*x^2*(b*x^4+a)^(1/2)+3/4*d*b^(1/2)*a*ln(b
^(1/2)*x^2+(b*x^4+a)^(1/2))-1/2*d*a/x^2*(b*x^4+a)^(1/2)-e*a*(b*x^
4+a)^(1/2)/x+1/5*e*b*x^3*(b*x^4+a)^(1/2)+12/5*I*e*a^(3/2)*b^(1/2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-12/5*I*e*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*f
*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/6*f*b*x^4*(b*x
^4+a)^(1/2)+2/3*f*a*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^{4}+a\right)^{\frac{3}{2}}\left(fx^{3}+ex^{2}+dx+c\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bfx^{7}+bex^{6}+bdx^{5}+bcx^{4}+afx^{3}+aex^{2}+adx+ac\right)\sqrt{bx^{4}+a}}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^4, x)
```

Sympy [A] time = 13.299, size = 381, normalized size = 0.93

$$\frac{a^{\frac{3}{2}}c\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\left(\frac{1}{4}\right)}-\frac{a^{\frac{3}{2}}d}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}e\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\left(\frac{3}{4}\right)}-\frac{a^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}bcx\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\left(\frac{5}{4}\right)}+\frac{\sqrt{a}bdx^2\sqrt{1+\frac{bx^4}{a}}}{4}-\frac{\sqrt{a}bdx^2}{2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bex^3\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\left(\frac{7}{4}\right)}+\frac{a^2f}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{3a\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4}+\frac{a\sqrt{b}fx^2}{2\sqrt{\frac{a}{bx^4}+1}}+bf\left(\begin{cases}\frac{\sqrt{a}x^4}{4}&\text{for }b=0\\\frac{(a+bx^4)^{\frac{3}{2}}}{6b}&\text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**4,x)
```

```
[Out] a**(3/2)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*d/(2*x**2*sqrt(1 + b*x
**4/a)) + a**(3/2)*e*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**(3/2)*f*asinh(sqrt(a)
/(sqrt(b)*x**2))/2 + sqrt(a)*b*c*x*gamma(1/4)*hyper((-1/2, 1/4),
(5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*d*x*
*2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*d*x**2/(2*sqrt(1 + b*x**4/a))
 + sqrt(a)*b*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*
exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**2*f/(2*sqrt(b)*x**2*sqrt(a
/(b*x**4) + 1)) + 3*a*sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/4 + a
*sqrt(b)*f*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*f*Piecewise((sqrt(a)
*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

3.507 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=386

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$
$$-\frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$-\frac{1}{12}\left(a+bx^4\right)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)+\frac{3}{4}b\sqrt{a+bx^4}\left(c+ex^2\right)-\frac{3}{4}\sqrt{a}bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{2}{15}bx\sqrt{a+bx^4}\left(5d+9fx^2\right)+\frac{3}{4}a$$

```
[Out] (12*a*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) +
(3*b*(c + e*x^2)*Sqrt[a + b*x^4])/4 + (2*b*x*(5*d + 9*f*x^2)*Sqrt
[a + b*x^4])/15 - (((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)
*(a + b*x^4)^(3/2))/12 + (3*a*Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqr
t[a + b*x^4]])/4 - (3*Sqrt[a]*b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]
])/4 - (12*a^(5/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*b^(1/4)*(5*Sqrt[b]
*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(15*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.775501, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$
$$-\frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$-\frac{1}{12}\left(a+bx^4\right)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)+\frac{3}{4}b\sqrt{a+bx^4}\left(c+ex^2\right)-\frac{3}{4}\sqrt{a}bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{2}{15}bx\sqrt{a+bx^4}\left(5d+9fx^2\right)+\frac{3}{4}a$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```



```
[Out] (12*a*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) +
(3*b*(c + e*x^2)*Sqrt[a + b*x^4])/4 + (2*b*x*(5*d + 9*f*x^2)*Sqrt
[a + b*x^4])/15 - (((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)
*(a + b*x^4)^(3/2))/12 + (3*a*Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqr
t[a + b*x^4]])/4 - (3*Sqrt[a]*b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]
])/4 - (12*a^(5/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*b^(1/4)*(5*Sqrt[b]
*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(15*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Mathematica [C]   time = 1.51293, size = 329, normalized size = 0.85

$$\frac{144a^{3/2}\sqrt{b}fx^4\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left(a+bx^4\right)\left(5a\left(3c+4dx+6x^2(e+2fx)\right)-bx^4(30c+x(20a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]   Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(5*a*(3*c + 4*d*x + 6*x
^2*(e + 2*f*x)) - b*x^4*(30*c + x*(20*d + 3*x*(5*e + 4*f*x))))) +
 45*a*Sqrt[b]*e*x^4*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a
+ b*x^4]] - 45*Sqrt[a]*b*c*x^4*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b
*x^4]/Sqrt[a]]) + 144*a^(3/2)*Sqrt[b]*f*x^4*Sqrt[1 + (b*x^4)/a]*E
llipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 16*a*Sqrt[
b]*((5*I)*Sqrt[b]*d + 9*Sqrt[a]*f)*x^4*Sqrt[1 + (b*x^4)/a]*Ellipt
icF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sqrt
[b])/Sqrt[a]]*x^4*Sqrt[a + b*x^4])
```

Maple [C]   time = 0.028, size = 409, normalized size = 1.1

$$\begin{aligned}
&\frac{bc}{2}\sqrt{bx^4+a}-\frac{ac}{4x^4}\sqrt{bx^4+a}-\frac{3bc}{4}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)\\
&-\frac{ad}{3x^3}\sqrt{bx^4+a}+\frac{xbd}{3}\sqrt{bx^4+a}\\
&+\frac{4bda}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{bex^2}{4}\sqrt{bx^4+a}+\frac{3ae}{4}\sqrt{b}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{ae}{2x^2}\sqrt{bx^4+a}-\frac{af}{x}\sqrt{bx^4+a}+\frac{bfx^3}{5}\sqrt{bx^4+a}\\
&+\frac{12i}{5}fa^{\frac{3}{2}}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{12i}{5}fa^{\frac{3}{2}}\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x)
```

```
[Out] 1/2*c*b*(b*x^4+a)^(1/2)-1/4*c*a/x^4*(b*x^4+a)^(1/2)-3/4*c*a^(1/2)
*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/3*d*a*(b*x^4+a)^(1/2
)/x^3+1/3*d*b*x*(b*x^4+a)^(1/2)+4/3*d*a*b/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*e*
b*x^2*(b*x^4+a)^(1/2)+3/4*e*b^(1/2)*a*ln(b^(1/2)*x^2+(b*x^4+a)^(1
/2))-1/2*e*a/x^2*(b*x^4+a)^(1/2)-f*a*(b*x^4+a)^(1/2)/x+1/5*f*b*x^
3*(b*x^4+a)^(1/2)+12/5*I*f*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*
I*f*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elli
```

```
pticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^5, x)
```

Sympy [A] time = 15.644, size = 379, normalized size = 0.98

$$\frac{a^{\frac{3}{2}}d\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4}, -\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}e}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}f\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bdx\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bex^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bex^2}{2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bfx^3\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\left(\frac{7}{4}\right)} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}c}{2x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{b^{\frac{3}{2}}cx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**5,x)
```

```
[Out] a**(3/2)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*e/(2*x**2*sqrt(1 + b*x
**4/a)) + a**(3/2)*f*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*c*asinh(sqrt
(a)/(sqrt(b)*x**2))/4 + sqrt(a)*b*d*x*gamma(1/4)*hyper((-1/2, 1/4
), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*e
*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*e*x**2/(2*sqrt(1 + b*x**4/
a)) + sqrt(a)*b*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*gamma(7/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4
) + 1)/(4*x**2) + a*sqrt(b)*c/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a
```

```
*sqrt(b)*e*asinh(sqrt(b)*x**2/sqrt(a))/4 + b**(3/2)*c*x**2/(2*sqr
t(a/(b*x**4) + 1))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{2}} \left(f x^3 + ex^2 + dx + c\right)}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

3.508 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=387

$$\frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}+\frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)-\frac{2b\sqrt{a+bx^4}\left(9c-5ex^2\right)}{15x}+\frac{3}{4}b\sqrt{a+bx^4}\left(d+fx^2\right)-\frac{3}{4}\sqrt{a}bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+$$

```
[Out] (12*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2
*b*(9*c - 5*e*x^2)*Sqrt[a + b*x^4])/(15*x) + (3*b*(d + f*x^2)*Sqr
t[a + b*x^4])/4 - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)
/x^2)*(a + b*x^4)^(3/2))/60 + (3*a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*d*ArcTanh[Sqrt[a + b*x^4]/Sq
rt[a]])/4 - (12*a^(1/4)*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sq
rt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(15*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.809648, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}+\frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)-\frac{2b\sqrt{a+bx^4}\left(9c-5ex^2\right)}{15x}+\frac{3}{4}b\sqrt{a+bx^4}\left(d+fx^2\right)-\frac{3}{4}\sqrt{a}bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

```
[Out] (12*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2
*b*(9*c - 5*e*x^2)*Sqrt[a + b*x^4])/(15*x) + (3*b*(d + f*x^2)*Sqr
t[a + b*x^4])/4 - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)
/x^2)*(a + b*x^4)^(3/2))/60 + (3*a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*d*ArcTanh[Sqrt[a + b*x^4]/Sq
rt[a]])/4 - (12*a^(1/4)*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sq
rt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(15*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**6,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.51494, size = 331, normalized size = 0.86

$$\frac{144\sqrt{a}b^{3/2}cx^5\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(12ac+5ax\left(3d+4ex+6fx^2\right)+84bcx^4-5bx^5(6a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(12*a*c + 84*b*c*x^4 +
5*a*x*(3*d + 4*e*x + 6*f*x^2) - 5*b*x^5*(6*d + x*(4*e + 3*f*x)))
- 45*a*Sqrt[b]*f*x^5*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a
 + b*x^4]] + 45*Sqrt[a]*b*d*x^5*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a +
b*x^4]/Sqrt[a]])) + 144*Sqrt[a]*b^(3/2)*c*x^5*Sqrt[1 + (b*x^4)/a]
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - (16*I)*S
qrt[a]*b*((-9*I)*Sqrt[b]*c + 5*Sqrt[a]*e)*x^5*Sqrt[1 + (b*x^4)/a]
*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[
(I*Sqrt[b])/Sqrt[a]]*x^5*Sqrt[a + b*x^4])
```

Maple [C] time = 0.024, size = 409, normalized size = 1.1

$$\begin{aligned}
&-\frac{ac}{5\,x^5}\sqrt{bx^4+a}-\frac{7\,bc}{5\,x}\sqrt{bx^4+a}\\
&+\frac{12\,i}{5}cb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{12\,i}{5}cb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{bd}{2}\sqrt{bx^4+a}-\frac{ad}{4\,x^4}\sqrt{bx^4+a}-\frac{3\,bd}{4}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\\
&-\frac{ae}{3\,x^3}\sqrt{bx^4+a}+\frac{bex}{3}\sqrt{bx^4+a}\\
&+\frac{4\,bea}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{bfx^2}{4}\sqrt{bx^4+a}+\frac{3\,af}{4}\sqrt{b}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{af}{2\,x^2}\sqrt{bx^4+a}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x)
```

```
[Out] -1/5*c*a*(b*x^4+a)^(1/2)/x^5-7/5*c*b*(b*x^4+a)^(1/2)/x+12/5*I*c*b
^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*c*b^(3/2)*a^(1/2)/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)+1/2*d*b*(b*x^4+a)^(1/2)-1/4*d*a/x^4*(b*x^4+a)^(1/2)-3/4*
d*a^(1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/3*e*a*(b*x^
4+a)^(1/2)/x^3+1/3*e*b*x*(b*x^4+a)^(1/2)+4/3*e*a*b/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
```

```
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)+1/4*f*b*x^2*(b*x^4+a)^(1/2)+3/4*f*b^(1/2)*a*ln(b^(1/2)*x^2+(b*
x^4+a)^(1/2))-1/2*f*a/x^2*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^6, x)
```

Sympy [A] time = 16.0485, size = 386, normalized size = 1.

$$\frac{a^{\frac{3}{2}}c\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)}+\frac{a^{\frac{3}{2}}e\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\,\Gamma\left(\frac{1}{4}\right)}-\frac{a^{\frac{3}{2}}f}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bc\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)}-\frac{3\sqrt{a}bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4}+\frac{\sqrt{a}bex\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}bfx^2\sqrt{1+\frac{bx^4}{a}}}{4}-\frac{\sqrt{a}bfx^2}{2\sqrt{1+\frac{bx^4}{a}}}-\frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4x^2}+\frac{a\sqrt{b}d}{2x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{3a\sqrt{b}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4}+\frac{b^{\frac{3}{2}}dx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**6,x)
```

```
[Out] a**(3/2)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*x**5*gamma(-1/4)) + a**(3/2)*e*gamma(-3/4)*hyper(
(-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4
)) - a**(3/2)*f/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*gamma(-
1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*g
amma(3/4)) - 3*sqrt(a)*b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqrt
(a)*b*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(
```

```
I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*f*x**2*sqrt(1 + b*x**4/a)/4 -
 sqrt(a)*b*f*x**2/(2*sqrt(1 + b*x**4/a)) - a*sqrt(b)*d*sqrt(a/(b*
x**4) + 1)/(4*x**2) + a*sqrt(b)*d/(2*x**2*sqrt(a/(b*x**4) + 1)) +
 3*a*sqrt(b)*f*asinh(sqrt(b)*x**2/sqrt(a))/4 + b**(3/2)*d*x**2/(2
*sqrt(a/(b*x**4) + 1))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

3.509 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=392

$$\frac{1}{2}b^{3/2}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+9\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}+\frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)-\frac{b\sqrt{a+bx^4}\left(2c-3ex^2\right)}{4x^2}-\frac{2b\sqrt{a+bx^4}\left(9d-5fx^2\right)}{15x}-\frac{3}{4}\sqrt{a}be\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

```
[Out] (12*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (b
*(2*c - 3*e*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (2*b*(9*d - 5*f*x^2)*
Sqrt[a + b*x^4])/(15*x) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4
+ (20*f)/x^3)*(a + b*x^4)^(3/2))/60 + (b^(3/2)*c*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b*e*ArcTanh[Sqrt[a + b*x^4
]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(
9*Sqrt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(15*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.784479, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{1}{2}b^{3/2}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+9\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}+\frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)-\frac{b\sqrt{a+bx^4}\left(2c-3ex^2\right)}{4x^2}-\frac{2b\sqrt{a+bx^4}\left(9d-5fx^2\right)}{15x}-\frac{3}{4}\sqrt{a}be\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```


```
[Out] (12*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (b
*(2*c - 3*e*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (2*b*(9*d - 5*f*x^2)*
Sqrt[a + b*x^4])/(15*x) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4
+ (20*f)/x^3)*(a + b*x^4)^(3/2))/60 + (b^(3/2)*c*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b*e*ArcTanh[Sqrt[a + b*x^4
]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(
9*Sqrt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(15*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**7,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.61651, size = 331, normalized size = 0.84

$$-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-30b^{3/2}cx^6\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\left(a+bx^4\right)\left(a(10c+x(12d+5x(3e+4fx)))+2bx^4(20c+x(42d-5x(3e+\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*b*x^4*(20*c + x*(42*
d - 5*x*(3*e + 2*f*x))) + a*(10*c + x*(12*d + 5*x*(3*e + 4*f*x)))
) - 30*b^(3/2)*c*x^6*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a
 + b*x^4]] + 45*Sqrt[a]*b*e*x^6*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a +
b*x^4]/Sqrt[a]])) + 144*Sqrt[a]*b^(3/2)*d*x^6*Sqrt[1 + (b*x^4)/a]
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - (16*I)*S
qrt[a]*b*((-9*I)*Sqrt[b]*d + 5*Sqrt[a]*f)*x^6*Sqrt[1 + (b*x^4)/a]
*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[
(I*Sqrt[b])/Sqrt[a]]*x^6*Sqrt[a + b*x^4])
```

Maple [C] time = 0.027, size = 408, normalized size = 1.

$$\frac{c}{2}b^{\frac{3}{2}}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{ac}{6x^6}\sqrt{bx^4+a}-\frac{2bc}{3x^2}\sqrt{bx^4+a}-\frac{ad}{5x^5}\sqrt{bx^4+a}-\frac{7bd}{5x}\sqrt{bx^4+a}$$
$$+\frac{12i}{5}db^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{12i}{5}db^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{be}{2}\sqrt{bx^4+a}-\frac{ae}{4x^4}\sqrt{bx^4+a}-\frac{3be}{4}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)$$
$$-\frac{af}{3x^3}\sqrt{bx^4+a}+\frac{fbx}{3}\sqrt{bx^4+a}$$
$$+\frac{4abf}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x)
```

```
[Out] 1/2*c*b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))-1/6*c*a/x^6*(b*x^4+
a)^(1/2)-2/3*c*b/x^2*(b*x^4+a)^(1/2)-1/5*d*a*(b*x^4+a)^(1/2)/x^5-
7/5*d*b*(b*x^4+a)^(1/2)/x+12/5*I*d*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-12/5*I*d*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*e*b*(b*x^4+a)^(1
/2)-1/4*e*a/x^4*(b*x^4+a)^(1/2)-3/4*e*a^(1/2)*b*ln((2*a+2*a^(1/2)
*(b*x^4+a)^(1/2))/x^2)-1/3*f*a*(b*x^4+a)^(1/2)/x^3+1/3*f*b*x*(b*x
^4+a)^(1/2)+4/3*f*a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*El
```

```
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac\right)\sqrt{bx^4 + a}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^7,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^7, x)
```

Sympy [A] time = 15.2135, size = 406, normalized size = 1.04

$$\frac{a^{\frac{3}{2}}d\,\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}}f\,\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\,\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bc}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bd\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bfx\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}e}{2x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{b^{\frac{3}{2}}ex^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^2cx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**7,x)
```

```
[Out] a**(3/2)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*x**5*gamma(-1/4)) + a**(3/2)*f*gamma(-3/4)*hyper(
(-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4
)) - sqrt(a)*b*c/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d*gamma(
-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*
gamma(3/4)) - 3*sqrt(a)*b*e*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqr
t(a)*b*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar
(I*pi)/a)/(4*gamma(5/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(6*x*
*4) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*e/(2*
x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/6 +
b**(3/2)*c*asinh(sqrt(b)*x**2/sqrt(a))/2 + b**(3/2)*e*x**2/(2*sqr
```

```
t(a/(b*x**4) + 1)) - b**2*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{2}} \left(fx^3 + ex^2 + dx + c\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

3.510 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=412

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}$$

$$+\frac{1}{2}b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{420}\left(a+b\right.$$

```
[Out] (-12*b*e*Sqrt[a + b*x^4])/(5*x) + (12*b^(3/2)*e*x*Sqrt[a + b*x^4]
)/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*b*(5*c - 21*e*x^2)*Sqrt[a + b*
x^4])/(35*x^3) - (b*(2*d - 3*f*x^2)*Sqrt[a + b*x^4])/(4*x^2) - ((
(60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*(a + b*x^4)^(
3/2))/420 + (b^(3/2)*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2
- (3*Sqrt[a]*b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4
)*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*
Sqrt[a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*c + 21*Sqrt[a]*e)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a +
b*x^4])
```

Rubi [A] time = 0.913646, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}$$

$$+\frac{1}{2}b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{420}\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```


```
[Out] (-12*b*e*Sqrt[a + b*x^4])/(5*x) + (12*b^(3/2)*e*x*Sqrt[a + b*x^4]
)/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*b*(5*c - 21*e*x^2)*Sqrt[a + b*
x^4])/(35*x^3) - (b*(2*d - 3*f*x^2)*Sqrt[a + b*x^4])/(4*x^2) - ((
(60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*(a + b*x^4)^(
3/2))/420 + (b^(3/2)*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2
- (3*Sqrt[a]*b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4
)*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*
Sqrt[a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*c + 21*Sqrt[a]*e)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a +
b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**8,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.12642, size = 330, normalized size = 0.8

$$-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-210b^{3/2}dx^7\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\left(a+bx^4\right)\left(a(60c+7x(10d+3x(4e+5fx)))+2bx^4(90c+7x(20d+3x(1\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*b*x^4*(90*c + 7*x*(2
0*d + 3*x*(14*e - 5*f*x))) + a*(60*c + 7*x*(10*d + 3*x*(4*e + 5*f
*x)))) - 210*b^(3/2)*d*x^7*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/
Sqrt[a + b*x^4]] + 315*Sqrt[a]*b*f*x^7*Sqrt[a + b*x^4]*ArcTanh[Sq
rt[a + b*x^4]/Sqrt[a]])) + 1008*Sqrt[a]*b^(3/2)*e*x^7*Sqrt[1 + (b
*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] -
48*b^(3/2)*((5*I)*Sqrt[b]*c + 21*Sqrt[a]*e)*x^7*Sqrt[1 + (b*x^4)/
a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(420*Sq
rt[(I*Sqrt[b])/Sqrt[a]]*x^7*Sqrt[a + b*x^4])
```

Maple [C] time = 0.026, size = 411, normalized size = 1.

$$\begin{aligned}
&-\frac{ac}{7x^7}\sqrt{bx^4+a}-\frac{3bc}{7x^3}\sqrt{bx^4+a}\\
&+\frac{4b^2c}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{d}{2}b^{\frac{3}{2}}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{ad}{6x^6}\sqrt{bx^4+a}-\frac{2bd}{3x^2}\sqrt{bx^4+a}-\frac{ae}{5x^5}\sqrt{bx^4+a}-\frac{7be}{5x}\sqrt{bx^4+a}\\
&+\frac{12i}{5}eb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{12i}{5}eb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{fb}{2}\sqrt{bx^4+a}-\frac{af}{4x^4}\sqrt{bx^4+a}-\frac{3fb}{4}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x)
```

```
[Out] -1/7*c*a*(b*x^4+a)^(1/2)/x^7-3/7*c*b*(b*x^4+a)^(1/2)/x^3+4/7*c*b^
2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+1/2*d*b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))-
1/6*d*a/x^6*(b*x^4+a)^(1/2)-2/3*d*b/x^2*(b*x^4+a)^(1/2)-1/5*e*a*(
b*x^4+a)^(1/2)/x^5-7/5*b*e*(b*x^4+a)^(1/2)/x+12/5*I*e*b^(3/2)*a^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)-12/5*I*e*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1
/2*f*b*(b*x^4+a)^(1/2)-1/4*f*a/x^4*(b*x^4+a)^(1/2)-3/4*f*a^(1/2)*
b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^8, x)
```

Sympy [A] time = 16.3071, size = 415, normalized size = 1.01

$$\frac{a^{\frac{3}{2}}c\,\Gamma\left(-\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\,\Gamma\left(-\frac{3}{4}\right)}+\frac{a^{\frac{3}{2}}e\,\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}bc\,\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\,\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}bd}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}be\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)}-\frac{3\sqrt{a}bf\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4}-\frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{6x^4}-\frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{4x^2}+\frac{a\sqrt{b}f}{2x^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{6}+\frac{b^{\frac{3}{2}}d\,\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}+\frac{b^{\frac{3}{2}}fx^2}{2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^2dx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**8,x)
```

```
[Out] a**(3/2)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*x**7*gamma(-3/4)) + a**(3/2)*e*gamma(-5/4)*hyper(
(-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1
/4)) + sqrt(a)*b*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4
*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*d/(2*x**2*sqr
t(1 + b*x**4/a)) + sqrt(a)*b*e*gamma(-1/4)*hyper((-1/2, -1/4), (3
/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*f*
asinh(sqrt(a)/(sqrt(b)*x**2))/4 - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1
)/(6*x**4) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b
)*f/(2*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) +
1)/6 + b**(3/2)*d*asinh(sqrt(b)*x**2/sqrt(a))/2 + b**(3/2)*f*x**2
/(2*sqrt(a/(b*x**4) + 1)) - b**2*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**
4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

3.511 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=377

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{3b^2c\tan}{}$$

```
[Out] -(b*((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)*Sqrt[a
+ b*x^4])/560 + (12*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sq
rt[b]*x^2)) - (((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)
/x^5)*(a + b*x^4)^(3/2))/840 + (b^(3/2)*e*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a + b*x^4]])/2 - (3*b^2*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(
16*Sqrt[a]) - (12*a^(1/4)*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*
d + 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(35*a^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.678711, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{3b^2c\tan}{}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```


```
[Out] -(b*((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)*Sqrt[a
+ b*x^4])/560 + (12*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sq
rt[b]*x^2)) - (((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)
/x^5)*(a + b*x^4)^(3/2))/840 + (b^(3/2)*e*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a + b*x^4]])/2 - (3*b^2*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(
16*Sqrt[a]) - (12*a^(1/4)*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*
d + 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(35*a^(1/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**9,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.97472, size = 309, normalized size = 0.82

$$-\frac{4b^{3/2}\sqrt{\frac{bx^4}{a}+1}\left(21\sqrt{a}f+5i\sqrt{b}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{35\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{\sqrt{a+bx^4}\left(a(210c+8x(30d+7x(5e+6fx)))+bx^4\left(525c+16x\left(45d+70ex+147fx^2\right)\right)\right)}{1680x^8}-\frac{12iabf\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(b*x^4*(525*c + 16*x*(45*d + 70*e*x + 147*f*x^2
)) + a*(210*c + 8*x*(30*d + 7*x*(5*e + 6*f*x)))))/(1680*x^8) + (b
^(3/2)*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*b^2*c*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (((12*I)/5)*a*Sqrt[
(I*Sqrt[b])/Sqrt[a]]*b*f*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[
Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/Sqrt[a + b*x^4] - (4*b^(3/2)*(
(5*I)*Sqrt[b]*d + 21*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*A
rcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(35*Sqrt[(I*Sqrt[b])/Sq
rt[a]]*Sqrt[a + b*x^4])
```

Maple [C] time = 0.027, size = 416, normalized size = 1.1

$$-\frac{ac}{8\,x^8}\sqrt{bx^4+a}-\frac{5\,bc}{16\,x^4}\sqrt{bx^4+a}-\frac{3\,b^2c}{16}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{ad}{7\,x^7}\sqrt{bx^4+a}-\frac{3\,bd}{7\,x^3}\sqrt{bx^4+a}+\frac{4\,b^2d}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{e}{2}b^{\frac{3}{2}}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{ae}{6\,x^6}\sqrt{bx^4+a}-\frac{2\,be}{3\,x^2}\sqrt{bx^4+a}-\frac{af}{5\,x^5}\sqrt{bx^4+a}-\frac{7\,fb}{5\,x}\sqrt{bx^4+a}+\frac{12\,i}{5}fb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{12\,i}{5}fb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x)
```

```
[Out] -1/8*c*a/x^8*(b*x^4+a)^(1/2)-5/16*c*b/x^4*(b*x^4+a)^(1/2)-3/16*c/
a^(1/2)*b^2*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/7*d*a*(b*x^
4+a)^(1/2)/x^7-3/7*d*b*(b*x^4+a)^(1/2)/x^3+4/7*d*b^2/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)+1/2*e*b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))-1/6*e*a/x^6*(b
*x^4+a)^(1/2)-2/3*e*b/x^2*(b*x^4+a)^(1/2)-1/5*f*a*(b*x^4+a)^(1/2)
```

```
/x^5-7/5*f*b*(b*x^4+a)^(1/2)/x+12/5*I*f*b^(3/2)*a^(1/2)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)-12/5*I*f*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac\right)\sqrt{bx^4 + a}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^9,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^9, x)
```

Sympy [A] time = 21.2665, size = 444, normalized size = 1.18

$$\frac{a^{\frac{3}{2}}d\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\,\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}}f\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}bd\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\,\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}be}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bf\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)} - \frac{a^2c}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3a\sqrt{b}c}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}c}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}e\,\text{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{3b^2c\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}} - \frac{b^2ex^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**9,x)
```

```
[Out] a**(3/2)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*x**7*gamma(-3/4)) + a**(3/2)*f*gamma(-5/4)*hyper(
(-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1
/4)) + sqrt(a)*b*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4
*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*e/(2*x**2*sqr
t(1 + b*x**4/a)) + sqrt(a)*b*f*gamma(-1/4)*hyper((-1/2, -1/4), (3
```

```
/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**2*c/(8*sqrt
(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*c/(16*x**6*sqrt(a/(
b*x**4) + 1)) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3
/2)*c*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c/(16*x**2*sqrt(a/
(b*x**4) + 1)) - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*e*a
sinh(sqrt(b)*x**2/sqrt(a))/2 - 3*b**2*c*asinh(sqrt(a)/(sqrt(b)*x*
*2))/(16*sqrt(a)) - b**2*e*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^9,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

3.512 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=405

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}e+7\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{2}b^{3/2}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{4b^2c\sqrt{a+bx^4}}{15ax}-\frac{3b^2d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{b\sqrt{a+bx^4}\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)}{1680}-\frac{1}{504}(a+bx$$

```
[Out] -(b*((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*Sqrt[
a + b*x^4])/1680 - (4*b^2*c*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2
)*c*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((56*c)/
x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*(a + b*x^4)^(3/2))/50
4 + (b^(3/2)*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*b^2
*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)*c*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqr
t[a + b*x^4]) + (2*b^(7/4)*(7*Sqrt[b]*c + 15*Sqrt[a]*e)*(Sqrt[a]
+ Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(3/4)*Sqrt[a + b*
x^4])
```

Rubi [A] time = 0.874215, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}e+7\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{2}b^{3/2}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{4b^2c\sqrt{a+bx^4}}{15ax}-\frac{3b^2d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{b\sqrt{a+bx^4}\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)}{1680}-\frac{1}{504}(a+bx$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

```
[Out] -(b*((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*Sqrt[
a + b*x^4])/1680 - (4*b^2*c*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2
)*c*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((56*c)/
x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*(a + b*x^4)^(3/2))/50
4 + (b^(3/2)*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*b^2
*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)*c*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqr
t[a + b*x^4]) + (2*b^(7/4)*(7*Sqrt[b]*c + 15*Sqrt[a]*e)*(Sqrt[a]
+ Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(3/4)*Sqrt[a + b*
x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**10,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.33716, size = 351, normalized size = 0.87

$$-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(10a^2\left(56c+63dx+72ex^2+84fx^3\right)+abx^4(1232c+15x(105d+16x(9e+14fx)))+1344b^2cx^8\right)-2520\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(1344*b^2*c*x^8 + 10*a^
2*(56*c + 63*d*x + 72*e*x^2 + 84*f*x^3) + a*b*x^4*(1232*c + 15*x*
(105*d + 16*x*(9*e + 14*f*x)))) - 2520*a*b^(3/2)*f*x^9*Sqrt[a + b
*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] + 945*Sqrt[a]*b^2*d*
x^9*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 1344*Sqr
t[a]*b^(5/2)*c*x^9*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(
I*Sqrt[b])/Sqrt[a]]*x], -1] - (192*I)*Sqrt[a]*b^2*((-7*I)*Sqrt[b]
*c + 15*Sqrt[a]*e)*x^9*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sq
rt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(5040*a*Sqrt[(I*Sqrt[b])/Sqrt[a]
]*x^9*Sqrt[a + b*x^4])
```

Maple [C] time = 0.03, size = 437, normalized size = 1.1

$$-\frac{ac}{9x^9}\sqrt{bx^4+a}-\frac{11\,bc}{45\,x^5}\sqrt{bx^4+a}-\frac{4\,b^2c}{15\,ax}\sqrt{bx^4+a}$$
$$+\frac{4\,i}{15}cb^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{4\,i}{15}cb^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{ad}{8\,x^8}\sqrt{bx^4+a}-\frac{5\,bd}{16\,x^4}\sqrt{bx^4+a}$$
$$-\frac{3\,b^2d}{16}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{ae}{7\,x^7}\sqrt{bx^4+a}-\frac{3\,be}{7\,x^3}\sqrt{bx^4+a}$$
$$+\frac{4\,b^2e}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{f}{2}b^{\frac{3}{2}}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)-\frac{af}{6\,x^6}\sqrt{bx^4+a}-\frac{2\,fb}{3\,x^2}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x)
```

```
[Out] -1/9*c*a*(b*x^4+a)^(1/2)/x^9-11/45*c*b*(b*x^4+a)^(1/2)/x^5-4/15*b
^2*c*(b*x^4+a)^(1/2)/a/x+4/15*I*c/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)-4/15*I*c/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/8*d*a/x^8*(b*x^4+a)
^(1/2)-5/16*d*b/x^4*(b*x^4+a)^(1/2)-3/16*d/a^(1/2)*b^2*ln((2*a+2*
```

```
a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/7*e*a*(b*x^4+a)^(1/2)/x^7-3/7*e*b
*(b*x^4+a)^(1/2)/x^3+4/7*e*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*f*b^(3/2)*ln(
b^(1/2)*x^2+(b*x^4+a)^(1/2))-1/6*f*a/x^6*(b*x^4+a)^(1/2)-2/3*f*b/
x^2*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac)\sqrt{bx^4+a}}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^10, x)
```

Sympy [A] time = 23.0875, size = 449, normalized size = 1.11

$$\frac{a^{\frac{3}{2}}c\,\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\,\Gamma\left(-\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}e\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\,\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}bc\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}be\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\,\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}bf}{2x^2\sqrt{1+\frac{bx^4}{a}}}-\frac{a^2d}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{3a\sqrt{b}d}{16x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{6x^4}-\frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{4x^2}-\frac{b^{\frac{3}{2}}d}{16x^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{6}+\frac{b^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}-\frac{3b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}}-\frac{b^2fx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**10,x)
```

```
[Out] a**(3/2)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*x**9*gamma(-5/4)) + a**(3/2)*e*gamma(-7/4)*hyper(
(-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3
/4)) + sqrt(a)*b*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**
4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*e*gamma(-3/
```

```
4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*
gamma(1/4)) - sqrt(a)*b*f/(2*x**2*sqrt(1 + b*x**4/a)) - a**2*d/(8
*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*d/(16*x**6*sqr
t(a/(b*x**4) + 1)) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(6*x**4) -
b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d/(16*x**2*sq
rt(a/(b*x**4) + 1)) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/6 + b**(3/2
)*f*asinh(sqrt(b)*x**2/sqrt(a))/2 - 3*b**2*d*asinh(sqrt(a)/(sqrt(
b)*x**2))/(16*sqrt(a)) - b**2*f*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{2}}\left(fx^3 + ex^2 + dx + c\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

3.513 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}} dx$

Optimal. Leaf size=399

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}f+7\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} - \frac{3b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{b\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)}{1680} - \frac{\left(a+bx^4\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

```
[Out] -(b*((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/x^3)*Sqrt[
a + b*x^4])/1680 - (b^2*c*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*d*
Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*d*x*Sqrt[a + b*x^4])/(15*a
*(Sqrt[a] + Sqrt[b]*x^2)) - (((252*c)/x^10 + (280*d)/x^9 + (315*e
)/x^8 + (360*f)/x^7)*(a + b*x^4)^(3/2))/2520 - (3*b^2*e*ArcTanh[S
qrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]
) + (2*b^(7/4)*(7*Sqrt[b]*d + 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.864589, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}f+7\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} - \frac{3b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{b\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)}{1680} - \frac{\left(a+bx^4\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```


```
[Out] -(b*((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/x^3)*Sqrt[
a + b*x^4])/1680 - (b^2*c*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*d*
Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*d*x*Sqrt[a + b*x^4])/(15*a
*(Sqrt[a] + Sqrt[b]*x^2)) - (((252*c)/x^10 + (280*d)/x^9 + (315*e
)/x^8 + (360*f)/x^7)*(a + b*x^4)^(3/2))/2520 - (3*b^2*e*ArcTanh[S
qrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]
) + (2*b^(7/4)*(7*Sqrt[b]*d + 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**11,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.06158, size = 314, normalized size = 0.79

$$-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(a^2(504c+10x(56d+9x(7e+8fx)))+abx^4(1008c+x(1232d+45x(35e+48fx)))+168b^2x^8(3c+8dx)\right)\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(168*b^2*x^8*(3*c + 8*d
*x) + a^2*(504*c + 10*x*(56*d + 9*x*(7*e + 8*f*x))) + a*b*x^4*(10
08*c + x*(1232*d + 45*x*(35*e + 48*f*x)))) + 945*Sqrt[a]*b^2*e*x^
10*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 1344*Sqrt
[a]*b^(5/2)*d*x^10*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(
I*Sqrt[b])/Sqrt[a]]*x], -1] - (192*I)*Sqrt[a]*b^2*((-7*I)*Sqrt[b]
*d + 15*Sqrt[a]*f)*x^10*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[S
qrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(5040*a*Sqrt[(I*Sqrt[b])/Sqrt[a
]]*x^10*Sqrt[a + b*x^4])
```

Maple [C] time = 0.026, size = 417, normalized size = 1.1

$$
\begin{aligned}
&-\frac{c\left(b^2x^8+2\,abx^4+a^2\right)}{10\,x^{10}a}\sqrt{bx^4+a}-\frac{ad}{9\,x^9}\sqrt{bx^4+a}-\frac{11\,bd}{45\,x^5}\sqrt{bx^4+a}-\frac{4\,b^2d}{15\,ax}\sqrt{bx^4+a}\\
&+\frac{4\,i}{15}db^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{4\,i}{15}db^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{ae}{8\,x^8}\sqrt{bx^4+a}-\frac{5\,be}{16\,x^4}\sqrt{bx^4+a}\\
&-\frac{3\,b^2e}{16}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{af}{7\,x^7}\sqrt{bx^4+a}-\frac{3\,fb}{7\,x^3}\sqrt{bx^4+a}\\
&+\frac{4\,fb^2}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x)
```

```
[Out] -1/10*c*(b*x^4+a)^(1/2)/x^10/a*(b^2*x^8+2*a*b*x^4+a^2)-1/9*d*a*(b
*x^4+a)^(1/2)/x^9-11/45*d*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2*d*(b*x^4
+a)^(1/2)/a/x+4/15*I*d/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*d/
```

```
a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elliptic
E(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/8*e*a/x^8*(b*x^4+a)^(1/2)-5/16
*e*b/x^4*(b*x^4+a)^(1/2)-3/16*e/a^(1/2)*b^2*ln((2*a+2*a^(1/2)*(b*
x^4+a)^(1/2))/x^2)-1/7*f*a*(b*x^4+a)^(1/2)/x^7-3/7*f*b*(b*x^4+a)^
(1/2)/x^3+4/7*f*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elli
pticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bx^4+a\right)^{\frac{5}{2}}c}{10\,ax^{10}}+\int\frac{\left(bfx^6+bex^5+bdx^4+afx^2+aex+ad\right)\sqrt{bx^4+a}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*(b*x^4 + a)^(5/2)*c/(a*x^10) + integrate((b*f*x^6 + b*e*x^5
 + b*d*x^4 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^{11}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^11,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^11, x)
```

Sympy [A] time = 24.6385, size = 398, normalized size = 1.

$$\frac{a^{\frac{3}{2}}d\,\Gamma\left(-\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\,\Gamma\left(-\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}f\,\Gamma\left(-\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\,\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}bd\,\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\,\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}bf\,\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\,\Gamma\left(\frac{1}{4}\right)}-\frac{a^2e}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{10x^8}-\frac{3a\sqrt{b}e}{16x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{5x^4}-\frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{4x^2}-\frac{b^{\frac{3}{2}}e}{16x^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{5}{2}}c\sqrt{\frac{a}{bx^4}+1}}{10a}-\frac{3b^2e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**11,x)
```

```
[Out] a**(3/2)*d*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*x**9*gamma(-5/4)) + a**(3/2)*f*gamma(-7/4)*hyper(
```

```
(-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3
/4)) + sqrt(a)*b*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**
4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*f*gamma(-3/
4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*
gamma(1/4)) - a**2*e/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - a*s
qrt(b)*c*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*e/(16*x**6*
sqrt(a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(5*x**4)
- b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*e/(16*x**2*
sqrt(a/(b*x**4) + 1)) - b**(5/2)*c*sqrt(a/(b*x**4) + 1)/(10*a) -
3*b**2*e*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^11,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^11, x)
```

3.514 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=424

$$-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}c-77\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{4b^{5/2}ex\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{4b^2c\sqrt{a+bx^4}}{77ax^3}-\frac{b^2d\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2e\sqrt{a+bx^4}}{15ax}-\frac{3b^2f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}$$
$$-\frac{b\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)}{18480}-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

```
[Out] -(b*((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (3465*f)/x^4)*S
qrt[a + b*x^4])/18480 - (4*b^2*c*Sqrt[a + b*x^4])/(77*a*x^3) - (b
^2*d*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*e*Sqrt[a + b*x^4])/(15*
a*x) + (4*b^(5/2)*e*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x
^2)) - (((360*c)/x^11 + (396*d)/x^10 + (440*e)/x^9 + (495*f)/x^8)
*(a + b*x^4)^(3/2))/3960 - (3*b^2*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[
a]])/(16*Sqrt[a]) - (4*b^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) - (2*b^(9/4)*(15*S
qrt[b]*c - 77*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(1155*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.98442, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}c-77\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{4b^{5/2}ex\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{4b^2c\sqrt{a+bx^4}}{77ax^3}-\frac{b^2d\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2e\sqrt{a+bx^4}}{15ax}-\frac{3b^2f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}$$
$$-\frac{b\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)}{18480}-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

```
[Out] -(b*((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (3465*f)/x^4)*S
qrt[a + b*x^4])/18480 - (4*b^2*c*Sqrt[a + b*x^4])/(77*a*x^3) - (b
^2*d*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*e*Sqrt[a + b*x^4])/(15*
a*x) + (4*b^(5/2)*e*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x
^2)) - (((360*c)/x^11 + (396*d)/x^10 + (440*e)/x^9 + (495*f)/x^8)
*(a + b*x^4)^(3/2))/3960 - (3*b^2*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[
a]])/(16*Sqrt[a]) - (4*b^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) - (2*b^(9/4)*(15*S
qrt[b]*c - 77*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
```

```
)], 1/2])/(1155*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**12,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.994866, size = 317, normalized size = 0.75

$$-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(14a^2(360c+11x(36d+5x(8e+9fx)))+abx^4\left(9360c+77x\left(144d+176ex+225fx^2\right)\right)+24b^2x^8(120c+\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(24*b^2*x^8*(120*c + 77
*x*(3*d + 8*e*x)) + a*b*x^4*(9360*c + 77*x*(144*d + 176*e*x + 225
*f*x^2)) + 14*a^2*(360*c + 11*x*(36*d + 5*x*(8*e + 9*f*x)))) + 10
395*Sqrt[a]*b^2*f*x^11*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sq
rt[a]])) + 14784*Sqrt[a]*b^(5/2)*e*x^11*Sqrt[1 + (b*x^4)/a]*Ellip
ticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 192*b^(5/2)*((
-15*I)*Sqrt[b]*c + 77*Sqrt[a]*e)*x^11*Sqrt[1 + (b*x^4)/a]*Ellipti
cF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(55440*a*Sqrt[(I*
Sqrt[b])/Sqrt[a]]*x^11*Sqrt[a + b*x^4])
```

Maple [C] time = 0.031, size = 441, normalized size = 1.

$$\begin{aligned}
&-\frac{ac}{11\,x^{11}}\sqrt{bx^4+a}-\frac{13\,bc}{77\,x^7}\sqrt{bx^4+a}-\frac{4\,b^2c}{77\,ax^3}\sqrt{bx^4+a}\\
&-\frac{4\,b^3c}{77\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{d\left(b^2x^8+2\,abx^4+a^2\right)}{10\,x^{10}a}\sqrt{bx^4+a}-\frac{ae}{9\,x^9}\sqrt{bx^4+a}-\frac{11\,be}{45\,x^5}\sqrt{bx^4+a}-\frac{4\,b^2e}{15\,ax}\sqrt{bx^4+a}\\
&+\frac{4\,i}{15}eb^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{4\,i}{15}eb^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{af}{8\,x^8}\sqrt{bx^4+a}-\frac{5\,fb}{16\,x^4}\sqrt{bx^4+a}-\frac{3\,fb^2}{16}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x)
```

```
[Out] -1/11*c*a*(b*x^4+a)^(1/2)/x^11-13/77*c*b*(b*x^4+a)^(1/2)/x^7-4/77
*b^2*c*(b*x^4+a)^(1/2)/a/x^3-4/77*c/a*b^3/(I/a^(1/2)*b^(1/2))^(1/
```

```
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/10*d
*(b*x^4+a)^(1/2)/x^10/a*(b^2*x^8+2*a*b*x^4+a^2)-1/9*e*a*(b*x^4+a)
^(1/2)/x^9-11/45*e*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2*e*(b*x^4+a)^(1/
2)/a/x+4/15*I*e/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*e/a^(1/2)
*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)-1/8*f*a/x^8*(b*x^4+a)^(1/2)-5/16*f*b/x^
4*(b*x^4+a)^(1/2)-3/16*f/a^(1/2)*b^2*ln((2*a+2*a^(1/2)*(b*x^4+a)^
(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac)\sqrt{bx^4+a}}{x^{12}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^12, x)
```

Sympy [A] time = 27.6757, size = 401, normalized size = 0.95

$$\frac{a^{\frac{3}{2}}c\,\Gamma\left(-\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}e\,\Gamma\left(-\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}bc\,\Gamma\left(-\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}be\,\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{5}\Gamma\left(-\frac{1}{4}\right)}-\frac{a^2f}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{10x^8}-\frac{3a\sqrt{b}f}{16x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{5x^4}-\frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{4x^2}-\frac{b^{\frac{3}{2}}f}{16x^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{5}{2}}d\sqrt{\frac{a}{bx^4}+1}}{10a}-\frac{3b^2f\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**12,x)
```

```
[Out] a**(3/2)*c*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_
polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*e*gamma(-9/4)*hyp
er((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma
(-5/4)) + sqrt(a)*b*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*
x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*e*gamma(
-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x
**5*gamma(-1/4)) - a**2*f/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1))
- a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*f/(16*
x**6*sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(5*x
**4) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*f/(16*
x**2*sqrt(a/(b*x**4) + 1)) - b**(5/2)*d*sqrt(a/(b*x**4) + 1)/(10*
a) - 3*b**2*f*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

3.515 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=449

$$-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}d-77\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{b^3c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}+\frac{4b^{5/2}fx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b^2c\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2d\sqrt{a+bx^4}}{77ax^3}-\frac{b^2e\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2f\sqrt{a+bx^4}}{15ax}-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}-\frac{b\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)}{18480}$$

```
[Out] -(b*((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2464*f)/x^5)*S
qrt[a + b*x^4])/18480 - (b^2*c*Sqrt[a + b*x^4])/(32*a*x^4) - (4*b
^2*d*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*e*Sqrt[a + b*x^4])/(10*a*
x^2) - (4*b^2*f*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*f*x*Sqrt[a
 + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((165*c)/x^12 + (180
*d)/x^11 + (198*e)/x^10 + (220*f)/x^9)*(a + b*x^4)^(3/2))/1980 +
(b^3*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) - (4*b^(9/4
)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)
*Sqrt[a + b*x^4]) - (2*b^(9/4)*(15*Sqrt[b]*d - 77*Sqrt[a]*f)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*a^(5/4)*Sqrt[
a + b*x^4])
```

Rubi [A] time = 1.06846, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}d-77\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{b^3c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}+\frac{4b^{5/2}fx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b^2c\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2d\sqrt{a+bx^4}}{77ax^3}-\frac{b^2e\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2f\sqrt{a+bx^4}}{15ax}-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}-\frac{b\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)}{18480}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```


```
[Out] -(b*((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2464*f)/x^5)*S
qrt[a + b*x^4])/18480 - (b^2*c*Sqrt[a + b*x^4])/(32*a*x^4) - (4*b
^2*d*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*e*Sqrt[a + b*x^4])/(10*a*
x^2) - (4*b^2*f*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*f*x*Sqrt[a
 + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((165*c)/x^12 + (180
*d)/x^11 + (198*e)/x^10 + (220*f)/x^9)*(a + b*x^4)^(3/2))/1980 +
(b^3*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) - (4*b^(9/4
)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)
*Sqrt[a + b*x^4]) - (2*b^(9/4)*(15*Sqrt[b]*d - 77*Sqrt[a]*f)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
```

```
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*a^(5/4)*Sqrt[
a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**13,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.17393, size = 328, normalized size = 0.73

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(3465b^3cx^{12}\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\sqrt{a}\left(a+bx^4\right)\left(56a^2\left(165c+2x\left(90d+99ex+110fx^2\right)\right)+2abx^4(8085c+1\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-(Sqrt[a]*(a + b*x^4)*(56*a^2*(165*c
+ 2*x*(90*d + 99*e*x + 110*f*x^2)) + 3*b^2*x^8*(1155*c + 16*x*(12
0*d + 77*x*(3*e + 8*f*x))) + 2*a*b*x^4*(8085*c + 16*x*(585*d + 77
*x*(9*e + 11*f*x))))) + 3465*b^3*c*x^12*Sqrt[a + b*x^4]*ArcTanh[S
qrt[a + b*x^4]/Sqrt[a]]) + 29568*a*b^(5/2)*f*x^12*Sqrt[1 + (b*x^4
)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 384*
Sqrt[a]*b^(5/2)*((-15*I)*Sqrt[b]*d + 77*Sqrt[a]*f)*x^12*Sqrt[1 +
(b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])
/(110880*a^(3/2)*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^12*Sqrt[a + b*x^4])
```

Maple [C] time = 0.03, size = 462, normalized size = 1.

$$-\frac{ac}{12\,x^{12}}\sqrt{bx^4+a}-\frac{7\,bc}{48\,x^8}\sqrt{bx^4+a}-\frac{b^2c}{32\,ax^4}\sqrt{bx^4+a}+\frac{b^3c}{32}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}$$
$$-\frac{ad}{11\,x^{11}}\sqrt{bx^4+a}-\frac{13\,bd}{77\,x^7}\sqrt{bx^4+a}-\frac{4\,b^2d}{77\,ax^3}\sqrt{bx^4+a}$$
$$-\frac{4\,b^3d}{77\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{e\left(b^2x^8+2\,abx^4+a^2\right)}{10\,x^{10}a}\sqrt{bx^4+a}-\frac{af}{9\,x^9}\sqrt{bx^4+a}-\frac{11\,fb}{45\,x^5}\sqrt{bx^4+a}-\frac{4\,b^2f}{15\,ax}\sqrt{bx^4+a}$$
$$+\frac{4\,i}{15}fb^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{4\,i}{15}fb^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x)
```

```
[Out] -1/12*c*a/x^12*(b*x^4+a)^(1/2)-7/48*c*b/x^8*(b*x^4+a)^(1/2)-1/32*
b^2*c*(b*x^4+a)^(1/2)/a/x^4+1/32*c/a^(3/2)*b^3*ln((2*a+2*a^(1/2)*
```

```
(b*x^4+a)^(1/2))/x^2)-1/11*d*a*(b*x^4+a)^(1/2)/x^11-13/77*d*b*(b*
x^4+a)^(1/2)/x^7-4/77*b^2*d*(b*x^4+a)^(1/2)/a/x^3-4/77*d/a*b^3/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)-1/10*e*(b*x^4+a)^(1/2)/x^10/a*(b^2*x^8+2*a*b*x^4+a
^2)-1/9*f*a*(b*x^4+a)^(1/2)/x^9-11/45*f*b*(b*x^4+a)^(1/2)/x^5-4/1
5*b^2*f*(b*x^4+a)^(1/2)/a/x+4/15*I*f/a^(1/2)*b^(5/2)/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)-4/15*I*f/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^{13}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^13,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^13, x)
```

Sympy [A] time = 37.6741, size = 403, normalized size = 0.9

$$\frac{a^{\frac{3}{2}}d\,\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}f\,\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}bd\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}bf\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{5}\Gamma\left(-\frac{1}{4}\right)}-\frac{a^2c}{12\sqrt{b}x^{14}\sqrt{\frac{a}{bx^4}+1}}-\frac{11a\sqrt{b}c}{48x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{10x^8}-\frac{17b^{\frac{3}{2}}c}{96x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{5x^4}-\frac{b^{\frac{5}{2}}c}{32ax^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{5}{2}}e\sqrt{\frac{a}{bx^4}+1}}{10a}+\frac{b^3c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{32a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**13,x)
```

```
[Out] a**(3/2)*d*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_
polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*f*gamma(-9/4)*hyp
```

```
er((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma
(-5/4)) + sqrt(a)*b*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*
x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*f*gamma(
-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x
**5*gamma(-1/4)) - a**2*c/(12*sqrt(b)*x**14*sqrt(a/(b*x**4) + 1))
 - 11*a*sqrt(b)*c/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*e*s
qrt(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*c/(96*x**6*sqrt(a/(b*
x**4) + 1)) - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)
*c/(32*a*x**2*sqrt(a/(b*x**4) + 1)) - b**(5/2)*e*sqrt(a/(b*x**4)
+ 1)/(10*a) + b**3*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(32*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^13,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^13, x)
```

3.516 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=474

$$-\frac{2b^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(65\sqrt{a}e+77\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}+\frac{4b^{13/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65a^{7/4}\sqrt{a+bx^4}}+\frac{b^3d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}-\frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4b^3c\sqrt{a+bx^4}}{65a^2x}-\frac{4b^2c\sqrt{a+bx^4}}{195ax^5}-\frac{b^2d\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2e\sqrt{a+bx^4}}{77ax^3}-\frac{b^2f\sqrt{a+bx^4}}{10ax^2}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}-\frac{b\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)}{240240}$$

```
[Out] -(b*((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7 + (24024*f)/x^
6)*Sqrt[a + b*x^4])/240240 - (4*b^2*c*Sqrt[a + b*x^4])/(195*a*x^5
) - (b^2*d*Sqrt[a + b*x^4])/(32*a*x^4) - (4*b^2*e*Sqrt[a + b*x^4]
)/(77*a*x^3) - (b^2*f*Sqrt[a + b*x^4])/(10*a*x^2) + (4*b^3*c*Sqrt
[a + b*x^4])/(65*a^2*x) - (4*b^(7/2)*c*x*Sqrt[a + b*x^4])/(65*a^2
*(Sqrt[a] + Sqrt[b]*x^2)) - (((660*c)/x^13 + (715*d)/x^12 + (780*
e)/x^11 + (858*f)/x^10)*(a + b*x^4)^(3/2))/8580 + (b^3*d*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) + (4*b^(13/4)*c*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*a^(7/4)*Sqrt[a + b*x^
4]) - (2*b^(11/4)*(77*Sqrt[b]*c + 65*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b
]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*a^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 1.20802, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$-\frac{2b^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(65\sqrt{a}e+77\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}+\frac{4b^{13/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65a^{7/4}\sqrt{a+bx^4}}+\frac{b^3d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}-\frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4b^3c\sqrt{a+bx^4}}{65a^2x}-\frac{4b^2c\sqrt{a+bx^4}}{195ax^5}-\frac{b^2d\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2e\sqrt{a+bx^4}}{77ax^3}-\frac{b^2f\sqrt{a+bx^4}}{10ax^2}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}-\frac{b\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)}{240240}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```


```
[Out] -(b*((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7 + (24024*f)/x^
6)*Sqrt[a + b*x^4])/240240 - (4*b^2*c*Sqrt[a + b*x^4])/(195*a*x^5
) - (b^2*d*Sqrt[a + b*x^4])/(32*a*x^4) - (4*b^2*e*Sqrt[a + b*x^4]
)/(77*a*x^3) - (b^2*f*Sqrt[a + b*x^4])/(10*a*x^2) + (4*b^3*c*Sqrt
[a + b*x^4])/(65*a^2*x) - (4*b^(7/2)*c*x*Sqrt[a + b*x^4])/(65*a^2
*(Sqrt[a] + Sqrt[b]*x^2)) - (((660*c)/x^13 + (715*d)/x^12 + (780*
```

```
e)/x^11 + (858*f)/x^10)*(a + b*x^4)^(3/2))/8580 + (b^3*d*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) + (4*b^(13/4)*c*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*a^(7/4)*Sqrt[a + b*x^
4]) - (2*b^(11/4)*(77*Sqrt[b]*c + 65*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b
]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*a^(7/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**14,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.09918, size = 339, normalized size = 0.72

$$\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(15015\sqrt{a}b^3dx^{13}\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\left(a+bx^4\right)\left(56a^3\left(660c+13x\left(55d+60ex+66fx^2\right)\right)+2a^2bx^4(30800c\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(-29568*b^3*c*x^12 + 56
*a^3*(660*c + 13*x*(55*d + 60*e*x + 66*f*x^2)) + a*b^2*x^8*(9856*
c + 39*x*(385*d + 16*x*(40*e + 77*f*x))) + 2*a^2*b*x^4*(30800*c +
 13*x*(2695*d + 48*x*(65*e + 77*f*x))))) + 15015*Sqrt[a]*b^3*d*x^
13*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) - 29568*Sqrt
[a]*b^(7/2)*c*x^13*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(
I*Sqrt[b])/Sqrt[a]]*x], -1] + 384*Sqrt[a]*b^3*(77*Sqrt[b]*c + (65
*I)*Sqrt[a]*e)*x^13*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[
(I*Sqrt[b])/Sqrt[a]]*x], -1])/(480480*a^2*Sqrt[(I*Sqrt[b])/Sqrt[a
]]*x^13*Sqrt[a + b*x^4])
```

Maple [C] time = 0.034, size = 483, normalized size = 1.

$$-\frac{ac}{13x^{13}}\sqrt{bx^4+a}-\frac{5bc}{39x^9}\sqrt{bx^4+a}-\frac{4b^2c}{195ax^5}\sqrt{bx^4+a}+\frac{4b^3c}{65a^2x}\sqrt{bx^4+a}$$
$$-\frac{4i}{65}cb^{\frac{7}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{4i}{65}cb^{\frac{7}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{ad}{12x^{12}}\sqrt{bx^4+a}-\frac{7bd}{48x^8}\sqrt{bx^4+a}-\frac{b^2d}{32ax^4}\sqrt{bx^4+a}$$
$$+\frac{b^3d}{32}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}-\frac{ae}{11x^{11}}\sqrt{bx^4+a}-\frac{13be}{77x^7}\sqrt{bx^4+a}-\frac{4b^2e}{77ax^3}\sqrt{bx^4+a}$$
$$-\frac{4eb^3}{77a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{f\left(b^2x^8+2abx^4+a^2\right)}{10x^{10}a}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x)
```

```
[Out] -1/13*c*a*(b*x^4+a)^(1/2)/x^13-5/39*c*b*(b*x^4+a)^(1/2)/x^9-4/195
*b^2*c*(b*x^4+a)^(1/2)/a/x^5+4/65*b^3*c*(b*x^4+a)^(1/2)/a^2/x-4/6
5*I*c*b^(7/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*El
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4/65*I*c*b^(7/2)/a^(3/2)/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^
(1/2))^(1/2),I)-1/12*d*a/x^12*(b*x^4+a)^(1/2)-7/48*d*b/x^8*(b*x^4
+a)^(1/2)-1/32*b^2*d*(b*x^4+a)^(1/2)/a/x^4+1/32*d/a^(3/2)*b^3*ln(
(2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/11*e*a*(b*x^4+a)^(1/2)/x^1
1-13/77*e*b*(b*x^4+a)^(1/2)/x^7-4/77*b^2*e*(b*x^4+a)^(1/2)/a/x^3-
4/77*e/a*b^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/10*f*(b*x^4+a)^(1/2)/x^10/a*(b^2*
x^8+2*a*b*x^4+a^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac)\sqrt{bx^4+a}}{x^{14}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x
^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^14, x)
```

Sympy [A] time = 42.4552, size = 403, normalized size = 0.85

$$\frac{a^{\frac{3}{2}}c\,\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{13}{4},-\frac{1}{2}\\-\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{13}\Gamma\left(-\frac{9}{4}\right)}+\frac{a^{\frac{3}{2}}e\,\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}+\frac{\sqrt{a}bc\,\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}be\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)}-\frac{a^2d}{12\sqrt{b}x^{14}\sqrt{\frac{a}{bx^4}+1}}-\frac{11a\sqrt{b}d}{48x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{10x^8}-\frac{17b^{\frac{3}{2}}d}{96x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{5x^4}-\frac{b^{\frac{5}{2}}d}{32ax^2\sqrt{\frac{a}{bx^4}+1}}-\frac{b^{\frac{5}{2}}f\sqrt{\frac{a}{bx^4}+1}}{10a}+\frac{b^3d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{32a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**14,x)
```

```
[Out] a**(3/2)*c*gamma(-13/4)*hyper((-13/4, -1/2), (-9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*x**13*gamma(-9/4)) + a**(3/2)*e*gamma(-11/4)*hy
per((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**11*ga
mma(-7/4)) + sqrt(a)*b*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*e*gam
ma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(
4*x**7*gamma(-3/4)) - a**2*d/(12*sqrt(b)*x**14*sqrt(a/(b*x**4) +
1)) - 11*a*sqrt(b)*d/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*
f*sqrt(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*d/(96*x**6*sqrt(a/
(b*x**4) + 1)) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5
/2)*d/(32*a*x**2*sqrt(a/(b*x**4) + 1)) - b**(5/2)*f*sqrt(a/(b*x**
4) + 1)/(10*a) + b**3*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(32*a**(3/2
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

3.517 $\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=361

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ad\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt{a+bx^4}\left(4af-3bdx^2\right)}{12b^2}+\frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}$$

```
[Out] (c*x*Sqrt[a + b*x^4])/(3*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*
x^4*Sqrt[a + b*x^4])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)
*(Sqrt[a] + Sqrt[b]*x^2)) - ((4*a*f - 3*b*d*x^2)*Sqrt[a + b*x^4])
/(12*b^2) - (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/
2)) + (3*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*c + 9*Sqrt[
a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]
*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/
4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.810429, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ad\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt{a+bx^4}\left(4af-3bdx^2\right)}{12b^2}+\frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```


```
[Out] (c*x*Sqrt[a + b*x^4])/(3*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*
x^4*Sqrt[a + b*x^4])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)
*(Sqrt[a] + Sqrt[b]*x^2)) - ((4*a*f - 3*b*d*x^2)*Sqrt[a + b*x^4])
/(12*b^2) - (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/
2)) + (3*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*c + 9*Sqrt[
a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]
*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/
4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 85.7179, size = 330, normalized size = 0.91

$$\frac{3a^{\frac{5}{4}}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{a^{\frac{3}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(9\sqrt{a}e+5\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{30b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{ad\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{\frac{3}{2}}} - \frac{3aex\sqrt{a+bx^4}}{5b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{cx\sqrt{a+bx^4}}{3b} + \frac{ex^3\sqrt{a+bx^4}}{5b} + \frac{fx^4\sqrt{a+bx^4}}{6b} - \frac{\sqrt{a+bx^4}\left(4af-3bdx^2\right)}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] 3*a**(5/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt
(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/
(5*b**(7/4)*sqrt(a + b*x**4)) - a**(3/4)*sqrt((a + b*x**4)/(sqrt(
a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(9*sqrt(a)*e + 5*
sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(30*b**(7
/4)*sqrt(a + b*x**4)) - a*d*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/
(4*b**(3/2)) - 3*a*e*x*sqrt(a + b*x**4)/(5*b**(3/2)*(sqrt(a) + sq
rt(b)*x**2)) + c*x*sqrt(a + b*x**4)/(3*b) + e*x**3*sqrt(a + b*x**
4)/(5*b) + f*x**4*sqrt(a + b*x**4)/(6*b) - sqrt(a + b*x**4)*(4*a*
f - 3*b*d*x**2)/(12*b**2)
```

Mathematica [C] time = 1.01655, size = 259, normalized size = 0.72

$$\frac{-36a^{3/2}\sqrt{b}e\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left(a+bx^4\right)\left(20af-bx(20c+x(15d+2x(6e+5fx)))\right)-15a\sqrt{b}\right.}{60b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(20*a*f - b*x*(20*c + x
*(15*d + 2*x*(6*e + 5*f*x))))) - 15*a*Sqrt[b]*d*Sqrt[a + b*x^4]*A
rcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) - 36*a^(3/2)*Sqrt[b]*e*Sqr
t[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x]
, -1] + 4*a*Sqrt[b]*((5*I)*Sqrt[b]*c + 9*Sqrt[a]*e)*Sqrt[1 + (b*x
^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(60
*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.022, size = 335, normalized size = 0.9

$$\frac{cx}{3\,b}\sqrt{bx^4+a}-\frac{ac}{3\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} + \frac{dx^2}{4\,b}\sqrt{bx^4+a}-\frac{ad}{4}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{ex^3}{5\,b}\sqrt{bx^4+a} - \frac{3\,i}{5}ea^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} + \frac{3\,i}{5}ea^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} - \frac{f\left(-bx^4+2\,a\right)}{6\,b^2}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/3*c*x*(b*x^4+a)^(1/2)/b-1/3*c*a/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*d*x^2/b*
(b*x^4+a)^(1/2)-1/4*d*a/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))+1
/5*e*x^3*(b*x^4+a)^(1/2)/b-3/5*I*e*a^(3/2)/b^(3/2)/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)+3/5*I*e*a^(3/2)/b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/6*f*(b*x^4+a)^(1/2)
*(-b*x^4+2*a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^4}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^7 + ex^6 + dx^5 + cx^4}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x^7 + e*x^6 + d*x^5 + c*x^4)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 8.13287, size = 177, normalized size = 0.49

$$\frac{\sqrt{a}dx^2\sqrt{1 + \frac{bx^4}{a}}}{4b} - \frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f\left(\begin{cases} -\frac{a\sqrt{a+bx^4}}{3b^2} + \frac{x^4\sqrt{a+bx^4}}{6b} & \text{for}\ b \neq 0 \\ \frac{x^8}{8\sqrt{a}} & \text{otherwise} \end{cases}\right) + \frac{cx^5\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)} + \frac{ex^7\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*d*asinh(sqrt(b)*x**2/
sqrt(a))/(4*b**(3/2)) + f*Piecewise((-a*sqrt(a + b*x**4)/(3*b**2)
 + x**4*sqrt(a + b*x**4)/(6*b), Ne(b, 0)), (x**8/(8*sqrt(a)), Tru
e)) + c*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e*x**7*gamma(7/4)*hyper((1/2,
 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3 + ex^2 + dx + c\right)x^4}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

3.518 $\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=336

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ae\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\sqrt{a+bx^4}\left(2c+ex^2\right)}{4b}+\frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}$$

```
[Out] (d*x*Sqrt[a + b*x^4])/(3*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*
a*f*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*
c + e*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4
)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.707819, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ae\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\sqrt{a+bx^4}\left(2c+ex^2\right)}{4b}+\frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```



```
[Out] (d*x*Sqrt[a + b*x^4])/(3*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*
a*f*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*
c + e*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4
)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 79.0022, size = 304, normalized size = 0.9

$$\frac{3a^{\frac{5}{4}}f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{a^{\frac{3}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(9\sqrt{a}f+5\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{30b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{ae\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{\frac{3}{2}}} - \frac{3afx\sqrt{a+bx^4}}{5b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{dx\sqrt{a+bx^4}}{3b} + \frac{fx^3\sqrt{a+bx^4}}{5b} + \frac{\sqrt{a+bx^4}\left(2c+ex^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] 3*a**(5/4)*f*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt
(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/
(5*b**(7/4)*sqrt(a + b*x**4)) - a**(3/4)*sqrt((a + b*x**4)/(sqrt(
a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(9*sqrt(a)*f + 5*
sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(30*b**(7
/4)*sqrt(a + b*x**4)) - a*e*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/
(4*b**(3/2)) - 3*a*f*x*sqrt(a + b*x**4)/(5*b**(3/2)*(sqrt(a) + sq
rt(b)*x**2)) + d*x*sqrt(a + b*x**4)/(3*b) + f*x**3*sqrt(a + b*x**
4)/(5*b) + sqrt(a + b*x**4)*(2*c + e*x**2)/(4*b)
```

Mathematica [C] time = 0.877423, size = 241, normalized size = 0.72

$$\frac{-36a^{3/2}f\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\left(a+bx^4\right)\left(30c+x(20d+3x(5e+4fx))\right)-15ae\sqrt{a+bx^4}\tanh^{-1}\right.}{60b^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[b]*(a + b*x^4)*(30*c + x*(20*d +
 3*x*(5*e + 4*f*x))) - 15*a*e*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^
2)/Sqrt[a + b*x^4]]) - 36*a^(3/2)*f*Sqrt[1 + (b*x^4)/a]*EllipticE
[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + 4*a*((5*I)*Sqrt[b]
*d + 9*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[b])/Sqrt[a]]*x], -1])/(60*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^(3/2)
*Sqrt[a + b*x^4])
```

Maple [C] time = 0.01, size = 325, normalized size = 1.

$$\frac{dx}{3\,b}\sqrt{bx^4+a}-\frac{ad}{3\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} + \frac{c}{2\,b}\sqrt{bx^4+a}+\frac{ex^2}{4\,b}\sqrt{bx^4+a}-\frac{ae}{4}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{fx^3}{5\,b}\sqrt{bx^4+a} - \frac{3\,i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} + \frac{3\,i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/3*d*x*(b*x^4+a)^(1/2)/b-1/3*d*a/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*c/b*(b*x
^4+a)^(1/2)+1/4*e*x^2/b*(b*x^4+a)^(1/2)-1/4*e*a/b^(3/2)*ln(b^(1/2
)*x^2+(b*x^4+a)^(1/2))+1/5*f*x^3*(b*x^4+a)^(1/2)/b-3/5*I*f*a^(3/2
)/b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)+3/5*I*f*a^(3/2)/b^(3/2)/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{bx^4+ac}}{2\,b}+\int\frac{fx^6+ex^5+dx^4}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/sqrt(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)/sqrt(
b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^6+ex^5+dx^4+cx^3}{\sqrt{bx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^3/sqrt(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 6.90931, size = 156, normalized size = 0.46

$$\frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4b}-\frac{ae\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{dx^5\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{fx^7\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*e*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*e*asinh(sqrt(b)*x**2/
sqrt(a))/(4*b**(3/2)) + c*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)),
 (sqrt(a + b*x**4)/(2*b), True)) + d*x**5*gamma(5/4)*hyper((1/2,
5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) +
f*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*p
i)/a)/(4*sqrt(a)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^3}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/sqrt(b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/sqrt(b*x^4 + a), x)
```

3.519 $\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=308

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}c-\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} - \frac{af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} + \frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{\sqrt{a+bx^4}\left(2d+fx^2\right)}{4b} + \frac{ex\sqrt{a+bx^4}}{3b}$$

```
[Out] (e*x*Sqrt[a + b*x^4])/(3*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqr
t[a] + Sqrt[b]*x^2)) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/(4*b) - (a
*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)
*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqr
t[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.58183, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}c-\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} - \frac{af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} + \frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{\sqrt{a+bx^4}\left(2d+fx^2\right)}{4b} + \frac{ex\sqrt{a+bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```


```
[Out] (e*x*Sqrt[a + b*x^4])/(3*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqr
t[a] + Sqrt[b]*x^2)) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/(4*b) - (a
*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)
*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqr
t[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 61.0822, size = 275, normalized size = 0.89

$$-\frac{\sqrt[4]{a}c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}e-3\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{6b^{\frac{5}{4}}\sqrt{a+bx^4}}-\frac{af\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{\frac{3}{2}}}+\frac{ex\sqrt{a+bx^4}}{3b}+\frac{\sqrt{a+bx^4}\left(2d+fx^2\right)}{4b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] -a**(1/4)*c*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(
a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(
b**(3/4)*sqrt(a + b*x**4)) - a**(1/4)*sqrt((a + b*x**4)/(sqrt(a)
+ sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*e - 3*sqrt(
b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(6*b**(5/4)*sq
rt(a + b*x**4)) - a*f*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(4*b**
(3/2)) + e*x*sqrt(a + b*x**4)/(3*b) + sqrt(a + b*x**4)*(2*d + f*x
**2)/(4*b) + c*x*sqrt(a + b*x**4)/(sqrt(b)*(sqrt(a) + sqrt(b)*x**
2))
```

Mathematica [C] time = 0.658796, size = 245, normalized size = 0.8

$$\frac{4i\sqrt{a}\sqrt{b}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}e+3i\sqrt{b}c\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+12\sqrt{a}bc\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\right.}{12b^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[b]*(6*d + 4*e*x + 3*f*x^2)*(a +
b*x^4) - 3*a*f*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x
^4]]) + 12*Sqrt[a]*b*c*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sq
rt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + (4*I)*Sqrt[a]*Sqrt[b]*((3*I)*Sq
rt[b]*c + Sqrt[a]*e)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt
[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(12*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^(3
/2)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.01, size = 248, normalized size = 0.8

$$\frac{d}{2\,b}\sqrt{bx^4+a}+ic\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}+\frac{ex}{3\,b}\sqrt{bx^4+a}-\frac{ae}{3\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{fx^2}{4\,b}\sqrt{bx^4+a}-\frac{af}{4}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/2*d/b*(b*x^4+a)^(1/2)+I*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ell
ipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/3*e*x*(b*x^4+a)^(1/2)/b-
1/3*e*a/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)+1/4*f*x^2/b*(b*x^4+a)^(1/2)-1/4*f*a/b^
(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3 + ex^2 + dx + c\right)x^2}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^5 + ex^4 + dx^3 + cx^2}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x^5 + e*x^4 + d*x^3 + c*x^2)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 6.53185, size = 156, normalized size = 0.51

$$\frac{\sqrt{a}fx^2\sqrt{1 + \frac{bx^4}{a}}}{4b} - \frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + d\left(\begin{cases} \frac{x^4}{4\sqrt{a}} & \text{for}\ b = 0 \\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise} \end{cases}\right) + \frac{cx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*f*asinh(sqrt(b)*x**2/
sqrt(a))/(4*b**(3/2)) + d*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)),
 (sqrt(a + b*x**4)/(2*b), True)) + c*x**3*gamma(3/4)*hyper((1/2,
3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) +
e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi
)/a)/(4*sqrt(a)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3 + ex^2 + dx + c\right)x^2}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

3.520 $\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=299

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}$$

```
[Out] (e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sq
rt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4])
+ (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.496377, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```


```
[Out] (e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sq
rt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4])
+ (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 56.105, size = 267, normalized size = 0.89

$$-\frac{\sqrt[4]{a}d\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}f-3\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{6b^{\frac{5}{4}}\sqrt{a+bx^4}}+\frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{c\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] -a**(1/4)*d*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(
a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(
b**(3/4)*sqrt(a + b*x**4)) - a**(1/4)*sqrt((a + b*x**4)/(sqrt(a)
+ sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*f - 3*sqrt(
b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(6*b**(5/4)*sq
rt(a + b*x**4)) + e*sqrt(a + b*x**4)/(2*b) + f*x*sqrt(a + b*x**4)
/(3*b) + c*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(2*sqrt(b)) + d*x
*sqrt(a + b*x**4)/(sqrt(b)*(sqrt(a) + sqrt(b)*x**2))
```

Mathematica [C] time = 0.512844, size = 235, normalized size = 0.79

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(3\sqrt{b}c\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\left(a+bx^4\right)(3e+2fx)\right)+2i\sqrt{a}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}f+3i\sqrt{b}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-\right.}{6b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*((3*e + 2*f*x)*(a + b*x^4) + 3*Sqrt[b]
*c*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) + 6*Sq
rt[a]*Sqrt[b]*d*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*S
qrt[b])/Sqrt[a]]*x], -1] + (2*I)*Sqrt[a]*((3*I)*Sqrt[b]*d + Sqrt[
a]*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sq
rt[a]]*x], -1])/(6*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.008, size = 229, normalized size = 0.8

$$id\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}+\frac{c}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}+\frac{e}{2b}\sqrt{bx^4+a}+\frac{fx}{3b}\sqrt{bx^4+a}-\frac{af}{3b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] I*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1
/2))^(1/2),I))+1/2*c*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+1/2*
e*(b*x^4+a)^(1/2)/b+1/3*f*x*(b*x^4+a)^(1/2)/b-1/3*f*a/b/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x/sqrt(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^4 + ex^3 + dx^2 + cx}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x/sqrt(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x^4 + e*x^3 + d*x^2 + c*x)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 4.43123, size = 129, normalized size = 0.43

$$e\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for}\ b = 0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{dx^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b)
, True)) + c*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + d*x**3*gam
ma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(7/4)) + f*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b
*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3 + ex^2 + dx + c\right)x}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x/sqrt(b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x/sqrt(b*x^4 + a), x)
```

3.521 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=276

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\sqrt{a+bx^4}}{2b}$$

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[
a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(
2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)
], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a
] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/
4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.353414, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\sqrt{a+bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```


```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[
a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(
2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)
], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a
] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/
4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 41.2453, size = 248, normalized size = 0.9

$$-\frac{\sqrt[4]{a}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{2b}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}e+\sqrt{b}c\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] -a**(1/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(
a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(
b**(3/4)*sqrt(a + b*x**4)) + f*sqrt(a + b*x**4)/(2*b) + d*atanh(s
qrt(b)*x**2/sqrt(a + b*x**4))/(2*sqrt(b)) + e*x*sqrt(a + b*x**4)/
(sqrt(b)*(sqrt(a) + sqrt(b)*x**2)) + sqrt((a + b*x**4)/(sqrt(a) +
 sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*e + sqrt(b)*
c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*b**(3
/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.415079, size = 225, normalized size = 0.82

$$\frac{-2\sqrt{b}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}e+i\sqrt{b}c\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}d\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+af+bfx^4\right)+2\sqrt{a}\sqrt{b}}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(a*f + b*f*x^4 + Sqrt[b]*d*Sqrt[a + b*
x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) + 2*Sqrt[a]*Sqrt[b]*
e*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a
]]*x], -1] - 2*Sqrt[b]*(I*Sqrt[b]*c + Sqrt[a]*e)*Sqrt[1 + (b*x^4)
/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(2*Sqr
t[(I*Sqrt[b])/Sqrt[a]]*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.007, size = 208, normalized size = 0.8

$$c\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{d}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}$$
$$+ie\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$+\frac{f}{2b}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+1/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+
I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1
/2))^(1/2),I))+1/2*f*(b*x^4+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 3.68323, size = 128, normalized size = 0.46

$$f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b = 0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{d\,\text{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b)
, True)) + d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(
1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

3.522 $\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=285

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[Out] (f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt
[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1
/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.376243, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

```
[Out] (f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt
[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1
/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 50.6012, size = 258, normalized size = 0.91

$$-\frac{\sqrt[4]{a}f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{e\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{c\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}f+\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(1/2),x)
```

```
[Out] -a**(1/4)*f*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(
a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(
b**(3/4)*sqrt(a + b*x**4)) + e*atanh(sqrt(b)*x**2/sqrt(a + b*x**4
))/(2*sqrt(b)) + f*x*sqrt(a + b*x**4)/(sqrt(b)*(sqrt(a) + sqrt(b)
*x**2)) - c*atanh(sqrt(a + b*x**4)/sqrt(a))/(2*sqrt(a)) + sqrt((a
 + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*
(sqrt(a)*f + sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1
/2)/(2*a**(1/4)*b**(3/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.918944, size = 235, normalized size = 0.82

$$-\frac{i\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}\left(\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)-2\sqrt{a}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}f+i\sqrt{b}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i}{\cdot}}\right.\right.\cdots}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

```
[Out] ((-I/2)*Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt
[a + b*x^4]*(Sqrt[a]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - S
qrt[b]*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 2*a*f*Sqrt[1 + (b*x^
4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 2*S
qrt[a]*(I*Sqrt[b]*d + Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*
ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/(b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.01, size = 222, normalized size = 0.8

$$d\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+if\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$-\frac{c}{2}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{e}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x)
```

```
[Out] d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elliptic
E(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*c/a^(1/2)*ln((2*a+2*a^(1/2)
*(b*x^4+a)^(1/2))/x^2)+1/2*e*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

Sympy [A] time = 4.56604, size = 126, normalized size = 0.44

$$\frac{e \operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} - \frac{c \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{dx\,\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} + \frac{fx^3\,\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(1/2),x)
```

```
[Out] e*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - c*asinh(sqrt(a)/(sqrt
(b)*x**2))/(2*sqrt(a)) + d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + f*x**3*gamma(
3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

3.523 $\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=309

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

```
[Out] -((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(
Sqrt[a] + Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4
]])/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]
) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a +
 b*x^4])
```

Rubi [A] time = 0.564272, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```



```
[Out] -((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(
Sqrt[a] + Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4
]])/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]
) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a +
 b*x^4])
```

Rubi in Sympy [A] time = 62.9062, size = 275, normalized size = 0.89

$$\frac{f\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{c\sqrt{a+bx^4}}{ax}-\frac{d\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$
$$-\frac{\sqrt[4]{b}c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{3}{4}}\sqrt{a+bx^4}}$$
$$+\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}e+\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(1/2),x)
```

```
[Out] f*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(2*sqrt(b)) + sqrt(b)*c*x*
sqrt(a + b*x**4)/(a*(sqrt(a) + sqrt(b)*x**2)) - c*sqrt(a + b*x**4
)/(a*x) - d*atanh(sqrt(a + b*x**4)/sqrt(a))/(2*sqrt(a)) - b**(1/4
)*c*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqr
t(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(a**(3/4)
*sqrt(a + b*x**4)) + sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**
2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*e + sqrt(b)*c)*elliptic_f(2*
atan(b**(1/4)*x/a**(1/4)), 1/2)/(2*a**(3/4)*b**(1/4)*sqrt(a + b*x
**4))
```

Mathematica [C] time = 3.77512, size = 250, normalized size = 0.81

$$\frac{1}{2}\left(-\frac{2c\sqrt{a+bx^4}}{ax}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{\sqrt{b}}\right)$$
$$-\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}e-i\sqrt{b}c\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{a+bx^4}}$$
$$-\frac{ic\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

```
[Out] ((-2*c*Sqrt[a + b*x^4])/(a*x) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
 b*x^4]])/Sqrt[b] - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/Sqrt[a])
/2 - (I*Sqrt[(I*Sqrt[b])/Sqrt[a]]*c*Sqrt[1 + (b*x^4)/a]*EllipticE
[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/Sqrt[a + b*x^4] - (
Sqrt[(I*Sqrt[b])/Sqrt[a]]*((-I)*Sqrt[b]*c + Sqrt[a]*e)*Sqrt[1 + (
b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/
(Sqrt[b]*Sqrt[a + b*x^4])
```

Maple [C] time = 0.012, size = 299, normalized size = 1.

$$e\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{f}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}-\frac{c}{ax}\sqrt{bx^4+a}$$
$$+ic\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-ic\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{d}{2}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x)
```

```
[Out] e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+1/2*f*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)-
c*(b*x^4+a)^(1/2)/a/x+I*c*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-I*c*b^
(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d/a^(1/2)*ln((2*a+2*a^(1/2)*(b
*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Sympy [A] time = 4.66349, size = 128, normalized size = 0.41

$$\frac{f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{c\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\,\Gamma\left(\frac{3}{4}\right)}-\frac{d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}}+\frac{ex\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(1/2),x)
```

```
[Out] f*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*gamma(-1/4)*hyper((
-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(
3/4)) - d*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + e*x*gamma(1
/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

3.524 $\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=300

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -(c*Sqrt[a + b*x^4])/(2*a*x^2) - (d*Sqrt[a + b*x^4])/(a*x) + (Sqr
t[b]*d*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) +
 ((Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.547339, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] -(c*Sqrt[a + b*x^4])/(2*a*x^2) - (d*Sqrt[a + b*x^4])/(a*x) + (Sqr
t[b]*d*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) +
 ((Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 60.4383, size = 265, normalized size = 0.88

$$\frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}-\frac{e\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$
$$-\frac{\sqrt[4]{b}d\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{3}{4}}\sqrt{a+bx^4}}$$
$$+\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}f+\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(b)*d*x*sqrt(a + b*x**4)/(a*(sqrt(a) + sqrt(b)*x**2)) - c*sqr
t(a + b*x**4)/(2*a*x**2) - d*sqrt(a + b*x**4)/(a*x) - e*atanh(sqr
t(a + b*x**4)/sqrt(a))/(2*sqrt(a)) - b**(1/4)*d*sqrt((a + b*x**4)
/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_e
(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(a**(3/4)*sqrt(a + b*x**4)) +
sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)
*x**2)*(sqrt(a)*f + sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1
/4)), 1/2)/(2*a**(3/4)*b**(1/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.760966, size = 242, normalized size = 0.81

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)(c+2dx)+\sqrt{a}ex^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)-2i\sqrt{a}x^2\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}f-i\sqrt{b}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\right.}{2ax^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((c + 2*d*x)*(a + b*x^4) + Sqrt[a]*e
*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 2*Sqrt[
a]*Sqrt[b]*d*x^2*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*
Sqrt[b])/Sqrt[a]]*x], -1] - (2*I)*Sqrt[a]*((-I)*Sqrt[b]*d + Sqrt[
a]*f)*x^2*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]
)/Sqrt[a]]*x], -1])/(2*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^2*Sqrt[a + b
*x^4])
```

Maple [C] time = 0.021, size = 293, normalized size = 1.

$$f\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{c}{2\,ax^2}\sqrt{bx^4+a}-\frac{d}{ax}\sqrt{bx^4+a}$$
$$+id\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-id\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{e}{2}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x)
```

```
[Out] f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)-1/2*c*(b*x^4+a)^(1/2)/a/x^2-d*(b*x^4+a)^(1/2)/
a/x+I*d*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*
EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-I*d*b^(1/2)/a^(1/2)/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/
2))^(1/2),I)-1/2*e/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

Sympy [A] time = 4.40882, size = 126, normalized size = 0.42

$$-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\,\Gamma\left(\frac{3}{4}\right)} - \frac{e\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{fx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(2*a) + d*gamma(-1/4)*hyper((-1/4
, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)
) - e*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + f*x*gamma(1/4)*
hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*ga
mma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

3.525 $\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=323

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-3\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -(c*Sqrt[a + b*x^4])/(3*a*x^3) - (d*Sqrt[a + b*x^4])/(2*a*x^2) -
(e*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqr
t[a] + Sqrt[b]*x^2)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sq
rt[a]) - (b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*c - 3*Sqrt[a]*
e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*S
qrt[a + b*x^4])
```

Rubi [A] time = 0.654556, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-3\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] -(c*Sqrt[a + b*x^4])/(3*a*x^3) - (d*Sqrt[a + b*x^4])/(2*a*x^2) -
(e*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqr
t[a] + Sqrt[b]*x^2)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sq
rt[a]) - (b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*c - 3*Sqrt[a]*
e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*S
qrt[a + b*x^4])
```

Rubi in Sympy [A] time = 74.6426, size = 286, normalized size = 0.89

$$\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}-\frac{f\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{b}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{\sqrt[4]{b}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{a}e-\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{6a^{\frac{5}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(b)*e*x*sqrt(a + b*x**4)/(a*(sqrt(a) + sqrt(b)*x**2)) - c*sqr
t(a + b*x**4)/(3*a*x**3) - d*sqrt(a + b*x**4)/(2*a*x**2) - e*sqrt
(a + b*x**4)/(a*x) - f*atanh(sqrt(a + b*x**4)/sqrt(a))/(2*sqrt(a)
) - b**(1/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sq
rt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2
)/(a**(3/4)*sqrt(a + b*x**4)) + b**(1/4)*sqrt((a + b*x**4)/(sqrt(
a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(3*sqrt(a)*e - sq
rt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(6*a**(5/4)
*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.65337, size = 249, normalized size = 0.77

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(2c+3x(d+2ex)\right)+3\sqrt{a}fx^3\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)-2\sqrt{b}x^3\sqrt{\frac{bx^4}{a}+1}\left(3\sqrt{a}e-i\sqrt{b}c\right)F\left(i\sinh^{-1}\right.}{6ax^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*((a + b*x^4)*(2*c + 3*x*(d + 2*e*x))
 + 3*Sqrt[a]*f*x^3*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a
]])) + 6*Sqrt[a]*Sqrt[b]*e*x^3*Sqrt[1 + (b*x^4)/a]*EllipticE[I*Ar
cSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 2*Sqrt[b]*((-I)*Sqrt[b]
*c + 3*Sqrt[a]*e)*x^3*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqr
t[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(6*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^
3*Sqrt[a + b*x^4])
```

Maple [C] time = 0.014, size = 316, normalized size = 1.

$$-\frac{c}{3\,a x^{3}}\sqrt{b x^{4}+a}-\frac{b c}{3\,a}\sqrt{1-i x^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+i x^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{b x^{4}+a}}-\frac{d}{2\,a x^{2}}\sqrt{b x^{4}+a}-\frac{e}{a x}\sqrt{b x^{4}+a}+ie\sqrt{b}\sqrt{1-i x^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+i x^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{b x^{4}+a}}-ie\sqrt{b}\sqrt{1-i x^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+i x^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{b x^{4}+a}}-\frac{f}{2}\ln\left(\frac{1}{x^{2}}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{4}+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/3*c*(b*x^4+a)^(1/2)/a/x^3-1/3*c*b/a/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d*(b*
x^4+a)^(1/2)/a/x^2-e*(b*x^4+a)^(1/2)/a/x+I*e*b^(1/2)/a^(1/2)/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/
2))^(1/2),I)-I*e*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*f/a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{f x^{3}+e x^{2}+d x+c}{\sqrt{b x^{4}+a x^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{f x^{3}+e x^{2}+d x+c}{\sqrt{b x^{4}+a x^{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Sympy [A] time = 4.87073, size = 131, normalized size = 0.41

$$-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{2a}+\frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)}+\frac{e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)}-\frac{f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(2*a) + c*gamma(-3/4)*hyper((-3/4
, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1
/4)) + e*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(
I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - f*asinh(sqrt(a)/(sqrt(b)*x**2
))/(2*sqrt(a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

3.526 $\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=346

$$\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-3\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[Out] -(c*Sqrt[a + b*x^4])/(4*a*x^4) - (d*Sqrt[a + b*x^4])/(3*a*x^3) -
(e*Sqrt[a + b*x^4])/(2*a*x^2) - (f*Sqrt[a + b*x^4])/(a*x) + (Sqrt
[b]*f*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (b*c*ArcTa
nh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) - (b^(1/4)*f*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4])
- (b^(1/4)*(Sqrt[b]*d - 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.749794, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-3\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```


```
[Out] -(c*Sqrt[a + b*x^4])/(4*a*x^4) - (d*Sqrt[a + b*x^4])/(3*a*x^3) -
(e*Sqrt[a + b*x^4])/(2*a*x^2) - (f*Sqrt[a + b*x^4])/(a*x) + (Sqrt
[b]*f*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (b*c*ArcTa
nh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) - (b^(1/4)*f*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4])
- (b^(1/4)*(Sqrt[b]*d - 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 81.2337, size = 306, normalized size = 0.88

$$\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{bc\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}-\frac{\sqrt[4]{b}f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{\sqrt[4]{b}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{a}f-\sqrt{b}d\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{6a^{\frac{5}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/x**5/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(b)*f*x*sqrt(a + b*x**4)/(a*(sqrt(a) + sqrt(b)*x**2)) - c*sqr
t(a + b*x**4)/(4*a*x**4) - d*sqrt(a + b*x**4)/(3*a*x**3) - e*sqrt
(a + b*x**4)/(2*a*x**2) - f*sqrt(a + b*x**4)/(a*x) + b*c*atanh(sq
rt(a + b*x**4)/sqrt(a))/(4*a**(3/2)) - b**(1/4)*f*sqrt((a + b*x**
4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic
_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(a**(3/4)*sqrt(a + b*x**4))
+ b**(1/4)*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a
) + sqrt(b)*x**2)*(3*sqrt(a)*f - sqrt(b)*d)*elliptic_f(2*atan(b**
(1/4)*x/a**(1/4)), 1/2)/(6*a**(5/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.893223, size = 259, normalized size = 0.75

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(3bcx^4\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\sqrt{a}\left(a+bx^4\right)\left(3c+4dx+6x^2(e+2fx)\right)\right)-4\sqrt{a}\sqrt{b}x^4\sqrt{\frac{bx^4}{a}+1}\left(3\sqrt{a}f-i\sqrt{b}d\right)}{12a^{3/2}x^4\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-(Sqrt[a]*(a + b*x^4)*(3*c + 4*d*x +
6*x^2*(e + 2*f*x))) + 3*b*c*x^4*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a +
b*x^4]/Sqrt[a]]) + 12*a*Sqrt[b]*f*x^4*Sqrt[1 + (b*x^4)/a]*Ellipti
cE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - 4*Sqrt[a]*Sqrt[b
]*((-I)*Sqrt[b]*d + 3*Sqrt[a]*f)*x^4*Sqrt[1 + (b*x^4)/a]*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(12*a^(3/2)*Sqrt[(
I*Sqrt[b])/Sqrt[a]]*x^4*Sqrt[a + b*x^4])
```

Maple [C] time = 0.015, size = 335, normalized size = 1.

$$-\frac{c}{4\,ax^4}\sqrt{bx^4+a}+\frac{bc}{4}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}-\frac{d}{3\,ax^3}\sqrt{bx^4+a}-\frac{bd}{3\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{e}{2\,ax^2}\sqrt{bx^4+a}-\frac{f}{ax}\sqrt{bx^4+a}+if\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-if\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/4*c*(b*x^4+a)^(1/2)/a/x^4+1/4*c*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b
*x^4+a)^(1/2))/x^2)-1/3*d*(b*x^4+a)^(1/2)/a/x^3-1/3*d*b/a/(I/a^(1
/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)-1/2*e*(b*x^4+a)^(1/2)/a/x^2-f*(b*x^4+a)^(1/2)/a/x+I*f*b
^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-I*f*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^
(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5), x)
```

Sympy [A] time = 7.1351, size = 158, normalized size = 0.46

$$-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\,\Gamma\left(\frac{1}{4}\right)} + \frac{f\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\,\Gamma\left(\frac{3}{4}\right)} + \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/x**5/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*e*sqrt(a/(b*
x**4) + 1)/(2*a) + d*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**
4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + f*gamma(-1/4)*
hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x
*gamma(3/4)) + b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*a**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5), x)
```

3.527 $\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=377

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}$$

```
[Out] -(c*Sqrt[a + b*x^4])/(5*a*x^5) - (d*Sqrt[a + b*x^4])/(4*a*x^4) -
(e*Sqrt[a + b*x^4])/(3*a*x^3) - (f*Sqrt[a + b*x^4])/(2*a*x^2) + (
3*b*c*Sqrt[a + b*x^4])/(5*a^2*x) - (3*b^(3/2)*c*x*Sqrt[a + b*x^4]
)/(5*a^2*(Sqrt[a] + Sqrt[b]*x^2)) + (b*d*ArcTanh[Sqrt[a + b*x^4]/
Sqrt[a]])/(4*a^(3/2)) + (3*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4]) - (b^(3/4)*(9*S
qrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)
], 1/2])/(30*a^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.864639, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] -(c*Sqrt[a + b*x^4])/(5*a*x^5) - (d*Sqrt[a + b*x^4])/(4*a*x^4) -
(e*Sqrt[a + b*x^4])/(3*a*x^3) - (f*Sqrt[a + b*x^4])/(2*a*x^2) + (
3*b*c*Sqrt[a + b*x^4])/(5*a^2*x) - (3*b^(3/2)*c*x*Sqrt[a + b*x^4]
)/(5*a^2*(Sqrt[a] + Sqrt[b]*x^2)) + (b*d*ArcTanh[Sqrt[a + b*x^4]/
Sqrt[a]])/(4*a^(3/2)) + (3*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4]) - (b^(3/4)*(9*S
qrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)
], 1/2])/(30*a^(7/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 98.6803, size = 342, normalized size = 0.91

$$-\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}-\frac{3b^{\frac{3}{2}}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}$$
$$+\frac{bd\,\operatorname{atanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}+\frac{3b^{\frac{5}{4}}c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{7}{4}}\sqrt{a+bx^4}}$$
$$-\frac{b^{\frac{3}{4}}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(5\sqrt{a}e+9\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{30a^{\frac{7}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/x**6/(b*x**4+a)**(1/2),x)
```

```
[Out] -c*sqrt(a + b*x**4)/(5*a*x**5) - d*sqrt(a + b*x**4)/(4*a*x**4) -
e*sqrt(a + b*x**4)/(3*a*x**3) - f*sqrt(a + b*x**4)/(2*a*x**2) - 3
*b**(3/2)*c*x*sqrt(a + b*x**4)/(5*a**2*(sqrt(a) + sqrt(b)*x**2))
+ 3*b*c*sqrt(a + b*x**4)/(5*a**2*x) + b*d*atanh(sqrt(a + b*x**4)/
sqrt(a))/(4*a**(3/2)) + 3*b**(5/4)*c*sqrt((a + b*x**4)/(sqrt(a) +
 sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**
(1/4)*x/a**(1/4)), 1/2)/(5*a**(7/4)*sqrt(a + b*x**4)) - b**(3/4)*
sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)
*x**2)*(5*sqrt(a)*e + 9*sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a
**(1/4)), 1/2)/(30*a**(7/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.905436, size = 268, normalized size = 0.71

$$\frac{-36\sqrt{a}b^{3/2}cx^5\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(15\sqrt{a}bdx^5\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\left(a+bx^4\right)\left(12ac+5ax\right.\right.}{60a^2x^5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-((a + b*x^4)*(12*a*c - 36*b*c*x^4 +
5*a*x*(3*d + 4*e*x + 6*f*x^2))) + 15*Sqrt[a]*b*d*x^5*Sqrt[a + b*x
^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) - 36*Sqrt[a]*b^(3/2)*c*x^5*
Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]
*x], -1] + 4*Sqrt[a]*b*(9*Sqrt[b]*c + (5*I)*Sqrt[a]*e)*x^5*Sqrt[1
 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -
1])/(60*a^2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^5*Sqrt[a + b*x^4])
```

Maple [C] time = 0.015, size = 354, normalized size = 0.9

$$
\begin{aligned}
&-\frac{c}{5\,ax^5}\sqrt{bx^4+a}+\frac{3\,bc}{5\,a^2x}\sqrt{bx^4+a}\\
&-\frac{3\,i}{5}cb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{3\,i}{5}cb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{d}{4\,ax^4}\sqrt{bx^4+a}+\frac{bd}{4}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}-\frac{e}{3\,ax^3}\sqrt{bx^4+a}\\
&-\frac{be}{3\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{f}{2\,ax^2}\sqrt{bx^4+a}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/5*c*(b*x^4+a)^(1/2)/a/x^5+3/5*b*c*(b*x^4+a)^(1/2)/a^2/x-3/5*I*
c/a^(3/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/5*I*c/a^(3/2)*b^(3/2)/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-1/4*d*(b*x^4+a)^(1/2)/a/x^4+1/4*d*b/a^(3/2)*ln((2*a+2*
a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/3*e*(b*x^4+a)^(1/2)/a/x^3-1/3*e*b
/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)-1/2*f*(b*x^4+a)^(1/2)/a/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Sympy [A] time = 7.77864, size = 163, normalized size = 0.43

$$-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4ax^2}-\frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{2a}+\frac{c\,\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^5\,\Gamma\left(-\frac{1}{4}\right)}+\frac{e\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\,\Gamma\left(\frac{1}{4}\right)}+\frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/x**6/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*f*sqrt(a/(b*
x**4) + 1)/(2*a) + c*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**5*gamma(-1/4)) + e*gamma(-3/4
)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*x**3*gamma(1/4)) + b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*a**(3/2)
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

3.528 $\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=365

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}c-5\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}-\frac{3af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}+\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}$$

```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (
d*Sqrt[a + b*x^4])/b^2 + (e*x*Sqrt[a + b*x^4])/(3*b^2) + (f*x^2*S
qrt[a + b*x^4])/(4*b^2) + (3*c*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqr
t[a] + Sqrt[b]*x^2)) - (3*a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^
4]])/(4*b^(5/2)) - (3*a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b
]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(12*b^(9/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.878707, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}c-5\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}-\frac{3af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}+\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```


```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (
d*Sqrt[a + b*x^4])/b^2 + (e*x*Sqrt[a + b*x^4])/(3*b^2) + (f*x^2*S
qrt[a + b*x^4])/(4*b^2) + (3*c*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqr
t[a] + Sqrt[b]*x^2)) - (3*a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^
4]])/(4*b^(5/2)) - (3*a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b
]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(12*b^(9/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.901135, size = 267, normalized size = 0.73

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\left(a\left(12d+10ex+9fx^2\right)+bx^3\left(-6c+6dx+4ex^2+3fx^3\right)\right)-9af\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)\right)+2i\sqrt{a}\sqrt{b}\sqrt{\frac{bx^4}{a}+}}{12b^{5/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[b]*(a*(12*d + 10*e*x + 9*f*x^2)
+ b*x^3*(-6*c + 6*d*x + 4*e*x^2 + 3*f*x^3)) - 9*a*f*Sqrt[a + b*x^
4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) + 18*Sqrt[a]*b*c*Sqrt[
1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x],
-1] + (2*I)*Sqrt[a]*Sqrt[b]*((9*I)*Sqrt[b]*c + 5*Sqrt[a]*e)*Sqrt[
1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x],
-1])/(12*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^(5/2)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.019, size = 378, normalized size = 1.

$$\begin{aligned}
&-\frac{cx^3}{2\,b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}} \\
&+\frac{3\,i}{2}c\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
&-\frac{3\,i}{2}c\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
&+\frac{d\left(bx^4+2\,a\right)}{2\,b^2}\frac{1}{\sqrt{bx^4+a}}+\frac{aex}{2\,b^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{ex}{3\,b^2}\sqrt{bx^4+a} \\
&-\frac{5\,ae}{6\,b^2}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
&+\frac{fx^6}{4\,b}\frac{1}{\sqrt{bx^4+a}}+\frac{3\,x^2af}{4\,b^2}\frac{1}{\sqrt{bx^4+a}}-\frac{3\,af}{4}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{5}{2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*c/b*x^3/((x^4+a/b)*b)^(1/2)+3/2*I*c/b^(3/2)*a^(1/2)/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)-3/2*I*c/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d*(b*x^4+2*
a)/(b*x^4+a)^(1/2)/b^2+1/2*e/b^2*a*x/((x^4+a/b)*b)^(1/2)+1/3*e*x*
(b*x^4+a)^(1/2)/b^2-5/6*e*a/b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*f*x^6/b/(b*x
^4+a)^(1/2)+3/4*f*a/b^2*x^2/(b*x^4+a)^(1/2)-3/4*f*a/b^(5/2)*ln(b^
(1/2)*x^2+(b*x^4+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^6}{(bx^4 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^9 + ex^8 + dx^7 + cx^6}{(bx^4 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^9 + e*x^8 + d*x^7 + c*x^6)/(b*x^4 + a)^(3/2), x)
```

Sympy [A] time = 72.375, size = 202, normalized size = 0.55

$$d\left(\begin{cases} \frac{a}{b^2\sqrt{a+bx^4}} + \frac{x^4}{2b\sqrt{a+bx^4}} & \text{for}\ b \neq 0 \\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + f\left(\frac{3\sqrt{a}x^2}{4b^2\sqrt{1+\frac{bx^4}{a}}} - \frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{x^6}{4\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{cx^7\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{11}{4}\right)} + \frac{ex^9\,\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**
4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + f*(3*sqrt(a)*x**2/(4
*b**2*sqrt(1 + b*x**4/a)) - 3*a*asinh(sqrt(b)*x**2/sqrt(a))/(4*b*
*(5/2)) + x**6/(4*sqrt(a)*b*sqrt(1 + b*x**4/a))) + c*x**7*gamma(7
/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3
/2)*gamma(11/4)) + e*x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b
*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^6}{(bx^4 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

3.529 $\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=343

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}d-5\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+\frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}}+\frac{e\sqrt{a+bx^4}}{b^2}+\frac{fx\sqrt{a+bx^4}}{3b^2}$$

```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (
e*Sqrt[a + b*x^4])/b^2 + (f*x*Sqrt[a + b*x^4])/(3*b^2) + (3*d*x*S
qrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*d*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b
*x^4]) + (a^(1/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.713984, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}d-5\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+\frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}}+\frac{e\sqrt{a+bx^4}}{b^2}+\frac{fx\sqrt{a+bx^4}}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```


```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (
e*Sqrt[a + b*x^4])/b^2 + (f*x*Sqrt[a + b*x^4])/(3*b^2) + (3*d*x*S
qrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*d*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b
*x^4]) + (a^(1/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 140.344, size = 316, normalized size = 0.92

$$-\frac{3\sqrt[4]{a}d\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2b^{\frac{7}{4}}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(5\sqrt{a}f-9\sqrt{b}d\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{12b^{\frac{9}{4}}\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{\frac{3}{2}}} + \frac{3dx\sqrt{a+bx^4}}{2b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**5*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] -3*a**(1/4)*d*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(2*b**(7/4)*sqrt(a + b*x**4)) - a**(1/4)*sqrt((a + b*x**4)/(sqrt
(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(5*sqrt(a)*f - 9
*sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(12*b**(
9/4)*sqrt(a + b*x**4)) + e*sqrt(a + b*x**4)/b**2 + f*x*sqrt(a + b
*x**4)/(3*b**2) + x*(a*f - b*c*x - b*d*x**2 - b*e*x**3)/(2*b**2*s
qrt(a + b*x**4)) + c*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(2*b**(
3/2)) + 3*d*x*sqrt(a + b*x**4)/(2*b**(3/2)*(sqrt(a) + sqrt(b)*x**
2))
```

Mathematica [C] time = 0.648994, size = 255, normalized size = 0.74

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(3\sqrt{b}c\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+a(6e+5fx)+bx^2\left(-3c-3dx+3ex^2+2fx^3\right)\right)+i\sqrt{a}\sqrt{\frac{bx^4}{a}+1}\left(5\sqrt{a}f+9i\sqrt{b}d\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(a*(6*e + 5*f*x) + b*x^2*(-3*c - 3*d*x
 + 3*e*x^2 + 2*f*x^3) + 3*Sqrt[b]*c*Sqrt[a + b*x^4]*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]]) + 9*Sqrt[a]*Sqrt[b]*d*Sqrt[1 + (b*x^4)
/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + I*Sqr
t[a]*((9*I)*Sqrt[b]*d + 5*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(6*Sqrt[(I*Sqrt[b]
)/Sqrt[a]]*b^2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.011, size = 358, normalized size = 1.

$$-\frac{d x^{3}}{2 b}\frac{1}{\sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{3 i}{2} d \sqrt{a}\sqrt{1-i x^{2} \sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+i x^{2} \sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x \sqrt{i \sqrt{b}\frac{1}{\sqrt{a}}}, i\right) b^{-\frac{3}{2}}\frac{1}{\sqrt{i \sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{b x^{4}+a}}-\frac{3 i}{2} d \sqrt{a}\sqrt{1-i x^{2} \sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+i x^{2} \sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x \sqrt{i \sqrt{b}\frac{1}{\sqrt{a}}}, i\right) b^{-\frac{3}{2}}\frac{1}{\sqrt{i \sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{b x^{4}+a}}-\frac{c x^{2}}{2 b}\frac{1}{\sqrt{b x^{4}+a}}+\frac{c}{2}\ln\left(\sqrt{b} x^{2}+\sqrt{b x^{4}+a}\right) b^{-\frac{3}{2}}+\frac{e\left(b x^{4}+2 a\right)}{2 b^{2}}\frac{1}{\sqrt{b x^{4}+a}}+\frac{a f x}{2 b^{2}}\frac{1}{\sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{f x}{3 b^{2}}\sqrt{b x^{4}+a}-\frac{5 a f}{6 b^{2}}\sqrt{1-i x^{2} \sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+i x^{2} \sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x \sqrt{i \sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{i \sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{b x^{4}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*d/b*x^3/((x^4+a/b)*b)^(1/2)+3/2*I*d/b^(3/2)*a^(1/2)/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)-3/2*I*d/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*c*x^2/b/(b*
x^4+a)^(1/2)+1/2*c/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))+1/2*e*
(b*x^4+2*a)/(b*x^4+a)^(1/2)/b^2+1/2*f/b^2*a*x/((x^4+a/b)*b)^(1/2)
+1/3*f*x*(b*x^4+a)^(1/2)/b^2-5/6*f*a/b^2/(I/a^(1/2)*b^(1/2))^(1/2
)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^5/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f x^{8}+e x^{7}+d x^{6}+c x^{5}}{\left(b x^{4}+a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^5/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^8 + e*x^7 + d*x^6 + c*x^5)/(b*x^4 + a)^(3/2), x)
```

Sympy [A] time = 50.5377, size = 172, normalized size = 0.5

$$c\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right)+e\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$
$$+\frac{dx^7\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{11}{4}\right)}+\frac{fx^9\,\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*s
qrt(1 + b*x**4/a))) + e*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x*
*4/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True))
+ d*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I
*pi)/a)/(4*a**(3/2)*gamma(11/4)) + f*x**9*gamma(9/4)*hyper((3/2,
9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx^3+ex^2+dx+c\right)x^5}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^5/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^5/(b*x^4 + a)^(3/2), x)
```

3.530 $\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=314

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{b^2}-\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}}$$

```
[Out] -(x*(c + d*x + e*x^2 + f*x^3))/(2*b*Sqrt[a + b*x^4]) + (f*Sqrt[a
+ b*x^4])/b^2 + (3*e*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqr
t[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2
)) - (3*a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(2*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*S
qrt[a + b*x^4])
```

Rubi [A] time = 0.57713, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{b^2}-\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```



```
[Out] -(x*(c + d*x + e*x^2 + f*x^3))/(2*b*Sqrt[a + b*x^4]) + (f*Sqrt[a
+ b*x^4])/b^2 + (3*e*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqr
t[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2
)) - (3*a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(2*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*S
qrt[a + b*x^4])
```

Rubi in Sympy [A] time = 92.6243, size = 287, normalized size = 0.91

$$-\frac{3\sqrt[4]{a}e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2b^{\frac{7}{4}}\sqrt{a+bx^4}}-\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{b^2}+\frac{d\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{\frac{3}{2}}}+\frac{3ex\sqrt{a+bx^4}}{2b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{a}e+\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{\frac{7}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] -3*a**(1/4)*e*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(2*b**(7/4)*sqrt(a + b*x**4)) - x*(c + d*x + e*x**2 + f*x**3)/(2
*b*sqrt(a + b*x**4)) + f*sqrt(a + b*x**4)/b**2 + d*atanh(sqrt(b)*
x**2/sqrt(a + b*x**4))/(2*b**(3/2)) + 3*e*x*sqrt(a + b*x**4)/(2*b
**(3/2)*(sqrt(a) + sqrt(b)*x**2)) + sqrt((a + b*x**4)/(sqrt(a) +
sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(3*sqrt(a)*e + sqrt(b)
*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(1/4)*b**(
7/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.584146, size = 243, normalized size = 0.77

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}d\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+2af+bx\left(-c-dx-ex^2+fx^3\right)\right)-\sqrt{b}\sqrt{\frac{bx^4}{a}+1}\left(3\sqrt{a}e+i\sqrt{b}c\right)F\left(i\sinh^{-1}\left(\sqrt{i\right.\right.}{2b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(2*a*f + b*x*(-c - d*x - e*x^2 + f*x^3
) + Sqrt[b]*d*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^
4]]) + 3*Sqrt[a]*Sqrt[b]*e*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSin
h[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - Sqrt[b]*(I*Sqrt[b]*c + 3*Sq
rt[a]*e)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])
/Sqrt[a]]*x], -1])/(2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^2*Sqrt[a + b*x^
4])
```

Maple [C] time = 0.011, size = 340, normalized size = 1.1

$$-\frac{cx}{2b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{c}{2b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{dx^2}{2b}\frac{1}{\sqrt{bx^4+a}}+\frac{d}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}-\frac{ex^3}{2b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i}{2}e\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{3i}{2}e\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{f\left(bx^4+2a\right)}{2b^2}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*c/b*x/((x^4+a/b)*b)^(1/2)+1/2*c/b/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d*x^2
/b/(b*x^4+a)^(1/2)+1/2*d/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))-
1/2*e/b*x^3/((x^4+a/b)*b)^(1/2)+3/2*I*e/b^(3/2)*a^(1/2)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)-3/2*I*e/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*f*(b*x^4+2*a
)/(b*x^4+a)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx^3+ex^2+dx+c\right)x^4}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^7+ex^6+dx^5+cx^4}{\left(bx^4+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^7 + e*x^6 + d*x^5 + c*x^4)/(b*x^4 + a)^(3/2), x)
```

Sympy [A] time = 38.2042, size = 172, normalized size = 0.55

$$d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right)+f\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^5\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{9}{4}\right)}+\frac{ex^7\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*s
qrt(1 + b*x**4/a))) + f*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x*
*4/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True))
+ c*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*
pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**7*gamma(7/4)*hyper((3/2, 7/
4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx^3+ex^2+dx+c\right)x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

3.531 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=297

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}-\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(2*b*Sqrt[a + b*x^4]) + (3*f*x*Sqrt[a
+ b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*f*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4])
 + ((Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.420967, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}-\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```


```
[Out] -(c + d*x + e*x^2 + f*x^3)/(2*b*Sqrt[a + b*x^4]) + (3*f*x*Sqrt[a
+ b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*f*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4])
 + ((Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 51.6553, size = 270, normalized size = 0.91

$$-\frac{3\sqrt[4]{a}f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2b^{\frac{7}{4}}\sqrt{a+bx^4}}-\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{e\operatorname{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{\frac{3}{2}}}+\frac{3fx\sqrt{a+bx^4}}{2b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{a}f+\sqrt{b}d\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{\frac{7}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] -3*a**(1/4)*f*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)
/(2*b**(7/4)*sqrt(a + b*x**4)) - (c + d*x + e*x**2 + f*x**3)/(2*b
*sqrt(a + b*x**4)) + e*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(2*b*
*(3/2)) + 3*f*x*sqrt(a + b*x**4)/(2*b**(3/2)*(sqrt(a) + sqrt(b)*x
**2)) + sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) +
 sqrt(b)*x**2)*(3*sqrt(a)*f + sqrt(b)*d)*elliptic_f(2*atan(b**(1/
4)*x/a**(1/4)), 1/2)/(4*a**(1/4)*b**(7/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.541057, size = 224, normalized size = 0.75

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(e\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\sqrt{b}(c+x(d+x(e+fx)))\right)-\sqrt{\frac{bx^4}{a}+1}\left(3\sqrt{a}f+i\sqrt{b}d\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+}{2b^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-(Sqrt[b]*(c + x*(d + x*(e + f*x))))
+ e*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]) + 3*S
qrt[a]*f*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])
/Sqrt[a]]*x], -1] - (I*Sqrt[b]*d + 3*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/
a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(2*Sqrt
[(I*Sqrt[b])/Sqrt[a]]*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.011, size = 331, normalized size = 1.1

$$\begin{aligned}
&-\frac{dx}{2\,b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}\\
&+\frac{d}{2\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{c}{2\,b}\frac{1}{\sqrt{bx^4+a}}-\frac{ex^2}{2\,b}\frac{1}{\sqrt{bx^4+a}}+\frac{e}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}-\frac{fx^3}{2\,b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}\\
&+\frac{3\,i}{2}f\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&-\frac{3\,i}{2}f\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*d/b*x/((x^4+a/b)*b)^(1/2)+1/2*d/b/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*c/b/(
b*x^4+a)^(1/2)-1/2*e*x^2/b/(b*x^4+a)^(1/2)+1/2*e/b^(3/2)*ln(b^(1/
2)*x^2+(b*x^4+a)^(1/2))-1/2*f/b*x^3/((x^4+a/b)*b)^(1/2)+3/2*I*f/b
^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*I*f/b^(3/2)*a^(1/2)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c}{2\sqrt{bx^4+ab}}+\int\frac{fx^6+ex^5+dx^4}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*c/(sqrt(b*x^4 + a)*b) + integrate((f*x^6 + e*x^5 + d*x^4)/(b
*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^6+ex^5+dx^4+cx^3}{\left(bx^4+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)/(b*x^4 + a)^(3/2), x)
```

Sympy [A] time = 31.075, size = 156, normalized size = 0.53

$$c\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+e\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right)+\frac{dx^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{9}{4}\right)}+\frac{fx^7\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] c*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3
/2)), True)) + e*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2
/(2*sqrt(a)*b*sqrt(1 + b*x**4/a))) + d*x**5*gamma(5/4)*hyper((5/4
, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
 + f*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(
I*pi)/a)/(4*a**(3/2)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx^3+ex^2+dx+c\right)x^3}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^(3/2), x)
```

3.532 $\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}-\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}}-\frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{d\sqrt{a+bx^4}}{2ab}$$

```
[Out] -(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a*b*Sqrt[a + b*x^4]) -
(d*Sqrt[a + b*x^4])/(2*a*b) - (c*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^
4]])/(2*b^(3/2)) + (c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) - ((Sqrt[b]*c - Sqrt[a
]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(3/4)
*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.561107, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}-\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}}-\frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{d\sqrt{a+bx^4}}{2ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```



```
[Out] -(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a*b*Sqrt[a + b*x^4]) -
(d*Sqrt[a + b*x^4])/(2*a*b) - (c*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^
4]])/(2*b^(3/2)) + (c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) - ((Sqrt[b]*c - Sqrt[a
]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(3/4)
*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 90.4119, size = 294, normalized size = 0.88

$$\frac{f\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{\frac{3}{2}}}-\frac{d\sqrt{a+bx^4}}{2ab}-\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}}$$
$$-\frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}\sqrt{a+bx^4}}$$
$$+\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}e-\sqrt{b}c\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*atanh(sqrt(b)*x**2/sqrt(a + b*x**4))/(2*b**(3/2)) - d*sqrt(a +
b*x**4)/(2*a*b) - x*(a*e + a*f*x - b*c*x**2 - b*d*x**3)/(2*a*b*sq
rt(a + b*x**4)) - c*x*sqrt(a + b*x**4)/(2*a*sqrt(b)*(sqrt(a) + sq
rt(b)*x**2)) + c*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(
sqrt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1
/2)/(2*a**(3/4)*b**(3/4)*sqrt(a + b*x**4)) + sqrt((a + b*x**4)/(s
qrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*e -
sqrt(b)*c)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(3/
4)*b**(5/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.910017, size = 242, normalized size = 0.73

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\left(bcx^3-a(d+x(e+fx))\right)+af\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)\right)+\sqrt{a}\sqrt{b}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{b}c-i\sqrt{a}e\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{2ab^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(Sqrt[b]*(b*c*x^3 - a*(d + x*(e + f*x)
)) + a*f*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])
- Sqrt[a]*b*c*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqr
t[b])/Sqrt[a]]*x], -1] + Sqrt[a]*Sqrt[b]*(Sqrt[b]*c - I*Sqrt[a]*e
)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a
]]*x], -1])/(2*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^(3/2)*Sqrt[a + b*x^4
])
```

Maple [C] time = 0.012, size = 331, normalized size = 1.

$$-\frac{d}{2b}\frac{1}{\sqrt{bx^4+a}}+\frac{cx^3}{2a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}$$
$$-\frac{i}{2}c\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$+\frac{i}{2}c\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}$$
$$-\frac{ex}{2b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}$$
$$+\frac{e}{2b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{fx^2}{2b}\frac{1}{\sqrt{bx^4+a}}+\frac{f}{2}\ln\left(\sqrt{b}x^2+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*d/b/(b*x^4+a)^(1/2)+1/2*c/a*x^3/((x^4+a/b)*b)^(1/2)-1/2*I*c/
a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*I*c/a^(1/2)/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^
2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)-1/2*e/b*x/((x^4+a/b)*b)^(1/2)+1/2*e/b/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1
/2*f*x^2/b/(b*x^4+a)^(1/2)+1/2*f/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx^3+ex^2+dx+c\right)x^2}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^5+ex^4+dx^3+cx^2}{\left(bx^4+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^5 + e*x^4 + d*x^3 + c*x^2)/(b*x^4 + a)^(3/2), x)
```

Sympy [A] time = 28.3428, size = 156, normalized size = 0.47

$$d\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{cx^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3
/2)), True)) + f*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2
/(2*sqrt(a)*b*sqrt(1 + b*x**4/a))) + c*x**3*gamma(3/4)*hyper((3/4
, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
 + e*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I
*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3 + ex^2 + dx + c\right)x^2}{(bx^4 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

3.533 $\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=303

$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}-\frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\sqrt{a+bx^4}}{2ab}$$

```
[Out] -(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a*b*Sqrt[a + b*x^4]) -
(e*Sqrt[a + b*x^4])/(2*a*b) - (d*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x^2)) + (d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) - ((Sqrt[b]*d
- Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4
*a^(3/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.403118, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}-\frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\sqrt{a+bx^4}}{2ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```


```
[Out] -(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a*b*Sqrt[a + b*x^4]) -
(e*Sqrt[a + b*x^4])/(2*a*b) - (d*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x^2)) + (d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) - ((Sqrt[b]*d
- Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4
*a^(3/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 56.3881, size = 265, normalized size = 0.87

$$-\frac{e\sqrt{a+bx^4}}{2ab}-\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}-\frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}\sqrt{a+bx^4}}+\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}f-\sqrt{b}d\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] -e*sqrt(a + b*x**4)/(2*a*b) - x*(a*f - b*c*x - b*d*x**2 - b*e*x**
3)/(2*a*b*sqrt(a + b*x**4)) - d*x*sqrt(a + b*x**4)/(2*a*sqrt(b)*(
sqrt(a) + sqrt(b)*x**2)) + d*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)
*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_e(2*atan(b**(1/4)*x/
a**(1/4)), 1/2)/(2*a**(3/4)*b**(3/4)*sqrt(a + b*x**4)) + sqrt((a
+ b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(
sqrt(a)*f - sqrt(b)*d)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/
2)/(4*a**(3/4)*b**(5/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.433688, size = 197, normalized size = 0.65

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(bx^2(c+dx)-a(e+fx)\right)+\sqrt{a}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{b}d-i\sqrt{a}f\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-\sqrt{a}\sqrt{b}d\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\right.\cdots}{2ab\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(b*x^2*(c + d*x) - a*(e + f*x)) - Sqrt
[a]*Sqrt[b]*d*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqr
t[b])/Sqrt[a]]*x], -1] + Sqrt[a]*(Sqrt[b]*d - I*Sqrt[a]*f)*Sqrt[1
 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -
1])/(2*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.014, size = 250, normalized size = 0.8

$$d\left(\frac{x^3}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i}{2}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}\right)+\frac{cx^2}{2\,a}\frac{1}{\sqrt{bx^4+a}}-\frac{e}{2\,b}\frac{1}{\sqrt{bx^4+a}}+f\left(-\frac{x}{2\,b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{1}{2\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] d*(1/2/a*x^3/((x^4+a/b)*b)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+1/2*c*x^2/a/(b
*x^4+a)^(1/2)-1/2*e/b/(b*x^4+a)^(1/2)+f*(-1/2/b*x/((x^4+a/b)*b)^(
1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^2}{2\sqrt{bx^4+a}a}+\int\frac{fx^4+ex^3+dx^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2/(sqrt(b*x^4 + a)*a) + integrate((f*x^4 + e*x^3 + d*x^2)
/(b*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^4+ex^3+dx^2+cx}{(bx^4+a)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^4 + e*x^3 + d*x^2 + c*x)/(b*x^4 + a)^(3/2), x)
```

Sympy [A] time = 26.5585, size = 133, normalized size = 0.44

$$e\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}+\frac{dx^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{fx^5\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3
/2)), True)) + c*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + d*x**3*ga
mma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(3/2)*gamma(7/4)) + f*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(fx^3+ex^2+dx+c)x}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*x/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x/(b*x^4 + a)^(3/2), x)
```

3.534 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=275

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{af-bx\left(c+dx+ex^2\right)}{2ab\sqrt{a+bx^4}}-\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[Out] -(e*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) - (a
*f - b*x*(c + d*x + e*x^2))/(2*a*b*Sqrt[a + b*x^4]) + (e*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt
[a + b*x^4]) + ((Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.27311, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{af-bx\left(c+dx+ex^2\right)}{2ab\sqrt{a+bx^4}}-\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```


```
[Out] -(e*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) - (a
*f - b*x*(c + d*x + e*x^2))/(2*a*b*Sqrt[a + b*x^4]) + (e*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt
[a + b*x^4]) + ((Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 35.6164, size = 241, normalized size = 0.88

$$-\frac{af-bx\left(c+dx+ex^2\right)}{2ab\sqrt{a+bx^4}}-\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}e-\sqrt{b}c\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{5}{4}}b^{\frac{3}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] -(a*f - b*x*(c + d*x + e*x**2))/(2*a*b*sqrt(a + b*x**4)) - e*x*sq
rt(a + b*x**4)/(2*a*sqrt(b)*(sqrt(a) + sqrt(b)*x**2)) + e*sqrt((a
 + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*
elliptic_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(2*a**(3/4)*b**(3/4)
*sqrt(a + b*x**4)) - sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**
2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*e - sqrt(b)*c)*elliptic_f(2*
atan(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(5/4)*b**(3/4)*sqrt(a + b*x
**4))
```

Mathematica [C] time = 0.392899, size = 195, normalized size = 0.71

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}(bx(c+x(d+ex))-af)+\sqrt{b}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}e-i\sqrt{b}c\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)-\sqrt{a}\sqrt{b}e\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\cdots}\right.\right.}{2ab\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-(a*f) + b*x*(c + x*(d + e*x))) - Sqr
t[a]*Sqrt[b]*e*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sq
rt[b])/Sqrt[a]]*x], -1] + Sqrt[b]*((-I)*Sqrt[b]*c + Sqrt[a]*e)*Sq
rt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x
], -1])/(2*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.007, size = 250, normalized size = 0.9

$$c\left(\frac{x}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{1}{2\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\right)$$
$$+\frac{dx^2}{2\,a}\frac{1}{\sqrt{bx^4+a}}+e\left(\frac{x^3}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}\right.$$
$$\left.-\frac{i}{2}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}\right)$$
$$-\frac{f}{2\,b}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] c*(1/2/a*x/((x^4+a/b)*b)^(1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*d*x^2/
a/(b*x^4+a)^(1/2)+e*(1/2/a*x^3/((x^4+a/b)*b)^(1/2)-1/2*I/a^(1/2)/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)))-1/2*f/b/(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

Sympy [A] time = 27.3559, size = 131, normalized size = 0.48

$$f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\,\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{5}{4}\right)} + \frac{dx^2}{2a^{\frac{3}{2}}\sqrt{1 + \frac{bx^4}{a}}} + \frac{ex^3\,\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3
/2)), True)) + c*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + d*x**2/(2*a**(3/2)*sqr
t(1 + b*x**4/a)) + e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

3.535 $\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=323

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}}+\frac{c\sqrt{a+bx^4}}{2a^2}-\frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[Out] (x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (
c*Sqrt[a + b*x^4])/(2*a^2) - (f*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(
Sqrt[a] + Sqrt[b]*x^2)) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2
*a^(3/2)) + (f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d - Sqrt[a]*f)*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4
)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.559231, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}}+\frac{c\sqrt{a+bx^4}}{2a^2}-\frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```


```
[Out] (x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (
c*Sqrt[a + b*x^4])/(2*a^2) - (f*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(
Sqrt[a] + Sqrt[b]*x^2)) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2
*a^(3/2)) + (f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d - Sqrt[a]*f)*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4
)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 44.1479, size = 238, normalized size = 0.74

$$\frac{x\left(\frac{c}{x}+d+ex+fx^2\right)}{2a\sqrt{a+bx^4}}-\frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}b^{\frac{3}{4}}\sqrt{a+bx^4}}-\frac{\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}f-\sqrt{b}d\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{5}{4}}b^{\frac{3}{4}}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(3/2),x)
```

```
[Out] x*(c/x + d + e*x + f*x**2)/(2*a*sqrt(a + b*x**4)) - f*x*sqrt(a +
b*x**4)/(2*a*sqrt(b)*(sqrt(a) + sqrt(b)*x**2)) + f*sqrt((a + b*x*
*4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*ellipti
c_e(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(2*a**(3/4)*b**(3/4)*sqrt(a
 + b*x**4)) - sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqr
t(a) + sqrt(b)*x**2)*(sqrt(a)*f - sqrt(b)*d)*elliptic_f(2*atan(b*
*(1/4)*x/a**(1/4)), 1/2)/(4*a**(5/4)*b**(3/4)*sqrt(a + b*x**4))
```

Mathematica [C] time = 5.22536, size = 225, normalized size = 0.7

$$\frac{ia^{3/2}f\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{\frac{bx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{a}b(c+x(d+x(e+fx)))-bc\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{b\sqrt{\frac{bx^4}{a}+1}(\sqrt{b}}{}}{2a^{3/2}b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*b*(c + x*(d + x*(e + f*x))) - b*c*Sqrt[a + b*x^4]*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]] + I*a^(3/2)*Sqrt[(I*Sqrt[b])/Sqrt[a]]*
f*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a
]]*x], -1] + (b*(Sqrt[b]*d + I*Sqrt[a]*f)*Sqrt[1 + (b*x^4)/a]*Ell
ipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/((I*Sqrt[b])/
Sqrt[a])^(3/2))/(2*a^(3/2)*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.011, size = 336, normalized size = 1.

$$\begin{aligned}
&\frac{dx}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}\\
&+\frac{d}{2\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\
&+\frac{fx^3}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}\\
&-\frac{i}{2}f\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}\\
&+\frac{i}{2}f\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}\\
&+\frac{c}{2\,a}\frac{1}{\sqrt{bx^4+a}}-\frac{c}{2}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}+\frac{ex^2}{2\,a}\frac{1}{\sqrt{bx^4+a}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*d/a*x/((x^4+a/b)*b)^(1/2)+1/2*d/a/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*f/a*x^
3/((x^4+a/b)*b)^(1/2)-1/2*I*f/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/
2*I*f/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
```

```
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*El
lipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*c/a/(b*x^4+a)^(1/2)-1/
2*c/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/2*e*x^2/a/(
b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{(bx^5 + ax)\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/((b*x^5 + a*x)*sqrt(b*x^4 + a)
), x)
```

Sympy [A] time = 39.947, size = 289, normalized size = 0.89

$$c\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{dx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}+\frac{fx^3\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a
**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(s
qrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*
x**4*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x*
*4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4))
+ d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)
/a)/(4*a**(3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/
a)) + f*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

3.536 $\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{3\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{d\sqrt{a+bx^4}}{2a^2}$$

```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (
d*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqr
t[b]*c*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (d*Ar
cTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*c*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b
*x^4]) + ((3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.696423, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{3\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{d\sqrt{a+bx^4}}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```


```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (
d*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqr
t[b]*c*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (d*Ar
cTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*c*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b
*x^4]) + ((3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 23.9362, size = 109, normalized size = 0.32

$$\frac{x\left(\frac{c}{x^2}+\frac{d}{x}+e+fx\right)}{2a\sqrt{a+bx^4}} + \frac{e\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{5}{4}}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(3/2),x)
```

```
[Out] x*(c/x**2 + d/x + e + f*x)/(2*a*sqrt(a + b*x**4)) + e*sqrt((a + b
*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*elli
ptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(5/4)*b**(1/4)*sqr
t(a + b*x**4))
```

Mathematica [C] time = 0.733707, size = 245, normalized size = 0.71

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}dx\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-2ac+ax(d+x(e+fx))-3bcx^4\right)-i\sqrt{a}x\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}e-3i\sqrt{b}c\right)F\left(i\sinh^{-1}\right.}{2a^2x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[a]]*(-2*a*c - 3*b*c*x^4 + a*x*(d + x*(e +
f*x)) - Sqrt[a]*d*x*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[
a]]) + 3*Sqrt[a]*Sqrt[b]*c*x*Sqrt[1 + (b*x^4)/a]*EllipticE[I*ArcS
inh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - I*Sqrt[a]*((-3*I)*Sqrt[b]
*c + Sqrt[a]*e)*x*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[b])/Sqrt[a]]*x], -1])/(2*a^2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x*Sq
rt[a + b*x^4])
```

Maple [C] time = 0.015, size = 355, normalized size = 1.

$$\begin{aligned}
&\frac{ex}{2a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}} \\
&+\frac{e}{2a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
&+\frac{fx^2}{2a}\frac{1}{\sqrt{bx^4+a}}-\frac{x^3bc}{2a^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c}{a^2x}\sqrt{bx^4+a} \\
&+\frac{3i}{2}c\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
&-\frac{3i}{2}c\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
&+\frac{d}{2a}\frac{1}{\sqrt{bx^4+a}}-\frac{d}{2}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*e/a*x/((x^4+a/b)*b)^(1/2)+1/2*e/a/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*f*x^2/
a/(b*x^4+a)^(1/2)-1/2*c*b/a^2*x^3/((x^4+a/b)*b)^(1/2)-c*(b*x^4+a)
^(1/2)/a^2/x+3/2*I*c*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*I*c*b^(
1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d/a/(b*x^4+a)^(1/2)-1/2*d/a^(3/
2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2),x, algorithm="maxima"
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{(bx^6 + ax^2)\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2),x, algorithm="fricas"
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/((b*x^6 + a*x^2)*sqrt(b*x^4 +
a)), x)
```

Sympy [A] time = 69.3578, size = 291, normalized size = 0.85

$$d\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4}\right) + \frac{c\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4}, \frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\,\Gamma\left(\frac{3}{4}\right)} + \frac{ex\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{5}{4}\right)} + \frac{fx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a
**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(s
qrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*
x**4*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x*
*4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4))
+ c*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)
/a)/(4*a**(3/2)*x*gamma(3/4)) + e*x*gamma(1/4)*hyper((1/4, 3/2),
(5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + f*x**
2/(2*a**(3/2)*sqrt(1 + b*x**4/a))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

3.537 $\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=367

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{2a^2x^2}-\frac{d\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\sqrt{a+bx^4}}{2a^2}$$

```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (
e*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(2*a^2*x^2) - (d
*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(2*a^
2*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/(2*a^(3/2)) - (3*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*d + Sqrt[a
]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)
*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.837959, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{2a^2x^2}-\frac{d\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\sqrt{a+bx^4}}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```


```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (
e*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(2*a^2*x^2) - (d
*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(2*a^
2*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/(2*a^(3/2)) - (3*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*d + Sqrt[a
]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)
*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 23.9306, size = 110, normalized size = 0.3

$$\frac{x\left(\frac{c}{x^3}+\frac{d}{x^2}+\frac{e}{x}+f\right)}{2a\sqrt{a+bx^4}}+\frac{f\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}+\sqrt{b}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{5}{4}}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(3/2),x)
```

```
[Out] x*(c/x**3 + d/x**2 + e/x + f)/(2*a*sqrt(a + b*x**4)) + f*sqrt((a
+ b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*e
lliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(5/4)*b**(1/4)*
sqrt(a + b*x**4))
```

Mathematica [C]   time = 0.710298, size = 259, normalized size = 0.71

$$-\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{a}ex^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+a\left(c+2dx+x^2(-(e+fx))\right)+bx^4(2c+3dx)\right)-i\sqrt{a}x^2\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a}f-3i\sqrt{b}\right.}{2a^2x^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*(b*x^4*(2*c + 3*d*x) + a*(c + 2*d*x
- x^2*(e + f*x)) + Sqrt[a]*e*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a +
 b*x^4]/Sqrt[a]])) + 3*Sqrt[a]*Sqrt[b]*d*x^2*Sqrt[1 + (b*x^4)/a]*
EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] - I*Sqrt[a]
*((-3*I)*Sqrt[b]*d + Sqrt[a]*f)*x^2*Sqrt[1 + (b*x^4)/a]*EllipticF
[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(2*a^2*Sqrt[(I*Sqrt
[b])/Sqrt[a]]*x^2*Sqrt[a + b*x^4])
```

Maple [C]   time = 0.024, size = 363, normalized size = 1.

$$\begin{aligned}&\frac{fx}{2a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}\\&+\frac{f}{2a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\&-\frac{c\left(2bx^4+a\right)}{2x^2a^2}\frac{1}{\sqrt{bx^4+a}}-\frac{x^3bd}{2a^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{d}{a^2x}\sqrt{bx^4+a}\\&+\frac{3i}{2}d\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\&-\frac{3i}{2}d\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\\&+\frac{e}{2a}\frac{1}{\sqrt{bx^4+a}}-\frac{e}{2}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*f/a*x/((x^4+a/b)*b)^(1/2)+1/2*f/a/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*c/x^2*
(2*b*x^4+a)/(b*x^4+a)^(1/2)/a^2-1/2*d*b/a^2*x^3/((x^4+a/b)*b)^(1/
2)-d*(b*x^4+a)^(1/2)/a^2/x+3/2*I*d*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-3/2*I*d*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
```

```
2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*e/a/(b*x^4+a)^(1/
2)-1/2*e/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3),x, algorithm="maxima"
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{(bx^7 + ax^3)\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3),x, algorithm="fricas"
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/((b*x^7 + a*x^3)*sqrt(b*x^4 +
a)), x)
```

Sympy [A] time = 84.4303, size = 316, normalized size = 0.86

$$c\left(-\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}} - \frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right) + e\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4}\right) + \frac{d\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{3}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\,\Gamma\left(\frac{3}{4}\right)} + \frac{fx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqr
t(a/(b*x**4) + 1))) + e*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) +
4*a**(7/2)*b*x**4) + a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*
b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(
7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)
*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2)
+ 4*a**(7/2)*b*x**4)) + d*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,),
b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + f*x*gamma(1
/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

3.538 $\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=387

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-9\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}-\frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{3a^2x^3}-\frac{d\sqrt{a+bx^4}}{2a^2x^2}-\frac{e\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}ex\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\sqrt{a+bx^4}}{2a^2}$$

```
[Out] -(x*(b*c + b*d*x + b*e*x^2 + b*f*x^3))/(2*a^2*Sqrt[a + b*x^4]) +
(f*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(3*a^2*x^3) - (
d*Sqrt[a + b*x^4])/(2*a^2*x^2) - (e*Sqrt[a + b*x^4])/(a^2*x) + (3
*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (
f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*e*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a
 + b*x^4]) - (b^(1/4)*(5*Sqrt[b]*c - 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*a^(9/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 1.04488, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-9\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}-\frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{3a^2x^3}-\frac{d\sqrt{a+bx^4}}{2a^2x^2}-\frac{e\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}ex\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\sqrt{a+bx^4}}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```


```
[Out] -(x*(b*c + b*d*x + b*e*x^2 + b*f*x^3))/(2*a^2*Sqrt[a + b*x^4]) +
(f*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(3*a^2*x^3) - (
d*Sqrt[a + b*x^4])/(2*a^2*x^2) - (e*Sqrt[a + b*x^4])/(a^2*x) + (3
*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (
f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*e*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a
 + b*x^4]) - (b^(1/4)*(5*Sqrt[b]*c - 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*a^(9/4)*Sqrt[a + b*x^4])
```

Rubi in Sympy [A] time = 15.818, size = 32, normalized size = 0.08

$$\frac{x\left(\frac{c}{x^4}+\frac{d}{x^3}+\frac{e}{x^2}+\frac{f}{x}\right)}{2a\sqrt{a+bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(3/2),x)
```

```
[Out] x*(c/x**4 + d/x**3 + e/x**2 + f/x)/(2*a*sqrt(a + b*x**4))
```

Mathematica [C] time = 0.719223, size = 267, normalized size = 0.69

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(3\sqrt{a}fx^3\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+2ac+3ax(d+x(2e-fx))+bx^4\left(5c+6dx+9ex^2\right)\right)-\sqrt{b}x^3\sqrt{\frac{bx^4}{a}+1}\left(9\sqrt{a}e\right.}{6a^2x^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[a]]*(2*a*c + b*x^4*(5*c + 6*d*x + 9*e*x^
2) + 3*a*x*(d + x*(2*e - f*x)) + 3*Sqrt[a]*f*x^3*Sqrt[a + b*x^4]*
ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])) + 9*Sqrt[a]*Sqrt[b]*e*x^3*Sqrt
[1 + (b*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x],
 -1] - Sqrt[b]*((-5*I)*Sqrt[b]*c + 9*Sqrt[a]*e)*x^3*Sqrt[1 + (b*x
^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(6*
a^2*Sqrt[(I*Sqrt[b])/Sqrt[a]]*x^3*Sqrt[a + b*x^4])
```

Maple [C] time = 0.014, size = 383, normalized size = 1.

$$-\frac{bcx}{2\,a^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c}{3\,a^2x^3}\sqrt{bx^4+a}$$
$$-\frac{5\,bc}{6\,a^2}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{d\left(2\,bx^4+a\right)}{2\,x^2a^2}\frac{1}{\sqrt{bx^4+a}}-\frac{x^3be}{2\,a^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{e}{a^2x}\sqrt{bx^4+a}$$
$$+\frac{3\,i}{2}e\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{3\,i}{2}e\sqrt{b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticE\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{f}{2\,a}\frac{1}{\sqrt{bx^4+a}}-\frac{f}{2}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*c*b/a^2*x/((x^4+a/b)*b)^(1/2)-1/3*c*(b*x^4+a)^(1/2)/a^2/x^3-
5/6*c*b/a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d/x^2*(2*b*x^4+a)/(b*x^4+a)^(1/2
```

```
)/a^2-1/2*e*b/a^2*x^3/((x^4+a/b)*b)^(1/2)-e*(b*x^4+a)^(1/2)/a^2/x
+3/2*I*e*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*I*e*b^(1/2)/a^(3/2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)+1/2*f/a/(b*x^4+a)^(1/2)-1/2*f/a^(3/2)*ln((2*a+2
*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4),x, algorithm="maxima"
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{(bx^8 + ax^4)\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4),x, algorithm="fricas"
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/((b*x^8 + a*x^4)*sqrt(b*x^4 +
a)), x)
```

Sympy [A] time = 114.883, size = 321, normalized size = 0.83

$$d\left(-\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}} - \frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right) + f\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right) + \frac{c\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^3\left(\frac{1}{4}\right)} + \frac{e\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqr
t(a/(b*x**4) + 1))) + f*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) +
4*a**(7/2)*b*x**4) + a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*
b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(
7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)
*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2)
```

```
+ 4*a**(7/2)*b*x**4)) + c*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,),
b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x**3*gamma(1/4)) + e*gamma(
-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*x*gamma(3/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

3.539 $\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=269

$$\frac{c(gx)^{m+1}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{4},-p;\frac{m+5}{4};-\frac{bx^4}{a}\right)}{g(m+1)}+\frac{d(gx)^{m+2}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{4},-p;\frac{m+6}{4};-\frac{bx^4}{a}\right)}{g^2(m+2)}+\frac{e(gx)^{m+3}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3}{4},-p;\frac{m+7}{4};-\frac{bx^4}{a}\right)}{g^3(m+3)}+\frac{f(gx)^{m+4}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+4}{4},-p;\frac{m+8}{4};-\frac{bx^4}{a}\right)}{g^4(m+4)}$$

```
[Out] (c*(g*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (
5 + m)/4, -((b*x^4)/a)])/(g*(1 + m)*(1 + (b*x^4)/a)^p) + (d*(g*x)
^(2 + m)*(a + b*x^4)^p*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4
, -((b*x^4)/a)])/(g^2*(2 + m)*(1 + (b*x^4)/a)^p) + (e*(g*x)^(3 +
m)*(a + b*x^4)^p*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4, -((b
*x^4)/a)])/(g^3*(3 + m)*(1 + (b*x^4)/a)^p) + (f*(g*x)^(4 + m)*(a
+ b*x^4)^p*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4, -((b*x^4)/
a)])/(g^4*(4 + m)*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.520783, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{c(gx)^{m+1}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{4},-p;\frac{m+5}{4};-\frac{bx^4}{a}\right)}{g(m+1)}+\frac{d(gx)^{m+2}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{4},-p;\frac{m+6}{4};-\frac{bx^4}{a}\right)}{g^2(m+2)}+\frac{e(gx)^{m+3}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3}{4},-p;\frac{m+7}{4};-\frac{bx^4}{a}\right)}{g^3(m+3)}+\frac{f(gx)^{m+4}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+4}{4},-p;\frac{m+8}{4};-\frac{bx^4}{a}\right)}{g^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (c*(g*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (
5 + m)/4, -((b*x^4)/a)])/(g*(1 + m)*(1 + (b*x^4)/a)^p) + (d*(g*x)
^(2 + m)*(a + b*x^4)^p*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4
, -((b*x^4)/a)])/(g^2*(2 + m)*(1 + (b*x^4)/a)^p) + (e*(g*x)^(3 +
m)*(a + b*x^4)^p*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4, -((b
*x^4)/a)])/(g^3*(3 + m)*(1 + (b*x^4)/a)^p) + (f*(g*x)^(4 + m)*(a
+ b*x^4)^p*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4, -((b*x^4)/
a)])/(g^4*(4 + m)*(1 + (b*x^4)/a)^p)
```

Rubi in Sympy [A] time = 58.1998, size = 218, normalized size = 0.81

$$\frac{c\,(gx)^{m+1}\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{g\,(m+1)}+\frac{d\,(gx)^{m+2}\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{m}{4}+\frac{1}{2}\\ \frac{m}{4}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{g^2\,(m+2)}+\frac{e\,(gx)^{m+3}\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{m}{4}+\frac{3}{4}\\ \frac{m}{4}+\frac{7}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{g^3\,(m+3)}+\frac{f\,(gx)^{m+4}\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{m}{4}+1\\ \frac{m}{4}+2\end{matrix}\middle|-\frac{bx^4}{a}\right)}{g^4\,(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x)**m*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] c*(g*x)**(m + 1)*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((-p,
m/4 + 1/4), (m/4 + 5/4,), -b*x**4/a)/(g*(m + 1)) + d*(g*x)**(m +
2)*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((-p, m/4 + 1/2), (m
/4 + 3/2,), -b*x**4/a)/(g**2*(m + 2)) + e*(g*x)**(m + 3)*(1 + b*x
**4/a)**(-p)*(a + b*x**4)**p*hyper((-p, m/4 + 3/4), (m/4 + 7/4,),
 -b*x**4/a)/(g**3*(m + 3)) + f*(g*x)**(m + 4)*(1 + b*x**4/a)**(-p
)*(a + b*x**4)**p*hyper((-p, m/4 + 1), (m/4 + 2,), -b*x**4/a)/(g*
*4*(m + 4))
```

Mathematica [A] time = 0.345973, size = 174, normalized size = 0.65

$$x(gx)^m\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(\frac{c\,{}_2F_1\left(\frac{m+1}{4},-p;\frac{m+5}{4};-\frac{bx^4}{a}\right)}{m+1}+x\left(\frac{d\,{}_2F_1\left(\frac{m+2}{4},-p;\frac{m+6}{4};-\frac{bx^4}{a}\right)}{m+2}+\frac{ex\,{}_2F_1\left(\frac{m+3}{4},-p;\frac{m+7}{4};-\frac{bx^4}{a}\right)}{m+3}\right)+\frac{fx^3\,{}_2F_1\left(\frac{m}{4}+1,-p;\frac{m}{4}+2;-\frac{bx^4}{a}\right)}{m+4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (x*(g*x)^m*(a + b*x^4)^p*((f*x^3*Hypergeometric2F1[1 + m/4, -p, 2
 + m/4, -((b*x^4)/a)])/(4 + m) + (c*Hypergeometric2F1[(1 + m)/4,
-p, (5 + m)/4, -((b*x^4)/a)])/(1 + m) + x*((d*Hypergeometric2F1[(
2 + m)/4, -p, (6 + m)/4, -((b*x^4)/a)])/(2 + m) + (e*x*Hypergeome
tric2F1[(3 + m)/4, -p, (7 + m)/4, -((b*x^4)/a)])/(3 + m))))/(1 +
(b*x^4)/a)^p
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (gx)^m\left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p (gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m,x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x)**m*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p (gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

3.540 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx$

Optimal. Leaf size=143

$$\frac{cx\left(a+bx^4\right)^{p+1}{}_2F_1\left(1,p+\frac{5}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{a}+\frac{dx^2\left(a+bx^4\right)^{p+1}{}_2F_1\left(1,p+\frac{3}{2};\frac{3}{2};-\frac{bx^4}{a}\right)}{2a}+\frac{ex^3\left(a+bx^4\right)^{p+1}{}_2F_1\left(1,p+\frac{7}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3a}+\frac{f\left(a+bx^4\right)^{p+1}}{4b(p+1)}$$

```
[Out] (f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^(1 + p)*
Hypergeometric2F1[1, 5/4 + p, 5/4, -((b*x^4)/a)])/a + (d*x^2*(a +
 b*x^4)^(1 + p)*Hypergeometric2F1[1, 3/2 + p, 3/2, -((b*x^4)/a)])
/(2*a) + (e*x^3*(a + b*x^4)^(1 + p)*Hypergeometric2F1[1, 7/4 + p,
 7/4, -((b*x^4)/a)])/(3*a)
```

Rubi [A] time = 0.275286, antiderivative size = 170, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{2}dx^2\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+\frac{f\left(a+bx^4\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^p*Hyperg
eometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (d*x
^2*(a + b*x^4)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)])/(
2*(1 + (b*x^4)/a)^p) + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4
, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p)
```

Rubi in Sympy [A] time = 36.2594, size = 136, normalized size = 0.95

$$cx\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)+\frac{dx^2\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{2}+\frac{ex^3\left(1+\frac{bx^4}{a}\right)^{-p}\left(a+bx^4\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|-\frac{bx^4}{a}\right)}{3}+\frac{f\left(a+bx^4\right)^{p+1}}{4b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] c*x*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((-p, 1/4), (5/4,),
 -b*x**4/a) + d*x**2*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((
-p, 1/2), (3/2,), -b*x**4/a)/2 + e*x**3*(1 + b*x**4/a)**(-p)*(a +
 b*x**4)**p*hyper((-p, 3/4), (7/4,), -b*x**4/a)/3 + f*(a + b*x**4
)**(p + 1)/(4*b*(p + 1))
```

Mathematica [A] time = 0.162862, size = 184, normalized size = 1.29

$$\frac{\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(12bc(p+1)x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+6bd(p+1)x^2\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^4}{a}\right)+4bex^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)\right.}{12b(p+1)}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] ((a + b*x^4)^p*(-3*a*f + 3*a*f*(1 + (b*x^4)/a)^p + 3*b*f*x^4*(1 +
 (b*x^4)/a)^p + 12*b*c*(1 + p)*x*Hypergeometric2F1[1/4, -p, 5/4,
-((b*x^4)/a)] + 6*b*d*(1 + p)*x^2*Hypergeometric2F1[1/2, -p, 3/2,
 -((b*x^4)/a)] + 4*b*e*x^3*Hypergeometric2F1[3/4, -p, 7/4, -((b*x
^4)/a)] + 4*b*e*p*x^3*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a
)]))/(12*b*(1 + p)*(1 + (b*x^4)/a)^p)
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \left(f x^3 + e x^2 + d x + c\right) \left(b x^4 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(f x^3 + e x^2 + d x + c\right) \left(b x^4 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(f x^3 + e x^2 + d x + c\right) \left(b x^4 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Sympy [A] time = 142.287, size = 141, normalized size = 0.99

$$\frac{a^p c x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, -p\\ \frac{5}{4}\end{matrix}\right|\frac{b x^4 e^{i\pi}}{a}\right)}{4 \Gamma\left(\frac{5}{4}\right)} + \frac{a^p d x^2 {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\right|\frac{b x^4 e^{i\pi}}{a}\right)}{2} + \frac{a^p e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, -p\\ \frac{7}{4}\end{matrix}\right|\frac{b x^4 e^{i\pi}}{a}\right)}{4 \Gamma\left(\frac{7}{4}\right)} + f \left(\begin{cases} \frac{a^p x^4}{4} & \text{for}\: b = 0 \\ \frac{\begin{cases} \frac{\left(a + b x^4\right)^{p+1}}{p+1} & \text{for}\: p \neq -1 \\ \log\left(a + b x^4\right) & \text{otherwise} \end{cases}}{4b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*p
i)/a)/(4*gamma(5/4)) + a**p*d*x**2*hyper((1/2, -p), (3/2,), b*x**
4*exp_polar(I*pi)/a)/2 + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p),
(7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + f*Piecewise((a
**p*x**4/4, Eq(b, 0)), (Piecewise(((a + b*x**4)**(p + 1)/(p + 1),
 Ne(p, -1)), (log(a + b*x**4), True))/(4*b), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

3.541 $\int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^p \, dx$

Optimal. Leaf size=175

$$\frac{c (a + bx^4)^{p+1}}{4b(p + 1)} + \frac{1}{5} dx^5 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^4}{a}\right) + \frac{1}{6} ex^6 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^4}{a}\right) + \frac{1}{7} fx^7 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -\frac{bx^4}{a}\right)$$

```
[Out] (c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hype
rgeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p)
+ (e*x^6*(a + b*x^4)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/
a)])/(6*(1 + (b*x^4)/a)^p) + (f*x^7*(a + b*x^4)^p*Hypergeometric2
F1[7/4, -p, 11/4, -((b*x^4)/a)])/(7*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.403432, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{c (a + bx^4)^{p+1}}{4b(p + 1)} + \frac{1}{5} dx^5 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^4}{a}\right) + \frac{1}{6} ex^6 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^4}{a}\right) + \frac{1}{7} fx^7 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hype
rgeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p)
+ (e*x^6*(a + b*x^4)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/
a)])/(6*(1 + (b*x^4)/a)^p) + (f*x^7*(a + b*x^4)^p*Hypergeometric2
F1[7/4, -p, 11/4, -((b*x^4)/a)])/(7*(1 + (b*x^4)/a)^p)
```

Rubi in Sympy [A] time = 46.9727, size = 139, normalized size = 0.79

$$\frac{dx^5 \left(1 + \frac{bx^4}{a}\right)^{-p} (a + bx^4)^p {}_2F_1\left(\begin{matrix} -p, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| -\frac{bx^4}{a}\right)}{5} + \frac{ex^6 \left(1 + \frac{bx^4}{a}\right)^{-p} (a + bx^4)^p {}_2F_1\left(\begin{matrix} -p, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| -\frac{bx^4}{a}\right)}{6} + \frac{fx^7 \left(1 + \frac{bx^4}{a}\right)^{-p} (a + bx^4)^p {}_2F_1\left(\begin{matrix} -p, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| -\frac{bx^4}{a}\right)}{7} + \frac{c (a + bx^4)^{p+1}}{4b (p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] d*x**5*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hyper((-p, 5/4), (9/4
,), -b*x**4/a)/5 + e*x**6*(1 + b*x**4/a)**(-p)*(a + b*x**4)**p*hy
per((-p, 3/2), (5/2,), -b*x**4/a)/6 + f*x**7*(1 + b*x**4/a)**(-p)
*(a + b*x**4)**p*hyper((-p, 7/4), (11/4,), -b*x**4/a)/7 + c*(a +
b*x**4)**(p + 1)/(4*b*(p + 1))
```

Mathematica [A] time = 0.239194, size = 160, normalized size = 0.91

$$\frac{(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(105c\left(bx^4\left(\frac{bx^4}{a}+1\right)^p+a\left(\left(\frac{bx^4}{a}+1\right)^p-1\right)\right)+84bd(p+1)x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+70be(p+1)x^6}{420b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] ((a + b*x^4)^p*(105*c*(b*x^4*(1 + (b*x^4)/a)^p + a*(-1 + (1 + (b*
x^4)/a)^p)) + 84*b*d*(1 + p)*x^5*Hypergeometric2F1[5/4, -p, 9/4,
-((b*x^4)/a)] + 70*b*e*(1 + p)*x^6*Hypergeometric2F1[3/2, -p, 5/2
, -((b*x^4)/a)] + 60*b*f*(1 + p)*x^7*Hypergeometric2F1[7/4, -p, 1
1/4, -((b*x^4)/a)]))/(420*b*(1 + p)*(1 + (b*x^4)/a)^p)
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int x^3\left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(fx^6+ex^5+dx^4+cx^3\right)\left(bx^4+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*x^3,x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*(b*x^4 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^3 + ex^2 + dx + c) (bx^4 + a)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*x^3,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*x^3, x)
```

3.542 $\int \frac{1+x+x^2+x^3+x^4}{1-x^5} \, dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -Log[1 - x]
```

Rubi [A] time = 0.012767, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] -Log[1 - x]
```

Rubi in Sympy [A] time = 4.34625, size = 5, normalized size = 0.62

$$-\log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**3+x**2+x+1)/(-x**5+1),x)
```

```
[Out] -log(-x + 1)
```

Mathematica [A] time = 0.00143416, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] -Log[1 - x]
```

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^3+x^2+x+1)/(-x^5+1),x)
```

```
[Out] -ln(-1+x)
```

Maxima [A] time = 1.42934, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 + x^3 + x^2 + x + 1)/(x^5 - 1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

Fricas [A] time = 0.201494, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 + x^3 + x^2 + x + 1)/(x^5 - 1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A] time = 0.045877, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**3+x**2+x+1)/(-x**5+1),x)
```

```
[Out] -log(x - 1)
```

GIAC/XCAS [A] time = 0.218168, size = 9, normalized size = 1.12

$$-\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 + x^3 + x^2 + x + 1)/(x^5 - 1),x, algorithm="giac")
```

```
[Out] -ln(abs(x - 1))
```

3.543 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\log(2x+3)$$

```
[Out] Log[3 + 2*x]/2
```

Rubi [A] time = 0.0162471, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$

$$\frac{1}{2}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6),x]
```

```
[Out] Log[3 + 2*x]/2
```

Rubi in Sympy [A] time = 8.25591, size = 7, normalized size = 0.7

$$\frac{\log(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729),x)
```

```
[Out] log(2*x + 3)/2
```

Mathematica [A] time = 0.00180022, size = 10, normalized size = 1.

$$\frac{1}{2}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6),x]
```

```
[Out] Log[3 + 2*x]/2
```

Maple [A] time = 0.002, size = 9, normalized size = 0.9

$$\frac{\ln(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x)
```

```
[Out] 1/2*ln(2*x+3)
```

Maxima [A] time = 1.4215, size = 11, normalized size = 1.1

$$\frac{1}{2}\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)/(64*x^6 - 729),x, alg
```

```
[Out] 1/2*log(2*x + 3)
```

Fricas [A] time = 0.201064, size = 11, normalized size = 1.1

$$\frac{1}{2}\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)/(64*x^6 - 729),x, alg
```

```
[Out] 1/2*log(2*x + 3)
```

Sympy [A] time = 0.058952, size = 7, normalized size = 0.7

$$\frac{\log(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729),x)
```

```
[Out] log(2*x + 3)/2
```

GIAC/XCAS [A] time = 0.216451, size = 12, normalized size = 1.2

$$\frac{1}{2}\ln(|2x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)/(64*x^6 - 729),x, alg
```

```
[Out] 1/2*ln(abs(2*x + 3))
```

3.544 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\log(3-2x)$$

```
[Out] -Log[3 - 2*x]/2
```

Rubi [A] time = 0.0158625, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057

$$-\frac{1}{2}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6),x]
```

```
[Out] -Log[3 - 2*x]/2
```

Rubi in Sympy [A] time = 7.47107, size = 8, normalized size = 0.8

$$-\frac{\log(-2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729),x)
```

```
[Out] -log(-2*x + 3)/2
```

Mathematica [A] time = 0.00179574, size = 10, normalized size = 1.

$$-\frac{1}{2}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6),x]
```

```
[Out] -Log[3 - 2*x]/2
```

Maple [A] time = 0.001, size = 9, normalized size = 0.9

$$-\frac{\ln(-3+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x)
```

```
[Out] -1/2*ln(-3+2*x)
```

Maxima [A] time = 1.40774, size = 11, normalized size = 1.1

$$-\frac{1}{2}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)/(64*x^6 - 729),x, al
```

```
[Out] -1/2*log(2*x - 3)
```

Fricas [A] time = 0.20057, size = 11, normalized size = 1.1

$$-\frac{1}{2}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)/(64*x^6 - 729),x, al
```

```
[Out] -1/2*log(2*x - 3)
```

Sympy [A] time = 0.062623, size = 8, normalized size = 0.8

$$-\frac{\log(2x-3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729),x)
```

```
[Out] -log(2*x - 3)/2
```

GIAC/XCAS [A] time = 0.217028, size = 12, normalized size = 1.2

$$-\frac{1}{2}\ln(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)/(64*x^6 - 729),x, al
```

```
[Out] -1/2*ln(abs(2*x - 3))
```

3.545 $\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx$

Optimal. Leaf size=10

$$\frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] ArcTanh[(2*x)/3]/6
```

Rubi [A] time = 0.0118707, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]

[Out] ArcTanh[(2*x)/3]/6
```

Rubi in Sympy [A] time = 3.89002, size = 7, normalized size = 0.7

$$\frac{\operatorname{atanh}\left(\frac{2x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((16*x**4+36*x**2+81)/(-64*x**6+729),x)

[Out] atanh(2*x/3)/6
```

Mathematica [B] time = 0.00429929, size = 21, normalized size = 2.1

$$\frac{1}{12}\log(2x+3)-\frac{1}{12}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]

[Out] -Log[3 - 2*x]/12 + Log[3 + 2*x]/12
```

Maple [B] time = 0.008, size = 18, normalized size = 1.8

$$\frac{\ln(2x+3)}{12}-\frac{\ln(-3+2x)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((16*x^4+36*x^2+81)/(-64*x^6+729),x)

[Out] 1/12*ln(2*x+3)-1/12*ln(-3+2*x)
```

Maxima [A] time = 1.36482, size = 23, normalized size = 2.3

$$\frac{1}{12}\,\log\left(2\,x+3\right)-\frac{1}{12}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(16*x^4 + 36*x^2 + 81)/(64*x^6 - 729),x, algorithm="maxima")
```

```
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Fricas [A] time = 0.205763, size = 23, normalized size = 2.3

$$\frac{1}{12}\,\log\left(2\,x+3\right)-\frac{1}{12}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(16*x^4 + 36*x^2 + 81)/(64*x^6 - 729),x, algorithm="fricas")
```

```
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Sympy [A] time = 0.099868, size = 15, normalized size = 1.5

$$-\frac{\log\left(x-\frac{3}{2}\right)}{12}+\frac{\log\left(x+\frac{3}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x**4+36*x**2+81)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/12 + log(x + 3/2)/12
```

GIAC/XCAS [A] time = 0.218568, size = 20, normalized size = 2.

$$\frac{1}{12}\,\ln\left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{12}\,\ln\left(\left|x-\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(16*x^4 + 36*x^2 + 81)/(64*x^6 - 729),x, algorithm="giac")
```

```
[Out] 1/12*ln(abs(x + 3/2)) - 1/12*ln(abs(x - 3/2))
```

3.546 $\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx$

Optimal. Leaf size=24

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Rubi [A] time = 0.0345201, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 6.38822, size = 22, normalized size = 0.92

$$-\frac{\sqrt{3}\,\operatorname{atan}\left(\sqrt{3}\left(-\frac{4x}{9}+\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729),x)
```

```
[Out] -sqrt(3)*atan(sqrt(3)*(-4*x/9 + 1/3))/9
```

Mathematica [A] time = 0.01145, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Maple [A] time = 0.006, size = 17, normalized size = 0.7

$$\frac{\sqrt{3}}{9}\arctan\left(\frac{(8\,x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x)
```

```
[Out] 1/9*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.54475, size = 22, normalized size = 0.92

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4 + 24*x^3 - 54*x - 81)/(64*x^6 - 729),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Fricas [A] time = 0.203516, size = 22, normalized size = 0.92

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4 + 24*x^3 - 54*x - 81)/(64*x^6 - 729),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Sympy [A] time = 0.128778, size = 24, normalized size = 1.

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729),x)
```

```
[Out] sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.215871, size = 22, normalized size = 0.92

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4 + 24*x^3 - 54*x - 81)/(64*x^6 - 729),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

3.547 $\int \frac{3-2x}{729-64x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 -
Log[9 - 6*x + 4*x^2]/972
```

Rubi [A] time = 0.0831348, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 -
Log[9 - 6*x + 4*x^2]/972
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-32x^5-48x^4-72x^3-108x^2-162x-243}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3-2*x)/(-64*x**6+729),x)
```

```
[Out] -Integral(1/(-32*x**5 - 48*x**4 - 72*x**3 - 108*x**2 - 162*x - 24
3), x)
```

Mathematica [A] time = 0.0284468, size = 50, normalized size = 1.

$$-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 -
Log[9 - 6*x + 4*x^2]/972
```

Maple [A] time = 0.014, size = 39, normalized size = 0.8

$$\frac{\ln(2x+3)}{486}+\frac{\sqrt{3}}{486}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)-\frac{\ln\left(4x^2-6x+9\right)}{972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3-2*x)/(-64*x^6+729),x)
```

```
[Out] 1/486*ln(2*x+3)+1/486*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/972*
ln(4*x^2-6*x+9)
```

Maxima [A] time = 1.56403, size = 51, normalized size = 1.02

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 3)/(64*x^6 - 729),x, algorithm="maxima")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6
*x + 9) + 1/486*log(2*x + 3)
```

Fricas [A] time = 0.211327, size = 61, normalized size = 1.22

$$-\frac{1}{2916}\sqrt{3}\left(\sqrt{3}\log\left(4x^2-6x+9\right)-2\sqrt{3}\log\left(2x+3\right)-6\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 3)/(64*x^6 - 729),x, algorithm="fricas")
```

```
[Out] -1/2916*sqrt(3)*(sqrt(3)*log(4*x^2 - 6*x + 9) - 2*sqrt(3)*log(2*x
 + 3) - 6*arctan(1/9*sqrt(3)*(4*x + 3)))
```

Sympy [A] time = 0.250292, size = 46, normalized size = 0.92

$$\frac{\log\left(x+\frac{3}{2}\right)}{486}-\frac{\log\left(4x^2-6x+9\right)}{972}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3-2*x)/(-64*x**6+729),x)
```

```
[Out] log(x + 3/2)/486 - log(4*x**2 - 6*x + 9)/972 + sqrt(3)*atan(4*sqr
t(3)*x/9 + sqrt(3)/3)/486
```

GIAC/XCAS [A] time = 0.220995, size = 53, normalized size = 1.06

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{972}\ln\left(4x^2-6x+9\right)+\frac{1}{486}\ln\left(|2x+3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x - 3)/(64*x^6 - 729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*ln(4*x^2 - 6*
x + 9) + 1/486*ln(abs(2*x + 3))
```

3.548 $\int \frac{3+2x}{729-64x^6}\,dx$

Optimal. Leaf size=50

$$\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) - Log[3 - 2*x]/486 +
 Log[9 + 6*x + 4*x^2]/972
```

Rubi [A] time = 0.0847005, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) - Log[3 - 2*x]/486 +
 Log[9 + 6*x + 4*x^2]/972
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-32x^5+48x^4-72x^3+108x^2-162x+243}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+2*x)/(-64*x**6+729),x)
```

```
[Out] Integral(1/(-32*x**5 + 48*x**4 - 72*x**3 + 108*x**2 - 162*x + 243
), x)
```

Mathematica [A] time = 0.0277928, size = 46, normalized size = 0.92

$$\frac{1}{972}\left(\log\left(4x^2+6x+9\right)-2\log(3-2x)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 2*Log[3 - 2*x] + Log[
9 + 6*x + 4*x^2])/972
```

Maple [A] time = 0.01, size = 39, normalized size = 0.8

$$-\frac{\ln(-3+2x)}{486}+\frac{\ln\left(4x^2+6x+9\right)}{972}+\frac{\sqrt{3}}{486}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x+3)/(-64*x^6+729),x)
```

```
[Out] -1/486*ln(-3+2*x)+1/972*ln(4*x^2+6*x+9)+1/486*3^(1/2)*arctan(1/18
*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.55074, size = 51, normalized size = 1.02

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x + 3)/(64*x^6 - 729),x, algorithm="maxima")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6
*x + 9) - 1/486*log(2*x - 3)
```

Fricas [A] time = 0.210587, size = 61, normalized size = 1.22

$$\frac{1}{2916}\sqrt{3}\left(\sqrt{3}\log\left(4x^2+6x+9\right)-2\sqrt{3}\log\left(2x-3\right)+6\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x + 3)/(64*x^6 - 729),x, algorithm="fricas")
```

```
[Out] 1/2916*sqrt(3)*(sqrt(3)*log(4*x^2 + 6*x + 9) - 2*sqrt(3)*log(2*x
- 3) + 6*arctan(1/9*sqrt(3)*(4*x - 3)))
```

Sympy [A] time = 0.264688, size = 46, normalized size = 0.92

$$-\frac{\log\left(x-\frac{3}{2}\right)}{486}+\frac{\log\left(4x^2+6x+9\right)}{972}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+2*x)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/486 + log(4*x**2 + 6*x + 9)/972 + sqrt(3)*atan(4*sq
rt(3)*x/9 - sqrt(3)/3)/486
```

GIAC/XCAS [A] time = 0.217976, size = 53, normalized size = 1.06

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{972}\ln\left(4x^2+6x+9\right)-\frac{1}{486}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x + 3)/(64*x^6 - 729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*ln(4*x^2 + 6*
x + 9) - 1/486*ln(abs(2*x - 3))
```

3.549 $\int \frac{9-6x+4x^2}{729-64x^6}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{324}\log\left(4x^2+6x+9\right)-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + L
og[3 + 2*x]/108 - Log[9 + 6*x + 4*x^2]/324
```

Rubi [A] time = 0.0884097, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{1}{324}\log\left(4x^2+6x+9\right)-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + L
og[3 + 2*x]/108 - Log[9 + 6*x + 4*x^2]/324
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-16x^4-24x^3+54x+81}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2-6*x+9)/(-64*x**6+729),x)
```

```
[Out] Integral(1/(-16*x**4 - 24*x**3 + 54*x + 81), x)
```

Mathematica [A] time = 0.0199033, size = 56, normalized size = 0.93

$$\frac{1}{324}\left(-\log\left(4x^2+6x+9\right)-\log(3-2x)+3\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - Log[3 - 2*x] + 3*Log[3
 + 2*x] - Log[9 + 6*x + 4*x^2])/324
```

Maple [A] time = 0.01, size = 47, normalized size = 0.8

$$\frac{\ln(2x+3)}{108}-\frac{\ln(-3+2x)}{324}-\frac{\ln\left(4x^2+6x+9\right)}{324}+\frac{\sqrt{3}}{162}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2-6*x+9)/(-64*x^6+729),x)
```

```
[Out] 1/108*ln(2*x+3)-1/324*ln(-3+2*x)-1/324*ln(4*x^2+6*x+9)+1/162*3^(1
/2)*arctan(1/18*(8*x+6)*3^(1/2))
```

Maxima [A]   time = 1.52214, size = 62, normalized size = 1.03

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log\left(2x+3\right)-\frac{1}{324}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x^2 - 6*x + 9)/(64*x^6 - 729),x, algorithm="maxima")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6
*x + 9) + 1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

Fricas [A]   time = 0.212307, size = 74, normalized size = 1.23

$$-\frac{1}{972}\sqrt{3}\left(\sqrt{3}\log\left(4x^2+6x+9\right)-3\sqrt{3}\log\left(2x+3\right)+\sqrt{3}\log\left(2x-3\right)-6\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x^2 - 6*x + 9)/(64*x^6 - 729),x, algorithm="fricas")
```

```
[Out] -1/972*sqrt(3)*(sqrt(3)*log(4*x^2 + 6*x + 9) - 3*sqrt(3)*log(2*x
+ 3) + sqrt(3)*log(2*x - 3) - 6*arctan(1/9*sqrt(3)*(4*x + 3)))
```

Sympy [A]   time = 0.284135, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{324}+\frac{\log\left(x+\frac{3}{2}\right)}{108}-\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2-6*x+9)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/324 + log(x + 3/2)/108 - log(x**2 + 3*x/2 + 9/4)/32
4 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/162
```

GIAC/XCAS [A]   time = 0.220924, size = 65, normalized size = 1.08

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\ln\left(4x^2+6x+9\right)+\frac{1}{108}\ln\left(|2x+3|\right)-\frac{1}{324}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x^2 - 6*x + 9)/(64*x^6 - 729),x, algorithm="giac")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*ln(4*x^2 + 6*
x + 9) + 1/108*ln(abs(2*x + 3)) - 1/324*ln(abs(2*x - 3))
```

3.550 $\int \frac{9+6x+4x^2}{729-64x^6}\,dx$

Optimal. Leaf size=60

$$\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/108 +
Log[3 + 2*x]/324 + Log[9 - 6*x + 4*x^2]/324
```

Rubi [A] time = 0.0876117, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/108 +
Log[3 + 2*x]/324 + Log[9 - 6*x + 4*x^2]/324
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-16x^4+24x^3-54x+81}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((4*x**2+6*x+9)/(-64*x**6+729),x)
```

```
[Out] Integral(1/(-16*x**4 + 24*x**3 - 54*x + 81), x)
```

Mathematica [A] time = 0.0191567, size = 52, normalized size = 0.87

$$\frac{1}{324}\left(\log\left(4x^2-6x+9\right)-3\log(3-2x)+\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + Log[
3 + 2*x] + Log[9 - 6*x + 4*x^2])/324
```

Maple [A] time = 0.01, size = 47, normalized size = 0.8

$$\frac{\ln(2x+3)}{324}-\frac{\ln(-3+2x)}{108}+\frac{\ln\left(4x^2-6x+9\right)}{324}+\frac{\sqrt{3}}{162}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2+6*x+9)/(-64*x^6+729),x)
```

```
[Out] 1/324*ln(2*x+3)-1/108*ln(-3+2*x)+1/324*ln(4*x^2-6*x+9)+1/162*3^(1
/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.51291, size = 62, normalized size = 1.03

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log\left(2x+3\right)-\frac{1}{108}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x^2 + 6*x + 9)/(64*x^6 - 729),x, algorithm="maxima")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6
*x + 9) + 1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

Fricas [A] time = 0.211458, size = 74, normalized size = 1.23

$$\frac{1}{972}\sqrt{3}\left(\sqrt{3}\log\left(4x^2-6x+9\right)+\sqrt{3}\log\left(2x+3\right)-3\sqrt{3}\log\left(2x-3\right)+6\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x^2 + 6*x + 9)/(64*x^6 - 729),x, algorithm="fricas")
```

```
[Out] 1/972*sqrt(3)*(sqrt(3)*log(4*x^2 - 6*x + 9) + sqrt(3)*log(2*x + 3
) - 3*sqrt(3)*log(2*x - 3) + 6*arctan(1/9*sqrt(3)*(4*x - 3)))
```

Sympy [A] time = 0.281761, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{108}+\frac{\log\left(x+\frac{3}{2}\right)}{324}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+6*x+9)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/108 + log(x + 3/2)/324 + log(x**2 - 3*x/2 + 9/4)/32
4 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/162
```

GIAC/XCAS [A] time = 0.219026, size = 65, normalized size = 1.08

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\ln\left(4x^2-6x+9\right)+\frac{1}{324}\ln\left(|2x+3|\right)-\frac{1}{108}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*x^2 + 6*x + 9)/(64*x^6 - 729),x, algorithm="giac")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*ln(4*x^2 - 6*
x + 9) + 1/324*ln(abs(2*x + 3)) - 1/108*ln(abs(2*x - 3))
```

3.551 $\int \frac{27-8x^3}{729-64x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - L
og[9 - 6*x + 4*x^2]/108
```

Rubi [A] time = 0.0523294, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - L
og[9 - 6*x + 4*x^2]/108
```

Rubi in Sympy [A] time = 7.66706, size = 42, normalized size = 0.84

$$\frac{\log(2x+3)}{54}-\frac{\log\left(4x^2-6x+9\right)}{108}+\frac{\sqrt{3}\,\operatorname{atan}\left(\sqrt{3}\left(\frac{4x}{9}-\frac{1}{3}\right)\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-8*x**3+27)/(-64*x**6+729),x)
```

```
[Out] log(2*x + 3)/54 - log(4*x**2 - 6*x + 9)/108 + sqrt(3)*atan(sqrt(3
)*(4*x/9 - 1/3))/54
```

Mathematica [A] time = 0.00931375, size = 50, normalized size = 1.

$$-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - L
og[9 - 6*x + 4*x^2]/108
```

Maple [A] time = 0.008, size = 39, normalized size = 0.8

$$\frac{\ln(2x+3)}{54}-\frac{\ln\left(4x^2-6x+9\right)}{108}+\frac{\sqrt{3}}{54}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-8*x^3+27)/(-64*x^6+729),x)
```

```
[Out] 1/54*ln(2*x+3)-1/108*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*
x-6)*3^(1/2))
```

Maxima [A] time = 1.51589, size = 51, normalized size = 1.02

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 - 27)/(64*x^6 - 729),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*
x + 9) + 1/54*log(2*x + 3)
```

Fricas [A] time = 0.20985, size = 61, normalized size = 1.22

$$-\frac{1}{324}\sqrt{3}\left(\sqrt{3}\log\left(4x^2-6x+9\right)-2\sqrt{3}\log\left(2x+3\right)-6\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 - 27)/(64*x^6 - 729),x, algorithm="fricas")
```

```
[Out] -1/324*sqrt(3)*(sqrt(3)*log(4*x^2 - 6*x + 9) - 2*sqrt(3)*log(2*x
+ 3) - 6*arctan(1/9*sqrt(3)*(4*x - 3)))
```

Sympy [A] time = 0.204576, size = 48, normalized size = 0.96

$$\frac{\log\left(x+\frac{3}{2}\right)}{54}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{108}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-8*x**3+27)/(-64*x**6+729),x)
```

```
[Out] log(x + 3/2)/54 - log(x**2 - 3*x/2 + 9/4)/108 + sqrt(3)*atan(4*sq
rt(3)*x/9 - sqrt(3)/3)/54
```

GIAC/XCAS [A] time = 0.219643, size = 47, normalized size = 0.94

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{108}\ln\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)+\frac{1}{54}\ln\left(\left|x+\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 - 27)/(64*x^6 - 729),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*ln(x^2 - 3/2*x
 + 9/4) + 1/54*ln(abs(x + 3/2))
```

3.552 $\int \frac{27+36x+24x^2+8x^3}{729-64x^6} \, dx$

Optimal. Leaf size=50

$$\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + L
og[9 - 6*x + 4*x^2]/36
```

Rubi [A] time = 0.072656, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + L
og[9 - 6*x + 4*x^2]/36
```

Rubi in Sympy [A] time = 20.4172, size = 44, normalized size = 0.88

$$-\frac{\log(-2x+3)}{18}+\frac{\log\left(2x+4(x-1)^2+5\right)}{36}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{4x}{9}-\frac{1}{3}\right)\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729),x)
```

```
[Out] -log(-2*x + 3)/18 + log(2*x + 4*(x - 1)**2 + 5)/36 + sqrt(3)*atan
(sqrt(3)*(4*x/9 - 1/3))/54
```

Mathematica [A] time = 0.0168301, size = 50, normalized size = 1.

$$\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)+\frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + L
og[9 - 6*x + 4*x^2]/36
```

Maple [A] time = 0.008, size = 39, normalized size = 0.8

$$-\frac{\ln(-3+2x)}{18}+\frac{\ln\left(4x^2-6x+9\right)}{36}+\frac{\sqrt{3}}{54}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x)
```

```
[Out] -1/18*ln(-3+2*x)+1/36*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8
*x-6)*3^(1/2))
```

Maxima [A] time = 1.52891, size = 51, normalized size = 1.02

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(8*x^3 + 24*x^2 + 36*x + 27)/(64*x^6 - 729),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x
 + 9) - 1/18*log(2*x - 3)
```

Fricas [A] time = 0.210262, size = 61, normalized size = 1.22

$$\frac{1}{108}\sqrt{3}\left(\sqrt{3}\log\left(4x^2-6x+9\right)-2\sqrt{3}\log\left(2x-3\right)+2\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(8*x^3 + 24*x^2 + 36*x + 27)/(64*x^6 - 729),x, algorithm="fricas")
```

```
[Out] 1/108*sqrt(3)*(sqrt(3)*log(4*x^2 - 6*x + 9) - 2*sqrt(3)*log(2*x -
 3) + 2*arctan(1/9*sqrt(3)*(4*x - 3)))
```

Sympy [A] time = 0.206795, size = 48, normalized size = 0.96

$$-\frac{\log\left(x-\frac{3}{2}\right)}{18}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{36}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/18 + log(x**2 - 3*x/2 + 9/4)/36 + sqrt(3)*atan(4*sq
rt(3)*x/9 - sqrt(3)/3)/54
```

GIAC/XCAS [A] time = 0.218279, size = 53, normalized size = 1.06

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\ln\left(4x^2-6x+9\right)-\frac{1}{18}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(8*x^3 + 24*x^2 + 36*x + 27)/(64*x^6 - 729),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*ln(4*x^2 - 6*x
+ 9) - 1/18*ln(abs(2*x - 3))
```

3.553 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\log\left(4x^2-6x+9\right)}{17496}-\frac{\log\left(4x^2+6x+9\right)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}$$

```
[Out] -1/(2916*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]
) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/1
7496 + (5*Log[3 + 2*x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[
9 + 6*x + 4*x^2]/17496
```

Rubi [A] time = 0.218052, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$-\frac{\log\left(4x^2-6x+9\right)}{17496}-\frac{\log\left(4x^2+6x+9\right)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] -1/(2916*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]
) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/1
7496 + (5*Log[3 + 2*x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[
9 + 6*x + 4*x^2]/17496
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729)**2
```

```
[Out] Timed out
```

Mathematica [A] time = 0.172915, size = 100, normalized size = 0.91

$$\frac{-3\log\left(4x^2-6x+9\right)-3\log\left(4x^2+6x+9\right)-\frac{18}{2x+3}-3\log(3-2x)+15\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488}$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] (-18/(3 + 2*x) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqr
t[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + 15*Log[3 +
2*x] - 3*Log[9 - 6*x + 4*x^2] - 3*Log[9 + 6*x + 4*x^2])/52488
```

Maple [A] time = 0.016, size = 85, normalized size = 0.8

$$-\frac{1}{5832x+8748}+\frac{5\ln(2x+3)}{17496}-\frac{\ln(-3+2x)}{17496}-\frac{\ln(4x^2+6x+9)}{17496}+\frac{\sqrt{3}}{8748}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)-\frac{\ln(4x^2-6x+9)}{17496}+\frac{\sqrt{3}}{26244}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x)
```

```
[Out] -1/2916/(2*x+3)+5/17496*ln(2*x+3)-1/17496*ln(-3+2*x)-1/17496*ln(4
*x^2+6*x+9)+1/8748*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/17496*l
n(4*x^2-6*x+9)+1/26244*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.51212, size = 113, normalized size = 1.03

$$\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x+3)}-\frac{1}{17496}\log(4x^2+6x+9)-\frac{1}{17496}\log(4x^2-6x+9)+\frac{5}{17496}\log(2x+3)-\frac{1}{17496}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)/(64*x^6 - 729)^2,x, 
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*ar
ctan(1/9*sqrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^
2 + 6*x + 9) - 1/17496*log(4*x^2 - 6*x + 9) + 5/17496*log(2*x + 3
) - 1/17496*log(2*x - 3)
```

Fricas [A] time = 0.214244, size = 169, normalized size = 1.54

$$-\frac{\sqrt{3}\left(\sqrt{3}(2x+3)\log(4x^2+6x+9)+\sqrt{3}(2x+3)\log(4x^2-6x+9)-5\sqrt{3}(2x+3)\log(2x+3)+\sqrt{3}(2x+3)\log(2x-\right.}{52488(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)/(64*x^6 - 729)^2,x, 
```

```
[Out] -1/52488*sqrt(3)*(sqrt(3)*(2*x + 3)*log(4*x^2 + 6*x + 9) + sqrt(3
)*(2*x + 3)*log(4*x^2 - 6*x + 9) - 5*sqrt(3)*(2*x + 3)*log(2*x +
3) + sqrt(3)*(2*x + 3)*log(2*x - 3) - 6*(2*x + 3)*arctan(1/9*sqrt
(3)*(4*x + 3)) - 2*(2*x + 3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 6*sq
rt(3))/(2*x + 3)
```

Sympy [A] time = 0.594917, size = 105, normalized size = 0.95

$$-\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{5\log\left(x+\frac{3}{2}\right)}{17496}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}-\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{26244}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{8748}-\frac{1}{5832x+8748}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729)**2,x)
```

```
[Out] -log(x - 3/2)/17496 + 5*log(x + 3/2)/17496 - log(x**2 - 3*x/2 + 9
/4)/17496 - log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3
)*x/9 - sqrt(3)/3)/26244 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3
)/8748 - 1/(5832*x + 8748)
```

GIAC/XCAS [A] time = 0.218589, size = 116, normalized size = 1.05

$$\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916\,(2x+3)}$$
$$-\frac{1}{17496}\ln\left(4x^2+6x+9\right)-\frac{1}{17496}\ln\left(4x^2-6x+9\right)+\frac{5}{17496}\ln\left(|2x+3|\right)-\frac{1}{17496}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)/(64*x^6 - 729)^2,x, a
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*ar
ctan(1/9*sqrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*ln(4*x^2
 + 6*x + 9) - 1/17496*ln(4*x^2 - 6*x + 9) + 5/17496*ln(abs(2*x +
3)) - 1/17496*ln(abs(2*x - 3))
```

3.554 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} dx$

Optimal. Leaf size=110

$$\frac{\log(4x^2-6x+9)}{17496}+\frac{\log(4x^2+6x+9)}{17496}+\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}$$

```
[Out] 1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3])
 + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x]
)/17496 + Log[3 + 2*x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9
 + 6*x + 4*x^2]/17496
```

Rubi [A] time = 0.206841, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$\frac{\log(4x^2-6x+9)}{17496}+\frac{\log(4x^2+6x+9)}{17496}+\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3])
 + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x]
)/17496 + Log[3 + 2*x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9
 + 6*x + 4*x^2]/17496
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729)**2,
```

```
[Out] Timed out
```

Mathematica [A] time = 0.135762, size = 97, normalized size = 0.88

$$\frac{3\left(\log(4x^2-6x+9)+\log(4x^2+6x+9)+\frac{6}{3-2x}-5\log(3-2x)+\log(2x+3)\right)+6\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488}$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] (6*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 +
 4*x)/(3*Sqrt[3])] + 3*(6/(3 - 2*x) - 5*Log[3 - 2*x] + Log[3 + 2*
x] + Log[9 - 6*x + 4*x^2] + Log[9 + 6*x + 4*x^2]))/52488
```

Maple [A] time = 0.015, size = 85, normalized size = 0.8

$$\frac{\ln(2x+3)}{17496}-\frac{1}{-8748+5832x}-\frac{5\ln(-3+2x)}{17496}+\frac{\ln(4x^2+6x+9)}{17496}+\frac{\sqrt{3}}{26244}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)+\frac{\ln(4x^2-6x+9)}{17496}+\frac{\sqrt{3}}{8748}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x)
```

```
[Out] 1/17496*ln(2*x+3)-1/2916/(-3+2*x)-5/17496*ln(-3+2*x)+1/17496*ln(4
*x^2+6*x+9)+1/26244*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/17496*
ln(4*x^2-6*x+9)+1/8748*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.50101, size = 113, normalized size = 1.03

$$\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right)+\frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{1}{17496}\log(2x+3)-\frac{5}{17496}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)/(64*x^6 - 729)^2,x, a
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*ar
ctan(1/9*sqrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^
2 + 6*x + 9) + 1/17496*log(4*x^2 - 6*x + 9) + 1/17496*log(2*x + 3
) - 5/17496*log(2*x - 3)
```

Fricas [A] time = 0.214526, size = 169, normalized size = 1.54

$$\frac{\sqrt{3}\left(\sqrt{3}(2x-3)\log\left(4x^2+6x+9\right)+\sqrt{3}(2x-3)\log\left(4x^2-6x+9\right)+\sqrt{3}(2x-3)\log(2x+3)-5\sqrt{3}(2x-3)\log(2x-3\right.}{52488(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)/(64*x^6 - 729)^2,x, a
```

```
[Out] 1/52488*sqrt(3)*(sqrt(3)*(2*x - 3)*log(4*x^2 + 6*x + 9) + sqrt(3)
*(2*x - 3)*log(4*x^2 - 6*x + 9) + sqrt(3)*(2*x - 3)*log(2*x + 3)
- 5*sqrt(3)*(2*x - 3)*log(2*x - 3) + 2*(2*x - 3)*arctan(1/9*sqrt(
3)*(4*x + 3)) + 6*(2*x - 3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 6*sqr
t(3))/(2*x - 3)
```

Sympy [A] time = 0.593222, size = 105, normalized size = 0.95

$$-\frac{5\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{8748}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{26244}-\frac{1}{5832x-8748}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729)**2,x)
```

```
[Out] -5*log(x - 3/2)/17496 + log(x + 3/2)/17496 + log(x**2 - 3*x/2 + 9
/4)/17496 + log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3
)*x/9 - sqrt(3)/3)/8748 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)
/26244 - 1/(5832*x - 8748)
```

GIAC/XCAS [A] time = 0.218703, size = 116, normalized size = 1.05

$$\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916\,(2x-3)}+\frac{1}{17496}\ln\left(4x^2+6x+9\right)+\frac{1}{17496}\ln\left(4x^2-6x+9\right)+\frac{1}{17496}\ln\left(|2x+3|\right)-\frac{5}{17496}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)/(64*x^6 - 729)^2,x, a
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*ar
ctan(1/9*sqrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*ln(4*x^2
 + 6*x + 9) + 1/17496*ln(4*x^2 - 6*x + 9) + 1/17496*ln(abs(2*x +
3)) - 5/17496*ln(abs(2*x - 3))
```

3.555 $\int \frac{81+36x^2+16x^4}{(729-64x^6)^2} dx$

Optimal. Leaf size=81

$$\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

```
[Out] 1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*S
qrt[3])]/(13122*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(13122*S
qrt[3]) + ArcTanh[(2*x)/3]/8748
```

Rubi [A] time = 0.126895, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

Antiderivative was successfully verified.

```
[In]  Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*S
qrt[3])]/(13122*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(13122*S
qrt[3]) + ArcTanh[(2*x)/3]/8748
```

Rubi in Sympy [A] time = 21.9605, size = 65, normalized size = 0.8

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{4x}{9}-\frac{1}{3}\right)\right)}{39366}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{4x}{9}+\frac{1}{3}\right)\right)}{39366}+\frac{\mathrm{atanh}\left(\frac{2x}{3}\right)}{8748}-\frac{1}{17496\,(2x+3)}+\frac{1}{17496\,(-2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((16*x**4+36*x**2+81)/(-64*x**6+729)**2,x)
```

```
[Out] sqrt(3)*atan(sqrt(3)*(4*x/9 - 1/3))/39366 + sqrt(3)*atan(sqrt(3)*
(4*x/9 + 1/3))/39366 + atanh(2*x/3)/8748 - 1/(17496*(2*x + 3)) +
1/(17496*(-2*x + 3))
```

Mathematica [C] time = 0.771274, size = 122, normalized size = 1.51

$$\frac{\frac{36x}{9-4x^2}-9\log(3-2x)+9\log(2x+3)+3\sqrt{3}\tan^{-1}\left(\frac{1}{3}\left(\sqrt{3}-i\right)x\right)+4i\sqrt{3}\tanh^{-1}\left(\frac{1}{3}\left(1-i\sqrt{3}\right)x\right)+\left(-3+\frac{2}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\right)\tanh^{-1}}{157464}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 4*x^2) + 3*Sqrt[3]*ArcTan[((-I + Sqrt[3])*x)/3] + (4
*I)*Sqrt[3]*ArcTanh[((1 - I*Sqrt[3])*x)/3] + (-3 + 2/Sqrt[(1 + I*
Sqrt[3])/6])*ArcTanh[(x + I*Sqrt[3]*x)/3] - 9*Log[3 - 2*x] + 9*Lo
g[3 + 2*x])/157464
```

Maple [A] time = 0.017, size = 68, normalized size = 0.8

$$-\frac{1}{34992\,x+52488}+\frac{\ln\left(2\,x+3\right)}{17496}-\frac{1}{-52488+34992\,x}-\frac{\ln\left(-3+2\,x\right)}{17496}+\frac{\sqrt{3}}{39366}\arctan\left(\frac{\left(8\,x+6\right)\sqrt{3}}{18}\right)+\frac{\sqrt{3}}{39366}\arctan\left(\frac{\left(8\,x-6\right)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x)
```

```
[Out] -1/17496/(2*x+3)+1/17496*ln(2*x+3)-1/17496/(-3+2*x)-1/17496*ln(-3
+2*x)+1/39366*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/39366*3^(1/2
)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.50652, size = 82, normalized size = 1.01

$$\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)-\frac{x}{4374\,(4\,x^2-9)}+\frac{1}{17496}\log\left(2\,x+3\right)-\frac{1}{17496}\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^4 + 36*x^2 + 81)/(64*x^6 - 729)^2,x, algorithm="maxima")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*a
rctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log
(2*x + 3) - 1/17496*log(2*x - 3)
```

Fricas [A] time = 0.205496, size = 131, normalized size = 1.62

$$\frac{\sqrt{3}\left(3\,\sqrt{3}\left(4\,x^2-9\right)\log\left(2\,x+3\right)-3\,\sqrt{3}\left(4\,x^2-9\right)\log\left(2\,x-3\right)+4\left(4\,x^2-9\right)\arctan\left(\frac{4}{81}\sqrt{3}\left(2\,x^3+9\,x\right)\right)+4\left(4\,x^2-9\right)\mathrm{arcta}\right.}{157464\,(4\,x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^4 + 36*x^2 + 81)/(64*x^6 - 729)^2,x, algorithm="fricas")
```

```
[Out] 1/157464*sqrt(3)*(3*sqrt(3)*(4*x^2 - 9)*log(2*x + 3) - 3*sqrt(3)*
(4*x^2 - 9)*log(2*x - 3) + 4*(4*x^2 - 9)*arctan(4/81*sqrt(3)*(2*x
^3 + 9*x)) + 4*(4*x^2 - 9)*arctan(2/9*sqrt(3)*x) - 12*sqrt(3)*x)/
(4*x^2 - 9)
```

Sympy [A] time = 0.271751, size = 70, normalized size = 0.86

$$-\frac{x}{17496x^2-39366}+\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{9}\right)+2\,\mathrm{atan}\left(\frac{8\sqrt{3}x^3}{81}+\frac{4\sqrt{3}x}{9}\right)\right)}{78732}-\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x**4+36*x**2+81)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(17496*x**2 - 39366) + sqrt(3)*(2*atan(2*sqrt(3)*x/9) + 2*atan
(8*sqrt(3)*x**3/81 + 4*sqrt(3)*x/9))/78732 - log(x - 3/2)/17496 +
```

```
log(x + 3/2)/17496
```

GIAC/XCAS [A] time = 0.21981, size = 85, normalized size = 1.05

$$\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374(4x^2-9)}+\frac{1}{17496}\ln\left(|2x+3|\right)-\frac{1}{17496}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4 + 36*x^2 + 81)/(64*x^6 - 729)^2,x, algorithm="giac")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*a
rctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*ln(
abs(2*x + 3)) - 1/17496*ln(abs(2*x - 3))
```

3.556 $\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=92

$$\frac{x}{4374\,(4x^2-6x+9)}-\frac{\log\left(4x^2-6x+9\right)}{157464}+\frac{\log\left(4x^2+6x+9\right)}{52488}-\frac{\log(3-2x)}{26244}+\frac{\log(2x+3)}{78732}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}$$

```
[Out] x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*
Sqrt[3]) - Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x
+ 4*x^2]/157464 + Log[9 + 6*x + 4*x^2]/52488
```

Rubi [A] time = 0.207291, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$\frac{x}{4374\,(4x^2-6x+9)}-\frac{\log\left(4x^2-6x+9\right)}{157464}+\frac{\log\left(4x^2+6x+9\right)}{52488}-\frac{\log(3-2x)}{26244}+\frac{\log(2x+3)}{78732}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*
Sqrt[3]) - Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x
+ 4*x^2]/157464 + Log[9 + 6*x + 4*x^2]/52488
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0593725, size = 84, normalized size = 0.91

$$\frac{\frac{36x}{4x^2-6x+9}-\log\left(4x^2-6x+9\right)+3\log\left(4x^2+6x+9\right)-6\log(3-2x)+2\log(2x+3)+12\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{157464}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 6*x + 4*x^2) + 12*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[
3])] - 6*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 3
*Log[9 + 6*x + 4*x^2])/157464
```

Maple [A] time = 0.019, size = 73, normalized size = 0.8

$$\frac{\ln(2\,x+3)}{78732}-\frac{\ln(-3+2\,x)}{26244}+\frac{\ln\left(4\,x^2+6\,x+9\right)}{52488}+\frac{x}{17496}\left(x^2-\frac{3\,x}{2}+\frac{9}{4}\right)^{-1}$$
$$-\frac{\ln\left(16\,x^2-24\,x+36\right)}{157464}+\frac{\sqrt{3}}{13122}\arctan\left(\frac{(32\,x-24)\sqrt{3}}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x)
```

```
[Out] 1/78732*ln(2*x+3)-1/26244*ln(-3+2*x)+1/52488*ln(4*x^2+6*x+9)+1/17
496*x/(x^2-3/2*x+9/4)-1/157464*ln(16*x^2-24*x+36)+1/13122*3^(1/2)
*arctan(1/72*(32*x-24)*3^(1/2))
```

Maxima [A] time = 1.58063, size = 100, normalized size = 1.09

$$\frac{1}{13122}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\,(4x^2-6x+9)}+\frac{1}{52488}\log\left(4x^2+6x+9\right)-\frac{1}{157464}\log\left(4x^2-6x+9\right)+\frac{1}{78732}\log\left(2x+3\right)-\frac{1}{26244}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(16*x^4 + 24*x^3 - 54*x - 81)/(64*x^6 - 729)^2,x, algorithm="maxima")
```

```
[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 -
 6*x + 9) + 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6
*x + 9) + 1/78732*log(2*x + 3) - 1/26244*log(2*x - 3)
```

Fricas [A] time = 0.21308, size = 190, normalized size = 2.07

$$\frac{\sqrt{3}\left(3\sqrt{3}\left(4x^2-6x+9\right)\log\left(4x^2+6x+9\right)-\sqrt{3}\left(4x^2-6x+9\right)\log\left(4x^2-6x+9\right)+2\sqrt{3}\left(4x^2-6x+9\right)\log\left(2x+3\right)-\right.}{472392\,(4x^2-6x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(16*x^4 + 24*x^3 - 54*x - 81)/(64*x^6 - 729)^2,x, algorithm="fricas")
```

```
[Out] 1/472392*sqrt(3)*(3*sqrt(3)*(4*x^2 - 6*x + 9)*log(4*x^2 + 6*x + 9
) - sqrt(3)*(4*x^2 - 6*x + 9)*log(4*x^2 - 6*x + 9) + 2*sqrt(3)*(4
*x^2 - 6*x + 9)*log(2*x + 3) - 6*sqrt(3)*(4*x^2 - 6*x + 9)*log(2*
x - 3) + 36*(4*x^2 - 6*x + 9)*arctan(1/9*sqrt(3)*(4*x - 3)) + 36*
sqrt(3)*x)/(4*x^2 - 6*x + 9)
```

Sympy [A] time = 0.462992, size = 82, normalized size = 0.89

$$\frac{x}{17496x^2-26244x+39366}-\frac{\log\left(x-\frac{3}{2}\right)}{26244}+\frac{\log\left(x+\frac{3}{2}\right)}{78732}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{157464}+\frac{\log\left(4x^2+6x+9\right)}{52488}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{13122}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729)**2,x)
```

```
[Out] x/(17496*x**2 - 26244*x + 39366) - log(x - 3/2)/26244 + log(x + 3
/2)/78732 - log(x**2 - 3*x/2 + 9/4)/157464 + log(4*x**2 + 6*x + 9
)/52488 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/13122
```

GIAC/XCAS [A] time = 0.219609, size = 103, normalized size = 1.12

$$\frac{1}{13122}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+\frac{x}{4374\,(4\,x^2-6\,x+9)}+\frac{1}{52488}\ln\left(4\,x^2+6\,x+9\right)-\frac{1}{157464}\ln\left(4\,x^2-6\,x+9\right)+\frac{1}{78732}\ln\left(|2\,x+3|\right)-\frac{1}{26244}\ln\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(16*x^4 + 24*x^3 - 54*x - 81)/(64*x^6 - 729)^2,x, algorithm="giac")
```

```
[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 -
 6*x + 9) + 1/52488*ln(4*x^2 + 6*x + 9) - 1/157464*ln(4*x^2 - 6*x
 + 9) + 1/78732*ln(abs(2*x + 3)) - 1/26244*ln(abs(2*x - 3))
```

3.557 $\int \frac{3-2x}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=148

$$\frac{3-x}{708588\left(4x^2-6x+9\right)}+\frac{x}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{8503056}-\frac{1}{708588(2x+3)}-\frac{\log(3-2x)}{4251528}+\frac{\log(2x+3)}{472392}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}$$

```
[Out] -1/(708588*(3 + 2*x)) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(2
36196*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176
*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[
3 - 2*x]/4251528 + Log[3 + 2*x]/472392 - Log[9 - 6*x + 4*x^2]/944
784 + Log[9 + 6*x + 4*x^2]/8503056
```

Rubi [A] time = 0.318964, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{3-x}{708588\left(4x^2-6x+9\right)}+\frac{x}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{8503056}-\frac{1}{708588(2x+3)}-\frac{\log(3-2x)}{4251528}+\frac{\log(2x+3)}{472392}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] -1/(708588*(3 + 2*x)) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(2
36196*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176
*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[
3 - 2*x]/4251528 + Log[3 + 2*x]/472392 - Log[9 - 6*x + 4*x^2]/944
784 + Log[9 + 6*x + 4*x^2]/8503056
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3-2*x)/(-64*x**6+729)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0966992, size = 119, normalized size = 0.8

$$\frac{-9\log\left(4x^2-6x+9\right)+\log\left(4x^2+6x+9\right)+\frac{1944x}{32x^5+48x^4+72x^3+108x^2+162x+243}-2\log(3-2x)+18\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-}{3\sqrt{3}}\right.}{8503056}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5) + 2*
Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4
```

```
*x)/(3*Sqrt[3])] - 2*Log[3 - 2*x] + 18*Log[3 + 2*x] - 9*Log[9 - 6
*x + 4*x^2] + Log[9 + 6*x + 4*x^2])/8503056
```

Maple [A] time = 0.027, size = 115, normalized size = 0.8

$$-\frac{1}{1417176x+2125764}+\frac{\ln(2x+3)}{472392}-\frac{\ln(-3+2x)}{4251528}+\frac{x}{944784}\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)^{-1}+\frac{\ln(16x^2+24x+36)}{8503056}+\frac{\sqrt{3}}{472392}\arctan\left(\frac{(32x+24)\sqrt{3}}{72}\right)-\frac{1}{708588}\left(\frac{x}{4}-\frac{3}{4}\right)\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)^{-1}-\frac{\ln(16x^2-24x+36)}{944784}+\frac{\sqrt{3}}{4251528}\arctan\left(\frac{(32x-24)\sqrt{3}}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3-2*x)/(-64*x^6+729)^2,x)
```

```
[Out] -1/708588/(2*x+3)+1/472392*ln(2*x+3)-1/4251528*ln(-3+2*x)+1/94478
4*x/(x^2+3/2*x+9/4)+1/8503056*ln(16*x^2+24*x+36)+1/472392*3^(1/2)
*arctan(1/72*(32*x+24)*3^(1/2))-1/708588*(1/4*x-3/4)/(x^2-3/2*x+9
/4)-1/944784*ln(16*x^2-24*x+36)+1/4251528*3^(1/2)*arctan(1/72*(32
*x-24)*3^(1/2))
```

Maxima [A] time = 1.6464, size = 142, normalized size = 0.96

$$\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\,(32x^5+48x^4+72x^3+108x^2+162x+243)}+\frac{1}{8503056}\log\left(4x^2+6x+9\right)-\frac{1}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{472392}\log(2x+3)-\frac{1}{4251528}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x - 3)/(64*x^6 - 729)^2,x, algorithm="maxima")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3
)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(32*x^5 + 48*x^4 + 72*
x^3 + 108*x^2 + 162*x + 243) + 1/8503056*log(4*x^2 + 6*x + 9) - 1
/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(2*x + 3) - 1/4251528*
log(2*x - 3)
```

Fricas [A] time = 0.21398, size = 362, normalized size = 2.45

$$\sqrt{3}\Big(\sqrt{3}(32x^5+48x^4+72x^3+108x^2+162x+243)\log\left(4x^2+6x+9\right)-9\sqrt{3}(32x^5+48x^4+72x^3+108x^2+162x+24$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x - 3)/(64*x^6 - 729)^2,x, algorithm="fricas")
```

```
[Out] 1/25509168*sqrt(3)*(sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 +
 162*x + 243)*log(4*x^2 + 6*x + 9) - 9*sqrt(3)*(32*x^5 + 48*x^4 +
 72*x^3 + 108*x^2 + 162*x + 243)*log(4*x^2 - 6*x + 9) + 18*sqrt(3
)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*log(2*x + 3)
 - 2*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*l
og(2*x - 3) + 54*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 24
3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 6*(32*x^5 + 48*x^4 + 72*x^3 +
```

```
108*x^2 + 162*x + 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1944*sqrt(
3)*x)/(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)
```

Sympy [A] time = 0.877677, size = 124, normalized size = 0.84

$$\frac{x}{139968x^5 + 209952x^4 + 314928x^3 + 472392x^2 + 708588x + 1062882} - \frac{\log\left(x - \frac{3}{2}\right)}{4251528} + \frac{\log\left(x + \frac{3}{2}\right)}{472392} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{944784} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{8503056} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{4251528} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9} + \frac{\sqrt{3}}{3}\right)}{472392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((3-2*x)/(-64*x**6+729)**2,x)
```

```
[Out] x/(139968*x**5 + 209952*x**4 + 314928*x**3 + 472392*x**2 + 708588
*x + 1062882) - log(x - 3/2)/4251528 + log(x + 3/2)/472392 - log(
x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/8503056 + sq
rt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/4251528 + sqrt(3)*atan(4*sq
rt(3)*x/9 + sqrt(3)/3)/472392
```

GIAC/XCAS [A] time = 0.221901, size = 150, normalized size = 1.01

$$\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{x}{4374\,(4x^2+6x+9)(4x^2-6x+9)(2x+3)} + \frac{1}{8503056}\ln\left(4x^2+6x+9\right) - \frac{1}{944784}\ln\left(4x^2-6x+9\right) + \frac{1}{472392}\ln\left(|2x+3|\right) - \frac{1}{4251528}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(2*x - 3)/(64*x^6 - 729)^2,x, algorithm="giac")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3
)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/((4*x^2 + 6*x + 9)*(4*
x^2 - 6*x + 9)*(2*x + 3)) + 1/8503056*ln(4*x^2 + 6*x + 9) - 1/944
784*ln(4*x^2 - 6*x + 9) + 1/472392*ln(abs(2*x + 3)) - 1/4251528*l
n(abs(2*x - 3))
```

3.558 $\int \frac{3+2x}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=146

$$\frac{x}{236196(4x^2-6x+9)}-\frac{x+3}{708588(4x^2+6x+9)}-\frac{\log(4x^2-6x+9)}{8503056}+\frac{\log(4x^2+6x+9)}{944784}+\frac{1}{708588(3-2x)}-\frac{\log(3-2x)}{472392}+\frac{\log(2x+3)}{4251528}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{1417176\sqrt{3}}$$

```
[Out] 1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(70
8588*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*S
qrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3
 - 2*x]/472392 + Log[3 + 2*x]/4251528 - Log[9 - 6*x + 4*x^2]/8503
056 + Log[9 + 6*x + 4*x^2]/944784
```

Rubi [A] time = 0.314212, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{x}{236196(4x^2-6x+9)}-\frac{x+3}{708588(4x^2+6x+9)}-\frac{\log(4x^2-6x+9)}{8503056}+\frac{\log(4x^2+6x+9)}{944784}+\frac{1}{708588(3-2x)}-\frac{\log(3-2x)}{472392}+\frac{\log(2x+3)}{4251528}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{1417176\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(70
8588*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*S
qrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3
 - 2*x]/472392 + Log[3 + 2*x]/4251528 - Log[9 - 6*x + 4*x^2]/8503
056 + Log[9 + 6*x + 4*x^2]/944784
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+2*x)/(-64*x**6+729)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.136821, size = 121, normalized size = 0.83

$$\frac{-\log(4x^2-6x+9)+9\log(4x^2+6x+9)+\frac{1944x}{-32x^5+48x^4-72x^3+108x^2-162x+243}-18\log(3-2x)+2\log(2x+3)+18\sqrt{3}\tan^{-1}\left(\right.}{8503056}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5) + 18
*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4
```

```
*x)/(3*Sqrt[3])] - 18*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x
 + 4*x^2] + 9*Log[9 + 6*x + 4*x^2])/8503056
```

Maple [A] time = 0.023, size = 115, normalized size = 0.8

$$\frac{\ln(2x+3)}{4251528}-\frac{1}{-2125764+1417176x}-\frac{\ln(-3+2x)}{472392}+\frac{1}{708588}\left(-\frac{x}{4}-\frac{3}{4}\right)\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)^{-1}+\frac{\ln\left(16x^2+24x+36\right)}{944784}+\frac{\sqrt{3}}{4251528}\arctan\left(\frac{(32x+24)\sqrt{3}}{72}\right)+\frac{x}{944784}\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)^{-1}-\frac{\ln\left(16x^2-24x+36\right)}{8503056}+\frac{\sqrt{3}}{472392}\arctan\left(\frac{(32x-24)\sqrt{3}}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((2*x+3)/(-64*x^6+729)^2,x)
```

```
[Out] 1/4251528*ln(2*x+3)-1/708588/(-3+2*x)-1/472392*ln(-3+2*x)+1/70858
8*(-1/4*x-3/4)/(x^2+3/2*x+9/4)+1/944784*ln(16*x^2+24*x+36)+1/4251
528*3^(1/2)*arctan(1/72*(32*x+24)*3^(1/2))+1/944784*x/(x^2-3/2*x+
9/4)-1/8503056*ln(16*x^2-24*x+36)+1/472392*3^(1/2)*arctan(1/72*(3
2*x-24)*3^(1/2))
```

Maxima [A] time = 1.72804, size = 142, normalized size = 0.97

$$\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)}+\frac{1}{944784}\log\left(4x^2+6x+9\right)-\frac{1}{8503056}\log\left(4x^2-6x+9\right)+\frac{1}{4251528}\log(2x+3)-\frac{1}{472392}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x + 3)/(64*x^6 - 729)^2,x, algorithm="maxima")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3
)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(32*x^5 - 48*x^4 + 72*
x^3 - 108*x^2 + 162*x - 243) + 1/944784*log(4*x^2 + 6*x + 9) - 1/
8503056*log(4*x^2 - 6*x + 9) + 1/4251528*log(2*x + 3) - 1/472392*
log(2*x - 3)
```

Fricas [A] time = 0.213924, size = 363, normalized size = 2.49

$$\frac{\sqrt{3}\left(9\sqrt{3}\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)\log\left(4x^2+6x+9\right)-\sqrt{3}\left(32x^5-48x^4+72x^3-108x^2+162x-2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x + 3)/(64*x^6 - 729)^2,x, algorithm="fricas")
```

```
[Out] 1/25509168*sqrt(3)*(9*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2
 + 162*x - 243)*log(4*x^2 + 6*x + 9) - sqrt(3)*(32*x^5 - 48*x^4 +
 72*x^3 - 108*x^2 + 162*x - 243)*log(4*x^2 - 6*x + 9) + 2*sqrt(3)
*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*log(2*x + 3)
- 18*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*l
og(2*x - 3) + 6*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243
)*arctan(1/9*sqrt(3)*(4*x + 3)) + 54*(32*x^5 - 48*x^4 + 72*x^3 -
```

```
108*x^2 + 162*x - 243)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1944*sqrt(
3)*x)/(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)
```

Sympy [A] time = 0.864815, size = 124, normalized size = 0.85

$$-\frac{x}{139968x^5-209952x^4+314928x^3-472392x^2+708588x-1062882}-\frac{\log\left(x-\frac{3}{2}\right)}{472392}+\frac{\log\left(x+\frac{3}{2}\right)}{4251528}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{8503056}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{4251528}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+2*x)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(139968*x**5 - 209952*x**4 + 314928*x**3 - 472392*x**2 + 70858
8*x - 1062882) - log(x - 3/2)/472392 + log(x + 3/2)/4251528 - log
(x**2 - 3*x/2 + 9/4)/8503056 + log(x**2 + 3*x/2 + 9/4)/944784 + s
qrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt(3)*atan(4*sq
rt(3)*x/9 + sqrt(3)/3)/4251528
```

GIAC/XCAS [A] time = 0.222503, size = 150, normalized size = 1.03

$$\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\,(4x^2+6x+9)(4x^2-6x+9)(2x-3)}+\frac{1}{944784}\ln\left(4x^2+6x+9\right)-\frac{1}{8503056}\ln\left(4x^2-6x+9\right)+\frac{1}{4251528}\ln\left(|2x+3|\right)-\frac{1}{472392}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(64*x^6 - 729)^2,x, algorithm="giac")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3
)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(4*
x^2 - 6*x + 9)*(2*x - 3)) + 1/944784*ln(4*x^2 + 6*x + 9) - 1/8503
056*ln(4*x^2 - 6*x + 9) + 1/4251528*ln(abs(2*x + 3)) - 1/472392*l
n(abs(2*x - 3))
```

3.559 $\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=142

$$\frac{4x+3}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}-\frac{5\log\left(4x^2+6x+9\right)}{2834352}+\frac{1}{472392(3-2x)}-\frac{1}{157464(2x+3)}-\frac{\log(3-2x)}{354294}+\frac{\log(2x+3)}{118098}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}$$

```
[Out] 1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(
9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]
) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/
354294 + Log[3 + 2*x]/118098 - Log[9 - 6*x + 4*x^2]/944784 - (5*L
og[9 + 6*x + 4*x^2])/2834352
```

Rubi [A] time = 0.267688, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{4x+3}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}-\frac{5\log\left(4x^2+6x+9\right)}{2834352}+\frac{1}{472392(3-2x)}-\frac{1}{157464(2x+3)}-\frac{\log(3-2x)}{354294}+\frac{\log(2x+3)}{118098}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(
9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]
) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/
354294 + Log[3 + 2*x]/118098 - Log[9 - 6*x + 4*x^2]/944784 - (5*L
og[9 + 6*x + 4*x^2])/2834352
```

Rubi in Sympy [A] time = 48.1266, size = 117, normalized size = 0.82

$$\frac{8x+6}{472392\left(4x^2+6x+9\right)}-\frac{\log\left(-2x+3\right)}{354294}+\frac{\log\left(2x+3\right)}{118098}-\frac{\log\left(4x^2-6x+9\right)}{944784}-\frac{5\log\left(4x^2+6x+9\right)}{2834352}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{4x}{9}-\frac{1}{3}\right)\right)}{1417176}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{4x}{9}+\frac{1}{3}\right)\right)}{157464}-\frac{1}{157464\left(2x+3\right)}+\frac{1}{472392\left(-2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2-6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] (8*x + 6)/(472392*(4*x**2 + 6*x + 9)) - log(-2*x + 3)/354294 + lo
g(2*x + 3)/118098 - log(4*x**2 - 6*x + 9)/944784 - 5*log(4*x**2 +
 6*x + 9)/2834352 + sqrt(3)*atan(sqrt(3)*(4*x/9 - 1/3))/1417176 +
 sqrt(3)*atan(sqrt(3)*(4*x/9 + 1/3))/157464 - 1/(157464*(2*x + 3)
) + 1/(472392*(-2*x + 3))
```

Mathematica [A] time = 0.117685, size = 111, normalized size = 0.78

$$\frac{-3\log\left(4x^2-6x+9\right)-5\log\left(4x^2+6x+9\right)+\frac{648x}{-16x^4-24x^3+54x+81}-8\log(3-2x)+24\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+18\sqrt{}}{2834352}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 + 54*x - 24*x^3 - 16*x^4) + 2*Sqrt[3]*ArcTan[(-3 + 4
*x)/(3*Sqrt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 8*L
og[3 - 2*x] + 24*Log[3 + 2*x] - 3*Log[9 - 6*x + 4*x^2] - 5*Log[9
+ 6*x + 4*x^2])/2834352
```

Maple [A] time = 0.022, size = 111, normalized size = 0.8

$$-\frac{1}{314928\,x+472392}+\frac{\ln(2\,x+3)}{118098}-\frac{1}{-1417176+944784\,x}-\frac{\ln(-3+2\,x)}{354294}-\frac{1}{708588}\left(-3\,x-\frac{9}{4}\right)\left(x^2+\frac{3\,x}{2}+\frac{9}{4}\right)^{-1}-\frac{5\,\ln\left(16\,x^2+24\,x+36\right)}{2834352}+\frac{\sqrt{3}}{157464}\arctan\left(\frac{(32\,x+24)\sqrt{3}}{72}\right)-\frac{\ln\left(4\,x^2-6\,x+9\right)}{944784}+\frac{\sqrt{3}}{1417176}\arctan\left(\frac{(8\,x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2-6*x+9)/(-64*x^6+729)^2,x)
```

```
[Out] -1/157464/(2*x+3)+1/118098*ln(2*x+3)-1/472392/(-3+2*x)-1/354294*l
n(-3+2*x)-1/708588*(-3*x-9/4)/(x^2+3/2*x+9/4)-5/2834352*ln(16*x^2
+24*x+36)+1/157464*3^(1/2)*arctan(1/72*(32*x+24)*3^(1/2))-1/94478
4*ln(4*x^2-6*x+9)+1/1417176*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.55608, size = 128, normalized size = 0.9

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)-\frac{x}{4374\,(16\,x^4+24\,x^3-54\,x-81)}-\frac{5}{2834352}\log\left(4\,x^2+6\,x+9\right)-\frac{1}{944784}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{118098}\log(2\,x+3)-\frac{1}{354294}\log(2\,x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 6*x + 9)/(64*x^6 - 729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3
)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 + 24*x^3 - 54*
x - 81) - 5/2834352*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6
*x + 9) + 1/118098*log(2*x + 3) - 1/354294*log(2*x - 3)
```

Fricas [A] time = 0.213703, size = 269, normalized size = 1.89

$$-\frac{\sqrt{3}\left(5\sqrt{3}\left(16\,x^4+24\,x^3-54\,x-81\right)\log\left(4\,x^2+6\,x+9\right)+3\sqrt{3}\left(16\,x^4+24\,x^3-54\,x-81\right)\log\left(4\,x^2-6\,x+9\right)-24\sqrt{3}(16\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 6*x + 9)/(64*x^6 - 729)^2,x, algorithm="fricas")
```

```
[Out] -1/8503056*sqrt(3)*(5*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*log(4
*x^2 + 6*x + 9) + 3*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*log(4*x
^2 - 6*x + 9) - 24*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*log(2*x
```

```
+ 3) + 8*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*log(2*x - 3) - 54*
(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x + 3)) - 6*(
16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x - 3)) + 648*
sqrt(3)*x)/(16*x^4 + 24*x^3 - 54*x - 81)
```

Sympy [A] time = 0.832613, size = 116, normalized size = 0.82

$$-\frac{x}{69984x^4+104976x^3-236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{354294}+\frac{\log\left(x+\frac{3}{2}\right)}{118098}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}-\frac{5\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{2834352}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{1417176}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2-6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 + 104976*x**3 - 236196*x - 354294) - log(x - 3/2)/
354294 + log(x + 3/2)/118098 - log(x**2 - 3*x/2 + 9/4)/944784 - 5
*log(x**2 + 3*x/2 + 9/4)/2834352 + sqrt(3)*atan(4*sqrt(3)*x/9 - s
qrt(3)/3)/1417176 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/15746
4
```

GIAC/XCAS [A] time = 0.219882, size = 143, normalized size = 1.01

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\,(4x^2+6x+9)(2x+3)(2x-3)}-\frac{5}{2834352}\ln\left(4x^2+6x+9\right)-\frac{1}{944784}\ln\left(4x^2-6x+9\right)+\frac{1}{118098}\ln\left(|2x+3|\right)-\frac{1}{354294}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 6*x + 9)/(64*x^6 - 729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3
)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(2*
x + 3)*(2*x - 3)) - 5/2834352*ln(4*x^2 + 6*x + 9) - 1/944784*ln(4
*x^2 - 6*x + 9) + 1/118098*ln(abs(2*x + 3)) - 1/354294*ln(abs(2*x
 - 3))
```

3.560 $\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=142

$$-\frac{3-4x}{236196\left(4x^2-6x+9\right)}+\frac{5\log\left(4x^2-6x+9\right)}{2834352}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-2x)}{118098}+\frac{\log(2x+3)}{354294}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{472392\sqrt{3}}$$

```
[Out] 1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(
9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3])
 + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/
118098 + Log[3 + 2*x]/354294 + (5*Log[9 - 6*x + 4*x^2])/2834352 +
 Log[9 + 6*x + 4*x^2]/944784
```

Rubi [A] time = 0.262404, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{3-4x}{236196\left(4x^2-6x+9\right)}+\frac{5\log\left(4x^2-6x+9\right)}{2834352}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-2x)}{118098}+\frac{\log(2x+3)}{354294}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{472392\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(
9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3])
 + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/
118098 + Log[3 + 2*x]/354294 + (5*Log[9 - 6*x + 4*x^2])/2834352 +
 Log[9 + 6*x + 4*x^2]/944784
```

Rubi in Sympy [A] time = 55.4866, size = 117, normalized size = 0.82

$$-\frac{-8x+6}{472392\left(4x^2-6x+9\right)}-\frac{\log\left(-2x+3\right)}{118098}+\frac{\log\left(2x+3\right)}{354294}+\frac{5\log\left(4x^2-6x+9\right)}{2834352}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{4x}{9}-\frac{1}{3}\right)\right)}{157464}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{4x}{9}+\frac{1}{3}\right)\right)}{1417176}-\frac{1}{472392\left(2x+3\right)}+\frac{1}{157464\left(-2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2+6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -(-8*x + 6)/(472392*(4*x**2 - 6*x + 9)) - log(-2*x + 3)/118098 +
log(2*x + 3)/354294 + 5*log(4*x**2 - 6*x + 9)/2834352 + log(4*x**
2 + 6*x + 9)/944784 + sqrt(3)*atan(sqrt(3)*(4*x/9 - 1/3))/157464
+ sqrt(3)*atan(sqrt(3)*(4*x/9 + 1/3))/1417176 - 1/(472392*(2*x +
3)) + 1/(157464*(-2*x + 3))
```

Mathematica [A] time = 0.10452, size = 111, normalized size = 0.78

$$\frac{5\log\left(4x^2-6x+9\right)+3\log\left(4x^2+6x+9\right)+\frac{648x}{-16x^4+24x^3-54x+81}-24\log(3-2x)+8\log(2x+3)+18\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+2\sqrt{3}}{2834352}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 - 54*x + 24*x^3 - 16*x^4) + 18*Sqrt[3]*ArcTan[(-3 +
4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 24*
Log[3 - 2*x] + 8*Log[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*Log[9
+ 6*x + 4*x^2])/2834352
```

Maple [A] time = 0.021, size = 111, normalized size = 0.8

$$-\frac{1}{944784\,x+1417176}+\frac{\ln(2\,x+3)}{354294}-\frac{1}{-472392+314928\,x}-\frac{\ln(-3+2\,x)}{118098}+\frac{\ln\left(4\,x^2+6\,x+9\right)}{944784}+\frac{\sqrt{3}}{1417176}\arctan\left(\frac{(8\,x+6)\,\sqrt{3}}{18}\right)+\frac{1}{708588}\left(3\,x-\frac{9}{4}\right)\left(x^2-\frac{3\,x}{2}+\frac{9}{4}\right)^{-1}+\frac{5\,\ln\left(16\,x^2-24\,x+36\right)}{2834352}+\frac{\sqrt{3}}{157464}\arctan\left(\frac{(32\,x-24)\,\sqrt{3}}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2+6*x+9)/(-64*x^6+729)^2,x)
```

```
[Out] -1/472392/(2*x+3)+1/354294*ln(2*x+3)-1/157464/(-3+2*x)-1/118098*l
n(-3+2*x)+1/944784*ln(4*x^2+6*x+9)+1/1417176*3^(1/2)*arctan(1/18*
(8*x+6)*3^(1/2))+1/708588*(3*x-9/4)/(x^2-3/2*x+9/4)+5/2834352*ln(
16*x^2-24*x+36)+1/157464*3^(1/2)*arctan(1/72*(32*x-24)*3^(1/2))
```

Maxima [A] time = 1.51937, size = 128, normalized size = 0.9

$$\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)-\frac{x}{4374\,(16\,x^4-24\,x^3+54\,x-81)}+\frac{1}{944784}\log\left(4\,x^2+6\,x+9\right)+\frac{5}{2834352}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{354294}\log(2\,x+3)-\frac{1}{118098}\log(2\,x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 6*x + 9)/(64*x^6 - 729)^2,x, algorithm="maxima")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3
)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 - 24*x^3 + 54*
x - 81) + 1/944784*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6
*x + 9) + 1/354294*log(2*x + 3) - 1/118098*log(2*x - 3)
```

Fricas [A] time = 0.215037, size = 269, normalized size = 1.89

$$\frac{\sqrt{3}\left(3\,\sqrt{3}\left(16\,x^4-24\,x^3+54\,x-81\right)\log\left(4\,x^2+6\,x+9\right)+5\,\sqrt{3}\left(16\,x^4-24\,x^3+54\,x-81\right)\log\left(4\,x^2-6\,x+9\right)+8\,\sqrt{3}\left(16\,x^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 6*x + 9)/(64*x^6 - 729)^2,x, algorithm="fricas")
```

```
[Out] 1/8503056*sqrt(3)*(3*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*log(4*
x^2 + 6*x + 9) + 5*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*log(4*x^
2 - 6*x + 9) + 8*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*log(2*x +
```

```
3) - 24*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*log(2*x - 3) + 6*(1
6*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x + 3)) + 54*(1
6*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x - 3)) - 648*s
qrt(3)*x)/(16*x^4 - 24*x^3 + 54*x - 81)
```

Sympy [A] time = 0.833748, size = 116, normalized size = 0.82

$$-\frac{x}{69984x^4-104976x^3+236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{118098}+\frac{\log\left(x+\frac{3}{2}\right)}{354294}+\frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{2834352}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{157464}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{1417176}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 - 104976*x**3 + 236196*x - 354294) - log(x - 3/2)/
118098 + log(x + 3/2)/354294 + 5*log(x**2 - 3*x/2 + 9/4)/2834352
+ log(x**2 + 3*x/2 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - s
qrt(3)/3)/157464 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/141717
6
```

GIAC/XCAS [A] time = 0.221159, size = 143, normalized size = 1.01

$$\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\,(4x^2-6x+9)(2x+3)(2x-3)}+\frac{1}{944784}\ln\left(4x^2+6x+9\right)+\frac{5}{2834352}\ln\left(4x^2-6x+9\right)+\frac{1}{354294}\ln\left(|2x+3|\right)-\frac{1}{118098}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 6*x + 9)/(64*x^6 - 729)^2,x, algorithm="giac")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3
)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*
x + 3)*(2*x - 3)) + 1/944784*ln(4*x^2 + 6*x + 9) + 5/2834352*ln(4
*x^2 - 6*x + 9) + 1/354294*ln(abs(2*x + 3)) - 1/118098*ln(abs(2*x
 - 3))
```

3.561 $\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=113

$$\frac{x}{4374(8x^3+27)}-\frac{7\log(4x^2-6x+9)}{944784}+\frac{\log(4x^2+6x+9)}{314928}-\frac{\log(3-2x)}{157464}+\frac{7\log(2x+3)}{472392}-\frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}$$

```
[Out] x/(4374*(27 + 8*x^3)) - (7*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464
*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3
 - 2*x]/157464 + (7*Log[3 + 2*x])/472392 - (7*Log[9 - 6*x + 4*x^2
])/944784 + Log[9 + 6*x + 4*x^2]/314928
```

Rubi [A] time = 0.167001, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529

$$\frac{x}{4374(8x^3+27)}-\frac{7\log(4x^2-6x+9)}{944784}+\frac{\log(4x^2+6x+9)}{314928}-\frac{\log(3-2x)}{157464}+\frac{7\log(2x+3)}{472392}-\frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(27 + 8*x^3)) - (7*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464
*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3
 - 2*x]/157464 + (7*Log[3 + 2*x])/472392 - (7*Log[9 - 6*x + 4*x^2
])/944784 + Log[9 + 6*x + 4*x^2]/314928
```

Rubi in Sympy [A] time = 25.8493, size = 100, normalized size = 0.88

$$\frac{x}{4374(8x^3+27)}-\frac{\log(-2x+3)}{157464}+\frac{7\log(2x+3)}{472392}-\frac{7\log(4x^2-6x+9)}{944784}+\frac{\log(4x^2+6x+9)}{314928}+\frac{7\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{4x}{9}-\frac{1}{3}\right)\right)}{472392}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{4x}{9}+\frac{1}{3}\right)\right)}{157464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-8*x**3+27)/(-64*x**6+729)**2,x)
```

```
[Out] x/(4374*(8*x**3 + 27)) - log(-2*x + 3)/157464 + 7*log(2*x + 3)/47
2392 - 7*log(4*x**2 - 6*x + 9)/944784 + log(4*x**2 + 6*x + 9)/314
928 + 7*sqrt(3)*atan(sqrt(3)*(4*x/9 - 1/3))/472392 + sqrt(3)*atan
(sqrt(3)*(4*x/9 + 1/3))/157464
```

Mathematica [A] time = 0.0930914, size = 103, normalized size = 0.91

$$\frac{\frac{216x}{8x^3+27}-7\log(4x^2-6x+9)+3\log(4x^2+6x+9)-6\log(3-2x)+14\log(2x+3)+14\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{4x+}{3\sqrt{3}}\right.}{944784}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 + 8*x^3) + 14*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])]
 + 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] + 14*
Log[3 + 2*x] - 7*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/9
44784
```

Maple [A] time = 0.019, size = 102, normalized size = 0.9

$$-\frac{1}{157464\,x+236196}+\frac{7\,\ln\left(2\,x+3\right)}{472392}-\frac{\ln\left(-3+2\,x\right)}{157464}+\frac{\ln\left(4\,x^{2}+6\,x+9\right)}{314928}+\frac{\sqrt{3}}{157464}\arctan\left(\frac{\left(8\,x+6\right)\sqrt{3}}{18}\right)-\frac{1}{118098}\left(-\frac{3\,x}{4}-\frac{9}{8}\right)\left(x^{2}-\frac{3\,x}{2}+\frac{9}{4}\right)^{-1}-\frac{7\,\ln\left(16\,x^{2}-24\,x+36\right)}{944784}+\frac{7\sqrt{3}}{472392}\arctan\left(\frac{\left(32\,x-24\right)\sqrt{3}}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-8*x^3+27)/(-64*x^6+729)^2,x)
```

```
[Out] -1/78732/(2*x+3)+7/472392*ln(2*x+3)-1/157464*ln(-3+2*x)+1/314928*
ln(4*x^2+6*x+9)+1/157464*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/1
18098*(-3/4*x-9/8)/(x^2-3/2*x+9/4)-7/944784*ln(16*x^2-24*x+36)+7/
472392*3^(1/2)*arctan(1/72*(32*x-24)*3^(1/2))
```

Maxima [A] time = 1.52106, size = 117, normalized size = 1.04

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+\frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)+\frac{x}{4374\left(8\,x^{3}+27\right)}+\frac{1}{314928}\log\left(4\,x^{2}+6\,x+9\right)-\frac{7}{944784}\log\left(4\,x^{2}-6\,x+9\right)+\frac{7}{472392}\log\left(2\,x+3\right)-\frac{1}{157464}\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(8*x^3 - 27)/(64*x^6 - 729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)
*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928
*log(4*x^2 + 6*x + 9) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*
log(2*x + 3) - 1/157464*log(2*x - 3)
```

Fricas [A] time = 0.21153, size = 193, normalized size = 1.71

$$\frac{\sqrt{3}\left(3\sqrt{3}\left(8\,x^{3}+27\right)\log\left(4\,x^{2}+6\,x+9\right)-7\sqrt{3}\left(8\,x^{3}+27\right)\log\left(4\,x^{2}-6\,x+9\right)+14\sqrt{3}\left(8\,x^{3}+27\right)\log\left(2\,x+3\right)-6\sqrt{3}\left(8\,x^{3}\right.\right.}{2834352\left(8\,x^{3}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(8*x^3 - 27)/(64*x^6 - 729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*sqrt(3)*(3*sqrt(3)*(8*x^3 + 27)*log(4*x^2 + 6*x + 9) -
7*sqrt(3)*(8*x^3 + 27)*log(4*x^2 - 6*x + 9) + 14*sqrt(3)*(8*x^3 +
 27)*log(2*x + 3) - 6*sqrt(3)*(8*x^3 + 27)*log(2*x - 3) + 18*(8*x
^3 + 27)*arctan(1/9*sqrt(3)*(4*x + 3)) + 42*(8*x^3 + 27)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 216*sqrt(3)*x)/(8*x^3 + 27)
```

Sympy [A] time = 0.679236, size = 110, normalized size = 0.97

$$\frac{x}{34992x^3+118098}-\frac{\log\left(x-\frac{3}{2}\right)}{157464}+\frac{7\log\left(x+\frac{3}{2}\right)}{472392}-\frac{7\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928}+\frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-8*x**3+27)/(-64*x**6+729)**2,x)
```

```
[Out] x/(34992*x**3 + 118098) - log(x - 3/2)/157464 + 7*log(x + 3/2)/47
2392 - 7*log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)
/314928 + 7*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt
(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/157464
```

GIAC/XCAS [A] time = 0.219819, size = 120, normalized size = 1.06

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\,(8x^3+27)}+\frac{1}{314928}\ln\left(4x^2+6x+9\right)-\frac{7}{944784}\ln\left(4x^2-6x+9\right)+\frac{7}{472392}\ln\left(|2x+3|\right)-\frac{1}{157464}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(8*x^3 - 27)/(64*x^6 - 729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)
*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928
*ln(4*x^2 + 6*x + 9) - 7/944784*ln(4*x^2 - 6*x + 9) + 7/472392*ln
(abs(2*x + 3)) - 1/157464*ln(abs(2*x - 3))
```

3.562 $\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} \, dx$

Optimal. Leaf size=131

$$-\frac{3-2x}{26244\left(4x^2-6x+9\right)}+\frac{17\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}+\frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(2x+3)}{472392}-\frac{11\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}$$

```
[Out] 1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*A
rcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)
/(3*Sqrt[3])]/(157464*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3
+ 2*x]/472392 + (17*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x +
4*x^2]/314928
```

Rubi [A] time = 0.263878, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$-\frac{3-2x}{26244\left(4x^2-6x+9\right)}+\frac{17\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}+\frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(2x+3)}{472392}-\frac{11\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*A
rcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)
/(3*Sqrt[3])]/(157464*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3
+ 2*x]/472392 + (17*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x +
4*x^2]/314928
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.110413, size = 111, normalized size = 0.85

$$\frac{17\log\left(4x^2-6x+9\right)+3\log\left(4x^2+6x+9\right)+\frac{216x}{-8x^3+24x^2-36x+27}-42\log(3-2x)+2\log(2x+3)+22\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)-2\sqrt{3}}{944784}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 - 36*x + 24*x^2 - 8*x^3) + 22*Sqrt[3]*ArcTan[(-3 + 4
*x)/(3*Sqrt[3])] - 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 42*L
```

```
og[3 - 2*x] + 2*Log[3 + 2*x] + 17*Log[9 - 6*x + 4*x^2] + 3*Log[9
+ 6*x + 4*x^2])/944784
```

Maple [A] time = 0.019, size = 102, normalized size = 0.8

$$\frac{\ln(2x+3)}{472392} - \frac{1}{-78732+52488x} - \frac{7\ln(-3+2x)}{157464} + \frac{\ln(4x^2+6x+9)}{314928} - \frac{\sqrt{3}}{472392}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right) + \frac{1}{118098}\left(\frac{9x}{4}-\frac{27}{8}\right)\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)^{-1} + \frac{17\ln(16x^2-24x+36)}{944784} + \frac{11\sqrt{3}}{472392}\arctan\left(\frac{(32x-24)\sqrt{3}}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x)
```

```
[Out] 1/472392*ln(2*x+3)-1/26244/(-3+2*x)-7/157464*ln(-3+2*x)+1/314928*
ln(4*x^2+6*x+9)-1/472392*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/1
18098*(9/4*x-27/8)/(x^2-3/2*x+9/4)+17/944784*ln(16*x^2-24*x+36)+1
1/472392*3^(1/2)*arctan(1/72*(32*x-24)*3^(1/2))
```

Maxima [A] time = 1.54312, size = 128, normalized size = 0.98

$$-\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374(8x^3-24x^2+36x-27)} + \frac{1}{314928}\log(4x^2+6x+9) + \frac{17}{944784}\log(4x^2-6x+9) + \frac{1}{472392}\log(2x+3) - \frac{7}{157464}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 + 24*x^2 + 36*x + 27)/(64*x^6 - 729)^2,x, algorithm="maxima")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(
3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(8*x^3 - 24*x^2 + 36*
x - 27) + 1/314928*log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6
*x + 9) + 1/472392*log(2*x + 3) - 7/157464*log(2*x - 3)
```

Fricas [A] time = 0.213829, size = 269, normalized size = 2.05

$$\sqrt{3}\Big(3\sqrt{3}(8x^3-24x^2+36x-27)\log(4x^2+6x+9) + 17\sqrt{3}(8x^3-24x^2+36x-27)\log(4x^2-6x+9) + 2\sqrt{3}(8x^3-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 + 24*x^2 + 36*x + 27)/(64*x^6 - 729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*sqrt(3)*(3*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x
^2 + 6*x + 9) + 17*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x^2
 - 6*x + 9) + 2*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x + 3)
 - 42*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x - 3) - 6*(8*x^
3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x + 3)) + 66*(8*x^3
 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x - 3)) - 216*sqrt(3
)*x)/(8*x^3 - 24*x^2 + 36*x - 27)
```

Sympy [A] time = 0.82442, size = 119, normalized size = 0.91

$$-\frac{x}{34992x^3-104976x^2+157464x-118098}-\frac{7\log\left(x-\frac{3}{2}\right)}{157464}+\frac{\log\left(x+\frac{3}{2}\right)}{472392}+\frac{17\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928}+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(34992*x**3 - 104976*x**2 + 157464*x - 118098) - 7*log(x - 3/2
)/157464 + log(x + 3/2)/472392 + 17*log(x**2 - 3*x/2 + 9/4)/94478
4 + log(x**2 + 3*x/2 + 9/4)/314928 + 11*sqrt(3)*atan(4*sqrt(3)*x/
9 - sqrt(3)/3)/472392 - sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/4
72392
```

GIAC/XCAS [A] time = 0.219799, size = 134, normalized size = 1.02

$$-\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\,(4x^2-6x+9)(2x-3)}+\frac{1}{314928}\ln\left(4x^2+6x+9\right)+\frac{17}{944784}\ln\left(4x^2-6x+9\right)+\frac{1}{472392}\ln\left(|2x+3|\right)-\frac{7}{157464}\ln\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3 + 24*x^2 + 36*x + 27)/(64*x^6 - 729)^2,x, algorithm="giac")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(
3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2
*x - 3)) + 1/314928*ln(4*x^2 + 6*x + 9) + 17/944784*ln(4*x^2 - 6*
x + 9) + 1/472392*ln(abs(2*x + 3)) - 7/157464*ln(abs(2*x - 3))
```

3.563 $\int \frac{x(27-2x^3)}{729-64x^6}\,dx$

Optimal. Leaf size=99

$$\frac{5}{576}\log\left(4x^2-6x+9\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(2x+3)-\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}}$$

```
[Out] (-5*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(96*Sqrt[3]) - ArcTan[(3 + 4*x
)/(3*Sqrt[3])]/(32*Sqrt[3]) - Log[3 - 2*x]/96 - (5*Log[3 + 2*x])/
288 + (5*Log[9 - 6*x + 4*x^2])/576 + Log[9 + 6*x + 4*x^2]/192
```

Rubi [A] time = 0.136423, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{5}{576}\log\left(4x^2-6x+9\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(2x+3)-\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```


```
[Out] (-5*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(96*Sqrt[3]) - ArcTan[(3 + 4*x
)/(3*Sqrt[3])]/(32*Sqrt[3]) - Log[3 - 2*x]/96 - (5*Log[3 + 2*x])/
288 + (5*Log[9 - 6*x + 4*x^2])/576 + Log[9 + 6*x + 4*x^2]/192
```

Rubi in Sympy [A] time = 19.2437, size = 92, normalized size = 0.93

$$-\frac{\log\left(-2x+3\right)}{96}-\frac{5\log\left(2x+3\right)}{288}+\frac{5\log\left(16x^2-24x+36\right)}{576}+\frac{\log\left(16x^2+24x+36\right)}{192}+\frac{5\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{4x}{9}-\frac{1}{3}\right)\right)}{288}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{4x}{9}+\frac{1}{3}\right)\right)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-2*x**3+27)/(-64*x**6+729),x)
```

```
[Out] -log(-2*x + 3)/96 - 5*log(2*x + 3)/288 + 5*log(16*x**2 - 24*x + 3
6)/576 + log(16*x**2 + 24*x + 36)/192 + 5*sqrt(3)*atan(sqrt(3)*(4
*x/9 - 1/3))/288 - sqrt(3)*atan(sqrt(3)*(4*x/9 + 1/3))/96
```

Mathematica [A] time = 0.0317212, size = 91, normalized size = 0.92

$$\frac{1}{576}\left(5\log\left(4x^2-6x+9\right)+3\log\left(4x^2+6x+9\right)-6\log(3-2x)-10\log(2x+3)+10\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)-6\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

```
[Out] (10*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*Sqrt[3]*ArcTan[(3
+ 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] - 10*Log[3 + 2*x] + 5*Log[9
- 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/576
```

Maple [A] time = 0.013, size = 76, normalized size = 0.8

$$-\frac{5\ln\left(2x+3\right)}{288}-\frac{\ln\left(-3+2x\right)}{96}+\frac{\ln\left(4x^{2}+6x+9\right)}{192}-\frac{\sqrt{3}}{96}\arctan\left(\frac{\left(8x+6\right)\sqrt{3}}{18}\right)+\frac{5\ln\left(4x^{2}-6x+9\right)}{576}+\frac{5\sqrt{3}}{288}\arctan\left(\frac{\left(8x-6\right)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-2*x^3+27)/(-64*x^6+729),x)
```

```
[Out] -5/288*ln(2*x+3)-1/96*ln(-3+2*x)+1/192*ln(4*x^2+6*x+9)-1/96*3^(1/
2)*arctan(1/18*(8*x+6)*3^(1/2))+5/576*ln(4*x^2-6*x+9)+5/288*3^(1/
2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.52351, size = 101, normalized size = 1.02

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(4x^{2}+6x+9\right)+\frac{5}{576}\log\left(4x^{2}-6x+9\right)-\frac{5}{288}\log\left(2x+3\right)-\frac{1}{96}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 - 27)*x/(64*x^6 - 729),x, algorithm="maxima")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arcta
n(1/9*sqrt(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log
(4*x^2 - 6*x + 9) - 5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

Fricas [A] time = 0.213124, size = 116, normalized size = 1.17

$$\frac{1}{1728}\sqrt{3}\left(3\sqrt{3}\log\left(4x^{2}+6x+9\right)+5\sqrt{3}\log\left(4x^{2}-6x+9\right)-10\sqrt{3}\log\left(2x+3\right)-6\sqrt{3}\log\left(2x-3\right)-18\arctan\left(\frac{1}{9}\sqrt{3}(4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 - 27)*x/(64*x^6 - 729),x, algorithm="fricas")
```

```
[Out] 1/1728*sqrt(3)*(3*sqrt(3)*log(4*x^2 + 6*x + 9) + 5*sqrt(3)*log(4*
x^2 - 6*x + 9) - 10*sqrt(3)*log(2*x + 3) - 6*sqrt(3)*log(2*x - 3)
 - 18*arctan(1/9*sqrt(3)*(4*x + 3)) + 30*arctan(1/9*sqrt(3)*(4*x
- 3)))
```

Sympy [A] time = 0.573778, size = 102, normalized size = 1.03

$$-\frac{\log\left(x-\frac{3}{2}\right)}{96}-\frac{5\log\left(x+\frac{3}{2}\right)}{288}+\frac{5\log\left(x^{2}-\frac{3x}{2}+\frac{9}{4}\right)}{576}+\frac{\log\left(x^{2}+\frac{3x}{2}+\frac{9}{4}\right)}{192}+\frac{5\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{288}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-2*x**3+27)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/96 - 5*log(x + 3/2)/288 + 5*log(x**2 - 3*x/2 + 9/4)
/576 + log(x**2 + 3*x/2 + 9/4)/192 + 5*sqrt(3)*atan(4*sqrt(3)*x/9
 - sqrt(3)/3)/288 - sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/96
```

GIAC/XCAS [A] time = 0.220171, size = 93, normalized size = 0.94

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\ln\left(x^2+\frac{3}{2}x+\frac{9}{4}\right)+\frac{5}{576}\ln\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)-\frac{5}{288}\ln\left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{96}\ln\left(\left|x-\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 - 27)*x/(64*x^6 - 729),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arcta
n(1/9*sqrt(3)*(4*x - 3)) + 1/192*ln(x^2 + 3/2*x + 9/4) + 5/576*ln
(x^2 - 3/2*x + 9/4) - 5/288*ln(abs(x + 3/2)) - 1/96*ln(abs(x - 3/
2))
```

3.564 $\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx$

Optimal. Leaf size=162

$$\frac{(cx)^{m+1}\left(a^2g-abf+b^2e\right)}{b^3c(m+1)}+\frac{(cx)^{m+1}\left(a^3(-g)+a^2bf-ab^2e+b^3d\right)\ {}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ab^3c(m+1)}+\frac{x^{n+1}(cx)^m(bf-ag)}{b^2(m+n+1)}+\frac{gx^{2n+1}(cx)^m}{b(m+2n+1)}$$

```
[Out] ((b*f - a*g)*x^(1 + n)*(c*x)^m)/(b^2*(1 + m + n)) + (g*x^(1 + 2*n
)*(c*x)^m)/(b*(1 + m + 2*n)) + ((b^2*e - a*b*f + a^2*g)*(c*x)^(1
+ m))/(b^3*c*(1 + m)) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*(c*x
)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n
)/a)])/(a*b^3*c*(1 + m))
```

Rubi [A] time = 0.312267, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{(cx)^{m+1}\left(a^2g-abf+b^2e\right)}{b^3c(m+1)}+\frac{(cx)^{m+1}\left(a^3(-g)+a^2bf-ab^2e+b^3d\right)\ {}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ab^3c(m+1)}+\frac{x^{n+1}(cx)^m(bf-ag)}{b^2(m+n+1)}+\frac{gx^{2n+1}(cx)^m}{b(m+2n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

```
[Out] ((b*f - a*g)*x^(1 + n)*(c*x)^m)/(b^2*(1 + m + n)) + (g*x^(1 + 2*n
)*(c*x)^m)/(b*(1 + m + 2*n)) + ((b^2*e - a*b*f + a^2*g)*(c*x)^(1
+ m))/(b^3*c*(1 + m)) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*(c*x
)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n
)/a)])/(a*b^3*c*(1 + m))
```

Rubi in Sympy [A] time = 53.3022, size = 190, normalized size = 1.17

$$\frac{fx^{-m}x^{m+2n+1}\,(cx)^m\ {}_2F_1\left(\begin{matrix}1,\frac{m+2n+1}{n}\\\frac{m+3n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a\,(m+2n+1)}+\frac{d\,(cx)^{m+1}\ {}_2F_1\left(\begin{matrix}1,\frac{m+1}{n}\\\frac{m+n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ac\,(m+1)}+\frac{ex^n\,(cx)^{-n}\,(cx)^{m+n+1}\ {}_2F_1\left(\begin{matrix}1,\frac{m+n+1}{n}\\\frac{m+2n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ac\,(m+n+1)}+\frac{gx^{3n}\,(cx)^{-3n}\,(cx)^{m+3n+1}\ {}_2F_1\left(\begin{matrix}1,\frac{m+3n+1}{n}\\\frac{m+4n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ac\,(m+3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n),x)
```

```
[Out] f*x**(-m)*x**(m + 2*n + 1)*(c*x)**m*hyper((1, (m + 2*n + 1)/n), (
(m + 3*n + 1)/n,), -b*x**n/a)/(a*(m + 2*n + 1)) + d*(c*x)**(m + 1
)*hyper((1, (m + 1)/n), ((m + n + 1)/n,), -b*x**n/a)/(a*c*(m + 1)
) + e*x**n*(c*x)**(-n)*(c*x)**(m + n + 1)*hyper((1, (m + n + 1)/n
), ((m + 2*n + 1)/n,), -b*x**n/a)/(a*c*(m + n + 1)) + g*x**(3*n)*
(c*x)**(-3*n)*(c*x)**(m + 3*n + 1)*hyper((1, (m + 3*n + 1)/n), ((
m + 4*n + 1)/n,), -b*x**n/a)/(a*c*(m + 3*n + 1))
```

Mathematica [A] time = 0.491568, size = 150, normalized size = 0.93

$$x(cx)^m\left(\frac{a^2g}{b^3(m+1)}+\frac{\left(a^3(-g)+a^2bf-ab^2e+b^3d\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ab^3(m+1)}-\frac{a\left(\frac{f}{m+1}+\frac{gx^n}{m+n+1}\right)}{b^2}+\frac{e}{bm+b}+\frac{fx^n}{b(m+n+1)}+\frac{gx^{2n}}{bm+2bn+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

```
[Out] x*(c*x)^m*((a^2*g)/(b^3*(1 + m)) + e/(b + b*m) + (f*x^n)/(b*(1 +
m + n)) + (g*x^(2*n))/(b + b*m + 2*b*n) - (a*(f/(1 + m) + (g*x^n)
/(1 + m + n)))/b^2 + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*Hyperge
ometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^3*(1
+ m)))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

```
[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(b^3c^md-ab^2c^me+a^2bc^mf-a^3c^mg\right)\int \frac{x^m}{b^4x^n+ab^3}\,dx$$
$$+\frac{\left(m^2+m(n+2)+n+1\right)b^2c^mgxe^{(m\log(x)+2n\log(x))}+\left(\left(m^2+m(3n+2)+2n^2+3n+1\right)b^2c^me-\left(m^2+m(3n+2)+2n^2+3\right.\right.}{\left(m^3+3m^2(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a),x, algorithm="m
```

```
[Out] (b^3*c^m*d - a*b^2*c^m*e + a^2*b*c^m*f - a^3*c^m*g)*integrate(x^m
/(b^4*x^n + a*b^3), x) + ((m^2 + m*(n + 2) + n + 1)*b^2*c^m*g*x*e
^(m*log(x) + 2*n*log(x)) + ((m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)
*b^2*c^m*e - (m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a*b*c^m*f + (m
^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a^2*c^m*g)*x*x^m + ((m^2 + 2*
m*(n + 1) + 2*n + 1)*b^2*c^m*f - (m^2 + 2*m*(n + 1) + 2*n + 1)*a*
b*c^m*g)*x*e^(m*log(x) + n*log(x)))/((m^3 + 3*m^2*(n + 1) + (2*n^
2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*b^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(gx^{3n}+fx^{2n}+ex^n+d\right)(cx)^m}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a),x, algorithm="f
```

```
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{3\,n} + fx^{2\,n} + ex^{n} + d\right)(cx)^{m}}{bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a),x, algorithm="g
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a)
, x)
```

3.565 $\int \left(c+dx^{-1+n}\right) \left(a+bx^n\right)^3 \, dx$

Optimal. Leaf size=84

$$a^3cx+\frac{3a^2bcx^{n+1}}{n+1}+\frac{3ab^2cx^{2n+1}}{2n+1}+\frac{d\left(a+bx^n\right)^4}{4bn}+\frac{b^3cx^{3n+1}}{3n+1}$$

```
[Out] a^3*c*x + (3*a^2*b*c*x^(1 + n))/(1 + n) + (3*a*b^2*c*x^(1 + 2*n))
/(1 + 2*n) + (b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x^n)^4)/(4
*b*n)
```

Rubi [A] time = 0.110754, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$a^3cx+\frac{3a^2bcx^{n+1}}{n+1}+\frac{3ab^2cx^{2n+1}}{2n+1}+\frac{d\left(a+bx^n\right)^4}{4bn}+\frac{b^3cx^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] a^3*c*x + (3*a^2*b*c*x^(1 + n))/(1 + n) + (3*a*b^2*c*x^(1 + 2*n))
/(1 + 2*n) + (b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x^n)^4)/(4
*b*n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3a^2bcx^{n+1}}{n+1}+\frac{3ab^2cx^{2n+1}}{2n+1}+\frac{b^3cx^{3n+1}}{3n+1}+c\int a^3\,dx+\frac{d\left(a+bx^n\right)^4}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**(-1+n))*(a+b*x**n)**3,x)
```

```
[Out] 3*a**2*b*c*x**(n + 1)/(n + 1) + 3*a*b**2*c*x**(2*n + 1)/(2*n + 1)
 + b**3*c*x**(3*n + 1)/(3*n + 1) + c*Integral(a**3, x) + d*(a + b
*x**n)**4/(4*b*n)
```

Mathematica [A] time = 0.185538, size = 162, normalized size = 1.93

$$\frac{4a^3\left(6n^3+11n^2+6n+1\right)\left(cnx+dx^n\right)+6a^2b\left(6n^2+5n+1\right)x^n\left(2cnx+d(n+1)x^n\right)+4ab^2\left(3n^2+4n+1\right)x^{2n}\left(3cnx+d(2n\right.}{4n(n+1)(2n+1)(3n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] (4*a^3*(1 + 6*n + 11*n^2 + 6*n^3)*(c*n*x + d*x^n) + 6*a^2*b*(1 +
5*n + 6*n^2)*x^n*(2*c*n*x + d*(1 + n)*x^n) + 4*a*b^2*(1 + 4*n + 3
*n^2)*x^(2*n)*(3*c*n*x + d*(1 + 2*n)*x^n) + b^3*(1 + 3*n + 2*n^2)
*x^(3*n)*(4*c*n*x + d*(1 + 3*n)*x^n))/(4*n*(1 + n)*(1 + 2*n)*(1 +
 3*n))
```

Maple [A] time = 0.029, size = 130, normalized size = 1.6

$$a^{3}cx+\frac{a^{3}d\,\mathrm{e}^{n\ln(x)}}{n}+\frac{ab^{2}d\left(\mathrm{e}^{n\ln(x)}\right)^{3}}{n}+\frac{b^{3}cx\left(\mathrm{e}^{n\ln(x)}\right)^{3}}{1+3n}+\frac{b^{3}d\left(\mathrm{e}^{n\ln(x)}\right)^{4}}{4n}+\frac{3a^{2}bd\left(\mathrm{e}^{n\ln(x)}\right)^{2}}{2n}+3\frac{acb^{2}x\left(\mathrm{e}^{n\ln(x)}\right)^{2}}{1+2n}+3\frac{a^{2}bcx\,\mathrm{e}^{n\ln(x)}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^(-1+n))*(a+b*x^n)^3,x)
```

```
[Out] a^3*c*x+a^3*d/n*exp(n*ln(x))+a*b^2*d/n*exp(n*ln(x))^3+b^3*c/(1+3*
n)*x*exp(n*ln(x))^3+1/4*b^3*d/n*exp(n*ln(x))^4+3/2*a^2*b*d/n*exp(
n*ln(x))^2+3*a*c*b^2/(1+2*n)*x*exp(n*ln(x))^2+3*a^2*b*c/(1+n)*x*e
xp(n*ln(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)*(b*x^n + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227749, size = 412, normalized size = 4.9

$$\frac{4\left(6a^{3}cn^{4}+11a^{3}cn^{3}+6a^{3}cn^{2}+a^{3}cn\right)x+\left(6b^{3}dn^{3}+11b^{3}dn^{2}+6b^{3}dn+b^{3}d\right)x^{4n}+4\left(6ab^{2}dn^{3}+11ab^{2}dn^{2}+6ab^{2}dn+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)*(b*x^n + a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(6*a^3*c*n^4 + 11*a^3*c*n^3 + 6*a^3*c*n^2 + a^3*c*n)*x + (
6*b^3*d*n^3 + 11*b^3*d*n^2 + 6*b^3*d*n + b^3*d)*x^(4*n) + 4*(6*a*
b^2*d*n^3 + 11*a*b^2*d*n^2 + 6*a*b^2*d*n + a*b^2*d + (2*b^3*c*n^3
 + 3*b^3*c*n^2 + b^3*c*n)*x)*x^(3*n) + 6*(6*a^2*b*d*n^3 + 11*a^2*
b*d*n^2 + 6*a^2*b*d*n + a^2*b*d + 2*(3*a*b^2*c*n^3 + 4*a*b^2*c*n^
2 + a*b^2*c*n)*x)*x^(2*n) + 4*(6*a^3*d*n^3 + 11*a^3*d*n^2 + 6*a^3
*d*n + a^3*d + 3*(6*a^2*b*c*n^3 + 5*a^2*b*c*n^2 + a^2*b*c*n)*x)*x
^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

Sympy [A] time = 10.984, size = 1251, normalized size = 14.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**(-1+n))*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*c*x - a**3*d/x + 3*a**2*b*c*log(x) - 3*a**2*b*d/(
2*x**2) - 3*a*b**2*c/x - a*b**2*d/x**3 - b**3*c/(2*x**2) - b**3*d
/(4*x**4), Eq(n, -1)), (a**3*c*x - 2*a**3*d/sqrt(x) + 6*a**2*b*c*
sqrt(x) - 3*a**2*b*d/x + 3*a*b**2*c*log(x) - 2*a*b**2*d/x**(3/2)
```

```
- 2*b**3*c/sqrt(x) - b**3*d/(2*x**2), Eq(n, -1/2)), (a**3*c*x - 3
*a**3*d/x**(1/3) + 9*a**2*b*c*x**(2/3)/2 - 9*a**2*b*d/(2*x**(2/3)
) + 9*a*b**2*c*x**(1/3) - 3*a*b**2*d/x + b**3*c*log(x) - 3*b**3*d
/(4*x**(4/3)), Eq(n, -1/3)), ((a + b)**3*(c*x + d*log(x)), Eq(n,
0)), (24*a**3*c*n**4*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a
**3*c*n**3*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*c*n**2
*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 4*a**3*c*n*x/(24*n**4 +
44*n**3 + 24*n**2 + 4*n) + 24*a**3*d*n**3*x**n/(24*n**4 + 44*n**3
 + 24*n**2 + 4*n) + 44*a**3*d*n**2*x**n/(24*n**4 + 44*n**3 + 24*n
**2 + 4*n) + 24*a**3*d*n*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n)
 + 4*a**3*d*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 72*a**2*b*
c*n**3*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 60*a**2*b*c*n
**2*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 12*a**2*b*c*n*x*
x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a**2*b*d*n**3*x**(2
*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 66*a**2*b*d*n**2*x**(2*
n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a**2*b*d*n*x**(2*n)/(
24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*a**2*b*d*x**(2*n)/(24*n**4
 + 44*n**3 + 24*n**2 + 4*n) + 36*a*b**2*c*n**3*x*x**(2*n)/(24*n**
4 + 44*n**3 + 24*n**2 + 4*n) + 48*a*b**2*c*n**2*x*x**(2*n)/(24*n*
*4 + 44*n**3 + 24*n**2 + 4*n) + 12*a*b**2*c*n*x*x**(2*n)/(24*n**4
 + 44*n**3 + 24*n**2 + 4*n) + 24*a*b**2*d*n**3*x**(3*n)/(24*n**4
+ 44*n**3 + 24*n**2 + 4*n) + 44*a*b**2*d*n**2*x**(3*n)/(24*n**4 +
 44*n**3 + 24*n**2 + 4*n) + 24*a*b**2*d*n*x**(3*n)/(24*n**4 + 44*
n**3 + 24*n**2 + 4*n) + 4*a*b**2*d*x**(3*n)/(24*n**4 + 44*n**3 +
24*n**2 + 4*n) + 8*b**3*c*n**3*x*x**(3*n)/(24*n**4 + 44*n**3 + 24
*n**2 + 4*n) + 12*b**3*c*n**2*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*
n**2 + 4*n) + 4*b**3*c*n*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2
+ 4*n) + 6*b**3*d*n**3*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*
n) + 11*b**3*d*n**2*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n)
+ 6*b**3*d*n*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + b**3*
d*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n), True))
```

GIAC/XCAS [A] time = 0.226922, size = 572, normalized size = 6.81

$$\frac{24\,a^{3}cn^{4}x + 8\,b^{3}cn^{3}xe^{\left(3\,n\ln\left(x\right)\right)} + 36\,ab^{2}cn^{3}xe^{\left(2\,n\ln\left(x\right)\right)} + 72\,a^{2}bcn^{3}xe^{\left(n\ln\left(x\right)\right)} + 44\,a^{3}cn^{3}x + 6\,b^{3}dn^{3}e^{\left(4\,n\ln\left(x\right)\right)} + 24\,ab^{2}dn^{3}e^{\left(3\,n\ln\left(x\right)\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)*(b*x^n + a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(24*a^3*c*n^4*x + 8*b^3*c*n^3*x*e^(3*n*ln(x)) + 36*a*b^2*c*n^
3*x*e^(2*n*ln(x)) + 72*a^2*b*c*n^3*x*e^(n*ln(x)) + 44*a^3*c*n^3*x
 + 6*b^3*d*n^3*e^(4*n*ln(x)) + 24*a*b^2*d*n^3*e^(3*n*ln(x)) + 12*
b^3*c*n^2*x*e^(3*n*ln(x)) + 36*a^2*b*d*n^3*e^(2*n*ln(x)) + 48*a*b
^2*c*n^2*x*e^(2*n*ln(x)) + 24*a^3*d*n^3*e^(n*ln(x)) + 60*a^2*b*c*
n^2*x*e^(n*ln(x)) + 24*a^3*c*n^2*x + 11*b^3*d*n^2*e^(4*n*ln(x)) +
 44*a*b^2*d*n^2*e^(3*n*ln(x)) + 4*b^3*c*n*x*e^(3*n*ln(x)) + 66*a^
2*b*d*n^2*e^(2*n*ln(x)) + 12*a*b^2*c*n*x*e^(2*n*ln(x)) + 44*a^3*d
*n^2*e^(n*ln(x)) + 12*a^2*b*c*n*x*e^(n*ln(x)) + 4*a^3*c*n*x + 6*b
^3*d*n*e^(4*n*ln(x)) + 24*a*b^2*d*n*e^(3*n*ln(x)) + 36*a^2*b*d*n*
e^(2*n*ln(x)) + 24*a^3*d*n*e^(n*ln(x)) + b^3*d*e^(4*n*ln(x)) + 4*
a*b^2*d*e^(3*n*ln(x)) + 6*a^2*b*d*e^(2*n*ln(x)) + 4*a^3*d*e^(n*ln
(x)))/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

3.566 $\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx$

Optimal. Leaf size=61

$$a^2cx+\frac{2abcx^{n+1}}{n+1}+\frac{d(a+bx^n)^3}{3bn}+\frac{b^2cx^{2n+1}}{2n+1}$$

```
[Out] a^2*c*x + (2*a*b*c*x^(1 + n))/(1 + n) + (b^2*c*x^(1 + 2*n))/(1 +
2*n) + (d*(a + b*x^n)^3)/(3*b*n)
```

Rubi [A] time = 0.0824043, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$a^2cx+\frac{2abcx^{n+1}}{n+1}+\frac{d(a+bx^n)^3}{3bn}+\frac{b^2cx^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

```
[Out] a^2*c*x + (2*a*b*c*x^(1 + n))/(1 + n) + (b^2*c*x^(1 + 2*n))/(1 +
2*n) + (d*(a + b*x^n)^3)/(3*b*n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2abcx^{n+1}}{n+1}+\frac{b^2cx^{2n+1}}{2n+1}+c\int a^2\,dx+\frac{d(a+bx^n)^3}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**(-1+n))*(a+b*x**n)**2,x)
```

```
[Out] 2*a*b*c*x**(n + 1)/(n + 1) + b**2*c*x**(2*n + 1)/(2*n + 1) + c*In
tegral(a**2, x) + d*(a + b*x**n)**3/(3*b*n)
```

Mathematica [A] time = 0.109347, size = 99, normalized size = 1.62

$$\frac{3a^2\left(2n^2+3n+1\right)(cnx+dx^n)+3ab(2n+1)x^n\left(2cnx+d(n+1)x^n\right)+b^2(n+1)x^{2n}\left(3cnx+d(2n+1)x^n\right)}{3n(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

```
[Out] (3*a^2*(1 + 3*n + 2*n^2)*(c*n*x + d*x^n) + 3*a*b*(1 + 2*n)*x^n*(2
*c*n*x + d*(1 + n)*x^n) + b^2*(1 + n)*x^(2*n)*(3*c*n*x + d*(1 + 2
*n)*x^n))/(3*n*(1 + n)*(1 + 2*n))
```

Maple [A] time = 0.024, size = 87, normalized size = 1.4

$$a^2cx+\frac{a^2de^{n\ln(x)}}{n}+\frac{bda\left(e^{n\ln(x)}\right)^2}{n}+\frac{b^2cx\left(e^{n\ln(x)}\right)^2}{1+2n}+\frac{b^2d\left(e^{n\ln(x)}\right)^3}{3n}+2\,\frac{abcxe^{n\ln(x)}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^(-1+n))*(a+b*x^n)^2,x)
```

```
[Out] a^2*c*x+a^2*d/n*exp(n*ln(x))+b*d*a/n*exp(n*ln(x))^2+b^2*c/(1+2*n)
*x*exp(n*ln(x))^2+1/3*b^2*d/n*exp(n*ln(x))^3+2*a*b*c/(1+n)*x*exp(
n*ln(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)*(b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227066, size = 216, normalized size = 3.54

$$\frac{3\left(2\,a^2cn^3+3\,a^2cn^2+a^2cn\right)x+\left(2\,b^2dn^2+3\,b^2dn+b^2d\right)x^{3\,n}+3\left(2\,abdn^2+3\,abdn+abd+\left(b^2cn^2+b^2cn\right)x\right)x^{2\,n}+3\left(2\,a^2d\right.}{3\left(2\,n^3+3\,n^2+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)*(b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(2*a^2*c*n^3 + 3*a^2*c*n^2 + a^2*c*n)*x + (2*b^2*d*n^2 + 3
*b^2*d*n + b^2*d)*x^(3*n) + 3*(2*a*b*d*n^2 + 3*a*b*d*n + a*b*d +
(b^2*c*n^2 + b^2*c*n)*x)*x^(2*n) + 3*(2*a^2*d*n^2 + 3*a^2*d*n + a
^2*d + 2*(2*a*b*c*n^2 + a*b*c*n)*x)*x^n)/(2*n^3 + 3*n^2 + n)
```

Sympy [A] time = 5.457, size = 552, normalized size = 9.05

$$\begin{cases} a^2cx-\frac{a^2d}{x}+2abc\log(x)-\frac{abd}{x^2}-\frac{b^2c}{x}-\frac{b^2d}{3x^3} \\ a^2cx-\frac{2a^2d}{\sqrt{x}}+4abc\sqrt{x}-\frac{2abd}{x}+b^2c\log(x)-\frac{2b^2d}{3x^{\frac{3}{2}}} \\ (a+b)^2\left(cx+d\log(x)\right) \\ \frac{6a^2cn^3x}{6n^3+9n^2+3n}+\frac{9a^2cn^2x}{6n^3+9n^2+3n}+\frac{3a^2cnx}{6n^3+9n^2+3n}+\frac{6a^2dn^2x^n}{6n^3+9n^2+3n}+\frac{9a^2dnx^n}{6n^3+9n^2+3n}+\frac{3a^2dx^n}{6n^3+9n^2+3n}+\frac{12abcn^2xx^n}{6n^3+9n^2+3n}+\frac{6abcnxx^n}{6n^3+9n^2+3n}+\frac{6abdn^2x^{2n}}{6n^3+9n^2+3n}+\frac{9abdn}{6n^3+9n}\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**(-1+n))*(a+b*x**n)**2,x)
```

```
[Out] Piecewise((a**2*c*x - a**2*d/x + 2*a*b*c*log(x) - a*b*d/x**2 - b*
*2*c/x - b**2*d/(3*x**3), Eq(n, -1)), (a**2*c*x - 2*a**2*d/sqrt(x
) + 4*a*b*c*sqrt(x) - 2*a*b*d/x + b**2*c*log(x) - 2*b**2*d/(3*x**
(3/2)), Eq(n, -1/2)), ((a + b)**2*(c*x + d*log(x)), Eq(n, 0)), (6
*a**2*c*n**3*x/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*c*n**2*x/(6*n**3
+ 9*n**2 + 3*n) + 3*a**2*c*n*x/(6*n**3 + 9*n**2 + 3*n) + 6*a**2*d
*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*d*n*x**n/(6*n**3 + 9*
n**2 + 3*n) + 3*a**2*d*x**n/(6*n**3 + 9*n**2 + 3*n) + 12*a*b*c*n*
*2*x*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x*x**n/(6*n**3 + 9*
n**2 + 3*n) + 6*a*b*d*n**2*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 9*a
*b*d*n*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 3*a*b*d*x**(2*n)/(6*n**
3 + 9*n**2 + 3*n) + 3*b**2*c*n**2*x*x**(2*n)/(6*n**3 + 9*n**2 + 3
*n) + 3*b**2*c*n*x*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 2*b**2*d*n*
*2*x**(3*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*d*n*x**(3*n)/(6*n**3
 + 9*n**2 + 3*n) + b**2*d*x**(3*n)/(6*n**3 + 9*n**2 + 3*n), True)
)
```

GIAC/XCAS [A] time = 0.224274, size = 289, normalized size = 4.74

$$\frac{6\,a^{2}cn^{3}x + 3\,b^{2}cn^{2}xe^{(2\,n\ln(x))} + 12\,abcn^{2}xe^{(n\ln(x))} + 9\,a^{2}cn^{2}x + 2\,b^{2}dn^{2}e^{(3\,n\ln(x))} + 6\,abdn^{2}e^{(2\,n\ln(x))} + 3\,b^{2}cnxe^{(2\,n\ln(x))} + 6\,a^{2}d\cdots}{3\,(2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)*(b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*c*n^3*x + 3*b^2*c*n^2*x*e^(2*n*ln(x)) + 12*a*b*c*n^2*x
*e^(n*ln(x)) + 9*a^2*c*n^2*x + 2*b^2*d*n^2*e^(3*n*ln(x)) + 6*a*b*
d*n^2*e^(2*n*ln(x)) + 3*b^2*c*n*x*e^(2*n*ln(x)) + 6*a^2*d*n^2*e^(
n*ln(x)) + 6*a*b*c*n*x*e^(n*ln(x)) + 3*a^2*c*n*x + 3*b^2*d*n*e^(3
*n*ln(x)) + 9*a*b*d*n*e^(2*n*ln(x)) + 9*a^2*d*n*e^(n*ln(x)) + b^2
*d*e^(3*n*ln(x)) + 3*a*b*d*e^(2*n*ln(x)) + 3*a^2*d*e^(n*ln(x)))/(
2*n^3 + 3*n^2 + n)
```

3.567 $\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx$

Optimal. Leaf size=41

$$acx+\frac{adx^n}{n}+\frac{bcx^{n+1}}{n+1}+\frac{bdx^{2n}}{2n}$$

```
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x^(1 + n))/(1 +
n)
```

Rubi [A] time = 0.0460881, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$acx+\frac{adx^n}{n}+\frac{bcx^{n+1}}{n+1}+\frac{bdx^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

```
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x^(1 + n))/(1 +
n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{adx^n}{n}+\frac{bcx^{n+1}}{n+1}+\frac{bdx^{2n}}{2n}+c\int a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**(-1+n))*(a+b*x**n),x)
```

```
[Out] a*d*x**n/n + b*c*x**(n + 1)/(n + 1) + b*d*x**(2*n)/(2*n) + c*Inte
gral(a, x)
```

Mathematica [A] time = 0.092453, size = 42, normalized size = 1.02

$$\frac{2a\left(cnx+dx^n\right)+bx^n\left(\frac{2cnx}{n+1}+dx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

```
[Out] (2*a*(c*n*x + d*x^n) + b*x^n*((2*c*n*x)/(1 + n) + d*x^n))/(2*n)
```

Maple [A] time = 0.021, size = 45, normalized size = 1.1

$$acx+\frac{ade^{n\ln(x)}}{n}+\frac{bcxe^{n\ln(x)}}{1+n}+\frac{bd\left(e^{n\ln(x)}\right)^2}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^(-1+n))*(a+b*x^n),x)
```

```
[Out] a*c*x+a*d/n*exp(n*ln(x))+b*c/(1+n)*x*exp(n*ln(x))+1/2*b*d/n*exp(n
*ln(x))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)*(b*x^n + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.224454, size = 76, normalized size = 1.85

$$\frac{2\left(acn^2+acn\right)x+(bdn+bd)x^{2\,n}+2\,(bcnx+adn+ad)x^n}{2\,(n^2+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)*(b*x^n + a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c*n^2 + a*c*n)*x + (b*d*n + b*d)*x^(2*n) + 2*(b*c*n*x +
 a*d*n + a*d)*x^n)/(n^2 + n)
```

Sympy [A] time = 2.41195, size = 163, normalized size = 3.98

$$\begin{cases} acx-\frac{ad}{x}+bc\log{(x)}-\frac{bd}{2x^2} & \text{for}\ n=-1\\ (a+b)\,(cx+d\log{(x)}) & \text{for}\ n=0\\ \frac{2acn^2x}{2n^2+2n}+\frac{2acnx}{2n^2+2n}+\frac{2adnx^n}{2n^2+2n}+\frac{2adx^n}{2n^2+2n}+\frac{2bcnxx^n}{2n^2+2n}+\frac{bdnx^{2n}}{2n^2+2n}+\frac{bdx^{2n}}{2n^2+2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**(-1+n))*(a+b*x**n),x)
```

```
[Out] Piecewise((a*c*x - a*d/x + b*c*log(x) - b*d/(2*x**2), Eq(n, -1)),
 ((a + b)*(c*x + d*log(x)), Eq(n, 0)), (2*a*c*n**2*x/(2*n**2 + 2*
n) + 2*a*c*n*x/(2*n**2 + 2*n) + 2*a*d*n*x**n/(2*n**2 + 2*n) + 2*a
*d*x**n/(2*n**2 + 2*n) + 2*b*c*n*x*x**n/(2*n**2 + 2*n) + b*d*n*x*
*(2*n)/(2*n**2 + 2*n) + b*d*x**(2*n)/(2*n**2 + 2*n), True))
```

GIAC/XCAS [A] time = 0.219808, size = 99, normalized size = 2.41

$$\frac{2\,acn^2x+2\,bcnxe^{(n\ln(x))}+2\,acnx+bdne^{(2\,n\ln(x))}+2\,adne^{(n\ln(x))}+bde^{(2\,n\ln(x))}+2\,ade^{(n\ln(x))}}{2\,(n^2+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)*(b*x^n + a),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*c*n^2*x + 2*b*c*n*x*e^(n*ln(x)) + 2*a*c*n*x + b*d*n*e^(2
*n*ln(x)) + 2*a*d*n*e^(n*ln(x)) + b*d*e^(2*n*ln(x)) + 2*a*d*e^(n*
ln(x)))/(n^2 + n)
```

3.568 $\int \left(c+dx^{-1+n}\right)\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^n}{n}$$

```
[Out] c*x + (d*x^n)/n
```

Rubi [A] time = 0.0100353, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$cx+\frac{dx^n}{n}$$

Antiderivative was successfully verified.

```
[In]  Int[c + d*x^(-1 + n),x]

[Out] c*x + (d*x^n)/n
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{dx^n}{n}+\int c\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(c+d*x**(-1+n),x)

[Out] d*x**n/n + Integral(c, x)
```

Mathematica [A] time = 0.00267154, size = 12, normalized size = 1.

$$cx+\frac{dx^n}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[c + d*x^(-1 + n),x]

[Out] c*x + (d*x^n)/n
```

Maple [A] time = 0.002, size = 13, normalized size = 1.1

$$cx+\frac{dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(c+d*x^(-1+n),x)

[Out] c*x+d*x^n/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d*x^(n - 1) + c,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.226963, size = 23, normalized size = 1.92

$$\frac{cnx + dxx^{n-1}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d*x^(n - 1) + c,x, algorithm="fricas")
```

```
[Out] (c*n*x + d*x*x^(n - 1))/n
```

Sympy [A] time = 0.037488, size = 15, normalized size = 1.25

$$cx + d\left(\begin{cases} \frac{x^{n}}{n} & \text{for}\ n - 1 \neq -1 \\ \log\left(x\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c+d*x**(-1+n),x)
```

```
[Out] c*x + d*Piecewise((x**n/n, Ne(n - 1, -1)), (log(x), True))
```

GIAC/XCAS [A] time = 0.213055, size = 16, normalized size = 1.33

$$cx + \frac{dx^{n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d*x^(n - 1) + c,x, algorithm="giac")
```

```
[Out] c*x + d*x^n/n
```

3.569 $\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx$

Optimal. Leaf size=42

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a +
(d*Log[a + b*x^n])/(b*n)
```

Rubi [A] time = 0.0630661, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a +
(d*Log[a + b*x^n])/(b*n)
```

Rubi in Sympy [A] time = 14.7596, size = 32, normalized size = 0.76

$$\frac{d\log\left(a+bx^n\right)}{bn}+\frac{cx{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**(-1+n))/(a+b*x**n),x)
```

```
[Out] d*log(a + b*x**n)/(b*n) + c*x*hyper((1, 1/n), (1 + 1/n,), -b*x**n
/a)/a
```

Mathematica [A] time = 0.0704356, size = 48, normalized size = 1.14

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\left(\log\left(a-ax^n\right)+n\log(x)\right)}{bn}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a +
(d*(n*Log[x] + Log[a - a*x^n]))/(b*n)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^(-1+n))/(a+b*x^n),x)

[Out] int((c+d*x^(-1+n))/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{d\log\left(x\right)}{b} + \int \frac{bcx - ad}{b^{2}xx^{n} + abx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)/(b*x^n + a),x, algorithm="maxima")

[Out] d*log(x)/b + integrate((b*c*x - a*d)/(b^2*x*x^n + a*b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx^{n-1} + c}{bx^{n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)/(b*x^n + a),x, algorithm="fricas")

[Out] integral((d*x^(n - 1) + c)/(b*x^n + a), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**(-1+n))/(a+b*x**n),x)

[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{n-1} + c}{bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)/(b*x^n + a),x, algorithm="giac")

[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a), x)
```

3.570 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=44

$$\frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}-\frac{d}{bn\,(a+bx^n)}$$

```
[Out] -(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^
(-1), -((b*x^n)/a)])/a^2
```

Rubi [A] time = 0.0596695, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}-\frac{d}{bn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

```
[Out] -(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^
(-1), -((b*x^n)/a)])/a^2
```

Rubi in Sympy [A] time = 8.99551, size = 32, normalized size = 0.73

$$-\frac{d}{bn\,(a+bx^n)}+\frac{cx{}_2F_1\left(\begin{matrix} 2,\frac{1}{n} \\ 1+\frac{1}{n} \end{matrix}\middle| -\frac{bx^n}{a}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**(-1+n))/(a+b*x**n)**2,x)
```

```
[Out] -d/(b*n*(a + b*x**n)) + c*x*hyper((2, 1/n), (1 + 1/n,), -b*x**n/a
)/a**2
```

Mathematica [A] time = 0.0879153, size = 56, normalized size = 1.27

$$\frac{\frac{a(bcx-ad)}{b(a+bx^n)}+c(n-1)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

```
[Out] ((a*(-(a*d) + b*c*x))/(b*(a + b*x^n)) + c*(-1 + n)*x*Hypergeometr
ic2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n)
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c(n-1)\int \frac{1}{abnx^n + a^2n}\,dx + \frac{bcx - ad}{ab^2nx^n + a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)/(b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] c*(n - 1)*integrate(1/(a*b*n*x^n + a^2*n), x) + (b*c*x - a*d)/(a*
b^2*n*x^n + a^2*b*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^{n-1} + c}{b^2x^{2\,n} + 2\,abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)/(b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^(n - 1) + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**(-1+n))/(a+b*x**n)**2,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{n-1} + c}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)/(b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^2, x)
```

3.571 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=46

$$\frac{cx\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}-\frac{d}{2bn\,(a+bx^n)^2}$$

```
[Out] -d/(2*b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 +
n^(-1), -((b*x^n)/a)])/a^3
```

Rubi [A] time = 0.0598445, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{cx\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}-\frac{d}{2bn\,(a+bx^n)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

```
[Out] -d/(2*b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 +
n^(-1), -((b*x^n)/a)])/a^3
```

Rubi in Sympy [A] time = 9.02079, size = 36, normalized size = 0.78

$$-\frac{d}{2bn\,(a+bx^n)^2}+\frac{cx{}_2F_1\left(\begin{matrix}3,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**(-1+n))/(a+b*x**n)**3,x)
```

```
[Out] -d/(2*b*n*(a + b*x**n)**2) + c*x*hyper((3, 1/n), (1 + 1/n,), -b*x
**n/a)/a**3
```

Mathematica [B] time = 0.13708, size = 108, normalized size = 2.35

$$\frac{x\left(c+dx^{n-1}\right)\left(\frac{a^2n(bcx-ad)}{b(a+bx^n)^2}+c\left(2n^2-3n+1\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+\frac{ac(2n-1)x}{a+bx^n}\right)}{2a^3n^2\,(cx+dx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

```
[Out] (x*(c + d*x^(-1 + n))*((a^2*n*(-(a*d) + b*c*x))/(b*(a + b*x^n)^2)
 + (a*c*(-1 + 2*n)*x)/(a + b*x^n) + c*(1 - 3*n + 2*n^2)*x*Hyperge
ometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(2*a^3*n^2*(c*x
 + d*x^n))
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,n^{2}-3\,n+1\right)c\int\frac{1}{2\left(a^{2}bn^{2}x^{n}+a^{3}n^{2}\right)}\,dx+\frac{b^{2}c(2\,n-1)xx^{n}+abc(3\,n-1)x-a^{2}dn}{2\left(a^{2}b^{3}n^{2}x^{2\,n}+2\,a^{3}b^{2}n^{2}x^{n}+a^{4}bn^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)/(b*x^n + a)^3,x, algorithm="maxima")
```

```
[Out] (2*n^2 - 3*n + 1)*c*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) +
 1/2*(b^2*c*(2*n - 1)*x*x^n + a*b*c*(3*n - 1)*x - a^2*d*n)/(a^2*b
^3*n^2*x^(2*n) + 2*a^3*b^2*n^2*x^n + a^4*b*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx^{n-1}+c}{b^{3}x^{3\,n}+3\,ab^{2}x^{2\,n}+3\,a^{2}bx^{n}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)/(b*x^n + a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^(n - 1) + c)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2
*b*x^n + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**(-1+n))/(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^{n-1}+c}{(bx^{n}+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^(n - 1) + c)/(b*x^n + a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^3, x)
```

3.572 $\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=305

$$\frac{d(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}}+\frac{ex^{n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}+\frac{fx^{2n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+2n+1}{n};\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}+\frac{gx^{3n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+3n+1}{n};\frac{m+4n+1}{n};-\frac{bx^n}{a}\right)}{(m+3n+1)\sqrt{a+bx^n}}$$

```
[Out] (d*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 +
m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*Sqrt[a + b*x^n]) +
 (e*x^(1 + n)*(c*x)^m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2,
(1 + m + n)/n, (1 + m + 2*n)/n, -((b*x^n)/a)])/((1 + m + n)*Sqrt[
a + b*x^n]) + (f*x^(1 + 2*n)*(c*x)^m*Sqrt[1 + (b*x^n)/a]*Hypergeo
metric2F1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -((b*x^n)/a)])/(
(1 + m + 2*n)*Sqrt[a + b*x^n]) + (g*x^(1 + 3*n)*(c*x)^m*Sqrt[1 +
(b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/
n, -((b*x^n)/a)])/((1 + m + 3*n)*Sqrt[a + b*x^n])
```

Rubi [A] time = 0.494093, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{d(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}}+\frac{ex^{n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}+\frac{fx^{2n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+2n+1}{n};\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}+\frac{gx^{3n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+3n+1}{n};\frac{m+4n+1}{n};-\frac{bx^n}{a}\right)}{(m+3n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In]  Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```

```
[Out] (d*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 +
m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*Sqrt[a + b*x^n]) +
 (e*x^(1 + n)*(c*x)^m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2,
(1 + m + n)/n, (1 + m + 2*n)/n, -((b*x^n)/a)])/((1 + m + n)*Sqrt[
a + b*x^n]) + (f*x^(1 + 2*n)*(c*x)^m*Sqrt[1 + (b*x^n)/a]*Hypergeo
metric2F1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -((b*x^n)/a)])/(
(1 + m + 2*n)*Sqrt[a + b*x^n]) + (g*x^(1 + 3*n)*(c*x)^m*Sqrt[1 +
(b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/
n, -((b*x^n)/a)])/((1 + m + 3*n)*Sqrt[a + b*x^n])
```

Rubi in Sympy [A] time = 57.7425, size = 292, normalized size = 0.96

$$\frac{d(cx)^{m+1}\sqrt{a+bx^n}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m+1}{n}\\ \frac{m+n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ac\sqrt{1+\frac{bx^n}{a}}(m+1)}+\frac{ex^n(cx)^{-n}(cx)^{m+n+1}\sqrt{a+bx^n}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m+n+1}{n}\\ \frac{m+2n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ac\sqrt{1+\frac{bx^n}{a}}(m+n+1)}+\frac{fx^{2n}(cx)^{-2n}(cx)^{m+2n+1}\sqrt{a+bx^n}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m+2n+1}{n}\\ \frac{m+3n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ac\sqrt{1+\frac{bx^n}{a}}(m+2n+1)}+\frac{gx^{3n}(cx)^{-3n}(cx)^{m+3n+1}\sqrt{a+bx^n}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m+3n+1}{n}\\ \frac{m+4n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ac\sqrt{1+\frac{bx^n}{a}}(m+3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n)**(1/2),x)
```

```
[Out] d*(c*x)**(m + 1)*sqrt(a + b*x**n)*hyper((1/2, (m + 1)/n), ((m + n
 + 1)/n,), -b*x**n/a)/(a*c*sqrt(1 + b*x**n/a)*(m + 1)) + e*x**n*(
c*x)**(-n)*(c*x)**(m + n + 1)*sqrt(a + b*x**n)*hyper((1/2, (m + n
 + 1)/n), ((m + 2*n + 1)/n,), -b*x**n/a)/(a*c*sqrt(1 + b*x**n/a)*
(m + n + 1)) + f*x**(2*n)*(c*x)**(-2*n)*(c*x)**(m + 2*n + 1)*sqrt
(a + b*x**n)*hyper((1/2, (m + 2*n + 1)/n), ((m + 3*n + 1)/n,), -b
*x**n/a)/(a*c*sqrt(1 + b*x**n/a)*(m + 2*n + 1)) + g*x**(3*n)*(c*x
)**(-3*n)*(c*x)**(m + 3*n + 1)*sqrt(a + b*x**n)*hyper((1/2, (m +
3*n + 1)/n), ((m + 4*n + 1)/n,), -b*x**n/a)/(a*c*sqrt(1 + b*x**n/
a)*(m + 3*n + 1))
```

Mathematica [A] time = 2.75986, size = 399, normalized size = 1.31

$$x(cx)^m\left(2(m+1)(a+bx^n)\left(4a^2g\left(m^2+m(3n+2)+2n^2+3n+1\right)-2ab\left(f\left(2m^2+m(7n+4)+5n^2+7n+2\right)+g\left(2m^2+m\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```

```
[Out] (x*(c*x)^m*(2*(1 + m)*(a + b*x^n)*(4*a^2*g*(1 + m^2 + 3*n + 2*n^2
 + m*(2 + 3*n)) - 2*a*b*(f*(2 + 2*m^2 + 7*n + 5*n^2 + m*(4 + 7*n)
) + g*(2 + 2*m^2 + 5*n + 2*n^2 + m*(4 + 5*n))*x^n) + b^2*(e*(4 +
4*m^2 + 16*n + 15*n^2 + 8*m*(1 + 2*n)) + (2 + 2*m + n)*x^n*(f*(2
+ 2*m + 5*n) + g*(2 + 2*m + 3*n)*x^n))) + (-2*a*b^2*e*(1 + m)*(4
+ 4*m^2 + 16*n + 15*n^2 + 8*m*(1 + 2*n)) - 8*a^3*g*(1 + m)*(1 + m
^2 + 3*n + 2*n^2 + m*(2 + 3*n)) + 4*a^2*b*f*(1 + m)*(2 + 2*m^2 +
7*n + 5*n^2 + m*(4 + 7*n)) + b^3*d*(8 + 8*m^3 + 36*n + 46*n^2 + 1
5*n^3 + 12*m^2*(2 + 3*n) + m*(24 + 72*n + 46*n^2)))*Sqrt[1 + (b*x
^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)
/a)])/(b^3*(1 + m)*(2 + 2*m + n)*(2 + 2*m + 3*n)*(2 + 2*m + 5*n)
*Sqrt[a + b*x^n])
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int (cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)\frac{1}{\sqrt{a+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{3\,n} + fx^{2\,n} + ex^{n} + d\right)(cx)^{m}}{\sqrt{bx^{n} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a),x, algorith
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n
+ a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a),x, algorith
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{3\,n} + fx^{2\,n} + ex^{n} + d\right)(cx)^{m}}{\sqrt{bx^{n} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a),x, algorith
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n
+ a), x)
```

3.573 $\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

```
[Out] (-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

Rubi [A] time = 0.655136, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^(-
1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-a*h*x**(-1+1/4*n)+b*f*x**(-1+1/2*n)+b*g*x**(-1+n)+b*h*x**(-
1+5/4*n))/(a+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.139775, size = 45, normalized size = 1.

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-
1 + n) + b*h*x^(-1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int 1\left(-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}\right)(a+bx^n)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b*x^
```

```
[Out] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/
4*n))/(a+b*x^n)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bhx^{\frac{5}{4}n-1} + bgx^{n-1} + bfx^{\frac{1}{2}n-1} - ahx^{\frac{1}{4}n-1}}{(bx^n + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1/4*n - 1
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1)
- a*h*x^(1/4*n - 1))/(b*x^n + a)^(3/2), x)
```

Fricas [A] time = 0.232558, size = 89, normalized size = 1.98

$$\frac{2\sqrt{bx^4x^{n-4} + a}\left(bfx^2x^{\frac{1}{2}n-2} - 2\,ahxx^{\frac{1}{4}n-1} - ag\right)}{abnx^4x^{n-4} + a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1/4*n - 1
```

```
[Out] 2*sqrt(b*x^4*x^(n - 4) + a)*(b*f*x^2*x^(1/2*n - 2) - 2*a*h*x*x^(1
/4*n - 1) - a*g)/(a*b*n*x^4*x^(n - 4) + a^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x**(-1+1/4*n)+b*f*x**(-1+1/2*n)+b*g*x**(-1+n)+b*h*x**(-
1+5/4*n))/(a+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bhx^{\frac{5}{4}n-1} + bgx^{n-1} + bfx^{\frac{1}{2}n-1} - ahx^{\frac{1}{4}n-1}}{(bx^n + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1/4*n - 1
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1)
- a*h*x^(1/4*n - 1))/(b*x^n + a)^(3/2), x)
```

3.574 $\int (cx)^m (d + ex + fx^2 + gx^3) (a + bx^n)^p \, dx$

Optimal. Leaf size=273

$$\frac{g(cx)^{m+4} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+4}{n}, -p; \frac{m+n+4}{n}; -\frac{bx^n}{a}\right)}{c^4(m + 4)} + \frac{f(cx)^{m+3} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+3}{n}, -p; \frac{m+n+3}{n}; -\frac{bx^n}{a}\right)}{c^3(m + 3)} + \frac{e(cx)^{m+2} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{n}, -p; \frac{m+n+2}{n}; -\frac{bx^n}{a}\right)}{c^2(m + 2)} + \frac{d(cx)^{m+1} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{c(m + 1)}$$

```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (
1 + m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*(
c*x)^(2 + m)*(a + b*x^n)^p*Hypergeometric2F1[(2 + m)/n, -p, (2 +
m + n)/n, -((b*x^n)/a)])/(c^2*(2 + m)*(1 + (b*x^n)/a)^p) + (f*(c*
x)^(3 + m)*(a + b*x^n)^p*Hypergeometric2F1[(3 + m)/n, -p, (3 + m
+ n)/n, -((b*x^n)/a)])/(c^3*(3 + m)*(1 + (b*x^n)/a)^p) + (g*(c*x)
^(4 + m)*(a + b*x^n)^p*Hypergeometric2F1[(4 + m)/n, -p, (4 + m +
n)/n, -((b*x^n)/a)])/(c^4*(4 + m)*(1 + (b*x^n)/a)^p)
```

Rubi [A] time = 0.376937, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{g(cx)^{m+4} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+4}{n}, -p; \frac{m+n+4}{n}; -\frac{bx^n}{a}\right)}{c^4(m + 4)} + \frac{f(cx)^{m+3} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+3}{n}, -p; \frac{m+n+3}{n}; -\frac{bx^n}{a}\right)}{c^3(m + 3)} + \frac{e(cx)^{m+2} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{n}, -p; \frac{m+n+2}{n}; -\frac{bx^n}{a}\right)}{c^2(m + 2)} + \frac{d(cx)^{m+1} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{c(m + 1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```


```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (
1 + m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*(
c*x)^(2 + m)*(a + b*x^n)^p*Hypergeometric2F1[(2 + m)/n, -p, (2 +
m + n)/n, -((b*x^n)/a)])/(c^2*(2 + m)*(1 + (b*x^n)/a)^p) + (f*(c*
x)^(3 + m)*(a + b*x^n)^p*Hypergeometric2F1[(3 + m)/n, -p, (3 + m
+ n)/n, -((b*x^n)/a)])/(c^3*(3 + m)*(1 + (b*x^n)/a)^p) + (g*(c*x)
^(4 + m)*(a + b*x^n)^p*Hypergeometric2F1[(4 + m)/n, -p, (4 + m +
n)/n, -((b*x^n)/a)])/(c^4*(4 + m)*(1 + (b*x^n)/a)^p)
```

Rubi in Sympy [A] time = 52.1569, size = 214, normalized size = 0.78

$$\frac{d(cx)^{m+1}\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{m+1}{n}\\\frac{m+n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{c(m+1)}+\frac{e(cx)^{m+2}\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{m+2}{n}\\\frac{m+n+2}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{c^2(m+2)}+\frac{f(cx)^{m+3}\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{m+3}{n}\\\frac{m+n+3}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{c^3(m+3)}+\frac{g(cx)^{m+4}\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{m+4}{n}\\\frac{m+n+4}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{c^4(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m*(g*x**3+f*x**2+e*x+d)*(a+b*x**n)**p,x)
```

```
[Out] d*(c*x)**(m + 1)*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*hyper((-p,
(m + 1)/n), ((m + n + 1)/n,), -b*x**n/a)/(c*(m + 1)) + e*(c*x)**(
m + 2)*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*hyper((-p, (m + 2)/n)
, ((m + n + 2)/n,), -b*x**n/a)/(c**2*(m + 2)) + f*(c*x)**(m + 3)*
(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*hyper((-p, (m + 3)/n), ((m +
 n + 3)/n,), -b*x**n/a)/(c**3*(m + 3)) + g*(c*x)**(m + 4)*(1 + b*
x**n/a)**(-p)*(a + b*x**n)**p*hyper((-p, (m + 4)/n), ((m + n + 4)
/n,), -b*x**n/a)/(c**4*(m + 4))
```

Mathematica [A] time = 0.337604, size = 178, normalized size = 0.65

$$x(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{d\,{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{m+1}+x\left(\frac{e\,{}_2F_1\left(\frac{m+2}{n},-p;\frac{m+n+2}{n};-\frac{bx^n}{a}\right)}{m+2}+x\left(\frac{f\,{}_2F_1\left(\frac{m+3}{n},-p;\frac{m+n+3}{n};-\frac{bx^n}{a}\right)}{m+3}+\frac{gx\,{}_2F_1\left(\frac{m+4}{n},-p;\frac{m+n+4}{n};-\frac{bx^n}{a}\right)}{m+4}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```

```
[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1
+ m + n)/n, -((b*x^n)/a)])/(1 + m) + x*((e*Hypergeometric2F1[(2 +
 m)/n, -p, (2 + m + n)/n, -((b*x^n)/a)])/(2 + m) + x*((f*Hypergeo
metric2F1[(3 + m)/n, -p, (3 + m + n)/n, -((b*x^n)/a)])/(3 + m) +
(g*x*Hypergeometric2F1[(4 + m)/n, -p, (4 + m + n)/n, -((b*x^n)/a)
])/(4 + m)))))/(1 + (b*x^n)/a)^p
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int (cx)^m\left(gx^3+fx^2+ex+d\right)(a+bx^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

```
[Out] int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^3 + fx^2 + ex + d\right)\left(bx^n + a\right)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^3 + fx^2 + ex + d\right)\left(bx^n + a\right)^p (cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(g*x**3+f*x**2+e*x+d)*(a+b*x**n)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^3 + fx^2 + ex + d\right)\left(bx^n + a\right)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

3.575 $\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx$

Optimal. Leaf size=297

$$\frac{d(cx)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)}+\frac{ex^{n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{m+n+1}+\frac{fx^{2n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2n+1}{n},-p;\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{m+2n+1}+\frac{gx^{3n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3n+1}{n},-p;\frac{m+4n+1}{n};-\frac{bx^n}{a}\right)}{m+3n+1}$$

```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (
1 + m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*x
^(1 + n)*(c*x)^m*(a + b*x^n)^p*Hypergeometric2F1[(1 + m + n)/n, -
p, (1 + m + 2*n)/n, -((b*x^n)/a)])/((1 + m + n)*(1 + (b*x^n)/a)^p
) + (f*x^(1 + 2*n)*(c*x)^m*(a + b*x^n)^p*Hypergeometric2F1[(1 + m
 + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/((1 + m + 2*n)*(1
+ (b*x^n)/a)^p) + (g*x^(1 + 3*n)*(c*x)^m*(a + b*x^n)^p*Hypergeome
tric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/((1
+ m + 3*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] time = 0.413251, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{d(cx)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)}+\frac{ex^{n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{m+n+1}+\frac{fx^{2n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2n+1}{n},-p;\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{m+2n+1}+\frac{gx^{3n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3n+1}{n},-p;\frac{m+4n+1}{n};-\frac{bx^n}{a}\right)}{m+3n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```


```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (
1 + m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*x
^(1 + n)*(c*x)^m*(a + b*x^n)^p*Hypergeometric2F1[(1 + m + n)/n, -
p, (1 + m + 2*n)/n, -((b*x^n)/a)])/((1 + m + n)*(1 + (b*x^n)/a)^p
) + (f*x^(1 + 2*n)*(c*x)^m*(a + b*x^n)^p*Hypergeometric2F1[(1 + m
 + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/((1 + m + 2*n)*(1
+ (b*x^n)/a)^p) + (g*x^(1 + 3*n)*(c*x)^m*(a + b*x^n)^p*Hypergeome
tric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/((1
+ m + 3*n)*(1 + (b*x^n)/a)^p)
```

Rubi in Sympy [A] time = 55.4421, size = 272, normalized size = 0.92

$$\frac{d\,(cx)^{m+1}\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{m+1}{n}\\\frac{m+n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{c\,(m+1)}+\frac{ex^n\,(cx)^{-n}\,(cx)^{m+n+1}\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{m+n+1}{n}\\\frac{m+2n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{c\,(m+n+1)}+\frac{fx^{2n}\,(cx)^{-2n}\,(cx)^{m+2n+1}\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{m+2n+1}{n}\\\frac{m+3n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{c\,(m+2n+1)}+\frac{gx^{3n}\,(cx)^{-3n}\,(cx)^{m+3n+1}\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,{}_2F_1\left(\begin{matrix}-p,\frac{m+3n+1}{n}\\\frac{m+4n+1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{c\,(m+3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x)**m*(a+b*x**n)**p*(d+e*x**n+f*x**(2*n)+g*x**(3*n)),x)
```

```
[Out] d*(c*x)**(m + 1)*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*hyper((-p,
(m + 1)/n), ((m + n + 1)/n,), -b*x**n/a)/(c*(m + 1)) + e*x**n*(c*
x)**(-n)*(c*x)**(m + n + 1)*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*
hyper((-p, (m + n + 1)/n), ((m + 2*n + 1)/n,), -b*x**n/a)/(c*(m +
 n + 1)) + f*x**(2*n)*(c*x)**(-2*n)*(c*x)**(m + 2*n + 1)*(1 + b*x
**n/a)**(-p)*(a + b*x**n)**p*hyper((-p, (m + 2*n + 1)/n), ((m + 3
*n + 1)/n,), -b*x**n/a)/(c*(m + 2*n + 1)) + g*x**(3*n)*(c*x)**(-3
*n)*(c*x)**(m + 3*n + 1)*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*hyp
er((-p, (m + 3*n + 1)/n), ((m + 4*n + 1)/n,), -b*x**n/a)/(c*(m +
3*n + 1))
```

Mathematica [A] time = 1.55533, size = 204, normalized size = 0.69

$$x(cx)^m\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{d\,{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{m+1}+x^n\left(\frac{e\,{}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{m+n+1}+x^n\left(\frac{f\,{}_2F_1\left(\frac{m+2n+1}{n},-p;\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{m+2n+1}+\frac{gx^n\,{}_2F_1\left(\frac{m+3n+1}{n},-p;\frac{m+4n+1}{n};-\frac{bx^n}{a}\right)}{m+3n+1}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```

```
[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1
+ m + n)/n, -((b*x^n)/a)])/(1 + m) + x^n*((e*Hypergeometric2F1[(1
 + m + n)/n, -p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + x^
n*((f*Hypergeometric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((
b*x^n)/a)])/(1 + m + 2*n) + (g*x^n*Hypergeometric2F1[(1 + m + 3*n
)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/(1 + m + 3*n)))))/(1 + (
b*x^n)/a)^p
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int (cx)^m\,(a+bx^n)^p\left(d+ex^n+fx^{2\,n}+gx^{3\,n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
```

```
[Out] int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^{3\,n}+fx^{2\,n}+ex^{n}+d\right)\left(bx^{n}+a\right)^{p}\left(cx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m,x, algorithm=
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)
^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^{3\,n}+fx^{2\,n}+ex^{n}+d\right)\left(bx^{n}+a\right)^{p}\left(cx\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m,x, algorithm=
```

```
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^
m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a+b*x**n)**p*(d+e*x**n+f*x**(2*n)+g*x**(3*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m,x, algorithm=
```

```
[Out] Exception raised: TypeError
```

3.576 $\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=162

$$\frac{x(ae-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}-\frac{x^{\frac{n+2}{2}}(bd(2-n)-af(n+2))\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{a^2bn(n+2)}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\left(a+bx^n\right)}$$

```
[Out] (x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) - ((b*d
*(2 - n) - a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2
/n)/2, (3 + 2/n)/2, -((b*x^n)/a)])/(a^2*b*n*(2 + n)) + ((a*e - b*
c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a
)])/(a^2*b*n)
```

Rubi [A] time = 0.264256, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$

$$\frac{x(ae-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}-\frac{x^{\frac{n+2}{2}}(bd(2-n)-af(n+2))\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{a^2bn(n+2)}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

```
[Out] (x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) - ((b*d
*(2 - n) - a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2
/n)/2, (3 + 2/n)/2, -((b*x^n)/a)])/(a^2*b*n*(2 + n)) + ((a*e - b*
c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a
)])/(a^2*b*n)
```

Rubi in Sympy [A] time = 39.1909, size = 122, normalized size = 0.75

$$-\frac{x\left(2ae-2bc+2x^{\frac{n}{2}}\left(af-bd\right)\right)}{2abn\left(a+bx^n\right)}+\frac{x\left(ae-bc\left(-n+1\right)\right){}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x^{\frac{n}{2}+1}\left(af\left(n+2\right)-bd\left(-n+2\right)\right){}_2F_1\left(\begin{matrix}1,\frac{n+2}{2n}\\\frac{3}{2}+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a^2bn\left(n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n)**2,x)
```

```
[Out] -x*(2*a*e - 2*b*c + 2*x**(n/2)*(a*f - b*d))/(2*a*b*n*(a + b*x**n
) + x*(a*e - b*c*(-n + 1))*hyper((1, 1/n), (1 + 1/n,), -b*x**n/a)
/(a**2*b*n) + x**(n/2 + 1)*(a*f*(n + 2) - b*d*(-n + 2))*hyper((1,
 (n + 2)/(2*n)), (3/2 + 1/n,), -b*x**n/a)/(a**2*b*n*(n + 2))
```

Mathematica [A] time = 0.32216, size = 151, normalized size = 0.93

$$\frac{x\left((n+2)\left(a\left(b\left(c+dx^{n/2}\right)-a\left(e+fx^{n/2}\right)\right)+(a+bx^n)(ae+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)+x^{n/2}\left(a+bx^n\right)(af(n+\right.}{a^2bn(n+2)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

```
[Out] (x*((b*d*(-2 + n) + a*f*(2 + n))*x^(n/2)*(a + b*x^n)*Hypergeometr
ic2F1[1, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x^n)/a)] + (2 + n)*(a*(
b*(c + d*x^(n/2)) - a*(e + f*x^(n/2))) + (a*e + b*c*(-1 + n))*(a
+ b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))
)/(a^2*b*n*(2 + n)*(a + b*x^n))
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2}\left(c+dx^{\frac{n}{2}}+ex^n+fx^{\frac{3n}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bd-af)xx^{\frac{1}{2}n}+(bc-ae)x}{ab^2nx^n+a^2bn}+\int \frac{2\,bc(n-1)+2\,ae+(af(n+2)+bd(n-2))x^{\frac{1}{2}n}}{2\,(ab^2nx^n+a^2bn)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2,x, algorithm="max
```

```
[Out] ((b*d - a*f)*x*x^(1/2*n) + (b*c - a*e)*x)/(a*b^2*n*x^n + a^2*b*n)
 + integrate(1/2*(2*b*c*(n - 1) + 2*a*e + (a*f*(n + 2) + b*d*(n -
 2))*x^(1/2*n))/(a*b^2*n*x^n + a^2*b*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^{\frac{3}{2}n}+dx^{\frac{1}{2}n}+ex^n+c}{b^2x^{2n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2,x, algorithm="fri
```

```
[Out] integral((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b^2*x^(2*n) + 2
*a*b*x^n + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^{\frac{3}{2}n} + dx^{\frac{1}{2}n} + ex^n + c}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2,x, algorithm="gia
```

```
[Out] integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2,
x)
```

3.577 $\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=24

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Rubi [A] time = 0.0226305, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]

[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Rubi in Sympy [A] time = 31.2192, size = 20, normalized size = 0.83

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*c+2*(a*d+b*c)*x**2+3*b*d*x**4)/(b*x**2+a)**(1/2)/(d*x**2+c)*

[Out] x*sqrt(a + b*x**2)*sqrt(c + d*x**2)
```

Mathematica [A] time = 0.076475, size = 24, normalized size = 1.

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])

[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Maple [A] time = 0.013, size = 21, normalized size = 0.9

$$x\sqrt{bx^2+a}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)

[Out] x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

Maxima [A] time = 1.54164, size = 27, normalized size = 1.12

$$\sqrt{bx^2 + a}\sqrt{dx^2 + c}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c))
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

Fricas [A] time = 0.218286, size = 27, normalized size = 1.12

$$\sqrt{bx^2 + a}\sqrt{dx^2 + c}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c))
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ac + 2adx^2 + 2bcx^2 + 3bdx^4}{\sqrt{a + bx^2}\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x**2+3*b*d*x**4)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2
```

```
[Out] Integral((a*c + 2*a*d*x**2 + 2*b*c*x**2 + 3*b*d*x**4)/(sqrt(a + b
*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\, bdx^4 + 2\,(bc + ad)x^2 + ac}{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c))
```

```
[Out] integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*
sqrt(d*x^2 + c)), x)
```

3.578 $\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4
)^(1/4)/2^(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4
)]/(2*2^(1/4)) + ArcTanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))
```

Rubi [A] time = 1.41132, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4
)^(1/4)/2^(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4
)]/(2*2^(1/4)) + ArcTanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: GeneratorsError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)/(-x**4+1)/(x**4+1)**(1/4),x)
```

```
[Out] Exception raised: GeneratorsError
```

Mathematica [C] time = 0.316525, size = 166, normalized size = 1.61

$$\frac{-\log\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}+1\right)+2\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}-\frac{2x^4F_1\left(1;\frac{1}{4},1;2;-x^4,x^4\right)}{(x^4-1)\sqrt[4]{x^4+1}\left(x^4\left(4F_1\left(2;\frac{1}{4},2;3;-x^4,x^4\right)-F_1\left(2;\frac{5}{4},1;3;-x^4,x^4\right)\right)+8F_1\left(1;\frac{1}{4},1;2;-x^4,x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] (-2*x^4*AppellF1[1, 1/4, 1, 2, -x^4, x^4])/((-1 + x^4)*(1 + x^4)^
(1/4)*(8*AppellF1[1, 1/4, 1, 2, -x^4, x^4] + x^4*(4*AppellF1[2, 1
/4, 2, 3, -x^4, x^4] - AppellF1[2, 5/4, 1, 3, -x^4, x^4]))) + (2*
ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)] - Log[1 - (2^(1/4)*x)/(1 + x^
4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 + x^4)^(1/4)])/(4*2^(1/4))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{x^3+1}{-x^4+1}\frac{1}{\sqrt[4]{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

```
[Out] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3+1}{(x^4+1)^{\frac{1}{4}}(x^4-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(-\frac{x}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\right)dx$$
$$-\int \frac{x^2}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx$$
$$-\int \frac{1}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)/(-x**4+1)/(x**4+1)**(1/4),x)
```

```
[Out] -Integral(-x/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x
*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4)), x) - Integral(x**2/(x**3
*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4)
 - (x**4 + 1)**(1/4)), x) - Integral(1/(x**3*(x**4 + 1)**(1/4) -
x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3+1}{(x^4+1)^{\frac{1}{4}}(x^4-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)),x, algorithm="giac")
```

```
[Out] integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

3.579 $\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx$

Optimal. Leaf size=28

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.150258, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2*n)),x]

[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Rubi in Sympy [A] time = 18.3048, size = 20, normalized size = 0.71

$$x(a+bx^n)^{-\frac{1}{n}}(c+dx^n)^{-\frac{1}{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a+b*x**n)**((-1-n)/n)*(c+d*x**n)**((-1-n)/n)*(a*c-
b*d*x**(2*n)),x)

[Out] x*(a + b*x**n)**(-1/n)*(c + d*x**n)**(-1/n)
```

Mathematica [A] time = 0.202057, size = 28, normalized size = 1.

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2*n)),x

[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Maple [F] time = 0.393, size = 0, normalized size = 0.

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2\,n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)

[Out] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x
)
```

Maxima [A] time = 2.75057, size = 41, normalized size = 1.46

$$xe^{\left(-\frac{\log(bx^n+a)}{n}-\frac{\log(dx^n+c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*d*x^(2*n) - a*c)*(b*x^n + a)^(-(n + 1)/n)*(d*x^n + c)^(-
(n + 1)/n),x, algorithm="maxima")
```

```
[Out] x*e^(-log(b*x^n + a)/n - log(d*x^n + c)/n)
```

Fricas [A] time = 0.241394, size = 82, normalized size = 2.93

$$\frac{bdxx^{2n}+acx+(bc+ad)xx^{n}}{(bx^n+a)^{\frac{n+1}{n}}(dx^n+c)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*d*x^(2*n) - a*c)/((b*x^n + a)^((n + 1)/n)*(d*x^n + c)^((n + 1)/n)),
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)/((b*x^n + a)^((n + 1)
/n)*(d*x^n + c)^((n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((-1-n)/n)*(c+d*x**n)**((-1-n)/n)*(a*c-b*d*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.256566, size = 354, normalized size = 12.64

$$\begin{aligned}&bdxe^{\left(2\,n\ln(x)-\frac{n\ln\left(be^{(n\ln(x))}+a\right)+\ln\left(be^{(n\ln(x))}+a\right)}{n}-\frac{n\ln\left(de^{(n\ln(x))}+c\right)+\ln\left(de^{(n\ln(x))}+c\right)}{n}\right)}\\&+bcxe^{\left(n\ln(x)-\frac{n\ln\left(be^{(n\ln(x))}+a\right)+\ln\left(be^{(n\ln(x))}+a\right)}{n}-\frac{n\ln\left(de^{(n\ln(x))}+c\right)+\ln\left(de^{(n\ln(x))}+c\right)}{n}\right)}\\&+adxe^{\left(n\ln(x)-\frac{n\ln\left(be^{(n\ln(x))}+a\right)+\ln\left(be^{(n\ln(x))}+a\right)}{n}-\frac{n\ln\left(de^{(n\ln(x))}+c\right)+\ln\left(de^{(n\ln(x))}+c\right)}{n}\right)}\\&+acxe^{\left(-\frac{n\ln\left(be^{(n\ln(x))}+a\right)+\ln\left(be^{(n\ln(x))}+a\right)}{n}-\frac{n\ln\left(de^{(n\ln(x))}+c\right)+\ln\left(de^{(n\ln(x))}+c\right)}{n}\right)}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*d*x^(2*n) - a*c)/((b*x^n + a)^((n + 1)/n)*(d*x^n + c)^((n + 1)/n)),
```

```
[Out] b*d*x*e^(2*n*ln(x) - (n*ln(b*e^(n*ln(x)) + a) + ln(b*e^(n*ln(x))
+ a))/n - (n*ln(d*e^(n*ln(x)) + c) + ln(d*e^(n*ln(x)) + c))/n) +
b*c*x*e^(n*ln(x) - (n*ln(b*e^(n*ln(x)) + a) + ln(b*e^(n*ln(x)) +
a))/n - (n*ln(d*e^(n*ln(x)) + c) + ln(d*e^(n*ln(x)) + c))/n) + a*
d*x*e^(n*ln(x) - (n*ln(b*e^(n*ln(x)) + a) + ln(b*e^(n*ln(x)) + a)
)/n - (n*ln(d*e^(n*ln(x)) + c) + ln(d*e^(n*ln(x)) + c))/n) + a*c*
x*e^(-(n*ln(b*e^(n*ln(x)) + a) + ln(b*e^(n*ln(x)) + a))/n - (n*ln
(d*e^(n*ln(x)) + c) + ln(d*e^(n*ln(x)) + c))/n)
```

3.580 $\int (hx)^{-1-n-np} (a + bx^n)^p (c + dx^n)^p \left(ac - bdx^{2n}\right) \, dx$

Optimal. Leaf size=45

$$-\frac{(hx)^{-n(p+1)} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hn(p + 1)}$$

```
[Out] -(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n
*(1 + p))))
```

Rubi [A] time = 0.241569, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$

$$-\frac{(hx)^{-n(p+1)} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hn(p + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*n)),x]
```

```
[Out] -(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n
*(1 + p))))
```

Rubi in Sympy [A] time = 38.0471, size = 36, normalized size = 0.8

$$-\frac{(hx)^{-n(p+1)} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hn\,(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x)**(-n*p-n-1)*(a+b*x**n)**p*(c+d*x**n)**p*(a*c-
b*d*x**(2*n)),x)
```

```
[Out] -(h*x)**(-n*(p + 1))*(a + b*x**n)**(p + 1)*(c + d*x**n)**(p + 1)/
(h*n*(p + 1))
```

Mathematica [A] time = 0.266415, size = 46, normalized size = 1.02

$$-\frac{(hx)^{n(-p)-n} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hnp + hn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*n)),x]
```

```
[Out] -(((h*x)^(-n - n*p)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n
 + h*n*p))
```

Maple [C] time = 0.677, size = 138, normalized size = 3.1

$$-\frac{(a + bx^n)^p \left(bd\,(x^n)^2 + adx^n + bcx^n + ac\right) x\,(c + dx^n)^p}{n\,(1 + p)} e^{-\frac{(np+n+1)\left(-i(\mathrm{csgn}(ihx))^3\pi + i(\mathrm{csgn}(ihx))^2\mathrm{csgn}(ih)\pi + i(\mathrm{csgn}(ihx))^2\mathrm{csgn}(ix)\pi - i\,\mathrm{csgn}(ihx)\mathrm{csgn}(ih)\mathrm{csgn}(ix}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x)
```

```
[Out] -(a+b*x^n)^p*exp(-1/2*(n*p+n+1)*(-I*csgn(I*h*x)^3*Pi+I*csgn(I*h*x
)^2*csgn(I*h)*Pi+I*csgn(I*h*x)^2*csgn(I*x)*Pi-I*csgn(I*h*x)*csgn(
I*h)*csgn(I*x)*Pi+2*ln(h)+2*ln(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+
a*c)*x/n/(1+p)*(c+d*x^n)^p
```

Maxima [A] time = 2.20546, size = 104, normalized size = 2.31

$$-\frac{\left(bdx^{2n}+ac+(bc+ad)x^n\right)h^{-np-n-1}e^{(-np\log(x)+p\log(bx^n+a)+p\log(dx^n+c)-n\log(x))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*d*x^(2*n) - a*c)*(b*x^n + a)^p*(d*x^n + c)^p*(h*x)^(-
n*p - n - 1),x, algorithm="maxima")
```

```
[Out] -(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n)*h^(-n*p - n - 1)*e^(-n*p*l
og(x) + p*log(b*x^n + a) + p*log(d*x^n + c) - n*log(x))/(n*(p + 1
))
```

Fricas [A] time = 0.246381, size = 161, normalized size = 3.58

$$-\frac{\left(bdxx^{2n}e^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}+acxe^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}+(bc+ad)xx^ne^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}\right)(}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*d*x^(2*n) - a*c)*(b*x^n + a)^p*(d*x^n + c)^p*(h*x)^(-
n*p - n - 1),x, algorithm="fricas")
```

```
[Out] -(b*d*x*x^(2*n)*e^(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x))
+ a*c*x*e^(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + (b*c +
 a*d)*x*x^n*e^(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)))*(b*
x^n + a)^p*(d*x^n + c)^p/(n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**(-n*p-n-1)*(a+b*x**n)**p*(c+d*x**n)**p*(a*c-b*d*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.25732, size = 323, normalized size = 7.18

$$-\frac{bdxe^{\left(-np\ln(h)-np\ln(x)+p\ln\left(be^{(n\ln(x))}+a\right)+p\ln\left(de^{(n\ln(x))}+c\right)-n\ln(h)+n\ln(x)-\ln(h)-\ln(x)\right)}+acxe^{\left(-np\ln(h)-np\ln(x)+p\ln\left(be^{(n\ln(x))}+a\right)+p\ln\left(de^{(n\ln(x)}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*d*x^(2*n) - a*c)*(b*x^n + a)^p*(d*x^n + c)^p*(h*x)^(-
n*p - n - 1),x, algorithm="giac")
```

```
[Out] -(b*d*x*e^(-n*p*ln(h) - n*p*ln(x) + p*ln(b*e^(n*ln(x)) + a) + p*l
n(d*e^(n*ln(x)) + c) - n*ln(h) + n*ln(x) - ln(h) - ln(x)) + a*c*x
*e^(-n*p*ln(h) - n*p*ln(x) + p*ln(b*e^(n*ln(x)) + a) + p*ln(d*e^(
n*ln(x)) + c) - n*ln(h) - n*ln(x) - ln(h) - ln(x)) + b*c*x*e^(-n*
p*ln(h) - n*p*ln(x) + p*ln(b*e^(n*ln(x)) + a) + p*ln(d*e^(n*ln(x)
) + c) - n*ln(h) - ln(h) - ln(x)) + a*d*x*e^(-n*p*ln(h) - n*p*ln(
x) + p*ln(b*e^(n*ln(x)) + a) + p*ln(d*e^(n*ln(x)) + c) - n*ln(h)
- ln(h) - ln(x)))/(n*p + n)
```

3.581 $\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx$

Optimal. Leaf size=31

$$\frac{ex (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{ac}$$

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Rubi [A] time = 0.313953, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014

$$\frac{ex (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n)/(a*c) + (b*c
```

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Rubi in Sympy [A] time = 75.3723, size = 26, normalized size = 0.84

$$\frac{ex (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+n+1)*x**n/a/c+b
```

```
[Out] e*x*(a + b*x**n)**(p + 1)*(c + d*x**n)**(p + 1)/(a*c)
```

Mathematica [A] time = 0.292104, size = 31, normalized size = 1.

$$\frac{ex (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n)/(a*c)
```

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Maple [A] time = 0.185, size = 52, normalized size = 1.7

$$\frac{(a+bx^n)^p \left(bd (x^n)^2 + adx^n + bcx^n + ac\right) ex (c+dx^n)^p}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2*n*p+2*n+
```

```
[Out] (a+b*x^n)^p*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c*(c+d*x^n)^p
```

Maxima [A] time = 1.93932, size = 80, normalized size = 2.58

$$\frac{\left(bdexx^{2n} + acex + (bce + ade)xx^{n}\right)e^{(p\log(bx^{n}+a)+p\log(dx^{n}+c))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2*n*p + 2*n + 1)*b*d*e*x^(2*n)/(a*c) + (b*c + a*d)*(n*p + n + 1)*e*x^n
```

```
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c*e + a*d*e)*x*x^n)*e^(p*log(b*x^
n + a) + p*log(d*x^n + c))/(a*c)
```

Fricas [A] time = 0.248956, size = 73, normalized size = 2.35

$$\frac{\left(bdexx^{2n} + acex + (bc + ad)exx^{n}\right)(bx^{n} + a)^{p}(dx^{n} + c)^{p}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2*n*p + 2*n + 1)*b*d*e*x^(2*n)/(a*c) + (b*c + a*d)*(n*p + n + 1)*e*x^n
```

```
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c + a*d)*e*x*x^n)*(b*x^n + a)^p*(
d*x^n + c)^p/(a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+n+1)*x**n/a/c+b*d*e*
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.264626, size = 196, normalized size = 6.32

$$\frac{bdxe^{\left(p\ln\left(be^{(n\ln(x))}+a\right)+p\ln\left(de^{(n\ln(x))}+c\right)+2\,n\ln(x)+1\right)} + bcxe^{\left(p\ln\left(be^{(n\ln(x))}+a\right)+p\ln\left(de^{(n\ln(x))}+c\right)+n\ln(x)+1\right)} + adxe^{\left(p\ln\left(be^{(n\ln(x))}+a\right)+p\ln\left(de^{(n\ln(x}\right.\right.}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2*n*p + 2*n + 1)*b*d*e*x^(2*n)/(a*c) + (b*c + a*d)*(n*p + n + 1)*e*x^n
```

```
[Out] (b*d*x*e^(p*ln(b*e^(n*ln(x)) + a) + p*ln(d*e^(n*ln(x)) + c) + 2*n
*ln(x) + 1) + b*c*x*e^(p*ln(b*e^(n*ln(x)) + a) + p*ln(d*e^(n*ln(x
)) + c) + n*ln(x) + 1) + a*d*x*e^(p*ln(b*e^(n*ln(x)) + a) + p*ln(
d*e^(n*ln(x)) + c) + n*ln(x) + 1) + a*c*x*e^(p*ln(b*e^(n*ln(x)) +
 a) + p*ln(d*e^(n*ln(x)) + c) + 1))/(a*c)
```

3.582 $\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)}{ac(1+m)}\right.$

Optimal. Leaf size=45

$$\frac{e(hx)^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ach(m+1)}$$

```
[Out] (e*(h*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*h*
(1 + m))
```

Rubi [A] time = 0.839767, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 86, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$

$$\frac{e(hx)^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ach(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m + n + n*p)*x^n)/
```

```
[Out] (e*(h*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*h*
(1 + m))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x)**m*(a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+m+n+1)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.423018, size = 41, normalized size = 0.91

$$\frac{ex(hx)^m(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m + n + n*p)
```

```
[Out] (e*x*(h*x)^m*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*(1 + m
))
```

Maple [C] time = 0.605, size = 136, normalized size = 3.

$$\frac{(a+bx^n)^p\left(bd(x^n)^2+adx^n+bcx^n+ac\right)ex(c+dx^n)^p}{ac(1+m)}e^{\frac{m\left(-i(\mathrm{csgn}(ihx))^3\pi+i(\mathrm{csgn}(ihx))^2\mathrm{csgn}(ih)\pi+i(\mathrm{csgn}(ihx))^2\mathrm{csgn}(ix)\pi-i\,\mathrm{csgn}(ihx)\mathrm{csgn}(ih)\mathrm{csgn}(ix)\pi+2\ln(h)\right.}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/c/(1+m)+b
```

```
[Out] (a+b*x^n)^p*exp(1/2*m*(-I*csgn(I*h*x)^3*Pi+I*csgn(I*h*x)^2*csgn(I
*h)*Pi+I*csgn(I*h*x)^2*csgn(I*x)*Pi-I*csgn(I*h*x)*csgn(I*h)*csgn(
I*x)*Pi+2*ln(h)+2*ln(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a
/c/(1+m)*(c+d*x^n)^p
```

Maxima [A] time = 2.11185, size = 124, normalized size = 2.76

$$\frac{\left(aceh^m xx^m + bdeh^m xe^{(m\log(x)+2\,n\log(x))} + (bceh^m + adeh^m)xe^{(m\log(x)+n\log(x))}\right)e^{(p\log(bx^n+a)+p\log(dx^n+c))}}{ac(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2*n*p + m + 2*n + 1)*b*d*e*x^(2*n)/(a*c*(m + 1)) + e + (b*c + a*d)*(n*p
```

```
[Out] (a*c*e*h^m*x*x^m + b*d*e*h^m*x*e^(m*log(x) + 2*n*log(x)) + (b*c*e
*h^m + a*d*e*h^m)*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a)
+ p*log(d*x^n + c))/(a*c*(m + 1))
```

Fricas [A] time = 0.254125, size = 119, normalized size = 2.64

$$\frac{\left(bdexx^{2\,n}e^{(m\log(h)+m\log(x))} + acexe^{(m\log(h)+m\log(x))} + (bc+ad)exx^n e^{(m\log(h)+m\log(x))}\right)(bx^n+a)^p(dx^n+c)^p}{acm+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2*n*p + m + 2*n + 1)*b*d*e*x^(2*n)/(a*c*(m + 1)) + e + (b*c + a*d)*(n*p
```

```
[Out] (b*d*e*x*x^(2*n)*e^(m*log(h) + m*log(x)) + a*c*e*x*e^(m*log(h) +
m*log(x)) + (b*c + a*d)*e*x*x^n*e^(m*log(h) + m*log(x)))*(b*x^n +
 a)^p*(d*x^n + c)^p/(a*c*m + a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**m*(a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x**n
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274291, size = 244, normalized size = 5.42

$$\frac{bdxe^{\left(p\ln\left(be^{(n\ln(x))}+a\right)+p\ln\left(de^{(n\ln(x))}+c\right)+m\ln(h)+m\ln(x)+2\,n\ln(x)+1\right)} + bcxe^{\left(p\ln\left(be^{(n\ln(x))}+a\right)+p\ln\left(de^{(n\ln(x))}+c\right)+m\ln(h)+m\ln(x)+n\ln(x)+1\right)} + adxe}{acm+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2*n*p + m + 2*n + 1)*b*d*e*x^(2*n)/(a*c*(m + 1)) + e + (b*c + a*d)*(n*p
```

```
[Out] (b*d*x*e^(p*ln(b*e^(n*ln(x)) + a) + p*ln(d*e^(n*ln(x)) + c) + m*l
n(h) + m*ln(x) + 2*n*ln(x) + 1) + b*c*x*e^(p*ln(b*e^(n*ln(x)) + a
) + p*ln(d*e^(n*ln(x)) + c) + m*ln(h) + m*ln(x) + n*ln(x) + 1) +
a*d*x*e^(p*ln(b*e^(n*ln(x)) + a) + p*ln(d*e^(n*ln(x)) + c) + m*ln
(h) + m*ln(x) + n*ln(x) + 1) + a*c*x*e^(p*ln(b*e^(n*ln(x)) + a) +
 p*ln(d*e^(n*ln(x)) + c) + m*ln(h) + m*ln(x) + 1))/(a*c*m + a*c)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```